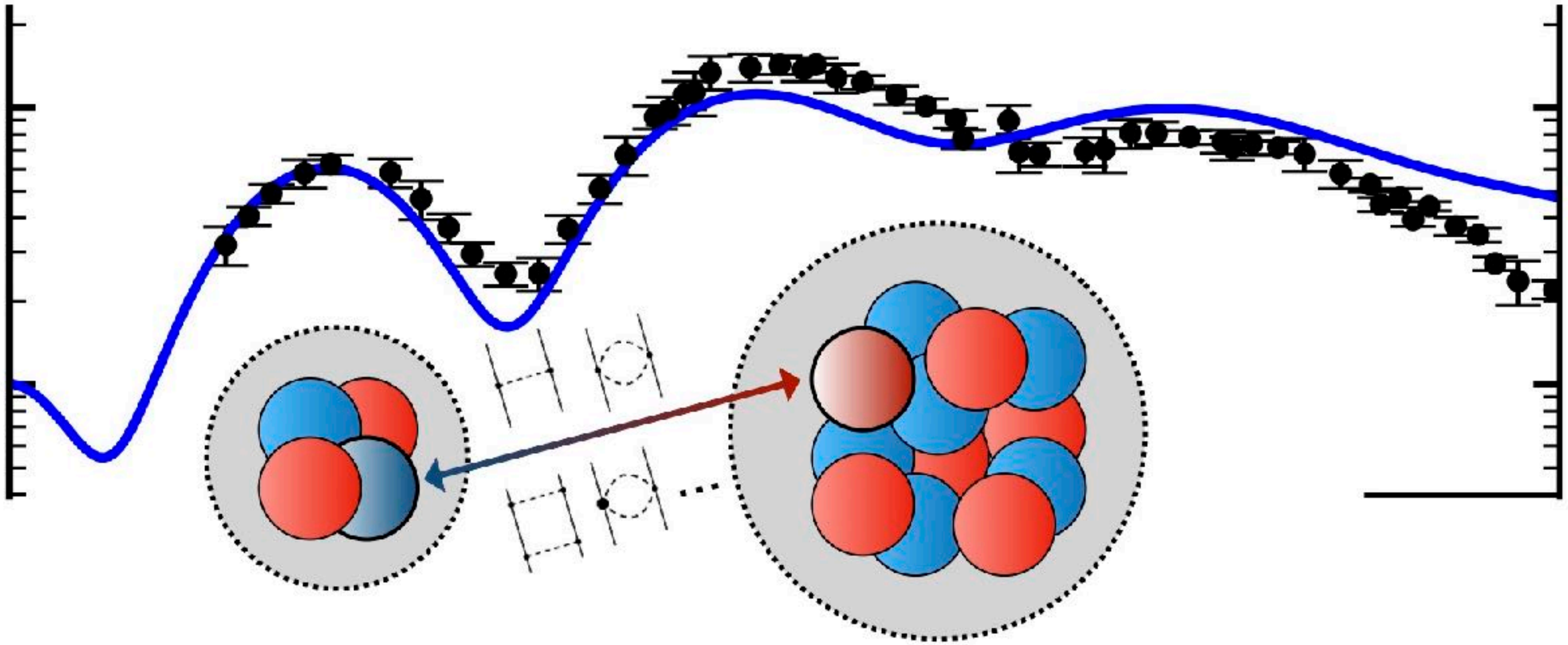
Microscopic Optical Potentials

New developments and achievements

Paolo Finelli

Theory and Phenomenology of Fundamental Interactions
UNIVERSITY AND INFN · BOLOGNA



<u>Collaborators</u>

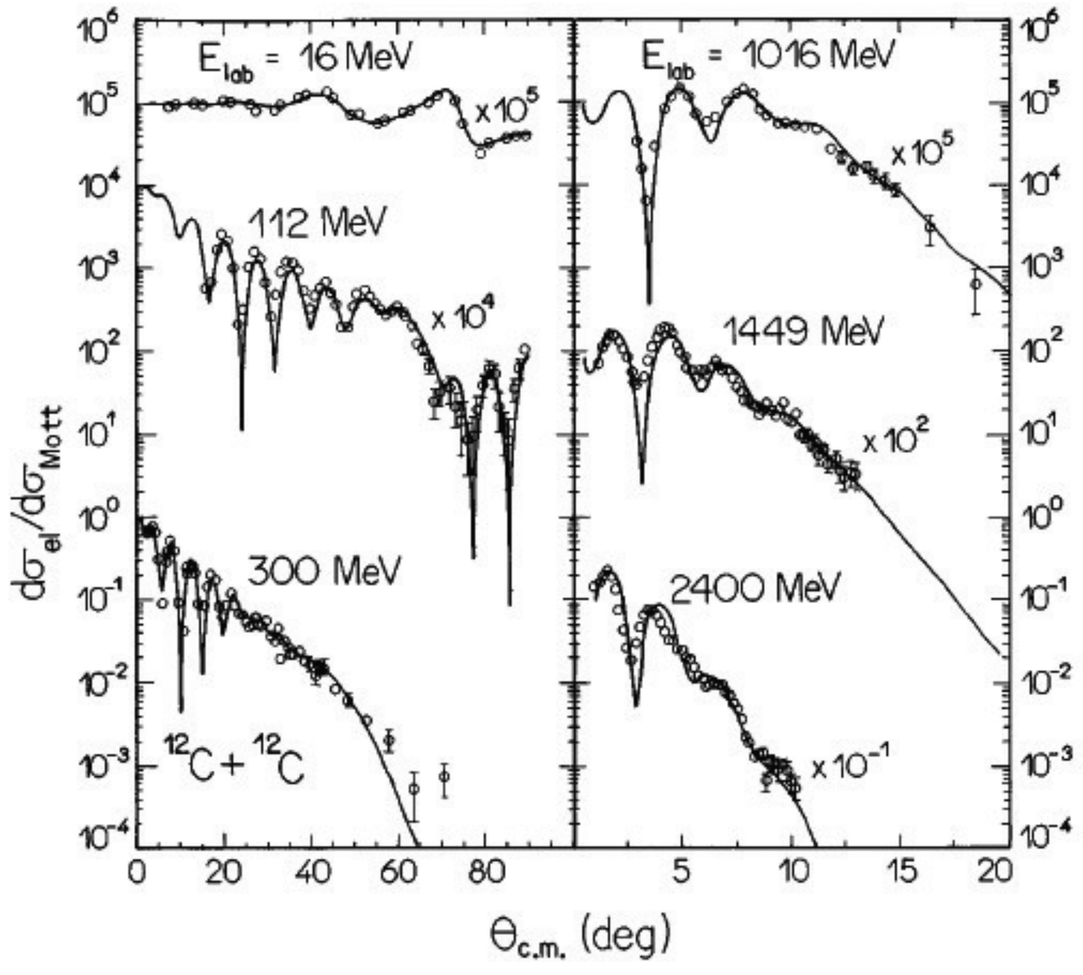
- Matteo Vorabbi
- Carlotta Giusti
- Carlo Barbieri
- Vittorio Somà
- Peter Návratil
- Michael Gennari

Optical potentials

Phenomenological approach

- ✓ Very successful (large community)
- ✓ Easy to use (but not always)
- ✓ Open-access codes (TALYS, FRESCO, EMPIRE, FLUKA,…)

- ? How to extrapolate to exotic nuclei
- ? How (and where) to improve the description of data



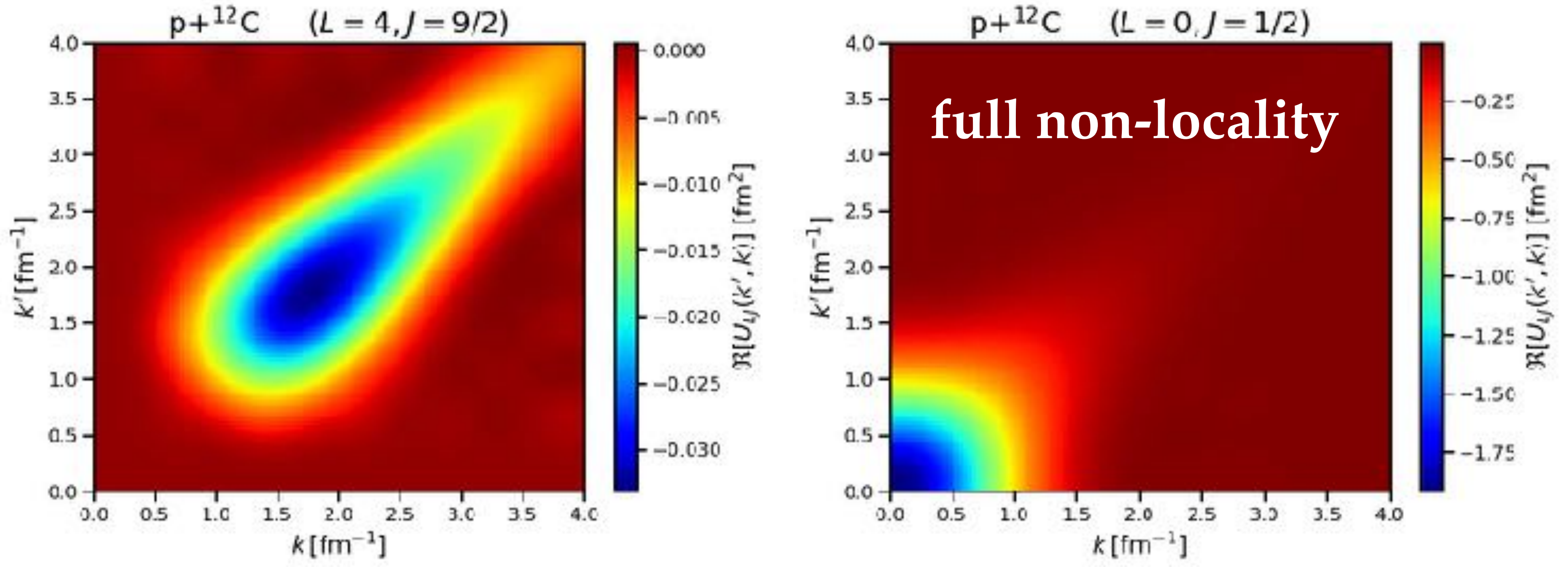
$^{12}C + ^{12}C$

Di Pietro, Eur. Phys. J. Plus (2018) 133: 404

Microscopic approach

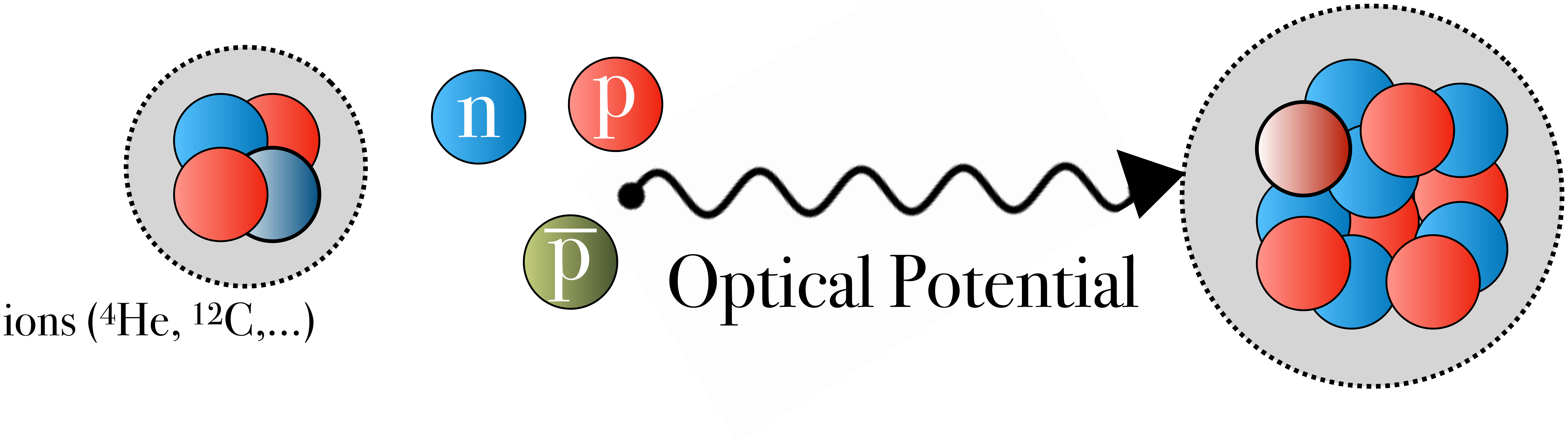
- ⚠ Difficult to manage
- ⚠ No open-access codes so far (only to generate NN potentials)
- ⚠ High-performance computers needed (depends on the reaction)

- ✓ Parameter free predictions (no fit)
- ✓ Reliable when extrapolate to exotic nuclei
- ✓ How (and where) to improve the description of data guided by theory (maybe difficult)

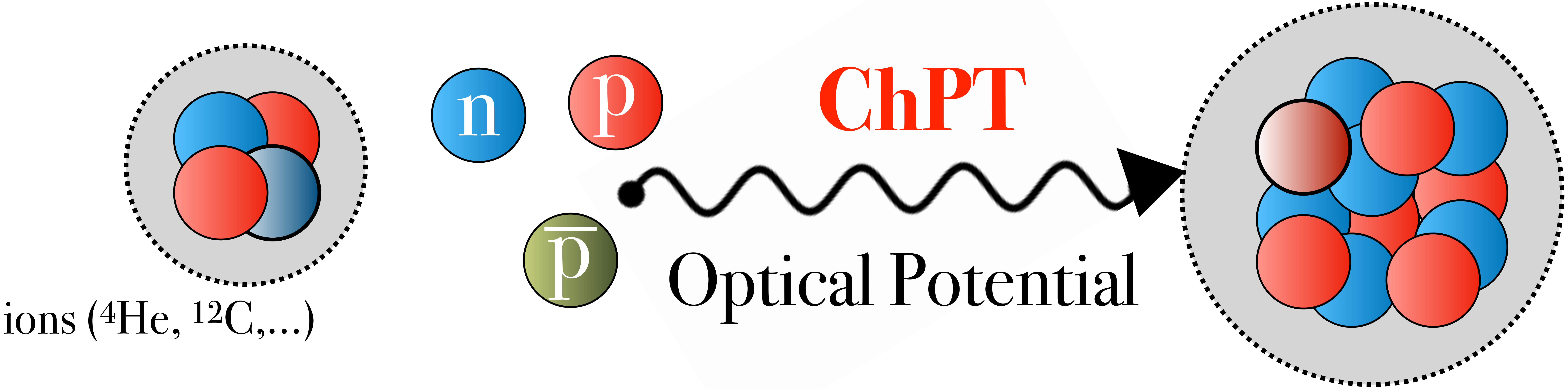


$p + ^{12}C$

Roadmap towards microscopic optical potentials



Roadmap towards microscopic optical potentials

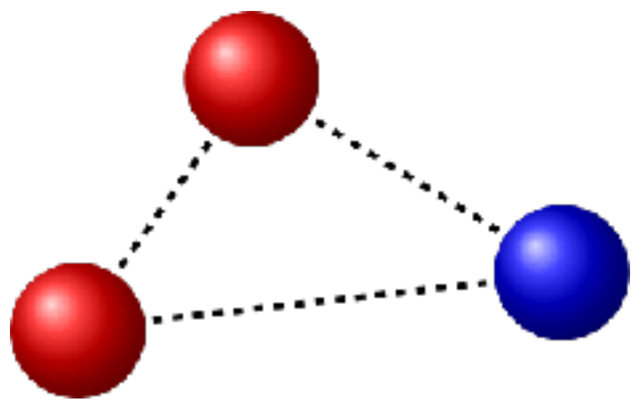


NN data

$$\left(\frac{d\sigma}{d\Omega}\right)_{NN}$$

$$\delta_L(E)$$

NN Body

\+ 

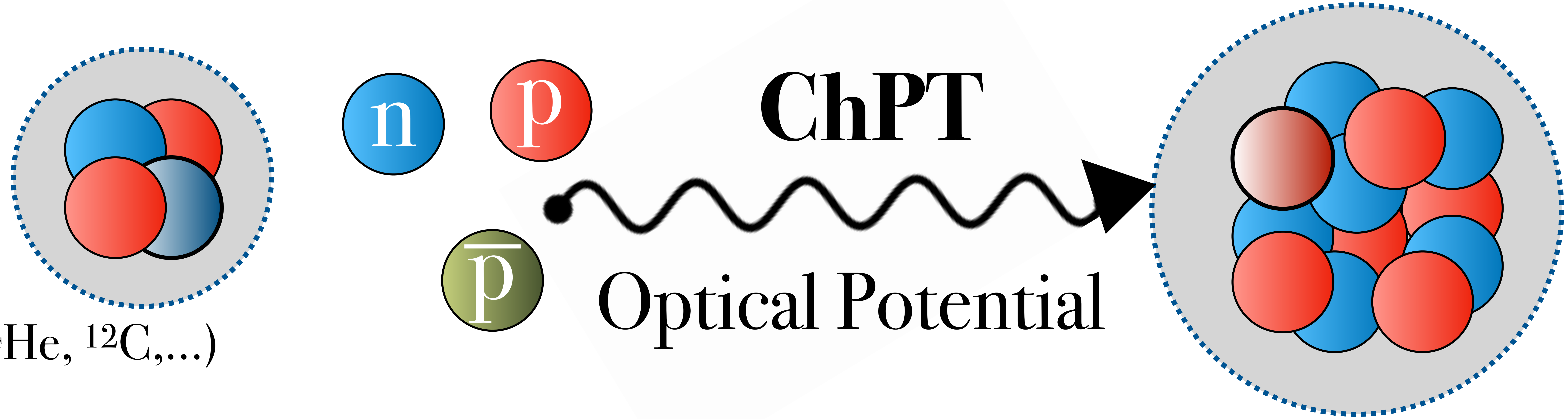
NNN Body

somehow...

<u>Starting point</u>

in principle everything we need to know

Roadmap towards microscopic optical potentials



ions (^{4}He, ^{12}C,...)

Effective Interaction

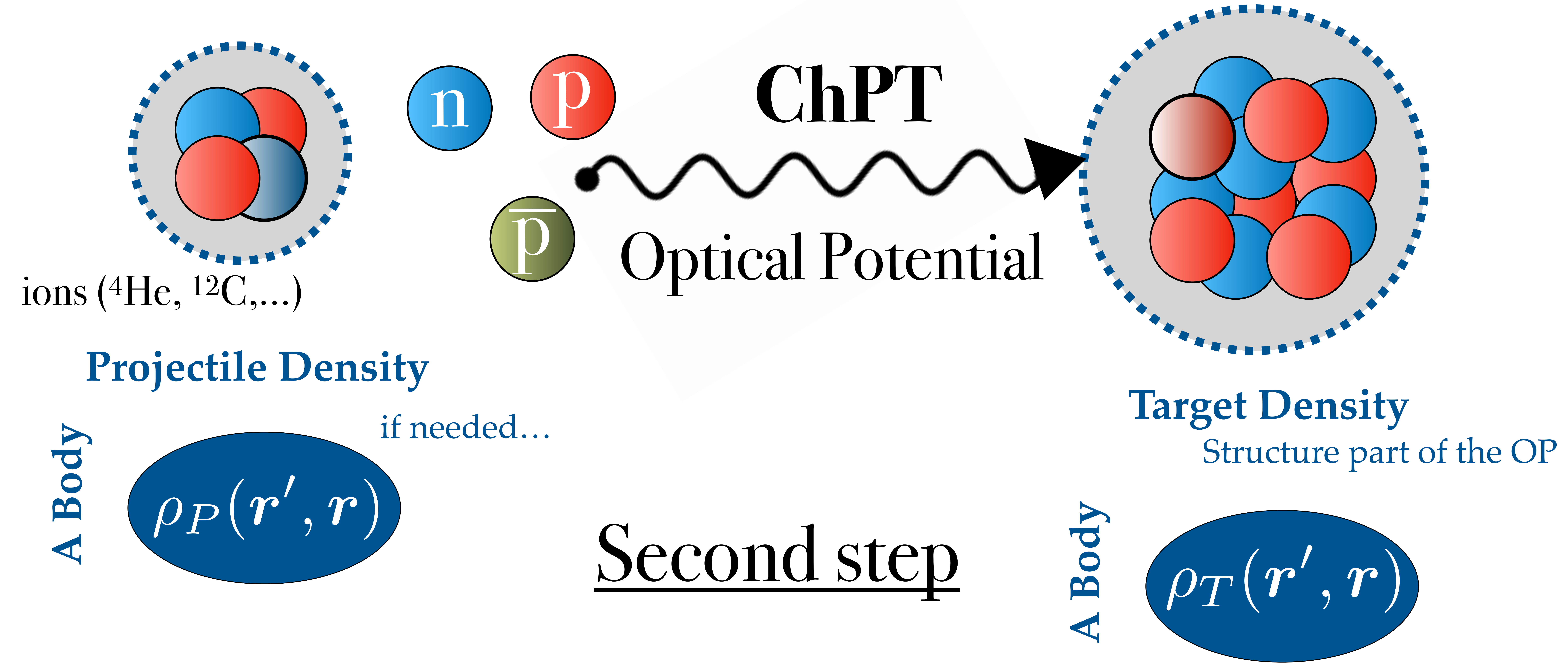
NN Body

$$V_{NN} = \begin{cases} t_{NN} \\ G_{NN} \end{cases}$$

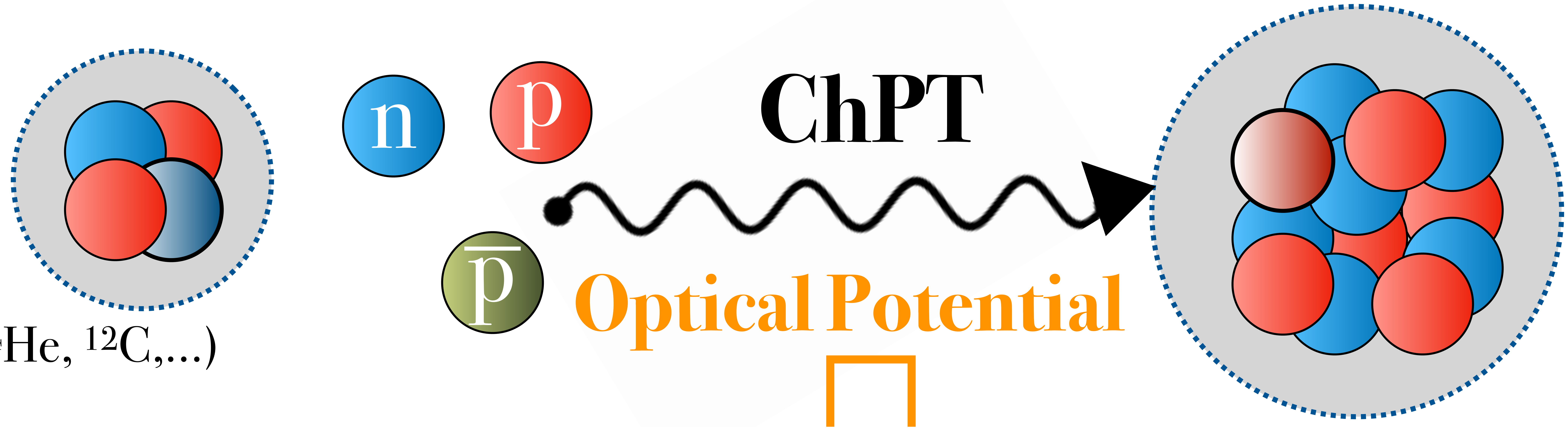
<u>First step</u>

Lippman-Schwinger equation, G-matrix,…

Roadmap towards microscopic optical potentials



Roadmap towards microscopic optical potentials



Multiple scattering theory - spectator expansion

Final step

putting everything together

$$U_{\text{opt}}(\boldsymbol{k}', \boldsymbol{k}; E)$$

$$U_{\text{opt}} \sim \int V_{NN}\ \rho$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{NA}$$

$$A_y$$

Multiple scattering approximation for NA

Lippmann-Schwinger equation for the nucleon-nucleus transition amplitude

$$T = V + V G_0(E) T$$

elastic case

<u>Projectile-target interaction</u>

$$V = \sum_{i=1}^{A} v_{0i} + \sum_{i<j}^{A} w_{0ij}$$

<u>Full</u> 3N interaction is still missing

<u>Many-body propagator</u>

$$G_0(E) = (E - H_0 + i\epsilon)^{-1}$$

Multiple scattering approximation for NA

Lippmann-Schwinger equation for the nucleon-nucleus transition amplitude

$$T = V + VG_0(E)T$$

elastic case

Let's introduce the **optical potential *U***

$$T = U + UG_0(E)PT$$

$$U = V + VG_0(E)QU$$

Projection operators $P + Q = 1$

P space (elastic)

$P = |\Psi_0\rangle\langle\Psi_0|$

Q Space

$Q = 1 - P$

Works by Thaler, Watson, Satchler,...

The **first-order** spectator expansion

Transition amplitude for elastic scattering

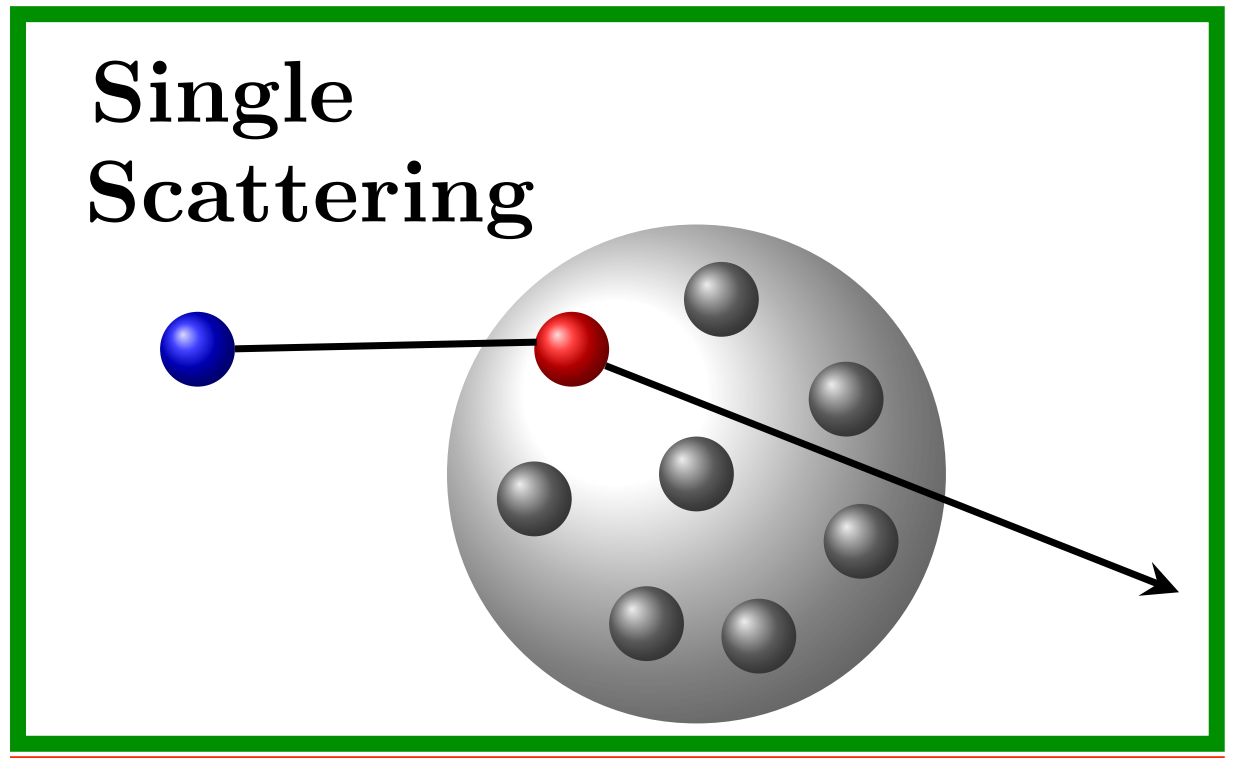
$$T_{\rm el} \equiv PTP = PUP + PUPG_0(E)T_{\rm el}$$

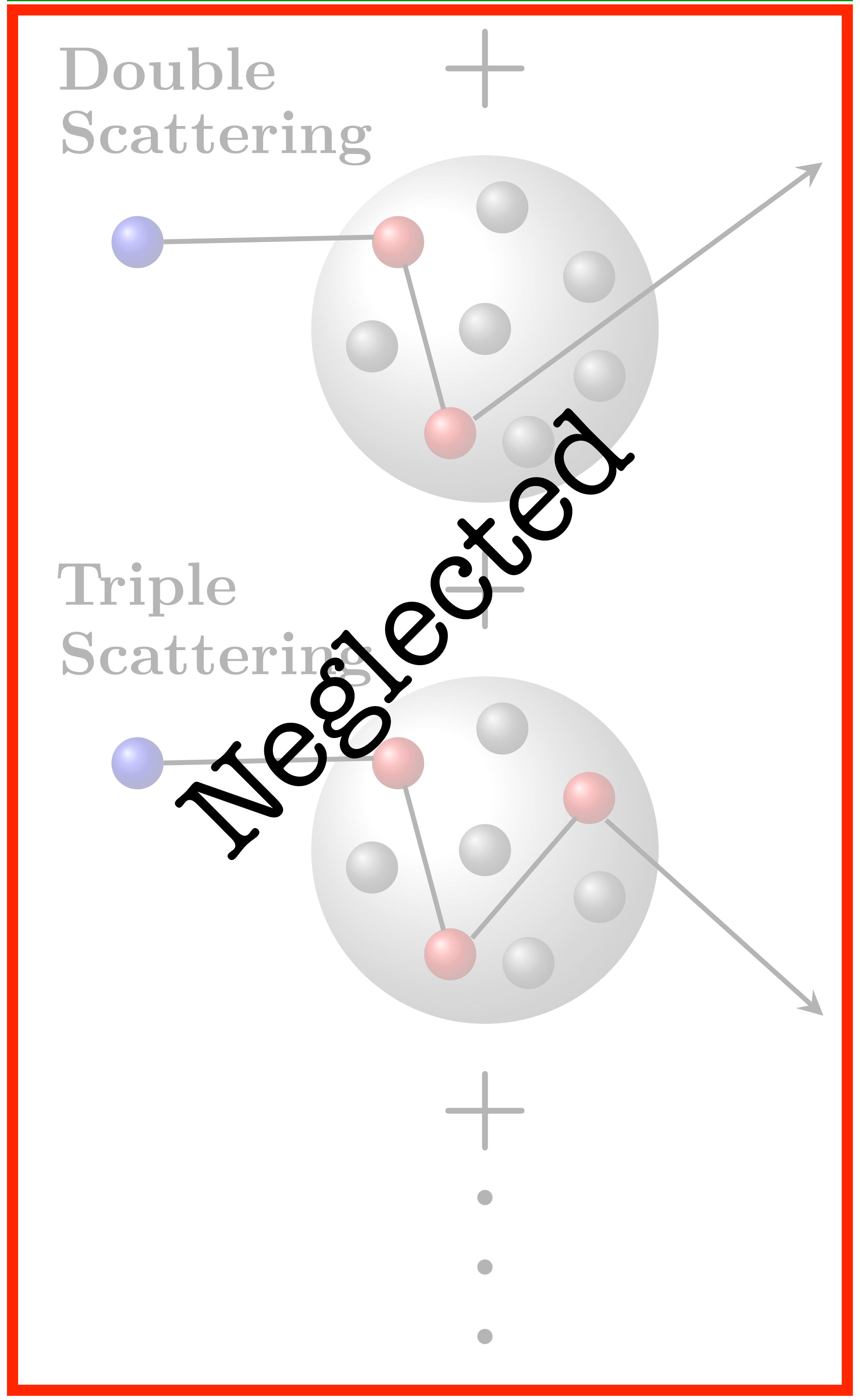
The spectator expansion

$$U = \underbrace{\sum_{i=1}^{A} \tau_{0i} + \sum_{i,j\neq i}^{A} \tau_{0ij} + \sum_{i,j\neq i,k\neq i,j}^{A} \tau_{0ijk} + \dots}_{\textbf{A terms}}$$

$$U \simeq \sum_{i=1}^{A} \tau_{0i} \longrightarrow \tau_{0i} \approx t_{0i} = v_{0i} + v_{0i}\, g_0(E)\, t_{0i}$$

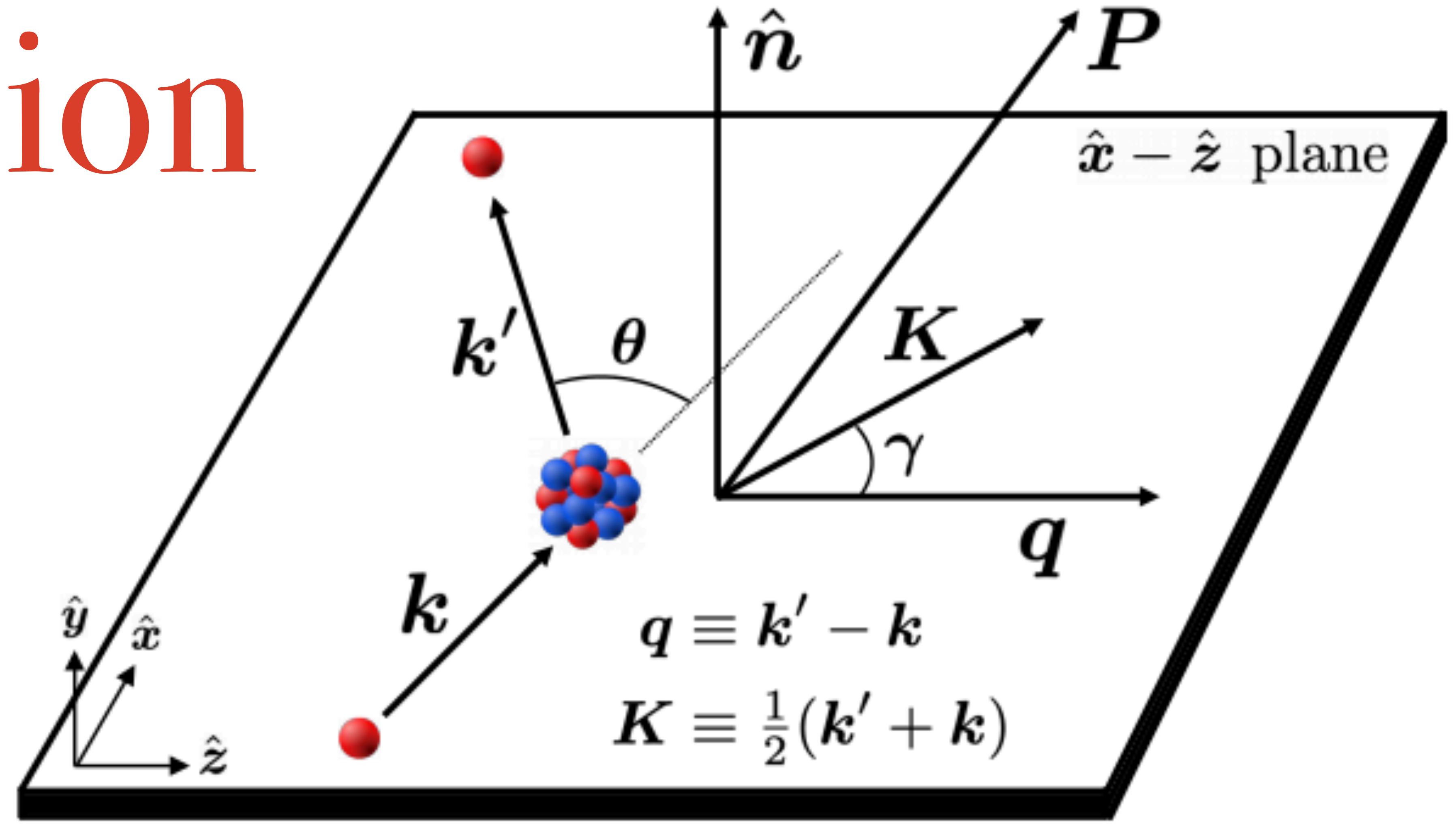
Impulse approximation   Two-body propagator





[Chinn, Elster, Thaler, Weppner, PRC **52**, 1992 (1995)]

The **first-order** spectator expansion



$$U_{\mathbf{p}}(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{\mathbf{p}N}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N(\boldsymbol{q}, \boldsymbol{P})$$

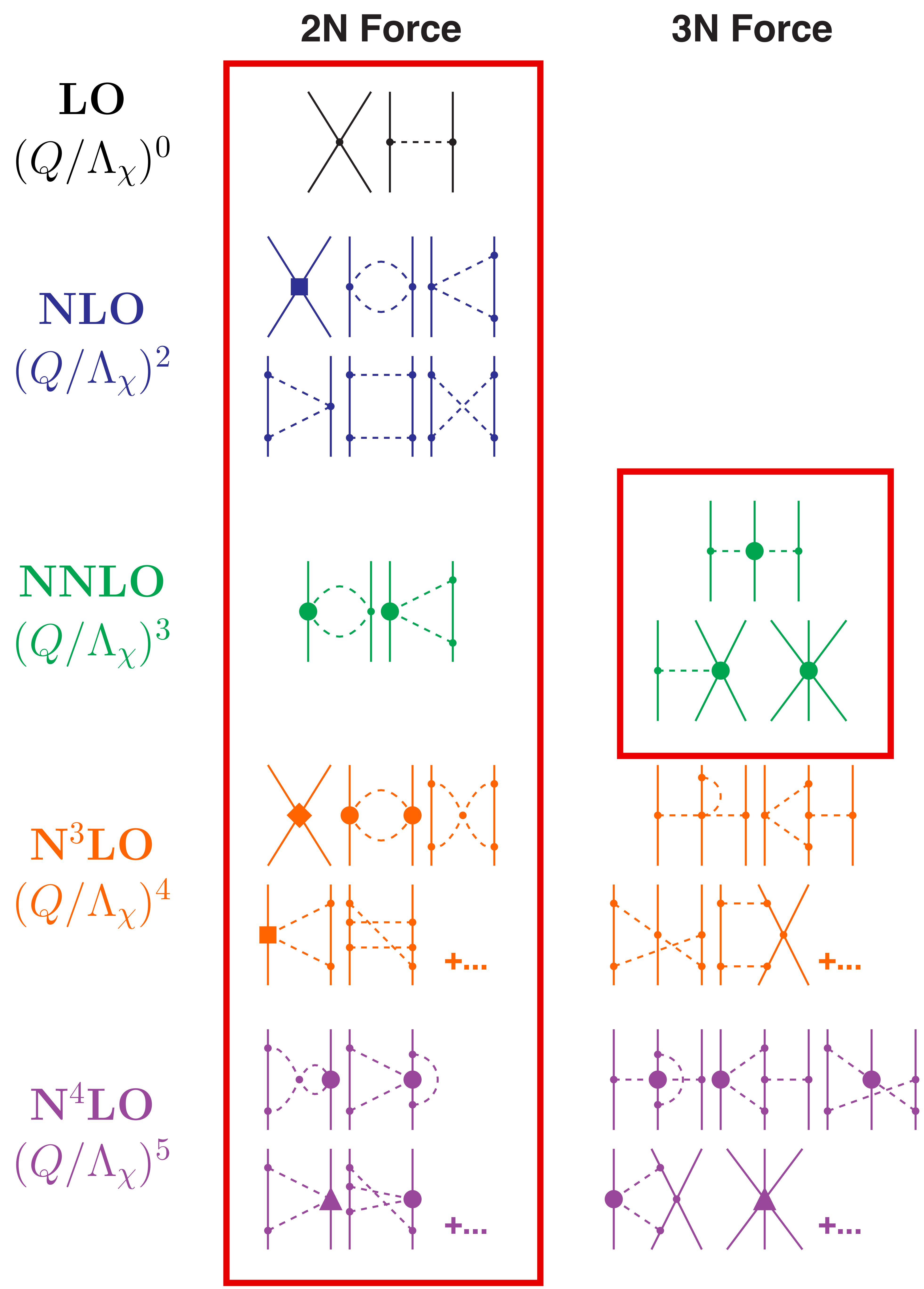
1. Folding procedure with full non-locality
2. Many-body complexity has been strongly reduced
3. Well suited for intermediate energies (70 - 350 MeV), we need improvements (work is in progress) for lower and higher energies
4. It does not require any correction/fit, approximations should be considered

Chiral interactions

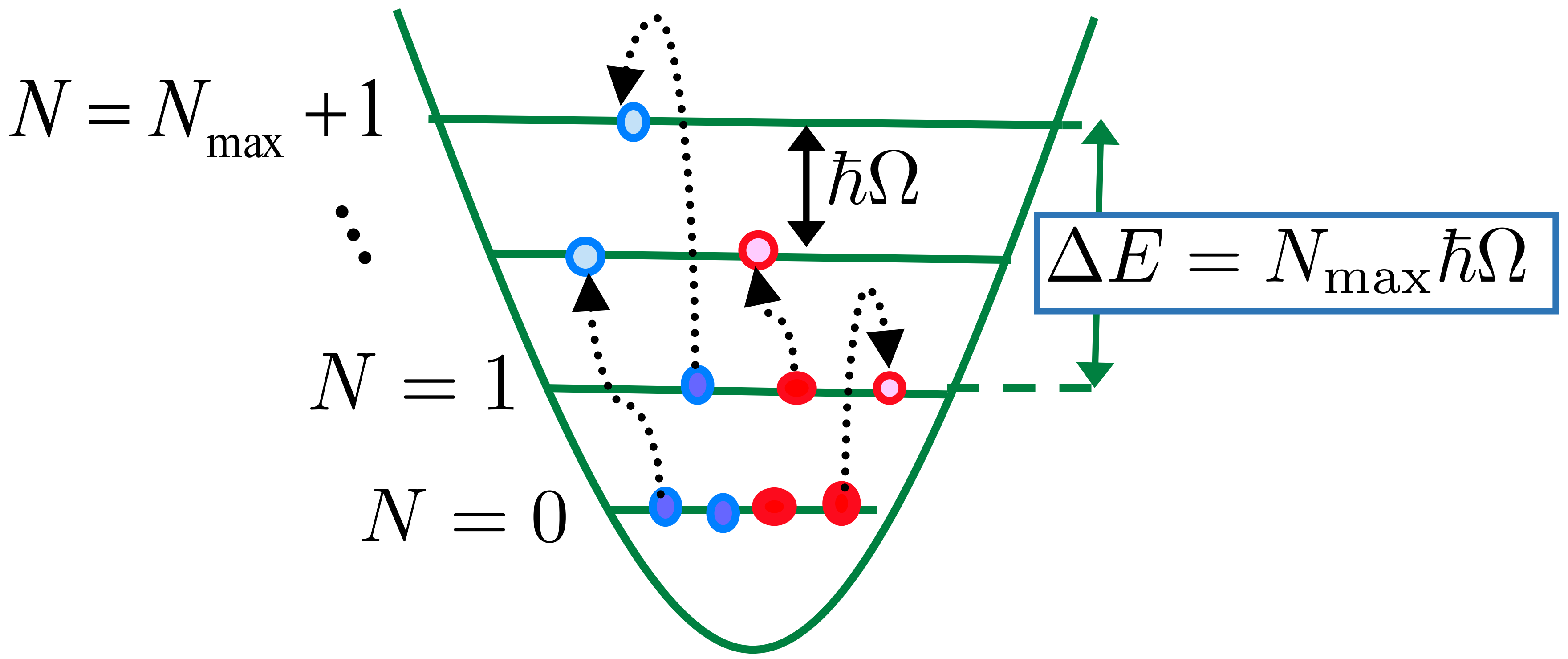
Advantages

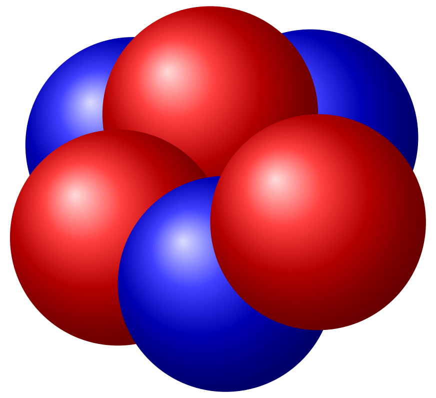
- ☑ QCD symmetries are consistently respected
- ☑ Systematic expansion (order by order we know exactly the terms to be included)
- ☑ Theoretical errors
- ☑ Two- and three-nucleon forces belong to the same framework

We use these interactions as **the only input** to calculate the effective interaction between projectile and the target and into the description of the target density



Target description - light nuclei





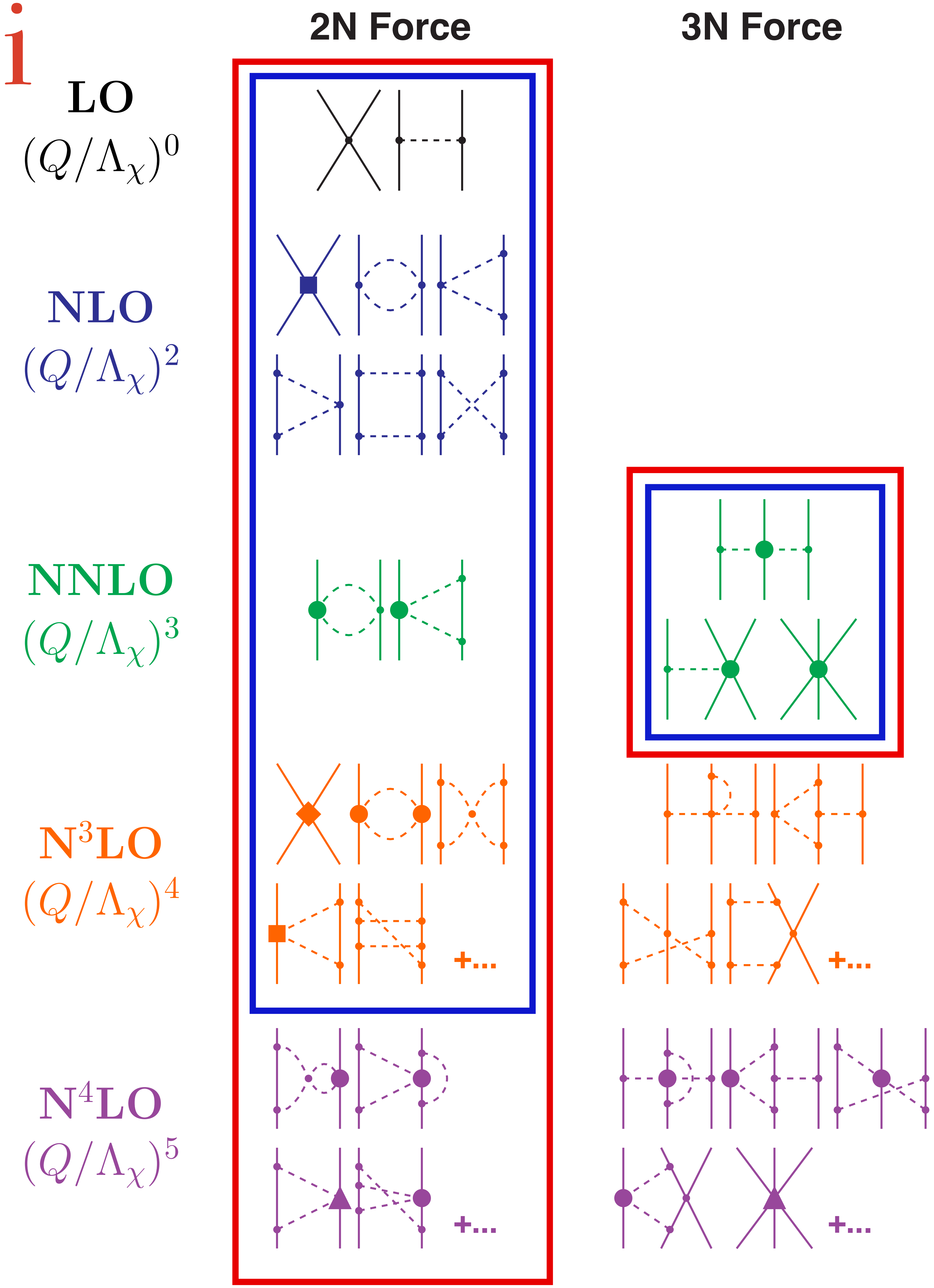
NCSM

- **NN-N⁴LO + 3Nlnl (¹²C, ¹⁶O)**

  - N⁴LO: Entem et al., Phys. Rev. C **96**, 024004 (2017)
  - 3Nlnl: Navrátil, Few-Body Syst. **41**, 117 (2007)
  - c_D & c_E: Kravvaris et al., Phys. Rev. C **102**, 024616 (2020)

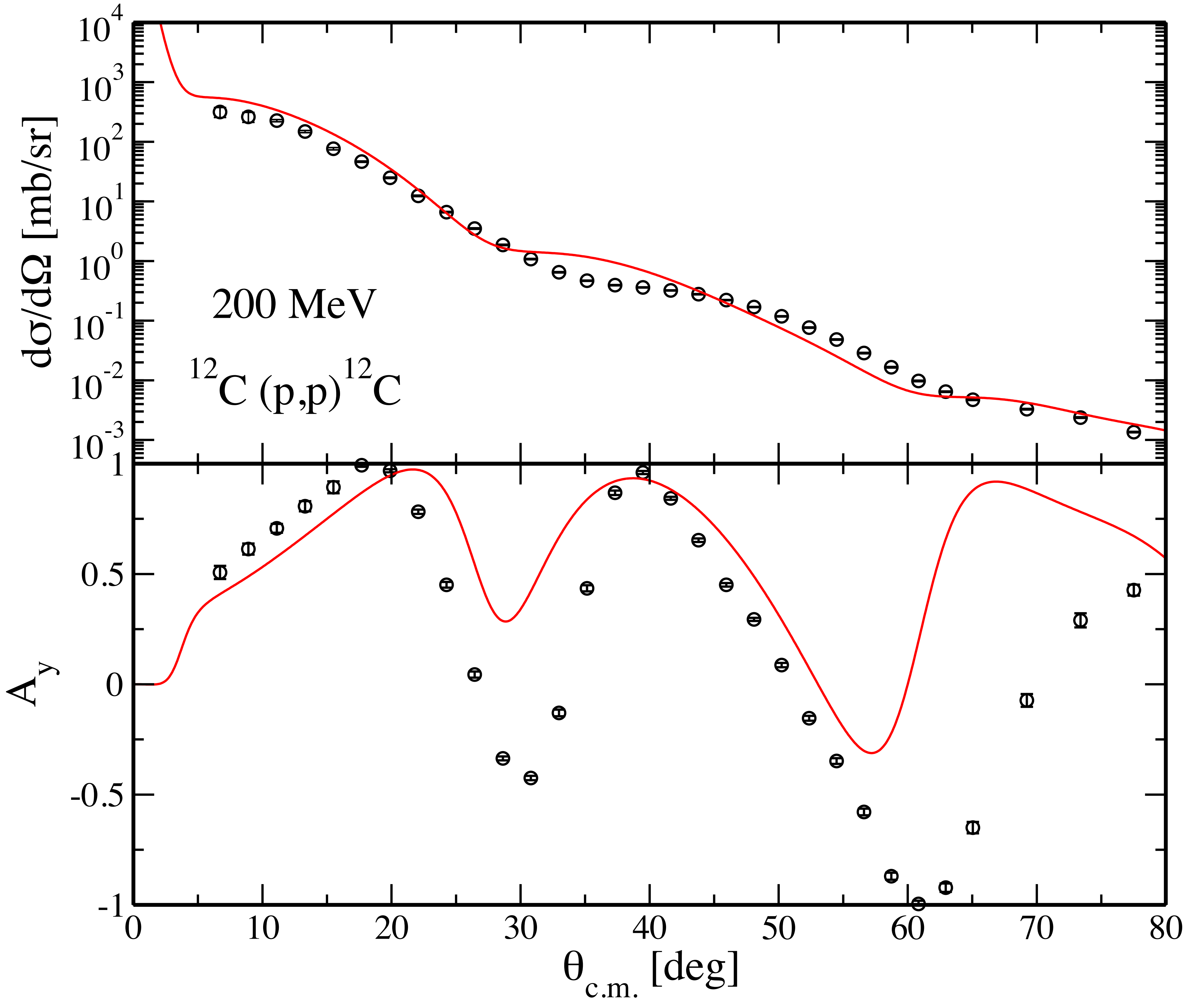
- **NN-N³LO + 3Nlnl (⁹˒¹³C, ⁶˒⁷Li, ¹⁰B)**

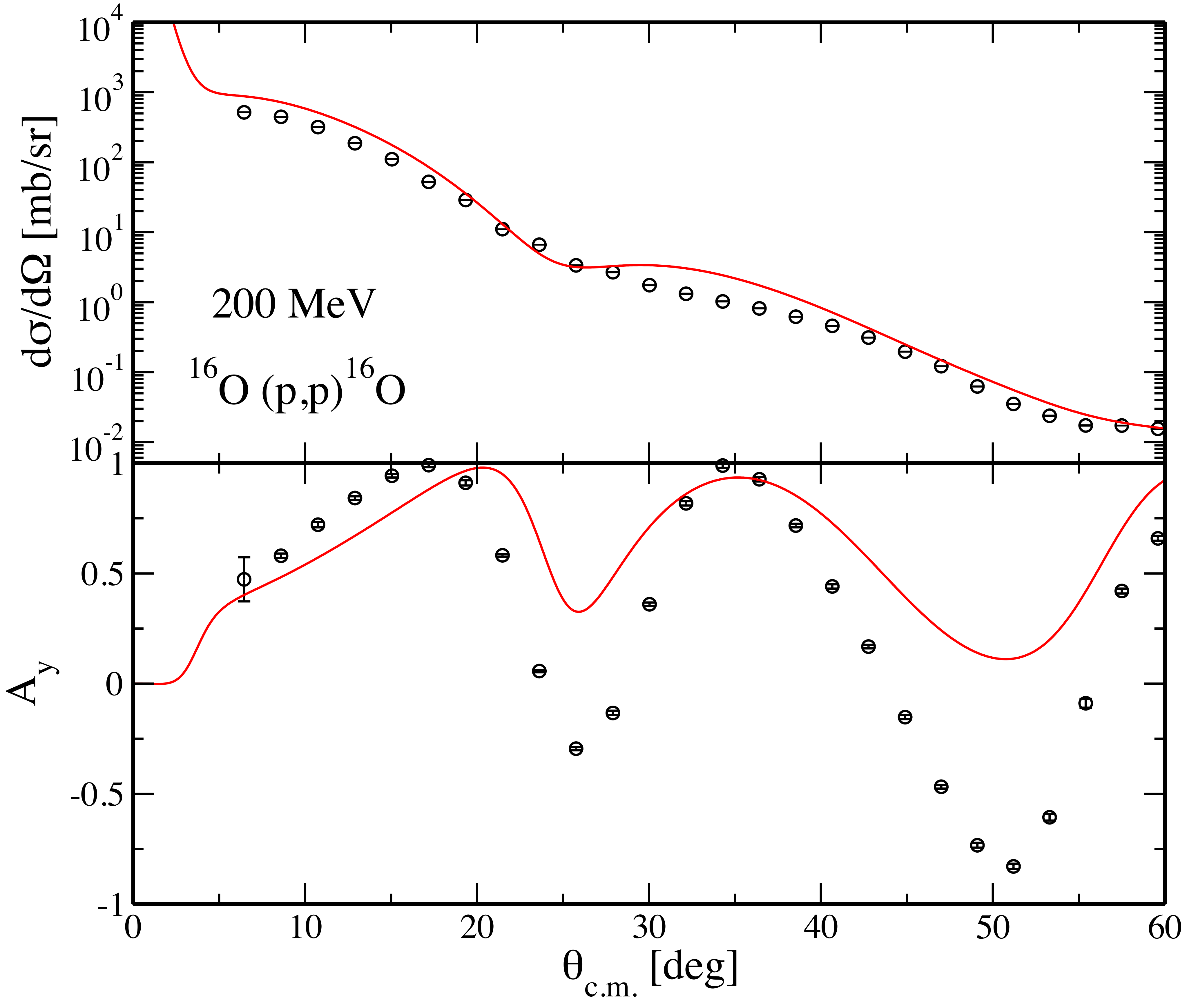
  - N³LO: E&M, Phys. Rev. C **68**, 041001(R) (2003)
  - 3Nlnl: Navrátil, Few-Body Syst. **41**, 117 (2007)
  - c_D & c_E: Somà et al., Phys. Rev. C **101**, 014318 (2020)



Elastic scattering: p+^{12}C and p+^{16}O

Vorabbi et al., PRC **103**, 024604 (2021)





$$U \sim \sum \int \eta \; t_{pN} \; \rho_N$$

- The t matrix is computed with the N^{4}LO interaction
- The density is computed with the N^{4}LO + 3Nlnl interaction

Assessing the impact of the 3N interaction

General equation for the optical potential

$$U = (V_{NN} + V_{3N}) + (V_{NN} + V_{3N})G_0(E)QU$$

Treatment of the 3N force

[Holt et al., Phys. Rev. C **81**, 024002 (2010)]

$$V_{3N} = \frac{1}{2}\sum_{i=1}^{A}\sum_{\substack{j=1\\ j\neq i}}^{A} w_{0ij} \approx \sum_{i=1}^{A}\langle w_{0i}\rangle$$

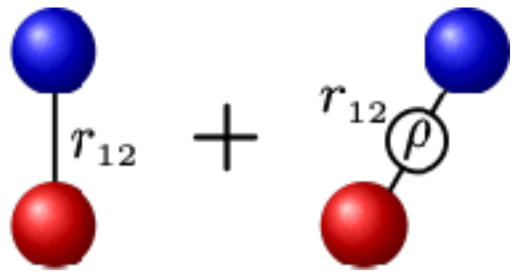
Density dependent

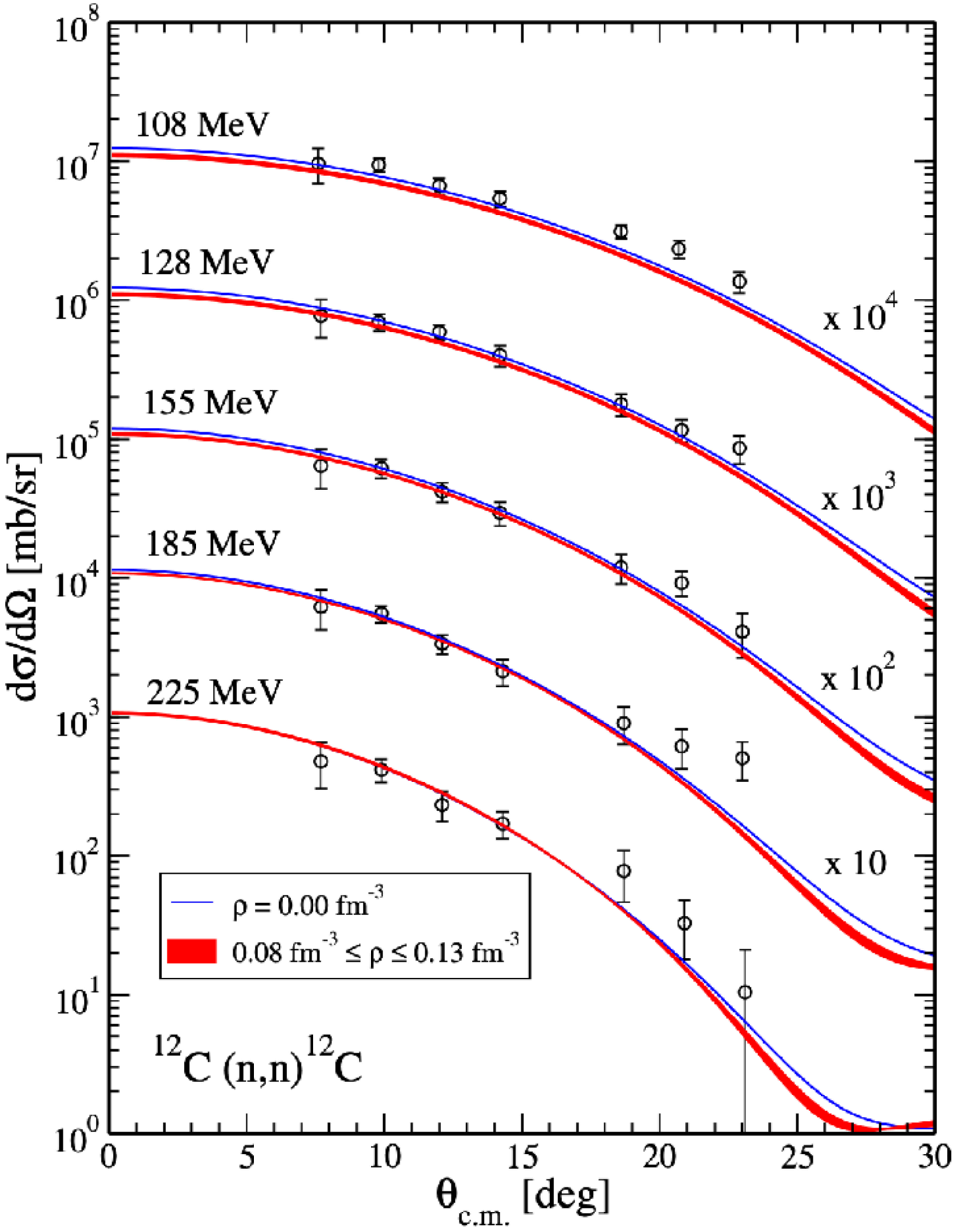
Modification of the *t* matrix

$$t_{0i} = v_{0i}^{(1)} + v_{0i}^{(2)} g_{0i}\, t_{0i}$$

$$v_{0i}^{(1)} = v_{0i} + \tfrac{1}{2}\langle w_{0i}\rangle$$

$$v_{0i}^{(2)} = v_{0i} + \langle w_{0i}\rangle$$

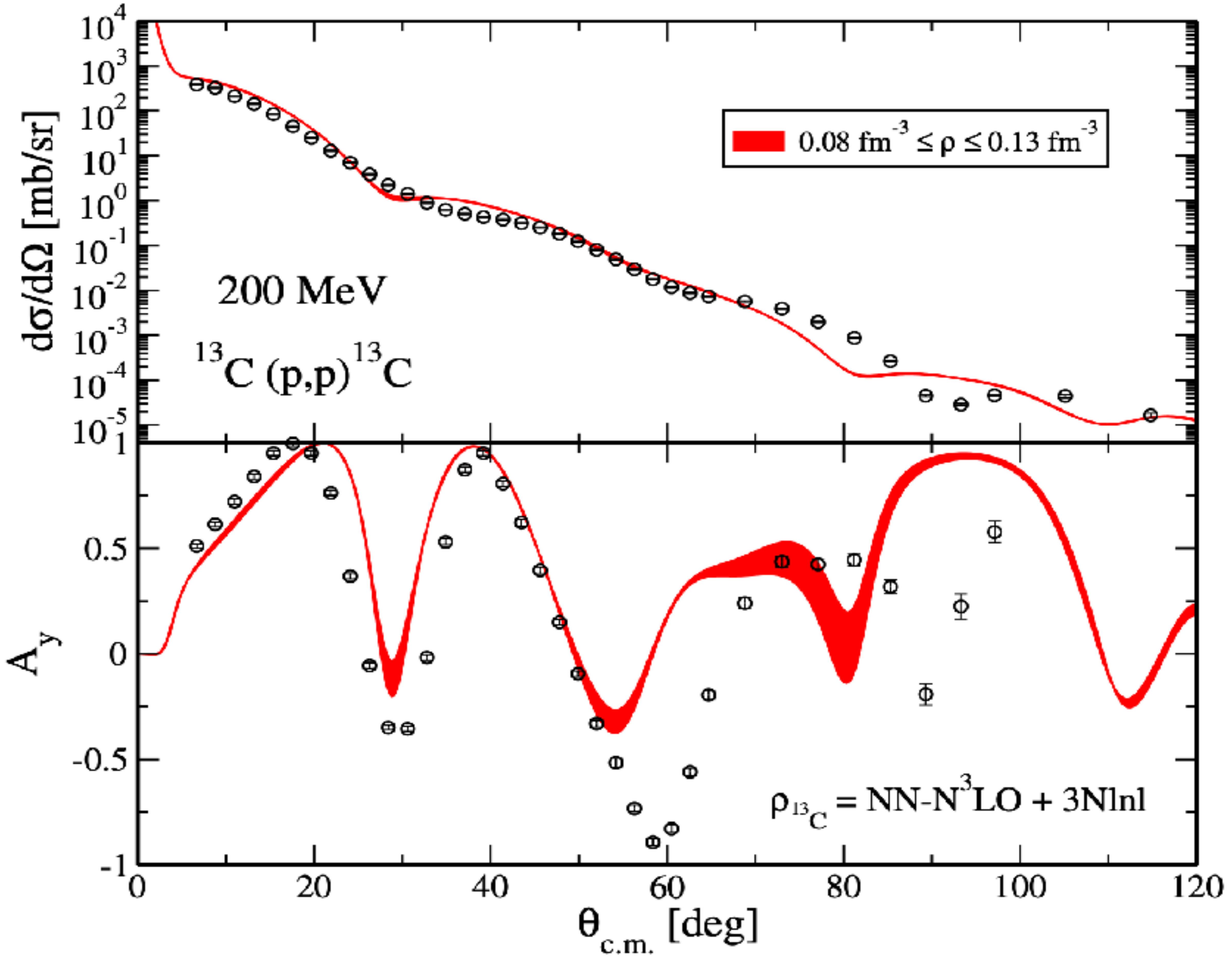
$v =$ 



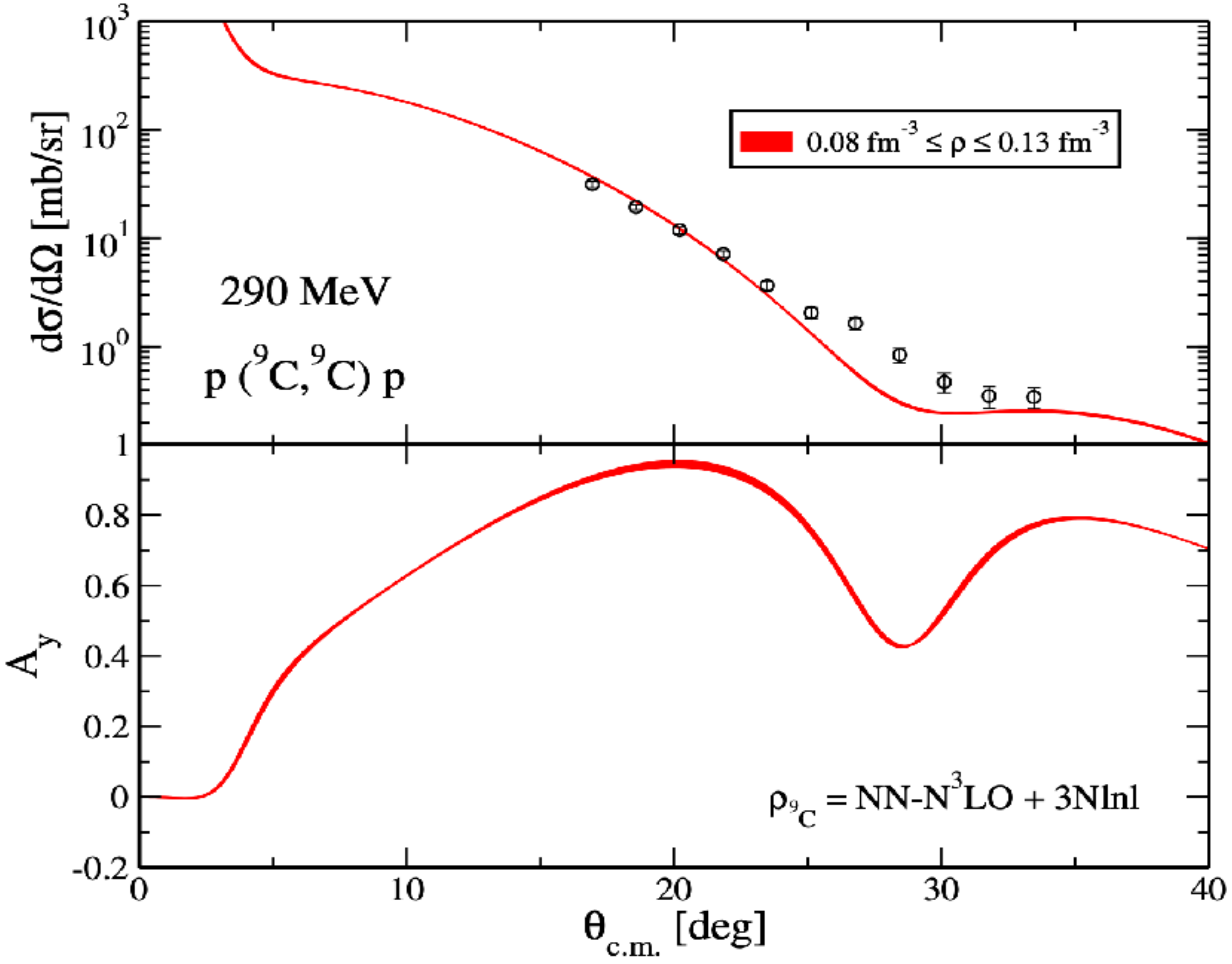
Vorabbi et al., PRC **103**, 024604 (2021)

Extension to non-zero spin targets

$J^\pi = 1/2^-$

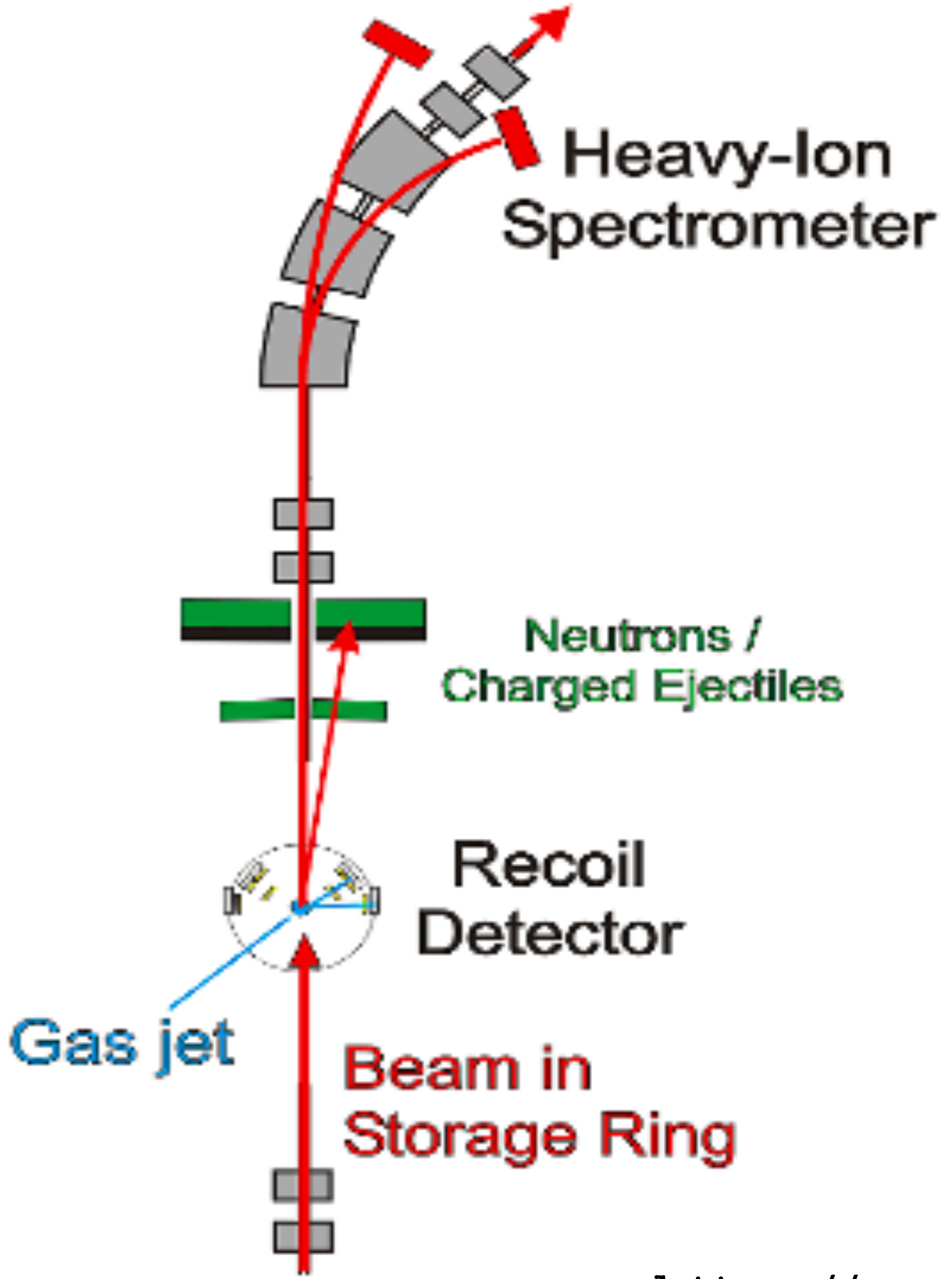


$J^\pi = 3/2^-$



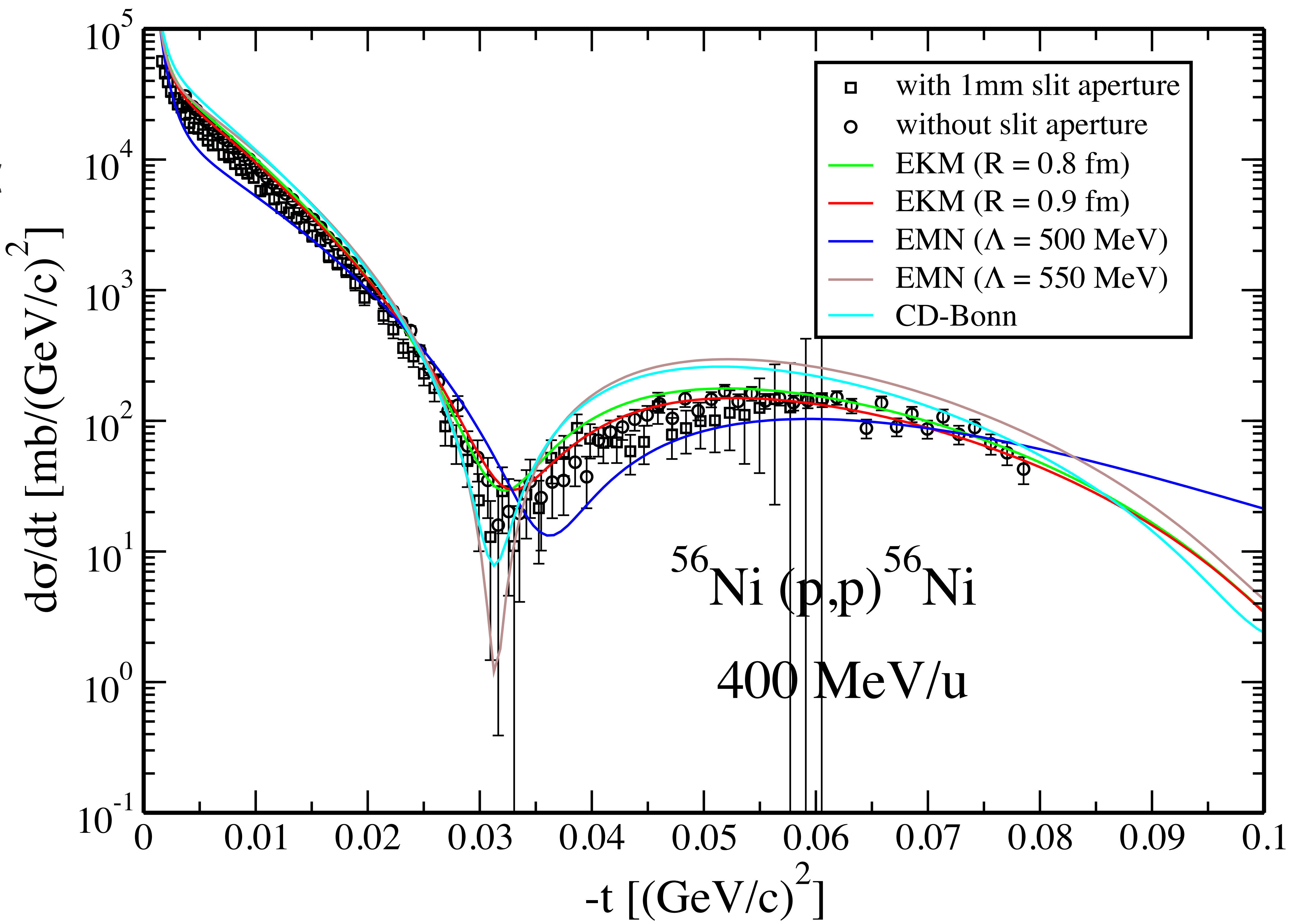
Vorabbi et al., Phys. Rev. C **105**, 014621 (2022)

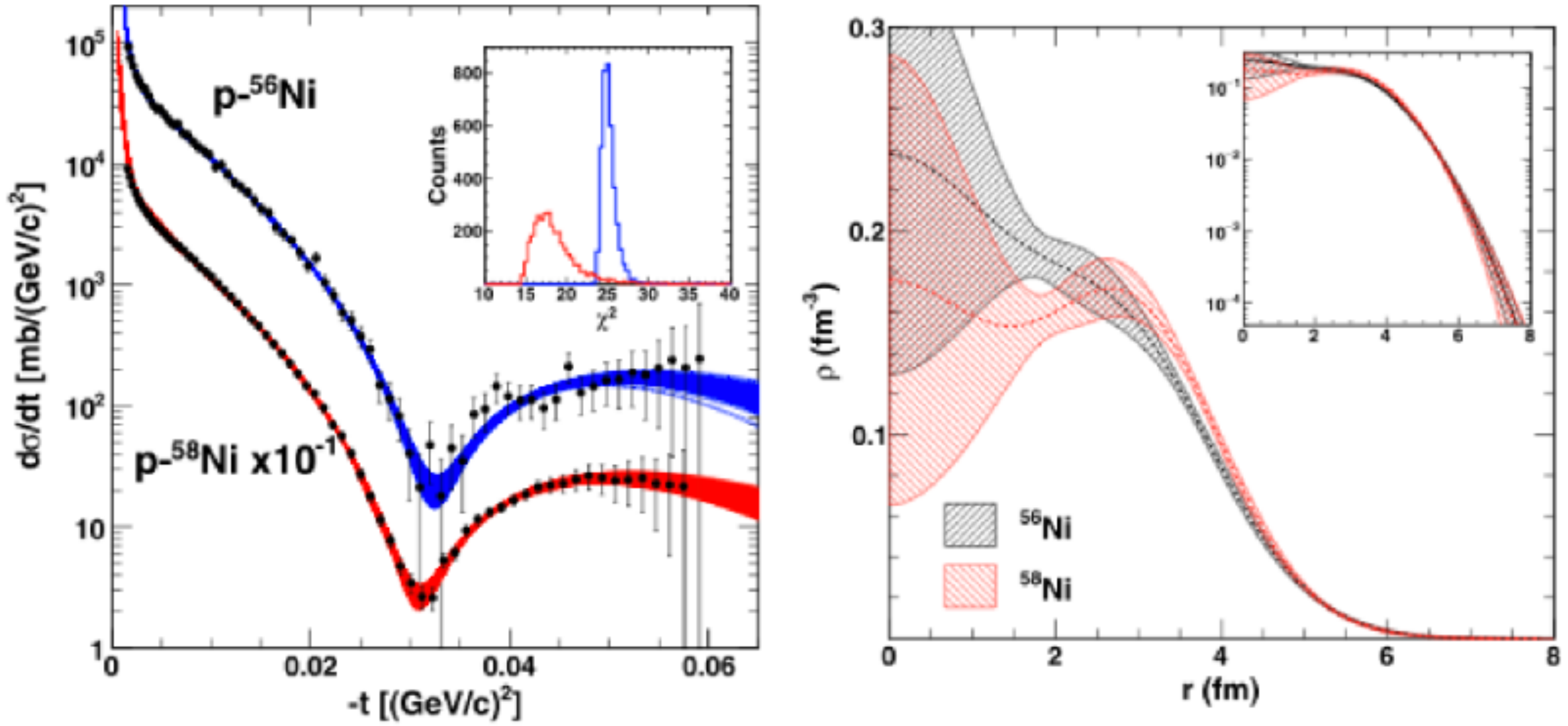
Theoretical predictions vs. Phenomenology



The experimental setup dedicated to studies of exotic nuclei with electromagnetic and ligh hadronic probes (EXL) is part of the **FAIR project**. The aim of the EXL experiment is to study the structure of unstable exotic nuclei in light-ion scattering experiments at intermediate energies.

https://www.rug.nl/kvi-cart/research/hnp/research/exl/

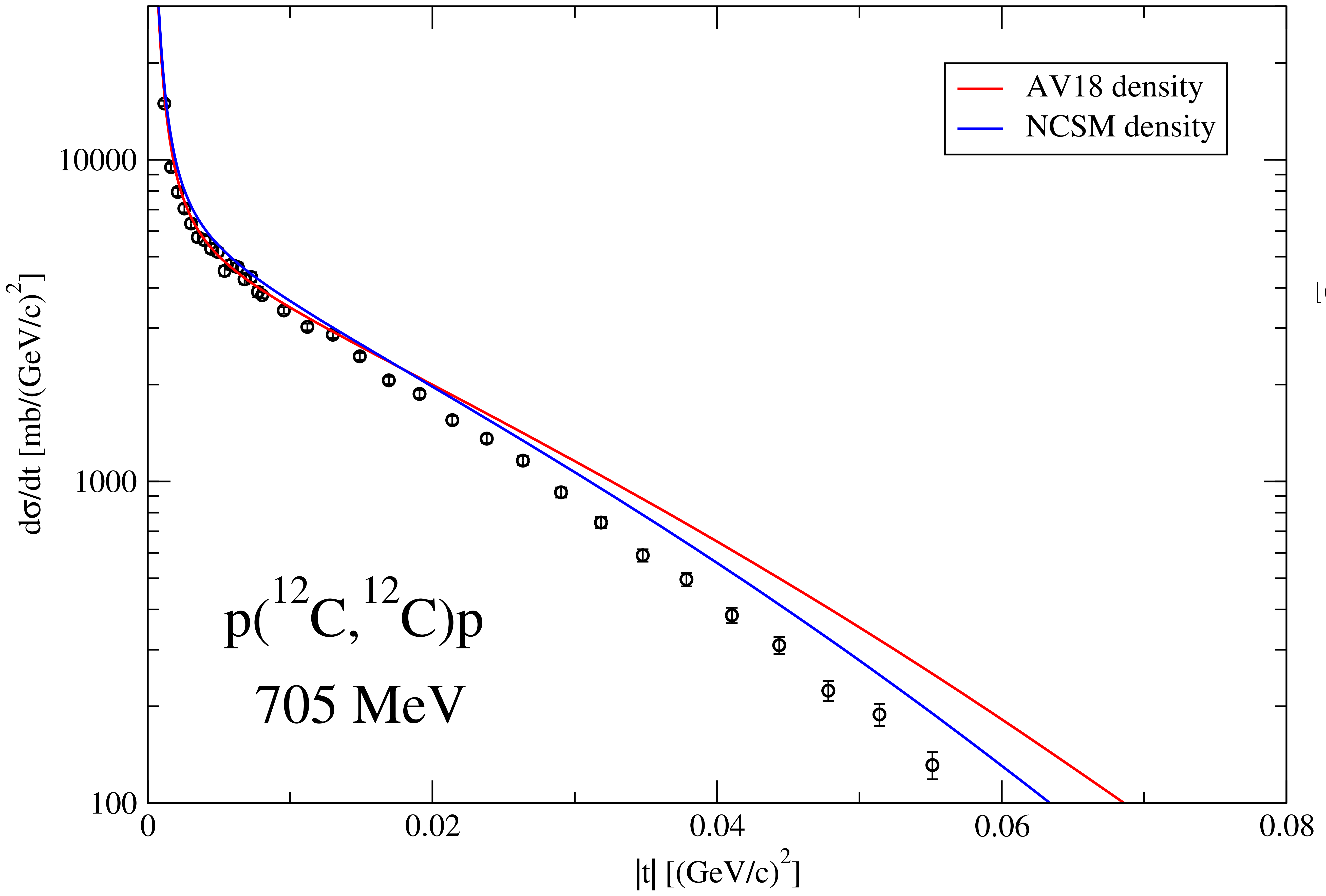


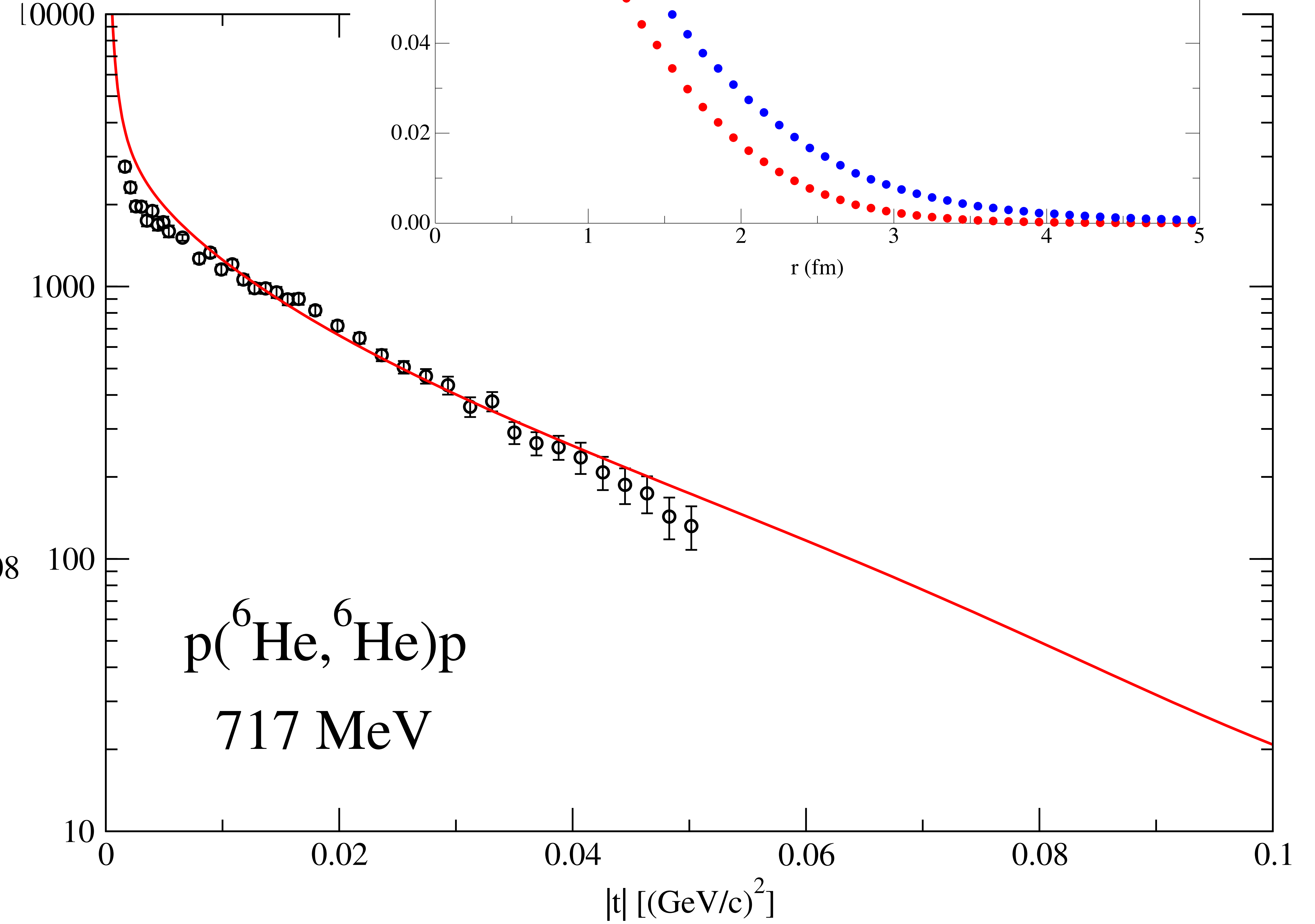


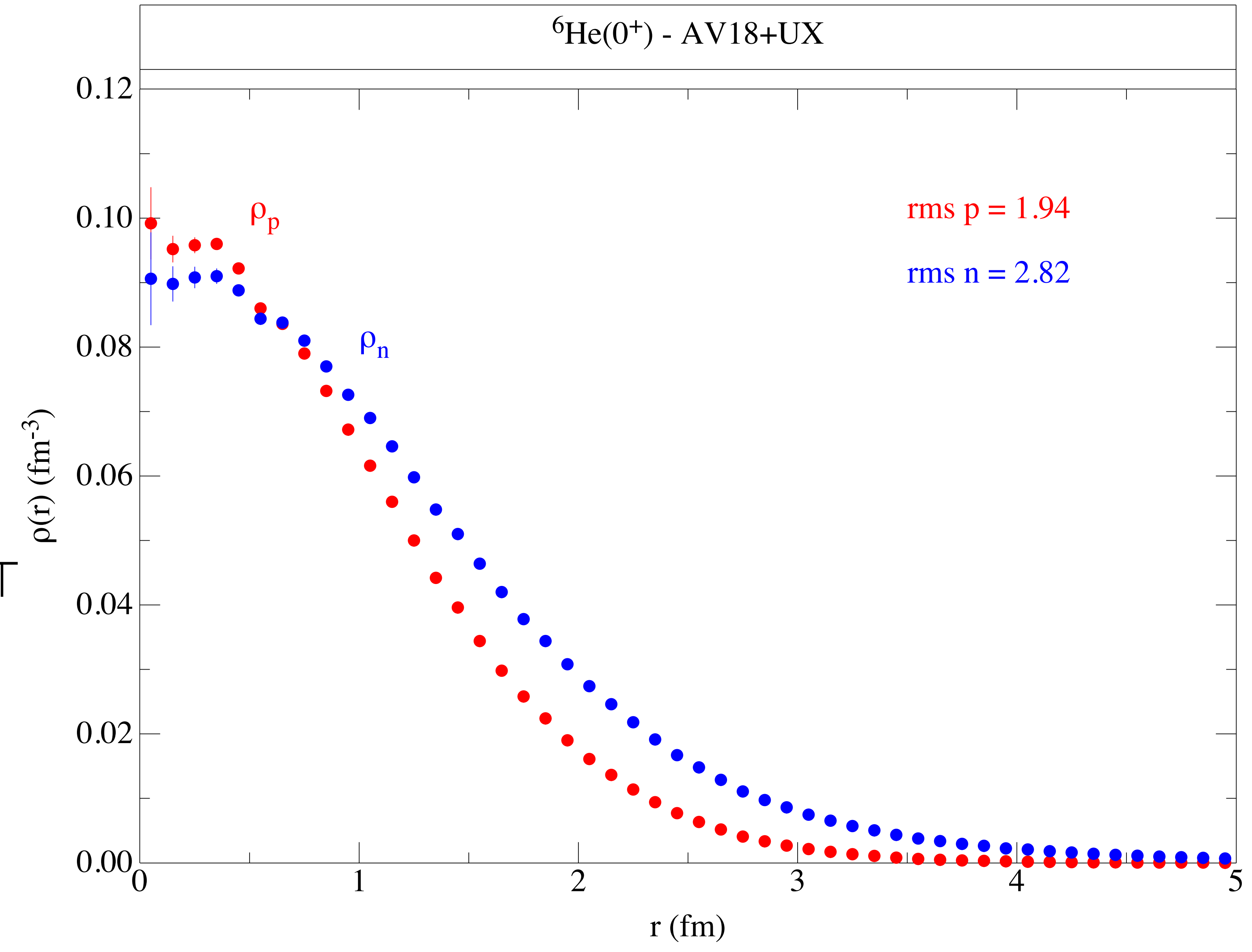
PLB 809 (2020) 135776

The example shown at 400 MeV suggests that, at this energy, the EKM potentials have not yet reached the limit after which the chiral expansion scheme breaks down.

To higher energies (AV18 potential)







Vorabbi et al., work in progress (2026)

Extension to antiproton-nucleus elastic scattering

$$U_p(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{pN}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N(\boldsymbol{q}, \boldsymbol{P})$$

- The projectile information only enters the t_{pN} matrix. For antiprotons we make the following replacement

$$t_{pN} \Rightarrow t_{\bar{p}N}$$

- An antiproton-nucleon interaction is needed. $\bar{p}N$ chiral interaction derived up to N^{3}LO

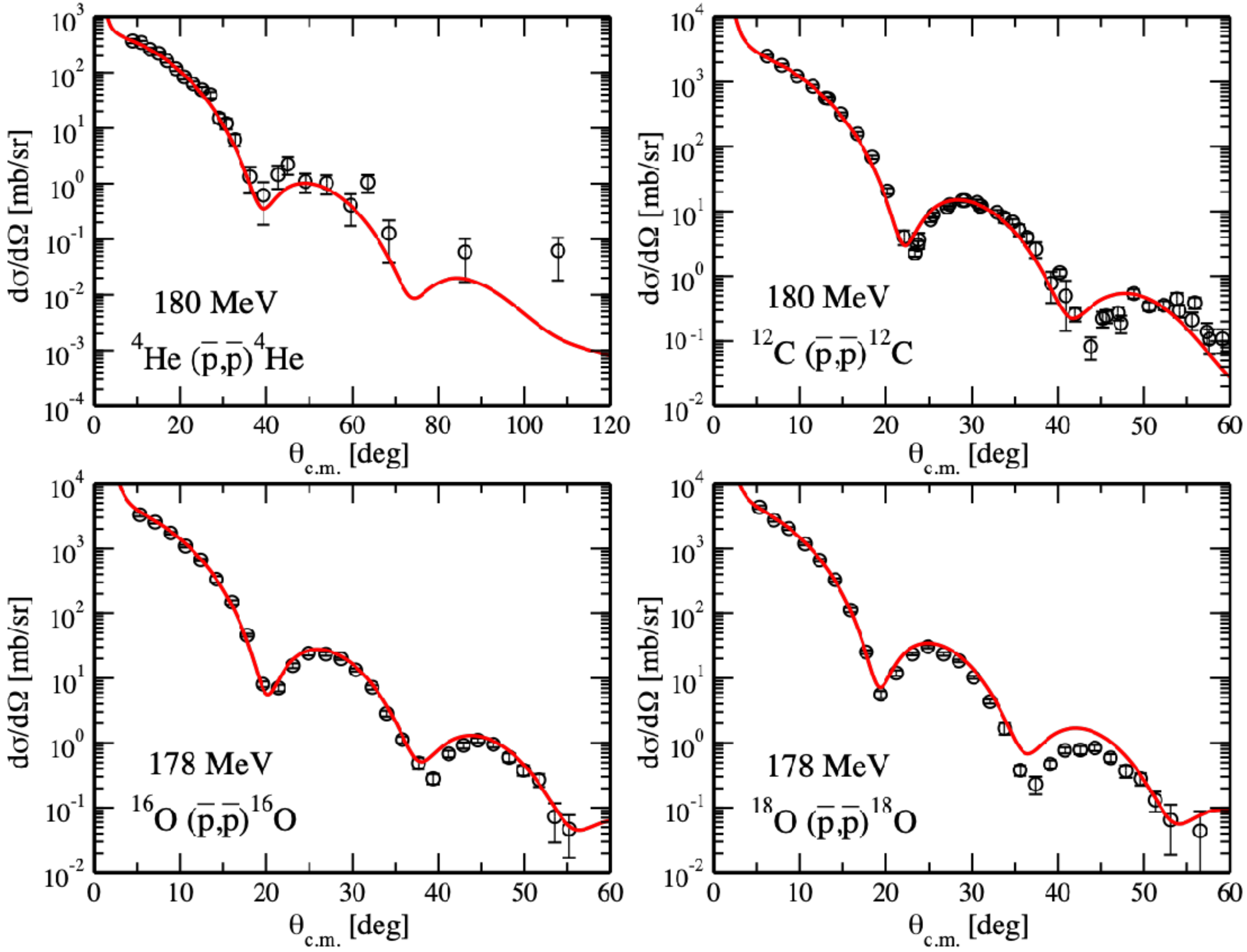
[Dai, Haidenbauer, Meißner, JHEP 07 (2017) 78]

- No projectile-target anti-symmetrisation

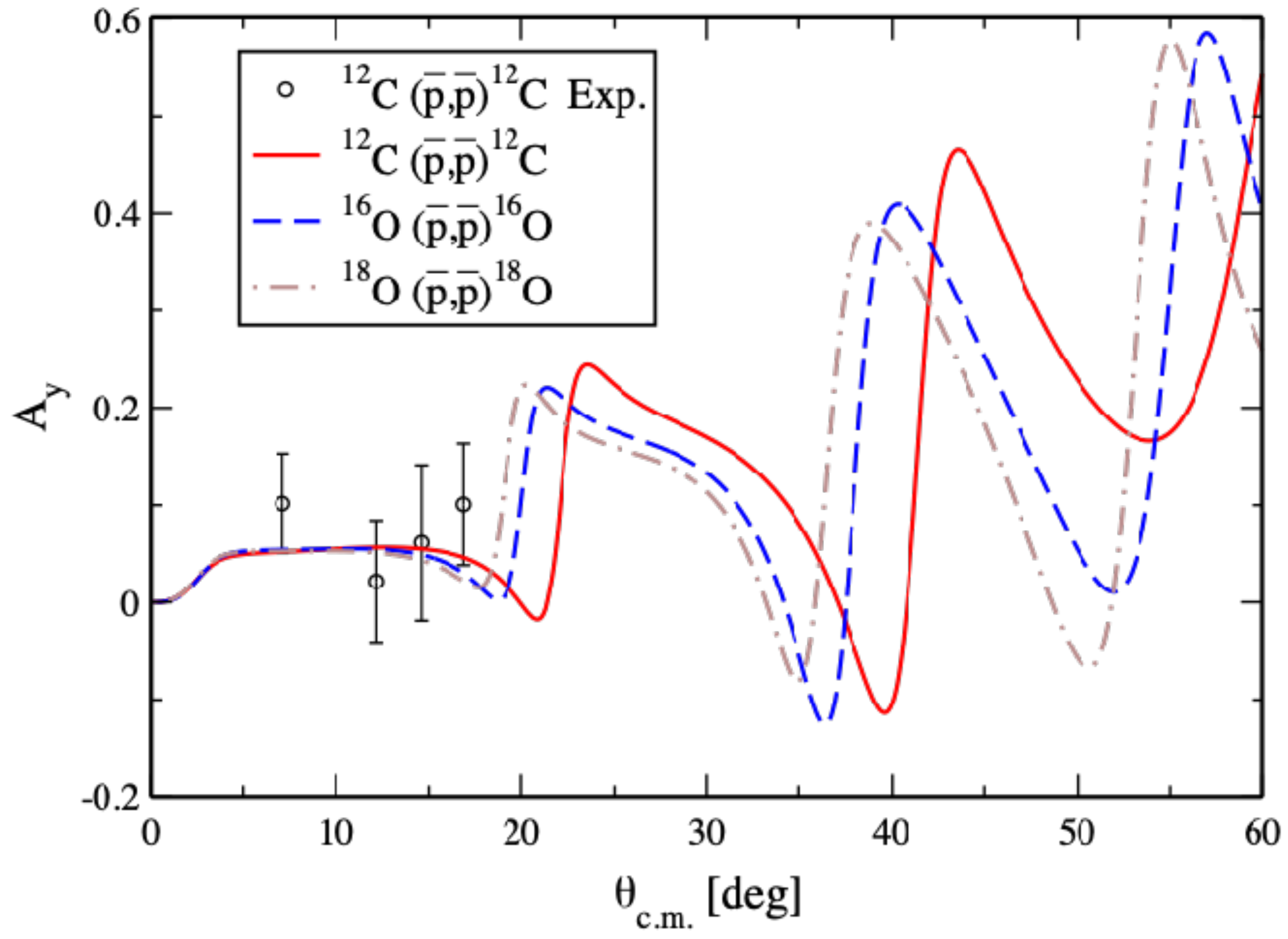
- Antiprotons are mostly absorbed at the surface of the nucleus so the first-order expansion should work better in this case!

Vorabbi et al., PRL **124**, 162501 (2020)

Extension to antiproton-nucleus elastic scattering



Antiprotons mostly interact at the surface of the nucleus so the first-order expansion should work better in this case.



Vorabbi et al., PRL **124**, 162501 (2020)

Application of MST to inelastic scattering

Vorabbi et al., arXiv:2603.26265, submitted to PRC (2026)

<u>The inelastic transition amplitude</u>

$$T_{fi}^{\rm inel} = \left\langle \psi_f^{(-)} \middle| U_{\rm tr} \middle| \psi_i^{(+)} \right\rangle$$

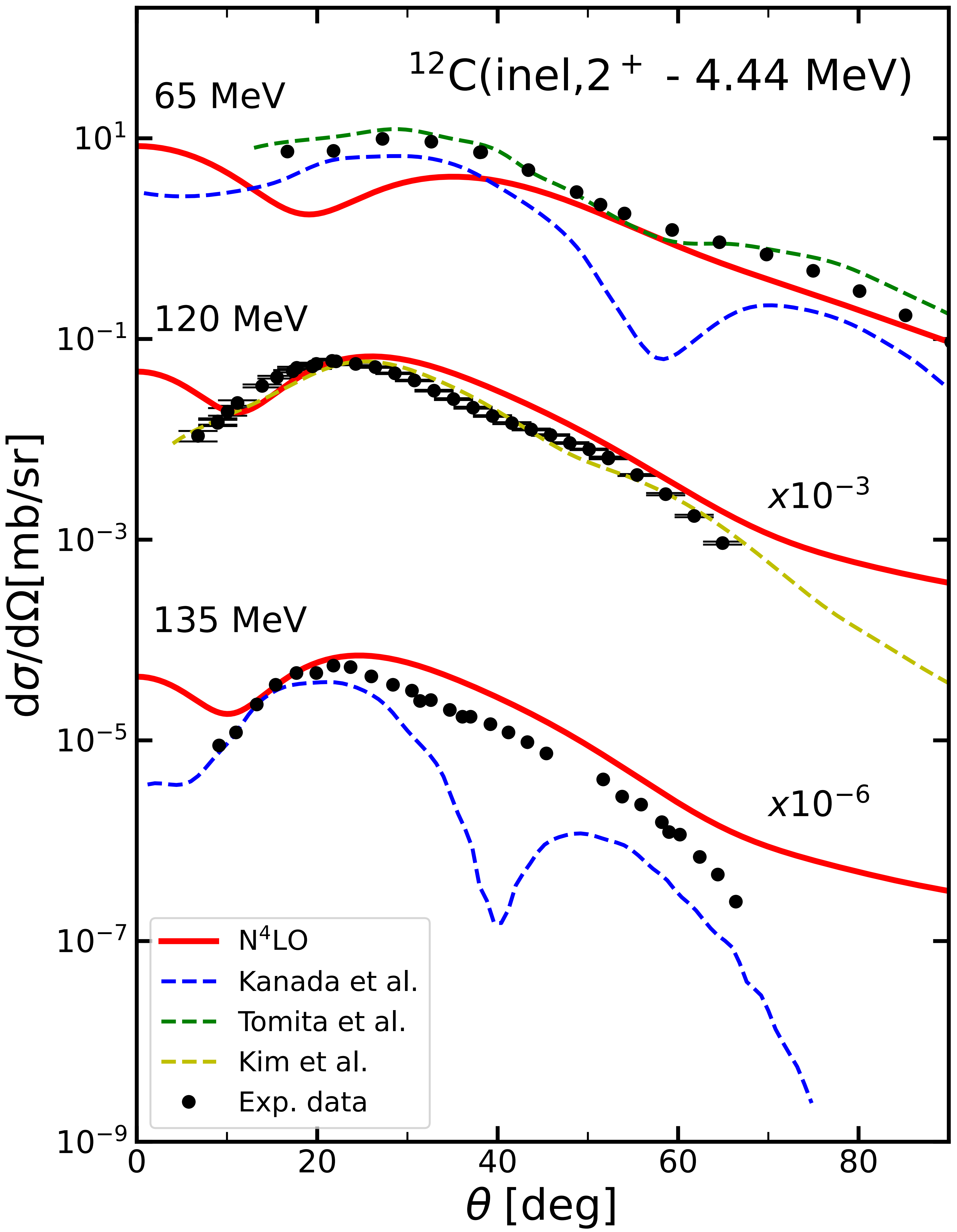
- The general shape of the data is reproduced
- The NN t matrix adopted for the calculation of the 3 potentials contains only two terms so far

$$A + i(\boldsymbol{\sigma} \cdot \hat{\boldsymbol{n}})C$$

- No adjustments, no free parameters
- Large angles behaviour needs improvement

Theoretical B(E2; $0^+ \to 2^+$) = 35.2987 e^2 fm^4

Experimental B(E2; $0^+ \to 2^+$) = 39.7(33) e^2 fm^4

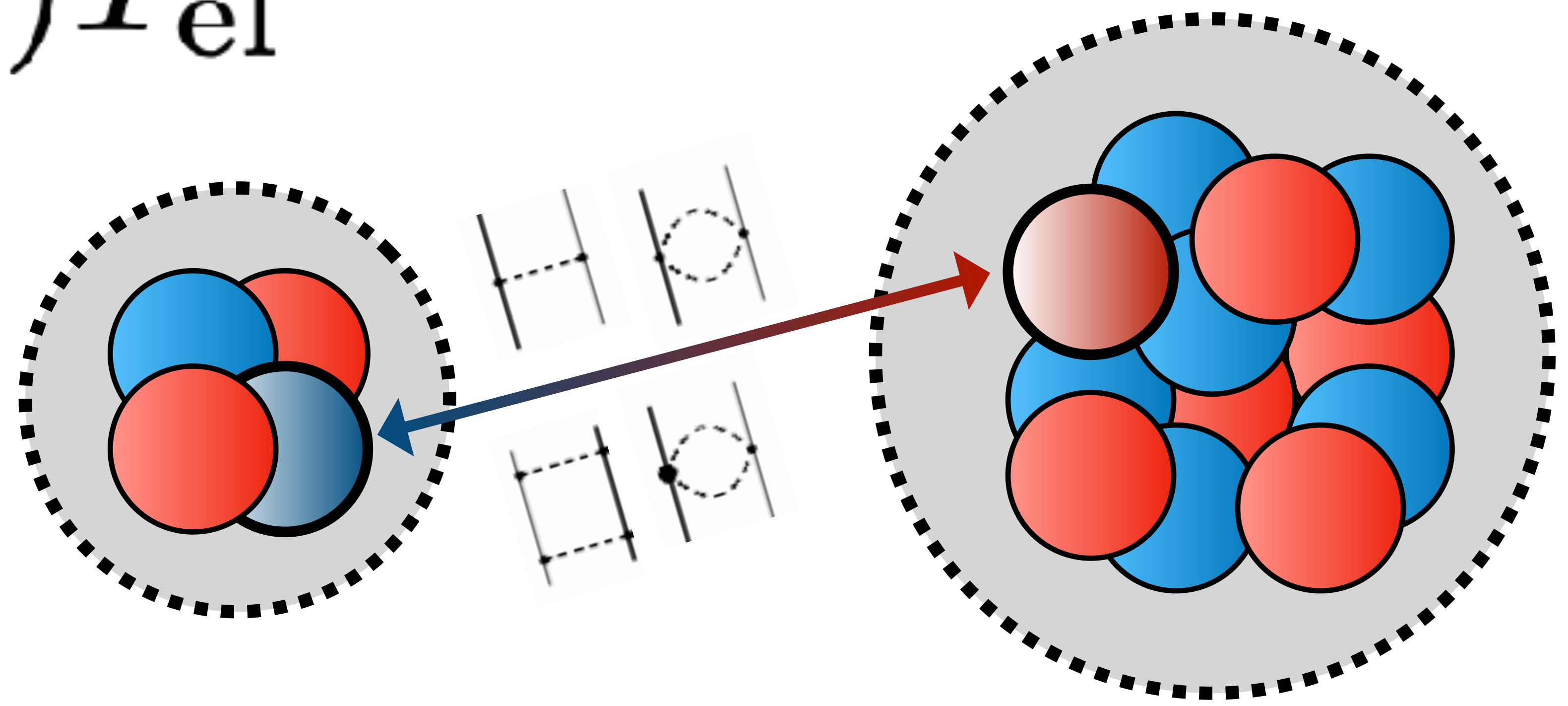


OP for nucleus-nucleus elastic scattering

Transition amplitude for elastic scattering

$$T_{\rm el} \equiv PTP = PUP + PUPG_0(E)T_{\rm el}$$

The spectator expansion



$$U \simeq \sum_{i=1}^{A} \sum_{j=A+1}^{A+B} \tau_{ij} \qquad \tau_{ij} = v_{ij} + v_{ij} G_0(E) \tau_{ij}$$

(A+B)-body propagator
More complicated than the NA case

$$\tau_{ij} \approx t_{ij} = v_{ij} + v_{ij}\, g_0(E)\, t_{ij}$$

Impulse approximation

Two-body propagator

Vorabbi et al., PRL 135 (2025) 172501

First-order OP for nucleus-nucleus elastic scattering

Møller factor

$$t^{(NA)}_{\mathbf{p}N} = \eta\, t^{(NN)}_{\mathbf{p}N}$$

It imposes the Lorentz invariance of flux when we pass from the NA to the NN frame where the t matrices are evaluated

Projectile density

Target density

$$U(\boldsymbol{q},\boldsymbol{K}) = \sum_{\alpha=n,p}\sum_{\beta=n,p}\int d\boldsymbol{P}\int d\boldsymbol{Q}\; \eta(\boldsymbol{q},\boldsymbol{K},\boldsymbol{P},\boldsymbol{Q})\, t_{\alpha\beta}(\boldsymbol{q},\boldsymbol{K},\boldsymbol{P},\boldsymbol{Q};\mathcal{E})\, \rho^{(\mathbb{P})}_{\alpha}(\boldsymbol{q},\boldsymbol{P})\, \rho^{(\mathbb{T})}_{\beta}(\boldsymbol{q},\boldsymbol{Q})$$

Free two-body scattering matrix

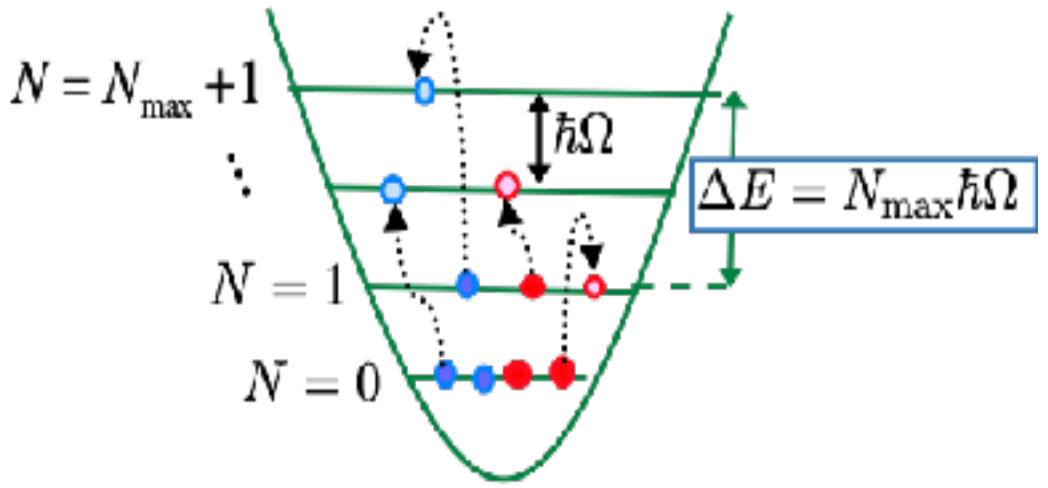
$$t_{0i} = v_{0i} + v_{0i}\, g_{0i}\, t_{0i}$$

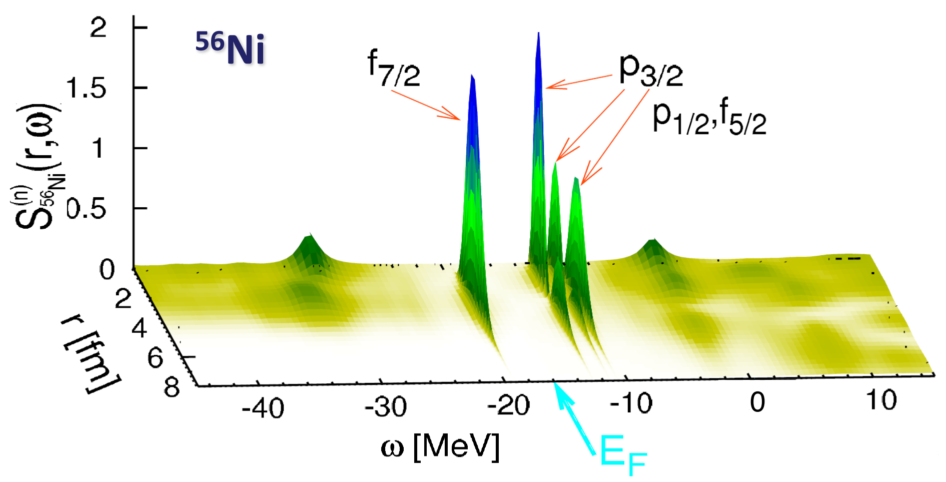
$$g_{0i} = (E - h_0 - h_i + i\epsilon)^{-1}$$

- Simple one-body equation
- Can be solved easily
- Only **NN** interaction

Nonlocal one-body density

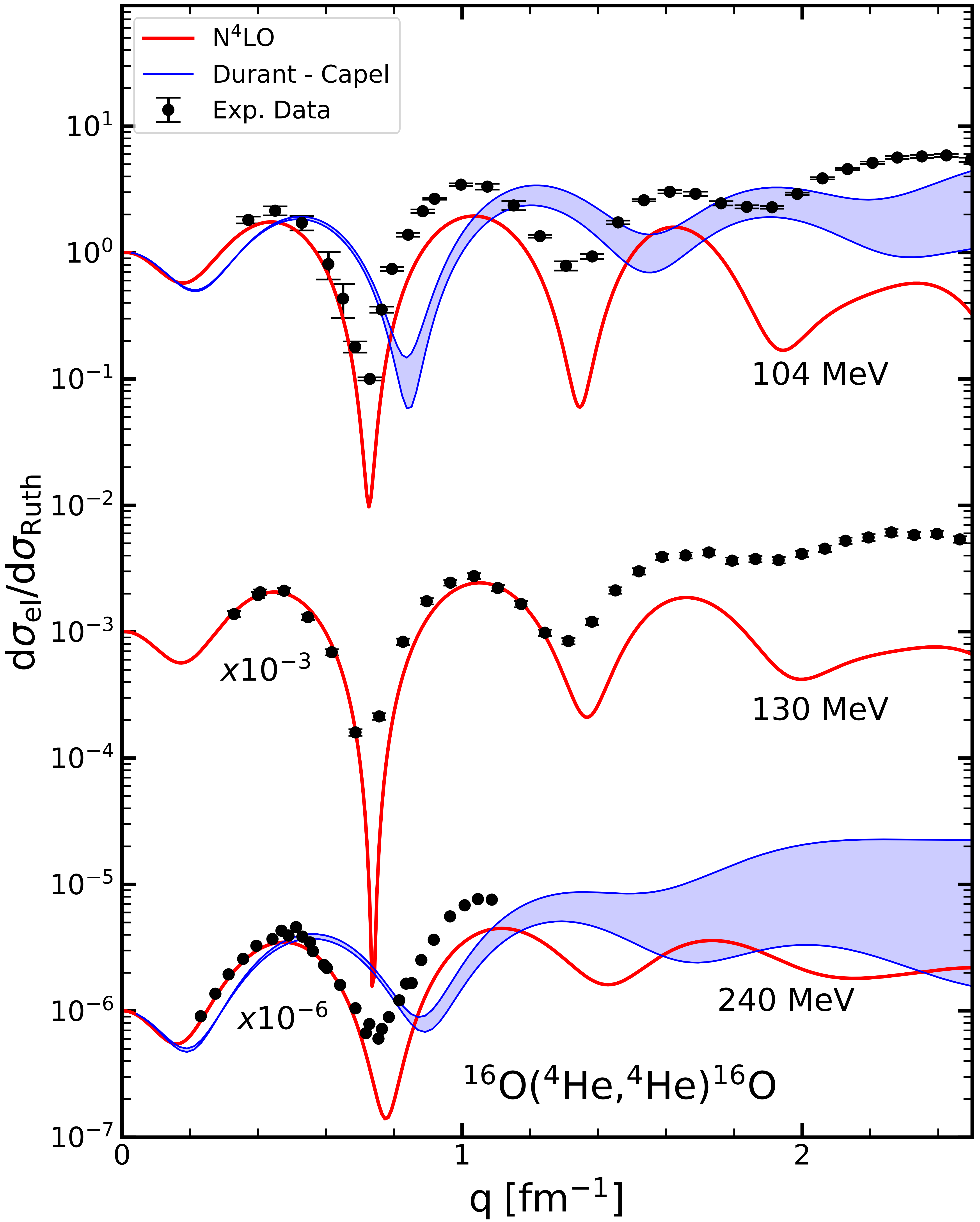
- Computationally expensive
- Obtained from the No-Core Shell Model or the Self-Consistent Green's Function
- Calculation performed with **NN** and **3N** interaction





Elastic ^{4}He-^{16}O scattering

- Parameter-free predictions
- Nice reproduction of experimental data at low-momentum transfer ($q \lesssim 1\ fm^{-1}$)
- Underestimates the high q behaviour, too much absorption?
- Competitive with a chiral microscopic *inspired* model by Durant and Capel



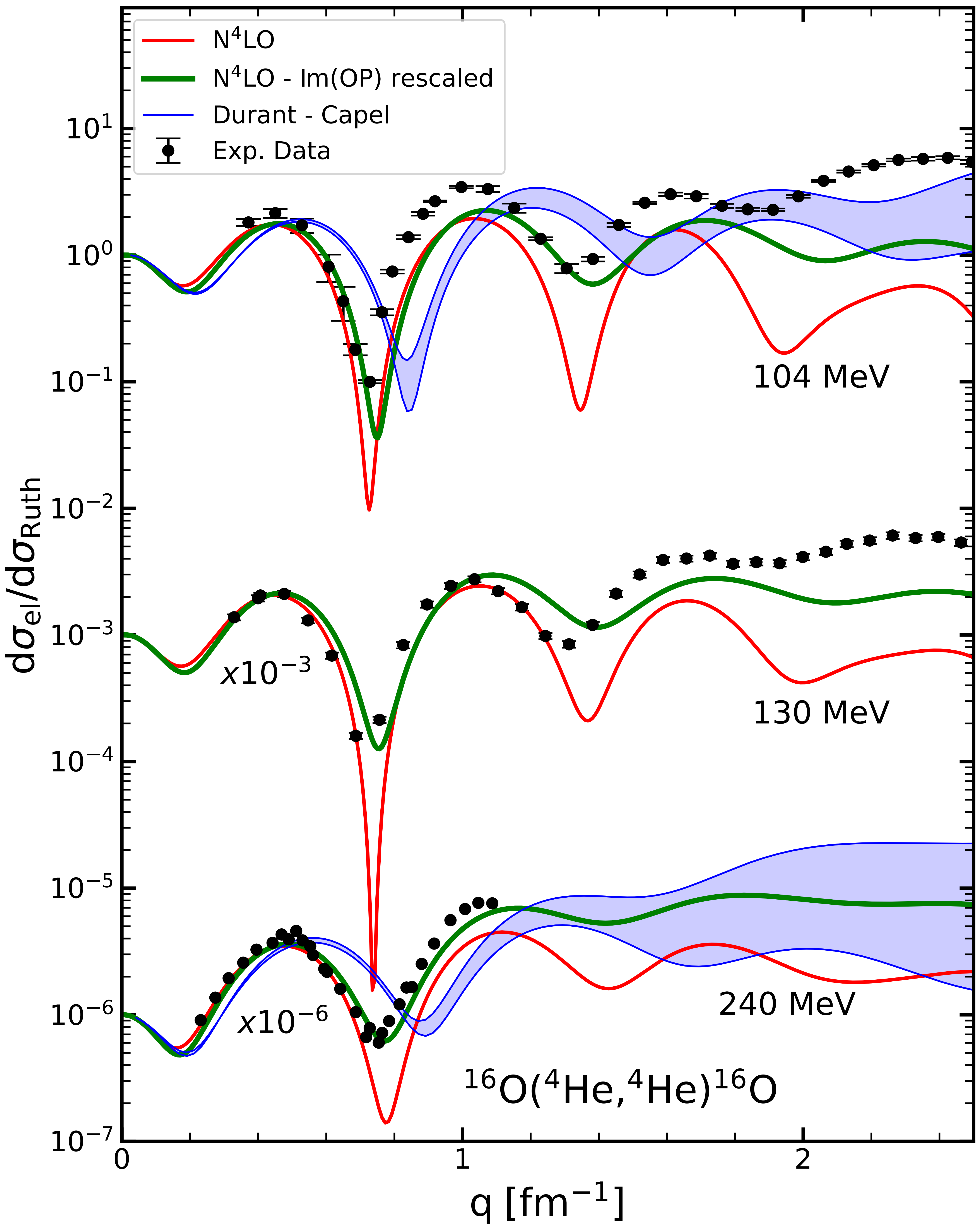
Vorabbi et al., PRL 135 (2025) 172501

Elastic ^{4}He-^{16}O scattering

How to solve the high-*q* behaviour? Very likely the introduction of higher-order effects would help

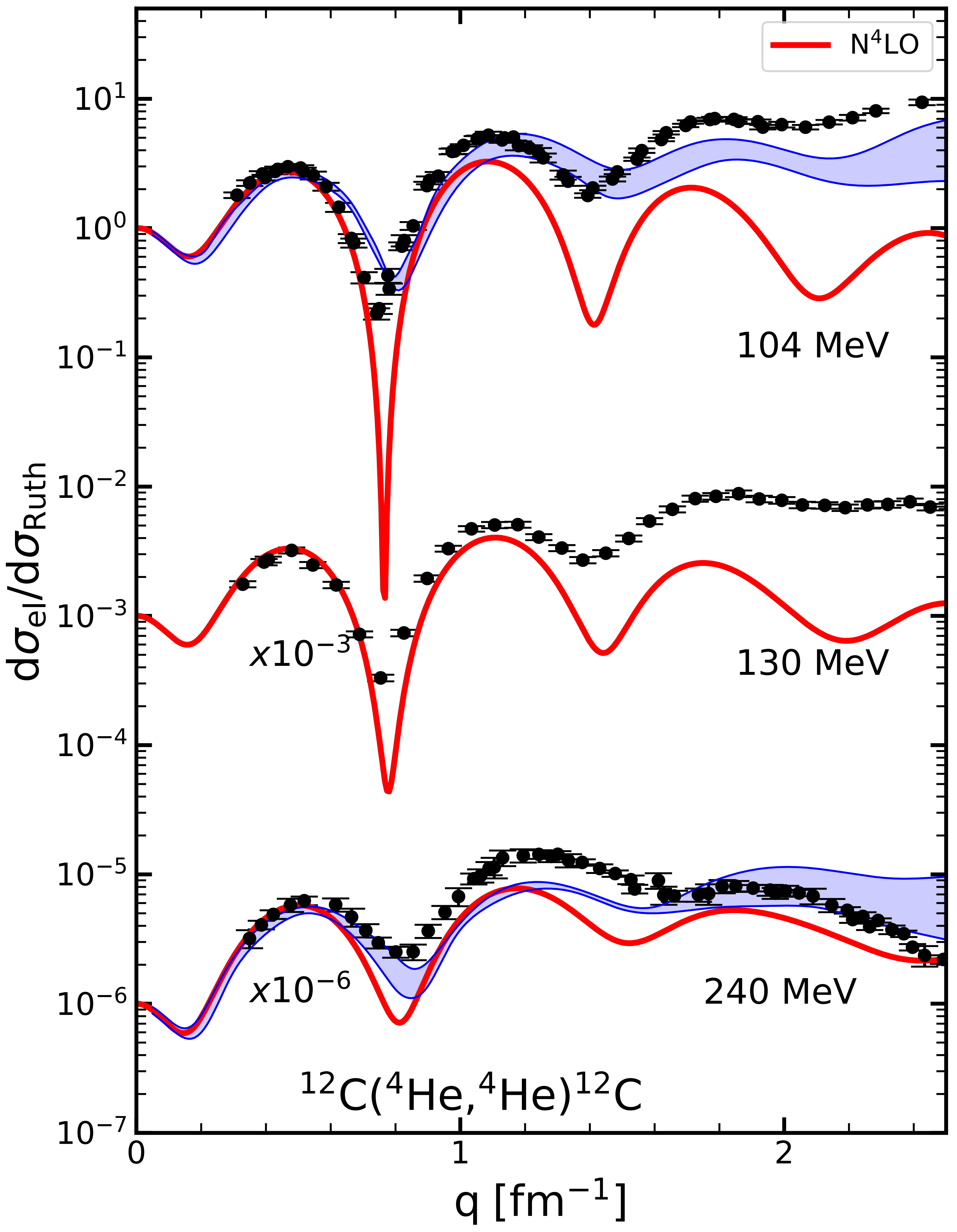
In the meantime, inspired by phenomenological optical potentials, we can reduce by hand the imaginary part

A guess in the range of 0.5/0.6 could be tested. We have chosen 0.5 (**green line**). The agreement is nicely improved.

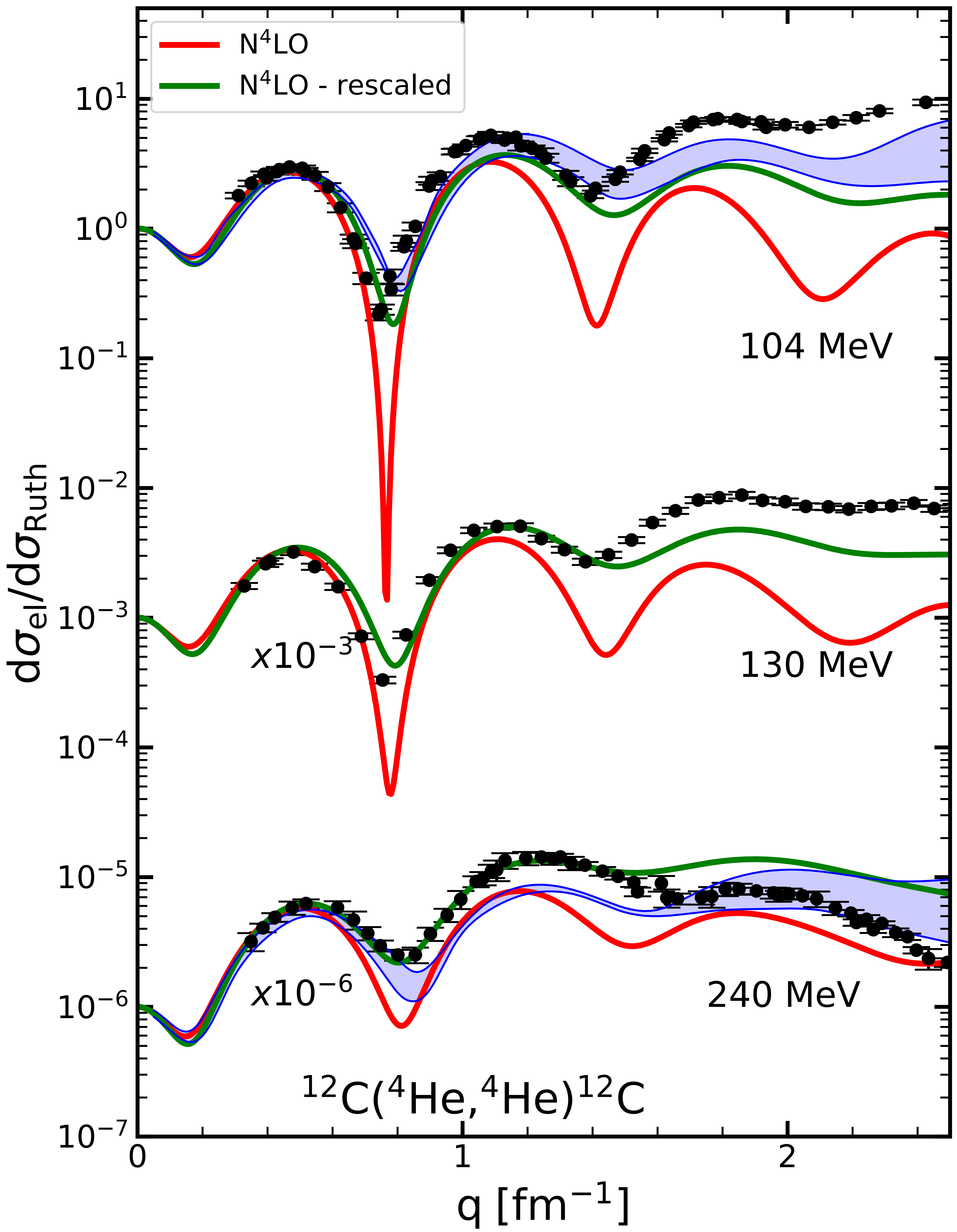


Vorabbi et al., PRL 135 (2025) 172501

Elastic ^{4}He-^{12}C scattering



- Same behaviour as in the oxygen case.
- A highly non-trivial fact is the quality of agreement along different energies without any other corrections aside the absoprtion reduction.



Vorabbi et al., PRL 135 (2025) 172501

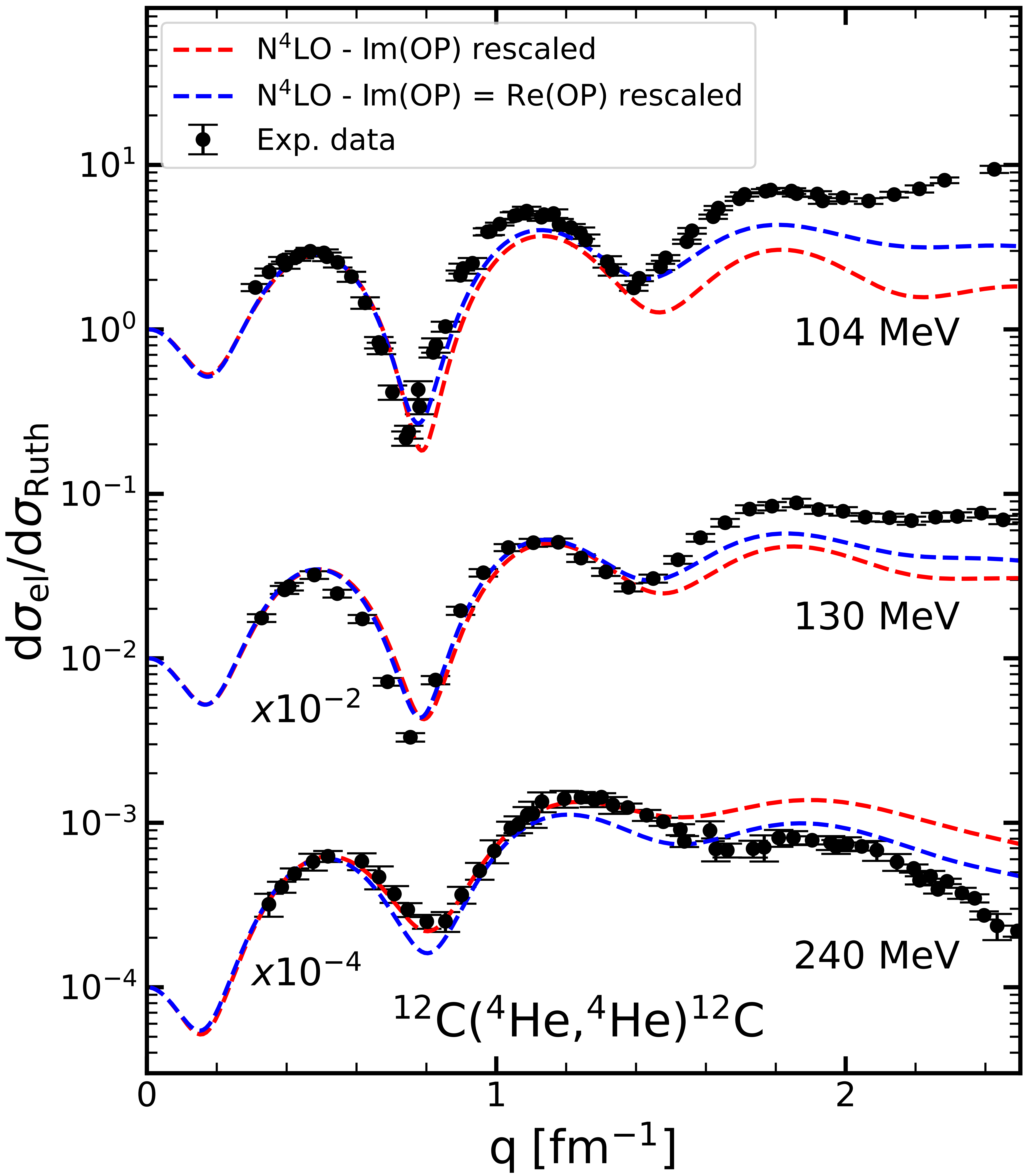
Elastic ^{4}He-^{12}C scattering

It is interesting to study a connection between phenomenology and theoretically inspired approaches.

We tested the usual adopted convention that the imaginary component of the optical potential could be taken as follows

$$Im\, \mathrm{U_{opt}} = 1/2\, Re\, \mathrm{U_{opt}}$$

meaning that from a phenomenogical point of view they have the same functional dependence. We conferm the prescription from a microscopic approach.



Vorabbi et al., PRL 135 (2025) 172501

Conclusions

- ☑ Achieved a satisfactory description of nucleon **elastic** scattering of light and medium-mass nuclei at the first order of the spectator expansion
- ☑ Extension of the model to nucleon-nucleus **inelastic** scattering
- ☑ Achieved a first step in the derivation of a **nucleus-nucleus** optical potential

- ☐ Inclusion of medium effects/higher order effects (preprint)
- ☐ Extend the high-energy limits of applicability of the optical potential
- ☐ Inclusion of the second-order term of the spectator expansion
- ☐ Consistent treatment of the full 3N interaction
- ☐ **Extend the formalism to transfer/charge-exchange** (long term goal)