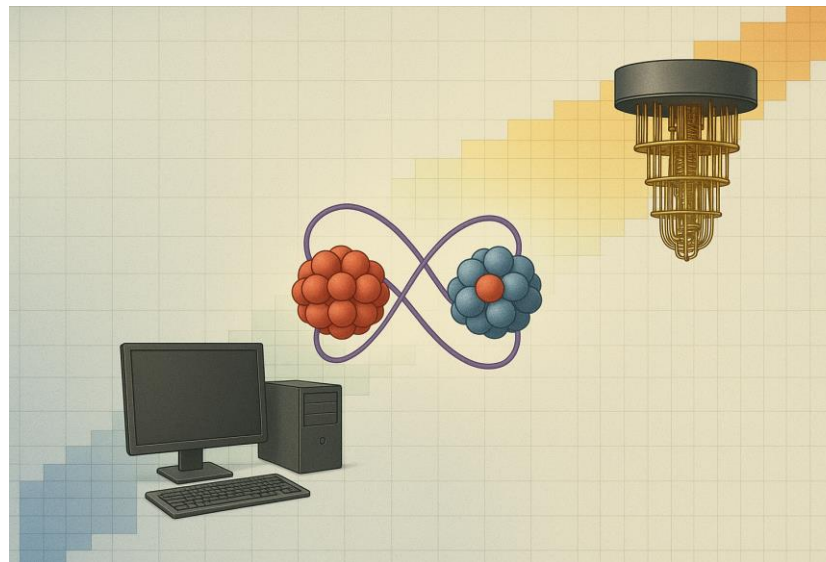




Using quantum computers for nuclear structure studies

Denis Lacroix



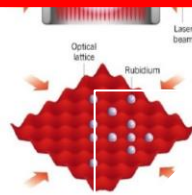
Many-body physics and QC - T. Ayral, P. Besserve, D. Lacroix, and E.A. Ruiz Guzman , Quantum computing with and for many-body physics, EPJA 59 (2023)
Symmetry and QC – D. Lacroix, A. Ruiz Guzman and P. Siwach, Symmetry breaking/symmetry preserving circuits and symmetry restoration on quantum computers, EPJA 59 (2023)
CERN Quantum Initiative – Di Meglio et al., Quantum Computing for High-Energy Physics: State of the Art and Challenges, PRX Quantum 5, 037001 (2024)



What means quantum devices today?

There are many types of quantum computers: ***analog versus digital quantum computers***

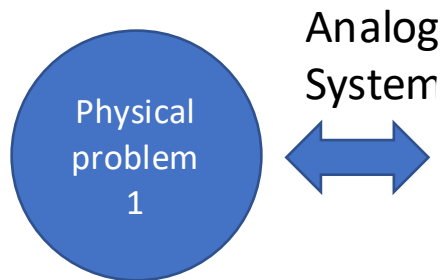
Quantum machines are often themselves many-body systems



Atomes de Rydberg dans des pinces optiques

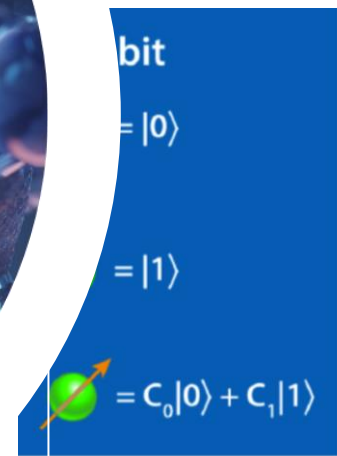
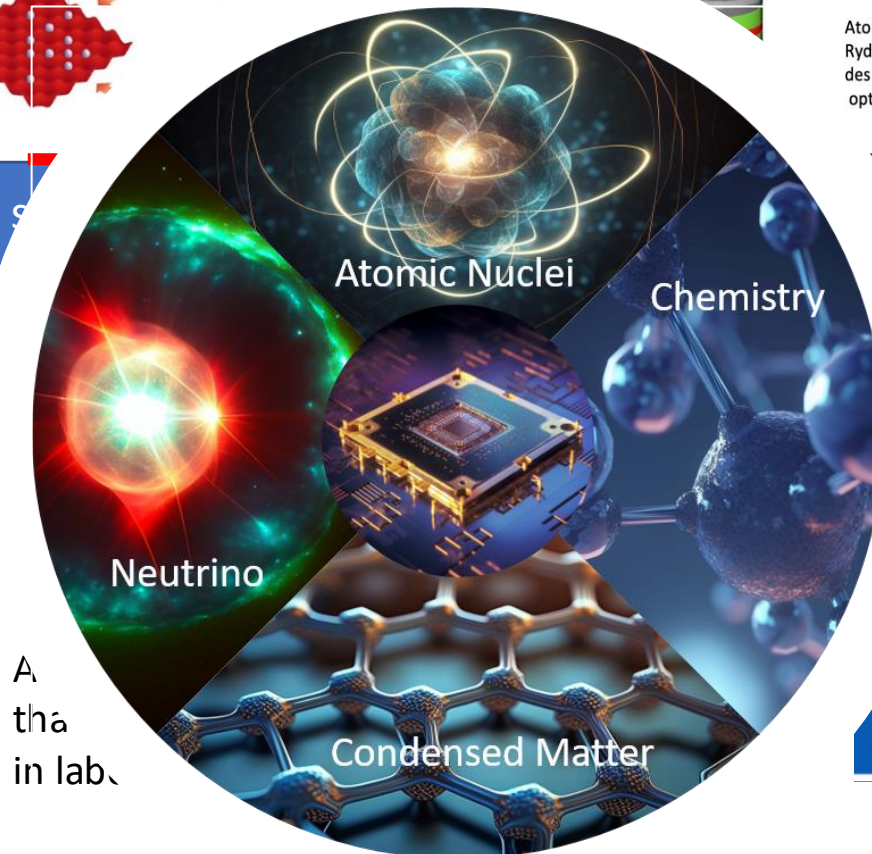
Analog quantum simulation

Digital quantum simulator



Complex problem that cannot or hardly be simulated on classical computers

➡ Non-universal



➡ Universal Quantum simulation

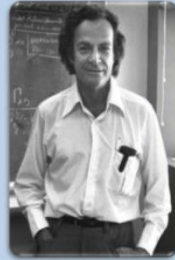
Why quantum computing is becoming mature now ?

Quantum technologies/devices

Simulating physics with computers-1982

Richard P. Feynman (Nobel Prize in Physics 1965)

"Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."



Quantum Theory

1927

Quantum Compute

1982

7 qubits
Los Alamos

2000

12 qubits
MIT

2006

128 qubits
DWave

2011

50 qubits
IBM

2015

17 qubits
IBM

2017

128 qubits
Rigetti

2018

128 qubits
Rigetti

72 qubits
Google

1152 qubits
DWave

2048 qubits
DWave

512 qubits
DWave

Quantum Computing Cloud

General Quantum mechanics

1940
Turing

1985
Deutsch

1992
Deutsch-Josza, Simon, Shor, Bernstein-Varizani, Kitaev ...

time

General Quantum algorithmic faster than classical ones.

Quantum computing is democratizing

What can be done on a *perfect* digital quantum computer?

We can essentially solve the Schroedinger equation

Practical solution on a classical computer

$$i\hbar \frac{d}{dt} |\Psi\rangle = H |\Psi\rangle$$

In a basis

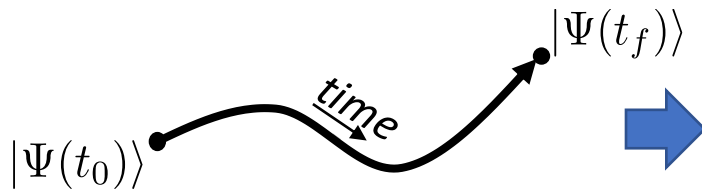
$\{|j\rangle\}_{j=0,\dots}$

$$F_j(t) = \langle j | \Psi(t) \rangle$$

$$i\hbar \dot{F}_j(t) = \sum_i \langle j | H | i \rangle F_i(t)$$

$$i\hbar \dot{\mathbf{F}}(t) = \mathbf{H} \times \mathbf{F}(t)$$

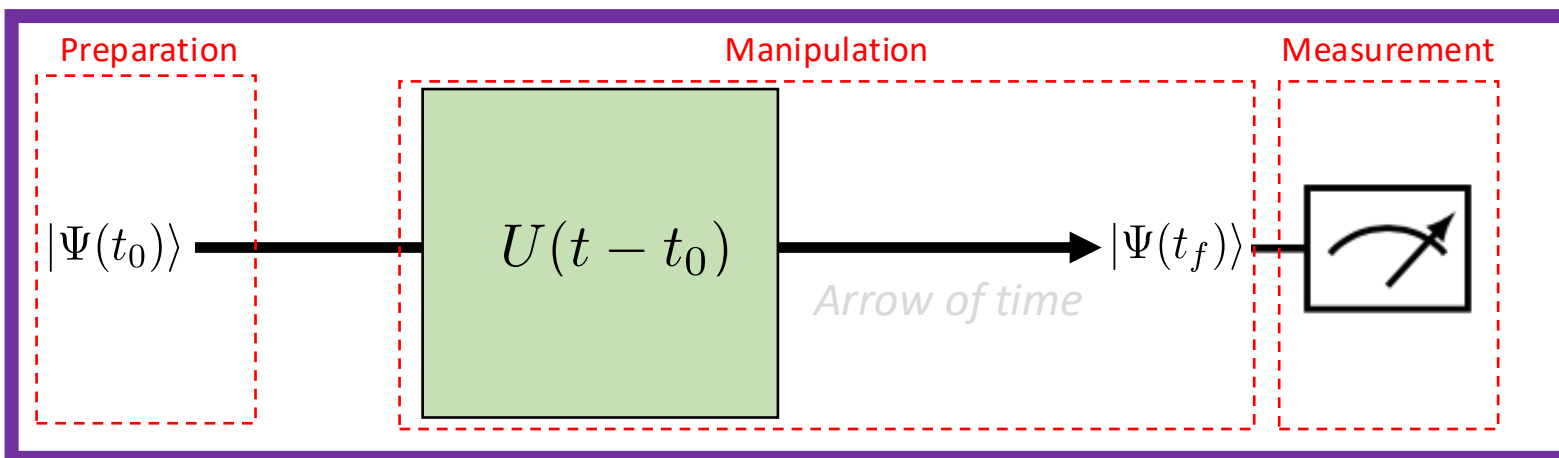
Formal solution



$$\begin{aligned} |\Psi(t)\rangle &= e^{\frac{1}{i\hbar}(t-t_0)H} |\Psi(t_0)\rangle \\ &= U(t - t_0) |\Psi(t_0)\rangle \end{aligned}$$

Schroedinger Eq. is a Linear algebra problems

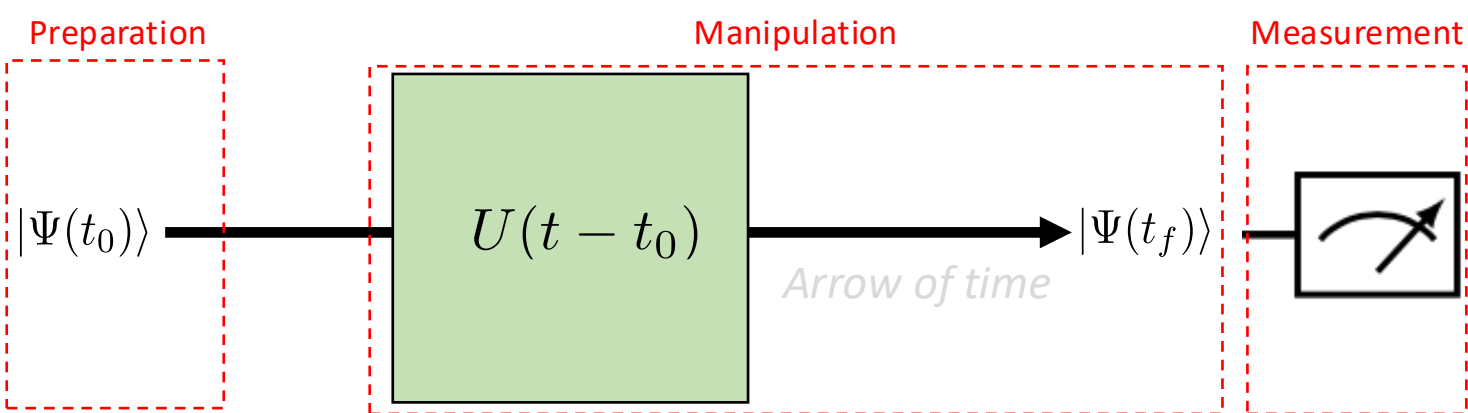
Having information on the w.f. means measuring it.



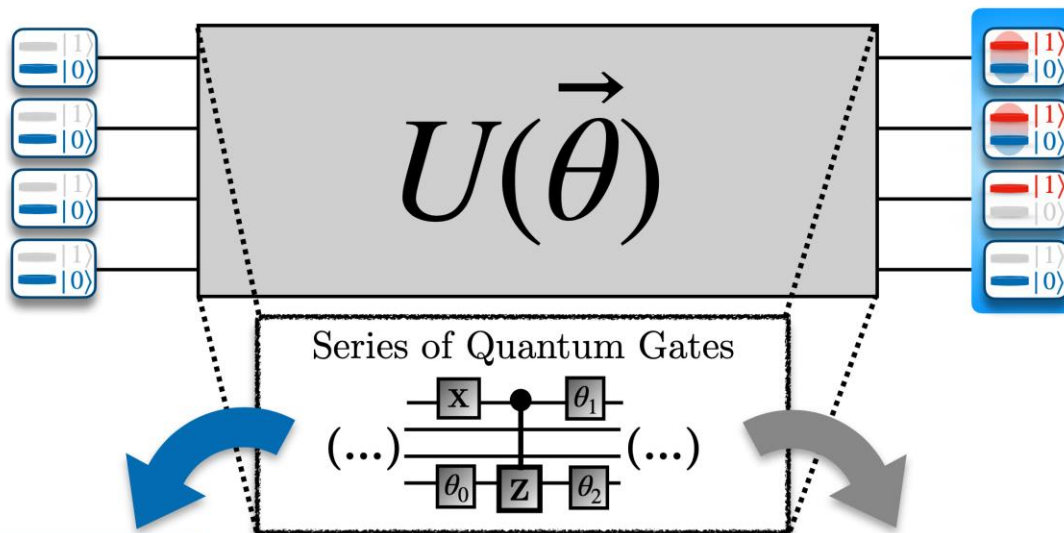
Quantum Circuit



Digital
Quantum Circuit



State preparation and manipulation are replaced by elementary quantum gates on one and two qubits



Near Term Era

Hardware Limitations

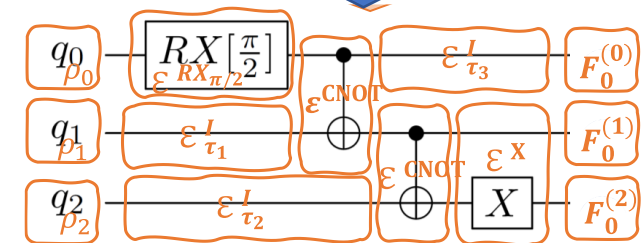
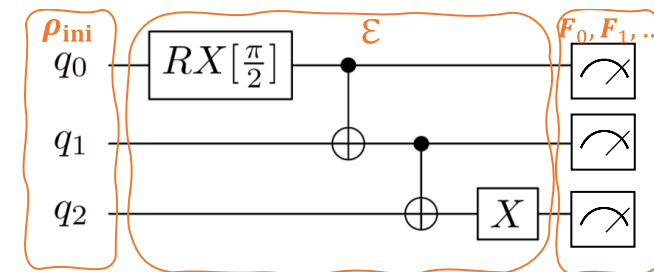
Variational Quantum Eigensolver

Fault Tolerant Era

No Hardware Limitations

Quantum Phase Estimation

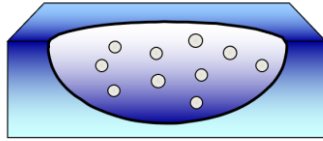
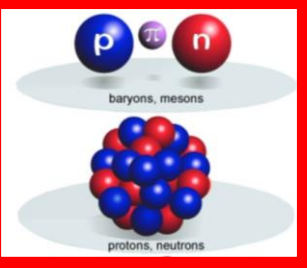
Ideal circuit



Quantum computing for the description

of static and dynamical properties of atomic nuclei

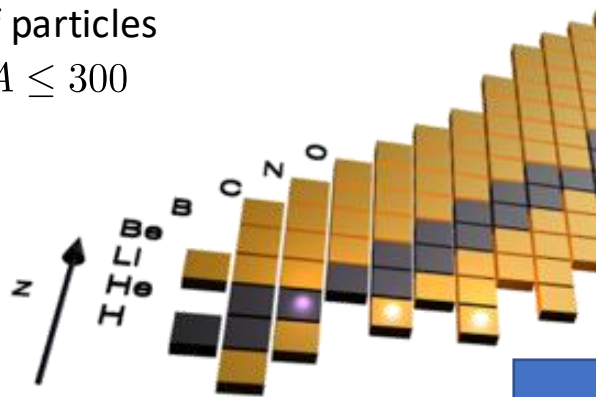
Problematic and challenges



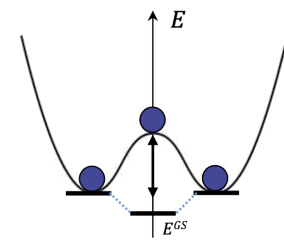
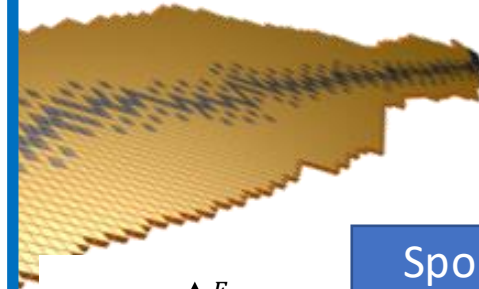
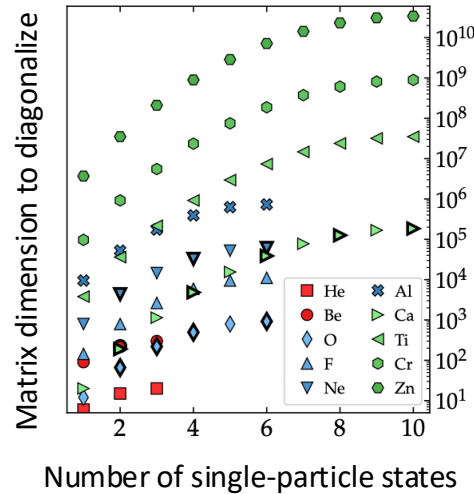
Nuclei are self-bound quantum mesoscopic systems

Nb of particles

$$2 \leq A \leq 300$$

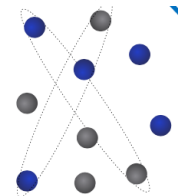


Finding eigenstates



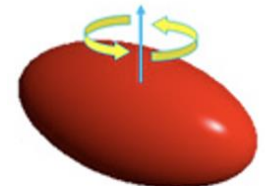
Spontaneous Broken symmetries (SB)

Small superfluid



(particle number SB)

Deformation can happen

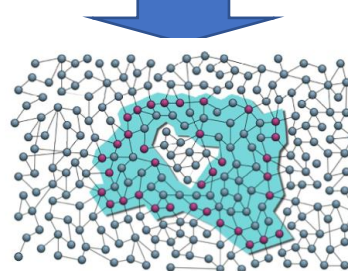


(rotational invariance SB)

Symmetries / Entanglement

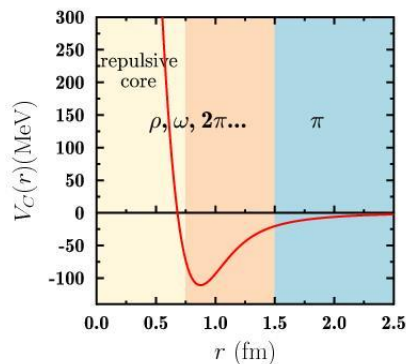
Global symmetries induce All-to-all entanglement

$$S, T, J, \pi$$



Nuclei are subject to entanglement volume law (bad candidate for Tensor Network)

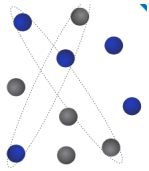
Interaction



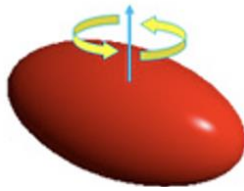
The problem is highly non-perturbative

Developing variational approaches based on symmetry-breaking (SB)/symmetry restoration (SR)

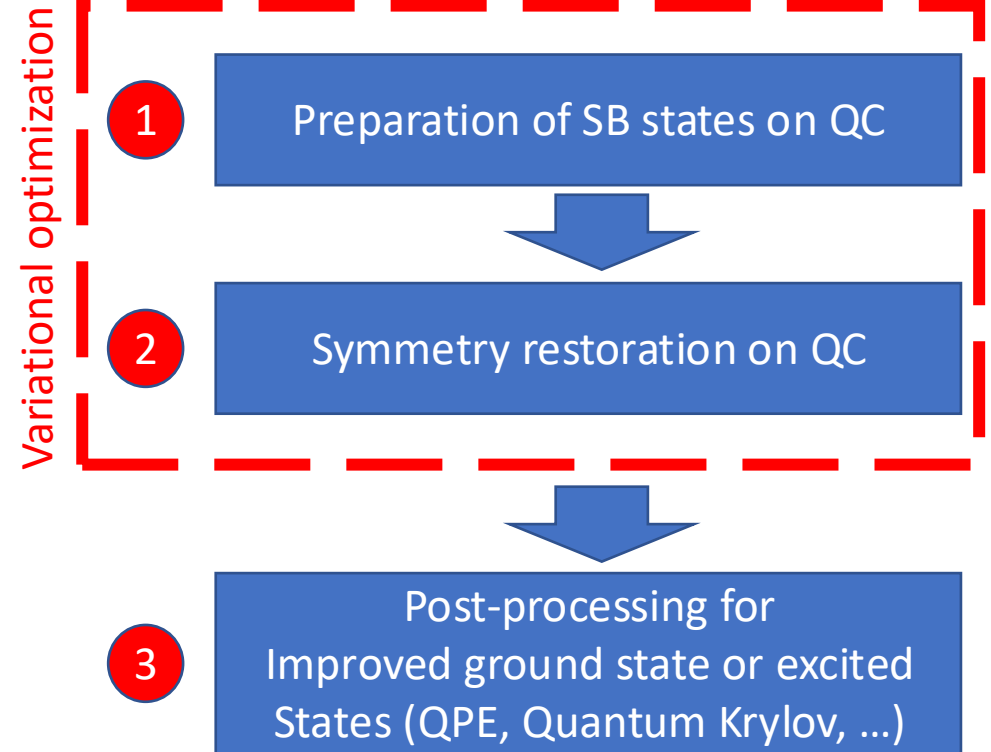
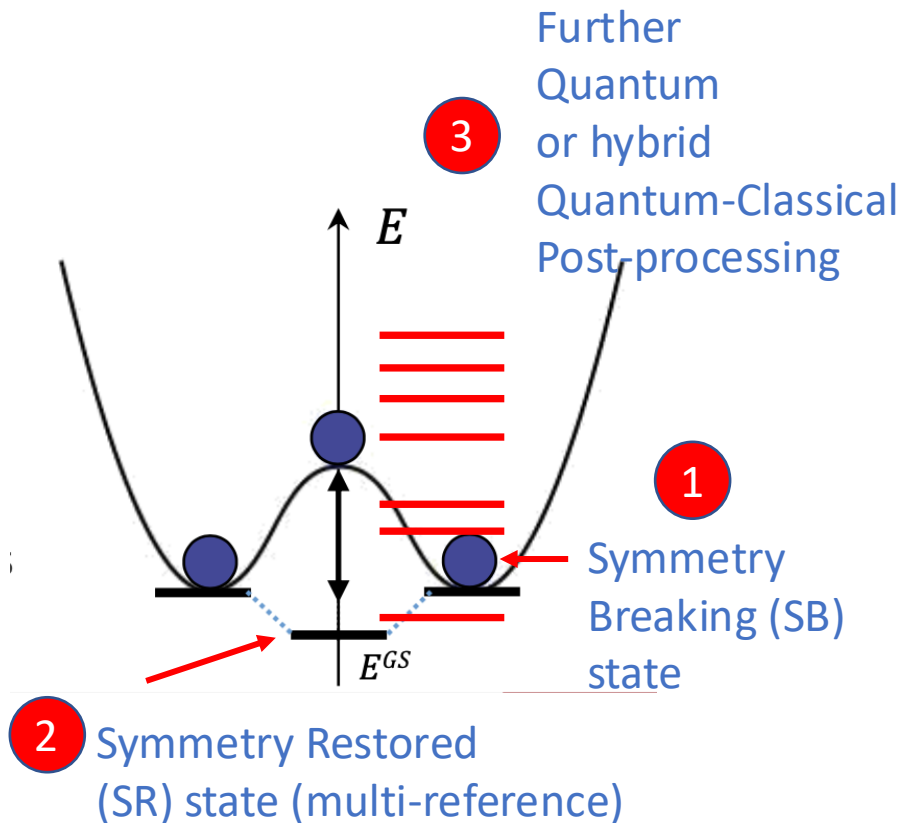
Pairing



Deformation



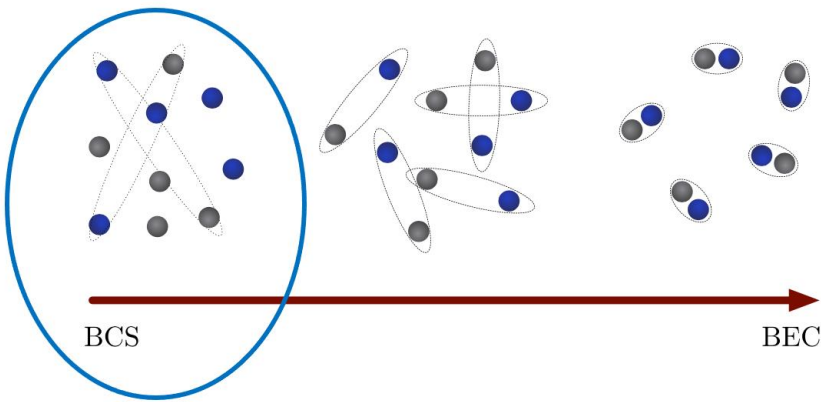
D. Lacroix, A. Ruiz Guzman and P. Siwach,
Symmetry breaking/symmetry preserving circuits
and symmetry restoration on quantum computers
EPJA 59 (2023)



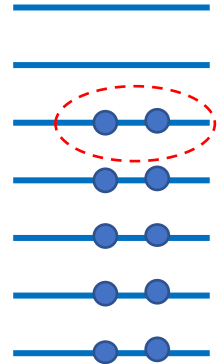
Which symmetries ?

Many-Body
Particle Number
Parity
Total Spin

Illustration with the pairing Hamiltonian



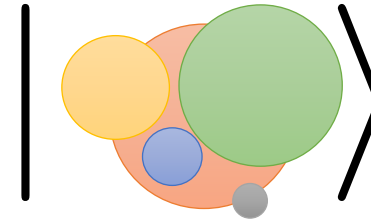
$$H_P = \sum_{i>0} \varepsilon_i (a_i^\dagger a_i + a_{\bar{i}}^\dagger a_{\bar{i}}) - g \sum_{i,j>0} a_i^\dagger a_{\bar{i}}^\dagger a_{\bar{j}} a_j$$



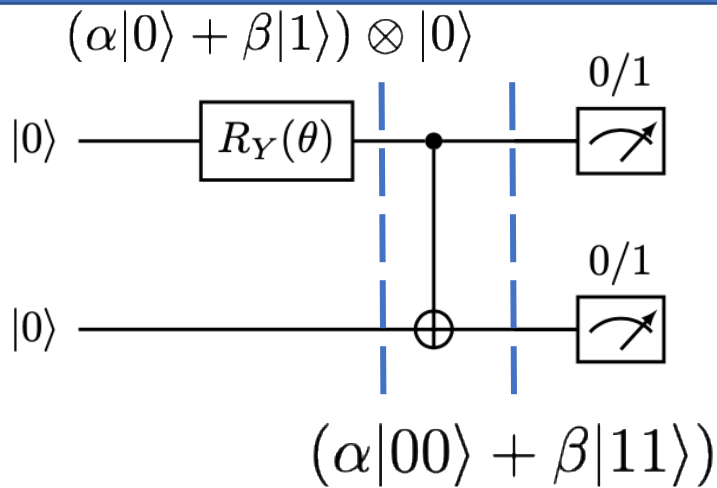
Example

$$|\Phi_0\rangle = \prod_i (u_i + v_i a_i^\dagger a_{\bar{i}}^\dagger) |-\rangle$$

➡ Mixes states with 0, 2, 4, ... particles

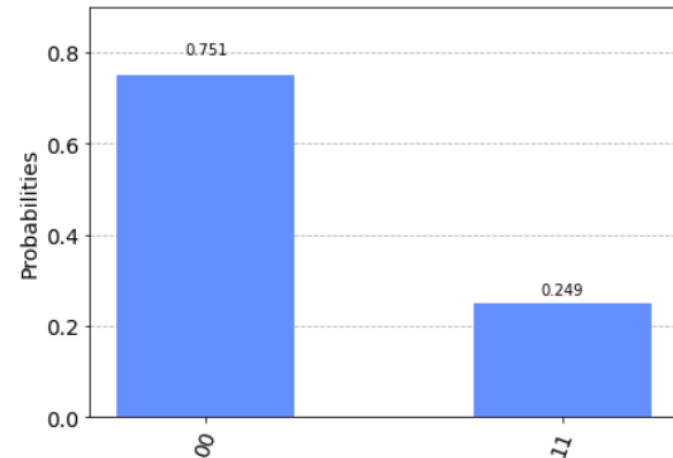


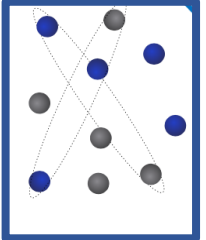
BCS states are generalized Bell states



Here I created a Bell state

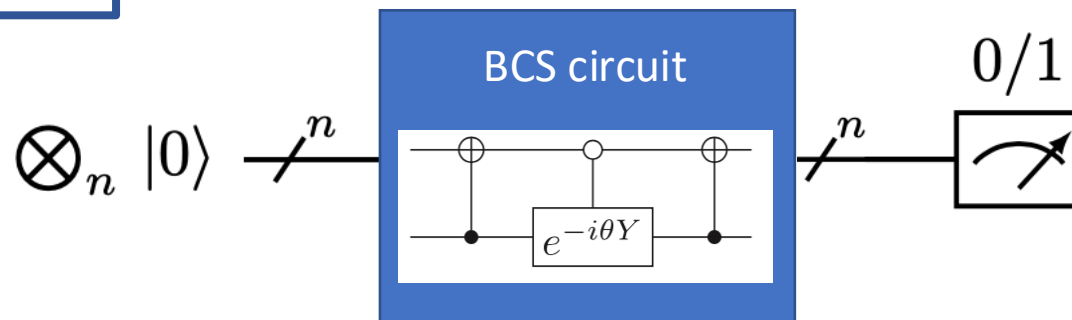
{'00': 7513, '11': 2487}



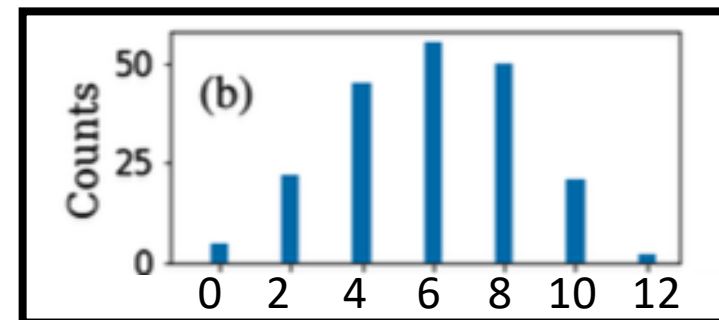


Quantum computing for atomic nuclei

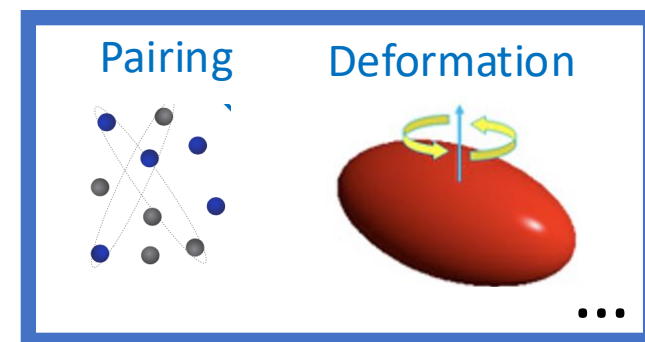
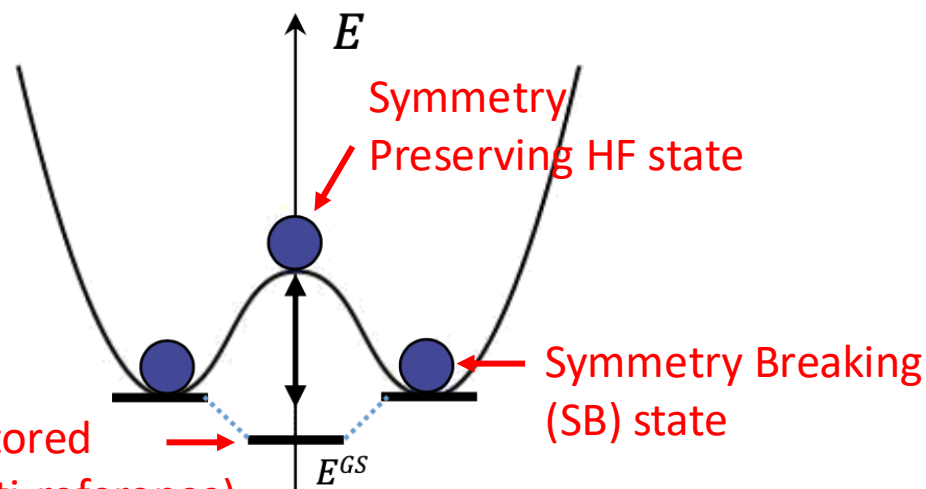
Illustration for small superfluids



Example of mixing for 12 qubits



Restoring symmetries on a quantum computer?



D. Lacroix, A. Ruiz Guzman and P. Siwach,
Symmetry breaking/symmetry preserving circuits
and symmetry restoration on quantum computers
EPJA 59 (2023)

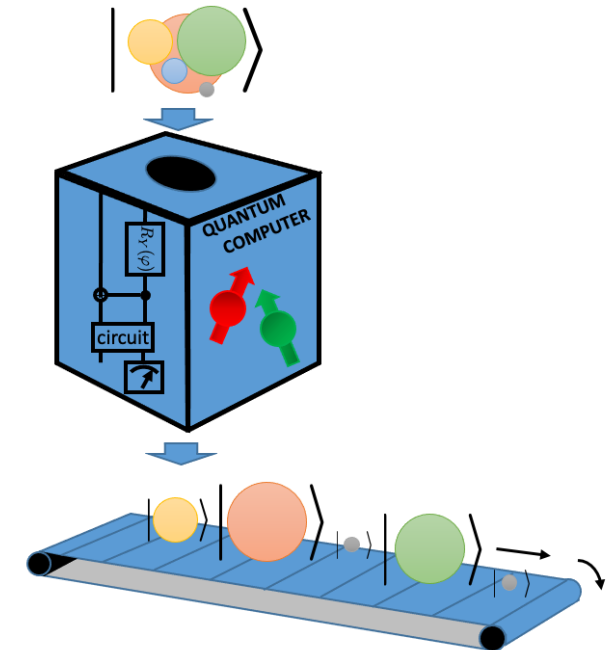
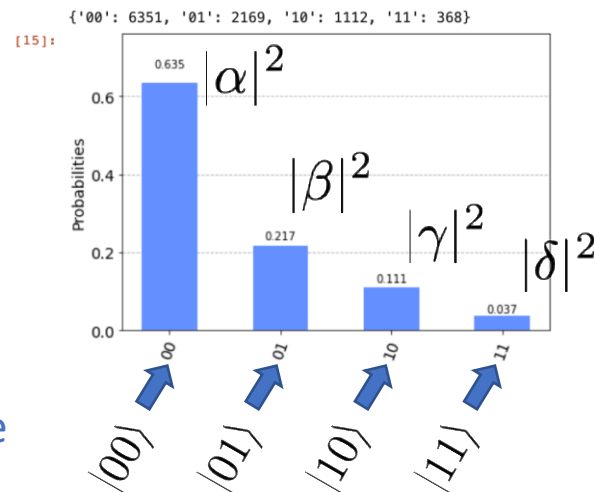
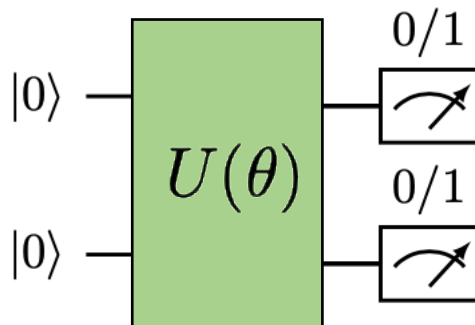
A brief overview of the symmetry restoration problem and its solution

The particle number case

$$|\Psi\rangle = \sum_N c_N |N\rangle \rightarrow |N\rangle \rightarrow \text{Use it for evolution, Energy minimization, ...}$$

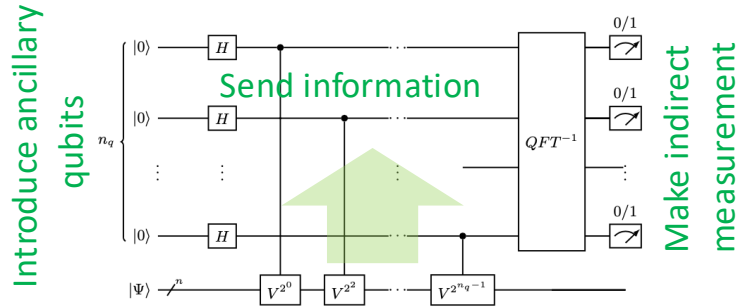
For 2 qubits

$$|\Psi\rangle = \alpha \underbrace{|00\rangle}_{|N=0\rangle} + \beta \underbrace{|01\rangle}_{\propto |N=1\rangle} + \gamma \underbrace{|10\rangle}_{|N=2\rangle} + \delta |11\rangle$$

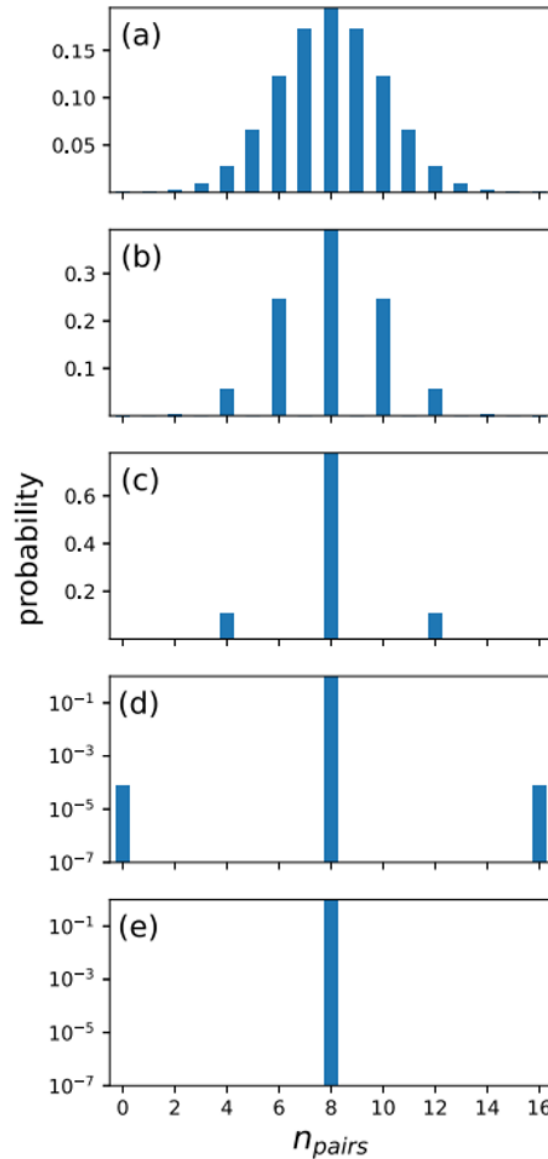


Difficulty – wave-function collapse

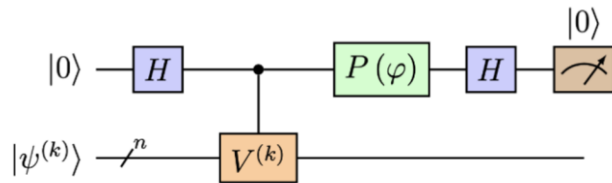
Standard Quantum Phase estimation



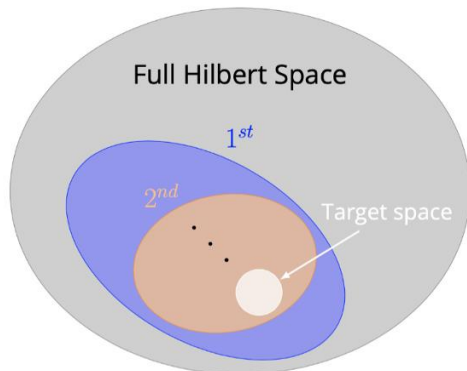
16 qubits, $N = 8$



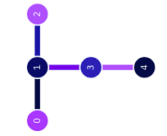
Iterative Quantum Phase estimation



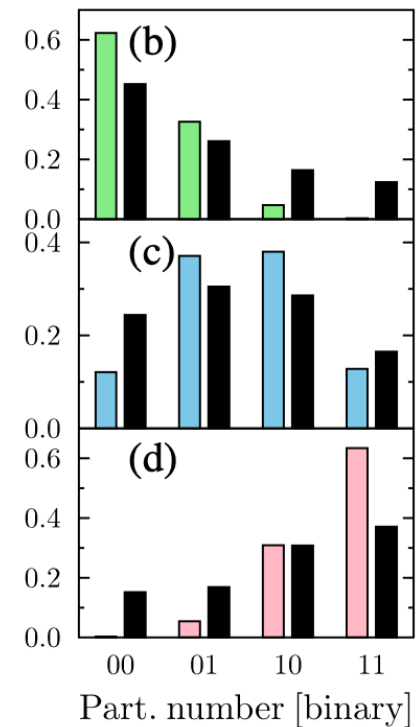
$$\hat{V}^{(k)} = e^{i\phi_k \hat{N}} \quad \phi_k = \frac{\pi}{2^k}$$



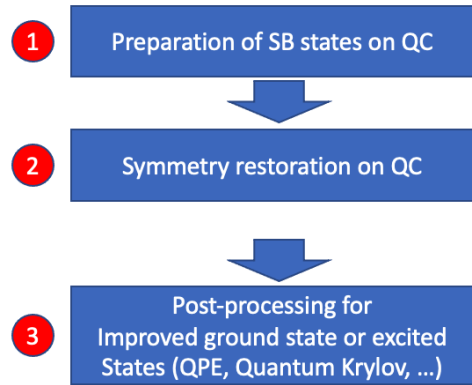
Test on real devices



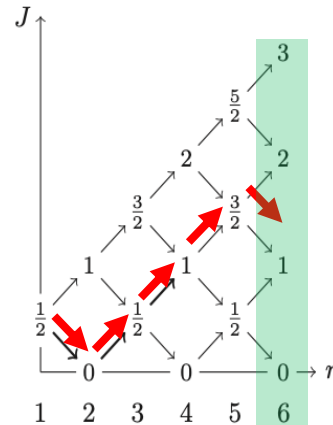
IBM Q5
(5 qubits)



Summary of achieved work on symmetry restoration

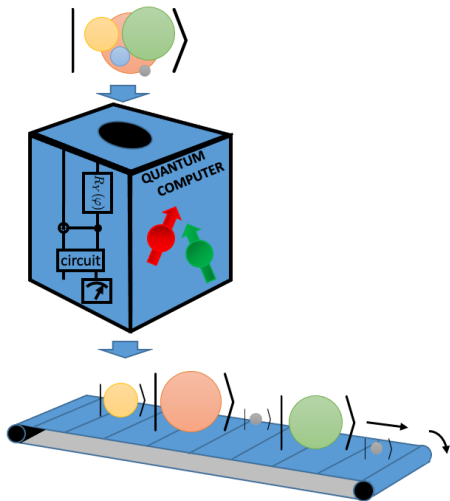


Total spin symmetry



P. Siwach and DL, Phys. Rev. A 104, (2021)
 Ruiz Guzman, DL, Eur. J. Phys. A 60 (2024)
 J. Zhang, PhD thesis Paris-Saclay Univ. (2025)

2 Restoring symmetries



Various methods being developed

-With phase-estimation

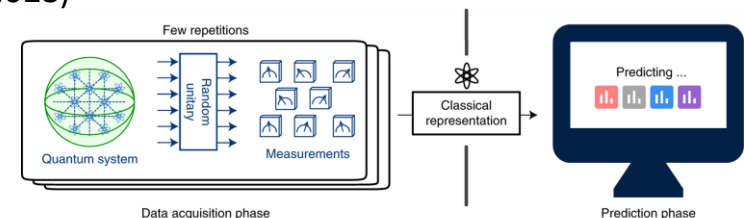
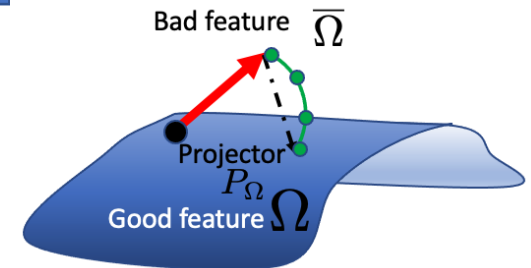
D. Lacroix, PRL 125, 230502 (2020).

-Using quantum Oracles

Ruiz Guzman, Lacroix, PRC 107 (2023)

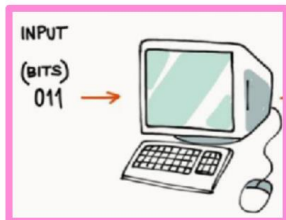
-Using classical post-processing (shadows)

Ruiz Guzman, Lacroix, Eur. J. Phys. A 60 (2024)





Classical optimization



Coming back to our superconducting problem

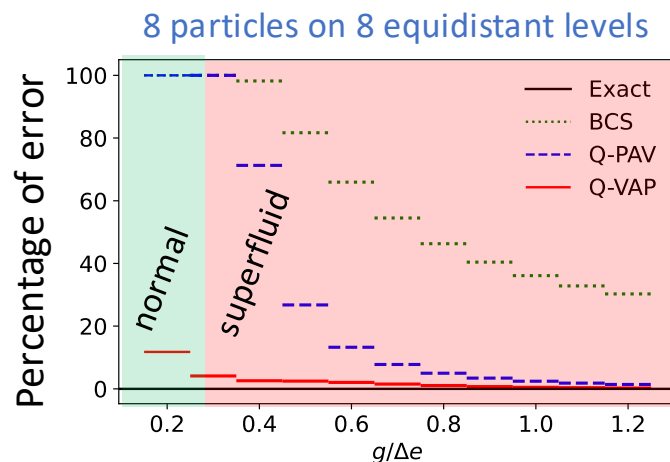
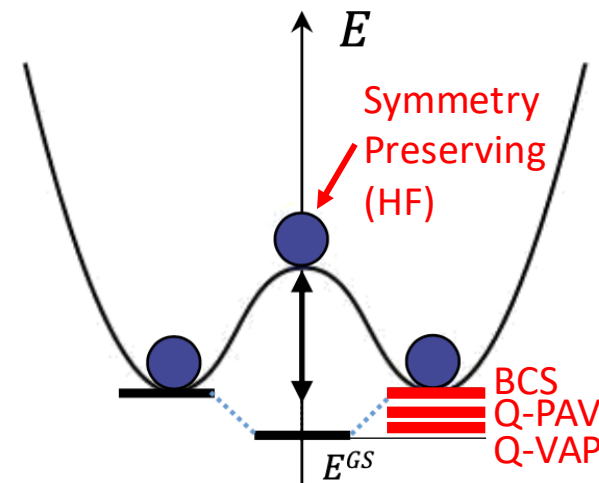
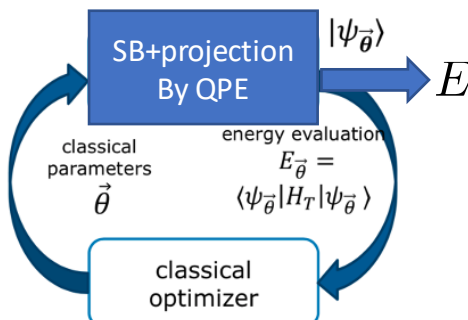
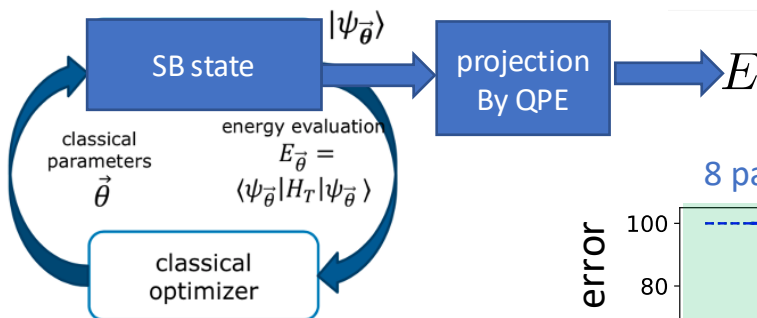
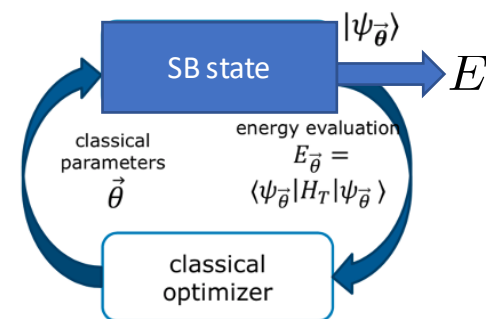
Combining projection with variational method

Quantum-Classical optimizers

→ Standard BCS theory

→ Project after optimization
Q-PAV: Quantum Projection
After Variation

→ The optimization is made on the
Symmetry restored state.
Q-VAP: Quantum Variation
After Projection



Ruiz Guzman and Lacroix, PRC 105 (2022)

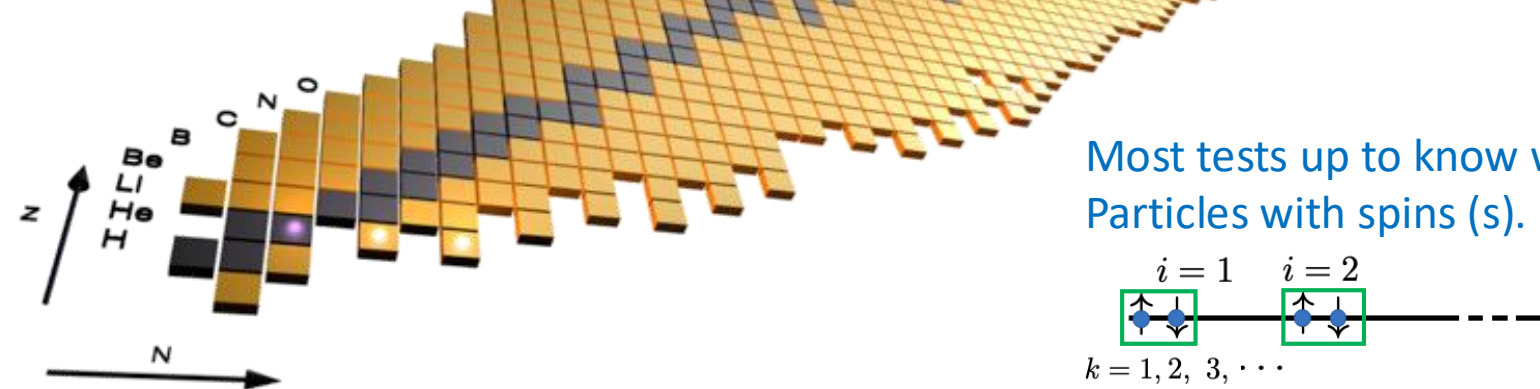
Getting closer to realistic problems

Is the breaking of symmetries always a good idea?

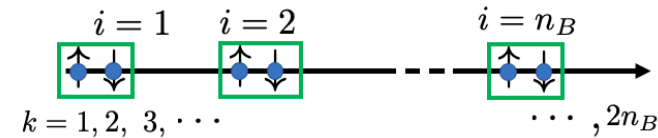
$0f_{5/2}$	19	18	17	16	15	14	
$1p_{1/2}$		13	12				pf
$1p_{3/2}$		11	10	9	8		
$0f_{7/2}$	7	6	5	4	3	2	1
							0

$0d_{3/2}$	11	10	9	8			
protons		7	6				sd
$1s_{1/2}$		5	4	3	2	1	0
$0d_{5/2}$							

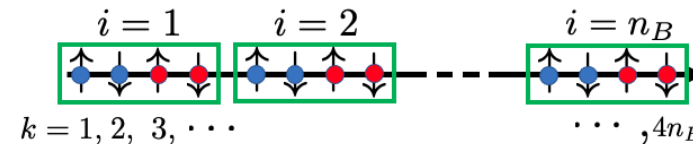
$0p_{1/2}$	5	4					
$0p_{3/2}$	3	2	1	0			p
m	$-\frac{7}{2}$	$-\frac{5}{2}$	$-\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$
							$\frac{7}{2}$



Most tests up to know were made on Particles with spins (s).



But nuclei have both spin (s) and isospin (t) (neutron/proton)



➡ This increases the number of qubits

$$S_z, S^2, \pi$$

➡ This increases the number of symmetries that could be broken

$$S_z, S^2, T_z, T^2, \pi$$

Symmetry-breaking states become extremely hard to control
Symmetry restoration becomes very demanding

Going beyond traditional nuclear physics methods: Use of adaptative methods

Iterative construction of the ansatz

Grimsley, et al, Nat. Commun. 10 (2019)

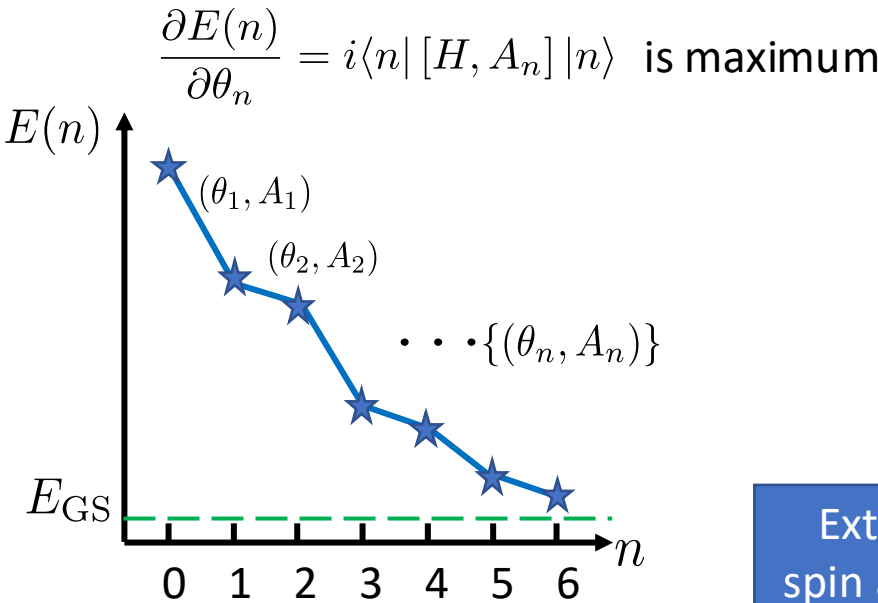
➡ Start from a state $|\Psi_0\rangle = |n = 0\rangle$

➡ Built iteratively the ansatz such as:

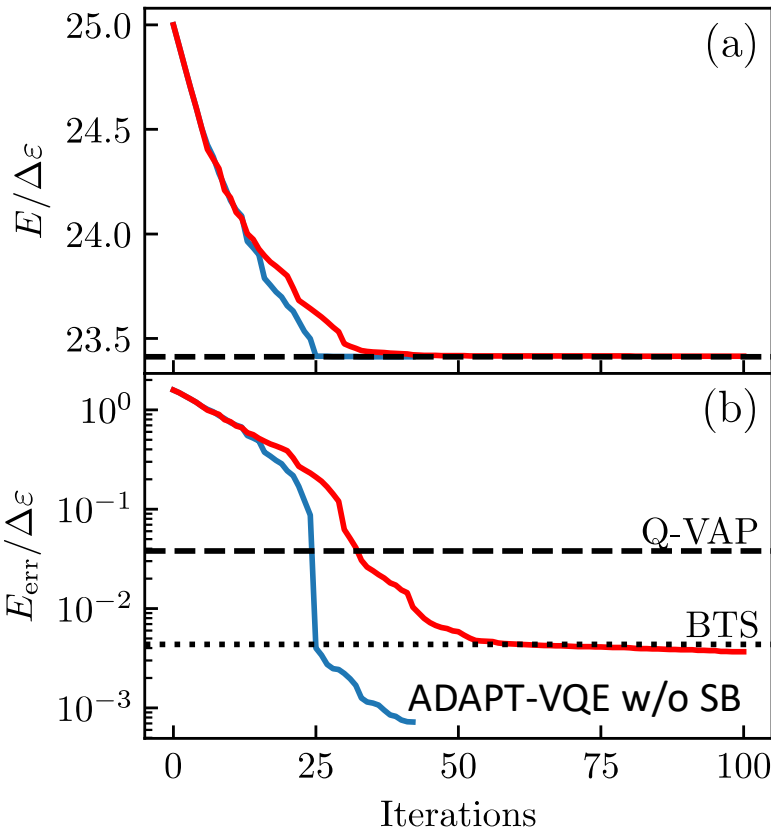
$$|n\rangle = e^{i\theta_n A_n} |n - 1\rangle = \prod_{k=1}^n e^{i\theta_k A_k} |0\rangle$$

$A_n \in \{O_1, \dots, O_\Omega\}$

Such that

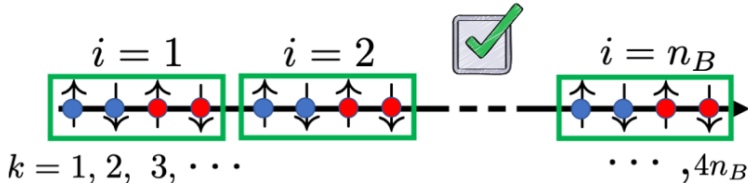


ADAPT-VQE applied to the Superfluid problems: only spins



J. Zhang, D. Lacroix, and Y. Beaujeault-Taudière, PRC (2025)

Extension to
spin and isospin



Is breaking symmetries always a good idea?

Extension to the proton-neutron pairing Hamiltonian problem

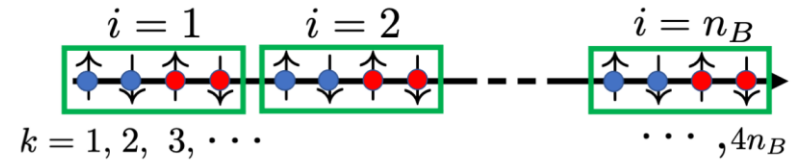
$$H = \sum_{i=1}^{n_B} \left[\varepsilon_{i,n} (\nu_i^\dagger \nu_i + \nu_i^\dagger \nu_i) + \varepsilon_{i,p} (\pi_i^\dagger \pi_i + \pi_i^\dagger \pi_i) \right] \\ - \sum_{T_z=-1,0,1} g_V(T_z) \mathcal{P}_{T_z}^\dagger \mathcal{P}_{T_z} \\ - \sum_{S_z=-1,0,1} g_S(S_z) \mathcal{D}_{S_z}^\dagger \mathcal{D}_{S_z}.$$

Different Hamiltonian limit

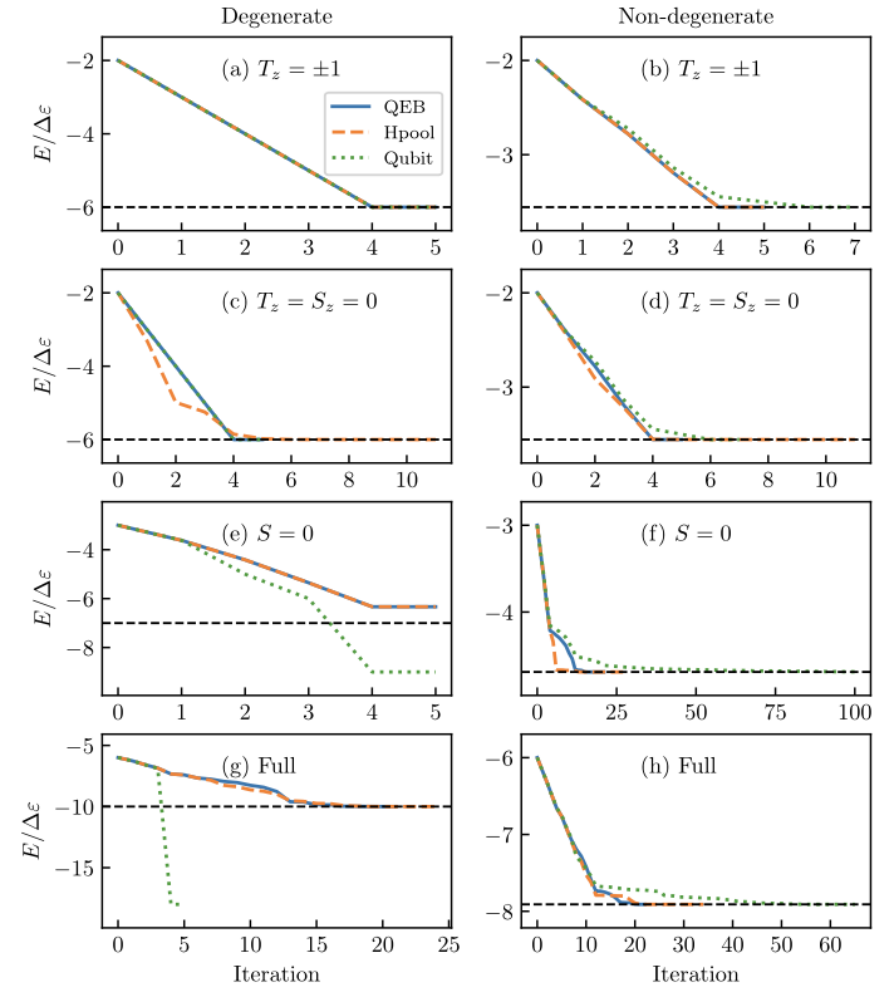
Case	S_z/T_z	Isoscalar			Isovector		
		-1	0	1	-1	0	1
1					✓		✓
2			✓			✓	
3					✓	✓	✓
4		✓	✓	✓	✓	✓	✓

Different operator pool in ADAPT-VQE breaking or not symmetries

	Particle number	Seniority	Parity
H-pool	✓	✓	✓
QEB-pool	✓	×	✓
Qubit-pool	×	×	✓



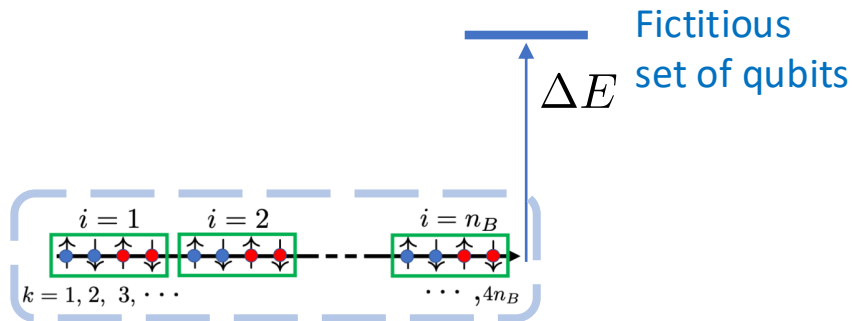
4 particles on 12 qubits



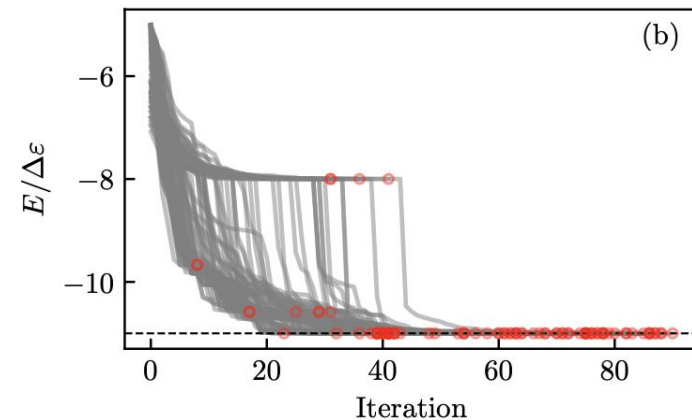
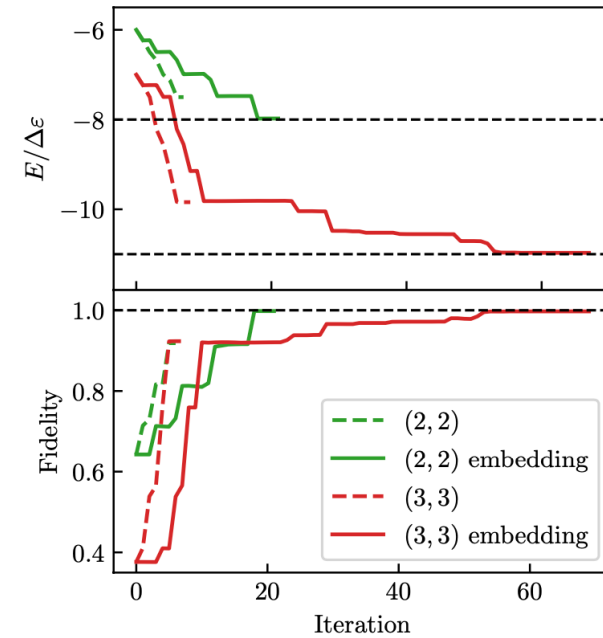
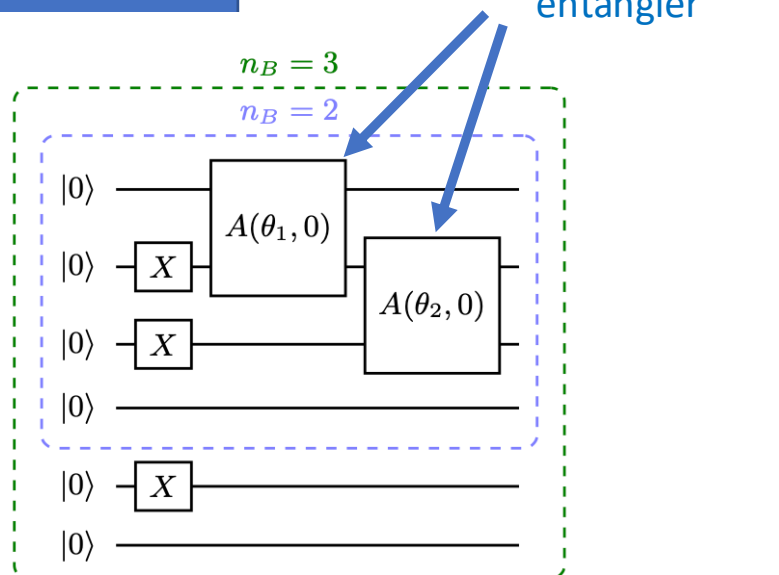
Specific methods to improving convergence

Going closer to nuclei: adding isospin

Embedding



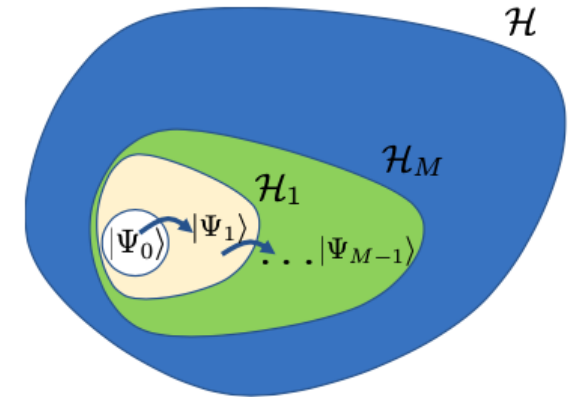
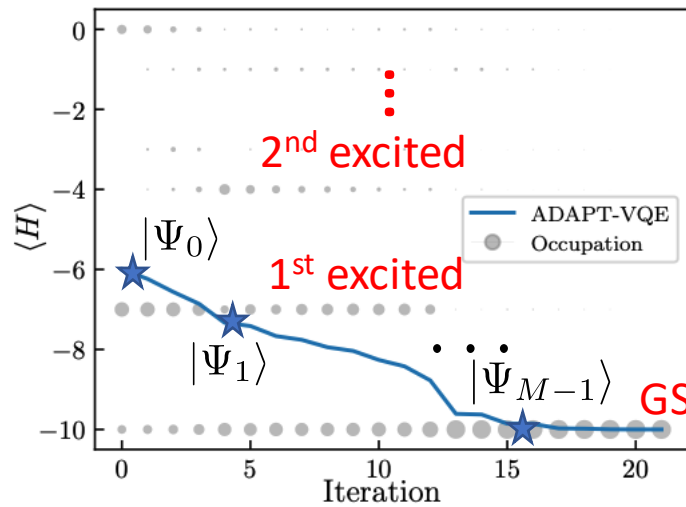
Initial condition
Randomization



From ground state only to excited states

The states generated during the convergence contain information on excited states

We used these states in a Quantum Subspace Diagonalization method (QSD)

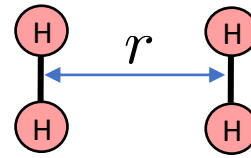
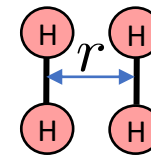
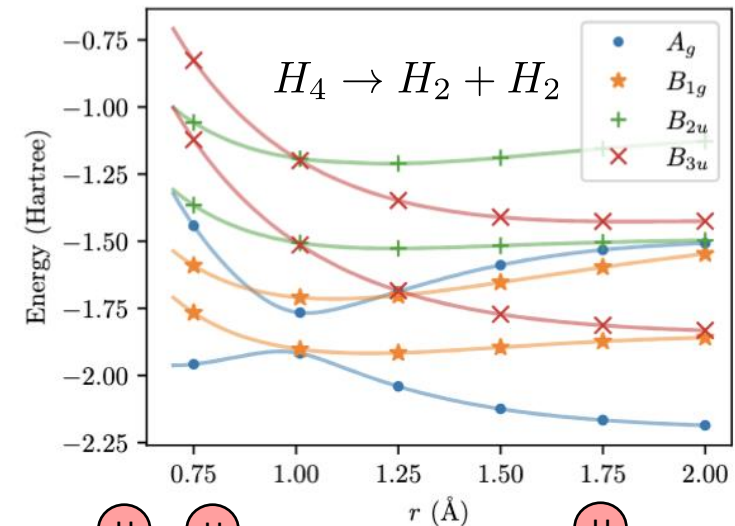
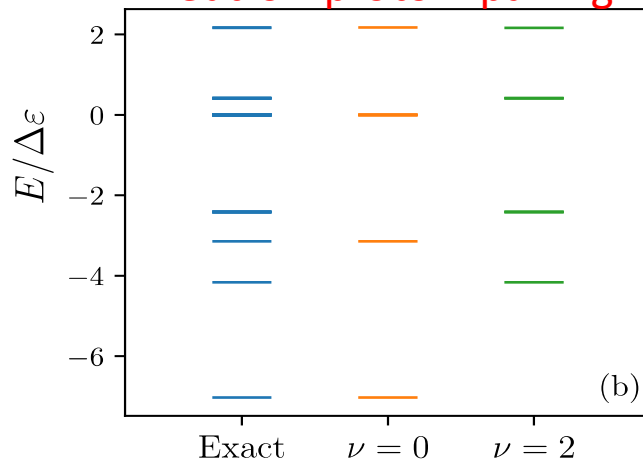


molecule dissociation

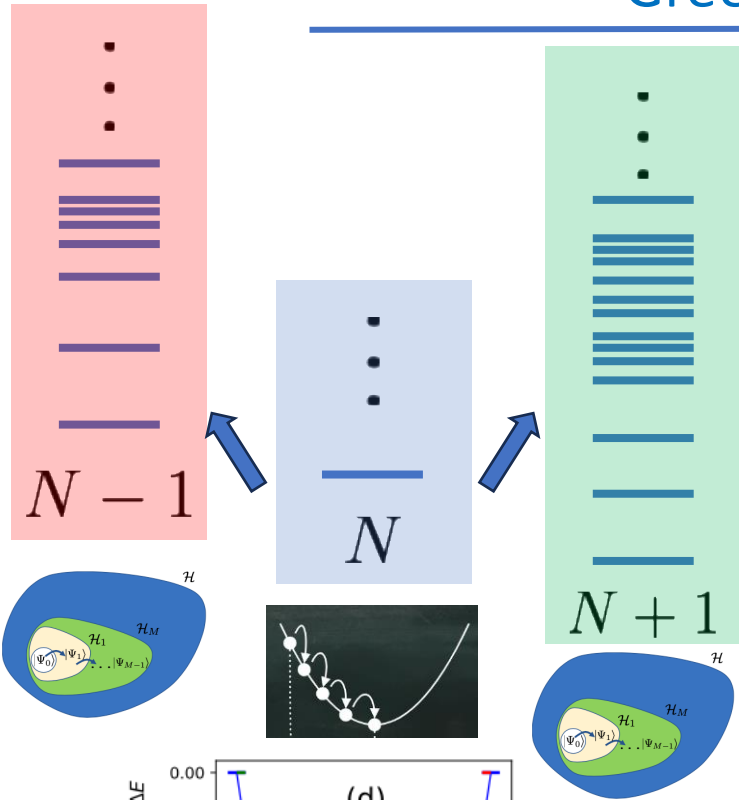
Applications

J. Zhang, D. Lacroix, Phys. Lett. B 869 (2025)

Neutron-proton pairing



Green's function estimated with a quantum computer

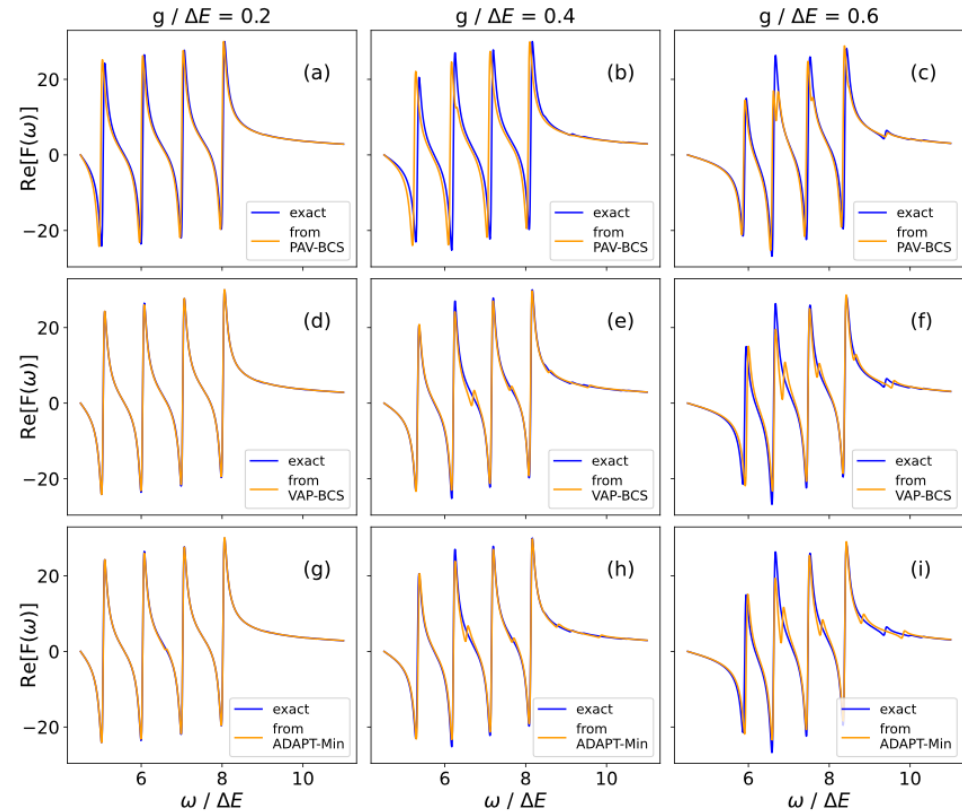
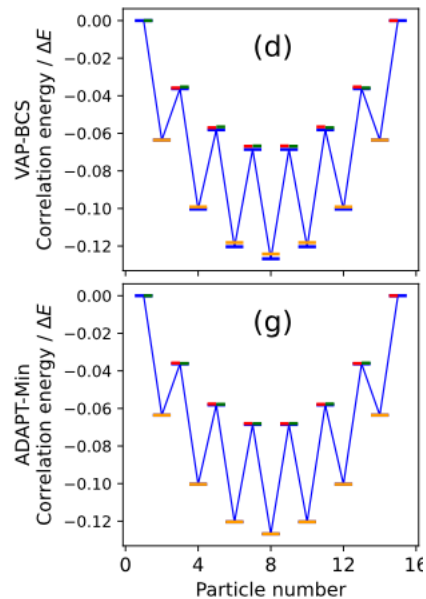


Green's function matrix elements $G_{ij}(t, t') = \langle \Psi_0 | T[a_j^\dagger(t) a_i(t')] | \Psi_0 \rangle$

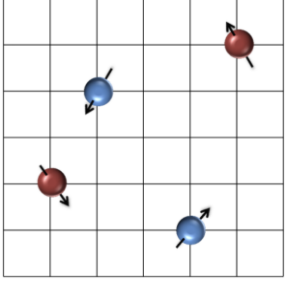
Lehman representation

$$G_{ij}(\omega) = \frac{\langle \Psi_0^N | a_i | \Psi_k^{N+1} \rangle \langle \Psi_k^{N+1} | a_j^\dagger | \Psi_0^N \rangle}{\omega - (E_k^{N+1} - E_0^N) + i\eta} + \sum_k \frac{\langle \Psi_0^N | a_j^\dagger | \Psi_k^{N-1} \rangle \langle \Psi_k^{N-1} | a_i | \Psi_0^N \rangle}{\omega - (E_0^N - E_k^{N-1}) - i\eta}.$$

Illustration with the pairing Hamiltonian

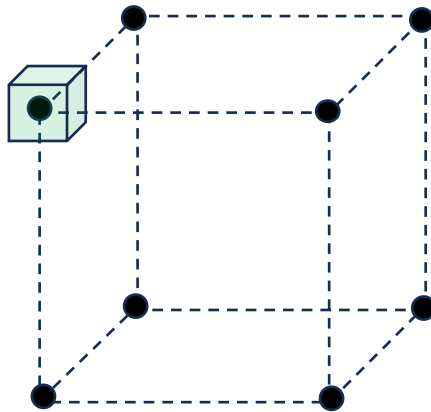
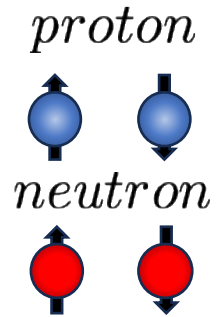


Ongoing work- Quantum simulation of nuclei on lattices



$L \times L \times L$ lattice
(here $L = 2$)

➡ Number of single-particle states: $4L^3$



Pionless EFT Hamiltonian

[Bedaque, van Kolck 2002]

$$\hat{H} = \sum_{\langle l, l' \rangle} \sum_{\tau s} T_{l'}^l \hat{a}_{l\tau s}^\dagger \hat{a}_{l'\tau s}$$

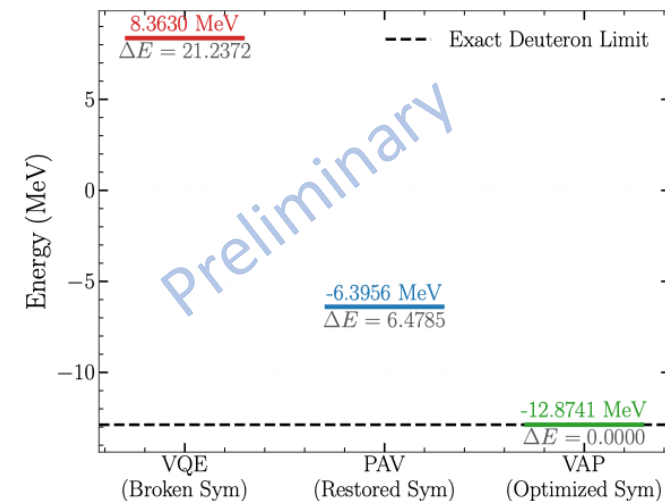
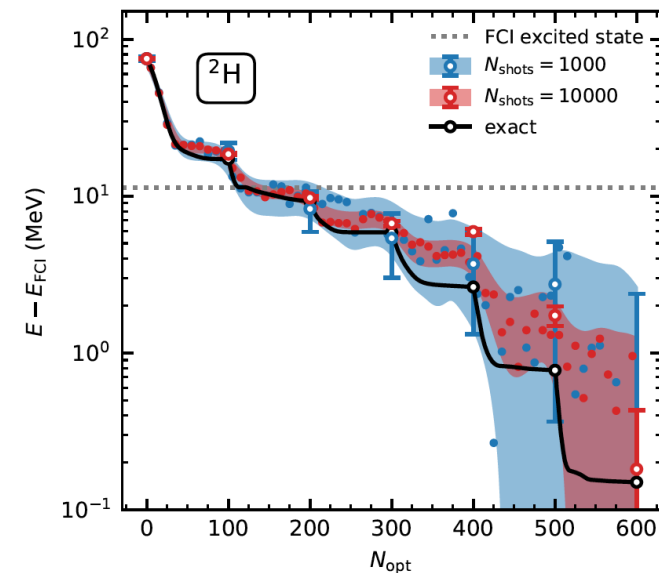
Kinetic-non local

$$+ \frac{V}{2} \sum_l \sum_{ss' \tau \tau'} \hat{a}_{l\tau s}^\dagger \hat{a}_{l\tau' s'}^\dagger \hat{a}_{l\tau' s} \hat{a}_{l\tau s}$$

Two-body local

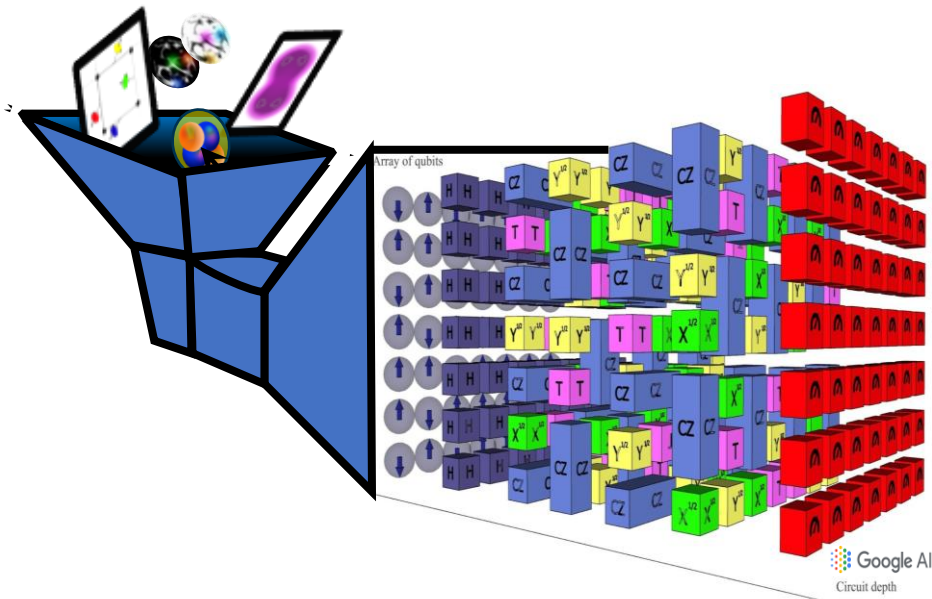
$$+ W \sum_l \sum_{\tau s} \hat{a}_{l\tau \uparrow}^\dagger \hat{a}_{l\tau \downarrow}^\dagger \hat{a}_{l-\tau s}^\dagger \hat{a}_{l-\tau s} \hat{a}_{l\tau \downarrow} \hat{a}_{l\tau \uparrow} .$$

Three-body local

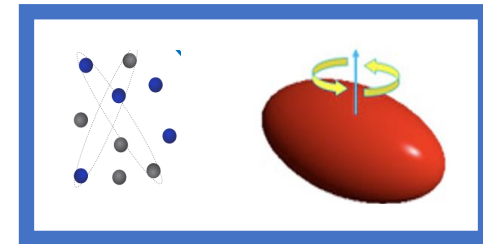


The field of quantum algorithms for many-body systems is evolving rapidly.

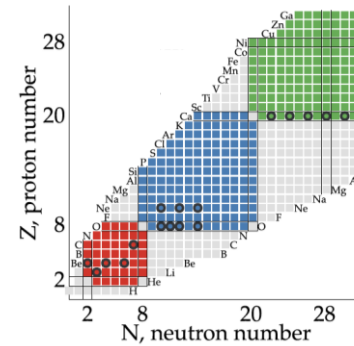
Experimental quantum computing platforms are improving constantly.



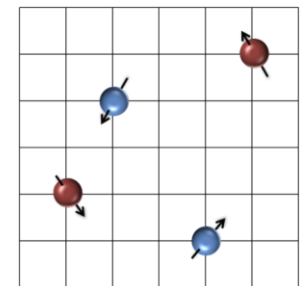
Symmetry problem



Traditional shell-model



Lattice pionless EFT

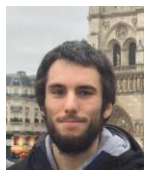


Special Thanks to

PhD students/postdocs



E. A. Ruiz Guzman



Y. Beaujeault-Taudiere



J. Zhang



S. Aychet Claisse



C. de Correç



R. Bocquet



A. Singh

Collaborators



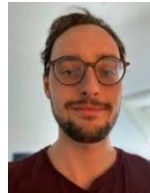
M. Mangin Brinet



B. Balantekin



A. Roggero



O. Kiss



F. Tavernelli



F. Tacchino



Vittorio Somà



P. Siwach



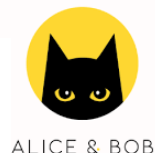
T. Ayral



P. Besserve



C. Bertrand



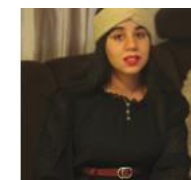
F. Jamet



B. Senjean



Laura Oliveira Atencio



S. Baid

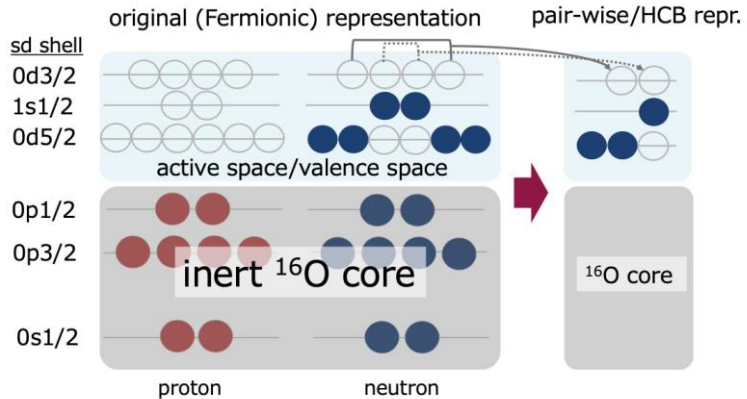


Jesus pascual Casado

Additional discussions
(applications to real quantum computers)

Simplified Shell Model applications on real quantum computers

Hard-core Boson mapping

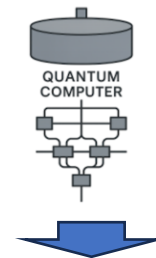


Wave preparation
+ Observ

Energy gradients
minimization

CLASSICAL
COMPUTER

After optimal
solution
Is found



Build the
Approximate GS
Compute observable

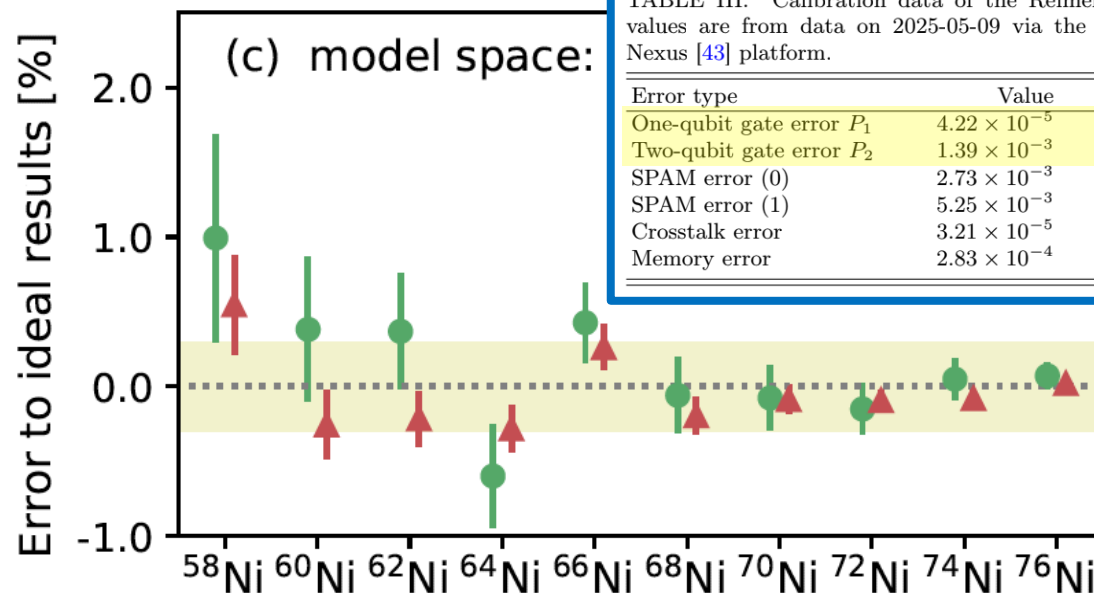
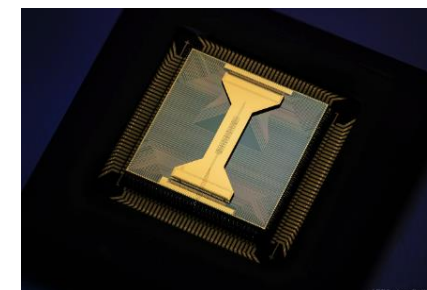


TABLE III. Calibration data of the Reimei device. The values are from data on 2025-05-09 via the Quantinuum's Nexus [43] platform.

Error type	Value	Uncertainty
One-qubit gate error P_1	4.22×10^{-5}	5.23×10^{-6}
Two-qubit gate error P_2	1.39×10^{-3}	5.92×10^{-5}
SPAM error (0)	2.73×10^{-3}	1.57×10^{-4}
SPAM error (1)	5.25×10^{-3}	2.18×10^{-4}
Crosstalk error	3.21×10^{-5}	1.89×10^{-6}
Memory error	2.83×10^{-4}	2.32×10^{-5}

RIKEN-Quantinuum's
Reimei device
(trapped Ion)

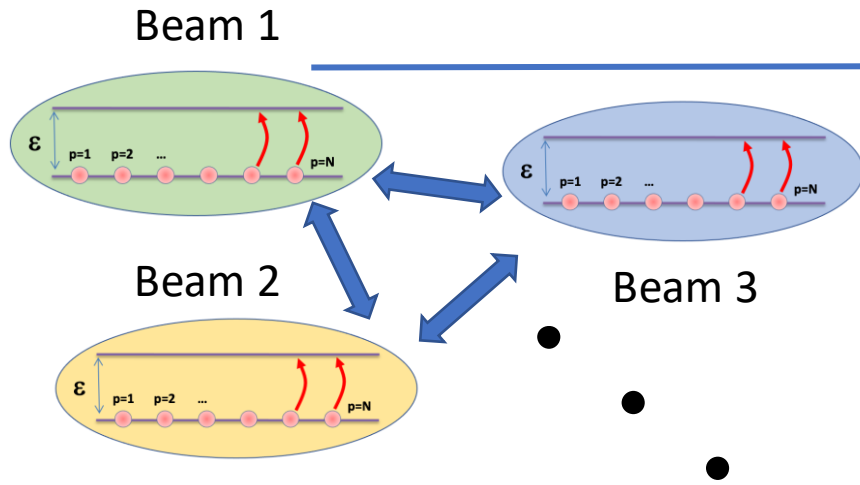


A focus on neutrino oscillation physics simulated on quantum computers

Illustration of the Hamiltonian (2 flavor approx)

Oscillation $H_\nu = \frac{1}{N} \sum_{i=0}^{N-1} \sin(2\theta_\nu) X_i - \cos(2\theta_\nu) Z_i$

Coupling $H_{\nu\nu} = \sum_{i<j}^{N-1} G_{i,j} [X_i X_j + Y_i Y_j + Z_i Z_j].$

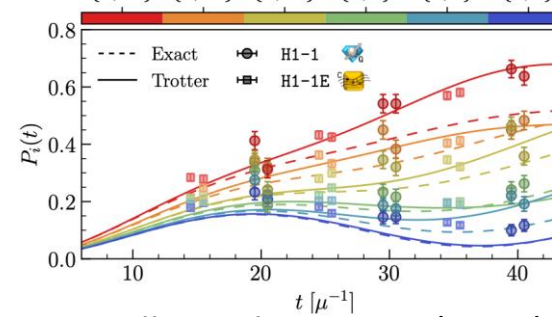
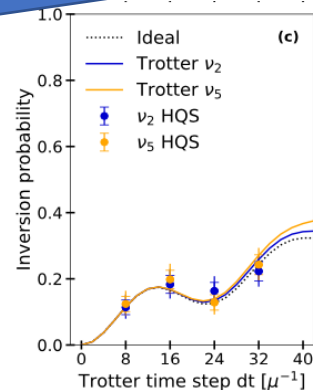
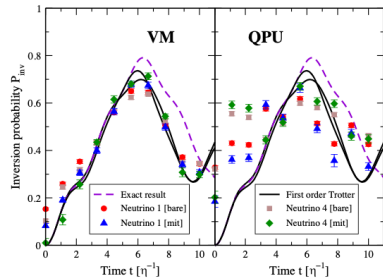


4 neutrinos
IBM-Vigo QPU

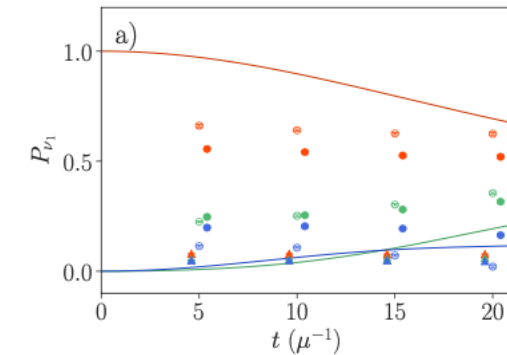
4 & 8 neutrinos
HQB-H1
Trapped Ion device

12 neutrinos
Quantinuum's H1-1
20 qubit trapped-ion

12 neutrinos / qutrits
H1-1 & ibm_torino



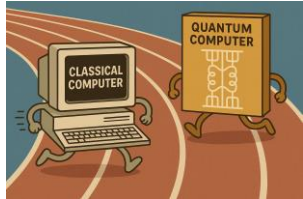
Illa et al, PRL 130 (2023)



Turro et al,
PRD D 111 (2025)

Hall et al, PRD 104 (2021)

Amitrano, et al, PRD 107, (2023)



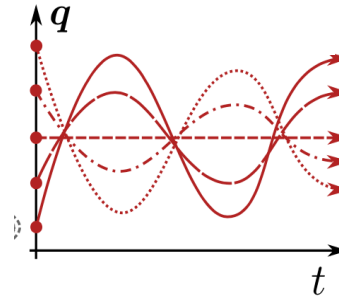
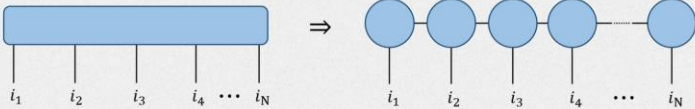
A focus on neutrino oscillation physics simulated on quantum computers

Phase-Space approach

Matrix Product State

Order-N tensor:
contains $\sim \exp(N)$ parameters

Network of low-order tensors:
contains $\sim \text{poly}(N)$ parameters



Lacroix et al, Phys Rev. D106 (2022),
Phys Rev D110 (2024).

Mangin Brinet, DL,
Phys. Rev. D 113 (2026)

De Correc et al,
APS Open science 1, (2026)

200 qubits



300 qutrits



2000 qubits

12 neutrinos / qutrits
H1-1 & ibm_torino

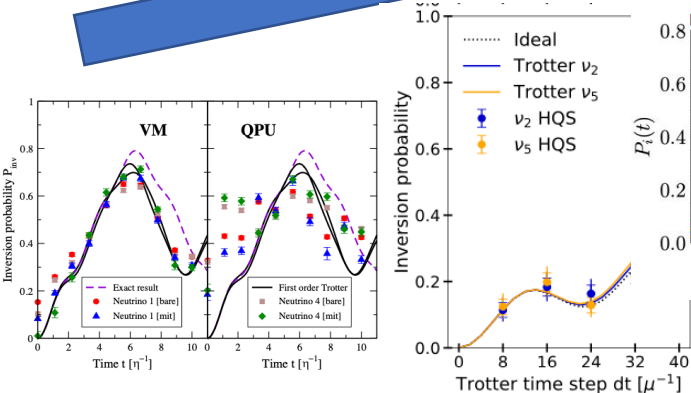
>100 neutrinos
IBM machines

4 & 8 neutrinos
HQD-H1

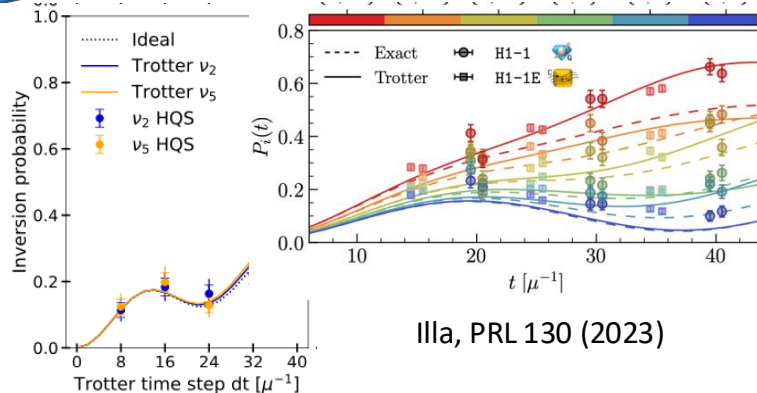
Trapped Ion device

4 neutrinos
IBM-Vigo QPU

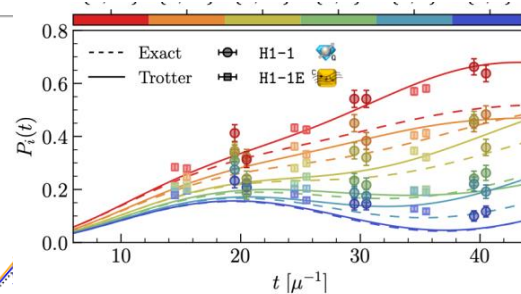
12 neutrinos
Quantinuum's H1-1



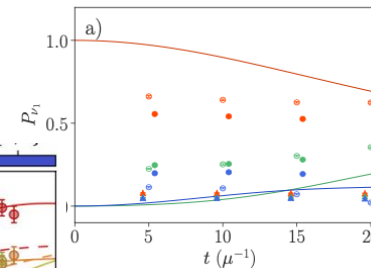
Hall, PRD 104 (2021)



Amitrano, PRD 107 (2023)

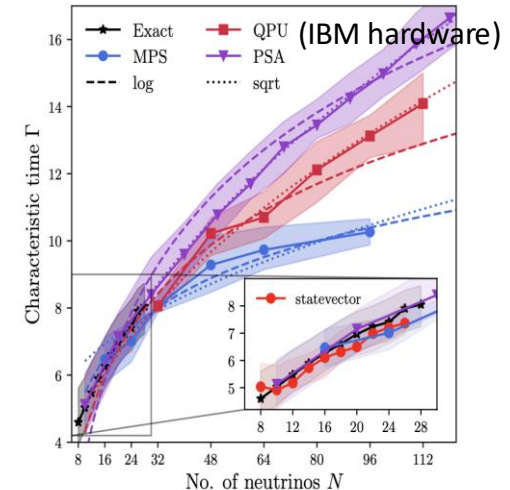


Illa, PRL 130 (2023)



Turro, PRD 111 (2025)

Neutrino thermalization time



Kiss, Tavernelli, Tacchino, Lacroix,
Roggero, nature com., arxiv:2510.24841