Ab-initio Renormalized Random Phase Approximation for closed-shells nuclear systems

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ZAKOPANE CONFERENCE on Nuclear Physics 2026
Extremes of the Nuclear Landscape

Charles University

Outline

- newly developed framework for HF + **renormalized RPA (RRPA)** calculations with modern chiral-based NN+NNN interactions

- calculations of selected closed shell nuclei with **chiral NN+NNN** potentials

  (g.s energies, low-lying spectra, dipole response)

- nature of low-lying dipole excitations

“Ab-initio” in nuclear theory

nuclear structure → many-body Schrödinger equation **→ approximations**

$$i\hbar \frac{\partial \Psi(1,2 \ldots A)}{\partial t} = \boldsymbol{H}\Psi(1,2 \ldots . A)$$

$$\boldsymbol{H} = \sum_{i=1}^{A} \frac{\boldsymbol{p}_i^2}{2m} + \sum_{i<j}^{A} \boldsymbol{V}(i,j) + \sum_{i<j<k}^{A} \boldsymbol{V}(i,j,k)$$

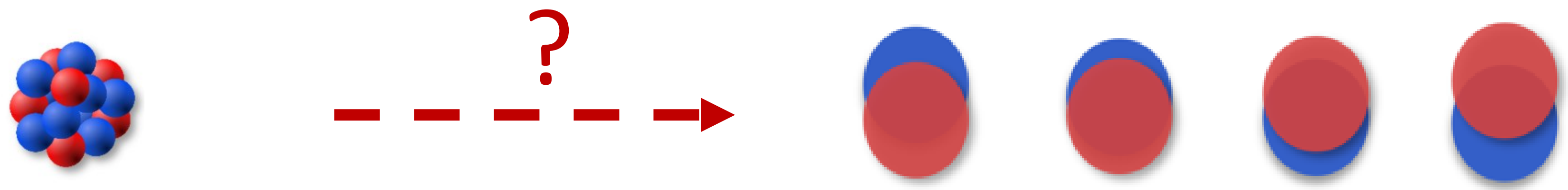
interaction models

NN and NNN interactions from chiral effective field theory

→ *ab initio* methods

phenomenological interactions

DFT, SM



Chiral EFT interactions

Chiral perturbation theory (ChpT)

- hierarchy of nuclear forces
- systematic improvements
- many-body forces

several fitting strategies

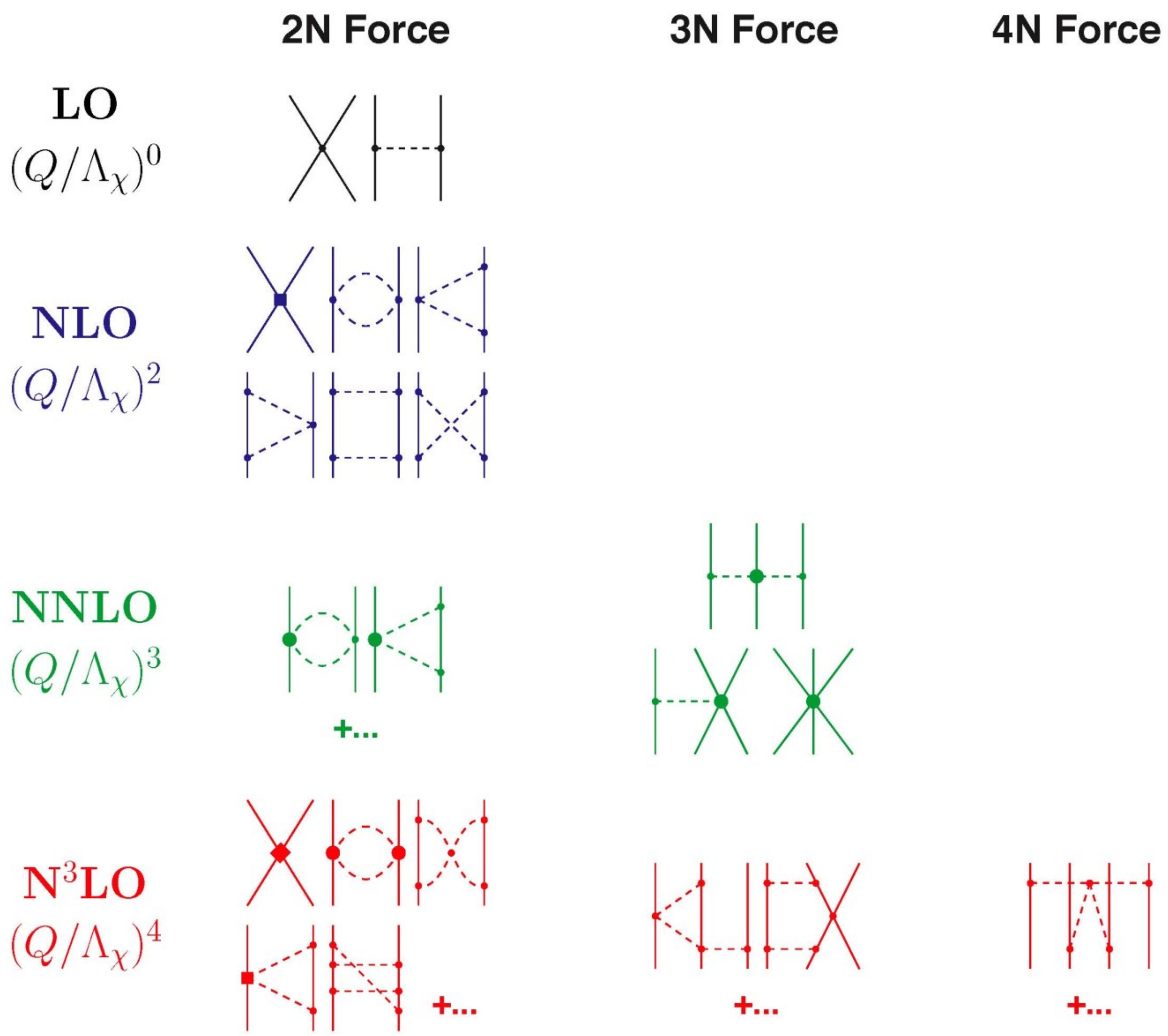
N^2LO_{sat}
Ekström A. et al., Phys. Rev C 91, 051301 (2015)

Δ -full nuclear interactions
(Gothenburg–Oak Ridge)

$\Delta N^2LOGO(394)$, $\Delta N^2LOGO(450)$
Jiang W. G. et al., Phys. Rev C102,054301(2020)

N^3LO_{texas}
Hu B. S. et al., arxiv:2512.11723 (2025)

→ different predictions for many-body systems



LEC's have to be fitted to selected set of experimental data

Mean-field and beyond …

- intrinsic Hamiltonian with NN+NNN interactions

$$H = \sum_{a,d} t_{ad} a_a^\dagger a_d + \frac{1}{4} \sum_{\substack{a,b,\\ d,e}} {}^{\mathrm{NN}}V_{abde} a_a^\dagger a_b^\dagger a_e a_d + \frac{1}{36} \sum_{\substack{a,b,c,\\ d,e,f}} {}^{\mathrm{NNN}}V_{abcdef} a_a^\dagger a_b^\dagger a_c^\dagger a_f a_e a_d$$

- reference state → Hartree-Fock approximation (HF)

$$H = E_0^{\mathrm{HF}} + \sum_a^{\mathrm{N}} \varepsilon_a : c_a^\dagger c_a : + \boxed{V^{\mathrm{Res}}}$$

- **residual interaction** → correlations

ground state properties → many-body Perturbation Theory (MBTP), Coupled cluster, Greens-function methods, In Medium Similarity Renormalisation Group (IMSRG)

excited states and response functions, GRs → Greens function methods, LIT-Coupled cluster
Renormalized Random Phase Approximation (RRPA)

Random Phase Approximation (RPA)

Equation of motion:

$$\left[H,\, Q_\nu^\dagger\right] |\mathrm{RPA}\rangle = (E_\nu - E_0)|\nu\rangle$$

$$|\nu\rangle \equiv Q_\nu^\dagger|0\rangle \equiv \sum_{ph} [X^\nu_{ph} B^\dagger_{ph} - Y^\nu_{ph} B_{ph}]|0\rangle$$

$$B^\dagger_{ph} \equiv a^\dagger_p a_h, \quad B_{ph} \equiv a^\dagger_h a_p$$

Quasiboson approximation : **valid if correlations are small**

$$\langle \mathrm{RPA}| \left[a^\dagger_h a_p,\, a^\dagger_q a_g\right] |\mathrm{RPA}\rangle \simeq \cancel{\langle \mathrm{HF}| \left[a^\dagger_h a_p,\, a^\dagger_q a_g\right] |\mathrm{HF}\rangle = \delta_{p,q}\delta_{h,g}}$$

What if correlations are strong? → correlated g.s. → fully self-consistent RPA (SCRPA)

D. J. Rowe, Rev. Mod. Phys 40, 153 (1968), P. Schuck et al., Physics Reports 929, 1-84 (2021)

Renormalized Random Phase Approximation (RRPA)

Renormalized RPA - simplified version of SCRPA

$$\langle 0|a_m^\dagger a_n^\dagger a_j a_i|0\rangle \simeq \langle 0|a_m^\dagger a_i|0\rangle\langle 0|a_n^\dagger a_j|0\rangle - \langle 0|a_m^\dagger a_j|0\rangle\langle 0|a_n^\dagger a_i|0\rangle$$

different names in literature: Improved RPA (IRPA), or Extended RPA (ERPA)

- metallic clusters

Catara F., Piccito G., Sambataro M. and Van Giai N., Phys. Rev. B 54, 17536 (1996)
Catara F. et al. , Phys. Rev. B ***58****, 16070 (1998)*
Voronov V. et al., Phys. Part. Nucl. ***31****, 904 (2000)*

- with UCOM potential

P. Papakonstantinou, R. Roth, and N. Paar, Phys. Rev. C 75, 014310 (2007)

- **application with chiral NN+NNN potentials**

R. Folprecht, F.K. et al., Phys. Rev. C 113, L041302 (2026)

Renormalized Random Phase Approximation (RRPA)

introduction of “renormalized” *ph* operators → expansion of one-body densities in terms of *Y* amplitudes

$$\boxed{a_p^\dagger a_h \rightarrow D_{ph}^{-1/2} a_p^\dagger a_h}$$

$$D_{ph} \equiv (n_h - n_p)$$

$$n_r = \langle 0 | a_r^\dagger a_r | 0 \rangle$$

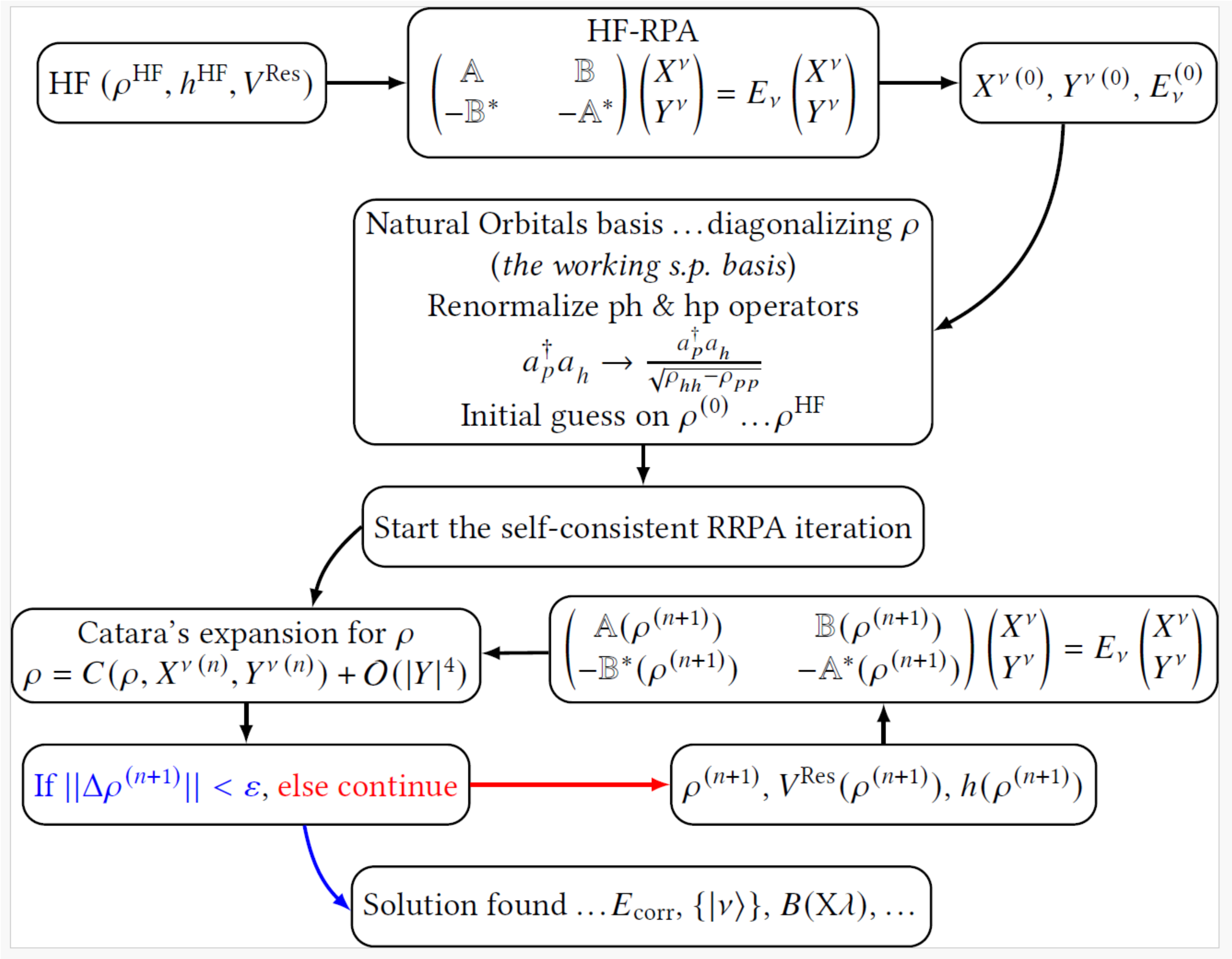
$$\begin{pmatrix} \mathbb{A} & \mathbb{B} \\ -\mathbb{B}^* & -\mathbb{A}^* \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = E_\nu \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix}$$

RPA-like eigenvalue problem with “renormalized” interaction

$$\mathbb{A}_{phqg} = \left(\sqrt{D_{ph}/D_{qg}} + \sqrt{D_{qg}/D_{ph}}\right)\left(\delta_{hg} h_{pq} - \delta_{pq} h_{hg}\right) + \sqrt{D_{ph} D_{qg}}\, V_{pgqh}$$

$$\mathbb{B}_{phqg} = \sqrt{D_{ph} D_{qg}}\, V_{pqhg}.$$

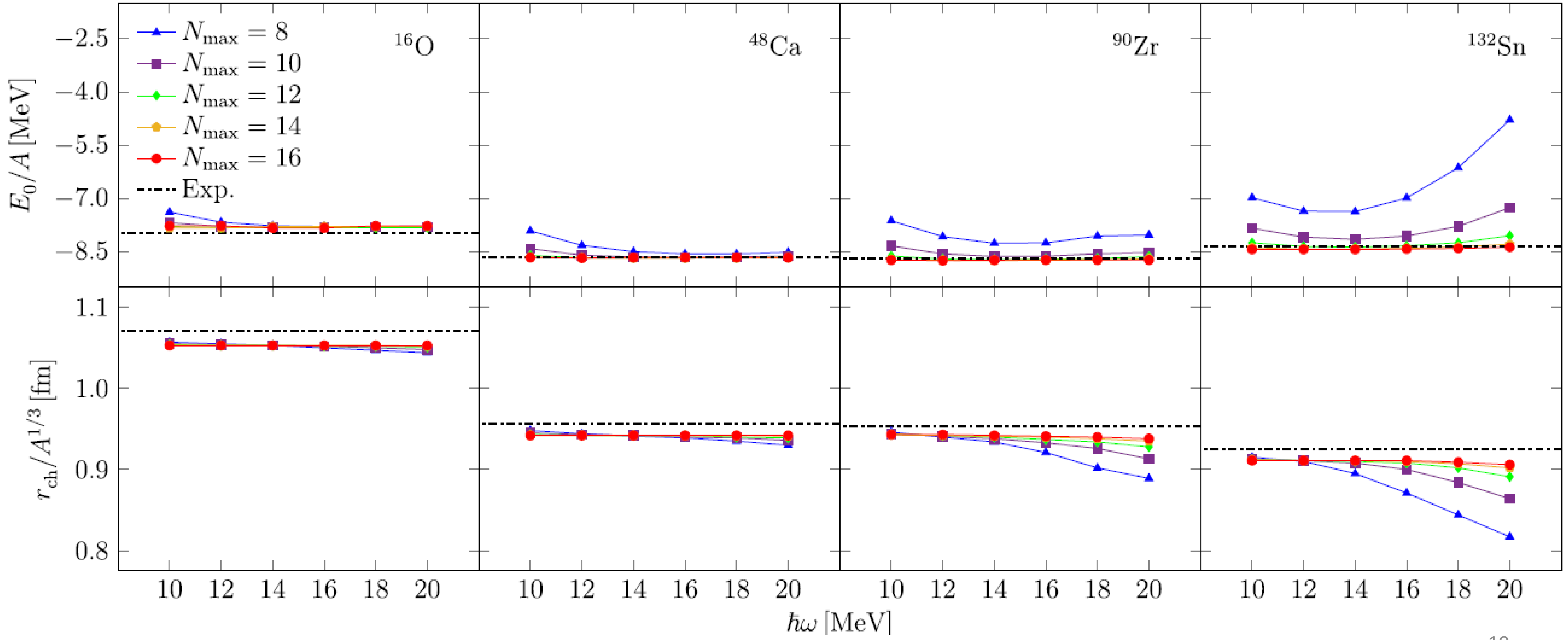
Renormalized Random Phase Approximation (RRPA)



Binding energies and radii

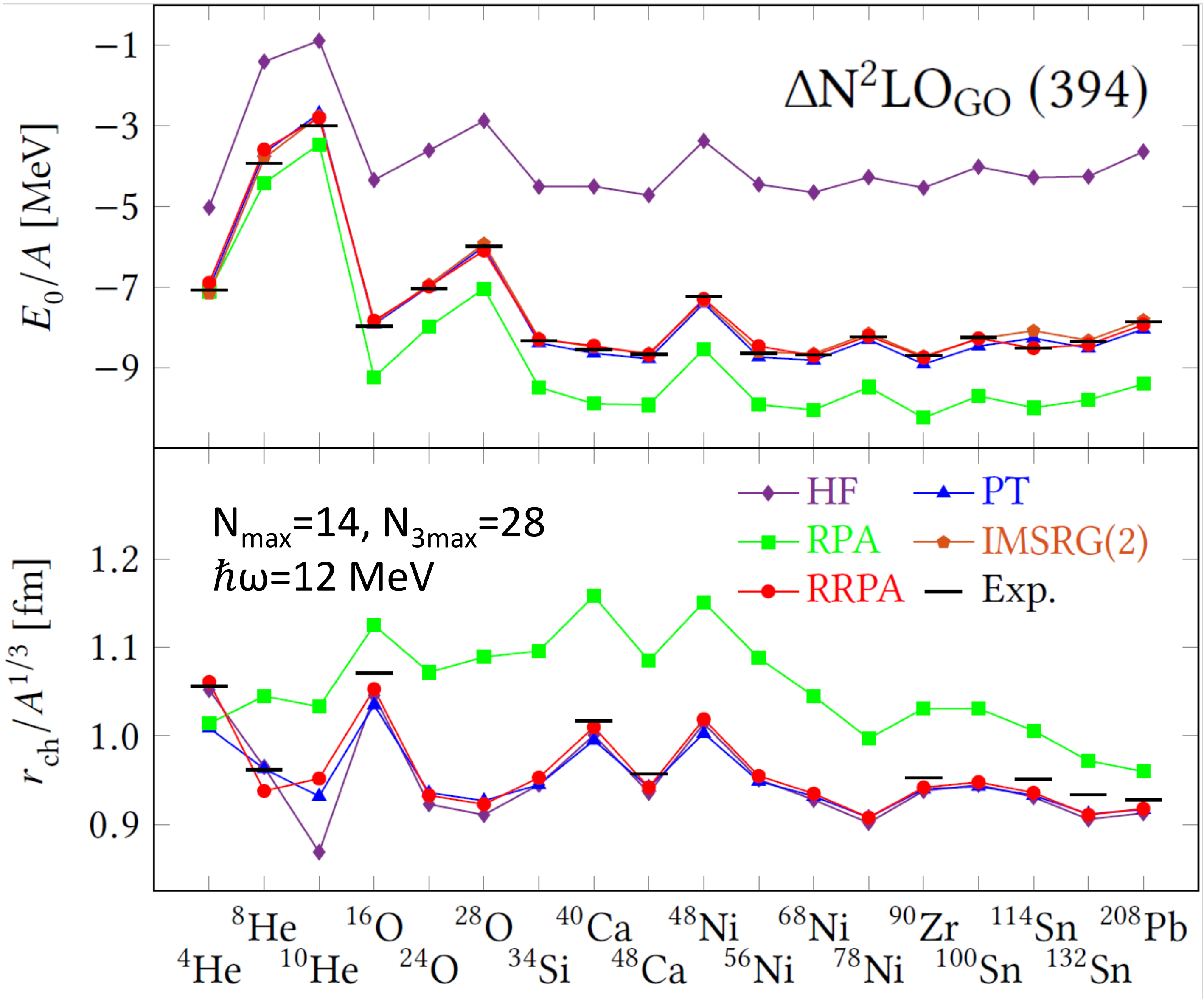
- convergence of RRPA results with increasing model space

ΔN^2LO_{GO} (394)



R. Folprecht, F.K. et al., Phys. Rev. C 113, L041302 (2026)

Binding energies and radii



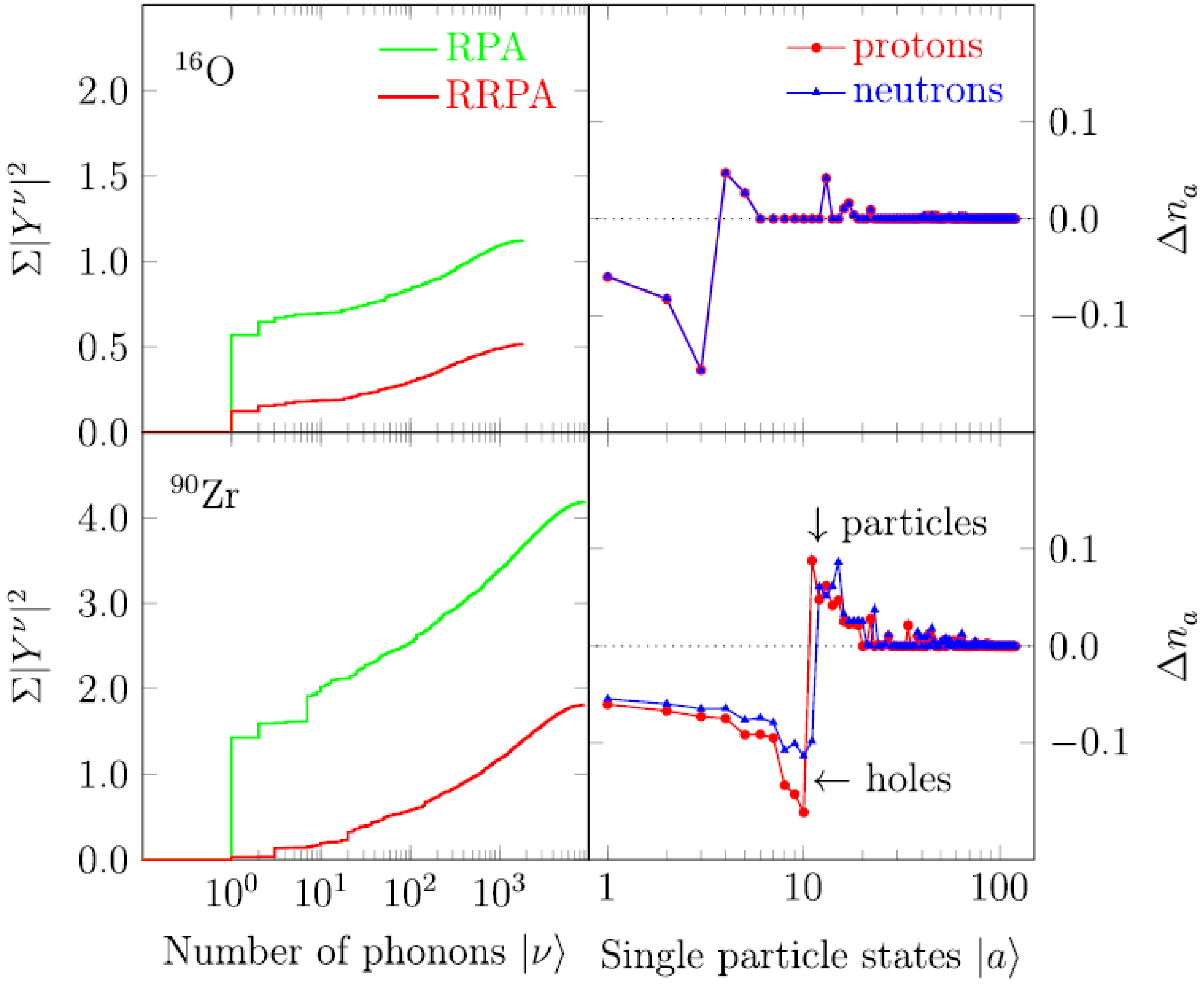
$$E_0^{\mathrm{RPA}} = E_0^{\mathrm{HF}} - \sum_{\nu} E_\nu \sum_{p>\mathrm{F}} \sum_{h<\mathrm{F}} |Y^{\nu}_{ph}|^2$$

	RRPA	PT	IMSRG(2)	CC	Exp.
^{16}O	124.7	124.4	126.2	127.5	127.62
^{24}O	168	164	167	169	168.96
^{40}Ca	338	338	339	346	342.05
^{48}Ca	416	410	415	420	416.00
^{78}Ni	634	624	629	639	641.55
^{90}Zr	773	764	768	782	783.90
^{100}Sn	807	793	804	818	825.30
^{132}Sn	1045	1021	1026	1043	1102.84

- RRPA results for binding energies and radii comparable to IMSRG(2) and PT calculations

R. Folprecht, F.K. et al., Phys. Rev. C 113, L041302 (2026)

Single-particle occupations



$$\Delta n_a = n_a^{\mathrm{RRPA}} - n_a^{\mathrm{HF}}$$

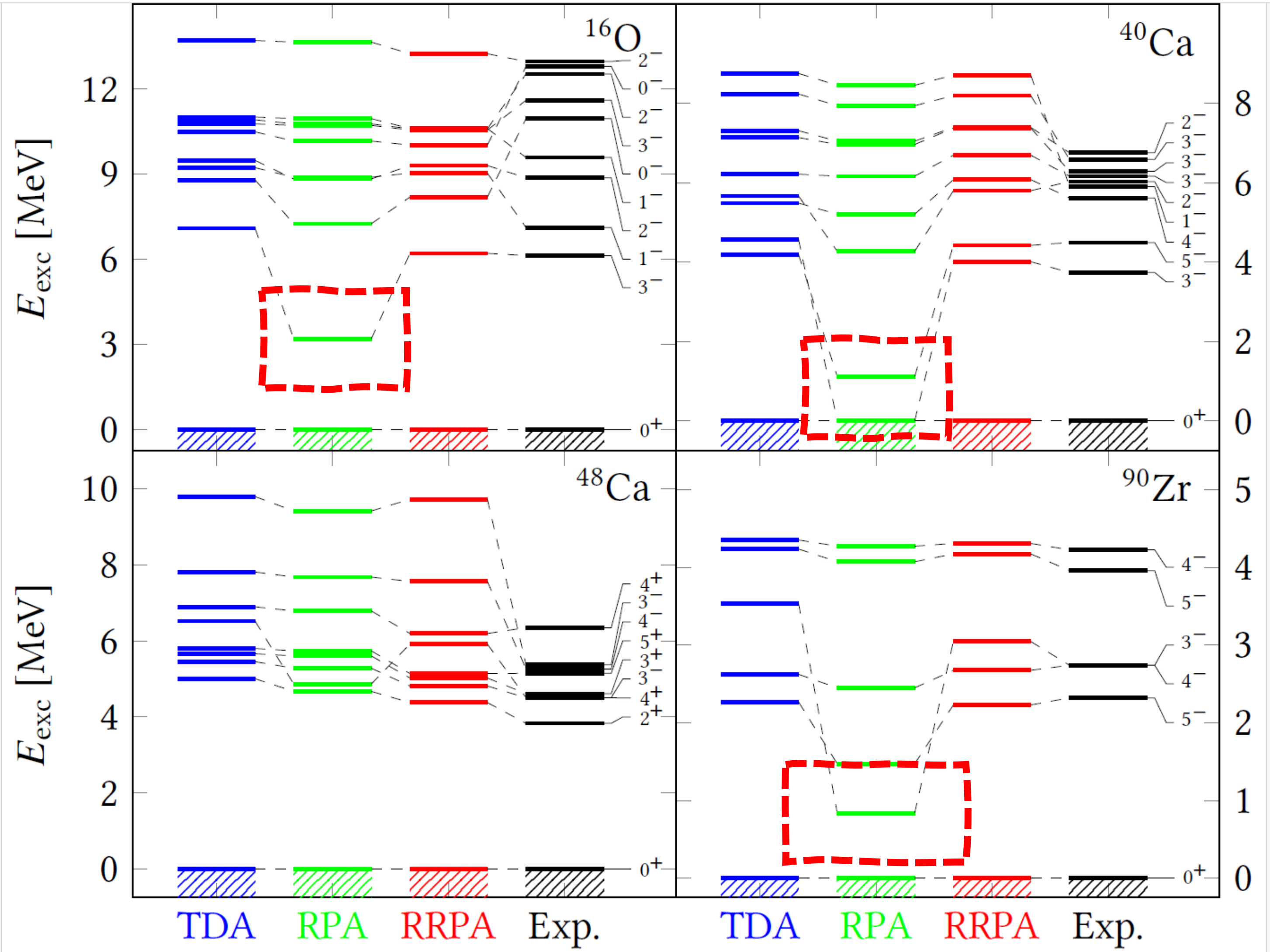
$$n_r = \langle 0 | a_r^\dagger a_r | 0 \rangle$$

- depletion of single-particle occupations
- renormalization of phonon amplitudes

R. Folprecht, F.K. et al., Phys. Rev. C 113, L041302 (2026)

Low-lying levels

- collapse of specific levels observed in RPA is cured in RRPA



R. Folprecht, F.K. et al., Phys. Rev. C 113, L041302 (2026)

Structure of dipole states

- expansion of transition dipole operator

$$\mathcal{M}^{\mathrm{E1}}(k) \simeq \mathcal{M}^{\mathrm{LO\,E1}} + k\,\mathcal{M}^{\mathrm{NLO\,E1}} + \mathcal{O}(k^2)$$

- general treatment of vortical, toroidal and compression modes
  J.Kvasil et al., Phys. Rev. C 84, 034303 (2011)

- toroidal E1 operator can be expressed as combination of vortical and compressional operators

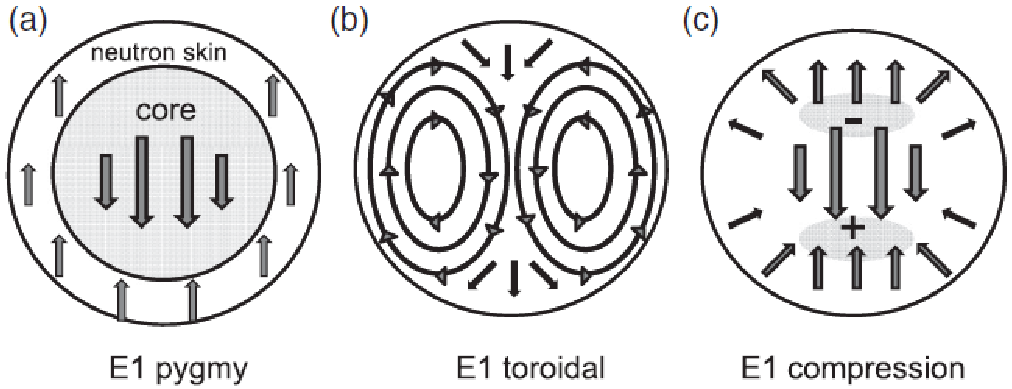
$$\mathcal{M}^{\mathrm{NLO\,E1}} \equiv \mathcal{M}^{\mathrm{Tor\,E1}} = \mathcal{M}^{\mathrm{Vor\,E1}} - \mathcal{M}^{\mathrm{Com\,E1}},$$
$$\mathcal{M}_{\mu}^{\mathrm{Tor\,E1}} \sim \int \mathrm{d}r^3\,\vec{j}(\vec{r}) \cdot r^2 \left(\frac{\sqrt{2}}{5}\,\vec{Y}_{12\mu}(\vec{n}) + \vec{Y}_{10\mu}(\vec{n})\right),$$
$$\mathcal{M}_{\mu}^{\mathrm{Vor\,E1}} \sim \int \mathrm{d}r^3\,\vec{j}(\vec{r}) \cdot r^2\,\vec{Y}_{12\mu}(\vec{n}).$$

Structure of dipole states

- **more details about structure we can see from** ”velocity fields” (current transition densities)

$$\vec{\boldsymbol{v}}^{\alpha} = \langle 0|\vec{\boldsymbol{j}}|\alpha\rangle/\rho^{0} \sim \boldsymbol{\nabla} M$$

- **toroidal (vortical) nature of low-energy E1 mode** in ^{208}Pb studied within RPA with Skyrme forces.
  A. Repko et al., Phys. Rev. C 87, 024305 (2013)



- candidates of toroidal modes in ^{58}Ni

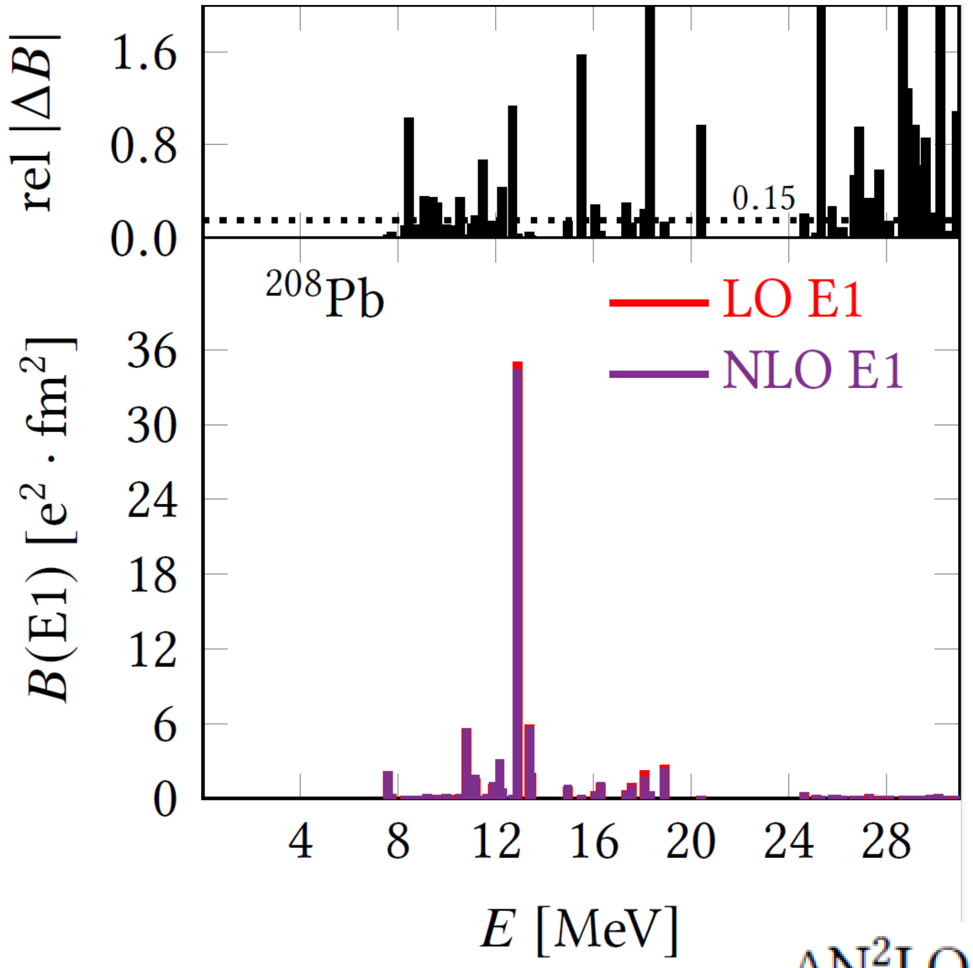
P. von Neumann-Cosel et al., Phys. Rev. Lett 133, 232502 (2024)

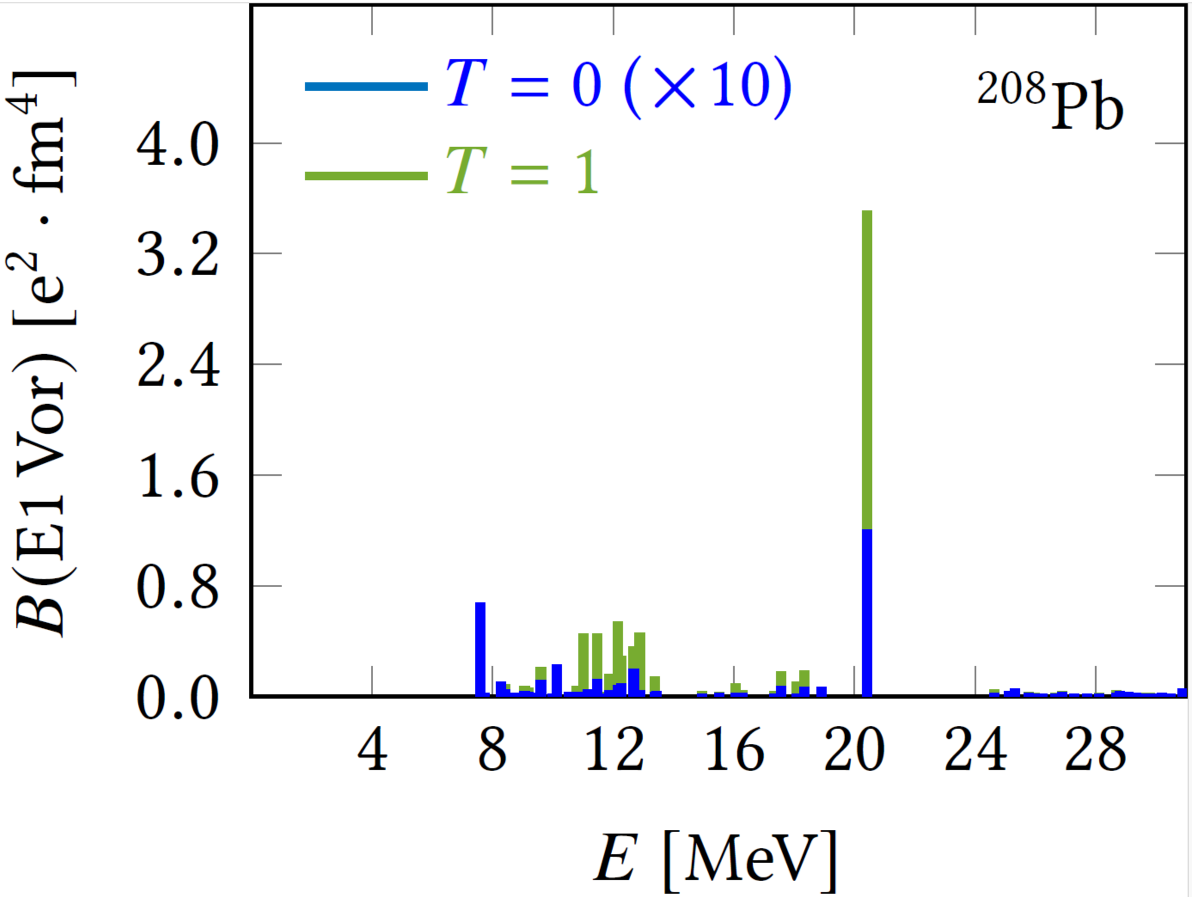
What we get within RRPA with chiral interactions?

Structure of dipole states

$$\mathcal{M}^{\mathrm{E1}}(k) \simeq \mathcal{M}^{\mathrm{LO\,E1}} + k\,\mathcal{M}^{\mathrm{NLO\,E1}} + \mathcal{O}(k^2)$$

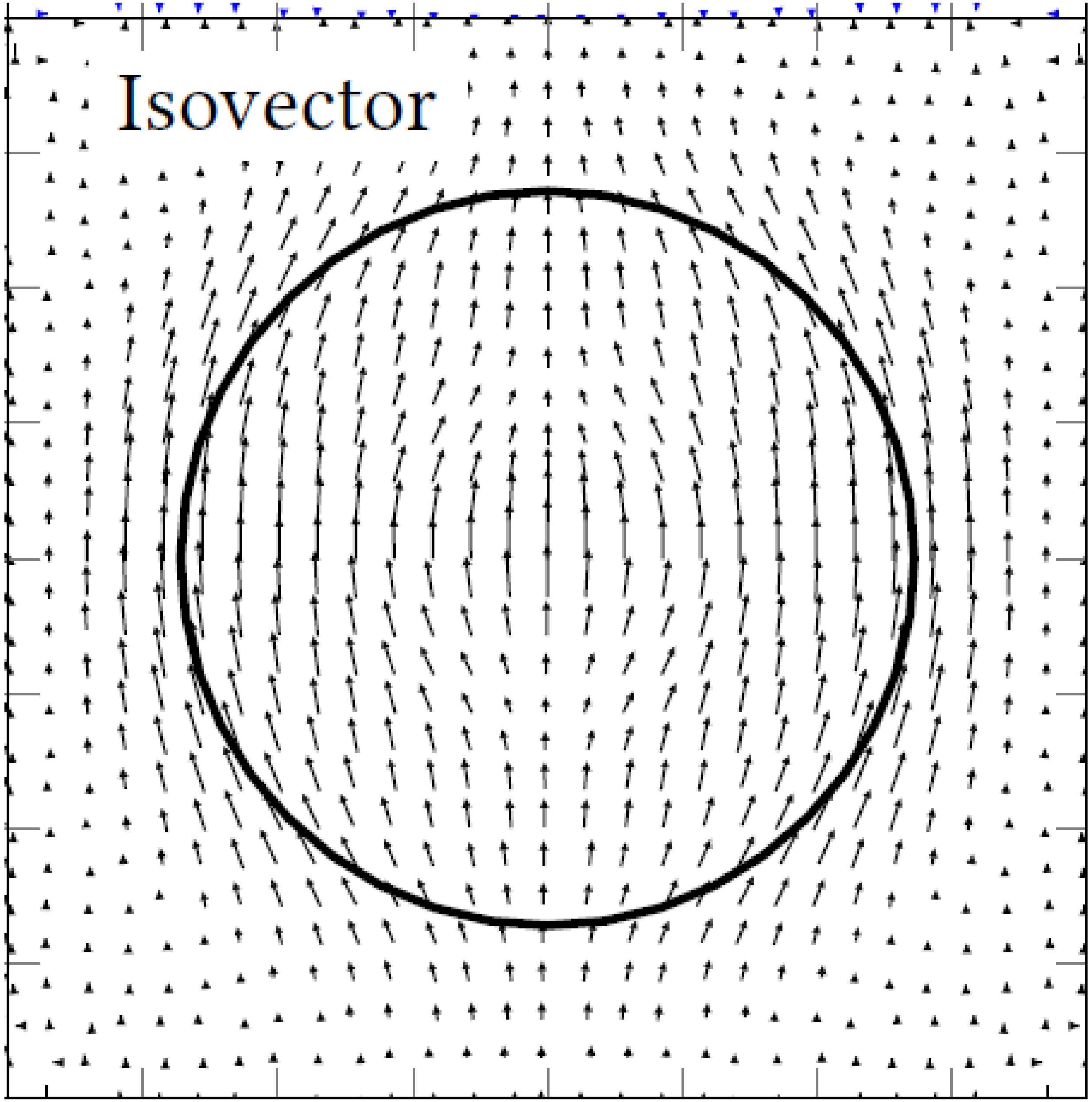
$$\mathcal{M}^{\mathrm{NLO\,E1}} \equiv \mathcal{M}^{\mathrm{Tor\,E1}} = \mathcal{M}^{\mathrm{Vor\,E1}} - \mathcal{M}^{\mathrm{Com\,E1}},$$
$$\mathcal{M}^{\mathrm{Tor\,E1}}_{\mu} \sim \int \mathrm{d}r^3\, \vec{j}(\vec{r}) \cdot r^2 \left(\frac{\sqrt{2}}{5}\, \vec{Y}_{12\mu}(\vec{n}) + \vec{Y}_{10\mu}(\vec{n}) \right),$$
$$\mathcal{M}^{\mathrm{Vor\,E1}}_{\mu} \sim \int \mathrm{d}r^3\, \vec{j}(\vec{r}) \cdot r^2\, \vec{Y}_{12\mu}(\vec{n}).$$

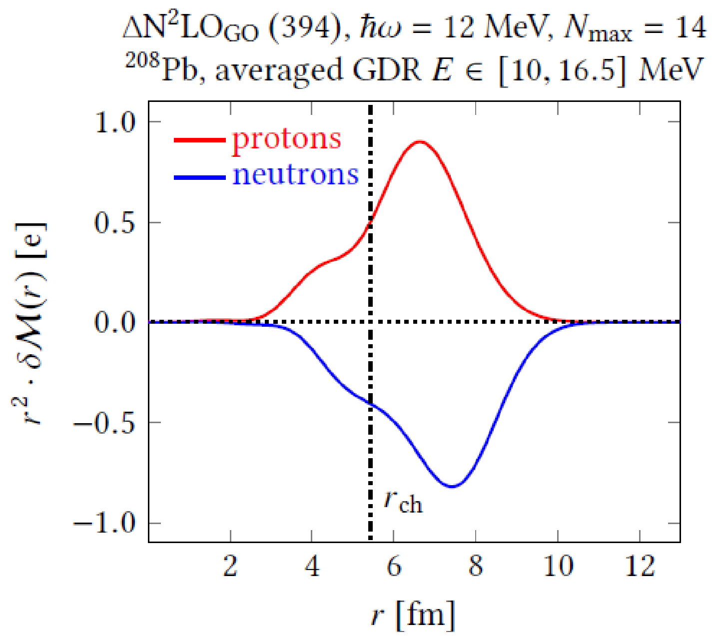


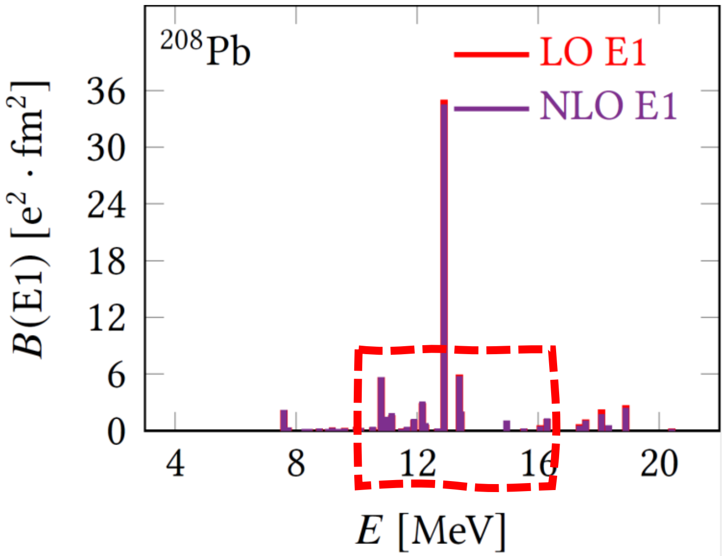


$\Delta\mathrm{N}^2\mathrm{LO}_{\mathrm{GO}}\,(394)$, $\hbar\omega = 12$ MeV, $N_{\max} = 14$

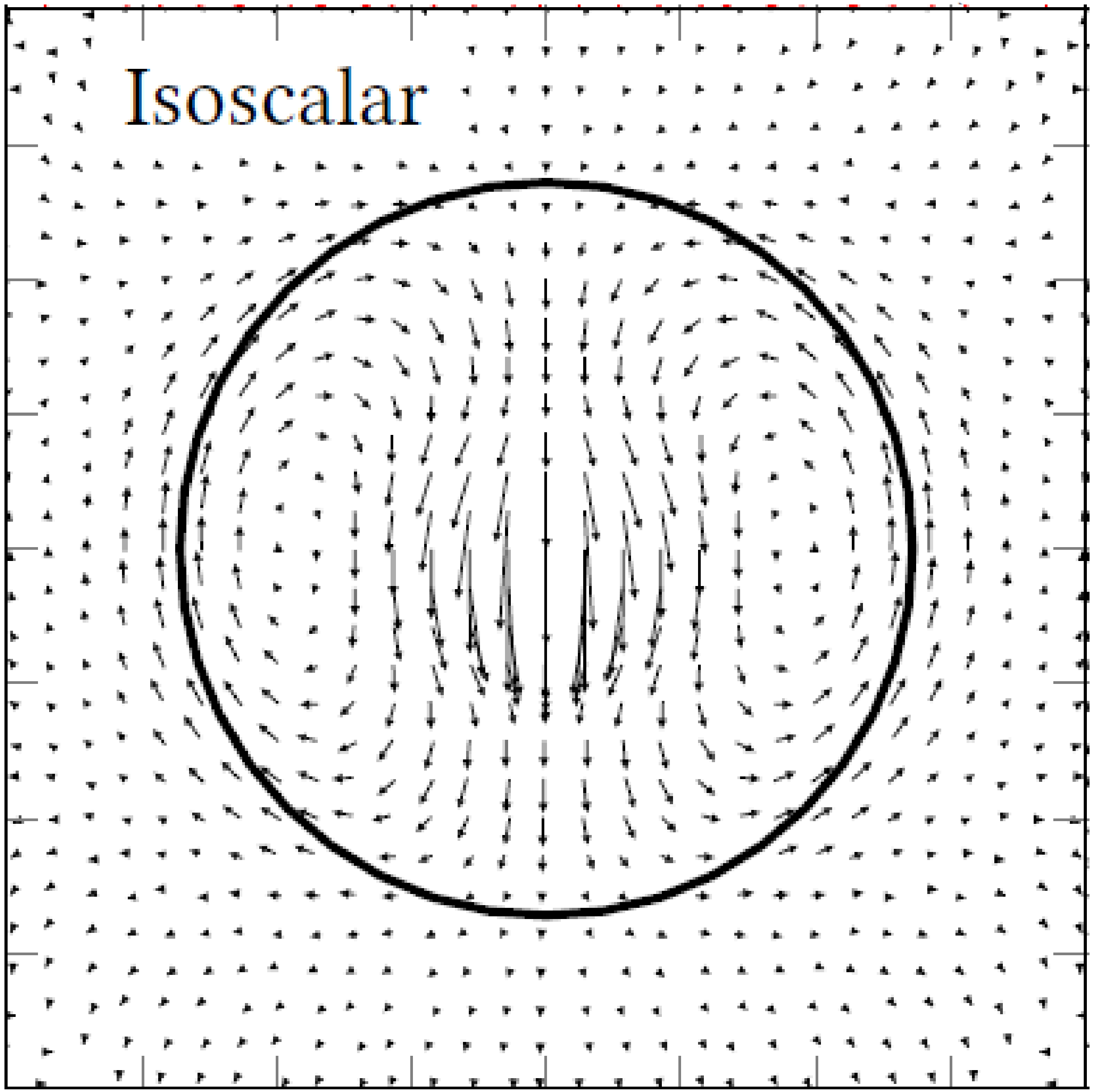
Structure of dipole states

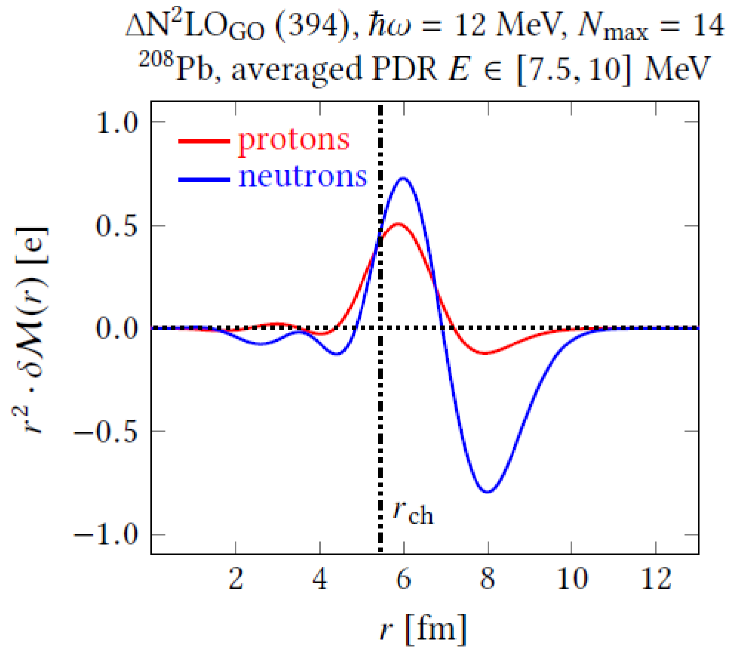


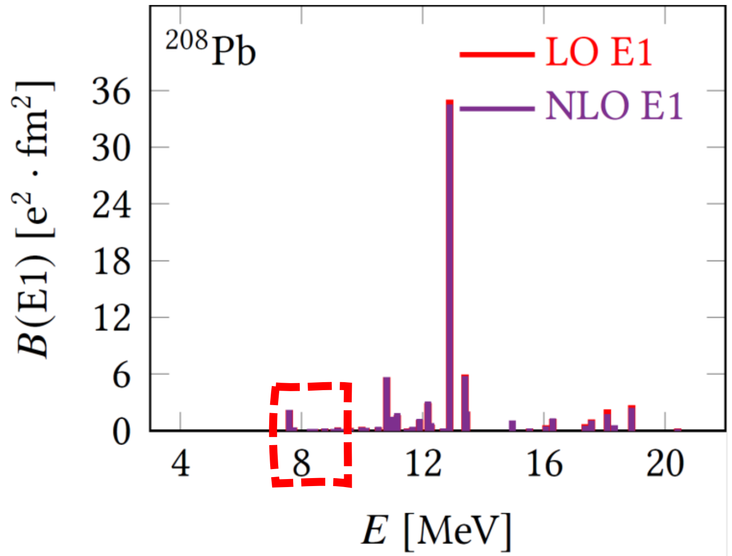




Structure of dipole states



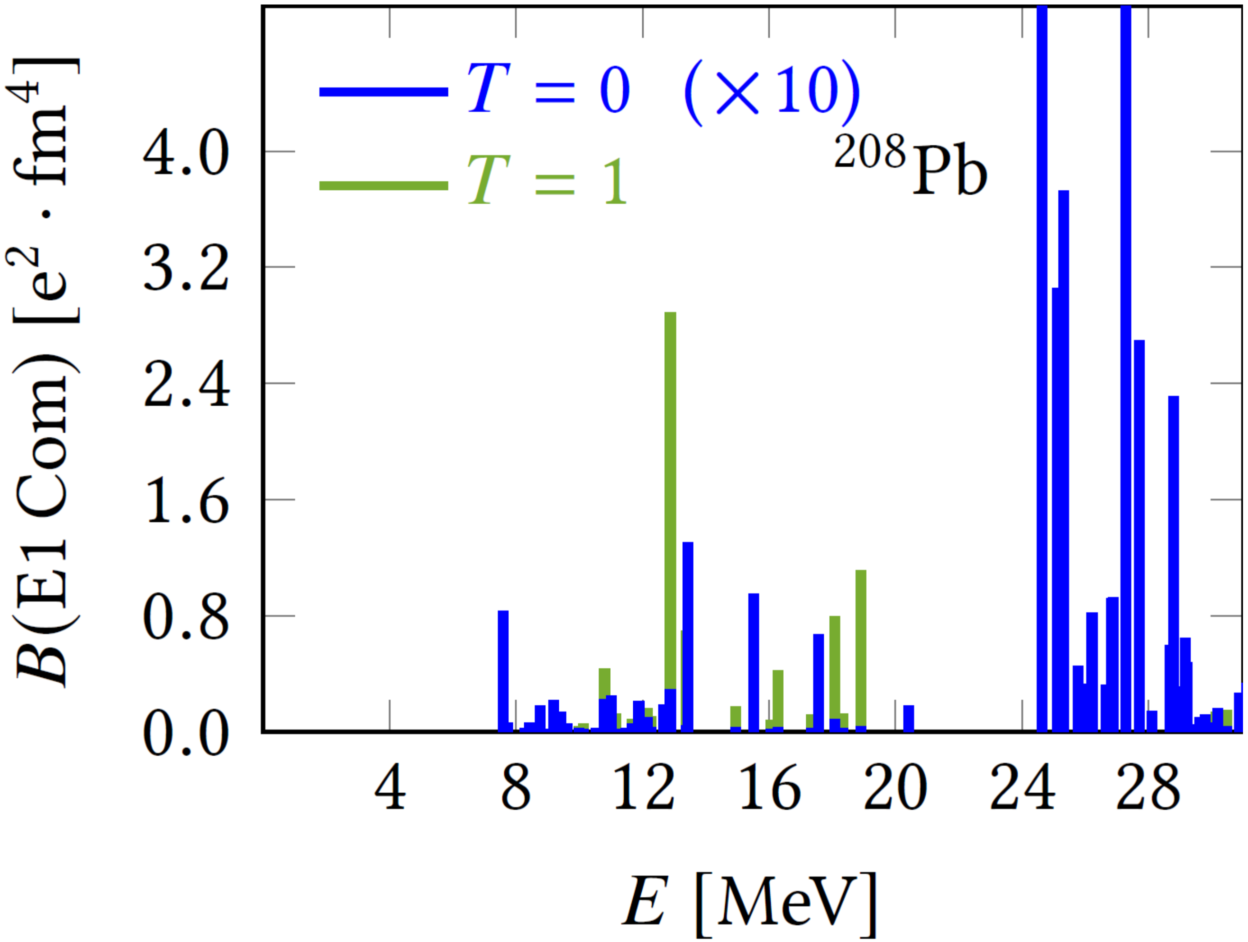




Structure of dipole states

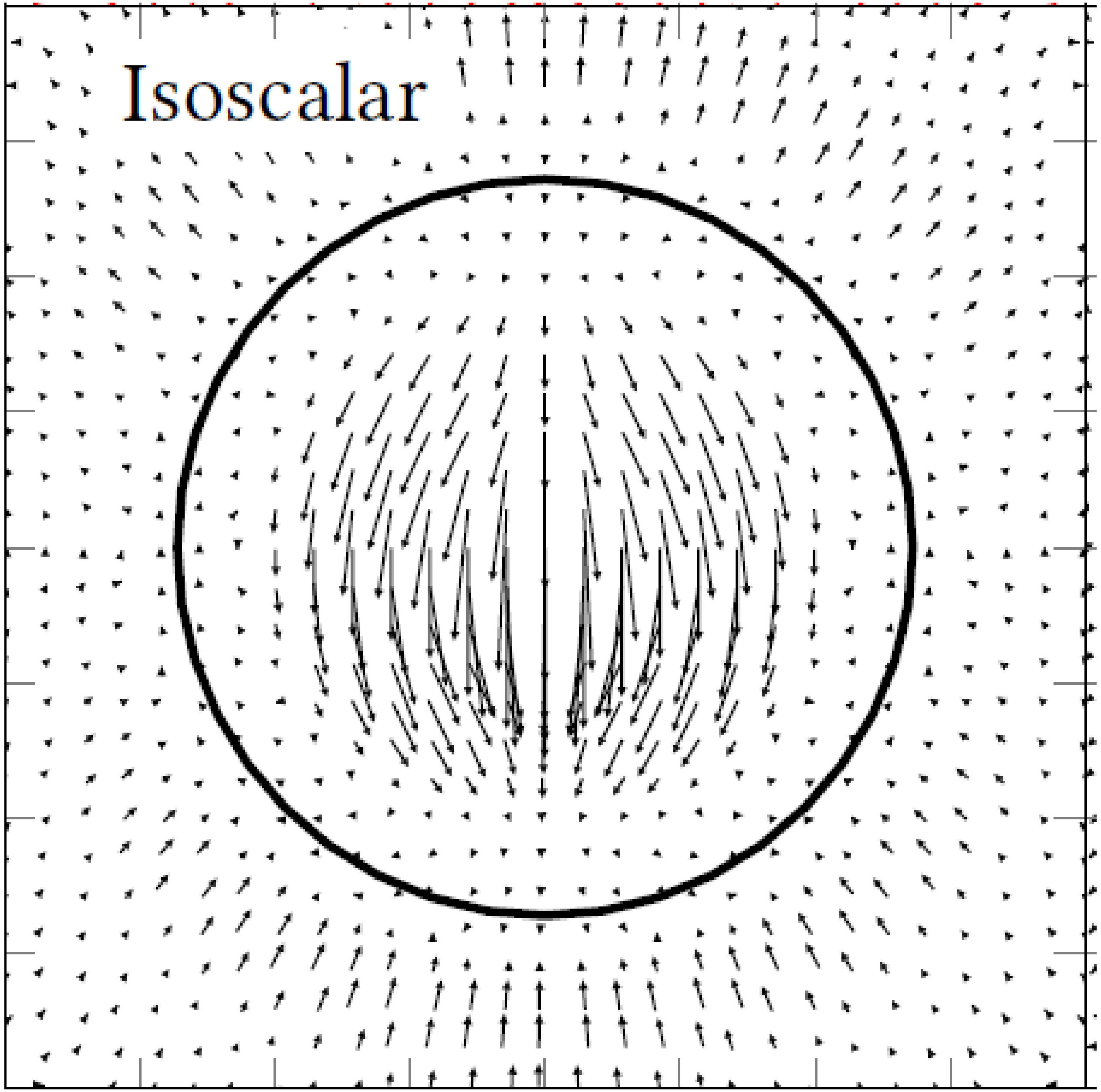
- compressional strength distribution

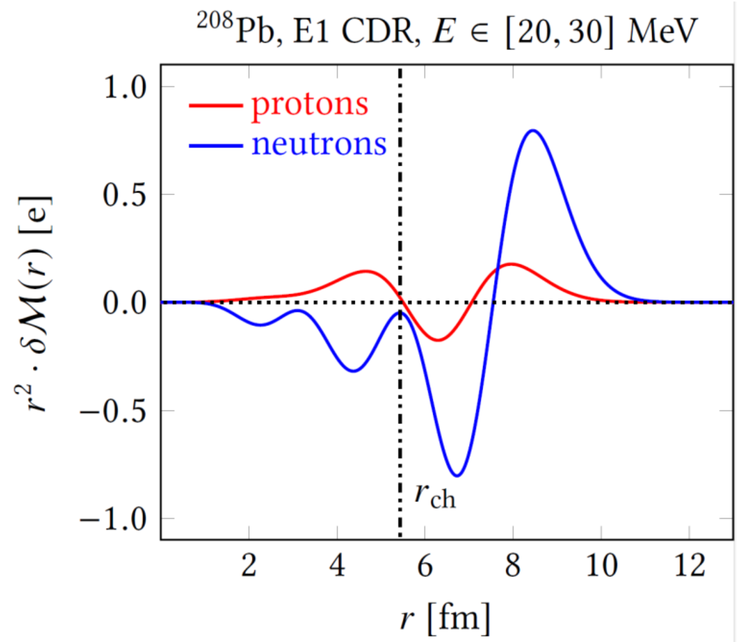
$$\mathcal{M}^{\mathrm{NLO\,E1}} \equiv \mathcal{M}^{\mathrm{Tor\,E1}} = \mathcal{M}^{\mathrm{Vor\,E1}} - \mathcal{M}^{\mathrm{Com\,E1}}$$

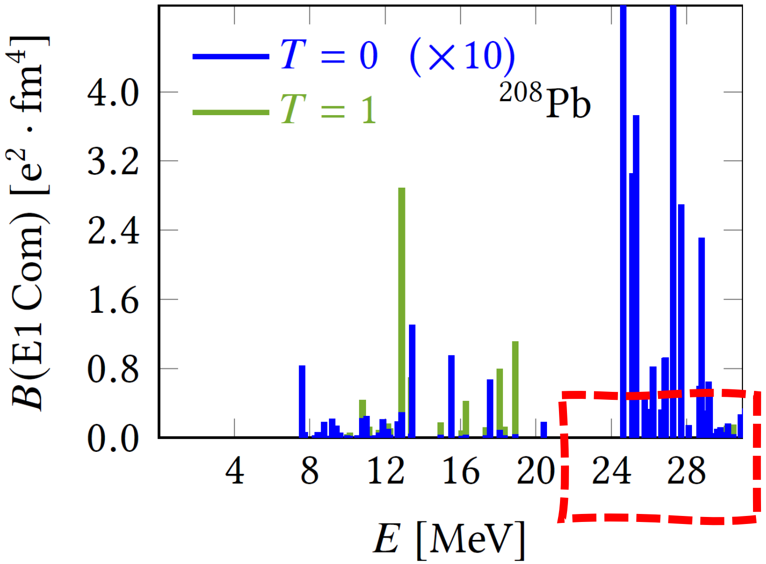


ΔN^2LO_{GO} (394), $\hbar\omega = 12$ MeV, $N_{max} = 14$

Structure of dipole states







Summary

- first implementation of RRPA with **NN+NNN chiral potentials**
- significant improvement with respect to RPA, reasonable description of **correlation energies** and some **low-lying levels** and gross properties of **GDRs** in spherical closed subshell nuclei
- **toroidal (vortical) character** of specific low-lying dipole states

Plans:

- systematic calculations with various NN+NNN potentials
- quasiparticle extension (**see poster of R. Folprecht**)
- inclusion of “complex” configurations

Collaborators

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Thank you for you attention!

NN and NNN interactions

- ***NuHamil* :** A numerical code to generate nuclear two- and three-body matrix elements from chiral effective field theory, *Miyagi T., Eur. Phys. J. A (2023) 59:150*

- spherical HO basis – model space given by the HO principal quantum number N_{max} (number of used major shells-1), typically N_{max}=14-16

- for practical reasons **NNN space must be truncated**

- **normal-ordered 2-body Approximation** → clever storage of NNN matrix elements up to $N_{3max}=N_1+N_2+N_3 \leq 28$

Miyagi T. , Stroberg S. R., Navrátil P., Hebeler K., and Holt J. D., Phys. Rev. C 105, 014302 (2022).

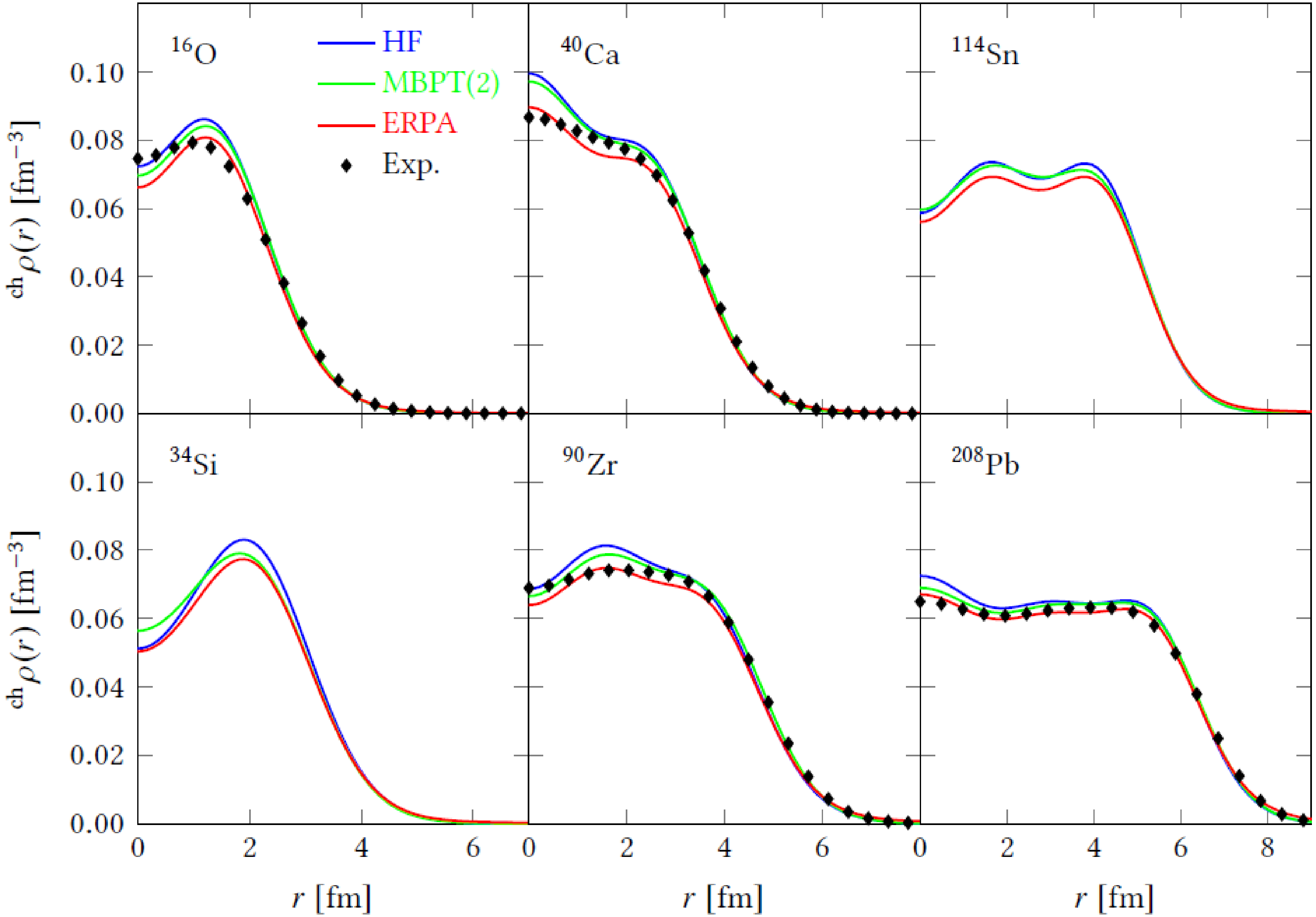
$$V^{\rm Res} = \frac{1}{4}\sum_{a,b,d,e} {}^{\rm NN}V_{abde} :c^\dagger_a c^\dagger_b c_e c_d: + \frac{1}{4}\sum_{a,b,c,d,e,f} {}^{\rm NNN}V_{abcdef}\, {}^{\rm N}\rho_{cf} :c^\dagger_a c^\dagger_b c_e c_d: +$$

$$\cancel{+\frac{1}{36}\sum_{a,b,c,d,e,f} {}^{\rm NNN}V_{abcdef} :c^\dagger_a c^\dagger_b c^\dagger_c c_f c_e c_d:} \, .$$

Backup slides

Charged densities

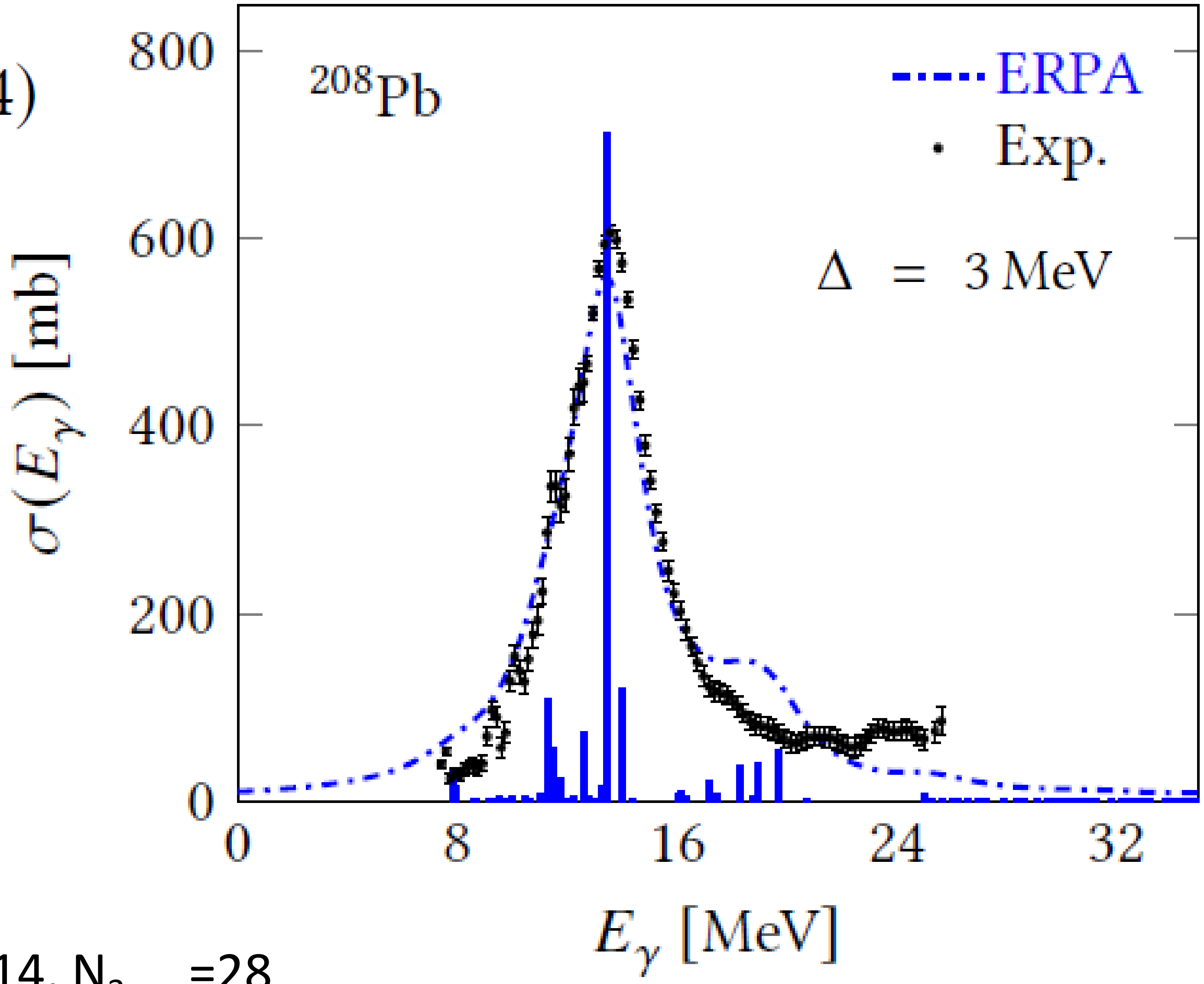
ΔN^2LO_{GO} (394)



$\hbar\omega$=12 MeV N_{max}=14, N_{3max}=28

Giant dipole resonance

ΔN^2LO_{GO} (394)



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Giant dipole resonance

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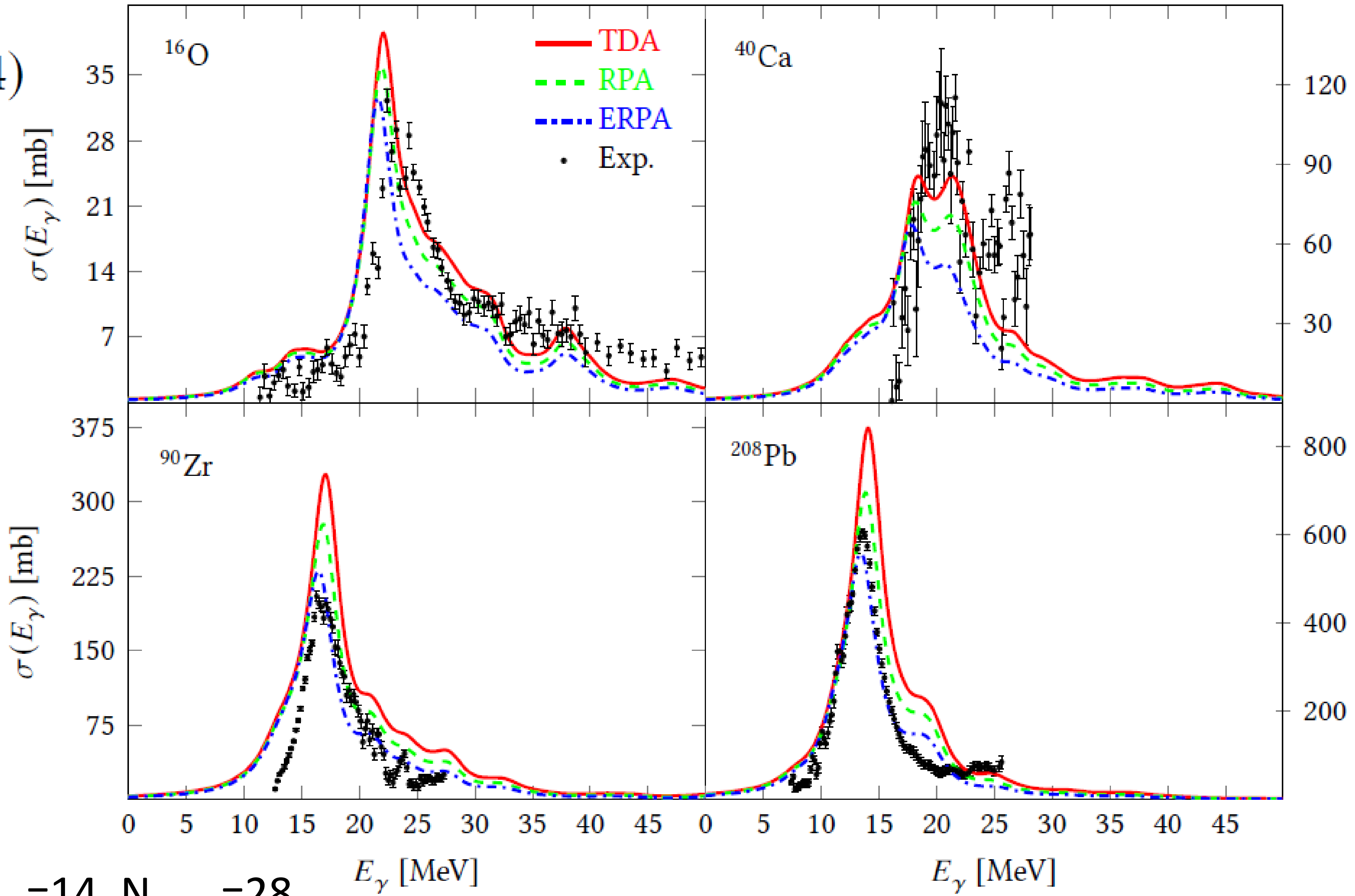

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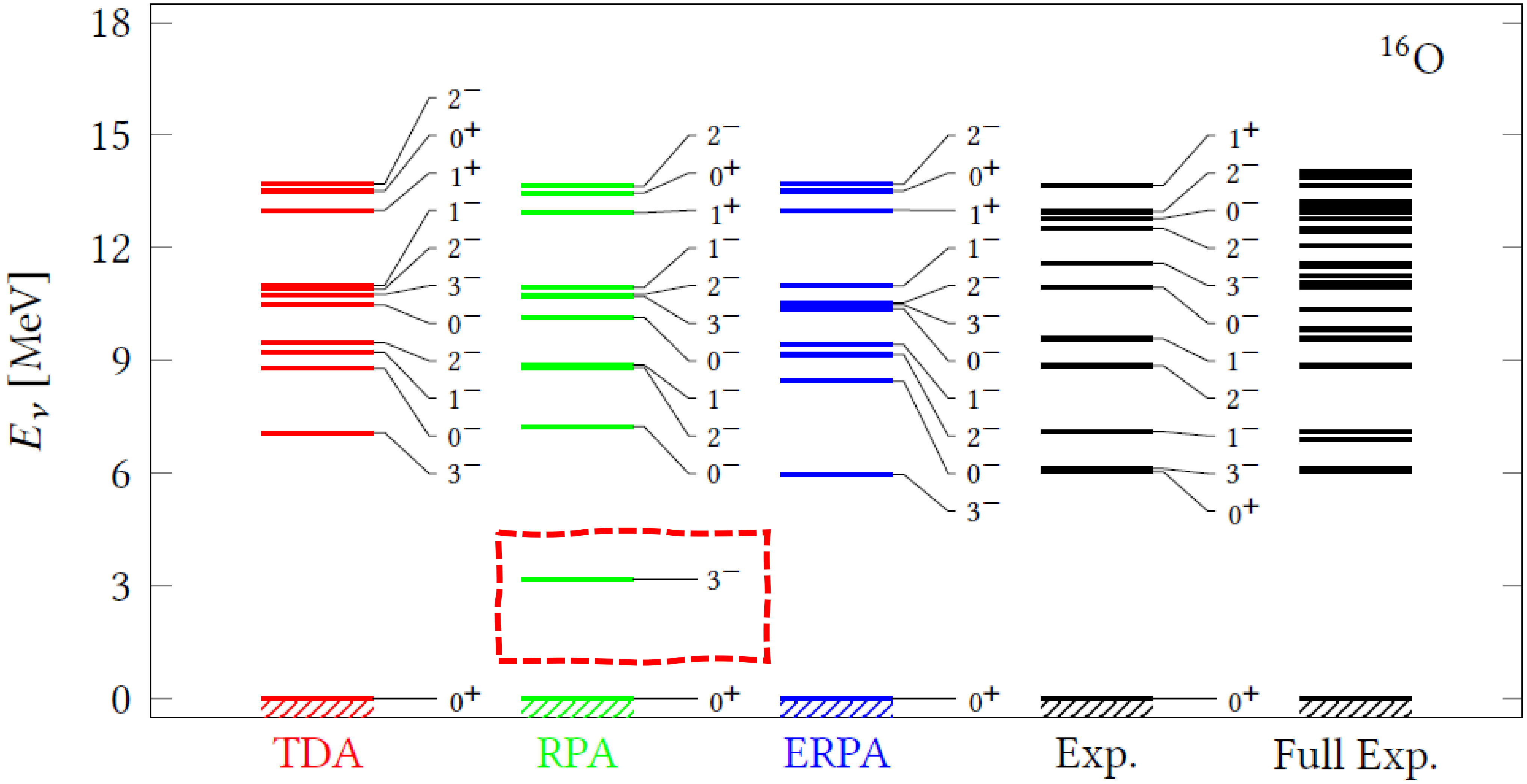
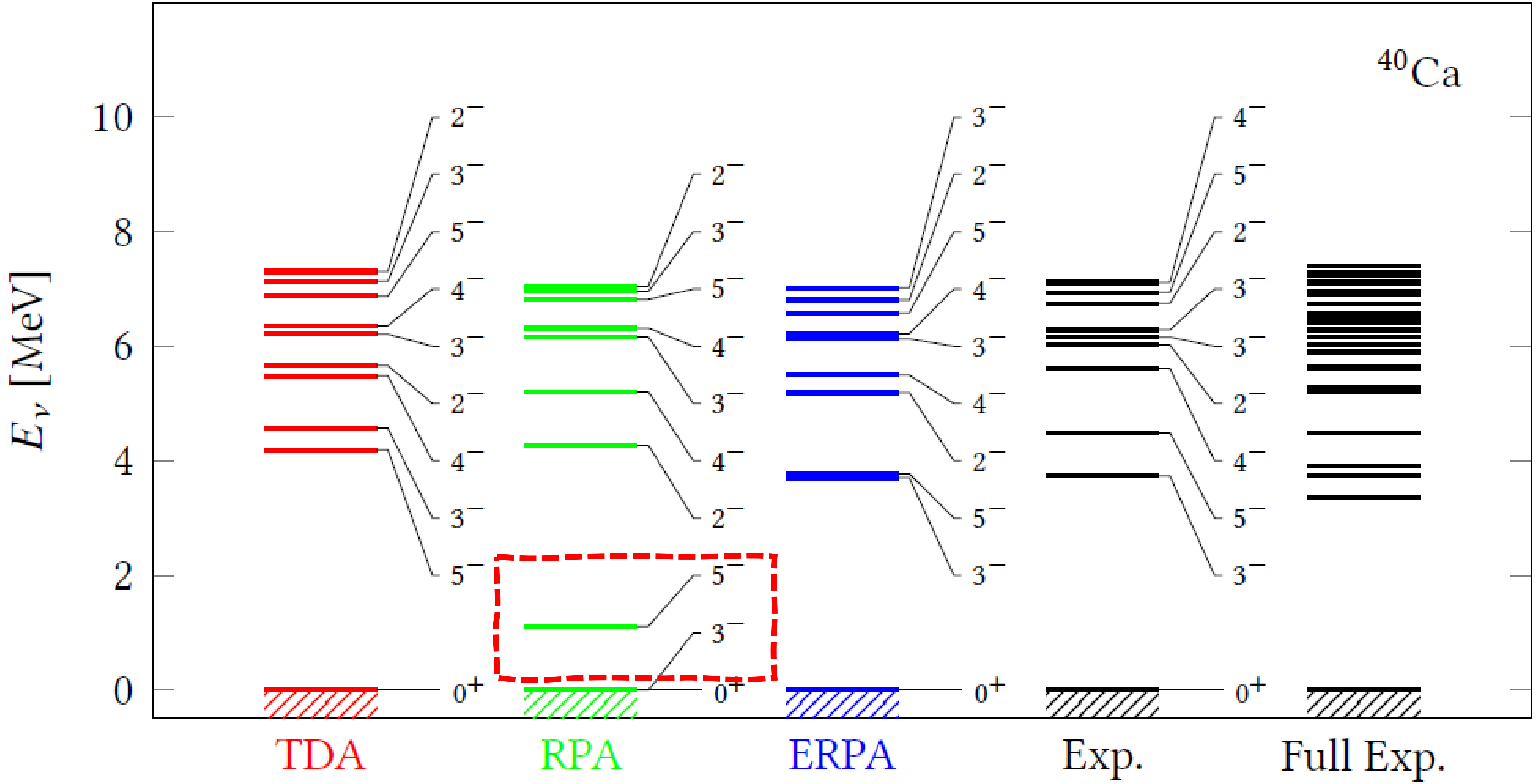
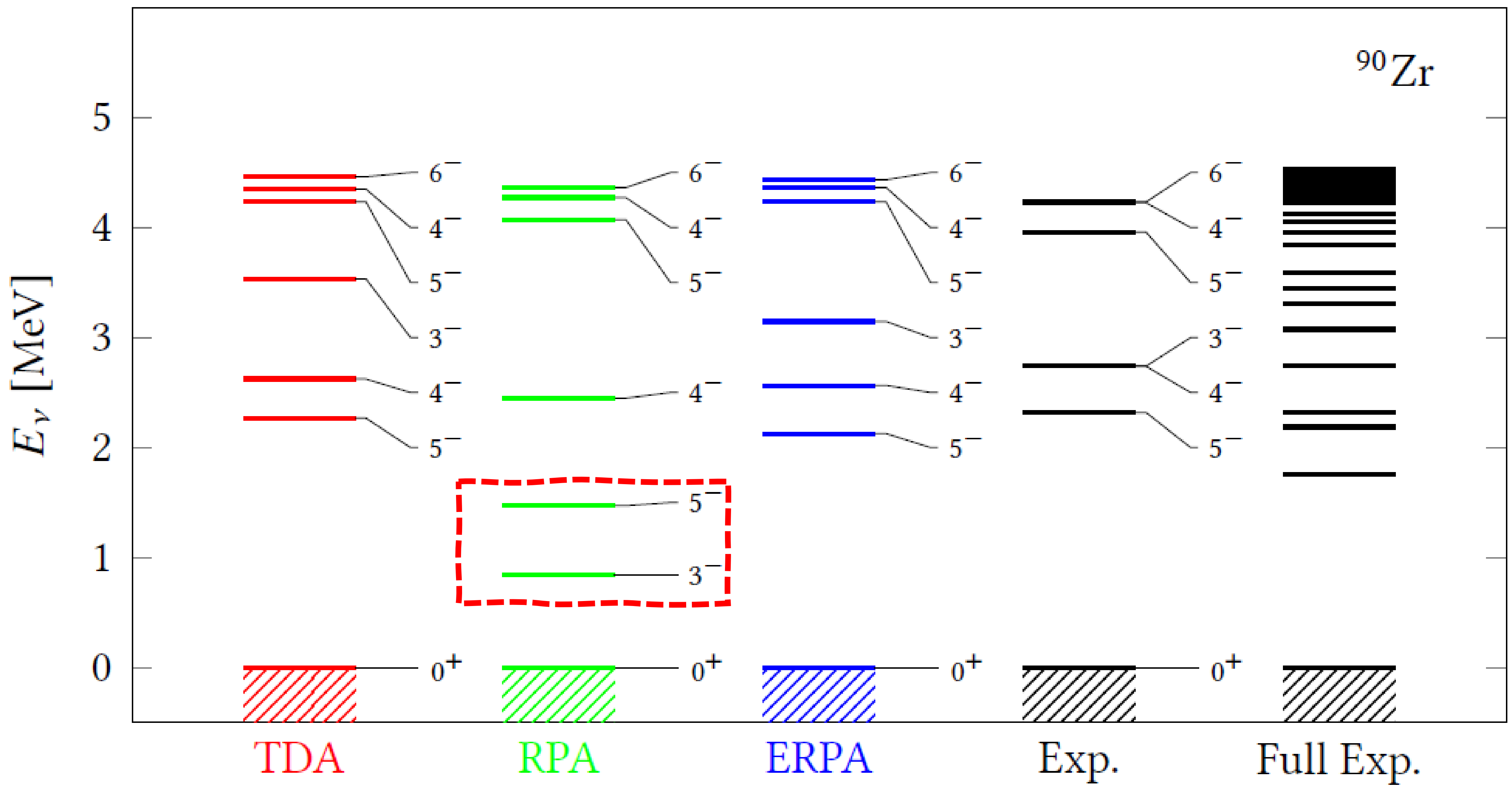
Low-energy spectra



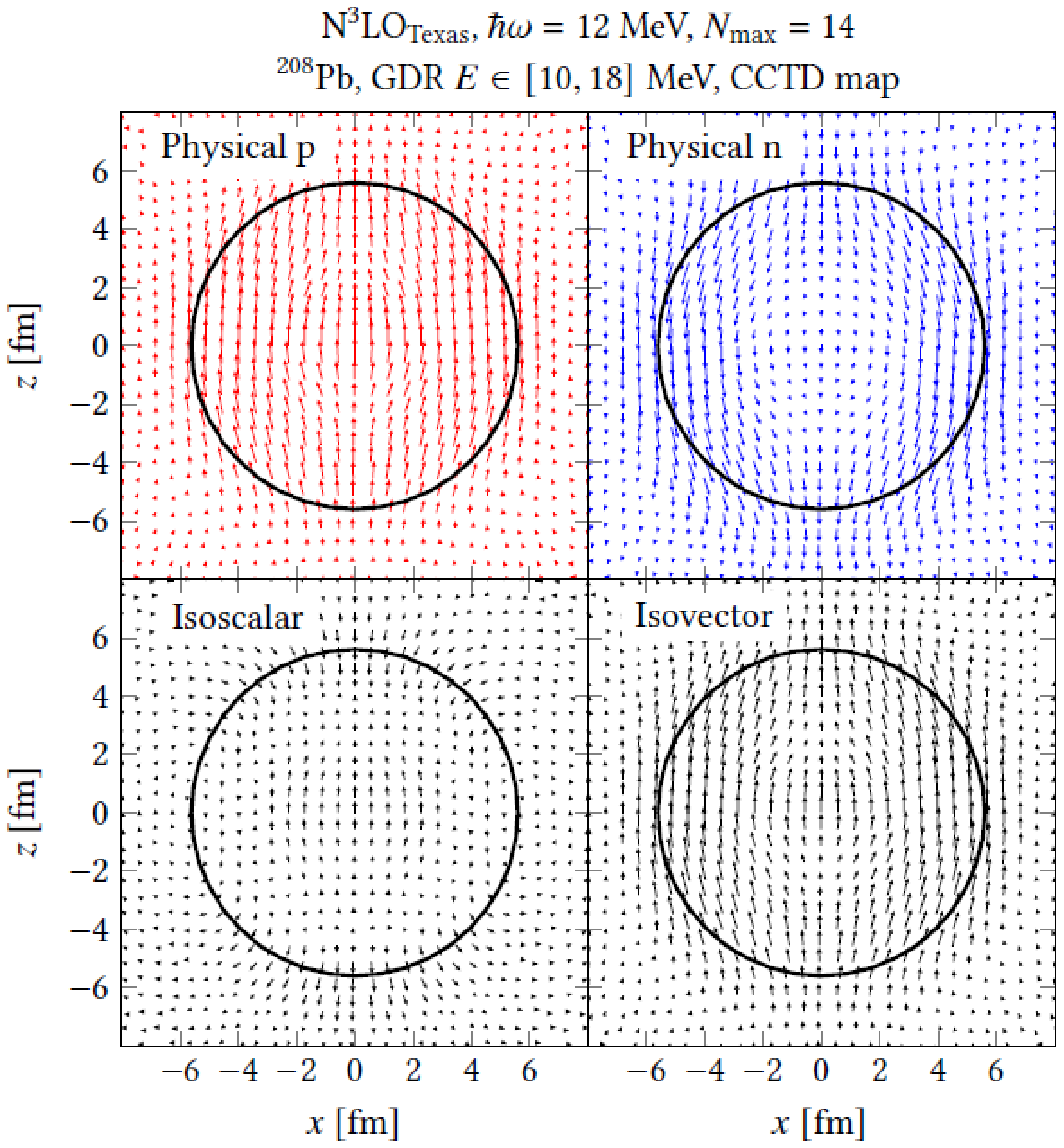
Low-energy spectra

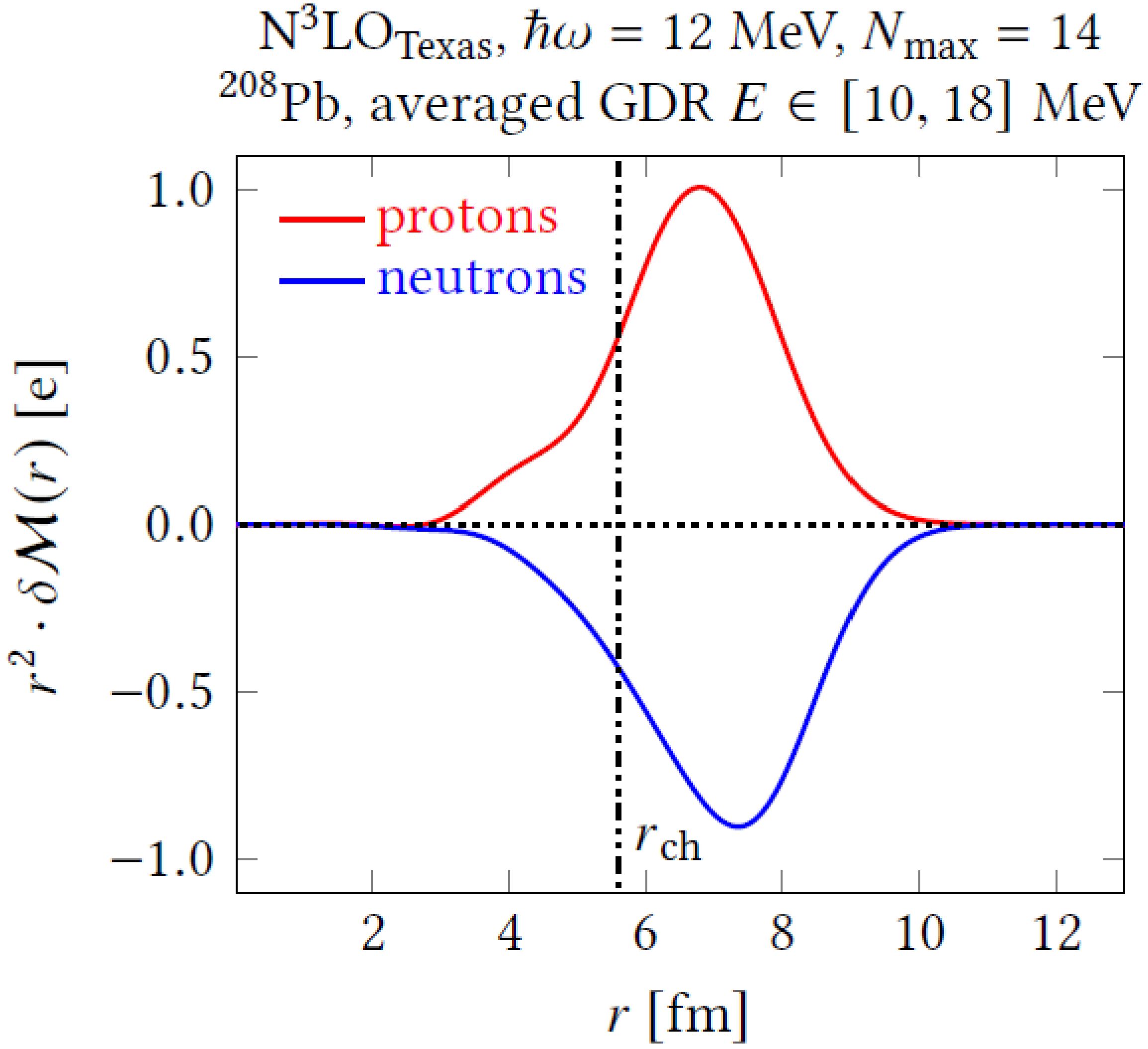


Low-energy spectra

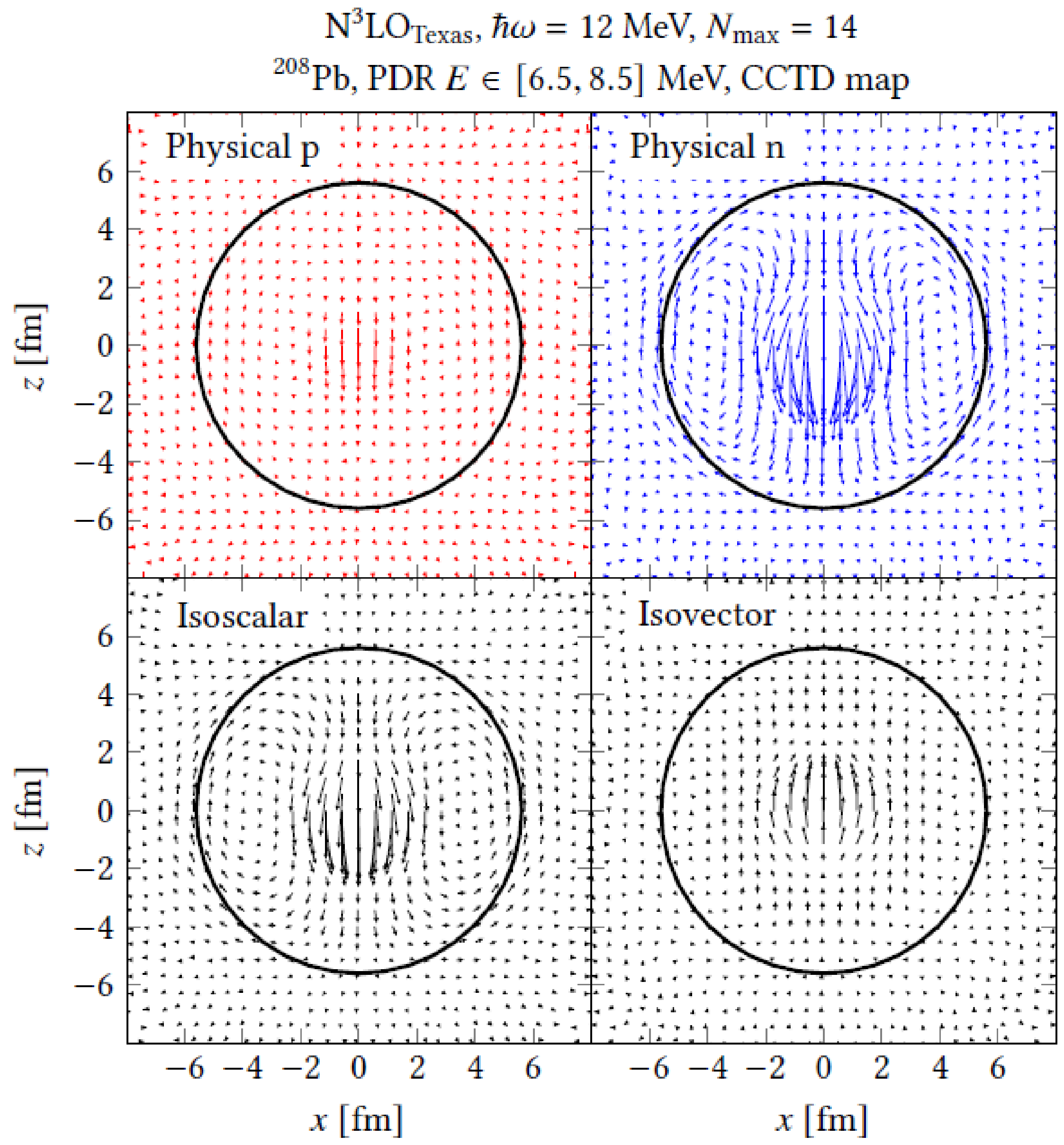


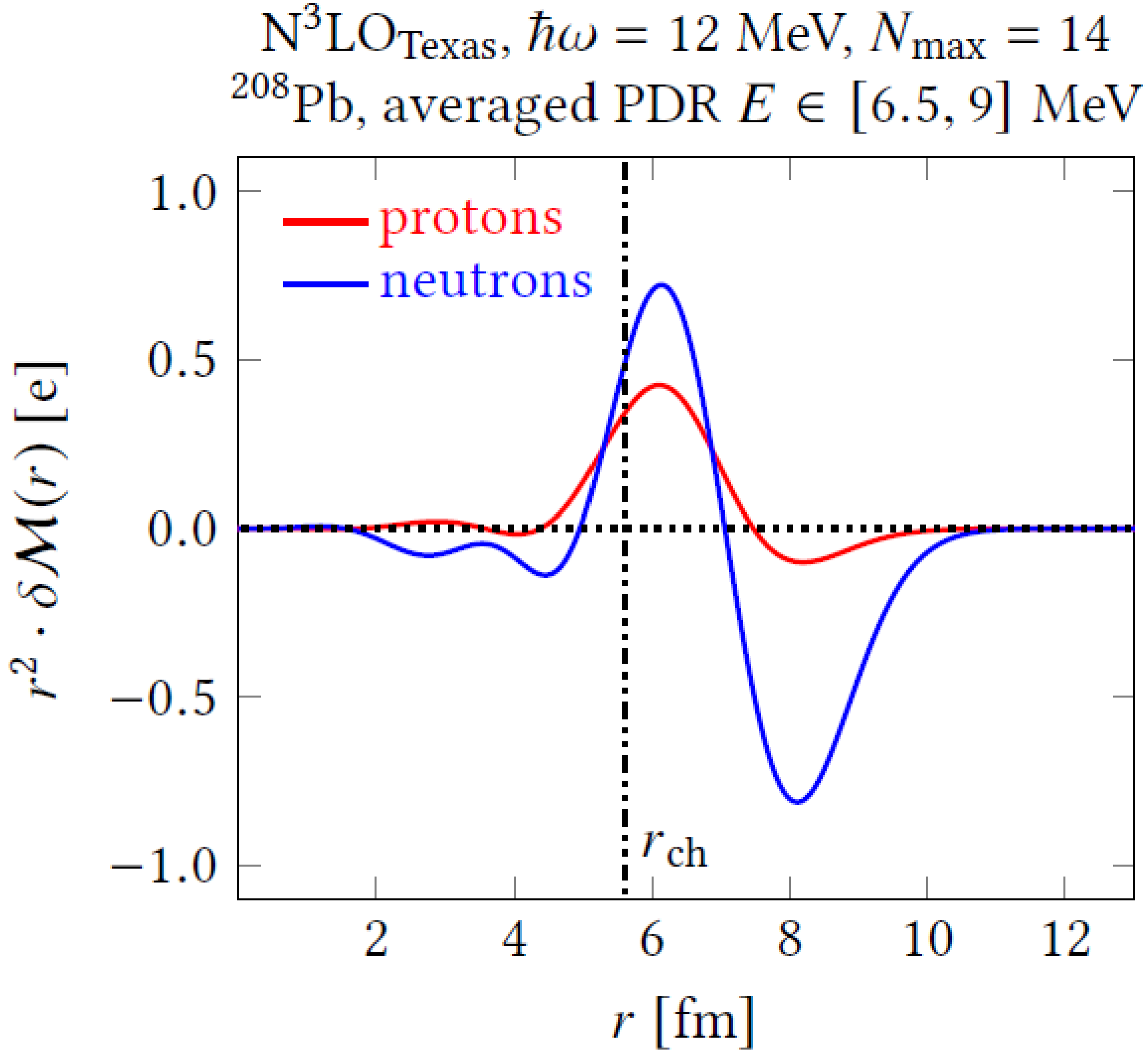
Structure of dipole states





Structure of dipole states





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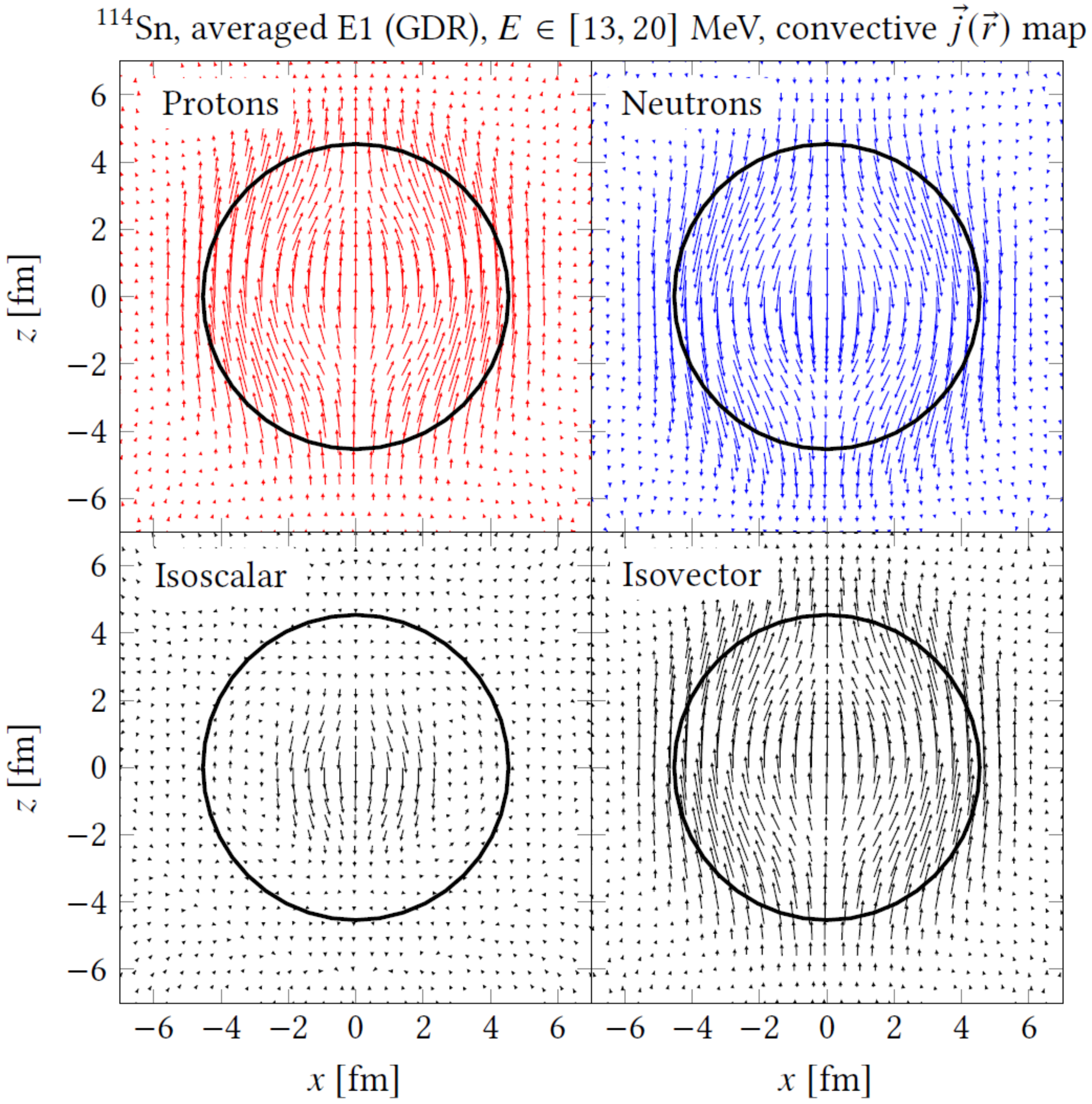

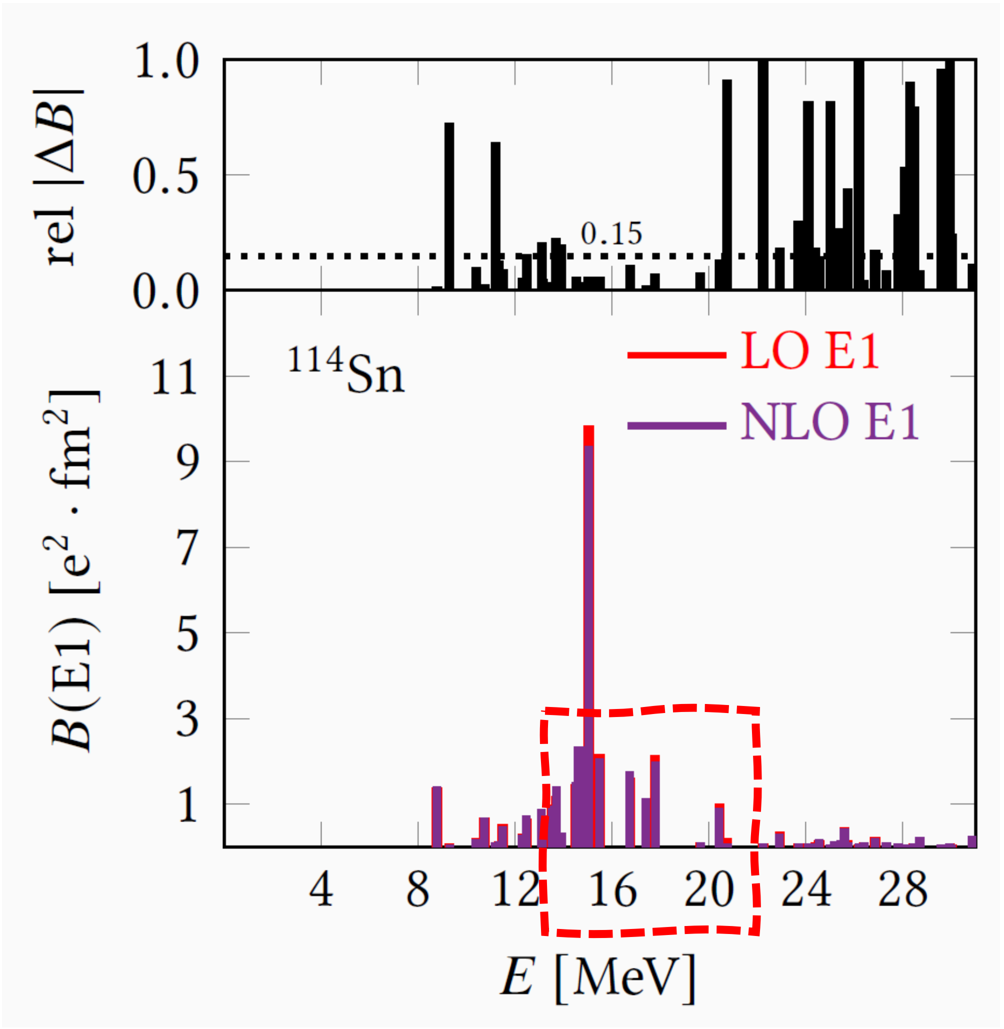

Structure of dipole states