

Evolution of the fission fragment angular momentum

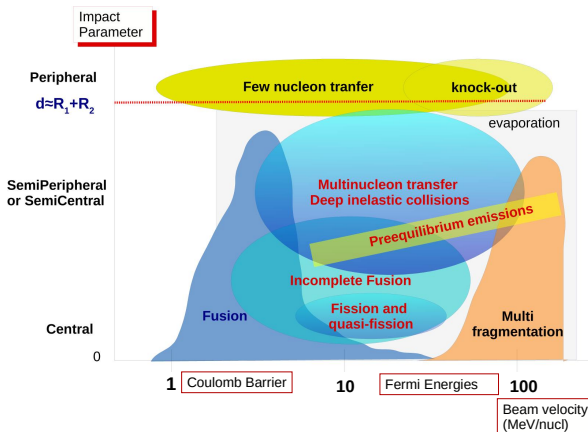
J. Randrup, P. Nadtochy, C. Schmitt, K. Mazurek

Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkley, USA
Istituto Nazionale di Fisica Nucleare, Sezione di Napoli, Naples, Italy
Institut Pluridisciplinaire Hubert Curien, Strasbourg, France
The Niewodniczanski Institute of Nuclear Physics - PAN, Kraków, Poland

September 3, 2026

Outline

- Langevin equations and fission dynamics
- Review of the results
- Current study
- Plans



Thanks to A. Maj, G. Casini

Stochastic approach

Dynamical effect

- path from equilibrium to scission slowed-down by the nuclear viscosity
- description of the time evolution of the collective variables like the evolution of Brownian particle that interacts stochastically with a "heat bath".
- Monte Carlo method for choosing the shape, initial angular momentum, type and energy of emitted particles....

Coupling to the evaporation

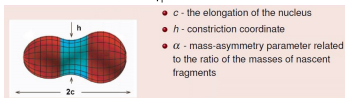
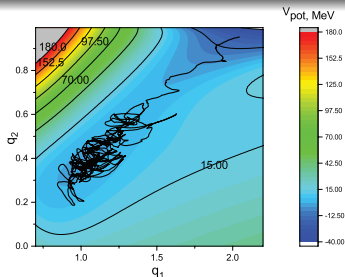
Pre and post- scission emission of neutrons, protons, α and γ .

Ingredients

Inertia ($[M^{-1}(\vec{q})]_{ij}$); Friction ($\gamma_i(t)$) and fluctuation (g_{ik})
 Macroscopic potential
 $(V(\vec{q}, K) \rightarrow F(\vec{q}, K) = V(\vec{q}, K) - \mathbf{a}(\vec{q}) T^2)$

Langevin equations

$$\begin{aligned}\frac{dq_i}{dt} &= \sum_j [M^{-1}(\vec{q})]_{ij} p_j \\ \frac{dp_i}{dt} &= -\frac{1}{2} \sum_{j,k} \frac{d[M^{-1}(\vec{q})]_{jk}}{dq_i} p_j p_k - \frac{dF(\vec{q}, K)}{dq_i} \\ &\quad - \sum_{j,k} \gamma_{ij}(\vec{q}) [M^{-1}(\vec{q})]_{jk} p_k + \sum_j g_{ij}(\vec{q}) \Gamma_j(t)\end{aligned}$$



Model Ingredients

Energies

- Potential energy in deformation space e.g: FRLDM + Wigner or LSD + Congruence

$$E_{\text{lsd}}(q) = b_{\text{vol}} \{1 - \kappa_{\text{vol}} I^2\} A + b_{\text{surf}} \{1 - \kappa_{\text{surf}} I^2\} A^{2/3} B_{\text{surf}}(q) \\ + b_{\text{curv}} \{1 - \kappa_{\text{curv}} I^2\} A^{1/3} B_{\text{curv}}(q) + \frac{3}{5} e^2 \frac{Z^2}{r_0^{\text{ch}} A^{1/3}} B_{\text{Coul}}(q)$$

$$I = (N - Z)/A$$

- Rotational energy:

$$E_{\text{rot}}(q, I, K) = \frac{\hbar^2 I(I+1)}{2J_{\perp}(q)} + \frac{\hbar^2 K^2}{2J_{\text{eff}}(q)} \quad \text{where } J_{\text{eff}}^{-1} = J_{\parallel}^{-1} - J_{\perp}^{-1} \text{ - the}$$

rigid-body moments of inertia.

- The friction parameter controlling coupling between K coordinate and the heat bath.

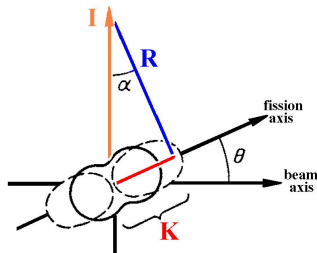
T. Dosing, J. Randrup, Nucl. Phys. A 433,215; J. Randrup, Nucl.Phys. A 383, 468

$$\gamma_K = \frac{1}{R_N R_{cm} \sqrt{2\pi^3 n_0}} \frac{J_{\parallel} |J_{\text{eff}}| J_R}{J_{\perp}^3}$$

where R_N - neck radius, R_{cm} - distance between the center of the nascent fragments, $n_0=0.0263 \text{ MeV zs fm}^{-4}$ - the bulk flux in standard nuclear mass and $J_R = M_0 R_{cm}^2/4$ for reflection symmetric shape.

Collective coordinates(4D)

- Description of the nuclear shape by elongation, neck and asymmetry – 3 parameters.
- K – spin about the fission (symmetry) axis



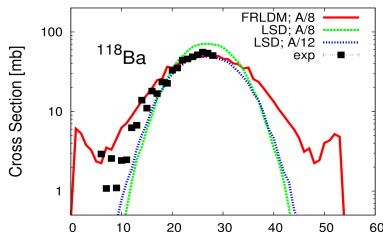
"Fission-fragment distributions within dynamical approach" K. M., P. N. Nadtochy, E. G. Ryabov, G. D. Adeev, Eur. Phys. J. A, 53 (2017) 79E

Energy Models study: LSD or FRLDM, Congruence

PHYSICAL REVIEW C 84, 014610 (2011)

Critical insight into the influence of the potential energy surface on fission dynamics

K. Mazurek,^{1,2,*} C. Schmitt,² J. P. Wieleccko,² P. N. Nadtochy,³ and G. Ademard²



EPJ Web of Conferences 17, 16006 (2011)

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Influence of the potential energy landscape on the fission dynamics

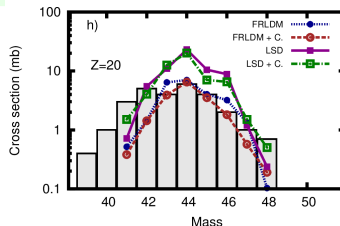
K. Mazurek^{1,2,*}, J.P. Wieleccko¹, C. Schmitt¹, and P. N. Nadtochy³

The change of the fission fragment mass distribution due to different PES taken to solve Langevin equations

PHYSICAL REVIEW C 88, 054614 (2013)

Examining fine potential energy effects in high-energy fission dynamics

K. Mazurek,^{1,*} C. Schmitt,² P. N. Nadtochy,³ M. Kmiecik,¹ A. Maj,¹ P. Wasiak,¹ and J. P. Wieleccko²



Vol. 44 (2013)

ACTA PHYSICA POLONICA B

FISSION FRAGMENT MASS DISTRIBUTION AS A PROBE OF THE SHAPE-DEPENDENT CONGRUENCE ENERGY TERM IN THE MACROSCOPIC MODELS* **

K. MAZUREK^a, C. SCHMITT^b, P. N. NADTOCHY^c, A. MAJ^a, P. WASIAK^a, M. KMECIK^a, B. WASILEWSKA^a

Exp: Y. Futami, et al., Nucl. Phys. A607, 85 (1996)

Nuclear level density

Importance

- Statistical models of decay
- Dynamical macroscopic models
- Astrophysics

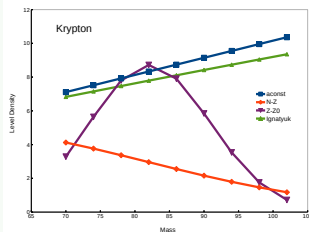
Level Density Formulas

- Mass dependent LD $a(A) = \alpha A$
- Deformation dependent (Ignatuk) $a(A) = \alpha A + \beta A^{2/3}$
- (N-Z) dependent $a(A, Z) = \alpha A / \exp[\beta(N - Z)^2]$
- $(Z - Z_0)$ dependent
 $a(A, Z) = \alpha A / \exp[\gamma(Z - Z_0)^2]$
 $Z_0 = 0.5042A / (1 + 0.0073A^{2/3})$
- Energy dependent (Reisdorf)
 $a(U, A, Z) = a(A) * [1 + dWg(Z, N) * f(U_{cor}) / (U_{cor})]$
 $f(x) = 1 - \exp(-\gamma * x)$

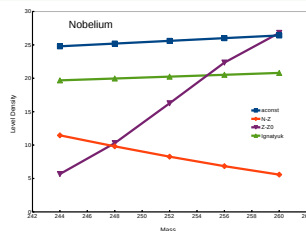
C. Schmitt, P.N. Nadtochy, K. Mazurek

Phys. Lett. B, 840 (2023) 137873

Light system



Heavy system

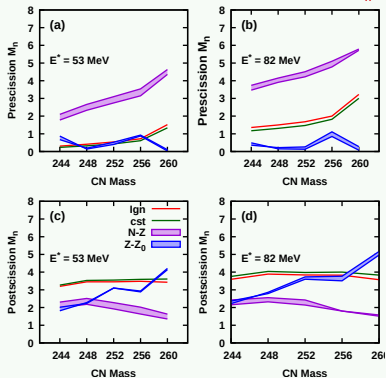


Nuclear level density - Testing both side of stability line

Fusion-fission ($^{40,48}\text{Ca} + ^{204,208}\text{Pb}$, $^{22}\text{Ne} + ^{238}\text{U}$)

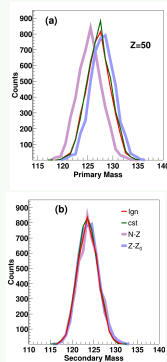
Pre and post scission neutron multiplicity as a LD probe

Heavy/superheavy compound nucleus - neutron deficient (M_n^{pre})



Mass distribution

Fission fragments -neutron rich (M_n^{post})



The $^{40}\text{Ca} + ^{208}\text{Pb}$ collisions which populate the No chain by fusion is very good candidates, since several projectile-target combinations can be readily accessible at current accelerator facilities. The further enhancement of this set of reactions with $^{22}\text{Ne} + ^{238}\text{U}$ involving a radioactive beam that is currently available. The PARIS+VAMOS detection system can be used

Angular momentum of fission fragments (with J. Randrup and P. Nadtochy)

Useful framework for discussion of fission fragment angular momenta:

Normal modes:

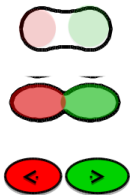
wriggling 

bending 

twisting 

Fission populates several distinct *rotational modes* whose presences reflect the physical *mechanisms* generating the angular momenta of the fragments

Generation of fission fragment angular momenta by nucleon exchange:



$$t_{\text{wrig}} < t_{\text{fiss}} \approx t_{\text{bend}} < t_{\text{twst}} \ll t_{\text{tilt}}$$

Wriggling will reach equilibrium

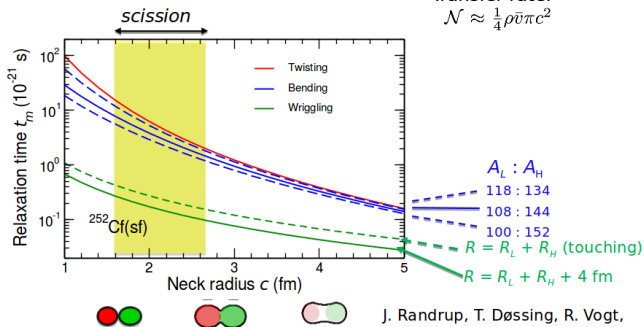
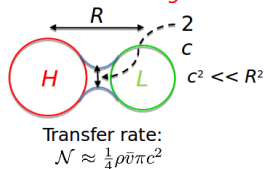
Bending probably has some presence;
it increases with the mass asymmetry

Twisting is unlikely to play a major role;
it grows more prominent with excitation

Angular momentum of fission fragments

Relaxation times
of dinuclear rotational modes caused by *nucleon exchange*

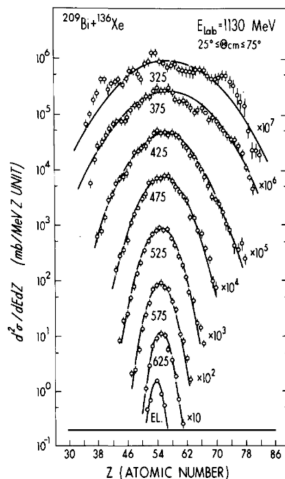
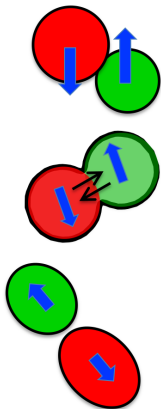
$$t_m = I_m / M_m \quad \begin{cases} M_{\text{wrig}} = m\mathcal{N}R^2 \\ M_{\text{bend}} = m\mathcal{N} \left[\left(\frac{\mathcal{I}_H R_L - \mathcal{I}_L R_H}{\mathcal{I}_L + \mathcal{I}_H} \right)^2 + c_{\text{ave}}^2 \right] \\ M_{\text{twst}} = m\mathcal{N}c_{\text{ave}}^2 \end{cases}$$



J. Randrup, T. Døssing, R. Vogt,
Phys. Rev. C **106**, 014609 (2022)

Multiple nucleon exchange was found to be a dominant mechanism in damped nuclear reactions

W.U. Schröder and J.R. Huizenga,
Damped Nuclear Reactions,
Treatise on Heavy-Ion Science II, p. 11:
(1984) Plenum Press, Ed: D.A. Bromley

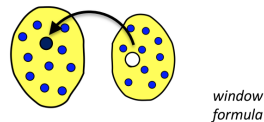


W.U. Schröder *et al.*, PRL **36**, 514 (1975)

The participating nucleons
are near the Fermi surface

=> The dissipation is **strong!**

W.U. Schröder *et al.*, PRL **44**, 308 (1980)



Multiple nucleon transfers
produce a dissipative force
affecting the linear and angular
momenta of the binary partners:

$Z_H, N_H, P_H, S_H, T_H, Z_L, N_L, P_L, S_L, T_L$

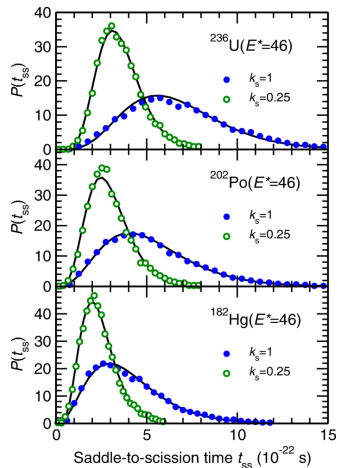
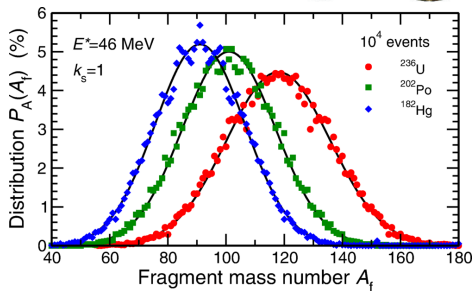
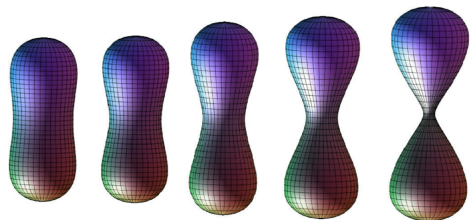
Nucleon Exchange Transport Theory

J. Randrup, Nucl. Phys. A **327**, 490 (1979)

J. Randrup, Nucl. Phys. A **383**, 468 (1983)

T. Døssing & J. Randrup, NPA **433**, 215 (1985)

Angular momentum of fission fragments



J. Randrup, P. Nadtochy, C. Schmitt, and K. Mazurek, Phys. Rev. C 113, 044605

Angular momentum of fission fragments

For each shape evolution, we wish to calculate the evolution of the distribution of the correlated fragment spins, $P(S_A, S_B)$

$$\begin{array}{c} \text{Rotational} \\ \text{energy:} \end{array}
 \quad
 \begin{array}{c} \text{overall} \\ \text{rotation} \end{array}
 \quad
 \begin{array}{c} \text{internal} \\ \text{rotations} \end{array}$$

$$E_{\text{rot}} = \frac{J_{\perp}^2}{2\mathcal{I}_{\perp}} + \frac{J_{\parallel}^2}{2\mathcal{I}_{\parallel}} + \frac{s_{\text{wrig}}^2}{2\mathcal{I}_{\text{wrig}}} + \frac{s_{\text{bend}}^2}{2\mathcal{I}_{\text{bend}}} + \frac{s_{\text{twst}}^2}{2\mathcal{I}_{\text{twst}}}$$

Normal spins: s_{wrig} , s_{bend} , s_{twst}

Fokker-Planck equation for $P(s_A, s_B; t)$

$$\frac{\partial}{\partial t} P = - \sum_i \frac{\partial}{\partial s_i} V_i P + \sum_{ij} \frac{\partial^2}{\partial s_i \partial s_j} D_{ij} P$$

Drift coefficient $V_i(\chi) = \sum_j M_{ij}(\chi) f_j$ determines the *mean* evolution of $s_i(t)$:

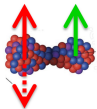
$$\frac{d\bar{s}_i}{dt} = V_i(\bar{\chi})$$

Mobility coefficients

Diffusion coefficient $D_{ij}(\chi) = M_{ij}(\chi) T$ generates *correlated spin fluctuations* $\sigma_{ij}(t)$:

$$\sigma_{ij} \equiv \langle (s_i - \bar{s}_i)(s_j - \bar{s}_j) \rangle : \quad \frac{d}{dt} \sigma_{ij} = 2D_{ij}(\bar{\chi}) - \sum_k [\nu_{ik} \sigma_{kj} + \nu_{jk} \sigma_{ki}]$$

Angular momentum of fission fragments



Results for a single shape evolution:

Spin components perpendicular to the fission axis

Wriggling & bending

$$\mathcal{I}_{\text{wrig}} = \frac{(\mathcal{I}_A^\perp + \mathcal{I}_B^\perp) \mathcal{I}_R}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp + \mathcal{I}_R}$$

$$\bar{\sigma}_{\text{wrig}}^2 = \mathcal{I}_{\text{wrig}} T$$

$$\bar{\sigma}_{\text{wrig}, \text{bend}} = 0$$

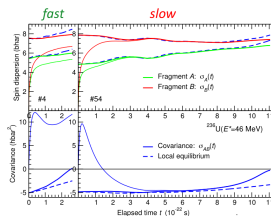
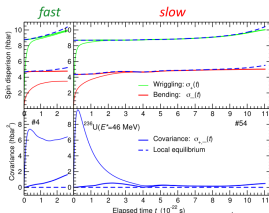
$$\mathcal{I}_{\text{bend}} = \frac{\mathcal{I}_A^\perp \mathcal{I}_B^\perp}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp}$$

$$\bar{\sigma}_{\text{bend}}^2 = \mathcal{I}_{\text{bend}} T$$

Individual fragments:

$$\mathbf{s}_A = \frac{\mathcal{I}_A^\perp}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp} \mathbf{s}_{\text{wrig}} + \mathbf{s}_{\text{bend}}$$

$$\mathbf{s}_B = \frac{\mathcal{I}_B^\perp}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp} \mathbf{s}_{\text{wrig}} - \mathbf{s}_{\text{bend}}$$



$$M_{\pm} = \frac{1}{4} m \rho \bar{v} \pi c^2 \begin{pmatrix} R^2 & R\delta \\ R\delta & \delta^2 + c_{\text{ave}}^2 \end{pmatrix}$$

$$M_{AB}^\perp = \frac{1}{4} m \rho \bar{v} \pi c^2 \begin{pmatrix} a^2 + c_{\text{ave}}^2 & ab - c_{\text{ave}}^2 \\ ab - c_{\text{ave}}^2 & b^2 + c_{\text{ave}}^2 \end{pmatrix}$$

$$\delta = (a\mathcal{I}_B - b\mathcal{I}_A)/(\mathcal{I}_A + \mathcal{I}_B) \text{ small}$$

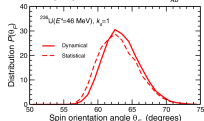
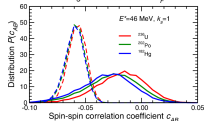
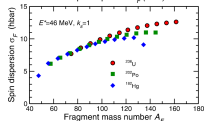
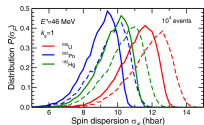
Jørgen Randrup

Fission 2026

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J. Randrup, P. Nadtochy, C. Schmitt, and K. Mazurek, Phys. Rev. C 113, 044605

Angular momentum of fission fragments



The fragment spins relax to the local equilibrium values throughout most of the evolution from saddle towards scission.

But near scission the spin equilibrium magnitudes grow rapidly (due to the increasing temperature), while the mobility coefficients decrease (because the neck closes), so the dynamical spin evolution effectively freezes out before scission, resulting in smaller magnitudes

Gating on fragment mass may be informative

The spin-spin correlation is quite small in equilibrium and it is even smaller when generated dynamically. The fragment spins are practically uncorrelated (J. Wilson et al., Nature 590 (2021))

The spin orientation angle θ_F is about 70° , both in equilibrium and dynamically
Can be measured !!!

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Summary and plans

- The fission description with Langevin type transport equation solving gives good estimation of various observables: mass, charge, TKE, angular distributions, emitted particles multiplicities and spectra and so on.
- The study of the correlation between mass and angular distributions allows to distinguish between fission and quasifission reactions.
- A single fission reaction makes available simultaneously two observables, viz. M_{pre} and M_{post} , which scan different regions of the (N , Z) phase space with the CN and fragments, respectively.
- **The angular momentum of the fission fragments is estimated, including the effects of exotic rotation on the path from the saddle to the scission point. The spin dispersion and space orientation have been tested**

Plans

A) Follow the study of angular momentum of fission fragments by adding particle emission.

B) Investigation of the GDR emitted from FF and comparison with U+Be of PARIS+VAMOS data.

