

QUANTUM COMPUTATION FOR NUCLEAR STRUCTURE WITH A LOW-DEPTH VARIATIONAL CIRCUIT

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BASIC INGREDIENTS OF QUANTUM COMPUTER



- Classical computer works on *bits*, quantum computers have *qubits*
- Qubits are two-level quantum systems of the form $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$
- α & β are complex numbers ($\alpha^*\alpha + \beta^*\beta = 1$) – in principle store much more information in these continuous numbers than in discrete bits.
- **Qubits** and **quantum gates** are the basic ingredients of QC
- Quantum gates changes the state of a qubit
- There are single-qubit gates and multi-qubit gates

Pauli-Y (Y)		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)		$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)		$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)		$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$
Controlled Not (CNOT, CX)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z (CZ)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
SWAP		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

VARIATIONAL QUANTUM EIGENSOLVER



- Quantum algorithms for many-body systems:

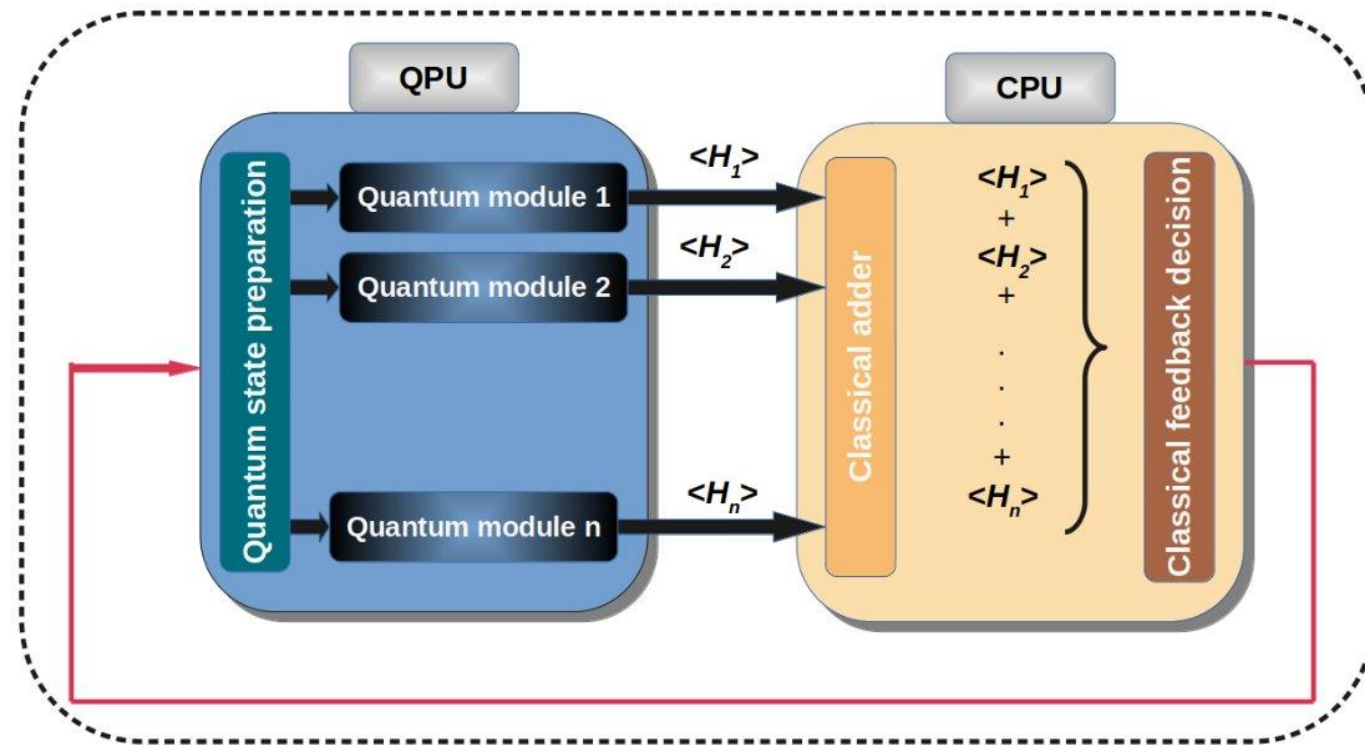
- Quantum Phase Estimation (QPE)
- Trotterized time evolution
- Variational Quantum Eigensolver (VQE)

- NISQ-era:

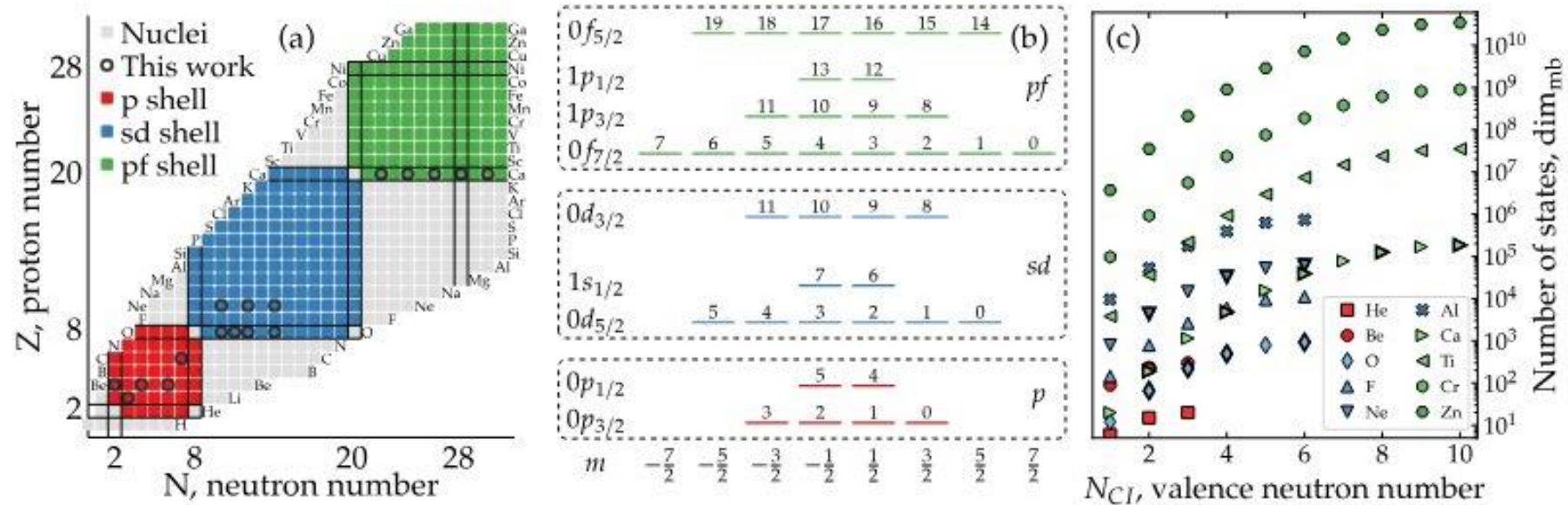
- Noisy qubits
- Intermediate number of qubits
- Limited circuit depth
- Variational algorithms are preferred

- VQE:

- Hybrid quantum–classical algorithm
- Uses a **parameterised quantum circuit (ansatz)** to prepare trial states.
- A classical optimiser updates circuit parameters to **minimise the expected energy**.



NUCLEAR SHELL MODEL



A. Pérez-Obiol et al., Sci. Rep. 13, 12291 (2023)

- Single particle state $(n, l, j, j_z, t_z) \rightarrow$ Qubit:
 - p -shell $\rightarrow$ 12 qubits
 - sd -shell $\rightarrow$ 24 qubits
 - pf -shell $\rightarrow$ 40 qubits

$$H = \sum_i \epsilon_i \hat{a}_i^\dagger \hat{a}_i + \frac{1}{2} \sum_{i,j} V_{ijkl} \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k \hat{a}_l$$

$$\hat{a}_i^\dagger = \frac{1}{2} \left(\prod_{j=0}^{i-1} -Z_j \right) (X_i - iY_i)$$

$$\hat{a}_i = \frac{1}{2} \left(\prod_{j=0}^{i-1} -Z_j \right) (X_i + iY_i)$$



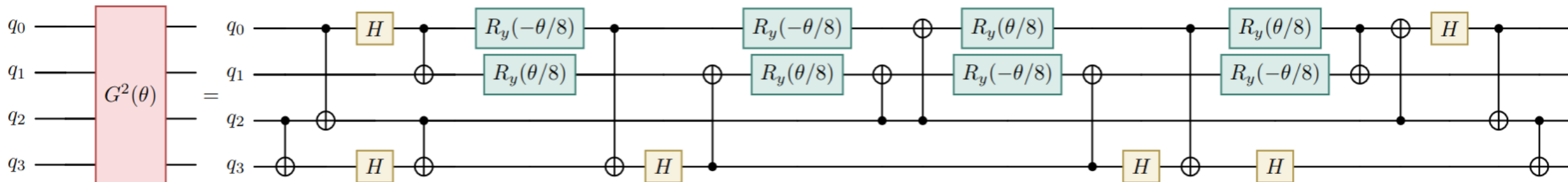
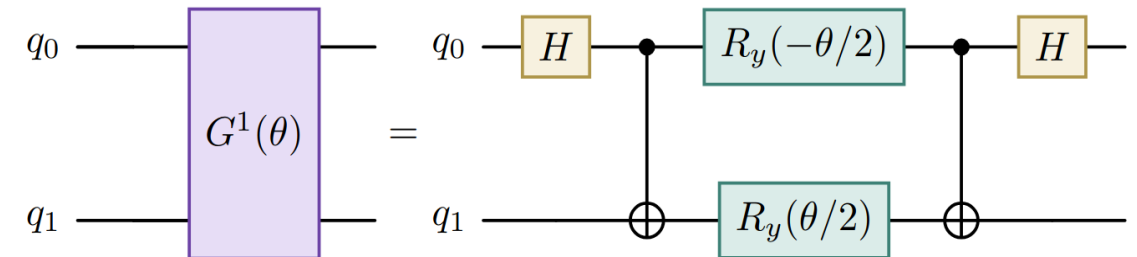
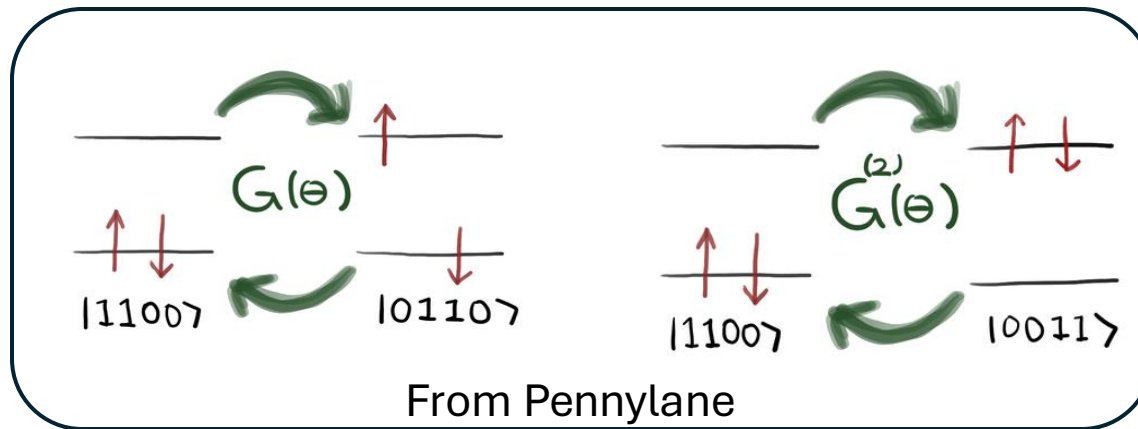
UNITARY COUPLED CLUSTER (UCC) ANSATZ:

- The UCC involves cluster operators that act on a reference state $|\psi_0\rangle$

$$|\psi(\theta)\rangle = e^{i(\hat{T}(\theta) - \hat{T}^\dagger(\theta))} |\psi_0\rangle$$

- These operators can be decomposed into singles, doubles,.. etc.

$$\hat{T} = \hat{T}_1 + \hat{T}_2 + \dots; \hat{T}_1 = \sum_{i,\alpha} \theta_i^\alpha a_i^\dagger a_\alpha; \hat{T}_2 = \sum_{ij,\alpha\beta} \theta_{ij}^{\alpha\beta} a_i^\dagger a_j^\dagger a_\alpha a_\beta$$

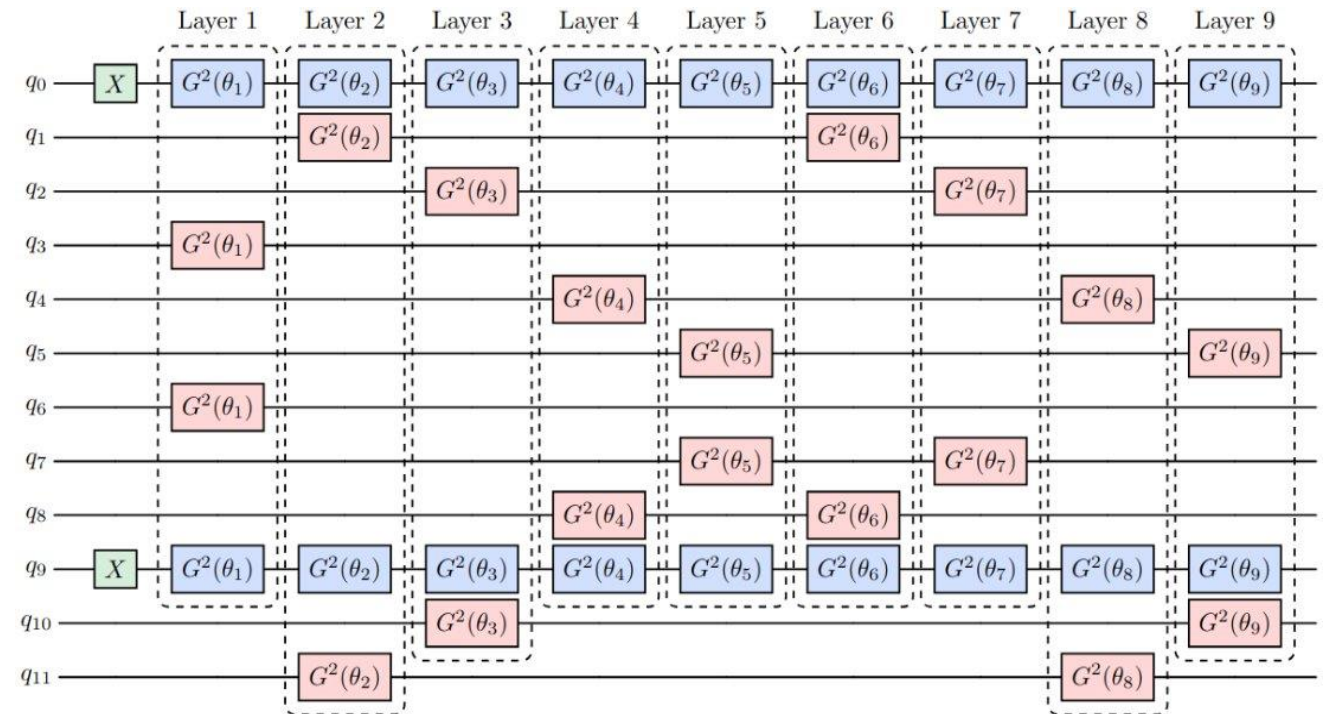


EXAMPLE OF UCC(D) ANSATZ



- $6\text{Li} (1^+)$: $M = 0 \rightarrow 10$ SDs
- p-shell: 12 single particle states

	Proton				Neutron			
$0p_{1/2}$		4	5		10	11		
$0p_{3/2}$	0	1	2	3	6	7	8	9
m	-3/2	-1/2	1/2	3/2	-3/2	-1/2	1/2	3/2

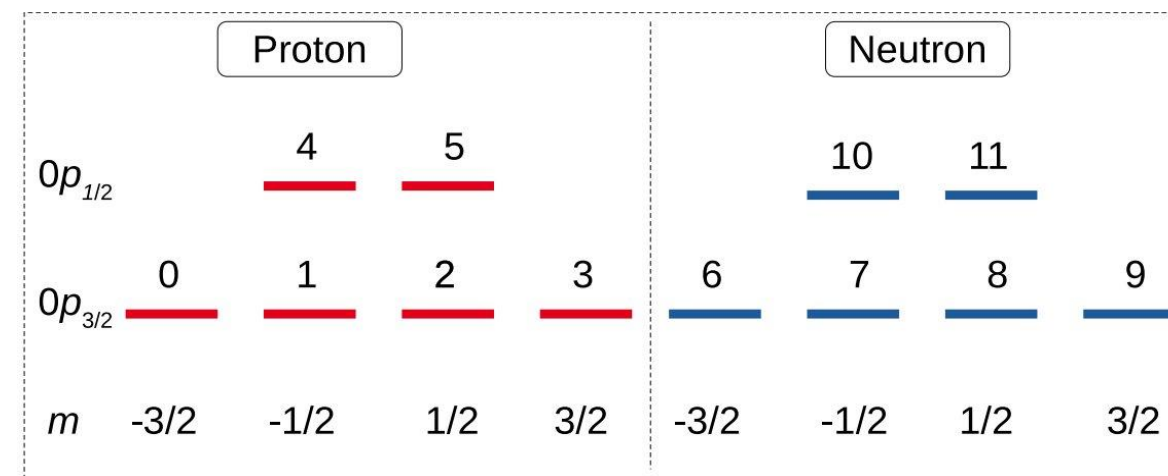


AN ALTERNATE QUBIT MAPPING STRATEGY

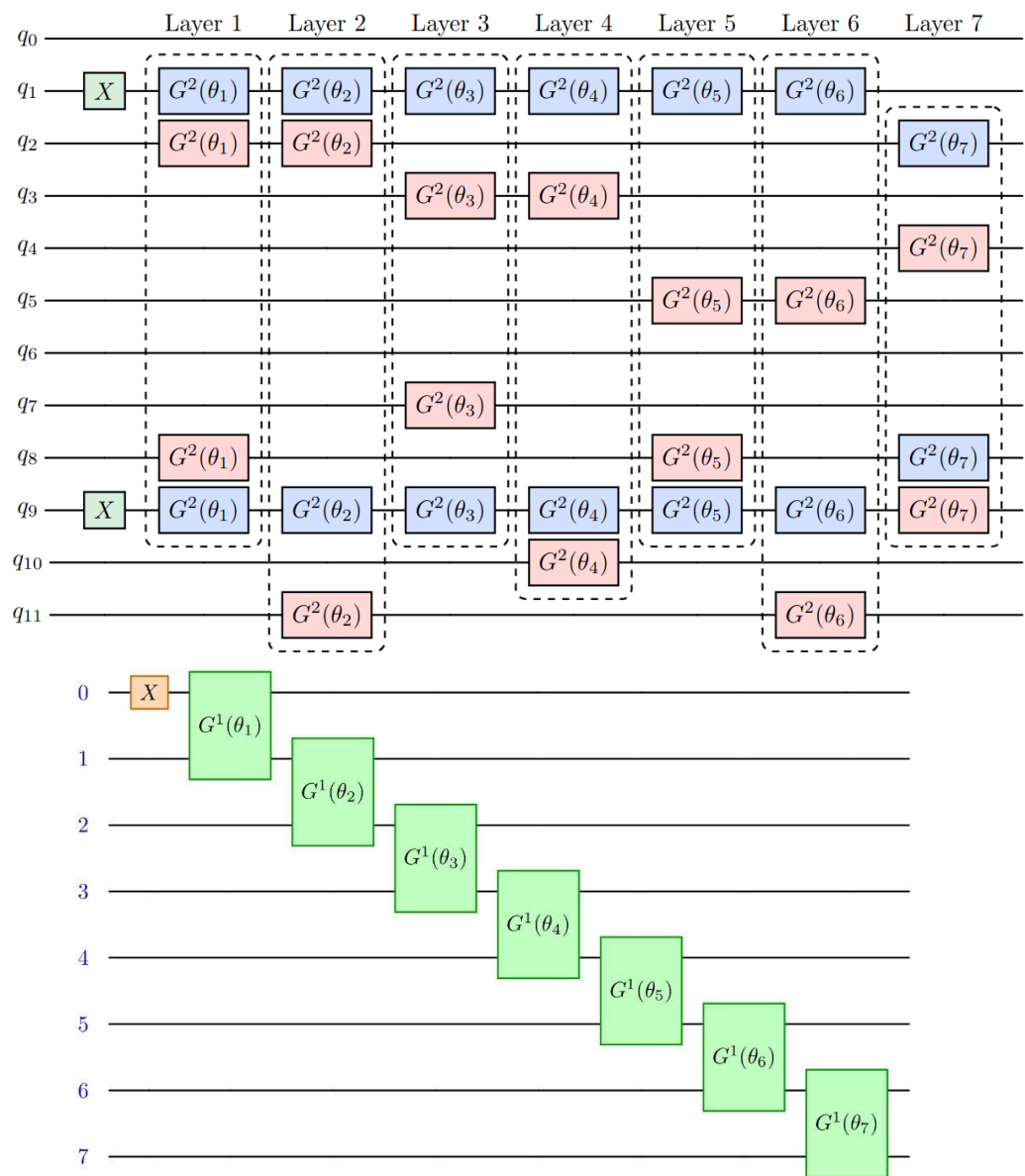


- Deep circuits are required for mid shell nuclei if we follow single particle state to qubit mapping.
- Many body configuration (SD) to qubit mapping can be done with low-depth circuit.

Qubits	${}^6\text{Li} (1^+)$	${}^7\text{Li} (3/2^-)$	${}^8\text{Li} (2^+)$	${}^9\text{Li} (3/2^-)$
0	$ 1, 9\rangle$	$ 1, 8, 9\rangle$	$ 1, 8, 9, 11\rangle$	$ 1, 7, 8, 9, 11\rangle$
1	$ 2, 8\rangle$	$ 1, 9, 11\rangle$	$ 2, 7, 8, 9\rangle$	$ 1, 8, 9, 10, 11\rangle$
2	$ 2, 11\rangle$	$ 2, 7, 9\rangle$	$ 2, 7, 9, 11\rangle$	$ 2, 6, 8, 9, 11\rangle$
3	$ 3, 7\rangle$	$ 2, 8, 11\rangle$	$ 2, 8, 9, 10\rangle$	$ 2, 7, 8, 9, 10\rangle$
4	$ 3, 10\rangle$	$ 2, 9, 10\rangle$	$ 2, 9, 10, 11\rangle$	$ 2, 7, 9, 10, 11\rangle$
5	$ 4, 9\rangle$	$ 3, 6, 9\rangle$	$ 3, 6, 8, 9\rangle$	$ 3, 6, 7, 8, 9\rangle$
6	$ 5, 8\rangle$	$ 3, 7, 8\rangle$	$ 3, 6, 9, 11\rangle$	$ 3, 6, 7, 9, 11\rangle$
7	$ 5, 11\rangle$	$ 3, 7, 11\rangle$	$ 3, 7, 8, 11\rangle$	$ 3, 6, 8, 9, 10\rangle$
8	–	$ 3, 8, 10\rangle$	$ 3, 7, 9, 10\rangle$	$ 3, 6, 9, 10, 11\rangle$
9	–	$ 3, 10, 11\rangle$	$ 3, 8, 10, 11\rangle$	$ 3, 7, 8, 10, 11\rangle$
10	–	$ 4, 8, 9\rangle$	$ 4, 8, 9, 11\rangle$	$ 4, 7, 8, 9, 11\rangle$
11	–	$ 4, 9, 11\rangle$	$ 5, 7, 8, 9\rangle$	$ 4, 8, 9, 10, 11\rangle$
12	–	$ 5, 7, 9\rangle$	$ 5, 7, 9, 11\rangle$	$ 5, 6, 8, 9, 11\rangle$
13	–	$ 5, 8, 11\rangle$	$ 5, 8, 9, 10\rangle$	$ 5, 7, 8, 9, 10\rangle$
14	–	$ 5, 9, 10\rangle$	$ 5, 9, 10, 11\rangle$	$ 5, 7, 9, 10, 11\rangle$



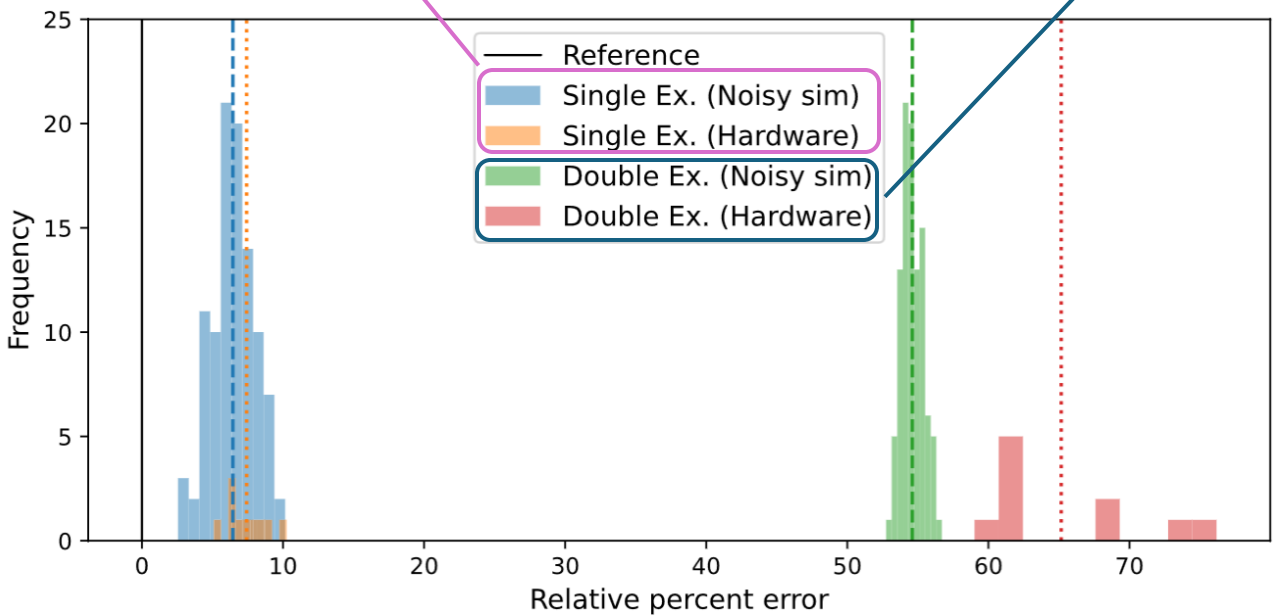
COMPARISON BETWEEN SP- AND SD-MAPPING



Nu- cle- us (J^π)	Ansatz	Qubits	Parameters	Pauli terms	1Q gates	2Q gates	Depth
${}^6\text{Li}$ (1^+)	Single Ex	8	7	65	29	14	36
	Single Ex. (Transpiled)	8	7	65	197	14	134
	Single Ex. (Optimized)	8	7	65	113	14	67
	Double Ex	12	11	207	100	98	132
	Double Ex. (Transpiled)	12	11	207	1776	488	1027
	Double Ex. (Optimized)	12	11	207	753	227	530

SD-mapping

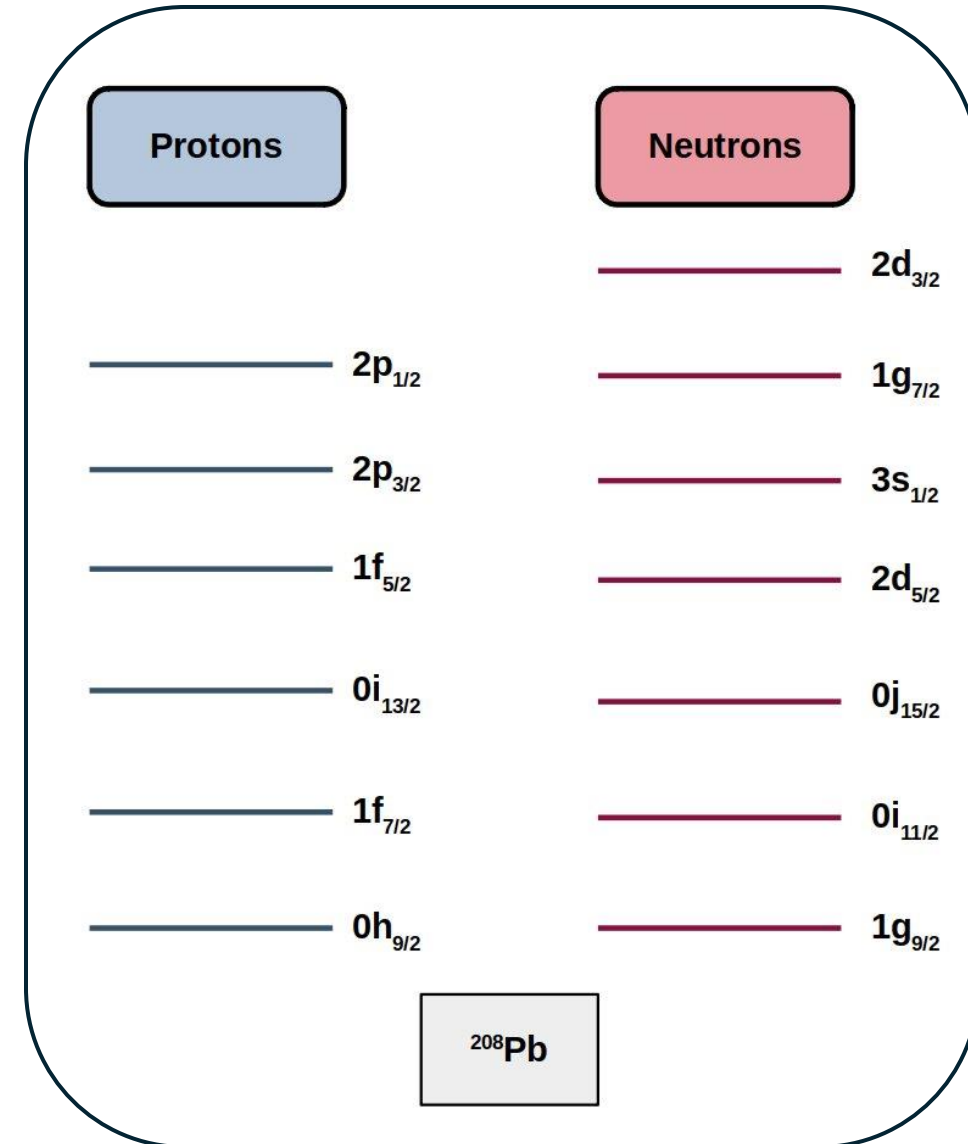
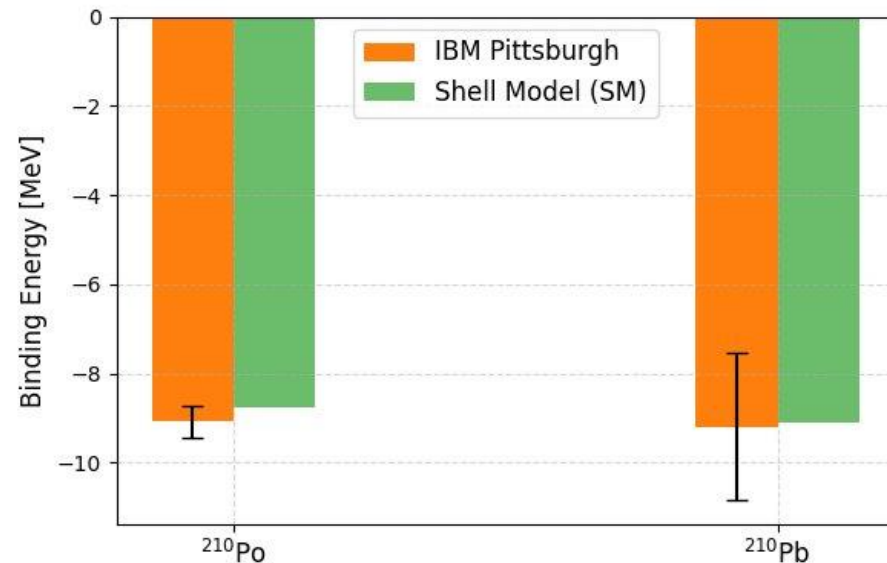
SP-mapping



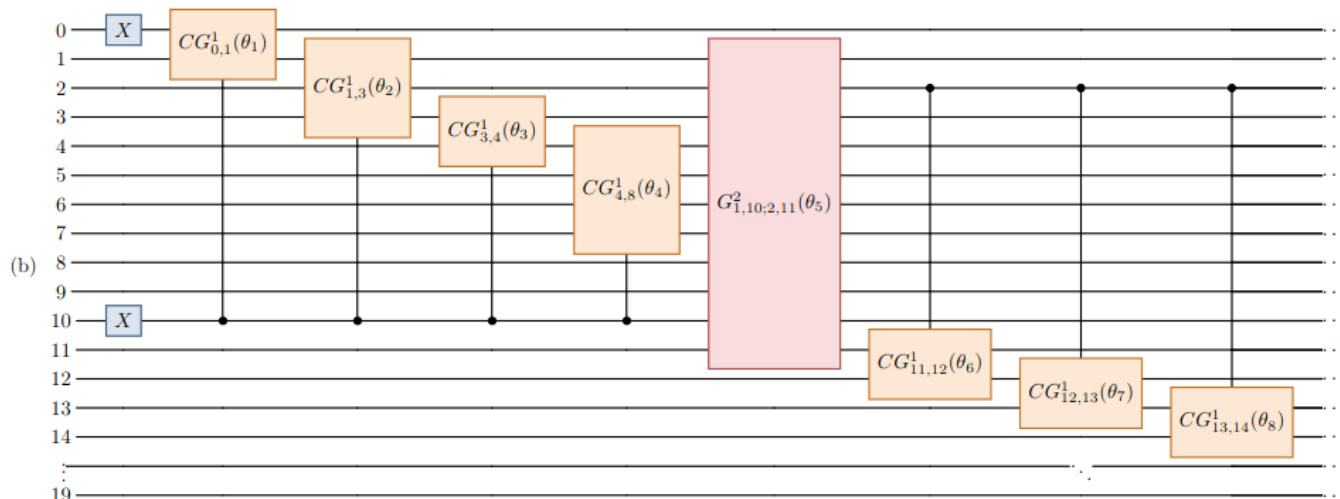
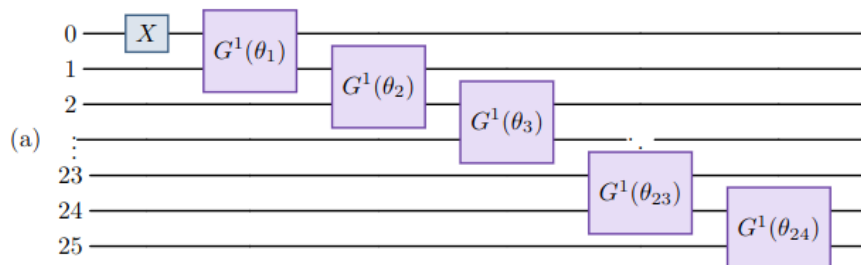
HEAVY TWO-NUCLEON SYSTEMS



- Two higher mass nuclei, ^{210}Po and ^{210}Pb , beyond ^{208}Pb core.
- Require 44 proton single proton states and 58 neutron single particle states: 44 and 58 qubits following SP --> qubit mapping
- Require 62 and 99 qubits following SD --> qubit mapping
- Considered SDs that are combinations of two time-reversed single particle states having the same $|jz|$ with opposite signs: **^{210}Po (22 qubit)** and **^{210}Pb (29 qubit)**

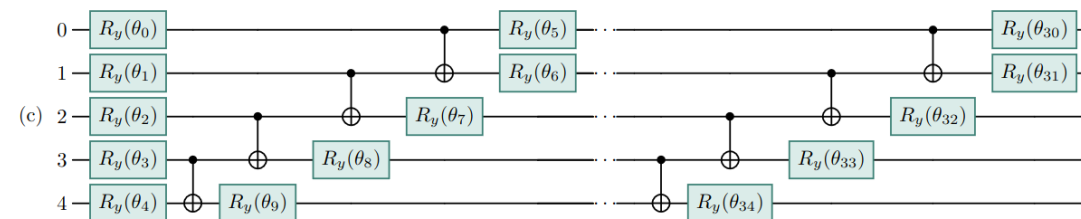


REDUCING NUMBER OF QUBIT WITHIN SD-MAPPING



- SD-mapping:
 - SD to qubit
 - pSD + nSD to qubit
 - SD to qubit state

- ^{10}B ($M = 3$) $\rightarrow$ number of SDs = 26
 - SD-mapping = 26 qubits
 - pnSD-mapping = 20 qubits
 - cSD-mapping = 5 qubits



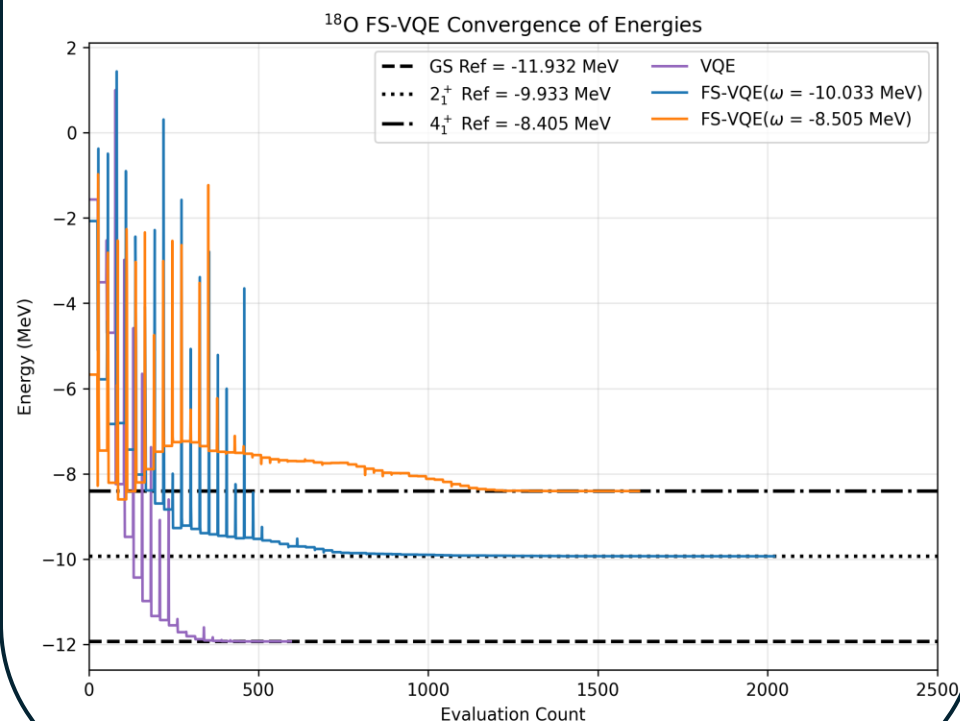
Nucleus (J^π)	Mapping	Qubits	Params	Pauli	Comm. Pauli	1Q	2Q	Depth
^{10}B (3^+)	SD (Noiseless sim.)	26	25	451	4	101	50	102
	SD (Noisy sim.)	26	25	451	4	391	50	229
	SD (Hardware)	26	25	451	4	364	50	211
	pnSD (Noiseless sim.)	20	25	1271	530	122	166	258
	pnSD (Noisy sim.)	20	25	1271	530	1071	305	746
	pnSD (Hardware)	20	25	1271	530	1143	314	762
	cSD (Noiseless sim.)	5	35	528	122	35	24	21
	cSD (Noisy sim.)	5	35	528	122	169	24	61
	cSD (Hardware)	5	35	528	122	169	24	61

C. Sarma and P. D. Stevenson,
arXiv:2605.30261

SOME OTHER WORKS



Extend the application of cSD mapping within FS-VQE method for nuclear excited states



Low T -count preparation of nuclear eigenstates with tensor networks

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(Dated: March 27, 2026)

arXiv:2603.11156

Fault-tolerant quantum algorithms for simulating atomic nuclei

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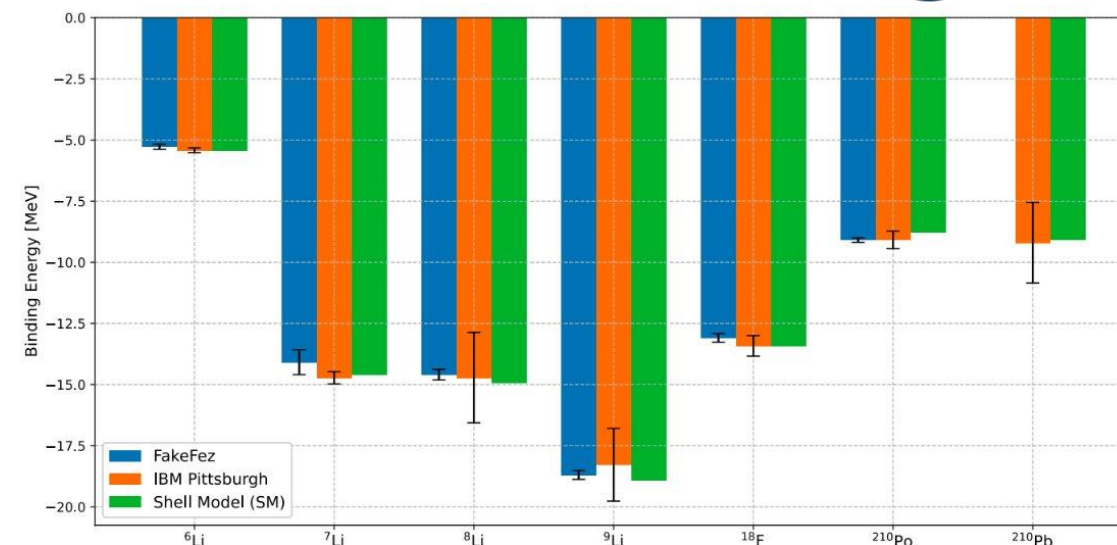
(Dated: July 24, 2026)

arXiv:2607.21563

S U M M A R Y



- Demonstrated feasibility of SD-based qubit mapping on several nuclei (from light Li isotopes to heavier nuclei such as ^{210}Po and ^{210}Pb) with successful simulations on both noisy simulators and IBM quantum hardware.
- Compact SD mapping is important from scaling point of view and can be extended to evaluate excited states.
- As quantum computation start transitioning from NISQ- to FTQC-era, new algorithms are being developed.



Thanks to: UK Gov agencies for funding

Collaborators: Paul Stevenson, Joe Gibbs, Lloyd La Ronde, PsiQuantum and STFC Hartree Team.

Thank you everyone for your kind attention!