Shape coexistence and deformation fluctuations from shape invariants

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Excelencia María de Maeztu 04/2025-03/2031

Deformation and shape coexistence

Deformation is everywhere:

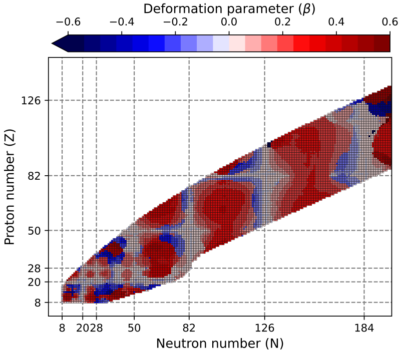
- Magic nuclei are spherical
- Deformation is enhanced mid-shell (most nuclei)

Quadrupole deformation:

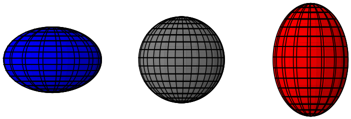
- Most extended shape
- Prolate: elongated sphere
- Oblate: flattened sphere

Shape coexistence:

- Different shapes coexist within the **same nucleus** at different energies
- Present in most nuclei



P. Möller, et al. Atom. Data Nucl. Data Tabl. 109-110 (2016)



Oblate Spherical Prolate

Shape invariants

Deformation is characterized in the intrinsic frame: (β, γ)

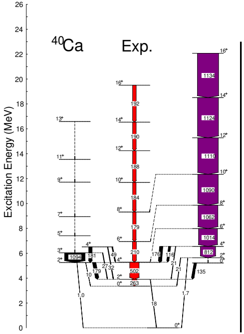
However, measurements are done in lab frame: $(B(E2), Q_s)$

Conversion from lab to intrinsic frame:

$$Q_s(J) \sim Q_{0,s}, \quad J > 1/2$$

$$B(E2, J_i \to J_f) \sim Q_{0,t}^2$$

$\to$ Perfect axial rotor assumption: $Q_{0,s} \simeq Q_{0,t}$



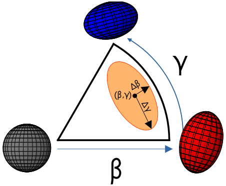
Shape invariants $\langle Q^n \rangle$: model independent

Couple Q_2 (tensor) to obtain a scalar:

- $\langle Q^2 \rangle = \langle [Q_2 \times Q_2]_0 \rangle \to \beta$
- $\langle Q^3 \rangle = \langle [[Q_2 \times Q_2]_2 \times Q_2]_0 \rangle \to \gamma$
- All $\langle Q^n \rangle$ up to $\langle Q^6 \rangle$ are needed for fluctuations of (β, γ)

K. Kumar, Phys. Rev. Lett. 28, 249 (1972)

D. Cline, Nuclear Structure 313–326 (1985)



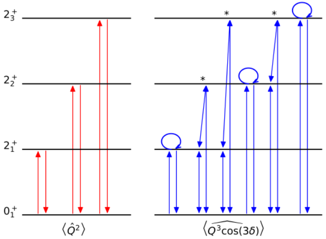
Shape invariants: Intermediate-state expansion

Intermediate-state expansion:

- (Infinite) expansion of $M_{if} = \langle i||Q||f\rangle \leftarrow Q_s$ or $B(E2)$
- Harder convergence for higher $\langle Q^n\rangle$

$$\langle Q^2\rangle_i \sim \sum M_{it} M_{ti} \to \beta$$

$$\langle Q^3\rangle_i \sim \sum M_{iu} M_{ut} M_{ti} \to \gamma$$



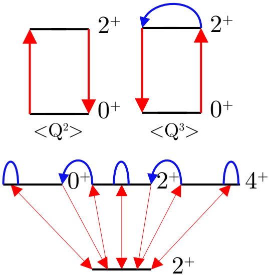
J. Henderson Phys Rev C 102, 054306 (2020)

Sum-rule method:

- Apply the $\hat{Q}_{20}$ operator to a state: $\hat{Q}_{20}|0^+\rangle = \mathrm{SR}(2^+)^{1/2}\overline{|2^+\rangle}$
  $\langle Q^2\rangle \sim \mathrm{SR}(2^+)$, $\langle Q^3\rangle \sim \overline{\langle 2^+|}Q\overline{|2^+\rangle}$
- Apply $\hat{Q}_{20}$ twice $\to \langle Q^4\rangle, \langle Q^6\rangle$

  Poves, A., Nowacki, F. & Alhassid, Y. Phys. Rev. C 101, 054307 (2020)

- **NEW!:** Extension for $J \neq 0$



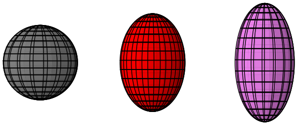
DF, A. Poves, J. Menéndez (in preparation)

Spectrum of ^{40}Ca

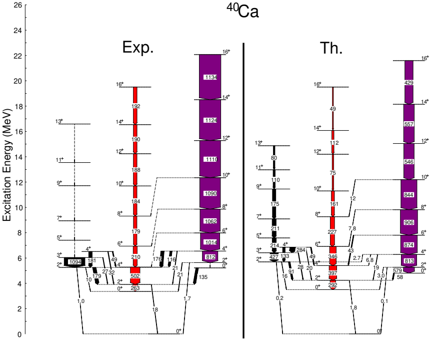
Doubly magic nucleus
$(Z = N = 20)$

Shape coexistence in ^{40}Ca:

1. 0_1^+ (0.0 MeV):
   Spherical 0p-0h
2. 0_2^+ (3.4 MeV):
   Deformed 4p-4h
3. 0_3^+ (5.2 MeV):
   Superdeformed 8p-8h



Spherical Normal Superdeformed



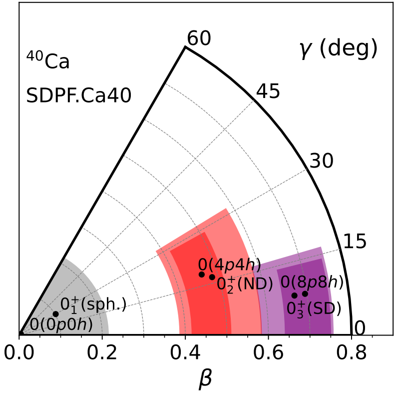
Accurate description of ^{40}Ca within the nuclear shell model

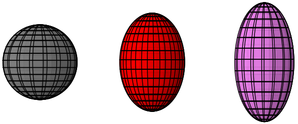
E. Caurier, J. Menéndez, F. Nowacki, and A. Poves
Phys. Rev. C **75**, 054317 (2007)

Shape coexistence in ^{40}Ca

^{40}Ca shape coexistence:

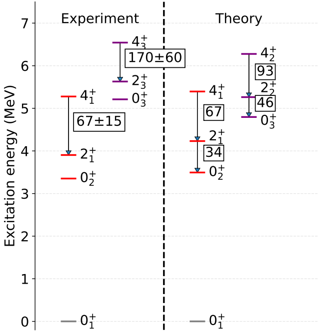
- "Spherical":
  $\beta = 0.10 \pm 0.11$
  $\gamma = -\ (0 - 60)$
  Compatible with $\beta = 0$ but reaches up to $\beta = 0.2$
  0p-0h: only 60%
- Normal deformed:
  $\beta = 0.49 \pm 0.10$
  $\gamma = 17\ (0 - 31)$
  4p-4h: $\simeq (\beta, \gamma)$ lower $\Delta\beta$
- Superdeformed:
  $\beta = 0.67 \pm 0.09$
  $\gamma = 8\ (0 - 16)$
  8p-8h: $\simeq (\beta, \gamma)$ lower $\Delta\beta$

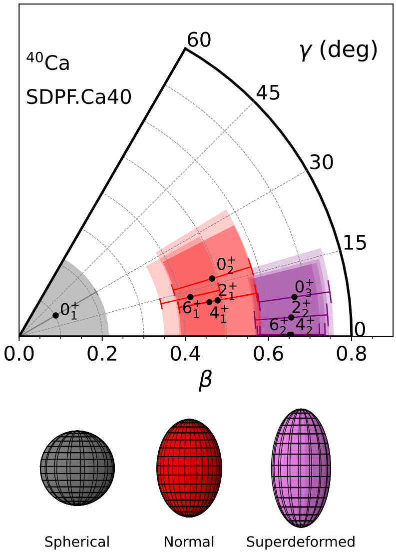




Band evolution of (β, γ):

- (β, γ) are consistent across the band (with fluctuations)
- Robust method to identify states with the **same intrinsic shape**



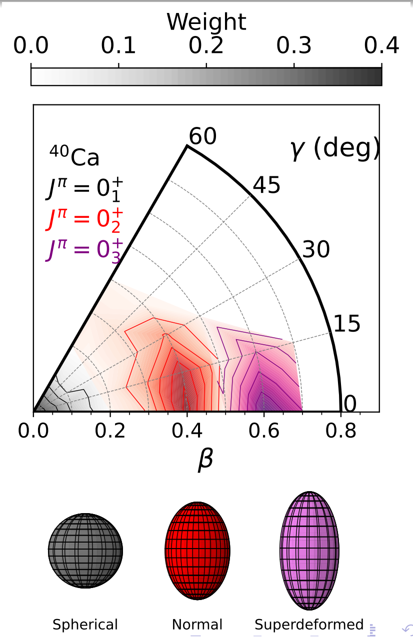


NSM: Slater determinant basis
$|\Psi_{\rm NSM}\rangle = \sum_i C_i |\Phi_i\rangle$

Projected-generator-coordinate method

- Quadrupole-constrained HFB basis $|\phi(q)\rangle$:
  $\mathcal{H}'_{\rm eff} = \mathcal{H}_{\rm eff} - \lambda \sum_\mu Q_{2,\mu}$
  - **Intrinsic frame!**
- Symmetry restoration
  $|\phi^{NZJ}\rangle = P^N P^Z P^J_{MK} |\phi\rangle$
- **Configuration mixing** of projected states (PGCM)
  $|\Psi_{\rm GCM}\rangle = \sum_q f_q |\phi^{NZJ}(q)\rangle$
- TAURUS code

B. Bally, et al. Eur. Phys. J. A **60**, 62 (2024)



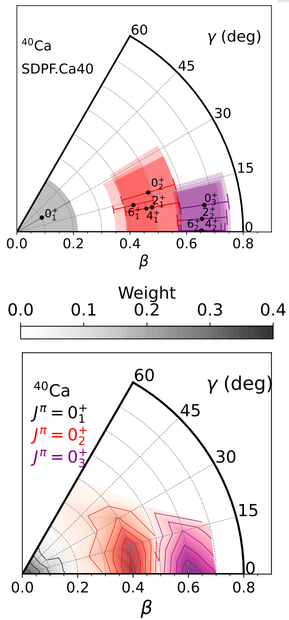
Conclusions

Shape invariants $\langle Q^n \rangle$:

- **Extension** of sum rule method for any J (up to $\langle Q^4 \rangle$)
- General method to **identify shapes**: $\langle Q^n \rangle \to (\beta, \gamma)$
- (β, γ) fluctuations are large
- (β, γ) are **constant** across the (rotational) bands
- Valid for odd nuclei
- **PGCM comparison**: $\langle Q^n \rangle$ provide an approximate distribution of (β, γ) in the intrinsic frame

Nuclear shapes: the Kumar-Cline lens

Dorian Frycz,[1,2,3,*] Alfredo Poves,[4,†] and Javier Menéndez[1,2,‡]



THANK YOU!!!

Feel free to ask any questions

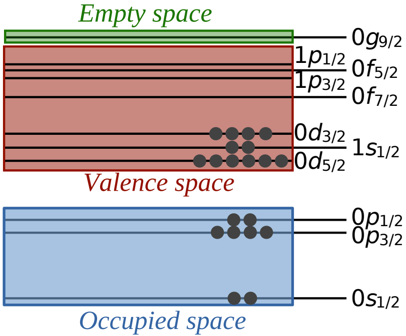
Shell model valence space

Schrödinger equation

$$\mathcal{H}|\Psi\rangle = E|\Psi\rangle$$

- **Interacting shell model**:
  $\mathcal{H}_{\text{eff}} = \mathcal{H}_0 + \mathcal{H}_{\text{res}}$
  $\mathcal{H}_{\text{res}}$: valence space
- Slater determinant basis $\{\Phi_i\}$
- Configuration mixing:
  $|\Psi_{\text{NSM}}\rangle = \sum_i C_i|\Phi_i\rangle$

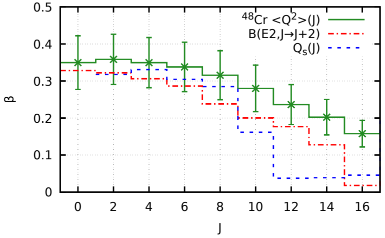
Caurier E., et al. Rev. Mod. Phys. **77**, 427 (2005)



- Phenomenological interaction:
  ^{40}Ca: SDPF-Ca40
  $sd + pf$ major shells

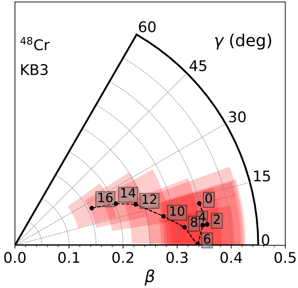
Comparison of β extracted from $\langle Q^2 \rangle$, Q_s, and $B(E2)$:

- $J \leq 6$ β are consistent
- $J = 8$ Q_s drops to half
- $J = 10 - 12$ Q_s drops to ~ 0
- $J = 8 - 12$ slow decrease in $\langle Q^2 \rangle$ and $B(E2)$



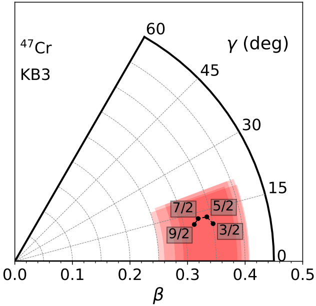
Band evolution of (β, γ):

- $J = 0$ $\gamma \simeq 12^\circ$ **triaxial**
- $J = 0 - 6$ γ decreases to 0°
- $J > 6$ γ increases to 17°
- Large $\Delta\gamma$: $0^\circ \leq \gamma \leq 18^\circ$
- Large β fluctuations $\Delta\beta/\beta \simeq 0.2$



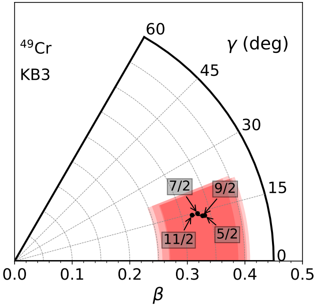
^{47}Cr=^{48}Cr-n (GS=$3/2^-$)

- Band members: $3/2^-, 5/2^-, 7/2^-, 9/2^-$



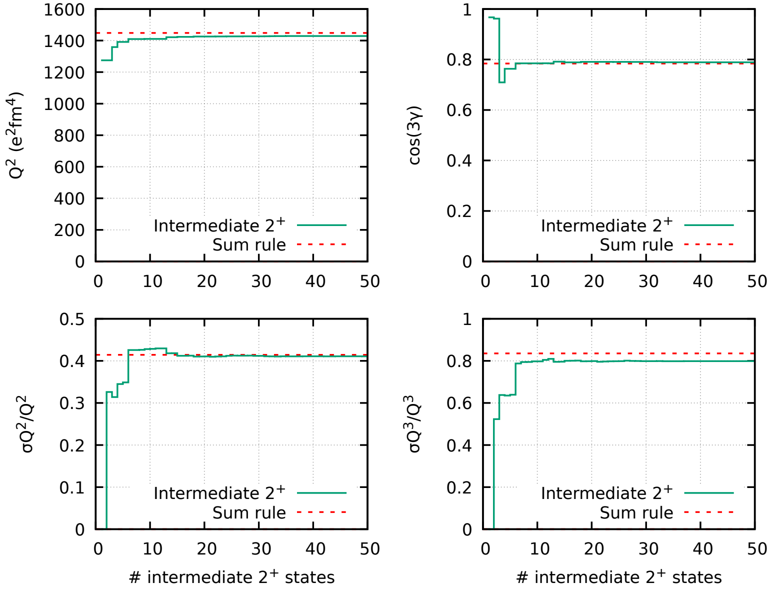
^{49}Cr=^{48}Cr+n (GS=$5/2^-$)

- Band members: $5/2^-, 7/2^-, 9/2^-, 11/2^-$



- Very consistent (β, γ) values across the bands
- Constant $\Delta\beta$ dispersion ($\Delta\gamma$ is taken from ^{48}Cr($J = 0$))
- Characterization of deformation of odd nuclei from $\langle Q^n \rangle$

Convergence of $\langle Q^n \rangle$



Intrinsic vs laboratory frame

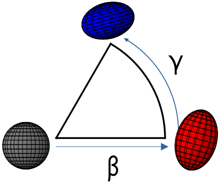
Deformation is characterized in the intrinsic frame: (β, γ)

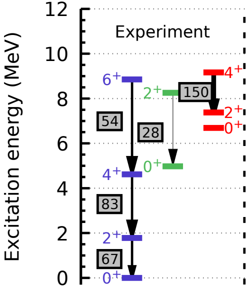
However, measurements are done in lab frame: $(B(E2), Q_s)$

Relation between lab and intrinsic frame:

$$Q_s(J) = \frac{3K^2 - J(J+1)}{(J+1)(2J+3)} Q_{0,s},$$

$$B(E2, J_i \to J_f) = \frac{5 {Q_{0,t}}^2}{16\pi} \langle J_i K 20 | J_f K \rangle$$





- If $Q_{0,s} \simeq Q_{0,t}$: well deformed rotor

1. For even-even nuclei (J=0) $\to$ $Q_s = 0$
2. Triaxial shapes diminish Q_s
3. $B(E2)$: scattered across out-band states

$\to$ Perfect axial rotor assumption!

Shape invariants $\langle Q^n \rangle$: model independent measure of deformation

Shape invariants: Sum rule method

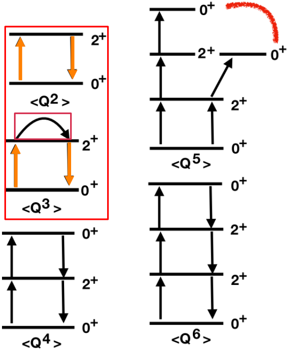
1). Apply the $\hat{Q}_{20}$ operator to a state:
$\hat{Q}_{20}|0^+\rangle = \text{SR1}(2^+)^{1/2}\overline{|2^+(1)\rangle}$
$\rightarrow \langle Q^2 \rangle = 5\text{SR1}(2^+)$

- $\overline{|2^+(1)\rangle}$ not an eigenstate but contains the whole quadrupole strenght

2). Evaluate $\langle\langle Q \rangle\rangle = \langle 2^+(1)|Q|2^+(1)\rangle/\sqrt{5}$
$\rightarrow \langle Q^3 \rangle = \langle Q^2 \rangle \langle\langle Q \rangle\rangle$

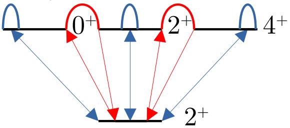
3). Apply $\hat{Q}_{20}$ a second and third time to obtain $\langle Q^4 \rangle$, $\langle Q^5 \rangle$, and $\langle Q^6 \rangle$

- **Exact** calculation (up to numeric)
- Computationally cheap
- Currently only for $J = 0$
- **New!:** extension for any J up to $\langle Q^4 \rangle$



Poves, A., Nowacki, F. & Alhassid, Y. Phys. Rev. C 101, 054307 (2020)

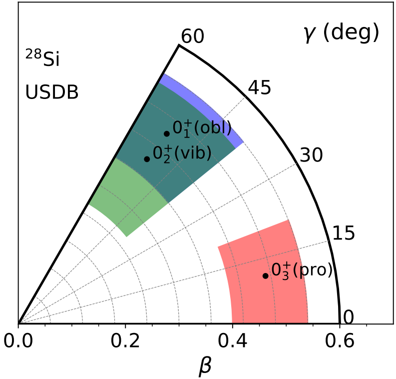
To be published:



^{28}Si shape coexistence (USDB):

- Oblate: $\beta = 0.45 \pm 0.09$
  $\gamma = 53\ (39 - 60)$
- Vibration: $\beta = 0.39 \pm 0.13$
  $\gamma = 53\ (39 - 60)$
- Prolate: $\beta = 0.47 \pm 0.07$
  $\gamma = 11\ (0 - 21)$

- Shapes are γ **and** β **soft**
- Vibration is **similar** in deformation to oblate GS
- Prolate shape has similar deformation as GS



Band evolution for ^{28}Si

Band evolution of (β, γ):

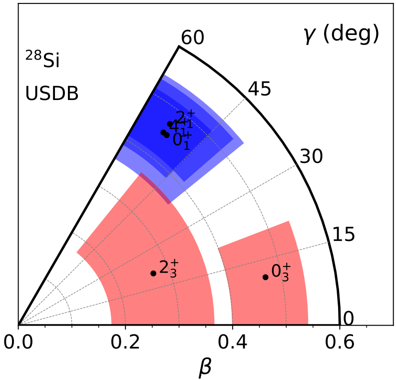
^{28}Si	$\beta \pm \Delta\beta$	γ $(\Delta\gamma)$
0_1^+	0.45 ± 0.09	52 (39-60)
2_1^+	0.47 ± 0.06	53 (41-60)
4_1^+	0.45 ± 0.06	53 (45-60)
0_3^+	0.47 ± 0.07	11 (0-21)
2_3^+	0.27 ± 0.10	21 (0-51)

Oblate (good rotational band):

- (β, γ) are consistent
- **Band states overlap**

Prolate (not really a band):

- 0_3^+ and 2_3^+ **different shapes**



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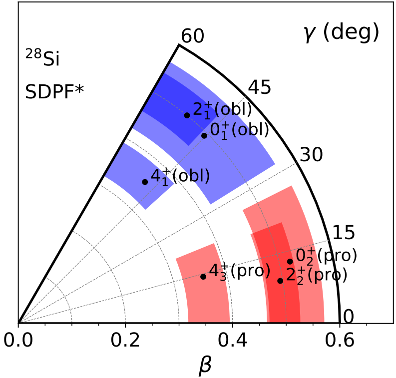
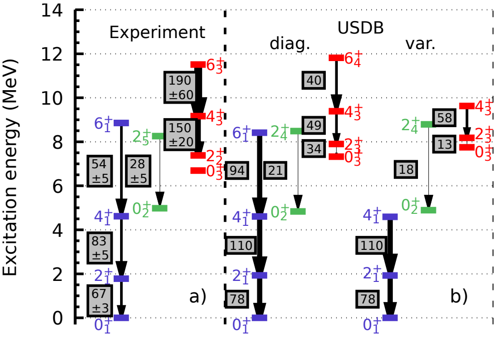
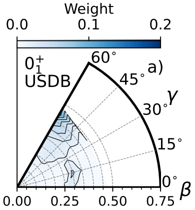


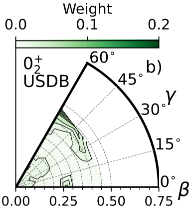
Oblate rotational band: **well described**, slightly more deformed

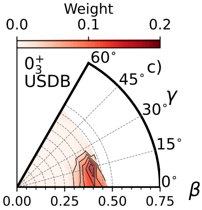
Vibrational band based on the ground state is also **well described**

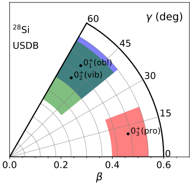
Prolate rotational band has too weak $B(E2)$











Shape invariants