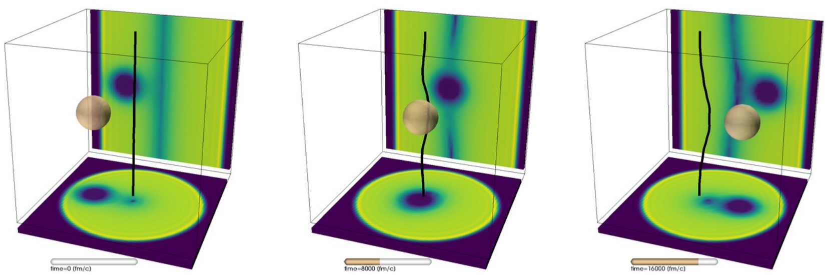
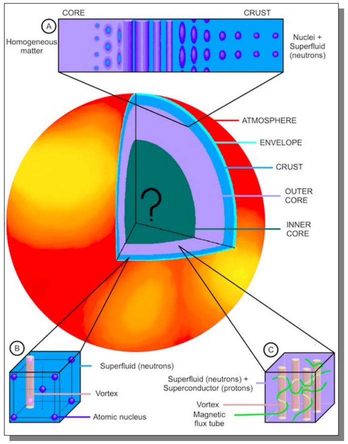
Microscopic dynamics of nuclear superfluids: vortices and nuclei in neutron star crusts

Gabriel Wlazłowski

Warsaw University of Technology
University of Washington





UCZELNIA BADAWCZA
INICJATYWA DOSKONAŁOŚCI

PL GRID

LUMI

NATIONAL SCIENCE CENTRE
POLAND

ZAKOPANE CONFERENCE ON NUCLEAR PHYSICS 2026
(August 30, 2026 to September 6, 2026)

Neutron star

Observables:

→ **Masses**

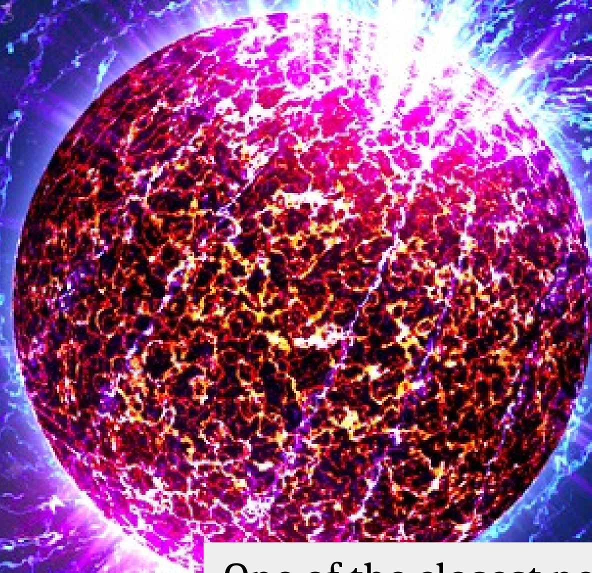
→ **Radii** (suffer from many systematic errors)

→ **EM Emission**

→ **Age** (from size of nebula)

→ **Gravitational waves**

→ **Rotation period**
(measured very accurately)



One of the closest neutron star discovered to date
RX J1856.5−3754:
distance= 400 light-years = 3.78 x 10^{15} km
size=20 km
size / distance ~ 10^{-14} – 10^{-15}

Nuclei: *tiny object → controlled experiment → indirect inference*
Neutron stars: *distant object → natural experiment → indirect inference*

Neutron star

Observables:

→ **Masses**

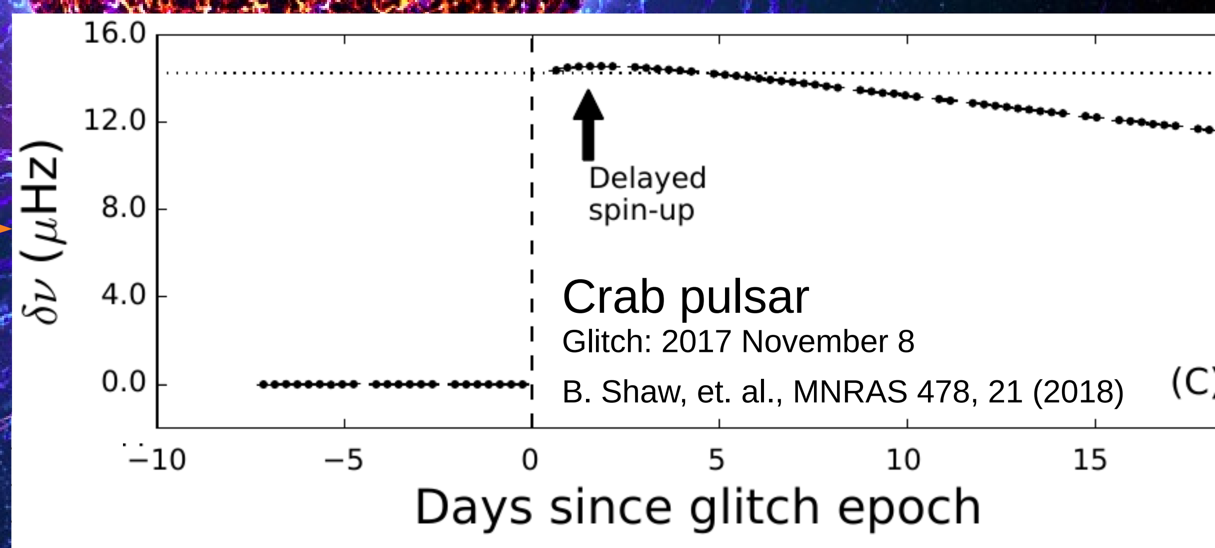
→ **Radii** (suffer from many systematic errors)

→ **EM Emission**

→ **Age** (from size of nebula)

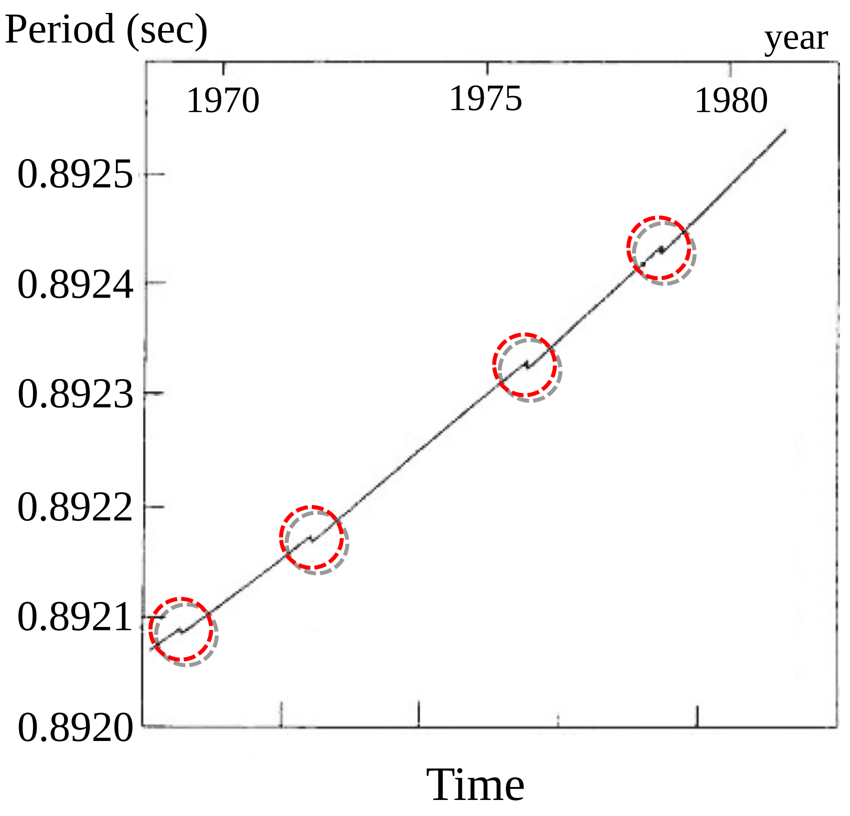
→ **Gravitational waves**

→ **Rotation period**
(measured very accurately)



Glitch: a sudden increase of the rotational frequency

- Glitches in the Vela pulsar



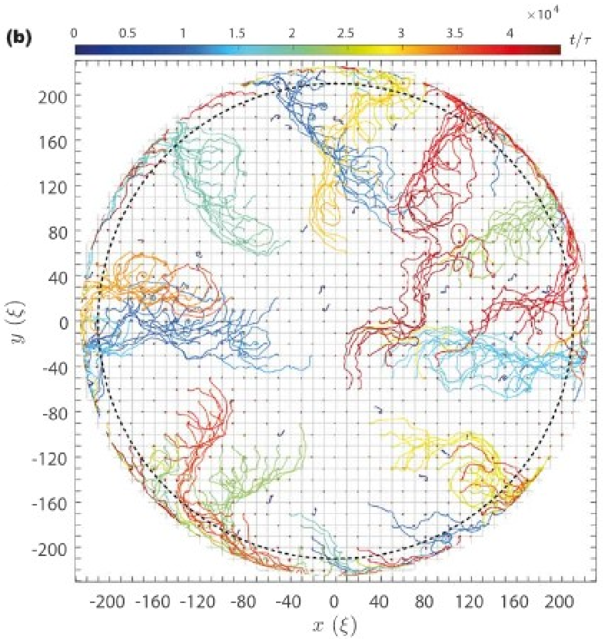
V.B. Bhatia, A Textbook of Astronomy and Astrophysics with Elements of Cosmology, Alpha Science, 2001.

Glitches due to quantum vortices

(P. W. Anderson and N. Itoh, Nature 256 (1975))

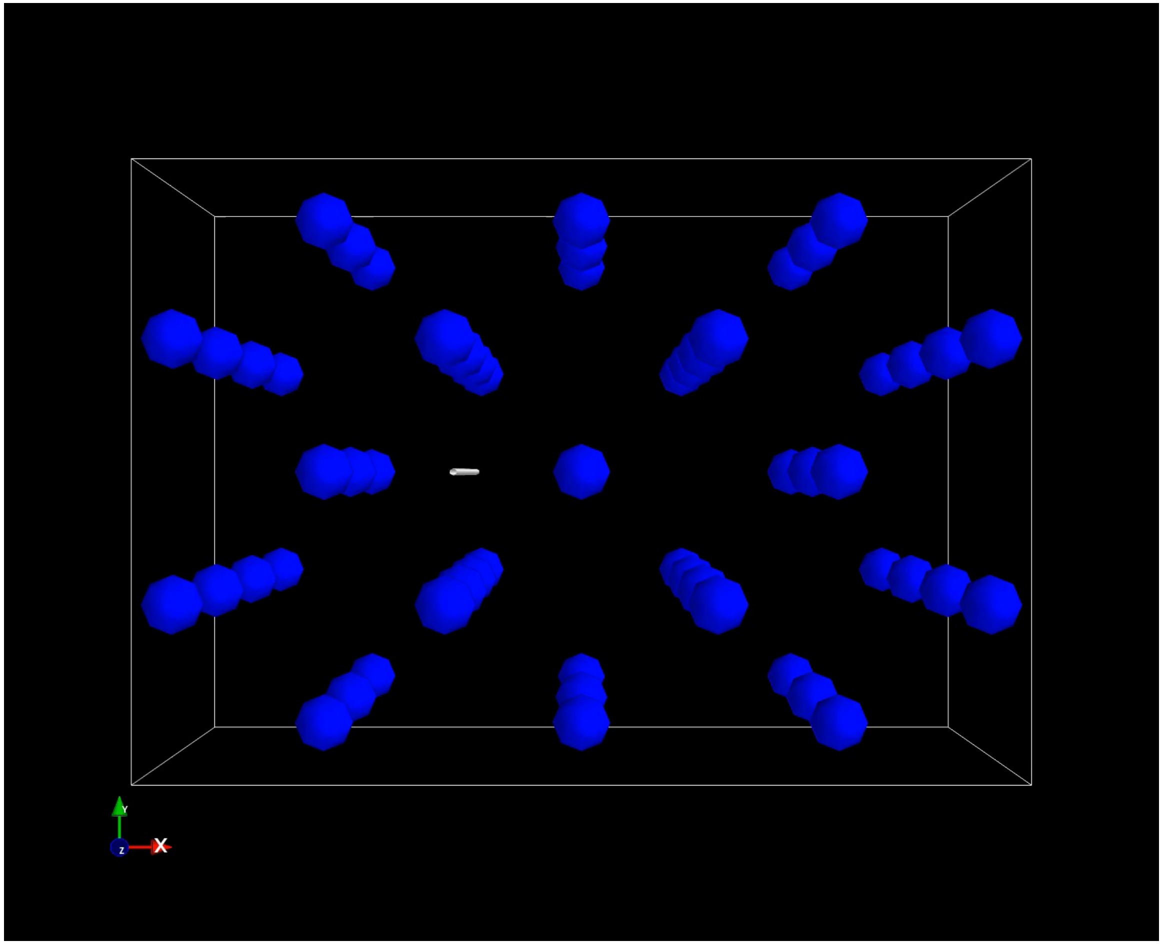
- Presently the standard picture for pulsar glitches
- Can explain: post-glitch relaxation, statistics of the glitching populations...
- Idea:
  - Superfluid interior contains quantized vortices pinned to the crustal lattice
  - Glitches are believed to occur when a large number of vortices simultaneously unpin and move outward

Simulation of vortex avalanches by I-Kang Liu et al 2025 ApJ 984 83



First observed in 1969: V. Radhakrishnan and R. N. Manchester, Nature 222, 228 (1969); P. E. Reichley and G. S. Downs, Nature 222, 229 (1969);

Providing microscopic inputs for mesoscopic models...



Movie 1

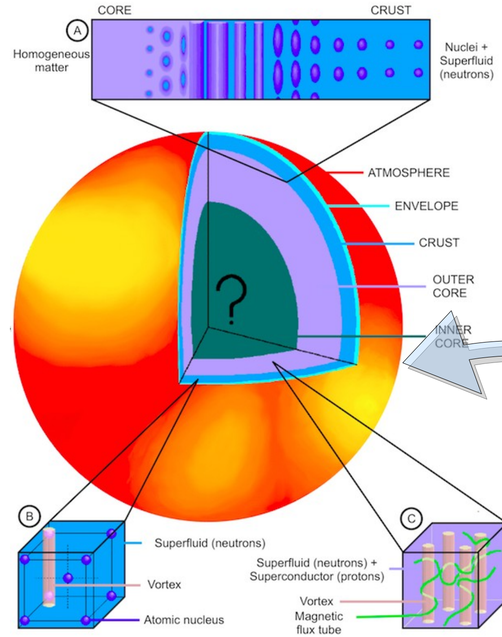
→ $v_{ext} < v_{crit}$

From diploma thesis of Konrad Kobuszewski, WUT 2018

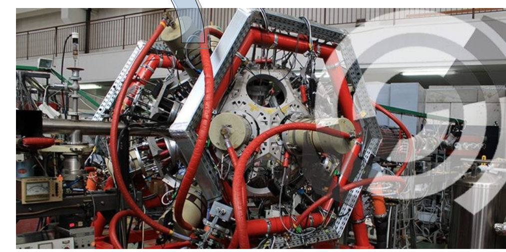
Glitches: the microscopic actors are quantized vortices interacting with <u>nuclear clusters</u>.

Density Functional Theory (DFT): Workhorse for ...

- Solid-state physics
- Quantum chemistry
- Condensed-matter physics

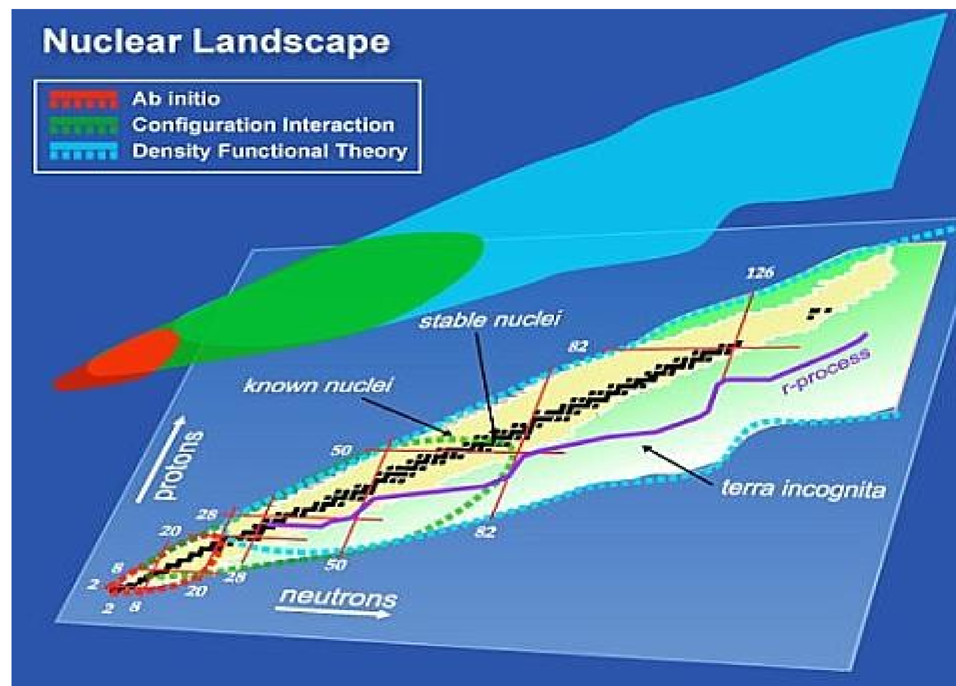


Our bridge between laboratory nuclear physics and neutron-star matter is the same framework: nuclear DFT.



… also important tool for

- Nuclear physics
- (Nuclear) astrophysics
- … plasma physics ...



Nature 514, 550 (2014)

… *Twelve papers on the top-100 list relate to it* [DFT], *including 2 of the top 10.*

For neutron star studies we use BSk

Brussels Skyrme functionals BSk(G)

We have fitted a series of nuclear energy-density functionals with full HFB calculations using extended Skyrme functionals

Experimental data/constraints:

- $\sim$ 2300 atomic masses (rms $\sim 0.5 - 0.6$ MeV$/c^2$)
- $\sim$ 900 nuclear charge radii (rms $\sim$ 0.03 fm)
- symmetry energy $29 \leq J \leq 32$ MeV
- incompressibility $K_v = 240 \pm 10$ MeV (giant resonances in nuclei)

Many-body ab initio calculations:

- equation of state of pure neutron matter
- $^1S_0$ pairing gaps in nuclear matter
- effective masses in nuclear matter (+giant resonances in nuclei)
- stability against spin and spin-isospin fluctuations

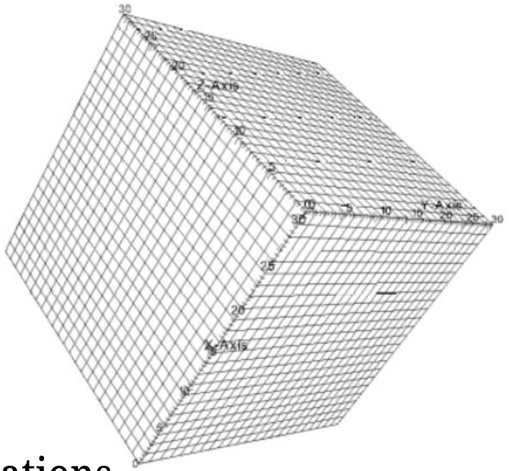
Grams et al., Eur. Phys. J. A 59, 270 (2023)

Slide from Nicolas Chamel's talk, ECT* Workshops, Trento, Apr. 2024

Time-dependent DFT as workhorse

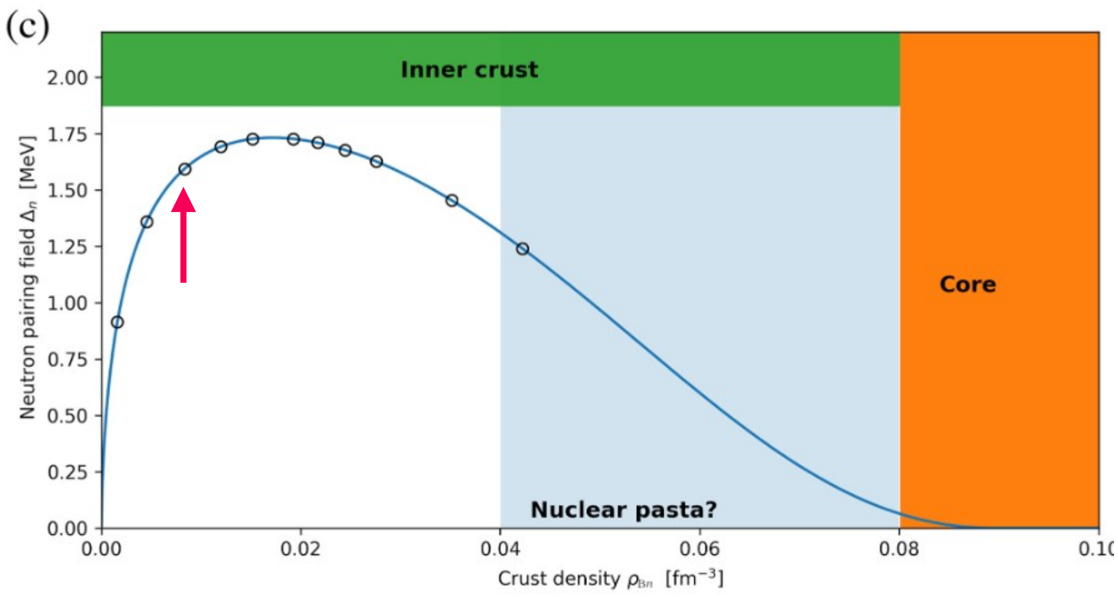
We solve self-consistently time-dependent HFB equations with BSk31 energy density functional (q=n,p):

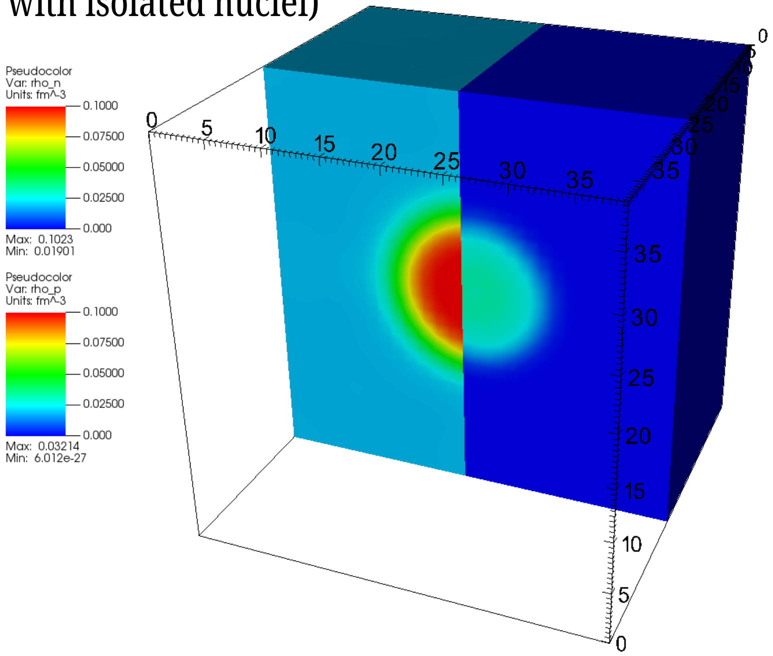
$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} u_{q,k\uparrow} \\ v_{q,k\downarrow} \end{pmatrix} = \begin{pmatrix} h_q & \Delta_q \\ \Delta_q^* & -h_q^* \end{pmatrix} \begin{pmatrix} u_{q,k\uparrow} \\ v_{q,k\downarrow} \end{pmatrix}$$



Main characteristics:

- Unconstrained 3D simulations on spatial grid
- Full self-consistency maintained throughout the time evolution.
- The evolution starts from a self-consistent solution of the static HFB equations.
- In simulations presented here there is no spin-orbit term included (its effect is expected to be reduced in the neutron-rich environment compared with isolated nuclei)

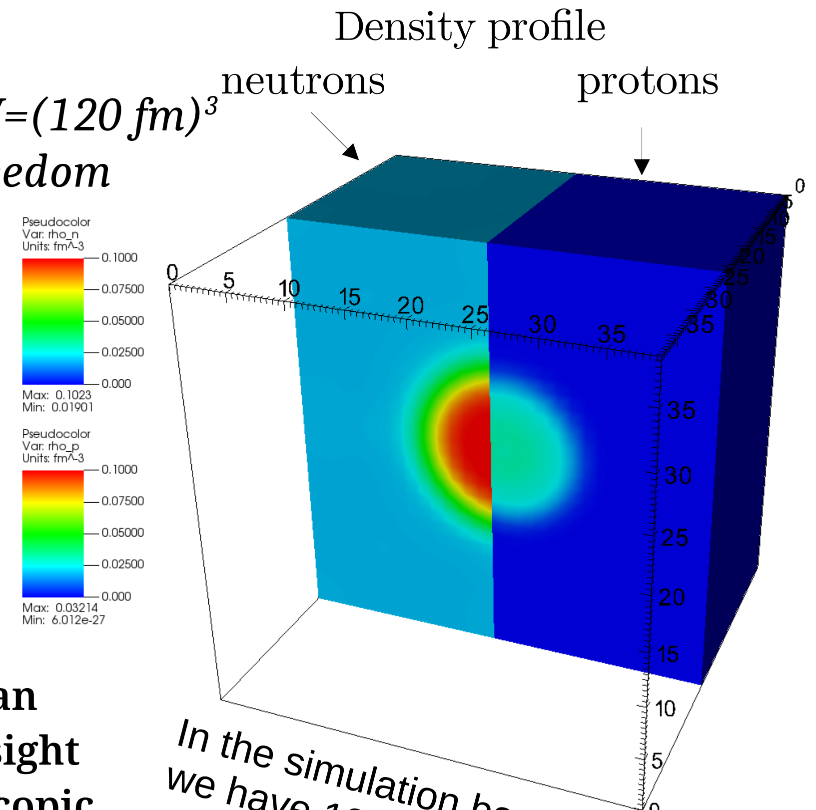




Today's capabilities of TDDFT (with nuclear functionals)

In context of nuclear applications

- *Unconstrained dynamics in 3D, volumes reaching V=(120 fm)^3*
- *Protons and neutrons as (dynamical) degrees of freedom*
- *Systems consisting of tens of thousands of particles*



In the simulation box (40 fm)^3 we have 1382 neutrons and 40 protons.

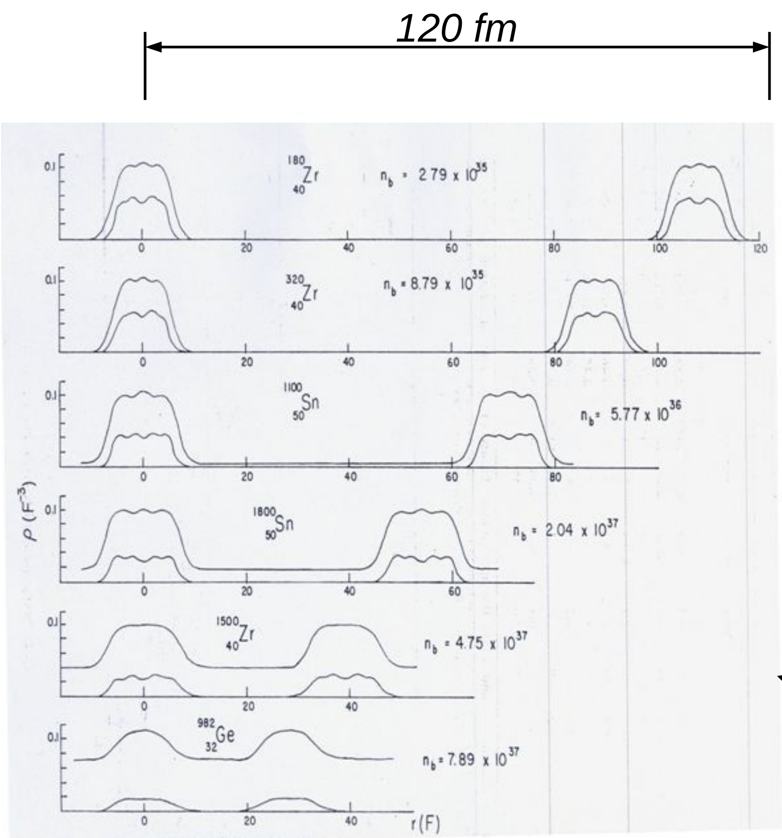
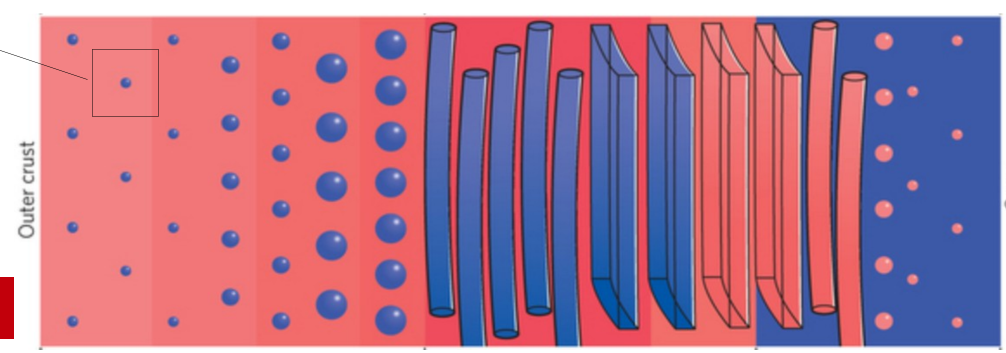
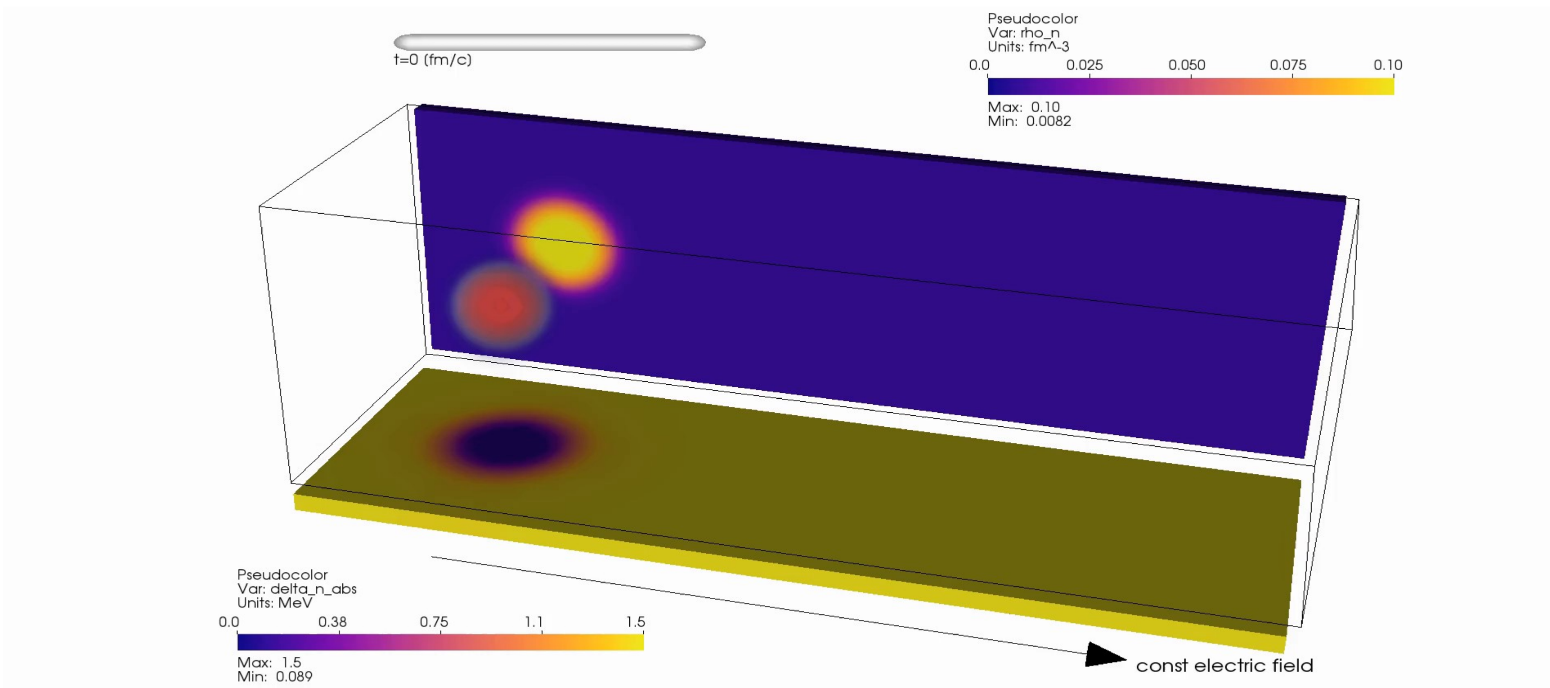


Fig. from: J.W. Negele, D. Vautherin, NPA207 (1973) 298

TDDFT can provide insight into microscopic dynamics at scales of Wigner-Seitz cell.





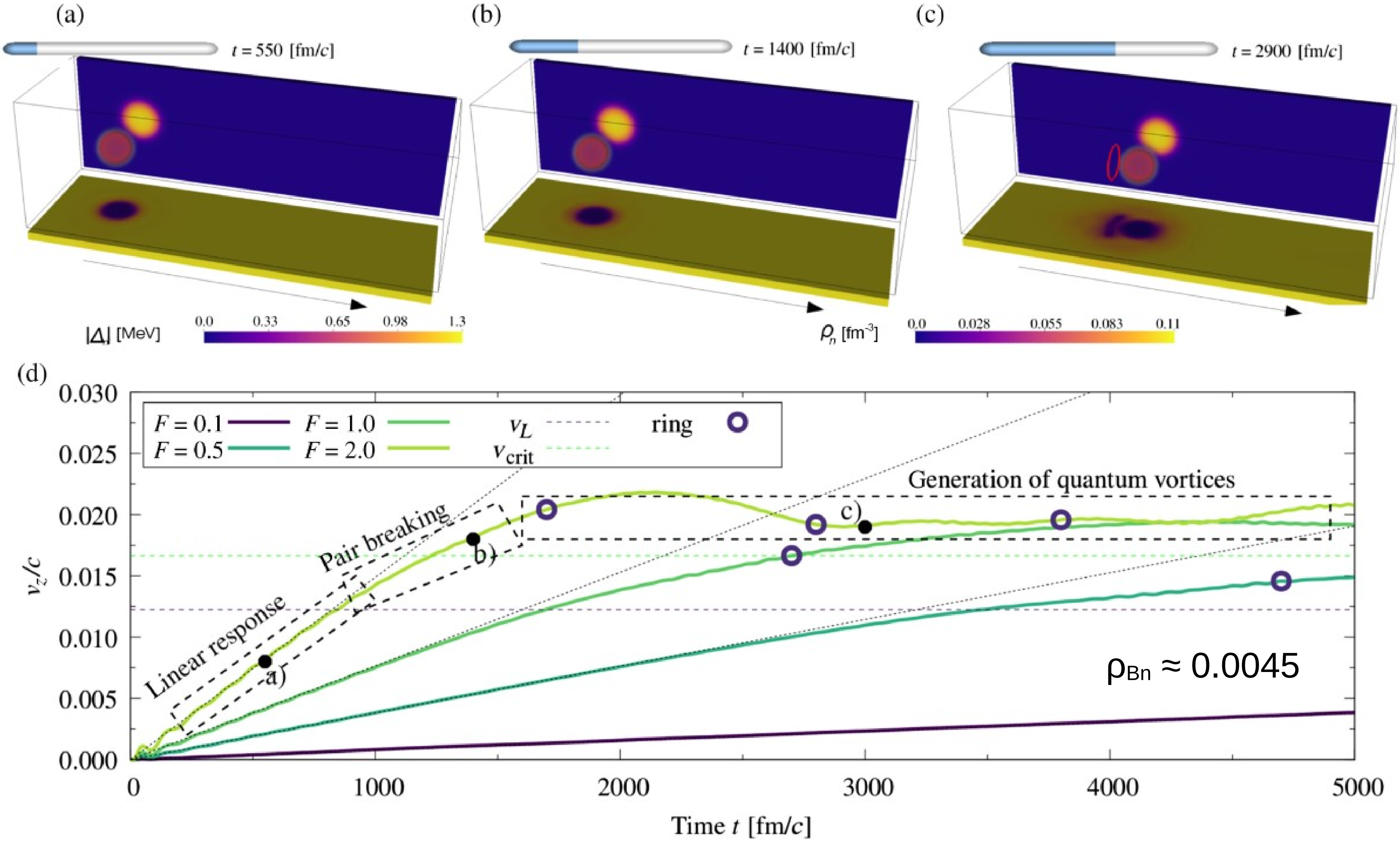
System: *nuclear matter,* 3D simulation 40 x 40 x 120 [fm]
number of neutrons: 1,736; number of protons: 40

Nuclear cluster moving through a superfluid neutron background: probing the coupling between the cluster and its environment.

response of nuclear impurity to uniform electric field

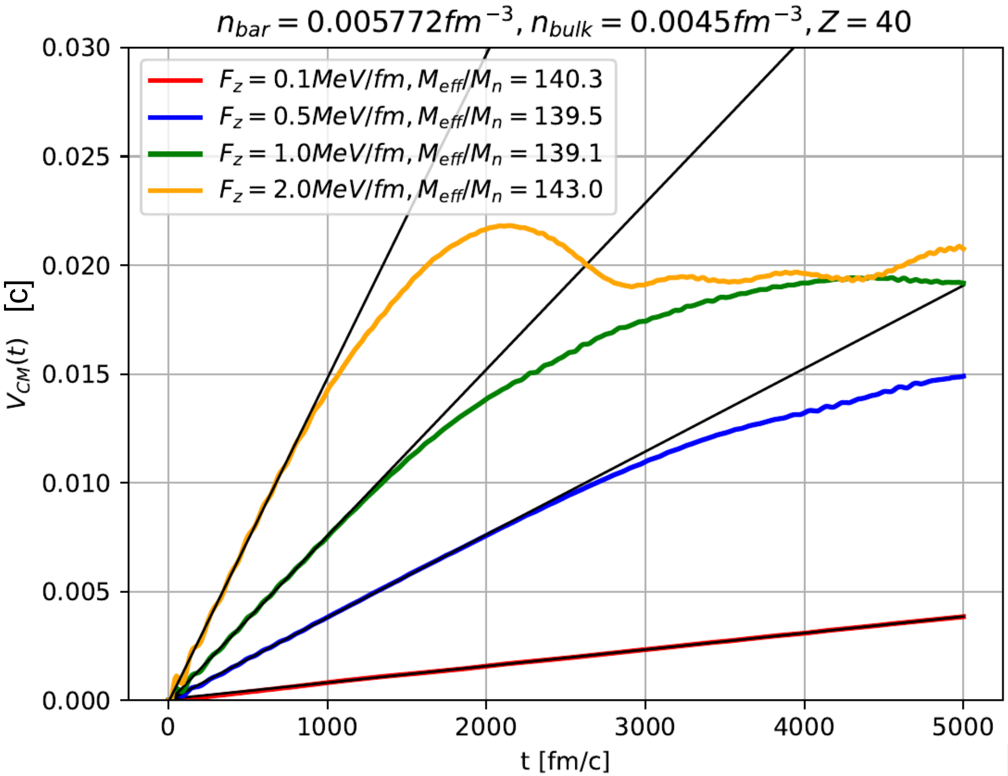
Movie 3

Phys. Rev. X 14, 041054 (2024)



$\rho_{Bn} \approx 0.0045$

Phys. Rev. X 14, 041054 (2024)

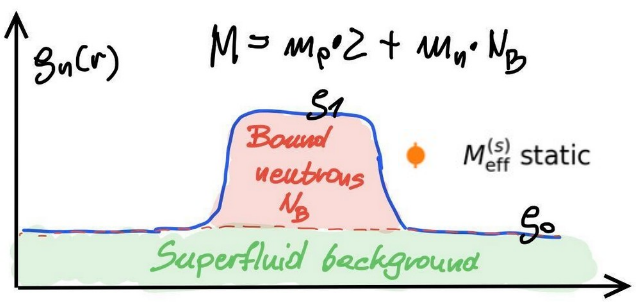


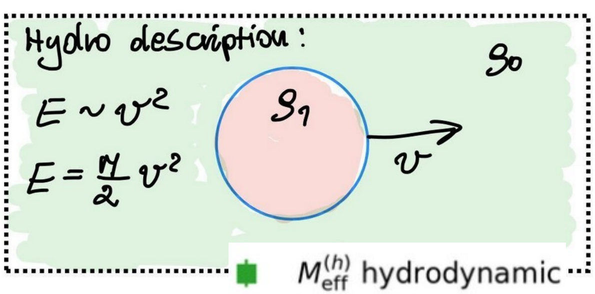
Example of application:
→ extraction of the effective mass

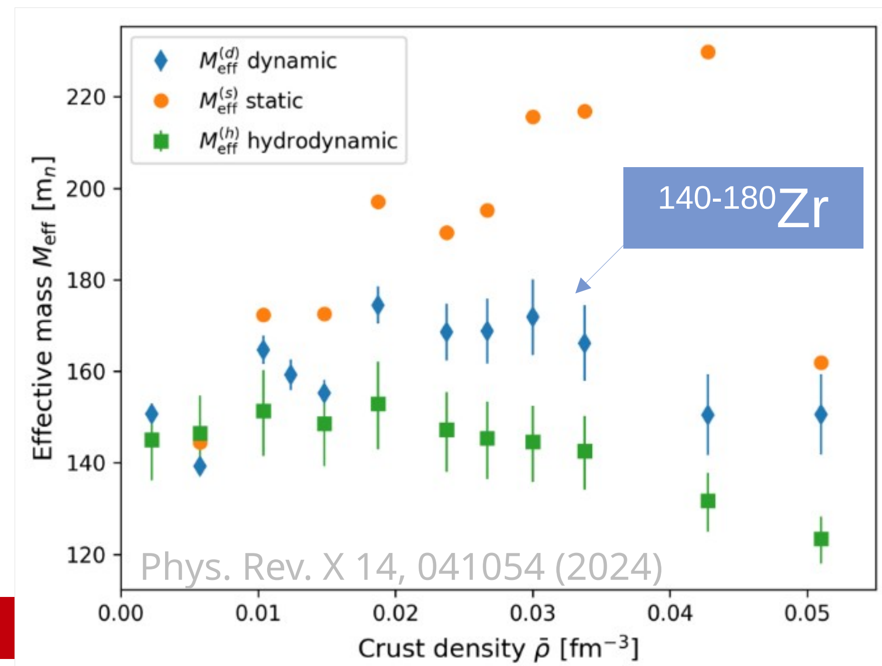
measure from simulation

$$ZeE = F = Ma$$

control parameter







Towards an effective collective model...

- Identify relevant degrees of freedom Q…

  (Vortex–nucleus interactions can excite not only translational motion of the nuclear cluster, but also its internal collective degrees of freedom.)

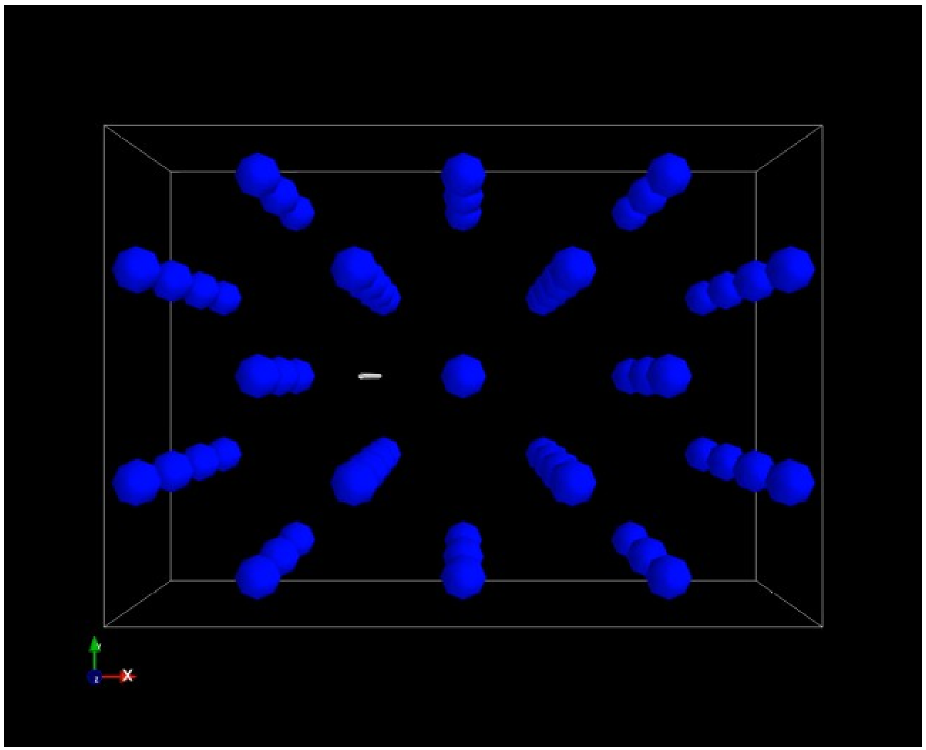
- Write effective Hamiltonian…

$$H_Q \approx \frac{M_Q \dot{Q}^2}{2} + V(Q),$$

Mass parameter

Potential energy surface

- Extract effective parameters..



Mass parameter:

- There are methods for extracting the effective mass parameters…

  [see for example Z. P. Li and D. Vretenar, in Handbook of Nucl. Phys. (2022)]

- ...they are typically designed to be applicable for extracting the mass parameters in “a vacuum”.
- This presentation: dynamical scheme of extracting the mass parameters applicable also to neutron stars.

Scheme: (prerequisite: TDHFB calculations are feasible)

1. Following the EDF philosophy, define a collective coordinate as a functional of the density:

$$Q(t) = Q[\rho(\boldsymbol{r}, t)]$$

2. Excite small-amplitude oscillations of Q(t). Their frequency determines the mode energy,

$$E_Q = \hbar\omega_Q$$

… but does not determine the collective mass M_Q.

3. Add a controllable harmonic constraint to the energy density functional:

$$E'(t) = E(t) + \frac{1}{2}k\left(Q(t) - Q_0\right)^2$$

4. The modified oscillation frequency satisfies

$$\omega^2(k) = \omega_Q^2 + \frac{k}{M_Q}$$

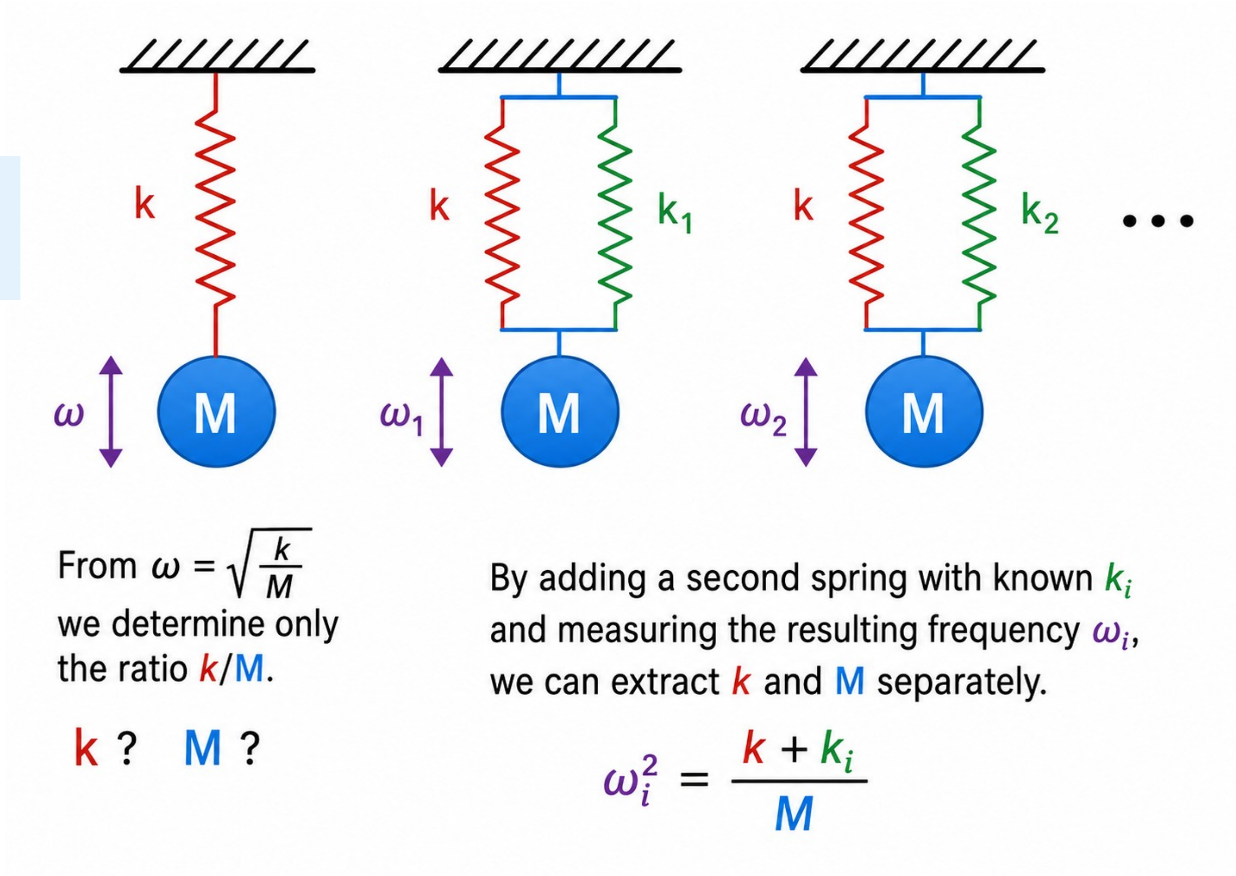
(The added harmonic constraint modifies the restoring force; since harmonic curvatures add, the oscillation frequency shifts accordingly.)

Run TDDFT for several values of k → extract ω(k) → fit $\omega^2(k)$ to determine ω_Q and M_Q.

It is general dynamical method for extracting the inertia associated with an arbitrary collective coordinate.

Scheme: (prerequisite: TDHFB calculations are feasible)

a classical analogy



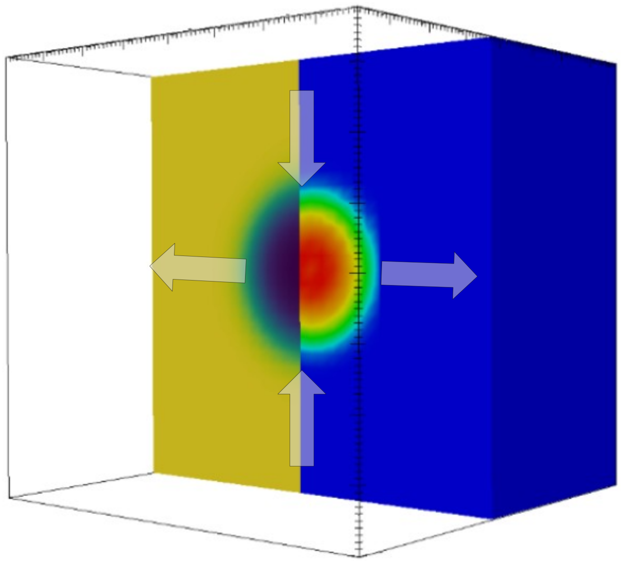
(The added harmonic constraint modifies the restoring force; since harmonic curvatures add, the oscillation frequency shifts accordingly.)

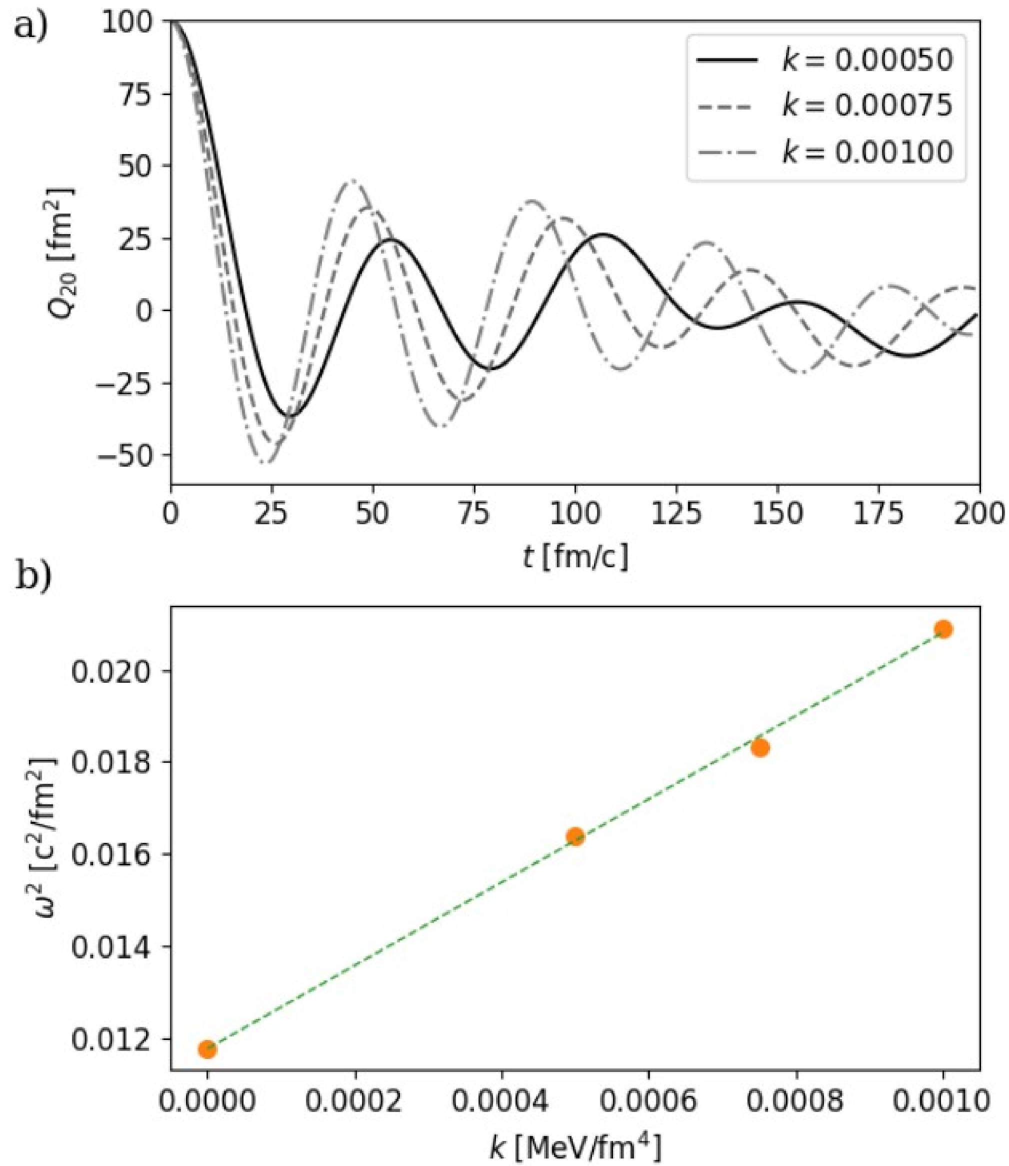
Run TDDFT for several values of k → extract ω(k) → fit $\omega^2(k)$ to determine ω_Q and M_Q.

It is general dynamical method for extracting the inertia associated with an arbitrary collective coordinate.

$$Q_{20}(t) = \int (3z^2 - r^2)\rho_p(\boldsymbol{r}, t)\, d\boldsymbol{r}$$

Giant-quadrupole mode



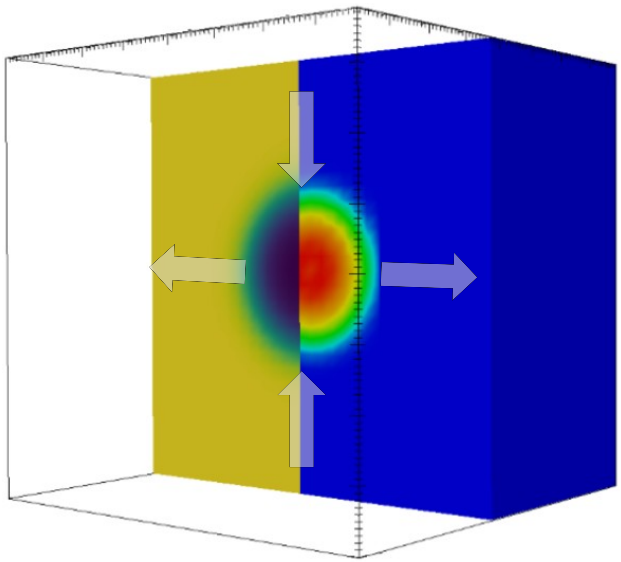


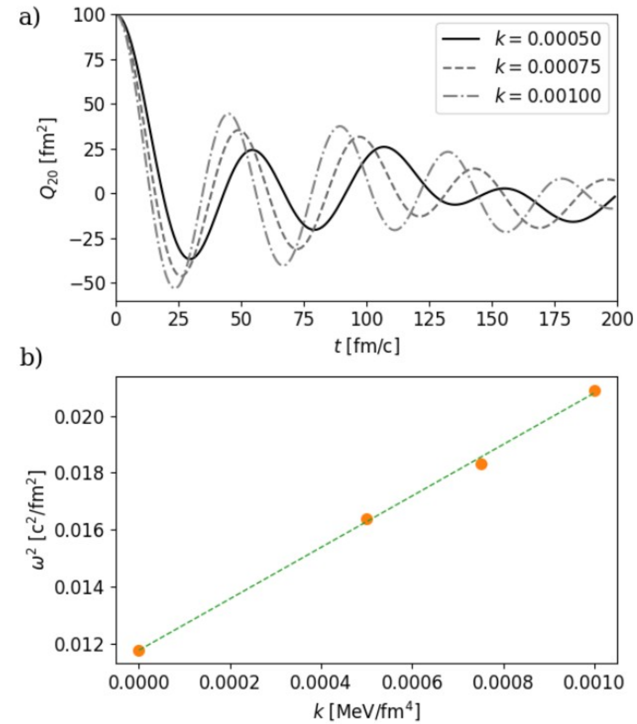
The unperturbed oscillation tells me the mode energy…

…by deliberately changing the restoring force with a known k, I can see how the frequency shifts.

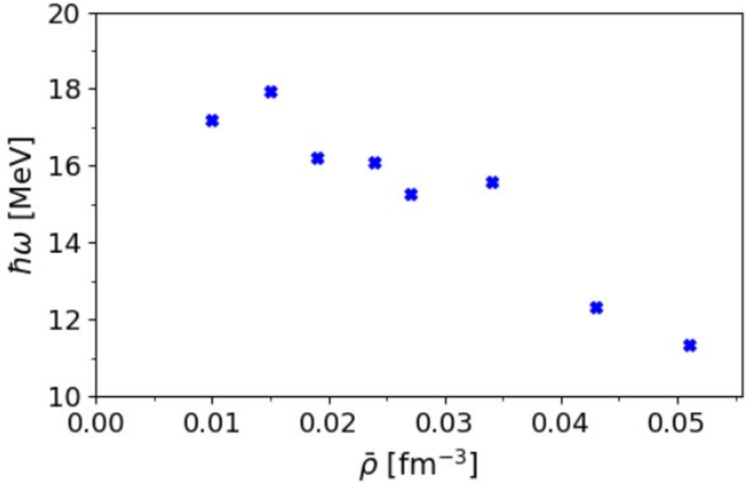
The slope of ω^2 versus k then gives the collective inertia.

$$Q_{20}(t) = \int (3z^2 - r^2)\rho_p(\boldsymbol{r}, t)\, d\boldsymbol{r}$$

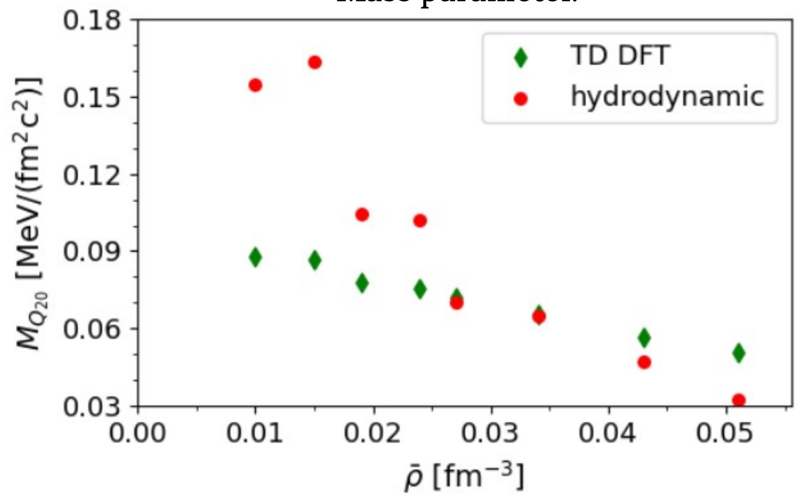




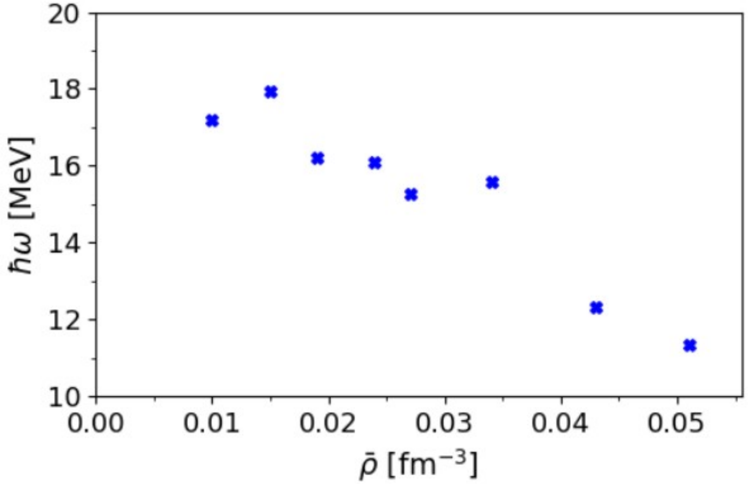
Quadrupole mode energy of Z=40 nuclei immersed in neutron matter from TDDFT



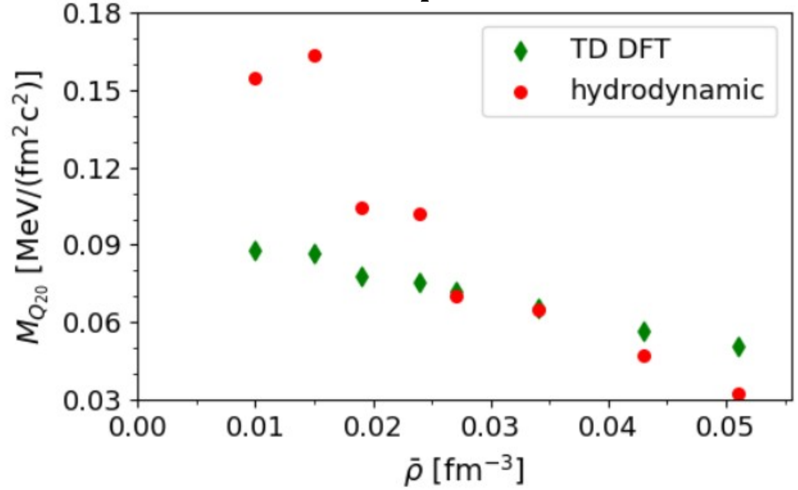
Mass parameter.



Quadrupole mode energy of Z=40 nuclei immersed in neutron matter from TDDFT



Mass parameter.

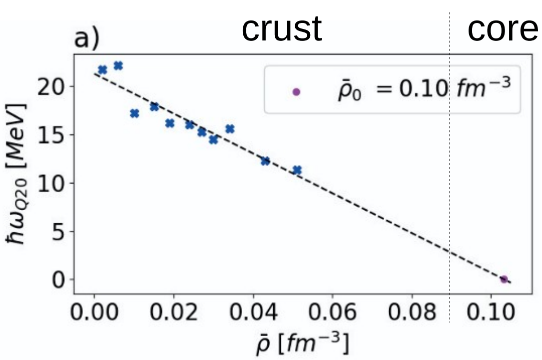


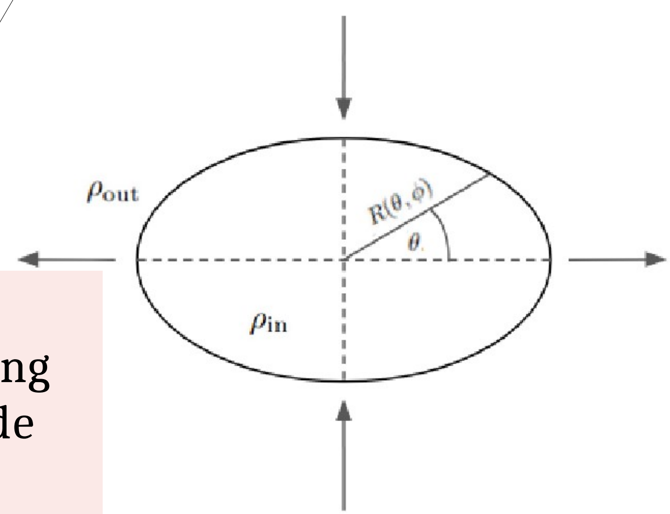
The mode remains on the giant-quadrupole energy scale of isolated Zr nuclei!

For example for ^{90}Zr → E_{ISGQR}=14MeV

$$M_{Q_{20}} = \frac{16\pi m_n^3}{5M_{\rm CM}^2}\rho_{\rm out}(\kappa-1)^2\frac{R_0}{2\kappa+3},$$

$$\kappa = \rho_{\rm in}/\rho_{\rm out}$$





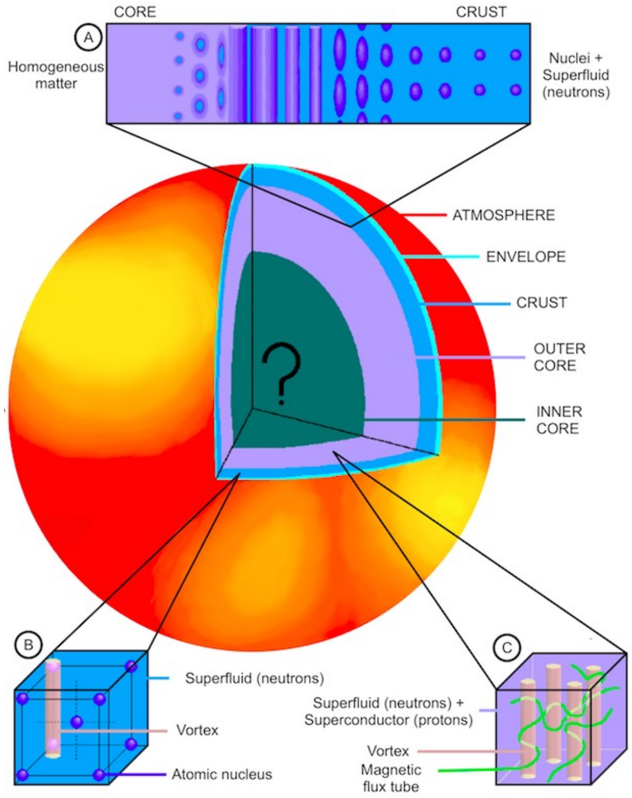
Hydrodynamic approach: from energy cost of deforming impurity of density ρ_{in} inside fluid of density ρ_{out}.

(following Ring & Schuck book)

Phys. Rev. C 112, 045804 (2025)

SUMMARY

- Microscopic TDDFT simulations provide direct insight into the dynamics of neutron-star crust matter
- Controlled real-time perturbations allow collective inertias to be extracted directly from microscopic dynamics.
- For Z=40 clusters embedded in neutron matter, quadrupole excitations remain on the giant-quadrupole energy scale, while their energy and inertia vary systematically with neutron density.
- These quantities provide microscopic input for effective descriptions of vortex–nucleus dynamics needed for models of *glitches*.

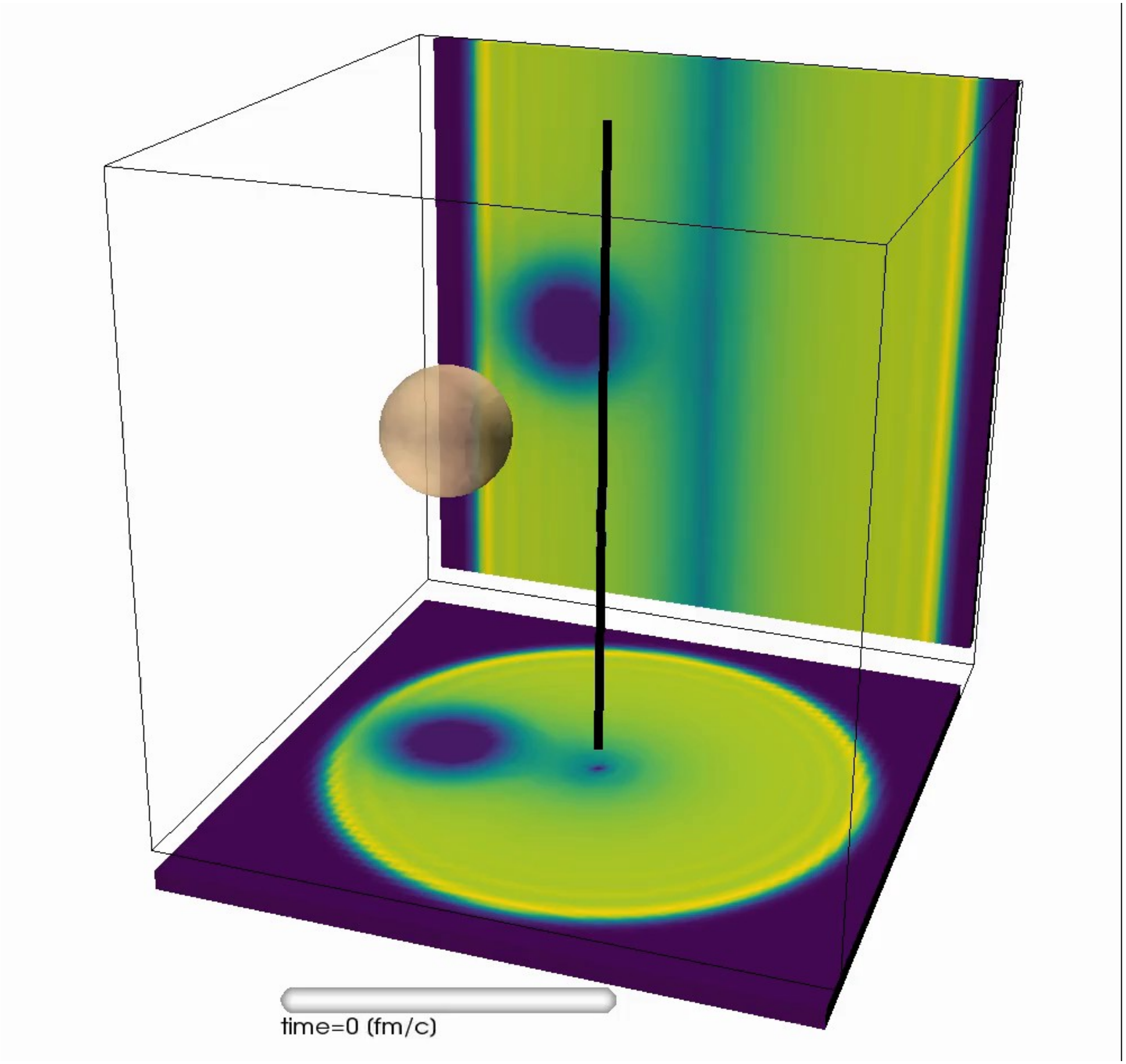


Thank you for your attention!

WUT Group: **P. Magierski**, G. Wlazłowski, **D. Pęcak**, A. Barresi, E. Alba, V. Allard, **A. Zdanowicz**, M. Śliwiński, D. Lazarou; A. Makowski
In collboration with: **N. Chamel** (U. Bruxelles);

physics wut

gabriel.wlazlowski@pw.edu.pl http://wlazlowski.fizyka.pw.edu.pl https://wslda.fizyka.pw.edu.pl/



TDDFT can also capture the coupled dynamics of a nuclear cluster and a quantum vortex, beyond the scope of this talk.

Functionalities Download Gallery Publications and Materials Related projects Info EN

Warsaw University of Technology | W-SLDA Toolkit W-BSk Toolkit

W-SLDA Toolkit

Self-consistent solver of mathematical problems which have structure formally equivalent to Bogoliubov-de Gennes equations.

static problems: st-wslda

$$\begin{pmatrix} h_a(\mathbf{r}) - \mu_a & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h_b^*(\mathbf{r}) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{pmatrix}$$

time-dependent problems: td-wslda

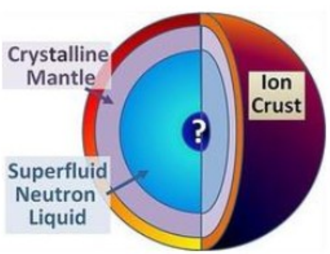
$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} u_n(\mathbf{r},t) \\ v_n(\mathbf{r},t) \end{pmatrix} = \begin{pmatrix} h_a(\mathbf{r},t) - \mu_a & \Delta(\mathbf{r},t) \\ \Delta^*(\mathbf{r},t) & -h_b^*(\mathbf{r},t) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r},t) \\ v_n(\mathbf{r},t) \end{pmatrix}$$

Extension to nuclear matter in neutron stars

Unified solvers for static and time-dependent problems

Dimensionalities of problems: 3D, 2D and 1D

Extension to nuclear matter in neutron stars



The W-SLDA Toolkit has been expanded to encompass nuclear systems, now available as the W-BSk Toolkit.

D. Pęcak, A. Zdanowicz, N. Chamel, P. Magierski, G. Wlazłowski, Phys. Rev. X 14, 041054 (2024)

ALL FUNCTIONALITIES →

Integration with VisIt: visualization, animation and analysis tool

Speed-up calculations by exploiting High Performance Computing

Functionals for studies of BCS and unitary regimes

can run on "small" computing clusters as well as leadership supercomputers
(depending on the problem size)

High Performance Computing

NVIDIA CUDA AMD ROCm open source

From theoretical framework to a usable computational instrument

ACKNOWLEDGMENTS

UCZELNIA BADAWCZA
INICJATYWA DOSKONAŁOŚCI

NATIONAL SCIENCE CENTRE
POLAND

This work was financially supported by the (Polish) National Science Center No. 2022/45/B/ST2/00358.

PL GRID

LUMI

We acknowledge Polish high-performance computing infrastructure PLGrid for awarding this project access to the LUMI supercomputer, owned by the EuroHPC Joint Undertaking, hosted by CSC (Finland) and the LUMI consortium through PLL/2024/07/017603