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CGC beyond Eikonal approximation

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Outline:

- Eikonal approximation and NEik corrections in the CGC
- $q\bar{q}$ dijet production in DIS at NEik accuracy in a pure gluon background field and gluon TMDs
- qg dijet production in DIS at NEik accuracy from quark background field and quark TMDs
- Summary and outlook

Dilute-dense scattering within CGC and Eikonal approximation

High energy scattering within the CGC relies on two pillars:

Semi-classical approximation → dense target is represented by strong semiclassical gluon field $\mathcal{A}_a^\mu(x) = O(1/g)$ at weak coupling g

Eikonal approximation → keeping only the leading power terms in the high energy limit

High energy limit can be achieved by boosting the target along x^-:

$$\mathcal{A}_a^\mu(x) \mapsto \begin{cases} \gamma_t\, \mathcal{A}_a^-\left(\gamma_t\, x^+,\, \frac{x^-}{\gamma_t},\, \mathbf{x}\right) \\ \frac{1}{\gamma_t}\mathcal{A}_a^+\left(\gamma_t\, x^+,\, \frac{x^-}{\gamma_t},\, \mathbf{x}\right) \\ \mathcal{A}_a^i\left(\gamma_t\, x^+,\, \frac{x^-}{\gamma_t},\, \mathbf{x}\right) \end{cases}$$

The Eikonal approximation can be understood as the limit of infinite boost of $\mathcal{A}_a^\mu(x)$

(i) Background field is independent of x^- due to Lorentz time dilation (static limit) → no p^+ transfer from the target

(ii) Background field is Lorentz contracted (shockwave limit) → no transverse motion within the target

(iii) Only the largest component of the background field is accounted for during the interaction

In the Eikonal limit the background field → $\mathcal{A}^\mu(x^+, x^-, \mathbf{x}) \simeq \delta^{\mu-}\mathcal{A}_a^-(x^+\mathbf{x}) \propto \delta(x^+)$

$\left(g\mathcal{A}^-(x^+, \mathbf{x})\right)^n$ are resumed to all orders which leads to Wilson lines along x^+

Next-to-Eikonal corrections to the CGC

Next-to-Eikonal (NEik) corrections are $\mathcal{O}(1/\gamma_t)$ at the level of the boosted background field.

NEik corrections arise from relaxing either of the three approximations:

1. Interactions with the suppressed components of background field (transverse component)
2. Finite longitudinal width of the target — transverse motion of the parton in the medium
3. x^- dependence of the background field beyond infinite Lorentz dilation

NEik corrections to quark and gluon propagators in a gluon background have been computed with applications to — TA, Beuf, *et al.* (2014-2025)

- forward parton-nucleus scattering at NEik (both dilute and dense limits)
- **DIS dijet production at NEik** (both dilute and dense limits)

An extra source of NEik corrections: interaction with the quark background of the target — TA, Beuf, *et al.* (2023-2025)

- interaction between the projectile parton and the target occurs via *t-channel quark exchange*
- application to **quark-gluon dijet production in DIS**, dijet production in forward pA collisions

see also

- quark and gluon helicity evolutions & single and/or double spin asymmetries — Kovchegov *et al.* (2016-2025)
- NEik corrections to quark and gluon propagators in high energy OPE formalism — Chirilli (2018-2021)
- subeikonal corrections via allowing longitudinal momentum exchange between projectile and target — Jalilian-Marian (2017-2020)
- formulation of inclusive DIS and exclusive Compton scattering that interpolates between small and moderate x — Boussarie *et al.* (2020-2023)
- NEik corrections in the CGC via an effective Hamiltonian approach — Li (2023-2024)

Power counting for quark background field

Under a boost of the target of parameter γ_t along the “-” direction, a current associated with the target should behave as

$$J^-(z) \propto \gamma_t\,, \quad J^j(z) \propto (\gamma_t)^0\,, \quad J^+(z) \propto (\gamma_t)^{-1}$$

The quark background field of the target can be split into good and bad components as

$$\Psi(z) = \Psi^{(-)}(z) + \Psi^{(+)}(z)$$ with $$\Psi^{(-)}(z) \equiv \frac{\gamma^+\gamma^-}{2}\Psi(z)\,, \quad \Psi^{(+)}(z) \equiv \frac{\gamma^-\gamma^+}{2}\Psi(z)$$

Then the components of quark background current reads

$$\begin{aligned}
\overline{\Psi}(z)\,\gamma^-\,\Psi(z) &= \overline{\Psi^{(-)}}(z)\,\gamma^-\,\Psi^{(-)}(z),\\
\overline{\Psi}(z)\,\gamma^j\,\Psi(z) &= \overline{\Psi^{(-)}}(z)\,\gamma^j\,\Psi^{(+)}(z) + \overline{\Psi^{(+)}}(z)\,\gamma^j\,\Psi^{(-)}(z),\\
\overline{\Psi}(z)\,\gamma^+\,\Psi(z) &= \overline{\Psi^{(+)}}(z)\,\gamma^-\,\Psi^{(+)}(z)\,.
\end{aligned}$$

Under a boost of the target, the projections $\Psi^{(-)}(z)$ and $\Psi^{(+)}(z)$, should scale as

$$\Psi^{(-)}(z) \propto (\gamma_t)^{\frac{1}{2}}\,, \quad \Psi^{(+)}(z) \propto (\gamma_t)^{-\frac{1}{2}}$$

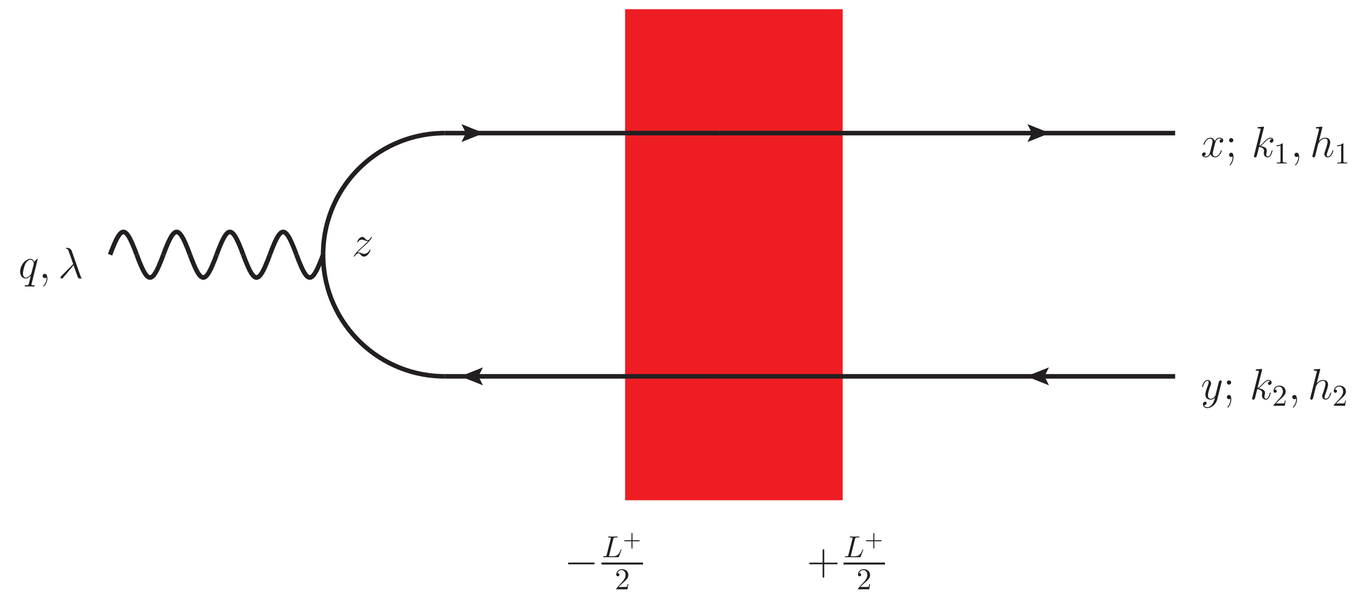
We keep only the leading component the quark background field: $\Psi^{(-)}(z)$

DIS dijet production at NEik accuracy

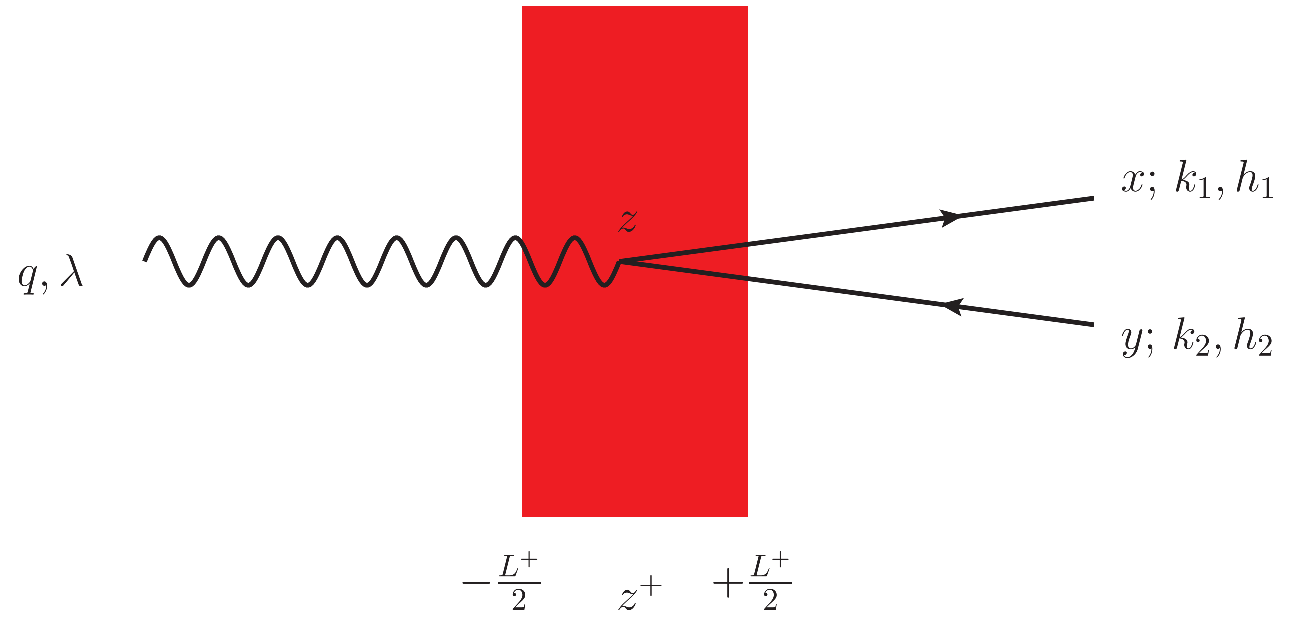
$q\bar{q}$ dijet production in pure gluon background field

TA, Beuf, Czajka, Tymowska (2023)
TA, Beuf, Czajka, Marquet (2025)

Splitting before the medium



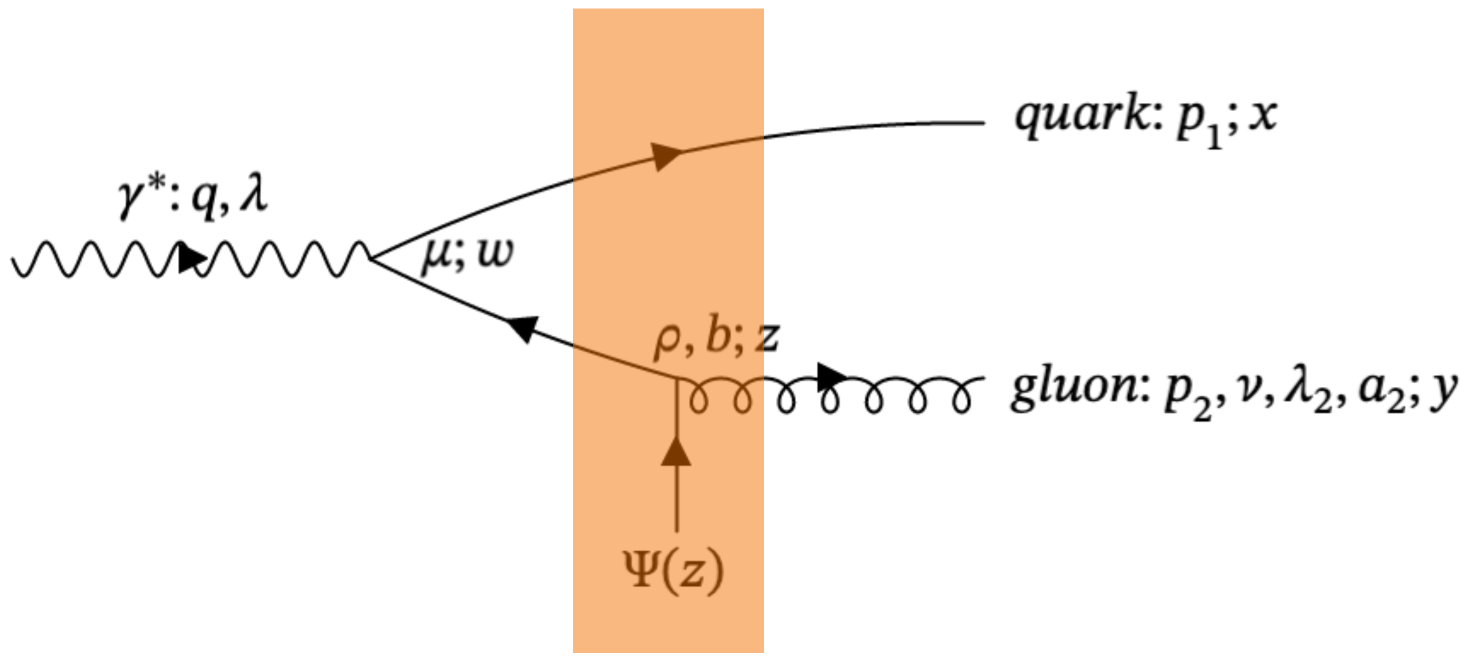
Splitting inside the medium

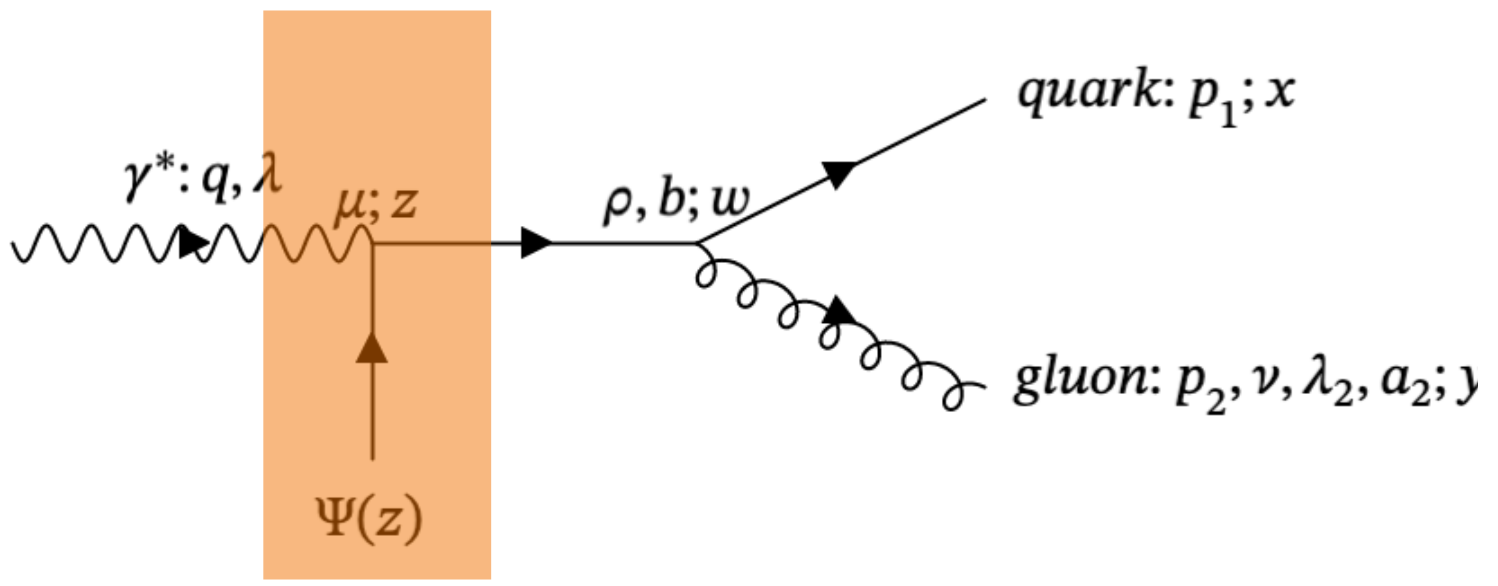


- Splitting inside the medium only contributes to the case of transversely polarized photon.
- DISCLAIMER: We will only consider the case of longitudinally polarized photon.

qg dijet production via t-channel quark exchange

TA, Armesto, Beuf (2023)





$q\bar{q}$ dijet production in DIS at NEik accuracy in pure gluon background field

TA, Beuf, Czajka, Tymowska - Phys. Rev. D 107 (2023) 7, 074016

TA, Beuf, Czajka, Marquet - Phys. Rev. D 111 (2025) 1, 014010

Quark propagator at NEik accuracy in gluon background

NEik quark propagator from before to after the medium

$$S_F(x,y) = \int \frac{dq^+ d^2\mathbf{q}}{(2\pi)^3} \int \frac{dk^+ d^2\mathbf{k}}{(2\pi)^3}\, \theta(q^+)\,\theta(k^+)\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}\, \frac{(\check{\not{q}}+m)}{2q^+}\gamma^+ \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \Big\{ \int dz^-\, e^{iz^-(q^+-k^+)}\, \mathcal{U}_F(\mathbf{z}, z^-)$$
$$+ 2\pi\delta(q^+-k^+) \Big[-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{2(q^++k^+)} \mathcal{U}^{(1)}_{F;j}(\mathbf{z}) - \frac{i}{(q^++k^+)} \mathcal{U}^{(2)}_F(\mathbf{z}) + \frac{[\gamma^i,\gamma^j]}{4(q^++k^+)} \mathcal{U}^{(3)}_{F;ij}(\mathbf{z}) \Big] \Big\} \frac{(\check{\not{k}}+m)}{2k^+} + \text{NNEik}$$

TA, Beuf (2023)

Generalized Eikonal Wilson line

$$\mathcal{U}_F(x^+, y^+; \mathbf{z}, z^-) \equiv \mathbf{1} + \sum_{N=1}^{+\infty} \frac{1}{N!}\, \mathcal{P}_+ \left[-ig \int_{y^+}^{x^+} dz^+\, t\cdot\mathcal{A}^-(z) \right]^N$$

NEik decorated Wilson lines

$$\mathcal{U}^{(1)}_{F;j}(\mathbf{z}) = -2\int dz^+\, z^+\, \mathcal{U}_F(+\infty, z^+; \mathbf{z}) \left[-igt\cdot\mathcal{F}_j^{\ -}(z^+, \mathbf{z})\right] \mathcal{U}_F(z'^+, -\infty; \mathbf{z})$$

$$\mathcal{U}^{(2)}_F(\mathbf{z}) = \int dz^+ \int dz'^+\, (z^+ - z'^+)\, \theta(z^+ - z'^+)\, \mathcal{U}_F(+\infty, z^+, \mathbf{z}) \left[-igt\cdot\mathcal{F}_j^{\ -}(z^+, \mathbf{z})\right] \mathcal{U}_F(z^+, z'^+; \mathbf{z}) \left[-igt\cdot\mathcal{F}_j^{\ -}(z'^+, \mathbf{z})\right] \mathcal{U}_F(z'^+, -\infty; \mathbf{z})$$

$$\mathcal{U}^{(3)}_{F;ij}(\mathbf{z}) = \int dz^+\, \mathcal{U}_F(+\infty, z^+; \mathbf{z}) [gt\cdot\mathcal{F}_{ij}(z^+, \mathbf{z})]\, \mathcal{U}_F(z^+, -\infty; \mathbf{z})$$

$\mathcal{U}^{(1)}_{F;j}(\mathbf{z})$ and $\mathcal{U}^{(2)}_F(\mathbf{z})$ NEik contributions beyond the shockwave approx or due to $\mathcal{A}_\perp$.

$\mathcal{U}^{(3)}_{F;ij}(\mathbf{z})$ quark helicity coupling with longitudinal chromomagnetic field of the target $\mathcal{F}_{ij}$.

S-matrix for longitudinal photon

S-matrix in terms of propagators

$$S_{q_1\bar{q}_2\leftarrow\gamma^*} = \lim_{x^+,y^+\to+\infty}\int d^2\mathbf{x}\int dx^-\int d^2\mathbf{y}\int dy^-\, e^{i\check{k}_1\cdot x}\, e^{i\check{k}_2\cdot y}$$
$$\times\,\epsilon^\lambda_\mu(q)\int d^4z\, e^{-iq\cdot z}\;\bar{u}(1)\gamma^+ S_F(x,z)\,(-iee_f\gamma^\mu)\,(-S_F(z,y))\,\gamma^+ v(2)$$

S-matrix element
for longitudinal photon

$$S_{q_1\bar{q}_2\leftarrow\gamma^*_L} = S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma^*_L}\Big|_{\text{Gen. Eik}} + S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma^*_L}\Big|_{\text{dyn. target}} + S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma^*_L}\Big|_{\text{dec. on }q} + S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma^*_L}\Big|_{\text{dec. on }\bar{q}}$$

$$S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma^*_L}\Big|_{\text{Gen. Eik}} = -2Q\,\frac{ee_f}{2\pi}\,\bar{u}(1)\gamma^+v(2)\,\frac{(q^++k_1^+-k_2^+)(q^++k_2^+-k_1^+)}{4(q^+)^2}\,\theta(q^++k_1^+-k_2^+)\,\theta(q^++k_2^+-k_1^+)$$
$$\times\int_{\mathbf{v},\mathbf{w}} e^{-i\mathbf{v}\cdot\mathbf{k}_1}\,e^{-i\mathbf{w}\cdot\mathbf{k}_2}\,\mathrm{K}_0\left(\hat{Q}\,|\mathbf{w}-\mathbf{v}|\right)\int db^-\,e^{ib^-(k_1^++k_2^+-q^+)}\left[\mathcal{U}_F\left(\mathbf{v},b^-\right)\mathcal{U}_F^\dagger\left(\mathbf{w},b^-\right)-1\right]$$

$$S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma^*_L}\Big|_{\text{dyn. target}} = 2\pi\delta(k_1^++k_2^+-q^+)\;iQ\,\frac{ee_f}{2\pi}\,\bar{u}(1)\gamma^+v(2)\,\frac{(k_1^+-k_2^+)}{(q^+)^2}\int d^2\mathbf{v}\,e^{-i\mathbf{v}\cdot\mathbf{k}_1}\int d^2\mathbf{w}\,e^{-i\mathbf{w}\cdot\mathbf{k}_2}$$
$$\times\left[\mathrm{K}_0\left(\bar{Q}\,|\mathbf{w}-\mathbf{v}|\right)-\frac{(\bar{Q}^2-m^2)}{2\bar{Q}}\,|\mathbf{w}-\mathbf{v}|\,\mathrm{K}_1\left(\bar{Q}\,|\mathbf{w}-\mathbf{v}|\right)\right]\left[\mathcal{U}_F\left(\mathbf{v},b^-\right)\overleftrightarrow{\partial_{b^-}}\mathcal{U}_F^\dagger\left(\mathbf{w},b^-\right)\right]\Big|_{b^-=0}$$

$$S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma^*_L}\Big|_{\text{dec. on }q} = 2\pi\delta(k_1^++k_2^+-q^+)\,\frac{ee_f}{2\pi}\,(-1)Q\,\frac{k_2^+}{(q^+)^2}\int d^2\mathbf{v}\,e^{-i\mathbf{v}\cdot\mathbf{k}_1}\int d^2\mathbf{w}\,e^{-i\mathbf{w}\cdot\mathbf{k}_2}\,\mathrm{K}_0\left(\bar{Q}\,|\mathbf{w}-\mathbf{v}|\right)$$
$$\times\,\bar{u}(1)\gamma^+\left[\frac{[\gamma^i,\gamma^j]}{4}\,\mathcal{U}^{(3)}_{F;ij}(\mathbf{v})-i\,\mathcal{U}^{(2)}_F(\mathbf{v})+\mathcal{U}^{(1)}_{F;j}(\mathbf{v})\left(\frac{(\mathbf{k}_2^j-\mathbf{k}_1^j)}{2}+\frac{i}{2}\,\partial_{\mathbf{w}^j}\right)\right]\mathcal{U}_F^\dagger(\mathbf{w})\,v(2)$$

DIS dijet production at NEik accuracy in general kinematics

Amplitude from S-matrix elements

$$S_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{\text{Gen. Eik}} = 2q^+\int db^- e^{ib^-(k_1^++k_2^+-q^+)}\, i\mathbf{M}^{\text{Gen. Eik}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}(b^-)$$

$$S_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{x^-\ \text{indep.}} = (2q^+)\,2\pi\,\delta(k_1^++k_2^+-q^+)\, i\mathcal{M}_{q_1\bar{q}_2\leftarrow\gamma_L^*}$$

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Big|_{\text{Gen. Eik}} = 2q^+\int d(\Delta b^-)e^{i\Delta b^-(k_1^++k_2^+-q^+)}\sum_{\text{hel., col.}}\left\langle\left(\mathbf{M}^{\text{Gen. Eik}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\left(-\frac{\Delta b^-}{2}\right)\right)^\dagger\mathbf{M}^{\text{Gen. Eik}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\left(\frac{\Delta b^-}{2}\right)\right\rangle$$

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Big|_{x^-\ \text{indep.}} = (2q^+)\,2\pi\,\delta(k_1^++k_2^+-q^+)\sum_{\text{hel., col.}}|\mathcal{M}_{q_1\bar{q}_2\leftarrow\gamma_L^*}|^2$$

Dijet production cross section

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}} = \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Big|_{\text{Gen. Eik}} + \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Big|^{\text{dec. on } q}_{\text{NEik corr.}} + \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Big|^{\text{dec. on } \bar{q}}_{\text{NEik corr.}} + \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Big|^{\text{dyn. target}}_{\text{NEik corr.}}$$

At NEik order cross sections written in terms of decorated dipole and quadrupole operators

$$d_j^{(1)}(\mathbf{v}_*,\mathbf{w}) = \left\langle\frac{1}{N_c}\text{Tr}\Big[\mathcal{U}^{(1)}_{F;j}(\mathbf{v})\mathcal{U}_F^\dagger(\mathbf{w})\Big]\right\rangle$$

$$d^{(2)}(\mathbf{v}_*,\mathbf{w}) = \left\langle\frac{1}{N_c}\text{Tr}\Big[\mathcal{U}^{(2)}_{F}(\mathbf{v})\mathcal{U}_F^\dagger(\mathbf{w})\Big]\right\rangle$$

$$\tilde{d}(\mathbf{v}_*,\mathbf{w}_*) = \left\langle\frac{1}{N_c}\text{Tr}\Big[\Big(\mathcal{U}_F(\mathbf{v},b^-)\overleftrightarrow{\partial_-}\mathcal{U}_F^\dagger(\mathbf{w},b^-)\Big)\Big|_{b^-=0}\Big]\right\rangle,$$

$$Q_j^{(1)}(\mathbf{w}',\mathbf{v}',\mathbf{v}_*,\mathbf{w}) = \left\langle\frac{1}{N_c}\text{Tr}\Big[\mathcal{U}_F(\mathbf{w}')\mathcal{U}_F^\dagger(\mathbf{v}')\mathcal{U}^{(1)}_{F;j}(\mathbf{v})\mathcal{U}_F^\dagger(\mathbf{w})\Big]\right\rangle$$

$$Q^{(2)}(\mathbf{w}',\mathbf{v}',\mathbf{v}_*,\mathbf{w}) = \left\langle\frac{1}{N_c}\text{Tr}\Big[\mathcal{U}_F(\mathbf{w}')\mathcal{U}_F^\dagger(\mathbf{v}')\mathcal{U}^{(2)}_{F}(\mathbf{v})\mathcal{U}_F^\dagger(\mathbf{w})\Big]\right\rangle$$

$$\tilde{Q}(\mathbf{w}',\mathbf{v}',\mathbf{v}_*,\mathbf{w}_*) = \left\langle\frac{1}{N_c}\text{Tr}\Big[\mathcal{U}_F(\mathbf{w}')\mathcal{U}_F^\dagger(\mathbf{v}')\Big(\mathcal{U}_F(\mathbf{v},b^-)\overleftrightarrow{\partial_-}\mathcal{U}_F^\dagger(\mathbf{w},b^-)\Big)\Big|_{b^-=0}\Big]\right\rangle$$

Back-to-back production limit

Back-to-back limit of dijets are conveniently expressed in terms of:

Dominguez, Marquet, Xiao, Yuan (2011)

(total dijet momentum) $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$ and (relative momentum) $\mathbf{P} = (z_2\mathbf{k}_1 - z_1\mathbf{k}_2)$

$z_1 = k_1^+/(k_1^+ + k_2^+)$ and $z_2 = k_2^+/(k_1^+ + k_2^+) = 1 - z_1$ such that

$\mathbf{k_1} = \mathbf{P} + z_1\mathbf{k}$ $\qquad$ $\mathbf{k}_2 = -\mathbf{P} + z_2\mathbf{k}$

Back-to-back correlation limit: $|\mathbf{k}| \ll |\mathbf{P}|$

In coordinate space:

(conjugate to $\mathbf{P}$) $\mathbf{r} = \mathbf{v} - \mathbf{w}$

(conjugate to $\mathbf{k}$) $\mathbf{b} = z_1\mathbf{v} + z_2\mathbf{w}$

such that

$\mathbf{w} = \mathbf{b} - z_1\mathbf{r}$

$\mathbf{v} = \mathbf{b} + z_2\mathbf{r}$

Back-to-back correlation limit: $|\mathbf{r}| \ll |\mathbf{b}|$

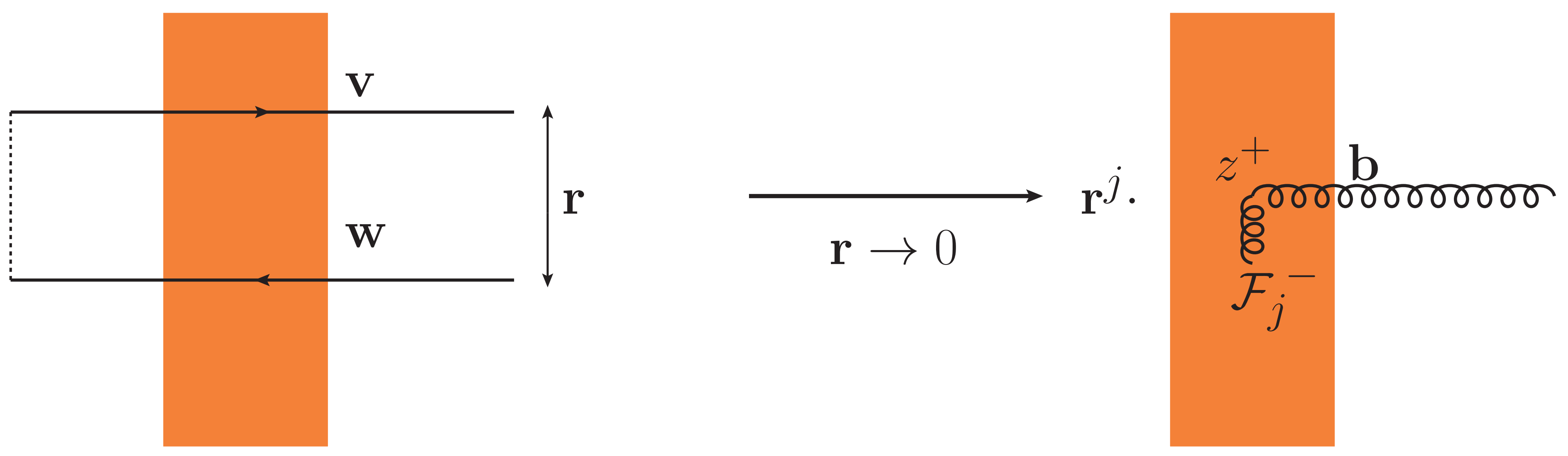
perform a small $\mathbf{r}$ expansion at the level of the squared amplitude

Small $\mathbf{r}$ expansion in the back-to-back limit (i)

perform small $\mathbf{r}$ expansion, keep appropriate order in the expansion (in order to capture the interplay between subleasing power terms)
use $\big[U_R T^a_R U^\dagger_R\big]_{ij} = \big[T^b_R\big]_{ij}\big(U_A\big)_{ab}$ to simplify the results

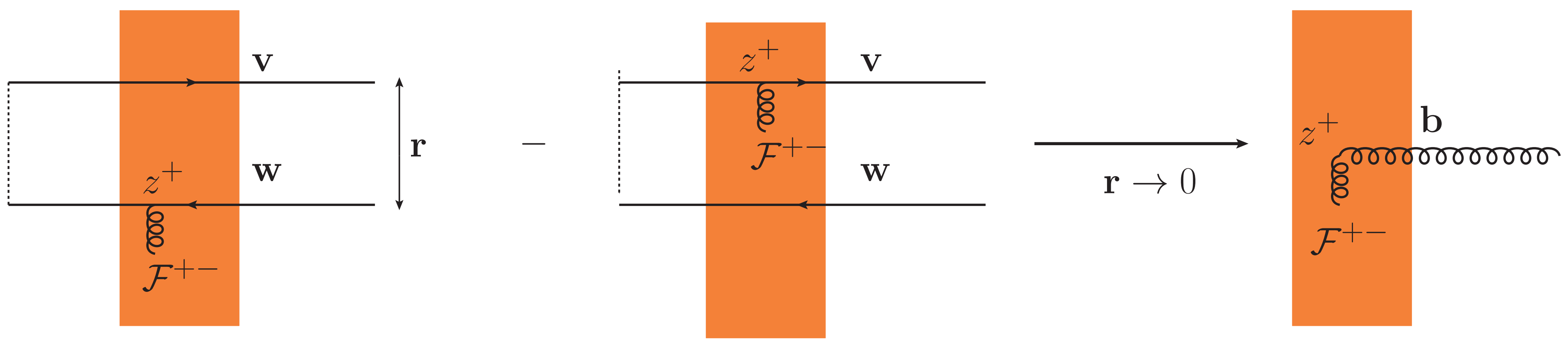
Open dipole from Gen. Eik.

$$\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}}\Big[\mathcal{U}_F\Big(\mathbf{b}+z_2\,\mathbf{r},b^-\Big)\mathcal{U}_F^\dagger\Big(\mathbf{b}-z_1\,\mathbf{r},b^-\Big)-1\Big]$$
$$=\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}}\Big[-i\Big(1+\frac{i(z_2-z_1)}{2}\mathbf{r}\cdot\mathbf{k}\Big)\mathbf{r}^j\,t^{a'}\int dv^+\,\mathcal{U}_A\left(+\infty,v^+;\mathbf{b},b^-\right)_{a'a}g\mathcal{F}_j{}^-{}_a(v^+,\mathbf{b},b^-)$$
$$-\frac{1}{2}\mathbf{r}^i\mathbf{r}^j\,t^{a'}t^{b'}\int dv^+\int dw^+\,\mathcal{U}_A\left(+\infty,v^+;\mathbf{b},b^-\right)_{a'a}g\mathcal{F}_i{}^-{}_a(v^+,\mathbf{b},b^-)$$
$$\times\mathcal{U}_A\left(+\infty,w^+;\mathbf{b},b^-\right)_{b'b}g\mathcal{F}_j{}^-{}_b(w^+,\mathbf{b},b^-)\;+O\left(|\mathbf{r}|^3\right)\Big]$$



Open dec. dipole from dynamics

$$\Big[\mathcal{U}_F\Big(\mathbf{v},b^-\Big)\overleftrightarrow{\partial}_{b^-}\mathcal{U}_F^\dagger\Big(\mathbf{w},b^-\Big)\Big]\Big|_{b^-=0}=\int_{z^+}\Big\{\mathcal{U}_F(\mathbf{v})\mathcal{U}_F^\dagger\Big(z^+,-\infty;\mathbf{w}\Big)igt\cdot\mathcal{F}^{+-}(z^+,\mathbf{w})\mathcal{U}_F^\dagger\Big(+\infty,z^+;\mathbf{w}\Big)$$
$$-\mathcal{U}_F\Big(+\infty,z^+;\mathbf{v}\Big)(-ig)t\cdot\mathcal{F}^{+-}(z^+,\mathbf{v})\mathcal{U}_F\Big(z^+,-\infty;\mathbf{v}\Big)\mathcal{U}_F^\dagger(\mathbf{w})\Big\}$$
$$=2igt^a\int_{z^+}\mathcal{U}_A\Big(+\infty,z^+;\mathbf{b}\Big)_{ab}\mathcal{F}_b^{+-}(z^+,\mathbf{b})+O(|\mathbf{r}|)$$



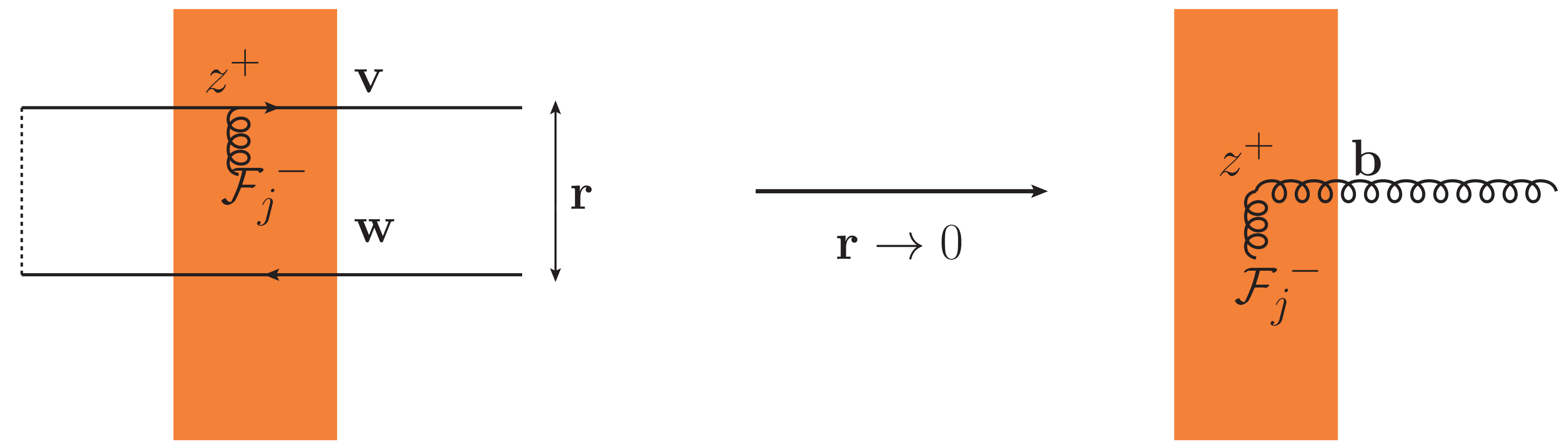
Small $\mathbf{r}$ expansion in the back-to-back limit (ii)

structures appear in decorations on quark line

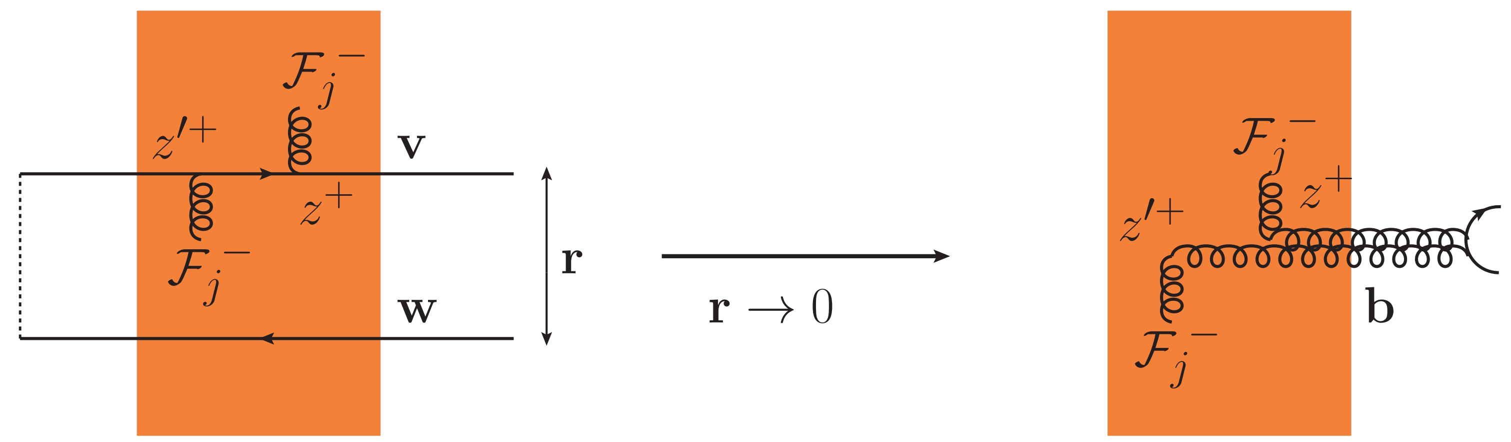
$$\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big[-\Big(\mathbf{P}^j + \frac{(z_1-z_2)}{2}\mathbf{k}^j\Big) \mathcal{U}^{(1)}_{F;j}(\mathbf{b}+z_2\mathbf{r})\mathcal{U}^\dagger_F(\mathbf{b}-z_1\mathbf{r}) + \frac{i}{2}\mathcal{U}^{(1)}_{F;j}(\mathbf{b}+z_2\mathbf{r})\,\partial_j\mathcal{U}^\dagger_F(\mathbf{b}-z_1\mathbf{r}) - i\mathcal{U}^{(2)}_F(\mathbf{b}+z_2\mathbf{r})\mathcal{U}^\dagger_F(\mathbf{b}-z_1\mathbf{r}) \Big]$$

$$= \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big\{ \Big[-\mathbf{P}^j + \frac{(z_2-z_1)}{2}\mathbf{k}^j - iz_2\mathbf{P}^j(\mathbf{r}\cdot\mathbf{k}) \Big] \mathcal{U}^{(1)}_{F;j}(\mathbf{b})\mathcal{U}^\dagger_F(\mathbf{b}) + \Big[\frac{i}{2}\delta^{ij} + \mathbf{P}^j\mathbf{r}^i\Big] \mathcal{U}^{(1)}_{F;j}(\mathbf{b})\,\partial_i\mathcal{U}^\dagger_F(\mathbf{b}) - i\mathcal{U}^{(2)}_F(\mathbf{b})\mathcal{U}^\dagger_F(\mathbf{b}) + O(|\mathbf{r}|) \Big\}$$

$$\mathcal{U}^{(1)}_{F;j}(\mathbf{b})\mathcal{U}^\dagger_F\big(\mathbf{b}\big) = 2it^{a'}\int_{z^+} z^+\,\mathcal{U}_A\big(+\infty, z^+;\mathbf{b}\big)_{a'a}\, g\mathcal{F}^{a}_{j}{}^{-}(z^+,\mathbf{b})$$



$$\mathcal{U}^{(2)}_F(\mathbf{b})\mathcal{U}^\dagger_F(\mathbf{b}) = -t^{a'}t^{b'}\int_{z^+,z'^+} (z^+-z'^+)\,\theta(z^+-z'^+)\mathcal{U}_A\big(+\infty, z^+;\mathbf{b}\big)_{a'a}\, g\mathcal{F}^{a}_{j}{}^{-}(z^+,\mathbf{b}) \times\mathcal{U}_A\big(+\infty, z'^+;\mathbf{b}\big)_{b'b}\, g\mathcal{F}^{b}_{j}{}^{-}(z'^+,\mathbf{b})$$



$$\mathcal{U}^{(1)}_{F;j}(\mathbf{b})\,\partial_i\mathcal{U}^\dagger_F(\mathbf{b}) = -2t^{a'}t^{b'}\int dz^+\int dz'^+\, z^+\,\mathcal{U}_A(+\infty,z^+;\mathbf{b})_{a'a}\; g\mathcal{F}^{a}_{j}{}^{-}(z^+,\mathbf{b})\mathcal{U}_A(+\infty,z'^+;\mathbf{b})_{b'b}\; g\mathcal{F}^{b}_{i}{}^{-}(z'^+,\mathbf{b})$$

Back-to-back cross section: Gen. Eik contribution

$\langle \mathcal{F}_\perp^{\ -} \mathcal{F}_\perp^{\ -} \rangle$ part of Gen. Eik. contribution

$$\frac{d\sigma_{\gamma_L^* \to q_1 \bar{q}_2}}{d\mathrm{P.S.}}\Big|^{\mathcal{F}_\perp^{\ -}\mathcal{F}_\perp^{\ -}}_{\mathrm{Gen.Eik}} = g^2 (ee_f)^2 Q^2 (q^+ + k_1^+ - k_2^+)^2 (q^+ - k_1^+ + k_2^+)^2 \frac{k_1^+ k_2^+}{4(q^+)^6} \Big[\frac{4\mathbf{P}^i \mathbf{P}^j}{(\mathbf{P}^2 + \hat{Q}^2)^4} - 2(z_2 - z_1) \frac{(\mathbf{P}^i \mathbf{k}^j + \mathbf{k}^i \mathbf{P}^j)}{[\mathbf{P}^2 + \hat{Q}^2]^4}$$
$$+ 16(z_2 - z_1) \frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i \mathbf{P}^j}{[\mathbf{P}^2 + \hat{Q}^2]^5} + O\Big(\frac{\mathbf{k}^2}{\mathbf{P}^8}\Big)\Big] (2q^+) \int d(\Delta b^-) e^{i\Delta b^- (k_1^+ + k_2^+ - q^+)} \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+, z'^+}$$
$$\times \Big\langle \mathcal{F}_i^{a\ -}\Big(z'^+, \mathbf{b}', -\frac{\Delta b^-}{2}\Big) \Big[\mathcal{U}_A^\dagger\Big(+\infty, z'^+; \mathbf{b}', -\frac{\Delta b^-}{2}\Big) \mathcal{U}_A\Big(+\infty, z^+; \mathbf{b}, \frac{\Delta b^-}{2}\Big)\Big]_{ab} \mathcal{F}_j^{b\ -}\Big(z^+, \mathbf{b}, \frac{\Delta b^-}{2}\Big) \Big\rangle$$

There is also $\langle \mathcal{F}_\perp^{\ -} \mathcal{F}_\perp^{\ -} \mathcal{F}_\perp^{\ -} \rangle$ which are twist 3 contributions that are not written here explicitly

Kinematic twist 3 corrections (suppressed by $|\mathbf{k}|/|\mathbf{P}|$)

Strict Eik. Limit

$$\frac{d\sigma_{\gamma_L^* \to q_1 \bar{q}_2}}{d\mathrm{P.S.}}\Big|^{\mathcal{F}_\perp^{\ -}\mathcal{F}_\perp^{\ -}}_{\mathrm{Strict.Eik}} = (2q^+) 2\pi \delta(k_1^+ + k_2^+ - q^+) (ee_f)^2 g^2 4 z_1^3 z_2^3 Q^2 \Big[\frac{4\mathbf{P}^i \mathbf{P}^j}{(\mathbf{P}^2 + \bar{Q}^2)^4} - 2(z_2 - z_1) \frac{(\mathbf{P}^i \mathbf{k}^j + \mathbf{k}^i \mathbf{P}^j)}{[\mathbf{P}^2 + \bar{Q}^2]^4} + 16(z_2 - z_1) \frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i \mathbf{P}^j}{[\mathbf{P}^2 + \bar{Q}^2]^5} \Big]$$
$$\times \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+, z'^+} \Big\langle \mathcal{F}_i^{a\ -}(z'^+, \mathbf{b}') \big[\mathcal{U}_A^\dagger(+\infty, z'^+; \mathbf{b}') \mathcal{U}_A(+\infty, z^+; \mathbf{b}) \big]_{ab} \mathcal{F}_j^{b\ -}(z^+, \mathbf{b}) \Big\rangle$$

Correlator related to twist 2 gluon TMD at x $= 0$

Difference between Gen. Eik and strict Eik. terms involve correlators of the type $\langle \mathcal{F}_\perp^{\ -} \mathcal{F}_\perp^{\ -} \mathcal{F}^{+-} \rangle$ or $\mathbf{k}\langle \mathcal{F}_\perp^{\ -} \mathcal{F}^{+-} \rangle$ These are NEik but twist 4 contributions.

Back-to-back cross section: Twist 2 gluon TMDs

Including all $\langle \mathcal{F}_\perp^{\;-}\mathcal{F}_\perp^{\;-}\rangle$ contributions

Kinematic twist 2 — Kinematic twist 3

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|^{\mathcal{F}_\perp^{\;-}\mathcal{F}_\perp^{\;-}} = (2q^+)2\pi\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 4z_1^3 z_2^3 Q^2\left[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4} - 2(z_2-z_1)\frac{(\mathbf{P}^i\mathbf{k}^j+\mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\bar{Q}^2]^4} + 16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\bar{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\right]$$

$$\times\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\left[1 + i(z^+ - z'^+)\frac{(\mathbf{P}^2+\bar{Q}^2)}{2q^+z_1z_2} + \text{NNEik}\right]\Big\langle \mathcal{F}_i^{a\;-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(+\infty,z'^+;\mathbf{b}')\mathcal{U}_A(+\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_j^{b\;-}(z^+,\mathbf{b})\Big\rangle$$

Eik — NEik — Correlator related to twist 2 gluon TMD at x $= 0$

NEik corrections and kinematic twist 3 corrections to $\langle \mathcal{F}_\perp^{\;-}\mathcal{F}_\perp^{\;-}\rangle$ contribution factorizes from each other!

TMD $\langle \mathcal{F}_\perp^{\;-}\mathcal{F}_\perp^{\;-}\rangle$ correlator in an unpolarized target:

$$\Phi^{\mu\nu;\rho\sigma}(\text{x},\mathbf{k}) \equiv \frac{1}{\text{x}P_{tar}^-}\frac{1}{(2\pi)^3}\int d^2\mathbf{z}\, e^{-i\mathbf{k}\cdot\mathbf{z}}\int dz^+\, e^{i\text{x}P_{tar}^- z^+}\Big\langle P_{tar}\Big|\mathcal{F}_a^{\mu\nu}(0)\left[\mathcal{U}_A^\dagger\left(+\infty,0;0\right)\mathcal{U}_A\left(+\infty,z^+;\mathbf{z}\right)\right]_{ab}\mathcal{F}_b^{\rho\sigma}(z^+,\mathbf{z})\Big|P_{tar}\Big\rangle$$

$$= \frac{1}{(2\pi)^3}\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+} e^{i\text{x}P_{tar}^-(z^+-z'^+)}\Big\langle \mathcal{F}_a^{\mu\nu}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(+\infty,z'^+;\mathbf{b}')\mathcal{U}_A(+\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_b^{\rho\sigma}(z^+,\mathbf{b})\Big\rangle,$$

Back-to-back cross section: twist 2 and twist 3 gluon TMDs

Twist 2 gluon TMD with non-zero momentum fraction x:

$$\frac{d\sigma_{\gamma^*_{L,T}\to q_1\bar{q}_2}}{dz_1\, d^2\mathbf{P}\, d^2\mathbf{k}}\bigg|^{\mathcal{F}_\perp^-\mathcal{F}_\perp^-} = \alpha_{\rm em}\,\alpha_s\, e_f^2\, \mathcal{C}^{ij}_{T,L}(z_1,\mathbf{P},\mathbf{k})\bigg\{\bigg[1+\frac{(\mathbf{P}^2+\bar{Q}^2)}{z_1 z_2 W^2}\,\partial_{\rm x} + \text{NNEik}\bigg]\bigg[{\rm x}\,\Phi^{i-;j-}({\rm x},\mathbf{k})\bigg]\bigg\}\bigg|_{{\rm x}=0}$$

$$= \alpha_{\rm em}\,\alpha_s\, e_f^2\, \mathcal{C}^{ij}_{T,L}(z_1,\mathbf{P},\mathbf{k})\bigg\{\bigg[{\rm x}\,\Phi^{i-;j-}({\rm x},\mathbf{k})\bigg]\bigg|_{{\rm x}=\frac{(\mathbf{P}^2+\bar{Q}^2)}{z_1 z_2 W^2}} + \text{NNEik}\bigg\}$$

with

$$\Phi^{i-;j-}({\rm x},\mathbf{k}) = \frac{\delta^{ij}}{2}\, f_1^g({\rm x},\mathbf{k}) + \bigg[\mathbf{k}^i\mathbf{k}^j - \frac{\mathbf{k}^2}{2}\,\delta^{ij}\bigg]\frac{1}{2M^2}\, h_1^{\perp g}({\rm x},\mathbf{k})$$

Non-zero values of momentum fraction x in the twist 2 gluon TMDs are recovered from NEik corrections!

Twist 3 gluon TMDs with momentum fraction x=0: Mulders, Rodrigues (2001) Lorce, Pasquini (2013)

$$\frac{d\sigma_{\gamma^*_{L,T}\to q_1\bar{q}_2}}{dz_1\, d^2\mathbf{P}\, d^2\mathbf{k}}\bigg|^{\mathcal{F}_\perp^-\mathcal{F}^{+-}}_{NEik} = \alpha_{\rm em}\,\alpha_s\, e_f^2\, \frac{1}{2q^+}\mathcal{C}^{j}_{T,L}(z_1,\mathbf{P})\bigg[{\rm x}\,\Phi^{j-;+-}({\rm x},\mathbf{k}) + {\rm x}\,\Phi^{+-;j-}({\rm x},\mathbf{k})\bigg]\bigg|_{{\rm x}=0}$$

with

$$\Phi^{j-;+-}({\rm x},\mathbf{k}) + \Phi^{+-;j-}({\rm x},\mathbf{k}) = \frac{2\mathbf{k}^j}{P^-_{tar}}\, f^{\perp g}({\rm x},\mathbf{k})$$

$\langle\mathcal{F}^{+-}\mathcal{F}_\perp^-\rangle$ terms →originate from the interference between the non-static NEik corrections and Eik amplitudes.

$$\frac{d\sigma_{\gamma^*_{L,T}\to q_1\bar{q}_2}}{dz_1\, d^2\mathbf{P}\, d^2\mathbf{k}}\bigg|^{\mathcal{F}_\perp^-\mathcal{F}^{ij}}_{NEik} = \alpha_{\rm em}\,\alpha_s\, e_f^2\, \frac{1}{2q^+}\mathcal{C}^{ijl}_{T,L}(z_1,\mathbf{P})\bigg[{\rm x}\,\Phi^{l-;ij}({\rm x},\mathbf{k}) + {\rm x}\,\Phi^{ij;l-}({\rm x},\mathbf{k})\bigg]\bigg|_{{\rm x}=0}$$

with

$$\Phi^{l-;ij}({\rm x},\mathbf{k}) + \Phi^{ij;l-}({\rm x},\mathbf{k}) = \epsilon^{ij}\,\epsilon^{ln}\,\frac{2\mathbf{k}^n}{P^-_{tar}}\,\bar{g}^{\perp g}({\rm x},\mathbf{k})$$

$\langle\mathcal{F}^{ij}\mathcal{F}_\perp^-\rangle \to$ originate from the interference between the NEik correction with $\mathcal{U}^{(3)}_{F;ij}$ and the strict Eikonal amplitude

absent in the γ^*_L: $\mathcal{C}^{ijl}_L(z_1,\mathbf{P}) = 0$ due to Dirac algebra.

qg dijet production in DIS at NEik accuracy from quark background field

TA, Armesto, Beuf - Phys. Rev. D 108 (2023) 7, 074023

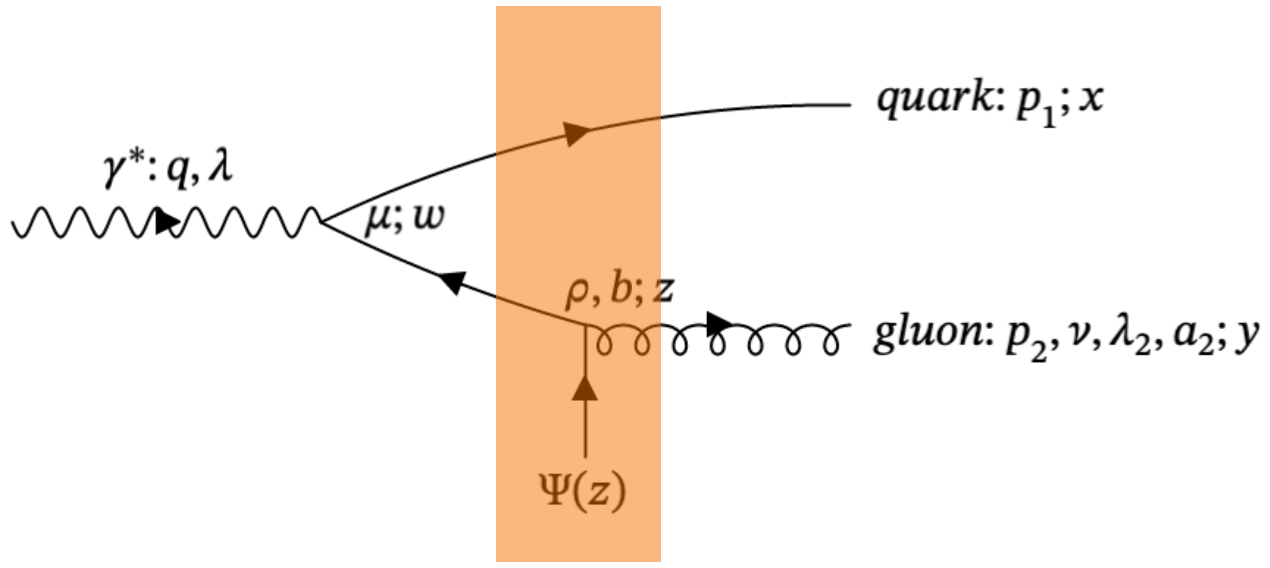
qg dijet production in DIS

TA, Armesto, Beuf (2023)

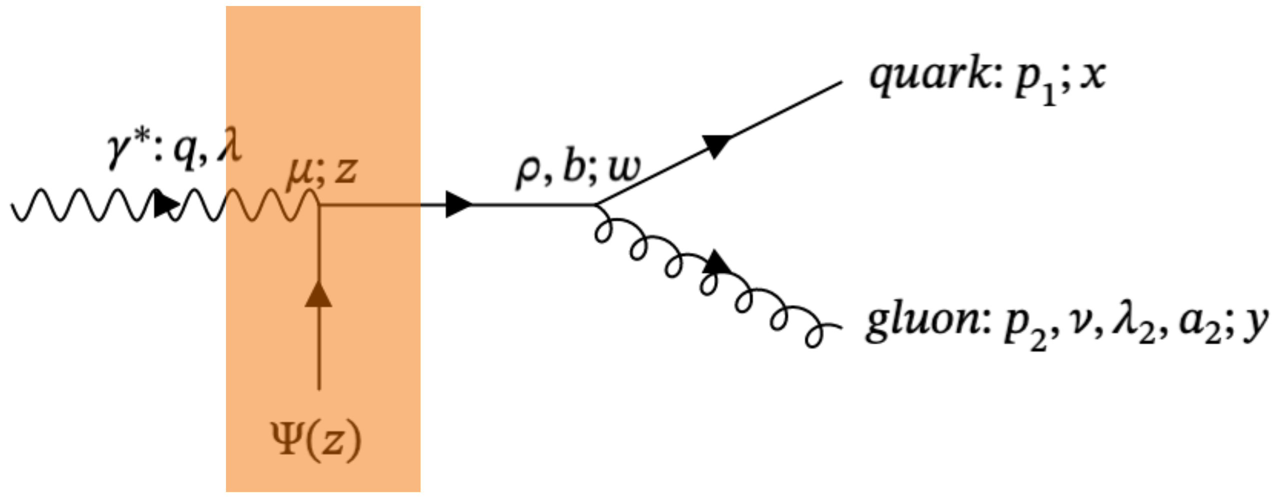
Gluon TMDs are dominant in the eikonal CGC.

Beyond eikonal accuracy quark background field is included in the CGC → **possibility to probe quark TMDs from non-eikonal CGC!**

Two mechanisms that contribute to the production of quark-gluon dijet:



$$S^{\rm bef}_{\gamma\to q_1 g_2} = \lim_{x^+,y^+\to+\infty}\int_{\mathbf{x},\mathbf{y}}\int_{x^-,y^-} e^{ip_1\cdot x}\,\bar{u}(1)\gamma^+\, e^{ip_2\cdot y}\epsilon^{\lambda_2}_\nu(p_2)^*(-2p_2^+) \times \int_{w,z} e^{-iq\cdot w}\epsilon^\lambda_\mu(q)\, G_F^{\nu\rho}(y,z)_{a_2 b} S_F(x,w)(-iee_f)\gamma^\mu S_F(w,z)(-ig)\gamma_\rho t^b \Psi(z)$$



$$S^{\rm in}_{\gamma\to q_1 g_2} = \lim_{x^+,y^+\to+\infty}\int_{\mathbf{x},\mathbf{y}}\int_{x^-,y^-} e^{ip_1\cdot x}\,\bar{u}(1)\gamma^+\, e^{ip_2\cdot y}\epsilon^{\lambda_2}_\nu(p_2)^*(-2p_2^+) \times \int_{w,z} e^{-iq\cdot z}\epsilon^\lambda_\mu(q)\, G_{F,0}^{\nu\rho}(y,w)_{a_2 b} S_{F,0}(x,w)(-ig)\gamma_\rho t^b S_F(w,z)(-iee_f)\gamma^\mu \Psi(z)$$

inside-after quark/gluon propagators together with before-inside and before-after quark propagators are computed at Eik. order.

Full set of propagators: before-inside, inside-after, before-after quark/gluon propogators are computed later. TA, Beuf, Mulani (2025)

Contribution from photon splitting before the medium

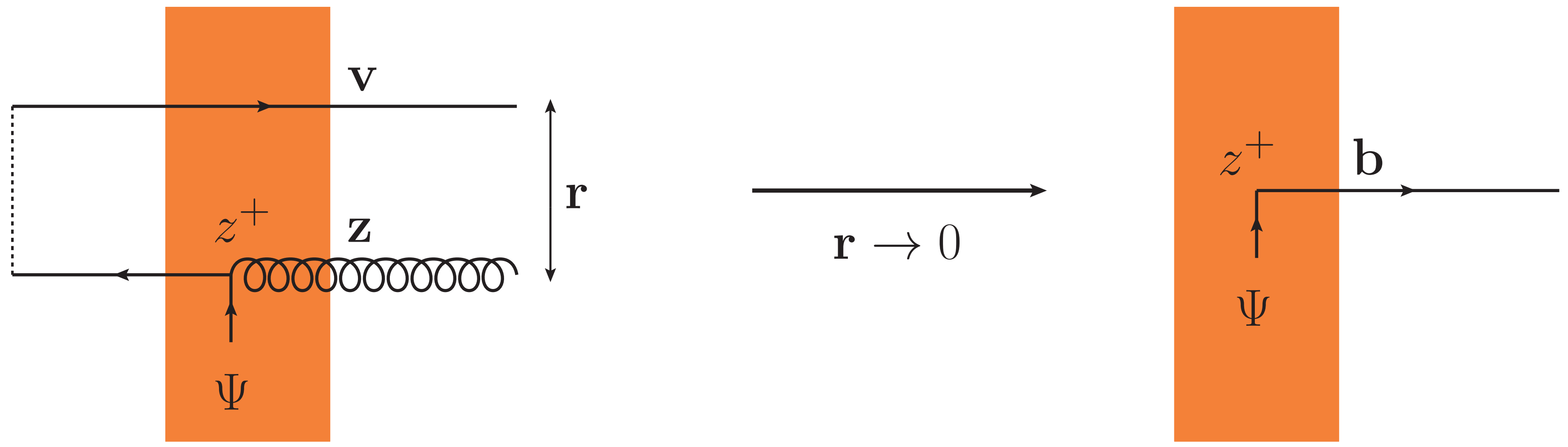
Using the explicit expressions for various propagators, one gets

$$S^{\text{bef}}_{\gamma_{T,L}\to q_1 g_2} = iee_f g 2\pi\delta(p_1^+ + p_2^+ - q^+)\int_{\mathbf{v},\mathbf{z}} e^{-i\mathbf{v}\cdot\mathbf{p}_1 - i\mathbf{z}\cdot\mathbf{p}_2}\int\frac{d^2\mathbf{K}}{(2\pi)^2}\frac{e^{i(\mathbf{v}-\mathbf{z})\cdot\mathbf{K}}}{\left[\mathbf{K}^2 + m^2 + \frac{p_1^+ p_2^+}{(q^+)^2}Q^2\right]}\bar{u}(1)\frac{\gamma^+\gamma^-}{2}\Gamma^{\text{bef}}_{T,L}\int_{z^+} U_A\left(+\infty, z^+;\mathbf{z}\right)_{a_2 b} U_F(\mathbf{v}) U_F^\dagger\left(z^+, -\infty;\mathbf{z}\right) t^b \Psi(z^+,\mathbf{z})$$

with $\Gamma^{\text{bef}}_L = 2\frac{p_1^+ p_2^+}{(q^+)^2} Q\varepsilon^{j*}_{\lambda_2}\gamma^j$ $\qquad \Gamma^{\text{bef}}_T = \varepsilon^i_\lambda\,\varepsilon^{j*}_{\lambda_2}\left\{\mathbf{K}^l\left[\left(\frac{p_1^+ - p_2^+}{q^+}\right)\delta^{il} - \frac{[\gamma^i,\gamma^l]}{2}\right] + m\gamma^i\right\}\gamma^j$

In the back-to-back limit:

$$U_A\left(+\infty, z^+;\mathbf{z}\right)_{a_2 b} U_F(\mathbf{v}) U_F^\dagger\left(z^+, -\infty;\mathbf{z}\right) t^b \Psi(z^+,\mathbf{z}) \;\to U_A\left(+\infty, z^+;\mathbf{b}\right)_{a_2 b} U_F(\mathbf{b}) U_F^\dagger\left(z^+, -\infty;\mathbf{b}\right) t^b \Psi(z^+,\mathbf{b}) = t^{a_2} U_F\left(+\infty, z^+;\mathbf{b}\right)\Psi(z^+,\mathbf{b})$$



$$S^{\text{bef}}_{\gamma_{T,L}\to q_1 g_2} \simeq i\frac{ee_f g\, 2\pi\delta(p_1^+ + p_2^+ - q^+)}{\left[\mathbf{P}^2 + \bar{Q}^2\right]}\,\bar{u}(1)\frac{\gamma^+\gamma^-}{2}\Gamma^{\text{bef}}_{T,L}\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}}\int_{z^+} t^{a_2} U_F\left(+\infty, z^+;\mathbf{b}\right)\Psi(z^+,\mathbf{b})$$

Contribution from photon conversion to quark inside medium

Using the explicit expressions for various propagators, one gets

$$S^{\text{in}}_{\gamma_T\to q_1 g_2} = i\frac{ee_f g 2\pi\delta(p_1^+ + p_2^+ - q^+)}{\left[\left(\mathbf{p}_1 - \frac{p_1^+}{p_2^+}\mathbf{p}_2\right)^2 + m^2\right]}\int_{\mathbf{z}} e^{-i\mathbf{z}\cdot(\mathbf{p}_1+\mathbf{p}_2)}\bar{u}(1)\frac{\gamma^+\gamma^-}{2}\Gamma^{\text{in}}_T\int_{z^+} t^{a_2}U_F\left(+\infty, z^+;\mathbf{z}\right)\Psi(z^+,\mathbf{z})$$

with $$\Gamma^{\text{in}}_T = \varepsilon^{l*}_{\lambda_2}\varepsilon^j_\lambda\left\{\left[\mathbf{p}_1^i - \frac{p_1^+}{p_2^+}\mathbf{p}_2^i\right]\left[-\left(\frac{2p_1^+ + p_2^+}{p_2^+}\right)\delta^{il} + \frac{[\gamma^i,\gamma^l]}{2}\right] + m\gamma^l\right\}\gamma^j$$

$S^{\text{in}}_{\gamma_L\to q_1 g_2} = 0$ at NEik in Light-cone gauge, because $\not{\epsilon}_L(q)\,\Psi^{(-)}(z) = \frac{Q}{q^+}\,\gamma^+\,\Psi^{(-)}(z) = 0.$

Back-to-back limit is not needed for this contribution. Upon changing $\mathbf{z}\to\mathbf{b}$:

$$S^{\text{in}}_{\gamma_T\to q_1 g_2} = i\frac{ee_f g\, 2\pi\delta(p_1^+ + p_2^+ - q^+)}{[\mathbf{P}^2 + (1-z)^2 m^2]}\,(1-z)^2\,\bar{u}(1)\frac{\gamma^+\gamma^-}{2}\Gamma^{\text{in}}_T\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}}\int_{z^+} t^{a_2}U_F\left(+\infty, z^+;\mathbf{b}\right)\Psi(z^+,\mathbf{b})$$

Quark TMDs from back-to-back dijet production in DIS

In the back-to-back limit, the cross section $$(2\pi)^6(2p_1^+)(2p_2^+)\frac{d\sigma^{\gamma_{T,L}\to q_1 g_2}}{dp_1^+ d^2\mathbf{p}_1 dp_2^+ d^2\mathbf{p}_2}\bigg|_{\text{corr.lim.}} = 2\pi\delta(p_1^+ + p_2^+ - q^+)\,(4\pi)^2\alpha_{\text{em}}\alpha_s C_F e_f^2\,\mathcal{H}_{T,L}(\mathbf{P},z,Q)\mathcal{T}(\mathbf{k})$$

with the hard factors $$\mathcal{H}_L = \frac{4Q^2z^3(1-z)^2}{[\mathbf{P}^2+\bar{Q}^2]^2}$$ and $$\mathcal{H}_T = z\left\{\frac{(1+z^2)\mathbf{P}^2+(1-z)^4m^2}{[\mathbf{P}^2+(1-z)^2m^2]^2} + \frac{[z^2+(1-z)^2]\mathbf{P}^2+m^2}{[\mathbf{P}^2+\bar{Q}^2]^2} - \frac{2z^2\mathbf{P}^2}{[\mathbf{P}^2+\bar{Q}^2][\mathbf{P}^2+(1-z)^2m^2]}\right\}$$

target averaged color operator $$\mathcal{T}(\mathbf{k}) = \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\Big\langle \bar{\Psi}(z'^+,\mathbf{b}')\gamma^- U_F^\dagger(+\infty,z'^+;\mathbf{b}')U_F(+\infty,z^+;\mathbf{b})\Psi(z^+,\mathbf{b})\Big\rangle$$

unpolarized quark TMD $$f_1^q(x,\mathbf{k}) = \frac{1}{(2\pi)^3}\int_{\mathbf{b}} e^{i\mathbf{k}\cdot\mathbf{b}}\int_{z^+} e^{-iz^+xP^-_{tar}}\langle P_{tar}|\bar{\Psi}(z^+,\mathbf{b})\frac{\gamma^-}{2}U_F^\dagger(+\infty,z^+;\mathbf{b})U_F(+\infty,0;\mathbf{0})\Psi(0,\mathbf{0})|P_{tar}\rangle$$

CGC average ↔ quantum expectation value in target state $$\langle\mathcal{O}\rangle = \lim_{P'_{tar}\to P_{tar}}\frac{\langle P'_{tar}|\hat{\mathcal{O}}|P_{tar}\rangle}{\langle P'_{tar}|P_{tar}\rangle} = \lim_{P'_{tar}\to P_{tar}}\frac{\langle P'_{tar}|\hat{\mathcal{O}}|P_{tar}\rangle}{2P^-_{tar}\,(2\pi)^3\,\delta(P'^-_{tar}-P^-_{tar})\,\delta^{(2)}(\mathbf{P}'_{tar}-\mathbf{P}_{tar})}$$ ➡ $$\boxed{\mathcal{T}(\mathbf{k}) = \frac{(2\pi)^3}{P^-_{tar}}f_1^q(x=0,\mathbf{k})}$$

Summary and outlook

$q\bar{q}$ production in the back-to-back limit in a pure gluon background field at NEik accuracy in DIS

- ⁕ probed Twist 2 gluon TMDs ➞ $\langle \mathcal{F}_i{}^{-} \mathcal{F}_j{}^{-} \rangle$
  - ◆ shown the factorization of the kinematic twist 3 and of NEik corrections
  - ◆ NEik correction is shown to be the first order correction to the Taylor expansion of TMD around x=0
  - ◆ Non-zero x dependance of the TMD is obtained by resuming corrections to all orders beyond Eik approximation
- ⁕ probed Twist 3 gluon TMDs ➞ $\langle \mathcal{F}_i{}^{-} \mathcal{F}^{+-} \rangle$ and $\langle \mathcal{F}_l{}^{-} \mathcal{F}_{ij} \rangle$ (for transversely polarized photon)
  - ◆ further contributions from NEik corrections
- ⁕ probed 3 body twist 3 correlators ➞ $\langle \mathcal{F}_i{}^{-} \mathcal{F}_j{}^{-} \mathcal{F}_l{}^{-} \rangle$
  - ◆ appear already at Eik. Order. How to resum and its parametrization? on going work — TA, Beuf, Czajka, Goslawski

qg production in the back-to-back limit in DIS at NEik accuracy via t-channel quark exchange

- ⁕probed unpolarized quark TMD at x=0

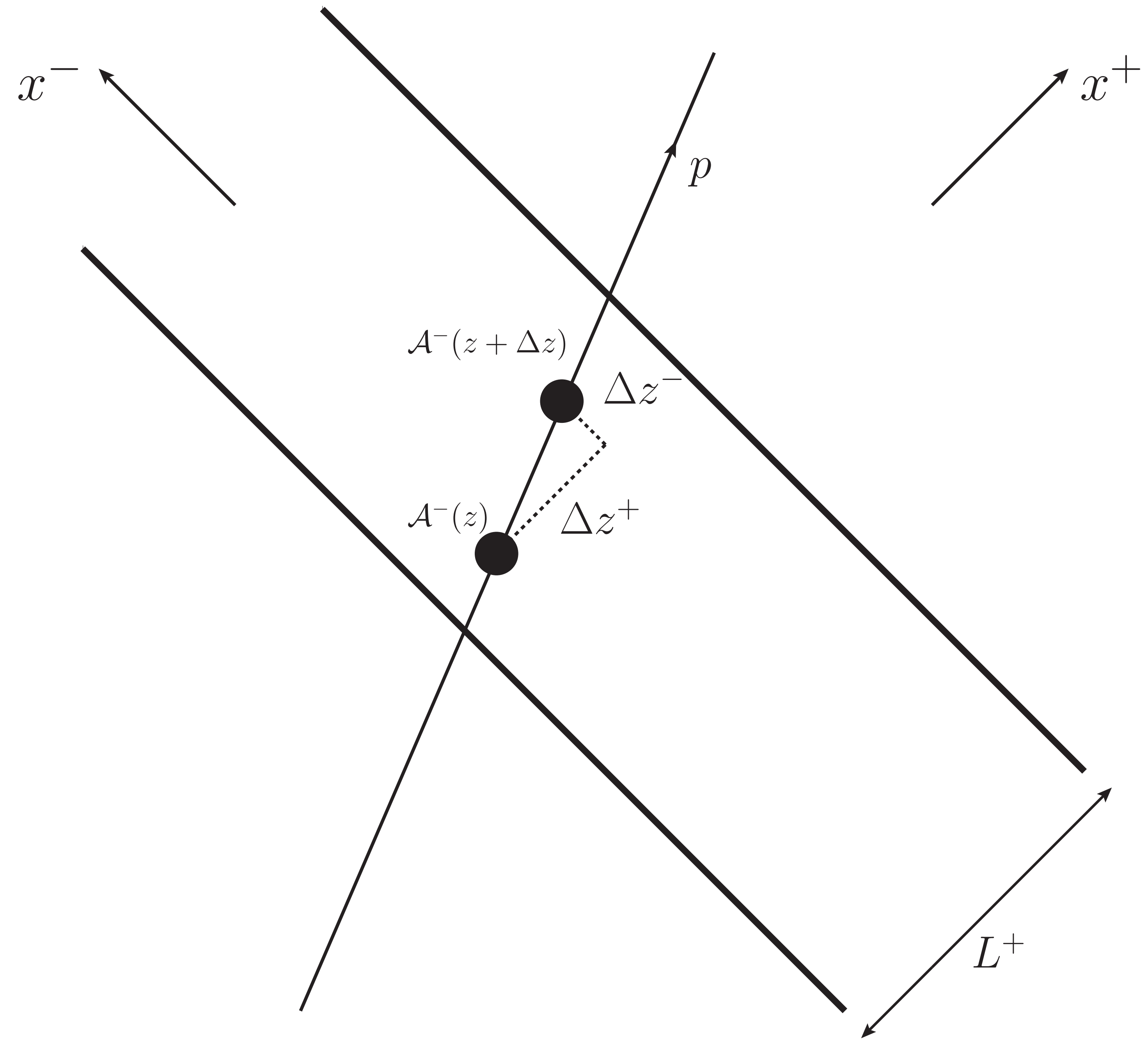
Inclusive DIS and SIDIS at NEik accuracy (both quark and gluon background contributions) with NLO corrections — on going work — TA, Beuf, Favrel, Fucilla

BACK UP

NEik correction from beyond static approximation

Effect of relative z^- dependence of $\mathcal{A}^-$ insertions along one propagator:

$$\mathcal{A}^-(z^- + \Delta z^-) - \mathcal{A}^-(z^-) \simeq \Delta z^- \, \partial_- \mathcal{A}^-(z^-)$$



- Slow z^- dependence from time dilation:

$$\partial_- \mathcal{A}^- \propto \frac{1}{\gamma_t} \mathcal{A}^-$$

- Small Δz^- displacement of the trajectory within the target width L^+:

$$\Delta z^- \sim \frac{p^-}{p^+} \Delta z^+ \leq \frac{p^-}{p^+} L^+ = O\left(\frac{1}{\gamma_t}\right)$$

Double power suppression, beyond static approx and beyond shockwave approx:

$\Rightarrow$ NNEik effect within a single propagator!

NEik correction from beyond static approximation

Effect of relative z^- dependence of $\mathcal{A}^-$ insertions along one propagator is NNEik.

However, dependence on average z^- is suppressed only once.

$\Rightarrow$ Use Wilson lines with overall z^- dependence

$$\partial_- \mathcal{U}_F(+\infty, -\infty; \mathbf{z}, z^-) \propto \frac{1}{\gamma_t} \mathcal{U}_F(+\infty, -\infty; \mathbf{z}, z^-)$$

$\rightarrow$ Accounts for NEik effects beyond static approx

In particular: NEik corrections induced by the difference in z^- between different Wilson lines.