Probing the Energy momentum tensor

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Based on Eur.Phys.J.C 85 (2025) 1, 105,
arXiv:2509.05059 and arXiv:2509.06669

Introduction

Hadron EMT in QCD

In QCD, the energy momentum tensor of the nucleon is a correlator of the EMT operator, evaluated between two nucleon states:

$$\langle p', s'| T_{q,g}^{\{\mu\nu\}} |p, s\rangle = \bar{u}\left[P^{\{\mu}\gamma^{\nu\}} A_{q,g}(t;\mu) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} C_{q,g}(t;\mu) \right.$$
$$\left. + M g^{\mu\nu}\bar{C}_{q,g}(t;\mu) + \frac{P^{\{\mu}i\sigma^{\nu\}\Delta}}{2M} B_{q,g}(t;\mu)\right] u$$

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- The total EMT is scale independent as it defines the dilatation current
- Different definitions exist for the EMT, we stick to the one above
- 4 form factors are needed to parameterise the (symmetric) EMT correlator in the spin-1/2 case
- Constraints exist on some of these form factors:

$$A(0) = 1, \quad B(0) = 0, \quad \bar{C}(t) = 0$$

- Note that there is **no** constraint on C.

Experimental access

Interestingly, EMT Form Factors A, B and C are connected to GPDs H and E through:

$$\int_{-1}^{1} \mathrm{d}x x H^q(x,\xi,t) = A^q(t) + 4\xi^2 C^q(t)$$

$$\int_{-1}^{1} \mathrm{d}x x E^q(x,\xi,t) = B^q(t) - 4\xi^2 C^q(t)$$

$$\int_{-1}^{1} \mathrm{d}x H^g(x,\xi,t) = A^g(t) + 4\xi^2 C^g(t)$$

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In principle, from GPDs extracted from experimental data, we would be able to get experimental information on these Form Factors.

Double Distributions

One of the convenient way to represent GPDs is to introduce the so-called Double Distribution

$$H(x,\xi,t) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \delta(x-\beta-\alpha\xi)\left[F(\beta,\alpha,t)+\xi\delta(\beta)D(\alpha,t)\right]$$

$$E(x,\xi,t) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \delta(x-\beta-\alpha\xi)\left[K(\beta,\alpha,t)-\xi\delta(\beta)D(\alpha,t)\right]$$

D. Mueller *et al.*, Forsth. Phys. 42 101 (1994)
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$$\int_{-1}^{1} \mathrm{d}\beta\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha F^q(\beta,\alpha,t) = A^q(t)$$

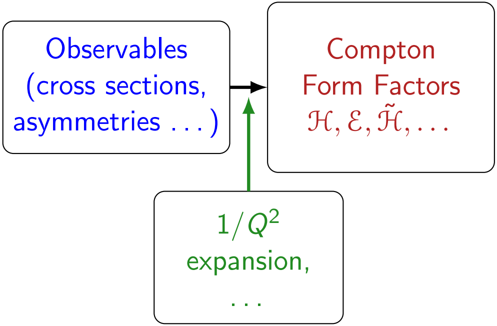
$$\int_{-1}^{1} \mathrm{d}\beta\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha K^q(\beta,\alpha,t) = B^q(t)$$

$$\int_{-1}^{1} \mathrm{d}\alpha\alpha D^q(\alpha,t) = 4C^q(t)$$

Experimental connection to GPDs

Observables
(cross sections,
asymmetries . . .)

Experimental connection to GPDs



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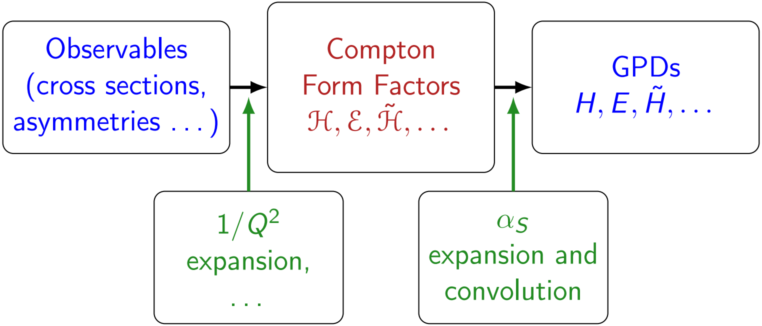

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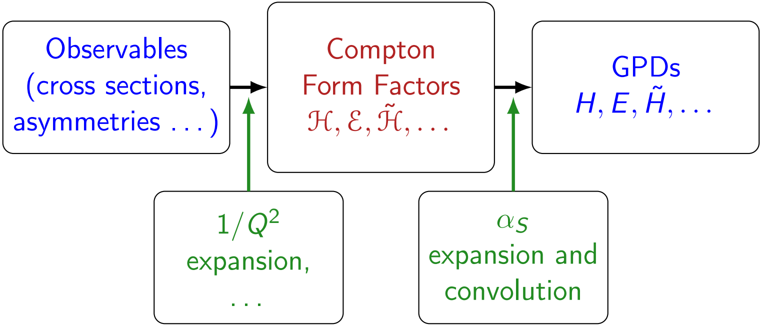
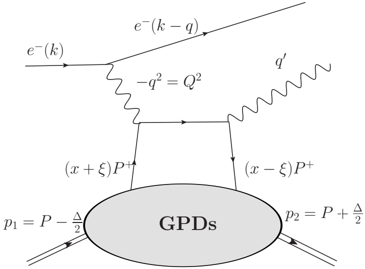
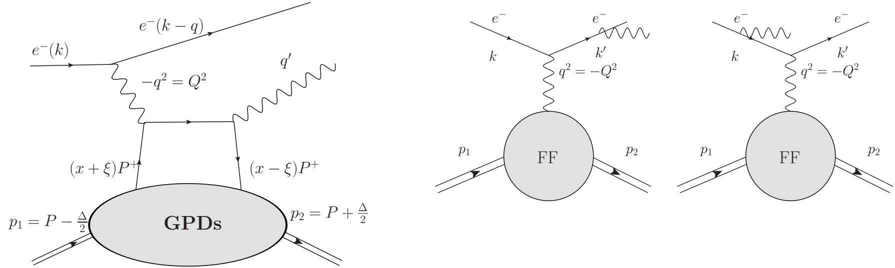


- CFFs play today a central role in our understanding of GPDs
- Extraction generally focused on CFFs

Deep Virtual Compton Scattering



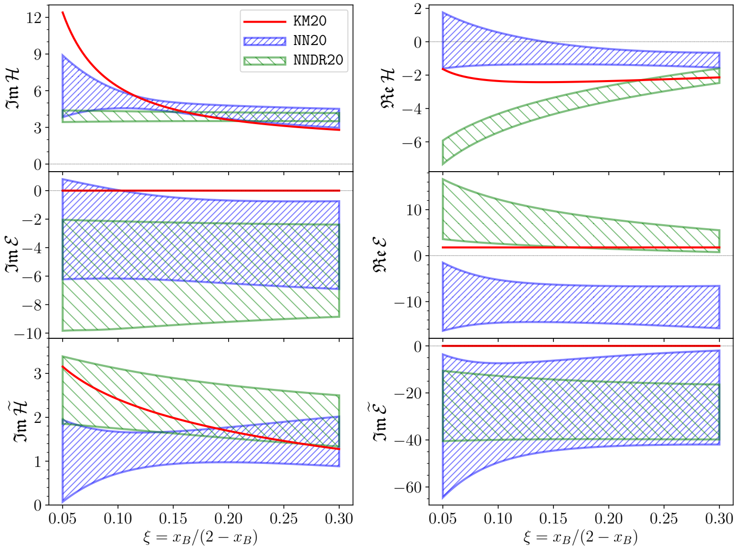
- Best studied experimental process connected to GPDs
  → Data taken at Hermes, Compass, JLab 6, JLab 12



- Best studied experimental process connected to GPDs
  → Data taken at Hermes, Compass, JLab 6, JLab 12
- Interferes with the Bethe-Heitler (BH) process
  - Blessing: Interference term boosted w.r.t. pure DVCS one
  - Curse: access to the angular modulation of the pure DVCS part difficult

M. Defurne *et al.*, Nature Commun. 8 (2017) 1, 1408

Examples of CFF extractions



M. Cuìc *et al.*, PRL 125, (2020), 232005

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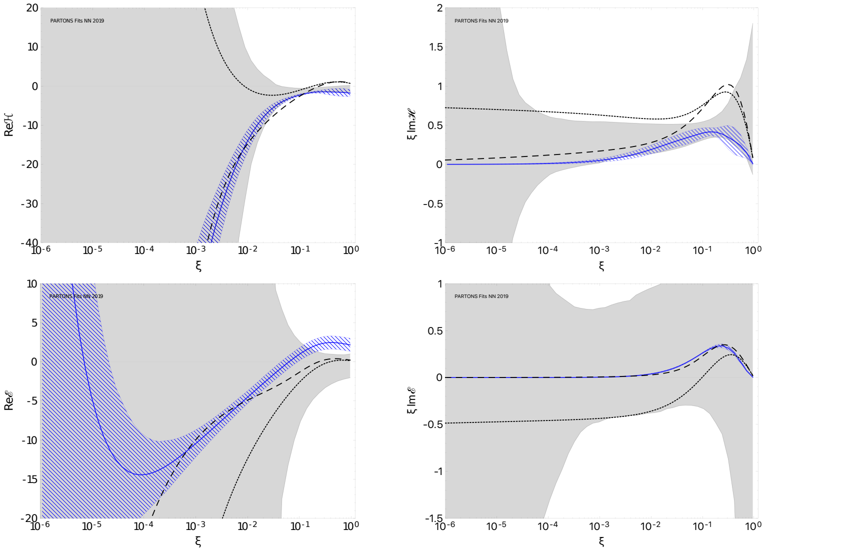
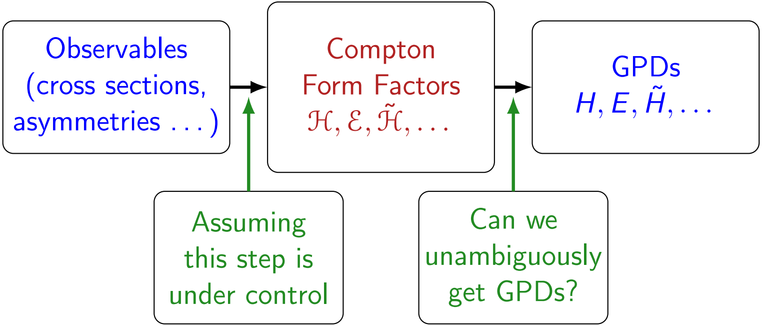


H. Moutarde *et al.*, EPJC 79, (2019), 614

The DVCS deconvolution problem

From CFF to GPDs



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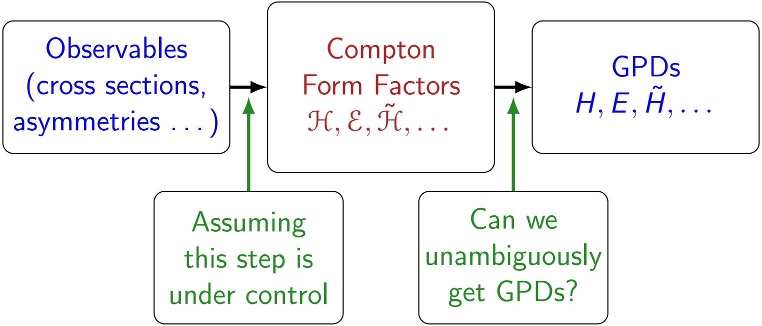

- It has been known for a long time that this is not the case at LO
  Due to dispersion relations, any GPD vanishing on $x = \pm\xi$ would not contribute to DVCS at LO (neglecting D-term contributions).

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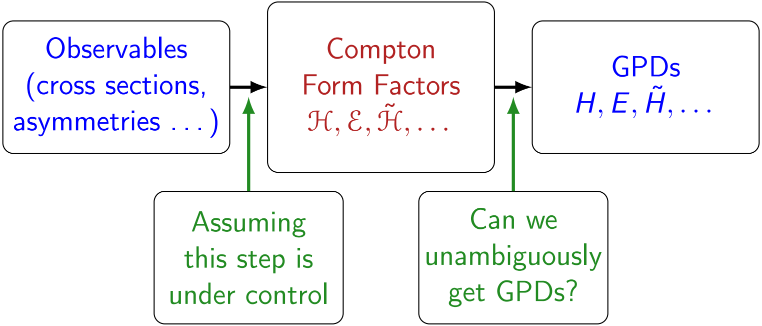

- It has been known for a long time that this is not the case at LO
  Due to dispersion relations, any GPD vanishing on $x = \pm\xi$ would not contribute to DVCS at LO (neglecting D-term contributions).
- QCD corrections are *not* improving the situation practically

Can we avoid the full deconvolution problem of GPDs to access the EMT form factors ?

D-term dispersion relation at LT

DVCS amplitude obeys the following dispersion relation at LT:

$$\mathcal{S} = \int_{-1}^{1} \mathrm{d}\alpha\, T(\alpha) D(\alpha) = \Re\mathcal{H}(\xi) - \frac{2}{\pi} \fint_{0}^{1} \frac{x^2 \Im\mathcal{H}(x)}{(\xi - x)(\xi + x)} \frac{\mathrm{d}x}{\xi}$$

with the subtraction constant expressed before any pQCD expansion (but using factorisation theorem).

M. Diehl and D. Ivanov, Eur.Phys.J.C 52 (2007) 919-932
H.Dutrieux *et al.*, EPJ C 85 (2025) 1, 105

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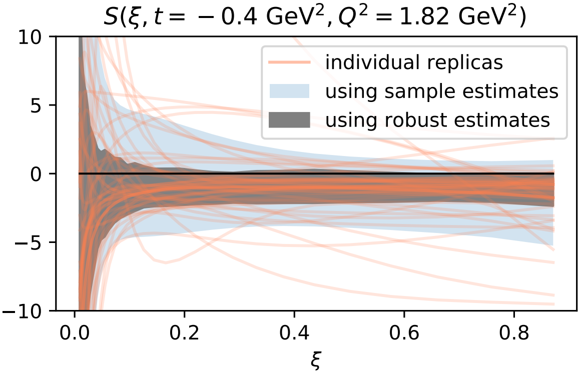
H.Dutrieux *et al.*, EPJ C 85 (2025) 1, 105

- Only the real part of the perturbative kernel contributes
- The end-point behaviour is regularised by the D-term going to zero (factorisation theorem)
- Since T is know at NNLO we could *naively* expect a fine extraction of the quarks and gluons contributions to the subtraction constant
- We need to know $\Im\mathcal{H}$ on the complete $(0, 1)$ domain.

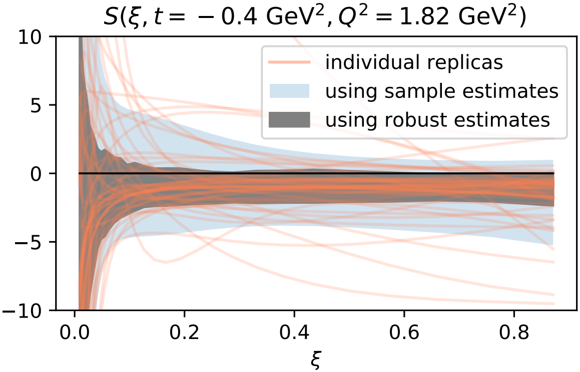
Experimental knowledge of $\mathcal{S}$



Replicas from H. Moutarde *et al.*, EPJC 79(7):614 (2019)
Estimator from H. Dutrieux *et al.*, EPJC 85 (2025) 1, 105

- Independent global fit of real and imaginary part of CFF
- 30 observables and 2500 kinematic points

Experimental knowledge of $\mathcal{S}$



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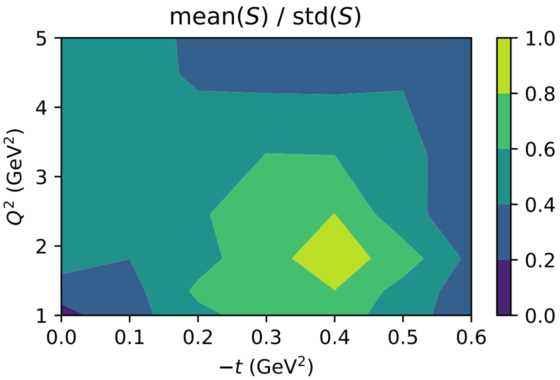
- Independent global fit of real and imaginary part of CFF
- 30 observables and 2500 kinematic points

- Noisy extraction with many outliers
- A signal is obtained after introducing robust statistical estimators

Kinematic map



1. The result of the extraction of the subtraction constant is compatible with 0 at 1σ level or below in the entire kinematics space.
2. The best constraint kinematic area is such that $M^2/(Q^2+t) \approx 1/2$ and $|t|/(Q^2+t) \approx 1/4$.

Taking into account power corrections, the DVCS DR is not modified, but the expression of $\mathcal{S}$ is:

$$\mathcal{S} = \Re\mathcal{H}(\xi) - \frac{2}{\pi} \fint_0^1 \frac{x^2 \Im\mathcal{H}(x)}{(\xi - x)(\xi + x)} \frac{\mathrm{d}x}{\xi}$$
$$= \int_{-1}^{1} \mathrm{d}\omega \, T_2\left(\omega; \frac{t}{Q^2+t}\right) D(\omega)$$
$$- 4\frac{M^2}{Q^2+t} \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \beta \, T_1(\alpha) \left[F(\beta,\alpha) + \frac{t}{4M^2} K(\beta,\alpha)\right]$$

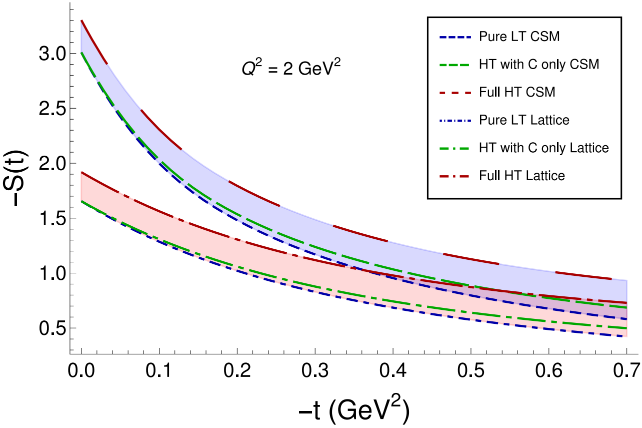
V. Braun *et al.*, Phys.Rev.D 89 (2014) 7, 074022
V. Martinez-Fernandez and C. Mezrag, arXiv:2509.05059

This can nicely be approximated as:

$$\mathcal{S}^q \approx 20 C^q(t)\left(1 - \frac{t}{3(Q^2+t)}\right) - \frac{4M^2 c_0}{Q^2+t}\left[\left(1 - \frac{t}{4M^2}\right) A^q(t) + \frac{t}{2M^2} J^q(t)\right]$$

V. Martinez-Fernandez, D. Binosi *et al.*, arXiv:2509.06669

Assessing the impact of power corrections



lattice data from Hackett *et al.*, PRL, 132(25):251904
CSM data from Yao *et al.*, EPJA, 61(5):92, 2025.

1. Power corrections account for 25-35% of the experimental signal
2. In principle, DR provide access to C^q, A^q and J^q

Including gluons

Higher-twist vs. NLO

- NLO corrections are available at LT only (no $\frac{\alpha_S}{Q^2}$ available)
- These corrections appear thus only at the level of the D-term

$$\mathcal{S} = \mathcal{D} + f(A^q, J^q)$$

$$\mathcal{D} = \int \mathrm{d}\omega T^q(\omega; \alpha_s) D^q(\omega) + \int \mathrm{d}\omega T^g(\omega; \alpha_s) D^g(\omega)$$

- Access only to C^g for now (A^g and J^g probably come with α_s/Q^2).

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$$C_a(t,\mu^2) = \frac{1}{4}\int_{-1}^{1} \mathrm{d}\omega\, \omega^{1-p_a} D^a(\omega, t, \mu^2)\,.$$

We still have a deconvolution problem to solve

Leading Order deconvolution problem

Taking a Gegenbauer expansion of the D-term as:

$$D^q(\alpha, t, \mu^2) = (1-\alpha^2) \sum_{\text{odd } n} d_n^q(t, \mu^2) C_n^{(3/2)}(\alpha)\,,$$

$$D^g(\alpha, t, \mu^2) = \frac{3}{2}(1-\alpha^2)^2 \sum_{\text{odd } n} d_n^g(t, \mu^2) C_{n-1}^{(5/2)}(\alpha)\,.$$

you obtain for quarks at LO:

$$\mathcal{S}(t, Q^2) = 4 \sum_q e_q^2 \sum_{\text{odd } n} d_n^q(t, \mu^2)\,,$$

and we are after

$$d_1(t, \mu^2) = 5\, C_a(t, \mu^2)$$

Quarks and gluons contributions at NLO

At NLO, gluons directly enter the description of the subtraction constant:

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Moreover, the relative weights of d_1 and d_3 are different:

$$S^q = d_1^q \left(4 - \frac{4}{9}\frac{\alpha_s C_F}{4\pi}\right) + d_3^q \left(4 + \frac{14759}{450}\frac{\alpha_s C_F}{4\pi}\right) \,,$$

$$S^g = \frac{\sum_q e_q^2 \alpha_s T_F}{4\pi}\left(-\frac{172}{9} d_1^g - \frac{3317}{150} d_3^g\right) \,,$$

At fixed scale, the solution is not unique.

NLO Fit -Radiative gluons

Fit of d_1^{uds} only at NLO with d_1^g radiatively generated

$$\frac{d_1^{uds}(t=0, 2\ \mathrm{GeV}^2) = -\,0.7 \pm 1.3}{d_1^{g}(t=0, 2\ \mathrm{GeV}^2) = -\,0.9 \pm 1.8}$$

(NLO n=1 radiative gluons)

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(NLO n=1 radiative gluons)

- Quark sector very similar to LO. This is because:

$$\frac{S^g}{S^q} \simeq \frac{d_1^g}{10 d_1^q}$$

- Sensitivity to d^g remains suppressed by roughly an order of magnitude (though we gained a factor 5 compared to LO)

NLO FIT - Radiative gluons

Same as fit 1, but allowing d_3^{uds} Gegenbauer mode

$$d_1^{uds}(t=0, 2\ \mathrm{GeV}^2) = -1.7 \pm 21$$

$$d_3^{uds}(t=0, 2\ \mathrm{GeV}^2) = 0.7 \pm 15$$

$$d_1^{g}(t=0, 2\ \mathrm{GeV}^2) = -2 \pm 30$$

$$d_3^{g}(t=0, 2\ \mathrm{GeV}^2) = 0.1 \pm 2.3 \qquad \text{(NLO n=3 radiative gluons)}$$

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$d_3^{g}(t=0, 2\ \mathrm{GeV}^2) =$	0.1 ± 2.3	(NLO n=3 radiative gluons)

- Uncertainties are of the same orders of magnitudes
- Correlations between d_1 and d_3 still exceed 99
- The shadow D-term issue has been “moved”, not solved by a different weighting of d_1 and d_3.

NLO Fit - Non radiative gluons

No d_3 modes but we extract d_1^q and d_1^g independently

$$d_1^{uds}(t = 0, 2\ \mathrm{GeV}^2) = -\,1.1 \pm 7.7$$
$$d_1^{g}(t = 0, 2\ \mathrm{GeV}^2) = -\,6 \pm 78$$

(NLO n=1 free gluons)

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(NLO n=1 free gluons)

- uncertainties on d_1^{uds} strongly increased
- There is now enough "gluon impact" to correlate both parameter and generate a new type of shadow D-term.
- These are not as painful as previously probably because of the bigger difference between their respective evolution operators.

What to do then ?

On the experimental side:

$$\sigma_{d_1} \simeq \sigma_{d_3} \propto \frac{\Delta \mathcal{S}}{1 - \frac{\alpha_s(Q^2_{max})}{\alpha_s(Q^2_{min})}}$$

- Reduce $\Delta \mathcal{S}$ by improving the measurement and extraction of DVCS amplitudes
- Increase the range in Q^2 of observables

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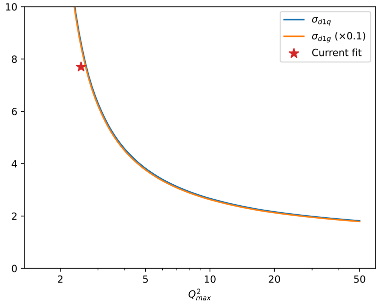
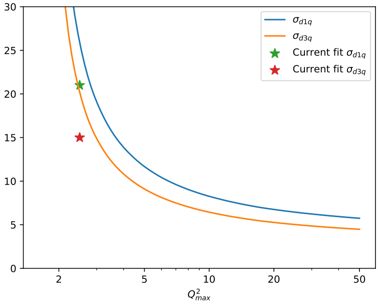
On the phenomenology side:

- NLO evolution may improve a bit the situation (greater scale sensitivity)
- One should start thinking at valuable theoretical bias to constrain the system
- **Are there other channels accessible ?**

Impact of an EIC

Assuming the current $\Delta\mathcal{S}$ can be extended at EIC kinematics the uncertainties reduction as a function of Q^2_{max} is:





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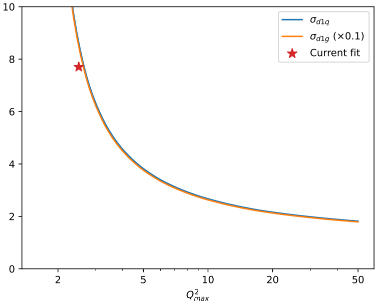

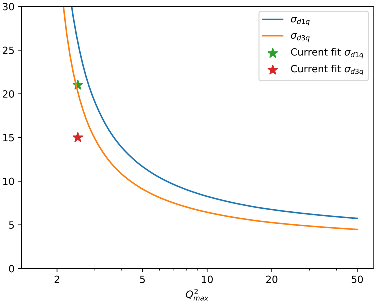

An extension to $20 GeV^2$ of the current precision would reduce the uncertainties by a factor 4. But this won't be enough by itself to solve the deconvolution problem.

Conclusions

Summary

- The theoretical precision achieved link the subtraction constant of DVCS to A^q, J^q, C^q and C^g
- A^g and J^g might be connected through α_s/Q^2 corrections
- Experimental knowledge of the subtraction constant is limited:
  - Q^2 allowing deconvolution is too small
  - $\Delta\mathcal{S}$ as the real part of the DVCS amplitude is poorly known
- Data driven extraction remains out-of-reach (deconvolution)

Perspectives

- Experimental efforts toward key observables/kinematic range
- Multichannel analysis is an crucial point.
  - Gluon sensitive probe such as J/Ψ are critical
  - LHC measurement of Exclusive J/Ψ photo-production
- Caveat: DVCS is probably better understood than meson exclusive production (on a theoretical point)

Thank you for your attention

Back up slides

A word about evolution

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$$\Gamma(\mu^2, \mu_0^2) = 1 + \alpha_s(\mu^2) K^{(0)} \ln\left(\frac{\mu^2}{\mu_0^2}\right) + \mathcal{O}(\alpha_s^2)$$

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- We expect CFF computed from evolved NLO shadow GPDs to exhibit an α_s^2 behaviour under evolution (provided that the logs remain small enough).

Shadow D-term

$$\mathcal{S}(t, Q^2) = 4 \sum_q e_q^2 \sum_{\text{odd } n} d_n^q(t, \mu^2)\,,$$

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Would evolution and higher order corrections improve the situation ?

Leading order evolution

If we take into account evolution, the contribution $\mathcal{S}_{shadow}$ of our shadow D-term is:

$$\mathcal{S}_{shadow}(Q^2) \propto \Gamma_1^{qq}(Q^2,\mu_0^2)d_1^q(\mu_0^2) + \Gamma_3^{qq}d_3(\mu_0^2)$$
$$\propto \lambda\left[\left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu_0^2)}\right)^{0.395} - \left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu_0^2)}\right)^{0.775}\right]$$

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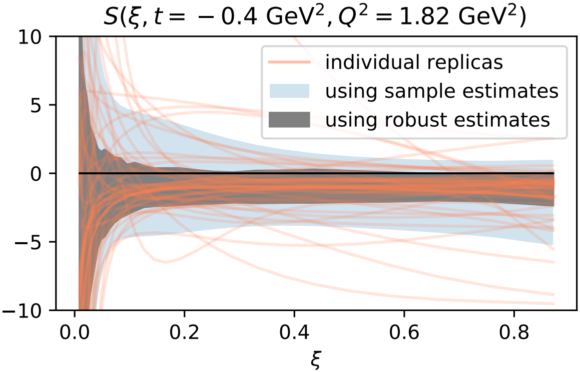
If experimental uncertainties are given by $\Delta\mathcal{S}$ on a range $[Q_{min}; Q_{max}]$:

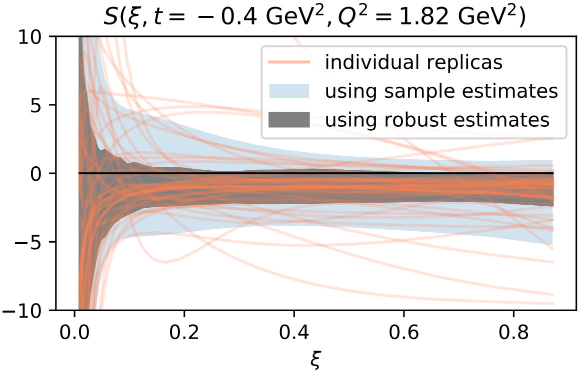
$$\begin{aligned}\sigma_{d_1} \simeq \sigma_{d_3} &\propto \frac{\Delta\mathcal{S}}{\Gamma_1^{qq}(Q^2_{max},Q^2_{min}) - \Gamma_3^{qq}(Q^2_{max},Q^2_{min})}\\ &\propto \frac{\Delta\mathcal{S}}{1-\frac{\alpha_s(Q^2_{max})}{\alpha_s(Q^2_{min})}}\end{aligned}$$



Replicas from H. Moutarde *et al.*, EPJC 79(7):614 (2019)

- Independent global fit of real and imaginary part of CFF
- 30 observables and 2500 kinematic points

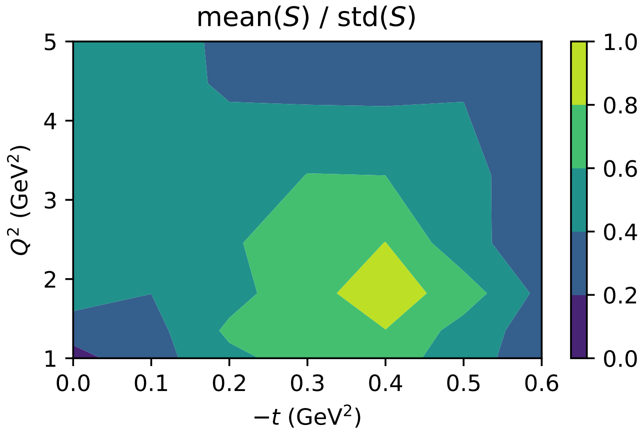
Dataset



Replicas from H. Moutarde *et al.*, EPJC 79(7):614 (2019)

- Independent global fit of real and imaginary part of CFF
- 30 observables and 2500 kinematic points
- Noisy extraction with many outliers
- A signal is obtained after introducing robust statistical estimators

Kinematic map



The result of the extraction of the subtraction constant is compatible with 0 at 1σ level or below in the entire kinematics space.

Modelling the D-term

With such a bad signal/noise ratio we need to introduce some theoretical bias:

- We restrict ourselves to LO order accuracy with Leading logarithm accuracy
- We assume that flavours are degenerated : $d_n^u = d_n^d = d_n^s = d_n^{uds}$
- We retain only $n = 1$ and $n = 3$ coefficient in Gegenbauer expansion of the D-term
- We assume a factorised t-dependence of the D-term:

$$D(\alpha, t, \mu^2) = \frac{D(\alpha, \mu^2)}{\left(1 - \frac{t}{M^2}\right)^3}$$

with $M = 0.8 GeV$. This is justified by the absence of distinctive t-dependence.

LO Fit - radiative gluons

First fit : we used LO hard kernel, a low scale of $\mu_g = 0.3GeV$ from which we generate purely radiative gluons.

$d_1^{uds}(t = 0, 2\ \mathrm{GeV}^2) =$	-0.6 ± 1.1
$d_1^{g}(t = 0, 2\ \mathrm{GeV}^2) =$	-0.8 ± 1.5

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Note : the initial scale has no impact on the extraction of d_1^{uds} at $2GeV$. This is due to the very weak radiation of quarks by gluons:

$$\Gamma_1^{qq}(2.5GeV^2, 1GeV^2) = 0.92, \quad \Gamma_1^{qg}(2.5GeV^2, 1GeV^2) = 0.015,$$

The contribution of purely radiative gluons is suppressed, and account for 2% of d_1^{uds}.

LO Fit -Radiative gluons

Same fit than previously, but allowing $d_3 \neq 0$.

$$d_1^{uds}(t = 0, 2\ \mathrm{GeV}^2) = -2.1 \pm 26.6$$
$$d_3^{uds}(t = 0, 2\ \mathrm{GeV}^2) = 1.5 \pm 26.5$$

$$d_1^{g}(t = 0, 2\ \mathrm{GeV}^2) = -2.9 \pm 37$$
$$d_3^{g}(t = 0, 2\ \mathrm{GeV}^2) = 0.2 \pm 4.1 \qquad \text{(LO n=3 radiative gluons)}$$

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- Uncertainties explode due to shadow D-term

$$\sigma_{d1q} \approx \sigma_{d3q} \approx \frac{\Delta S}{\left(1 - \frac{\alpha_s(Q^2_{max})}{\alpha_s(Q^2_{min})}\right)}$$
$$\approx 25 \text{ for } (Q^2_{min}, Q^2_{max}) = (1.4 GeV^2, 2.5 GeV^2)$$

- Again, radiative gluons play no role.

LO Fit - Non-radiative Gluons

This time we keep $n = 1$ in the Gegenbauer expansion but proceed fitting the gluon parameter as a free one.

$$d_1^{uds}(t = 0, 2\ \mathrm{GeV}^2) = \qquad -0.6 \pm 1.1$$
$$d_1^{g}(t = 0, 2\ \mathrm{GeV}^2) = \qquad -11 \pm 132$$

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- The quark result is unchanged
- The gluon uncertainties blow up by a factor 90.
- The reason is that gluons need to "fight" their evolution suppression:

$$\Gamma^{qq}(2.5, 1)/\Gamma^{qg}(2.5, 1) = 0.92/0.015 \approx 60$$