Exclusive photoproduction of $\pi^+\pi^-$ pairs in the tensor-pomeron approach

Piotr Lebiedowicz (IFJ PAN)

THE HENRYK NIEWODNICZAŃSKI
INSTITUTE OF NUCLEAR PHYSICS
POLISH ACADEMY OF SCIENCES

in collaboration with
Antoni Szczurek (IFJ PAN) and
Otto Nachtmann (U Heidelberg)

arXiv: 2508.06334

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Introduction

- We study the production of $\pi^+\pi^-$ pairs in photon-proton collisions and central exclusive production (CEP) of such pairs in proton-proton collisions,

$$\gamma^{(*)} + p \to \pi^+ + \pi^- + p\,,$$
$$p + p \to p + \pi^+ + \pi^- + p\,.$$

  We are interested in high energies and small momentum transfers, that is, in the regime of Regge exchanges.

- An old problem there is to understand the shape of the $\rho^0(770)$ resonance and the $\pi^+\pi^-$ invariant mass region below ρ^0. Compared to the ρ^0 shape measured in e^+e^- annihilation there is a skewing of the ρ^0 shape observed in the reactions above.

  Already a long time ago this skewing was attributed to the interference of the decay $\rho^0 \to \pi^+\pi^-$ with the non-resonant production of $\pi^+\pi^-$, the Drell-Söding (DS) term:

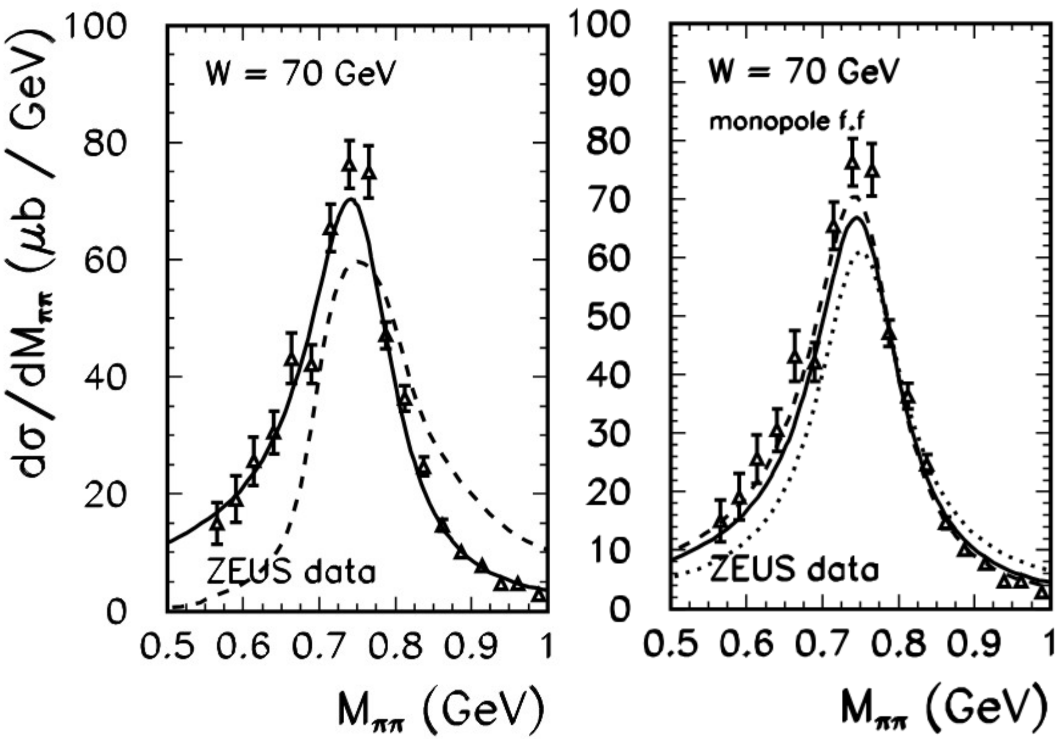
  S.D. Drell, *Production of particle beams at very high energies*, Phys. Rev. Lett. 5 (1960) 278,
  S.D. Drell, *Peripheral contributions to high-energy interaction processes,* Rev. Mod. Phys. 33 (1961) 458
  P. Söding, On the apparent shift of the *ρ meson mass in photoproduction,* Phys. Lett. 19 (1966) 702

- In practice, the calculation of the continuum (DS) term is a tricky problem, not the least due to requirements of gauge invariance.

Introduction

Results from A. Szczurek and A. Szczepaniak

PHYSICAL REVIEW D **71,** 054005 (2005)

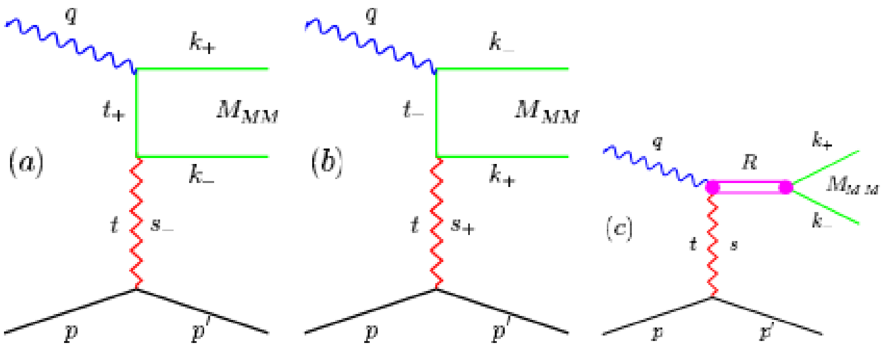


Left: ρ^0 + DS (solid line), ρ^0 (dashed line)
ZEUS data: M. Derrick et al., Z. Phys. C69 (1995) 39

Right: ρ^0 + DS contribution for three values of the off-shell form factor parameter, Λ = 0.5, 1.0, 2.0 GeV.

The exact form of the combined form factor (in two vertices with an off-shell pion meson) is not known. In principle, a good quality data would help to find the proper functional form.

- Gauge-invariant skewing mechanism



- The amplitude for the nonresonant (DS) component is written in factorised form

$$\mathcal{M}^{(\mathrm{DS})}_{\lambda_\gamma \lambda \to \lambda'}(s, t, s_+, t_+, s_-, t_-)$$
$$= V^{\gamma\pi^+}_{\lambda_\gamma} \frac{F(t_+)}{t_+ - m_\pi^2} \mathcal{M}^{\pi^- p}_{\lambda\lambda'}(s_-, t)$$
$$+ V^{\gamma\pi^-}_{\lambda_\gamma} \frac{F(t_-)}{t_- - m_\pi^2} \mathcal{M}^{\pi^+ p}_{\lambda\lambda'}(s_+, t) + \delta\mathcal{M}$$

$$V^{\gamma\pi^\pm}_{\lambda_\gamma} = \pm e(2k^\mu_\pm)\, \epsilon_\mu(\lambda_\gamma = \pm 1)$$

$$\mathcal{M}^{\pi^\pm p}_{\lambda\lambda'}(s_\pm, t) = i s_\pm\, \sigma^{\pi^\pm p}_{\mathrm{tot}}(s_\pm)\, \exp\left(\frac{B}{2} t\right) \delta_{\lambda\lambda'}$$

DL parametrisation

$$F(t_\pm) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - t_\pm}$$

- Except for the off-shell dependence determined by the the form factors $F(t_+)$ and $F(t_-)$ the DS model is essentially parameter free.

Tensor-pomeron model

- Tensor pomeron and vector odderon model [C. Ewerz, M. Maniatis, and O. Nachtmann, Annals Phys. 342 (2014) 31] has been constructed in order to describe soft high-energy hadronic reactions.

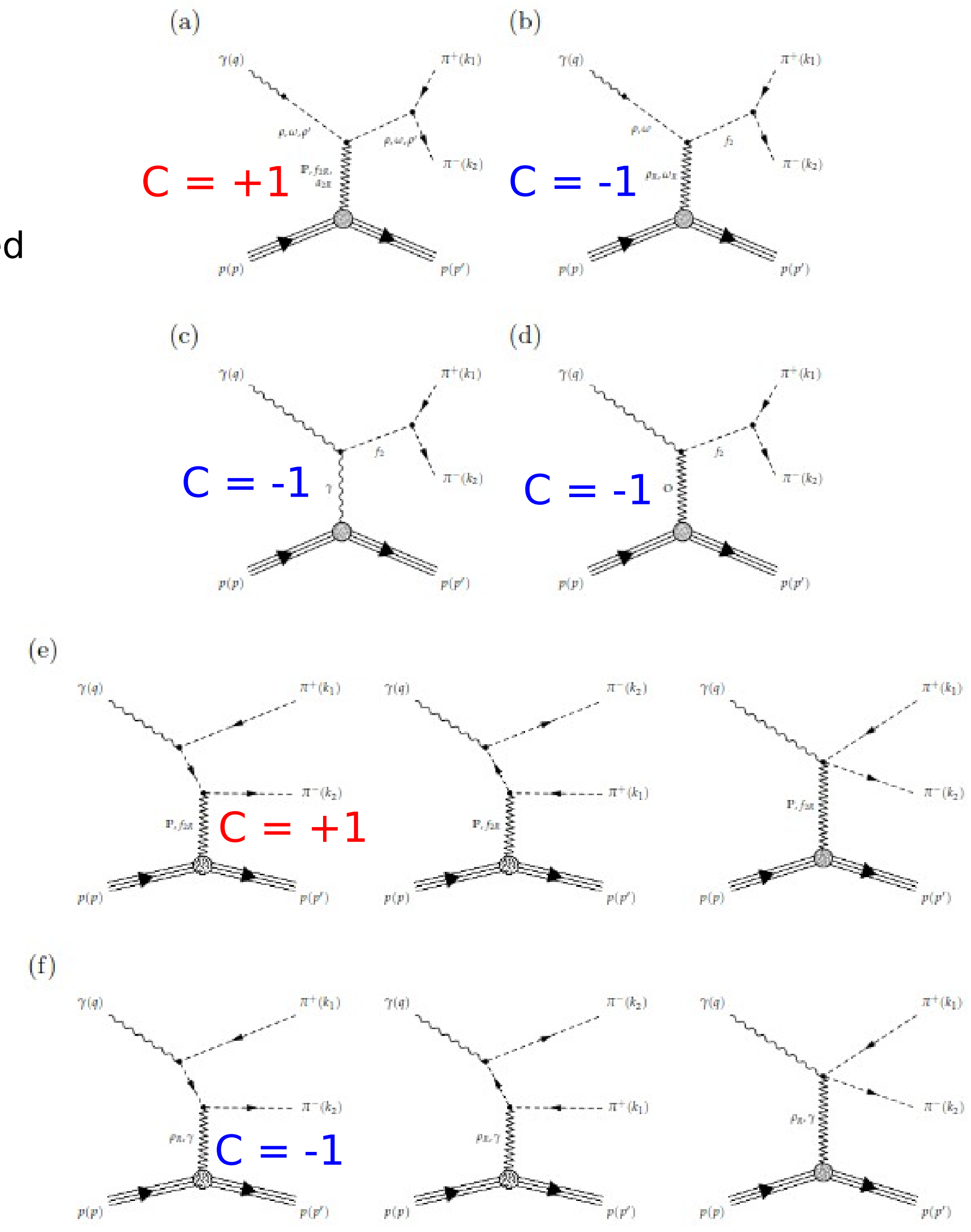
  The pomeron (IP) and C = +1 reggeons are described as effective tensor-exchange objects,
  the odderon (O) and C = -1 reggeons as effective vector-exchange objects.

- It was applied to $\pi^+\pi^-$ photoproduction [A. Bolz, C. Ewerz, M. Maniatis, O. Nachtmann, M. Sauter, A. Schöning, JHEP 01 (2015) 151]

  → gauge-invariant mechanism with effective vertices derived from coupling Lagrangians

  → a common energy variable (s) was used in the respective Regge factors of non-resonant amplitudes

  → in this way a skewing of the ρ^0 shape was achieved but compared to experiment it was not big enough



Tensor-pomeron model

Results from

A. Bolz, C. Ewerz, M. Maniatis, O. Nachtmann, M. Sauter, A. Schöning, JHEP 01 (2015) 151

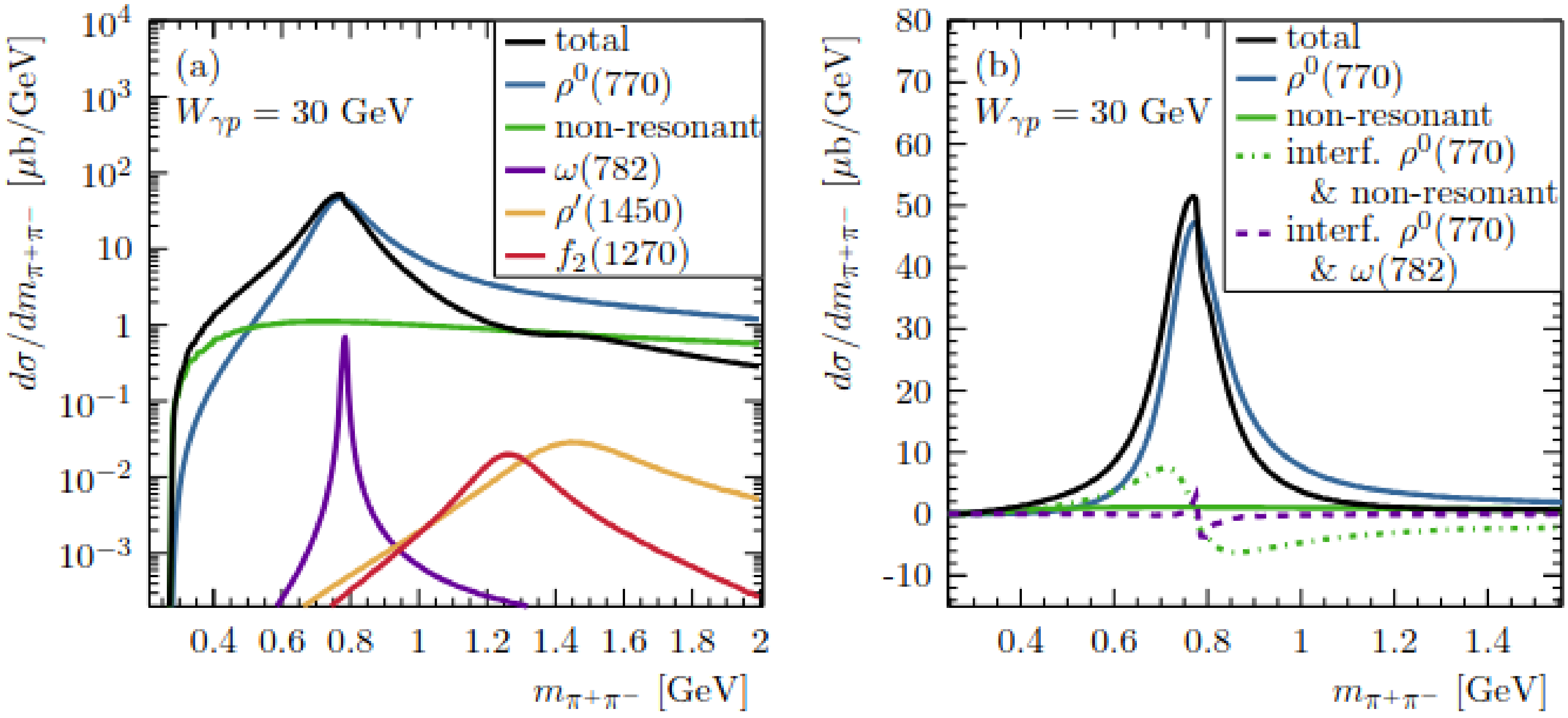
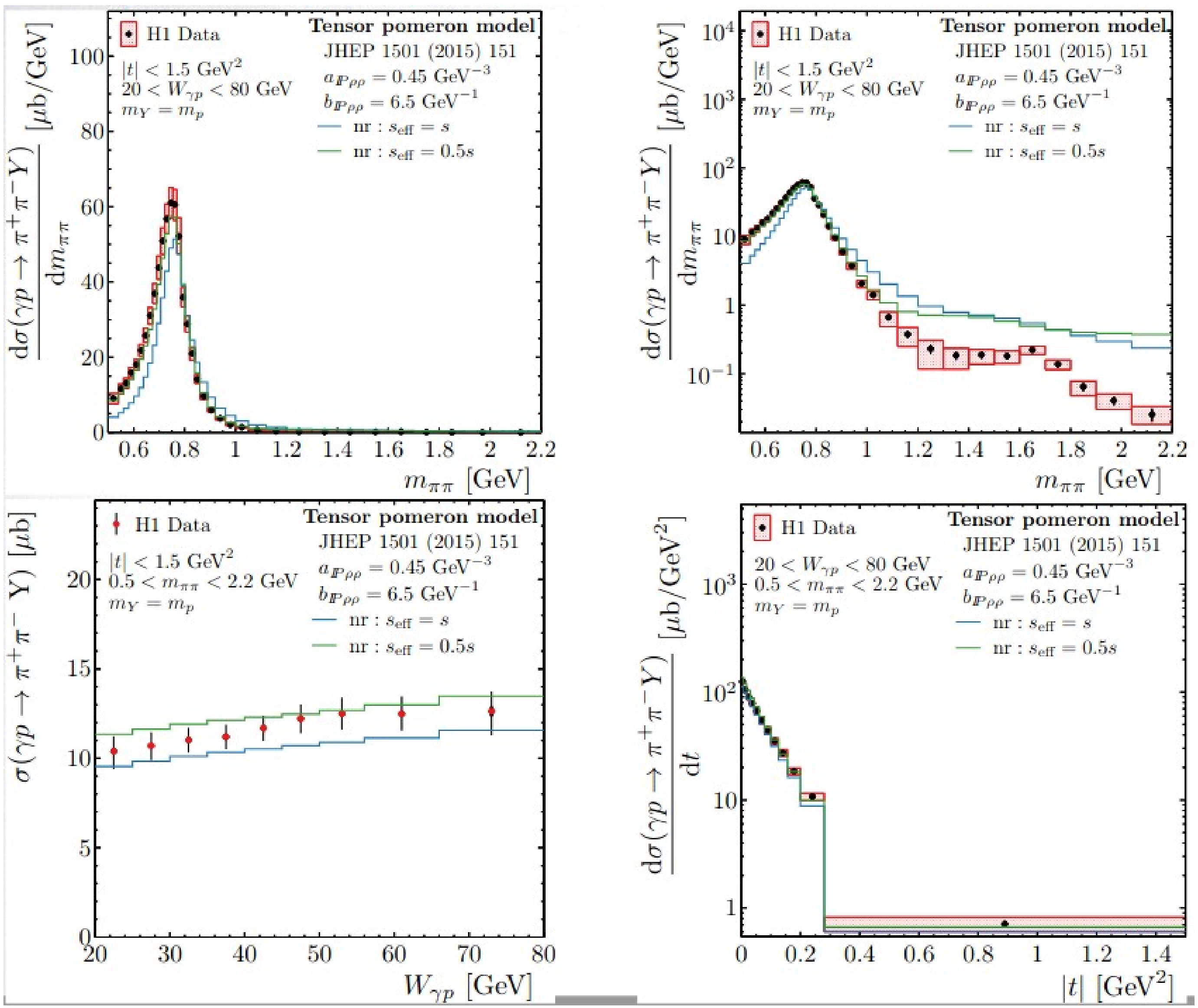
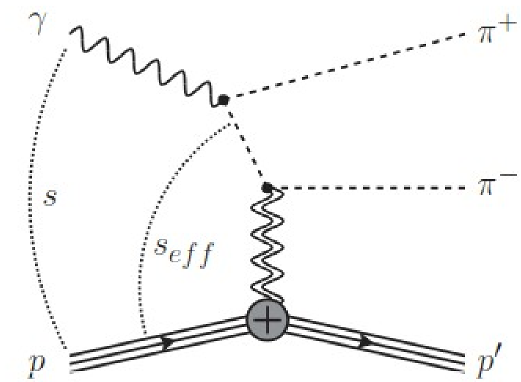


Figure 5. Differential cross sections $d\sigma/dm_{\pi^+\pi^-}$ ($\gamma p \to \pi^+\pi^- p$) as function of $m_{\pi^+\pi^-}$ for fixed $W_{\gamma p} = 30\,\mathrm{GeV}$ and integrated over the range $-1\,\mathrm{GeV}^2 \leq t \leq 0$. (a) The full model, non-resonant contributions and the contributions from the resonances $\rho^0(770)$, $\omega(782)$, $f_2(1270)$ and $\rho'(1450)$ are shown. (b) Dominant contributions in the ρ mass region including the leading interferences of $\rho^0(770)$ with the non-resonant $\pi^+\pi^-$ production and the $\omega(782)$ meson are shown.

Towards better modelling

- Comparison of modified model to H1 data [H1 Collaboration, EPJC 80 (2020) 1169]. Results from A. Bolz talk "*Measurement of Exclusive $\pi^+\pi^-$ and ρ^0 Meson Photoproduction at HERA*" presented at MESON 2021.





- modification in Regge propagators:
  $s \to s_{\rm eff} = s/2$
  "effective" average πp scattering energy
- good description of ρ^0 peak, $W_{\gamma p}$ and t shapes
- tuning of model parameters needed,
  especially at higher $\pi^+\pi^-$ masses
- dedicated data analysis needed to get further insight

Towards better modelling

arXiv: 2508.06334

→ improved calculation of the Drell-Söding contribution in the tensor-pomeron model
→ now we use for each diagram the appropriate energy variable in the Regge factors
→ we think that our new method is rather satisfactory from the point of view of QFT

Production of $\pi^+\pi^-$ pairs in diffractive photon-proton and in proton-proton collisions revisited, in particular concerning the Drell-Söding contribution

Piotr Lebiedowicz,[1,*] Otto Nachtmann,[2,†] and Antoni Szczurek[1,3,‡]

[1] *Institute of Nuclear Physics Polish Academy of Sciences, Radzikowskiego 152, PL-31342 Kraków, Poland*
[2] *Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg, Germany*
[3] *Institute of Physics, Faculty of Exact and Technical Sciences, University of Rzeszów, Pigonia 1, PL-35310 Rzeszów, Poland*

We discuss the central exclusive photoproduction of $\pi^+\pi^-$ pairs in photon-proton and in proton-proton collisions at high energies. The ρ^0, ω, $f_2(1270)$, and non-resonant (Drell-Söding) contributions are considered. The calculation is based on the tensor-pomeron model that includes not only the dominant pomeron exchange but also reggeon and odderon exchanges. In the Drell-Söding contribution we have different subenergies for the $\pi^+ p$ and $\pi^- p$ systems. In the method which we propose now we take this into account. Respecting the gauge-invariance constraints is then a nontrivial problem for which, however, we present a solution here. In this way we improve the corresponding calculations presented in JHEP 01, 151 (2015) and in Phys. Rev. D 91, 074023 (2015). The revised model leads to enhanced cross sections and gives an increased skewing of the ρ^0 spectral shape. For the $pp \to pp\pi^+\pi^-$ reaction, we calculate differential cross sections as function of the two-pion invariant mass, pion transverse momentum and pion pseudorapidity. Predictions of proton-pion and proton-pion-pion invariant mass distributions and the distribution in the proton-proton four-momentum transfer squared are also presented. This research is relevant in the context of ALICE, ATLAS, CMS, and LHCb measurements in pp collisions, even when the leading protons are not detected and instead only rapidity-gap conditions are checked experimentally. Our results can also serve as basis for the description of coherent $\pi^+\pi^-$ production in ultra-peripheral pA and AA collisions at the LHC. The formulas given in our paper can directly be used for the analysis of photoproduction and small-Q^2 electroproduction in ep collisions at high energies. Such data exist from the HERA experiments and will be obtained in the future at the electron-ion colliders.

a unique synergy between the LHC and the EIC

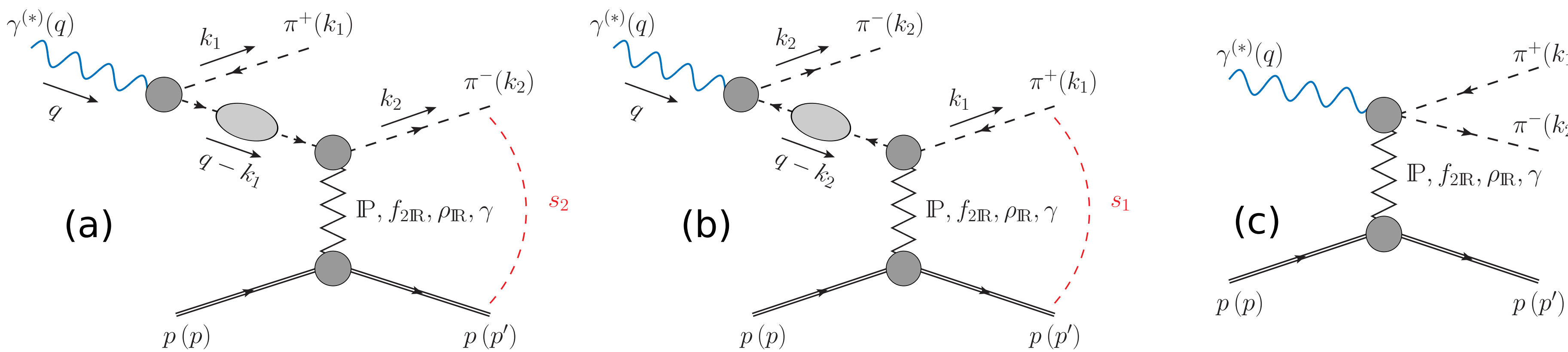
Theoretical formalism for $\gamma p \to \pi^+\pi^- p$

$$\gamma^{(*)}(q,\mu) + p(p,\mathfrak{s}) \to \pi^+(k_1) + \pi^-(k_2) + p(p',\mathfrak{s}')$$

$$\langle \pi^+(k_1), \pi^-(k_2), p(p',\mathfrak{s}') | \mathcal{T} | \gamma(q,\epsilon), p(p,\mathfrak{s}) \rangle = \epsilon^\mu \mathcal{M}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p)$$

for real photon $q^2 = 0$

Diagrams for non-resonant production of $\pi^+\pi^-$ pairs (Drell-Söding mechanism):



$$\mathcal{M}^{(\mathrm{DS})}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) = \mathcal{M}^{(a)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) + \mathcal{M}^{(b)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) + \mathcal{M}^{(c)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p)$$

$$\mathcal{M}^{(a)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) = e\widehat{\Gamma}^{(\gamma\pi\pi)}_{\mu}(k_1,k_1-q)\Delta_F[(k_1-q)^2]\mathcal{M}^{(0,\,a)}_{\mathfrak{s}',\mathfrak{s}}(k_2,p',q-k_1,p)$$

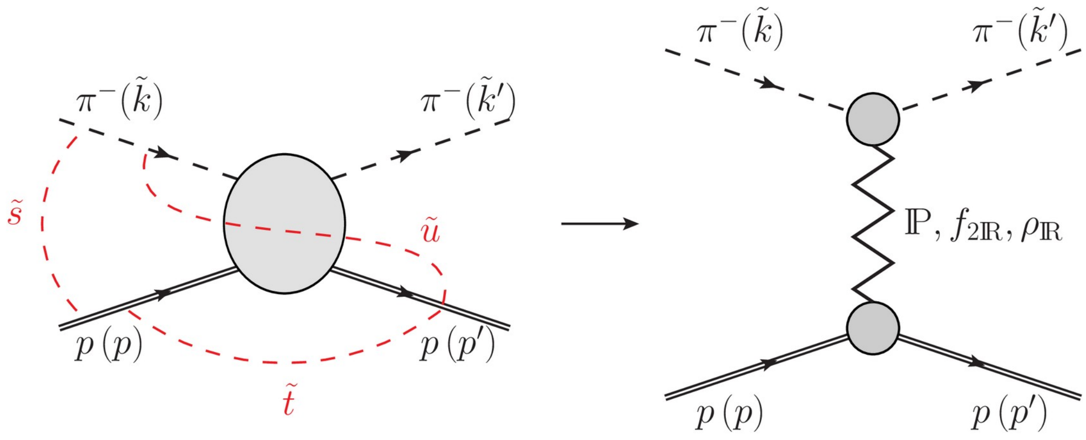
$e = \sqrt{4\pi\alpha_{\mathrm{em}}} > 0$

full pion-photon vertex function

full pion propagator

hadronic scattering amplitude denoted by $\mathcal{M}^{(\pi^-)}_{\mathfrak{s}',\mathfrak{s}}$

Pion-proton scattering (on shell)



$$\tilde{k} + p = \tilde{k}' + p'$$

$$\begin{aligned}
\tilde{s} &= (\tilde{k} + p)^2 = (\tilde{k}' + p')^2\,,\\
\tilde{t} &= (\tilde{k} - \tilde{k}')^2 = (p - p')^2\,,\\
\tilde{u} &= (\tilde{k} - p')^2 = (p - \tilde{k}')^2\,,\\
\tilde{\nu} &= \frac{1}{4}(\tilde{s} - \tilde{u})\,.
\end{aligned}$$

Amplitudes for pomeron exchange (considered as effective rank-2 symmetric tensor exchange)

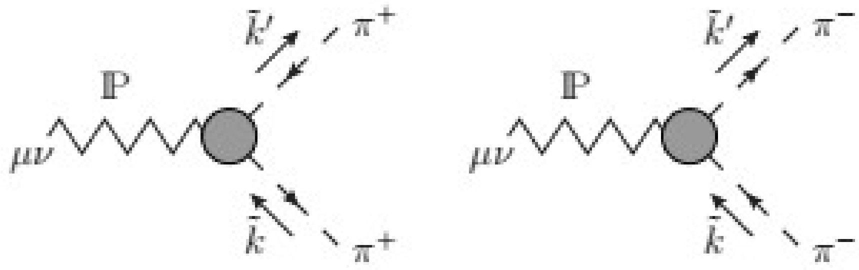
$$\begin{aligned}
\mathcal{M}^{(\pi^\pm)}_{\mathfrak{s}',\mathfrak{s}}(\tilde{k}', p', \tilde{k}, p)|_{\mathbb{P}} &= (-i) i\Gamma^{(\mathbb{P}\pi\pi)}_{\mu\nu}(\tilde{k}', \tilde{k})\, i\Delta^{(\mathbb{P})\mu\nu,\kappa\lambda}(2\tilde{\nu}, \tilde{t})\; \bar{u}_{\mathfrak{s}'}(p') i\Gamma^{(\mathbb{P}pp)}_{\kappa\lambda}(p', p) u_{\mathfrak{s}}(p)\\
&= i\mathcal{F}_{\mathbb{P}\pi p}(2\tilde{\nu}, \tilde{t})\; \bar{u}_{\mathfrak{s}'}(p') \Big[2(\tilde{k}' + \tilde{k})^\nu \gamma_\nu (\tilde{k}' + \tilde{k}, p' + p) - (\tilde{k}' + \tilde{k})^2 m_p\Big]\, u_{\mathfrak{s}}(p)
\end{aligned}$$

$$\mathcal{F}_{\mathbb{P}\pi p}(2\tilde{\nu}, \tilde{t}) = 2\beta_{\mathbb{P}\pi\pi}\, 3\beta_{\mathbb{P}NN}\, F_M(\tilde{t})\, F_1(\tilde{t})\, \underbrace{\frac{1}{8\tilde{\nu}}(-i\, 2\tilde{\nu}\, \alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(\tilde{t})-1}}_{\frac{1}{2}\left(-\frac{i}{2}\alpha'_{\mathbb{P}}\right)^{\alpha_{\mathbb{P}}(\tilde{t})-1} (16\tilde{\nu}_2^2)^{\frac{\alpha_{\mathbb{P}}(\tilde{t})-2}{2}}}$$

$$F_M(\tilde{t}) = \frac{m_0^2}{m_0^2 - \tilde{t}}\,, \qquad m_0^2 = 0.5\ \text{GeV}^2$$

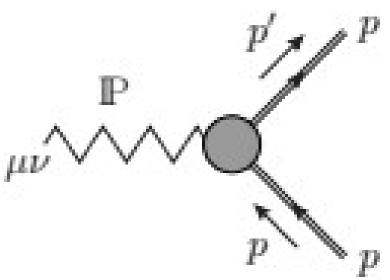
proton Dirac form factor

Effective vertices and pomeron propagator



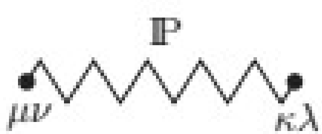
$$i\Gamma^{(\mathbb{P}\pi\pi)}_{\mu\nu}(\tilde{k}',\tilde{k}) = -i2\beta_{\mathbb{P}\pi\pi}F_M[(\tilde{k}'-\tilde{k})^2]\left[(\tilde{k}'+\tilde{k})_\mu(\tilde{k}'+\tilde{k})_\nu - \frac{1}{4}g_{\mu\nu}(\tilde{k}'+\tilde{k})^2\right],$$

$$\beta_{\mathbb{P}\pi\pi} = 1.76\ \mathrm{GeV}^{-1},$$



$$i\Gamma^{(\mathbb{P}pp)}_{\mu\nu}(p',p) = -i3\beta_{\mathbb{P}NN}F_1[(p'-p)^2]\left[\frac{1}{2}\gamma_\mu(p'+p)_\nu + \frac{1}{2}\gamma_\nu(p'+p)_\mu - \frac{1}{4}g_{\mu\nu}(\not{p}'+\not{p})\right],$$

$$\beta_{\mathbb{P}NN} = 1.87\ \mathrm{GeV}^{-1},$$



$$i\Delta^{(\mathbb{P})}_{\mu\nu,\kappa\lambda}(2\tilde{\nu},\tilde{t}) = \frac{1}{8\tilde{\nu}}\left(g_{\mu\kappa}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\kappa} - \frac{1}{2}g_{\mu\nu}g_{\kappa\lambda}\right)(-i\,2\tilde{\nu}\,\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(\tilde{t})-1},$$

$$\alpha_{\mathbb{P}}(\tilde{t}) = 1 + \epsilon_{\mathbb{P}} + \alpha'_{\mathbb{P}}\tilde{t},$$

$$\epsilon_{\mathbb{P}} = 0.0808, \qquad \alpha'_{\mathbb{P}} = 0.25\ \mathrm{GeV}^{-2};$$

$$\mathcal{M}^{(a)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) = e\widehat{\Gamma}^{(\gamma\pi\pi)}_{\mu}(k_1,k_1-q)\Delta_F[(k_1-q)^2]\mathcal{M}^{(0,\,a)}_{\mathfrak{s}',\mathfrak{s}}(k_2,p',q-k_1,p)$$

For photons of small virtuality, $Q^2 = -q^2 < 0.5\ \mathrm{GeV}^2$ we set

description of low-x DIS and DVCS HERA data
→ predominant soft-pomeron exchange;
Britzger *et al.*, PRD 100 (2019) 114007
Lebiedowicz *et al.*, PLB 835 (2022) 137947

$$\widehat{\Gamma}^{(\gamma\pi\pi)}_{\mu}(k_1,k_1-q)\,\Delta_F\left[(k_1-q)^2\right]|_{k_1^2=m_\pi^2} = \frac{(2k_1-q)_\mu}{-2k_1\cdot q+q^2+i\varepsilon}F_M(q^2) - q_\mu\frac{1-F_M(q^2)}{q^2}$$

where $F_M(q^2) = \dfrac{m_0^2}{m_0^2-q^2}\,, \quad m_0^2 = 0.5\ \mathrm{GeV}^2$

is a simple representation of the pion electromagnetic form factor.

For real photons ($q^2 = 0$) this is an exact result up to an irrelevant gauge term.
For small photon virtualities this represents our model assumption.

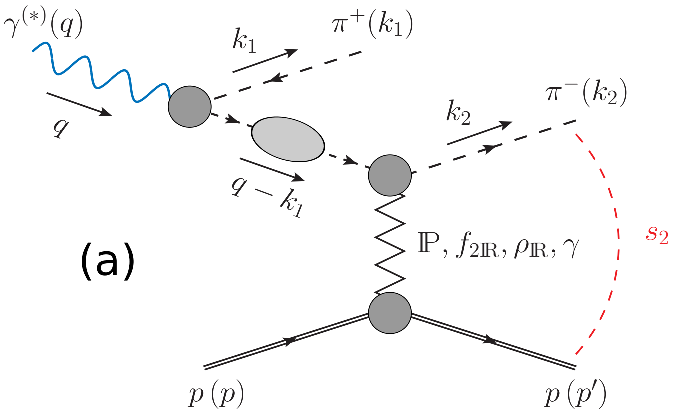
We get for diagram (a):

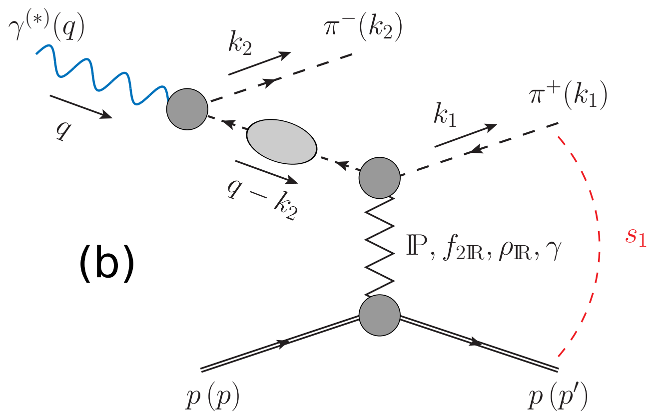
$$\mathcal{M}^{(a)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p)|_{\mathbb{P}} = ie\left[\frac{(2k_1-q)_\mu}{-2k_1\cdot q+q^2+i\varepsilon}F_M(q^2) - q_\mu\frac{1-F_M(q^2)}{q^2}\right]\mathcal{F}_{\mathbb{P}\pi p}(2\nu_2,t)$$
$$\times\left[2(k_2-k_1+q)^\nu(k_2-k_1+q,p'+p)\bar{u}_{\mathfrak{s}'}(p')\gamma_\nu u_{\mathfrak{s}}(p) - (k_2-k_1+q)^2 m_p\bar{u}_{\mathfrak{s}'}(p')u_{\mathfrak{s}}(p)\right]$$

$$\mathcal{F}_{\mathbb{P}\pi p}(2\nu_2,t) = \mathcal{F}_{\mathbb{P}\pi p}(2\bar{\nu},t)\left[1+(2-\alpha_{\mathbb{P}}(t))\frac{\varkappa}{2}g\left(\frac{2-\alpha_{\mathbb{P}}(t)}{2},\varkappa\right)\right]$$

$$g(\lambda,\varkappa) = \frac{(1-\varkappa)^{-\lambda}-1}{\lambda\varkappa}$$

In a completely analogous way we get amplitude for diagram (b) for IP exchange.





$$q+p=k_1+k_2+p'$$

$$\begin{aligned}
s &= (q+p)^2=(k_1+k_2+p')^2\,,\\
t &= (p-p')^2=(q-k_1-k_2)^2\,,\\
s_1 &= (p'+k_1)^2=(p+q-k_2)^2\,,\\
u_1 &= (p-k_1)^2=(p'-q+k_2)^2\,,\\
s_2 &= (p'+k_2)^2=(p+q-k_1)^2\,,\\
u_2 &= (p-k_2)^2=(p'-q+k_1)^2\,,\\
M_{\pi\pi}^2 &= (k_1+k_2)^2=(p-p'+q)^2\,;\\
\nu_1 &= \frac{1}{4}(s_1-u_1)\\
&= \frac{1}{4}\left[(p+p',k_1-k_2)+(p+p',q)\right]\,,\\
\nu_2 &= \frac{1}{4}(s_2-u_2)\\
&= \frac{1}{4}\left[(p+p',k_2-k_1)+(p+p',q)\right]\,.
\end{aligned}$$

We have

$$\begin{aligned}
\nu_1^2 &= \frac{1}{16}[(p+p',q)^2+(p+p',k_1-k_2)^2+2(p+p',k_1-k_2)(p+p',q)]\,,\\
\nu_2^2 &= \frac{1}{16}[(p+p',q)^2+(p+p',k_1-k_2)^2-2(p+p',k_1-k_2)(p+p',q)]\,,
\end{aligned}$$

and we define

$$\begin{aligned}
\bar{\nu}^2 &= \frac{1}{2}(\nu_1^2+\nu_2^2)\,,\\
\varkappa &= \frac{2(q,p+p')(p+p',k_1-k_2)}{16\bar{\nu}^2}\,.
\end{aligned}$$

$$\begin{aligned}
|\varkappa| &\leqslant 1\,,\\
16\nu_1^2 &= 16\bar{\nu}^2(1+\varkappa)\,,\\
16\nu_2^2 &= 16\bar{\nu}^2(1-\varkappa)\,.
\end{aligned}$$

Gauge invariance requires

$$q^{\mu}\mathcal{M}^{(\mathrm{DS})}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p)=0 \quad \Rightarrow \quad q^{\mu}\mathcal{M}^{(c)}_{\mu,\mathfrak{s}',\mathfrak{s}}=-q^{\mu}\mathcal{M}^{(a)}_{\mu,\mathfrak{s}',\mathfrak{s}}-q^{\mu}\mathcal{M}^{(b)}_{\mu,\mathfrak{s}',\mathfrak{s}}$$

Using the generalised Ward identity

$$(k'-k)^{\mu}\,\widehat{\Gamma}^{(\gamma\pi\pi)}_{\mu}(k',k)=\Delta_F^{-1}(k'^2)-\Delta_F^{-1}(k^2)$$ and normalisation conditions for pion propagator

$$\Delta_F^{-1}(m_\pi^2)=0\,,\ \frac{\partial}{\partial k^2}\Delta_F^{-1}(k^2)|_{k^2=m_\pi^2}=1$$

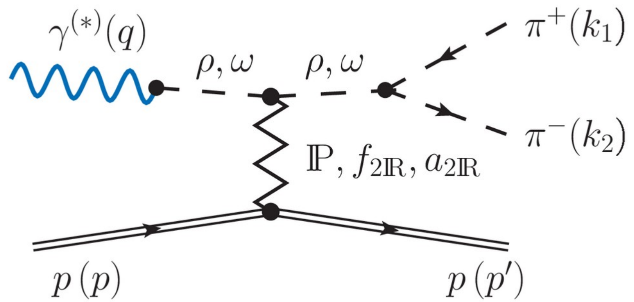
we find

$$q^{\mu}\mathcal{M}^{(c)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p)=e\Big[\mathcal{M}^{(0,\,a)}_{\mathfrak{s}',\mathfrak{s}}(k_2,p',q-k_1,p)-\mathcal{M}^{(0,\,b)}_{\mathfrak{s}',\mathfrak{s}}(k_1,p',q-k_2,p)\Big]$$

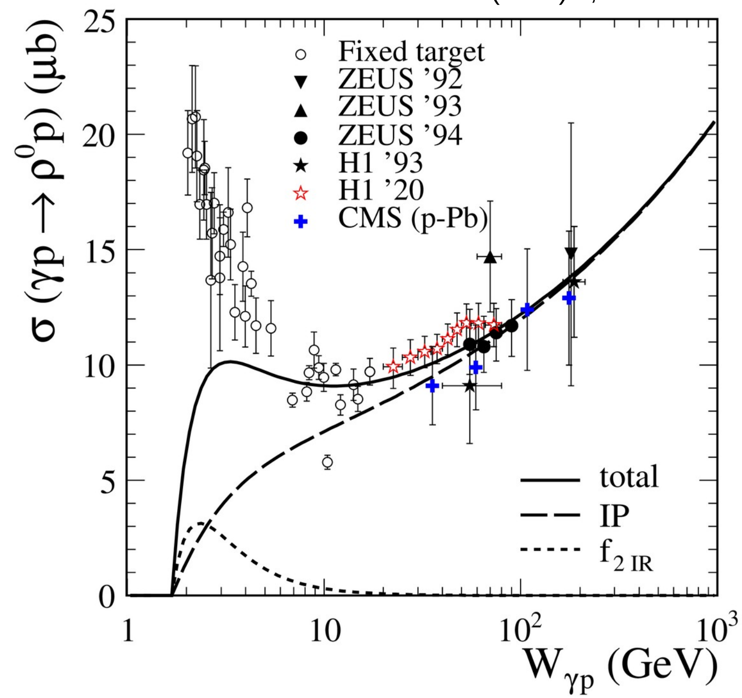
The r.h.s. of this equation is written in a way that is explicitly ∝ q^{μ}.

$$\begin{aligned}
\mathcal{M}^{(c)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p)|_{\mathbb{P}} = {}& 2ie\mathcal{F}_{\mathbb{P}\pi p}(2\bar{\nu},t)\Big\{2\delta_{\mu}{}^{\nu}(k_2-k_1,p'+p)+2(p'+p)_{\mu}(k_2-k_1)^{\nu}\\
&+(p'+p)_{\mu}(2-\alpha_{\mathbb{P}}(t))\frac{(p'+p,k_1-k_2)}{16\bar{\nu}^2}\Big[g\left(\frac{2-\alpha_{\mathbb{P}}(t)}{2},\varkappa\right)(k_2-k_1+q)^{\nu}(k_2-k_1+q,p'+p)\\
&+g\left(\frac{2-\alpha_{\mathbb{P}}(t)}{2},-\varkappa\right)(k_2-k_1-q)^{\nu}(k_2-k_1-q,p'+p)\Big]\Big\}\bar{u}_{\mathfrak{s}'}(p')\gamma_{\nu}u_{\mathfrak{s}}(p)\\
&+2ie\mathcal{F}_{\mathbb{P}\pi p}(2\bar{\nu},t)\Big\{-2(k_2-k_1)_{\mu}-(p'+p)_{\mu}\frac{2-\alpha_{\mathbb{P}}(t)}{2}\frac{(p'+p,k_1-k_2)}{16\bar{\nu}^2}\\
&\times\Big[g\left(\frac{2-\alpha_{\mathbb{P}}(t)}{2},\varkappa\right)(k_2-k_1+q)^2+g\left(\frac{2-\alpha_{\mathbb{P}}(t)}{2},-\varkappa\right)(k_2-k_1-q)^2\Big]\Big\}m_p\bar{u}_{\mathfrak{s}'}(p')u_{\mathfrak{s}}(p)
\end{aligned}$$

Resonant $\pi^+\pi^-$ production via ρ and ω scattering on the proton



ZEUS '94: EPJC 2 (1998) 247
H1 '20: EPJC 80 (2020) 12, 1189
CMS: EPJC 79 (2019) 8, 702



- Amplitudes includes IP and secondary C = +1 IR exchanges
- ρ-ω interference effect included only in the final state via propagator mixing and the explicit ω → $\pi^+\pi^-$ decay
- Numerical values of coupling constants occurring in vertices and vertex form factors were obtained from comparison of the model to experimental data; JHEP 01 (2015) 151, PRD 91 (2015) 074023, PRD 101 (2020) 094012.
- In JHEP 01 (2015) 151, a model for real photons was given. In arXiv:2508.06334 we extended the model for the case of slightly virtual photons.

$$\mathcal{M}^{(\mathrm{res})}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p)|_{\mathbb{P}+f_{2\mathbb{R}}+a_{2\mathbb{R}}} = \sum_{\substack{V=\rho,\,\omega,\\ V'=\rho,\,\omega}} \left[\mathcal{M}^{(V',V)}_{\mu,\mathfrak{s}',\mathfrak{s}}|_{\mathbb{P}} + \mathcal{M}^{(V',V)}_{\mu,\mathfrak{s}',\mathfrak{s}}|_{f_{2\mathbb{R}}}\right] + \mathcal{M}^{(\omega,\rho)}_{\mu,\mathfrak{s}',\mathfrak{s}}|_{a_{2\mathbb{R}}} + \mathcal{M}^{(\rho,\omega)}_{\mu,\mathfrak{s}',\mathfrak{s}}|_{a_{2\mathbb{R}}}$$

$$\mathcal{M}^{(V',V)}_{\mu,\mathfrak{s}',\mathfrak{s}}|_{\mathbb{P}} = \frac{i}{4}\, e\, s\, F_1(t)\, F_M(t)\, \tilde{F}^{(V)}(k^2)\, g_{V'\pi\pi} \left[\mathcal{K}^{(0,V',V)}_{\mu,\mathfrak{s}',\mathfrak{s}} V^{(0,V)}_{\mathbb{P}} - \mathcal{K}^{(2,V',V)}_{\mu,\mathfrak{s}',\mathfrak{s}} V^{(2,V)}_{\mathbb{P}}\right] \tilde{F}^{(V)}(q^2) \underbrace{(-m_V^2)\Delta^{(V,V)}_T(q^2)}_{\cong \frac{m_V^2}{m_V^2 - q^2}}$$

$$\mathcal{K}^{(i,V',V)}_{\mu,\mathfrak{s}',\mathfrak{s}} = \frac{1}{s^2}(k_1-k_2)^\nu \Delta^{(V',V)}_T(k^2)\, \Gamma^{(i)}_{\nu\mu\kappa\lambda}(k,-q)\, \bar{u}_{\mathfrak{s}'}(p')\gamma^\kappa\,(p'+p)^\lambda\, u_{\mathfrak{s}}(p)$$

Melikhov, Nachtmann, Nikonov, Paulus, EPJC 34 (2004) 345

$$V^{(0,V)}_{\mathbb{P}} = \gamma_V^{-1}\, 6\, \beta_{\mathbb{P}NN}\, a_{\mathbb{P}VV}(-i\, s\, \alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$$

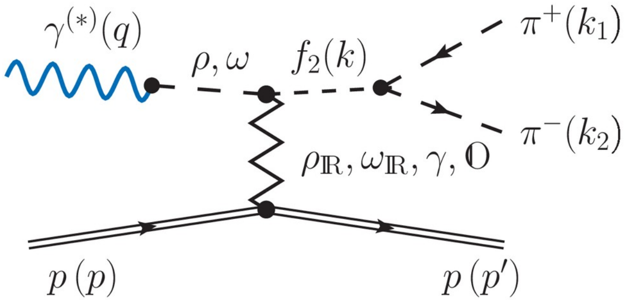
$$V^{(2,V)}_{\mathbb{P}} = \gamma_V^{-1}\, 3\, \beta_{\mathbb{P}NN}\, b_{\mathbb{P}VV}(-i\, s\, \alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$$

$$2m_\rho^2\, a_{I\!P\rho\rho} + b_{I\!P\rho\rho} = 4\beta_{I\!P\pi\pi} = 7.04\ \mathrm{GeV}^{-1}$$

$$a_{I\!P\rho\rho} = 0.7\ \mathrm{GeV}^{-3}$$

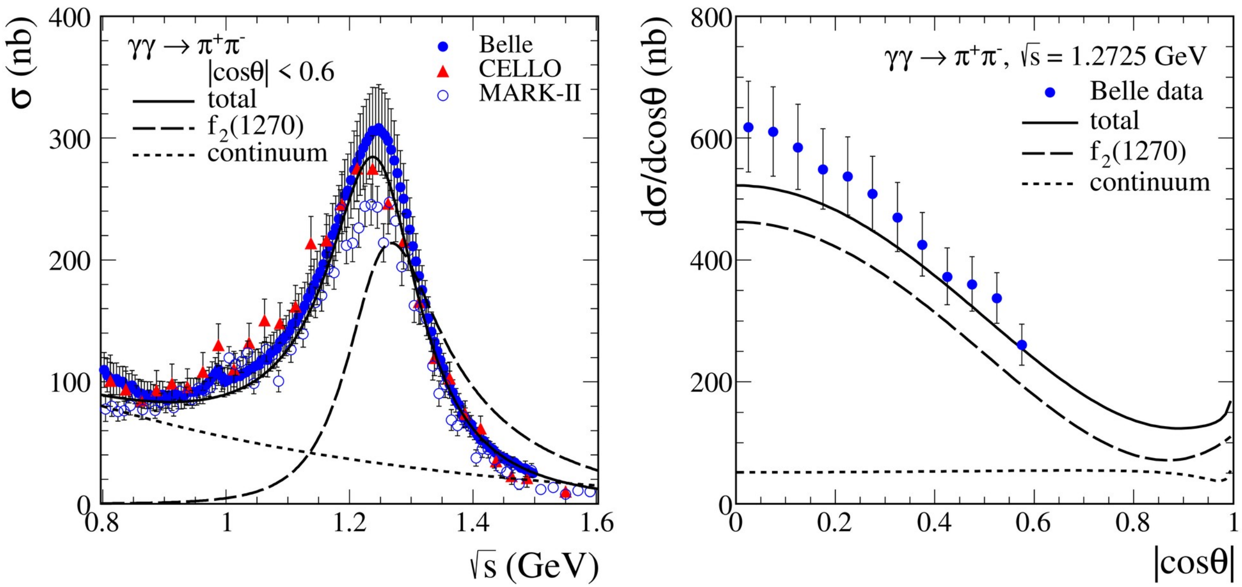
$$b_{I\!P\rho\rho} = 6.2\ \mathrm{GeV}^{-1}$$

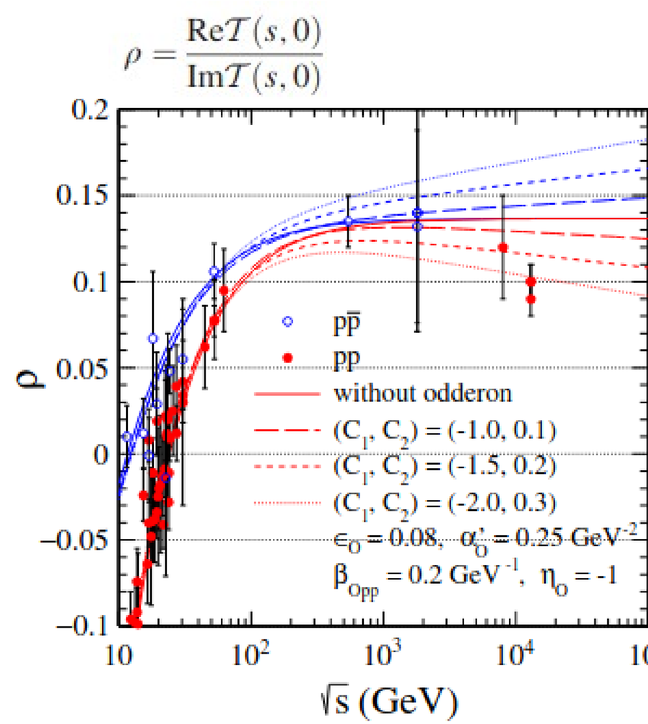
Resonant $\pi^+\pi^-$ production via $f_2(1270)$



- Production of f_2 resonance can occur by the C = -1 reggeons and odderon considered as effective vector exchanges and by the photon (Primakoff effect)
- $f_2\pi\pi$ and $f_2\gamma\gamma$ coupling constants and form factors with cut-off parameters were estimated by comparing model results for the $\gamma\gamma \to \pi^+\pi^-$ reaction with the Belle data
- For the coupling parameters of $V_{IR}Vf_2$ we have assumed here that are the same size as VVf_2
- From study of *pp* and $p\bar{p}$ scattering [PRD 106 (2022) 034023] we found that a double-pole ansatz for the odderon seems to be preferred → a better description of TOTEM data for ρ = 0.1

Belle data: T. Mori *et al.*, J. Phys. Soc. Jap. 76 (2007) 074102

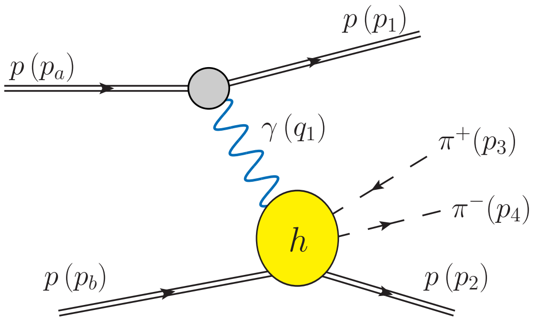


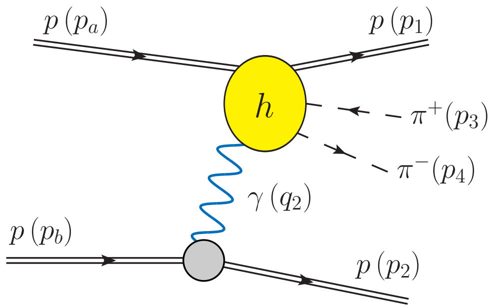


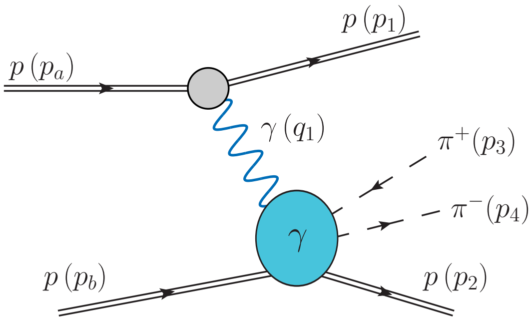
Lebiedowicz, Nachtmann, Szczurek, PRD 106 (2022) 034023

$pp \to pp\ \pi^+\pi^-$

$$p(p_a, \lambda_a) + p(p_b, \lambda_b) \to p(p_1, \lambda_1) + \pi^+(p_3) + \pi^-(p_4) + p(p_2, \lambda_2)$$







Complete amplitude for central exclusive photoproduction of $\pi^+\pi^-$ pairs:

$$\mathcal{M}_{pp\to pp\pi^+\pi^-} = \mathcal{M}^{(\gamma h)}_{pp\to pp\pi^+\pi^-} + \mathcal{M}^{(h\gamma)}_{pp\to pp\pi^+\pi^-} + \mathcal{M}^{(\gamma\gamma)}_{pp\to pp\pi^+\pi^-}$$

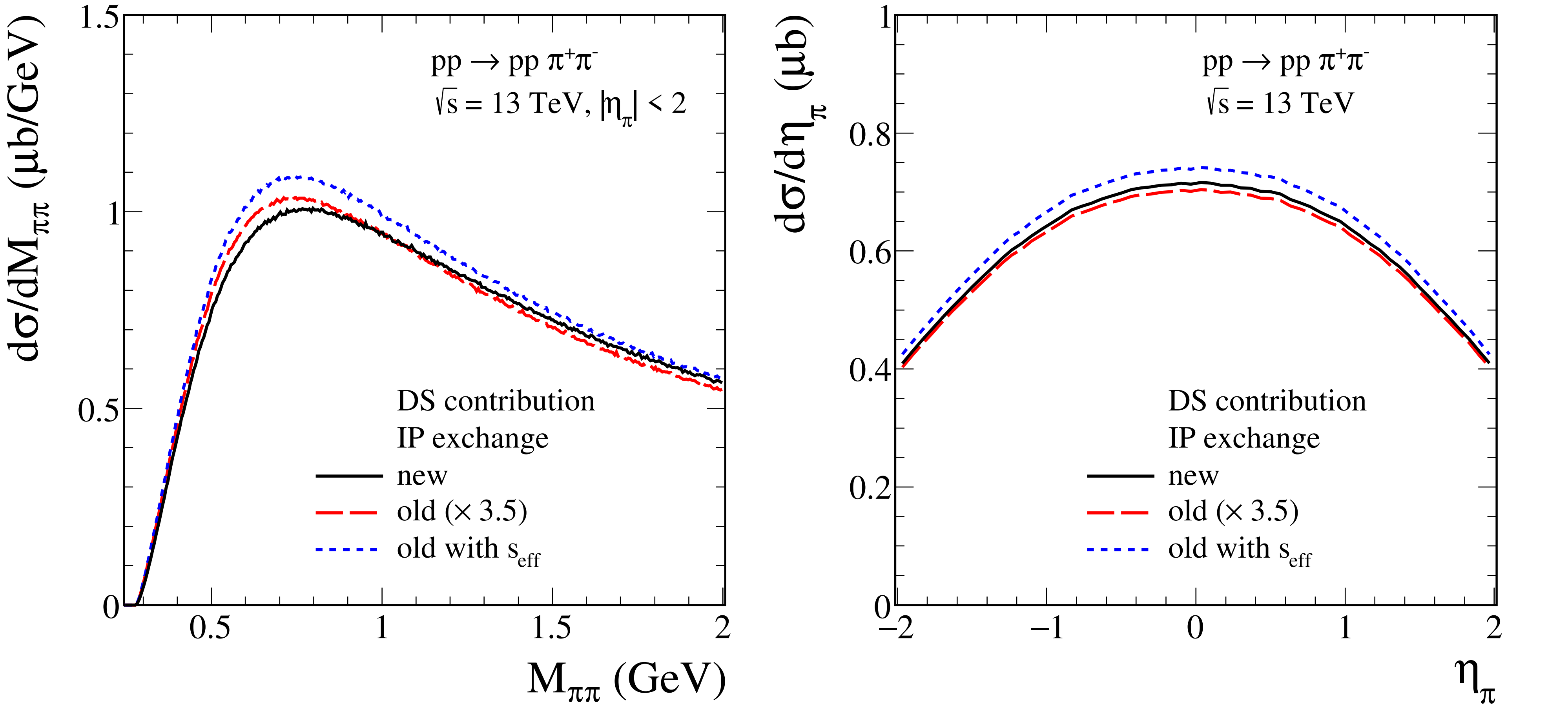
$$\mathcal{M}^{(\gamma h)}_{pp\to pp\pi^+\pi^-} = \bar{u}(p_1)\Gamma^{(\gamma pp)\mu}u(p_a)\frac{1}{t_1}\left(\mathcal{M}^{(\mathrm{res})}_{\mu}|_{\mathbb{P}+f_{2\mathbb{R}}+a_{2\mathbb{R}}} + \mathcal{M}^{(f_2)}_{\mu}|_{\rho_{\mathbb{R}}+\omega_{\mathbb{R}}+\mathbb{O}} + \mathcal{M}^{(\mathrm{DS})}_{\mu}|_{\mathbb{P}+f_{2\mathbb{R}}+\rho_{\mathbb{R}}}\right)$$

$$\mathcal{M}^{(\gamma\gamma)}_{pp\to pp\pi^+\pi^-} = \bar{u}(p_1)\Gamma^{(\gamma pp)\mu}u(p_a)\frac{1}{t_1}\left(\mathcal{M}^{(f_2)}_{\mu}|_{\gamma} + \mathcal{M}^{(\mathrm{DS})}_{\mu}|_{\gamma}\right)$$

- Eventually we should also include absorption corrections due to the proton-proton interactions to the Born amplitudes above. This absorption reduces the cross section for photoproduction processes by about 10% at LHC energies. These effects depend on the kinematic conditions in a particular experiment.

- We consider the kinematic regime where at least one photon exchange is involved, that is, at least one proton gets only a very small deviation. We neglect the diffractive IP IP, IP IR, and IR IR contributions that were discussed e.g. in PRD93 (2016) 054015.

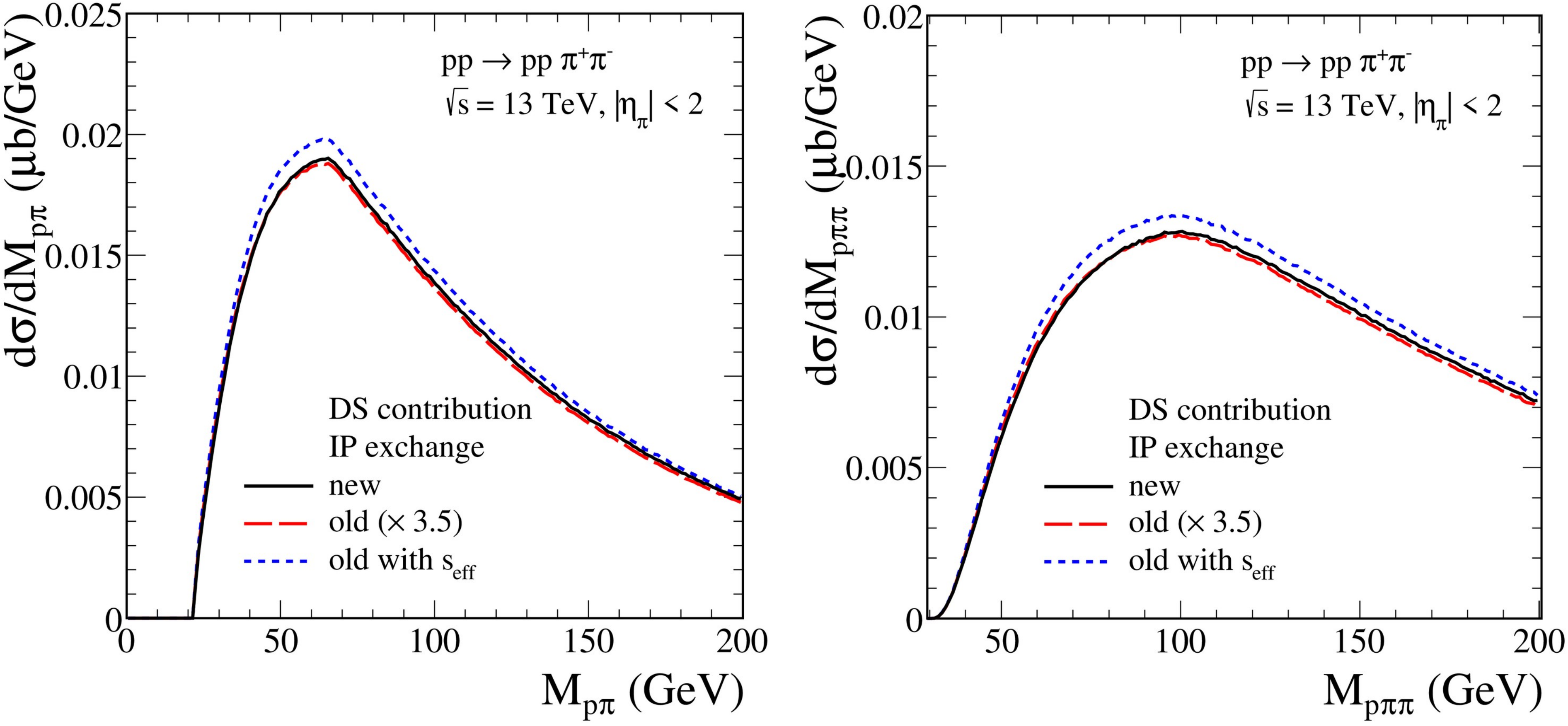
Results



- Cross section for our new Drell-Söding (DS) contribution is larger by a factor of 3.5 compared to old result with a common energy variable ($s_{eff} = s = M^2_{p\pi\pi}$) in the pomeron propagator.

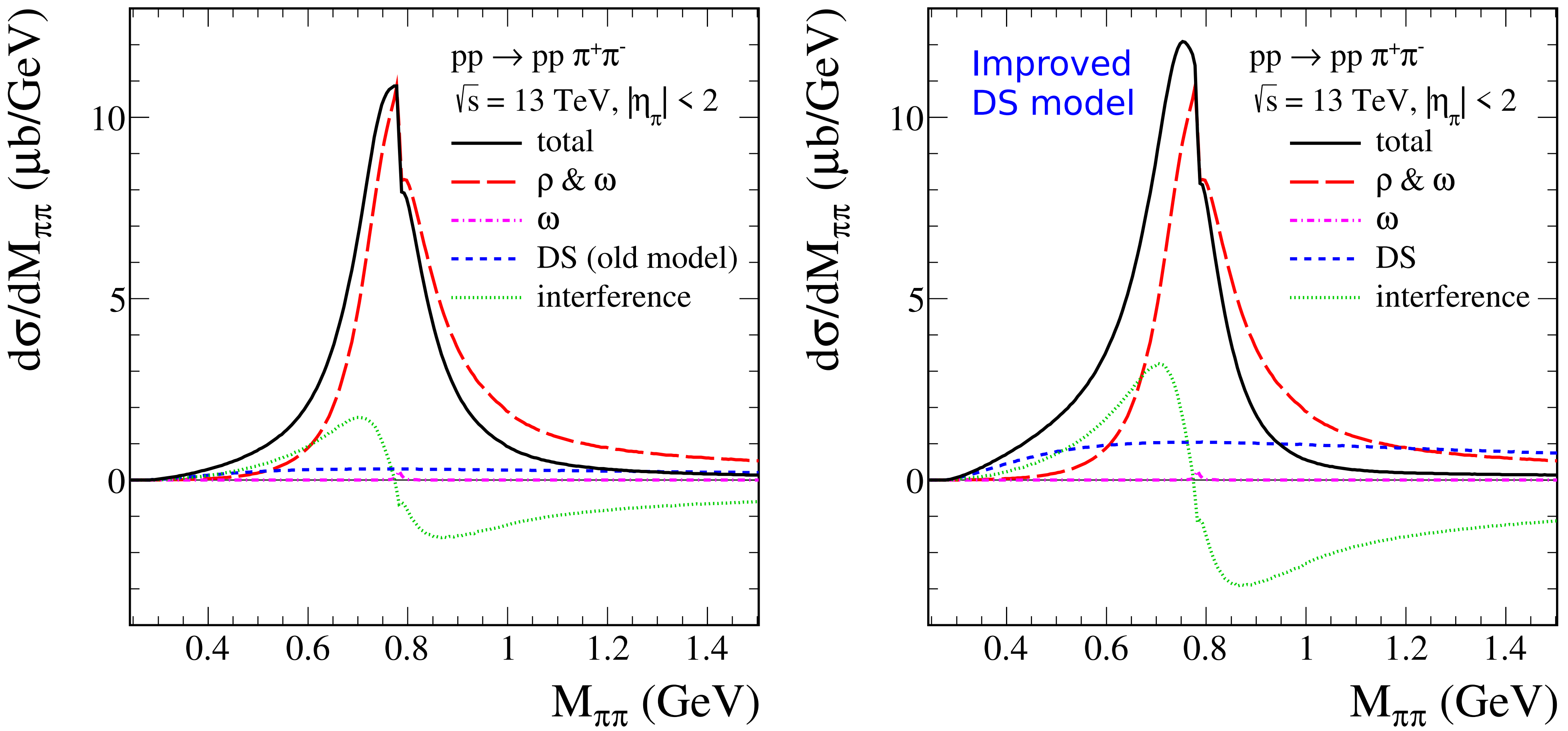
- In order to improve the old model, one can use $s_{eff} = s/2$ instead of s. This procedure leads to description of data for the $\gamma p \to \pi^+\pi^- p$ reaction measured by the H1 Collaboration.

Results



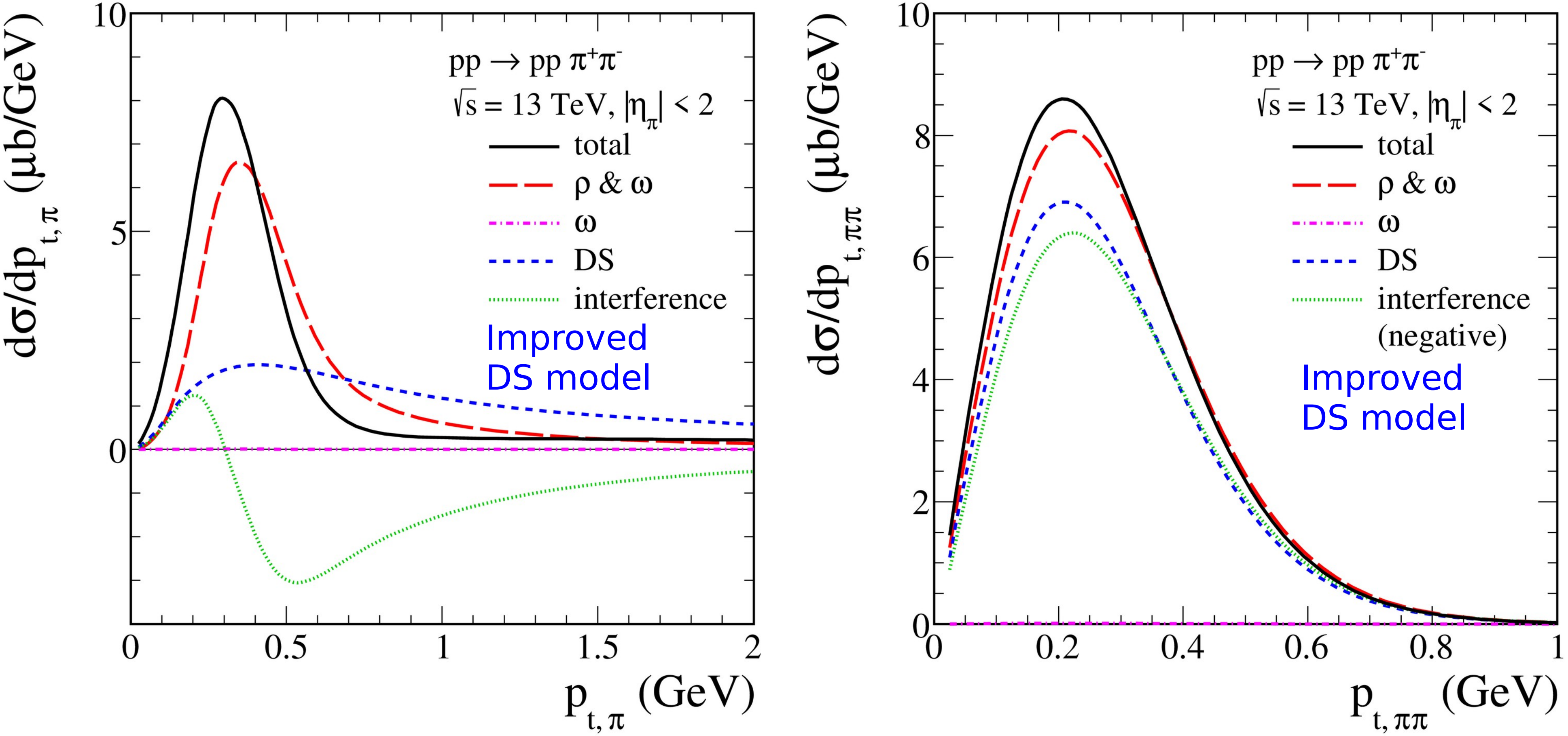
- Predictions of proton-pion and proton-pion-pion invariant mass distributions.

Results



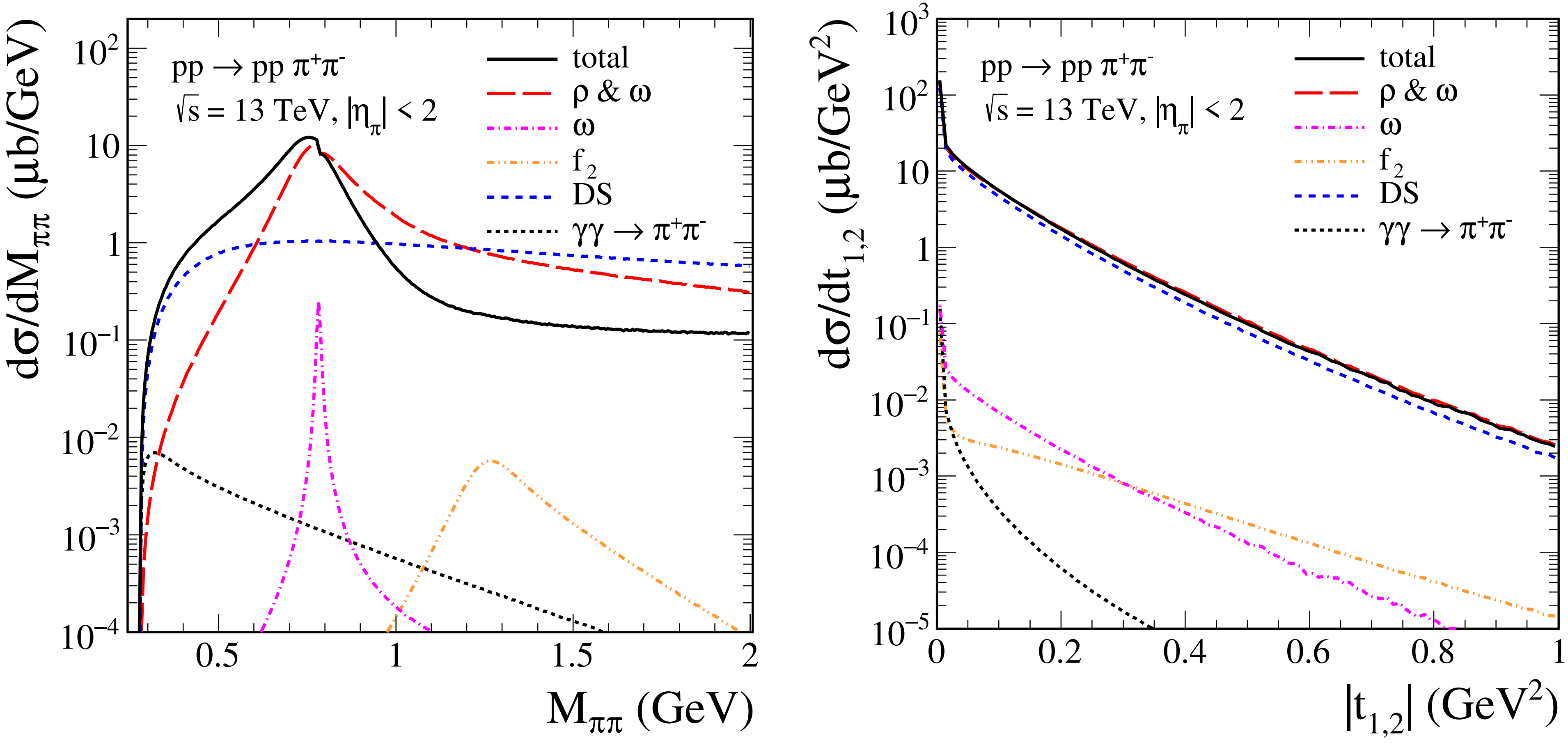
- Improved DS model leads to enhanced cross section and gives an increased skewing of the ρ^0 spectral shape (caused by the interference of the ρ^0 and $\pi^+\pi^-$ continuum)
- ρ-ω interference effect is also clearly exposed

Results



- Distributions in transverse momentum of the pion and in transverse momentum of the $\pi^+\pi^-$ pair.

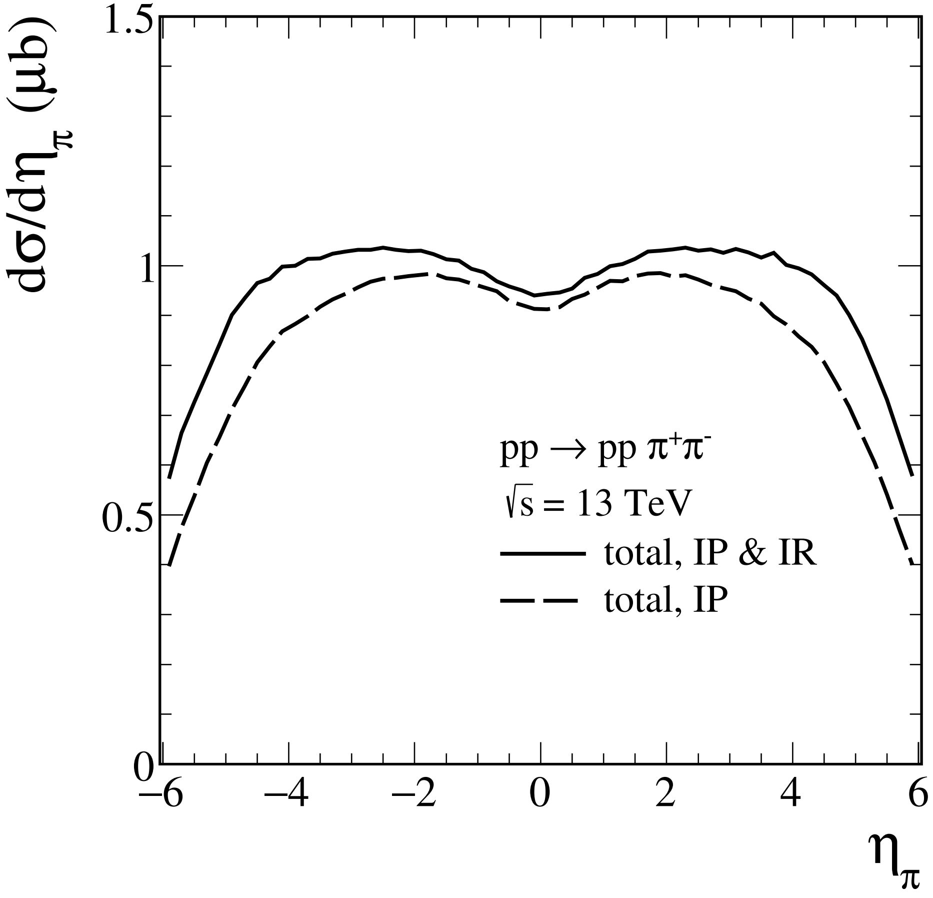
Results

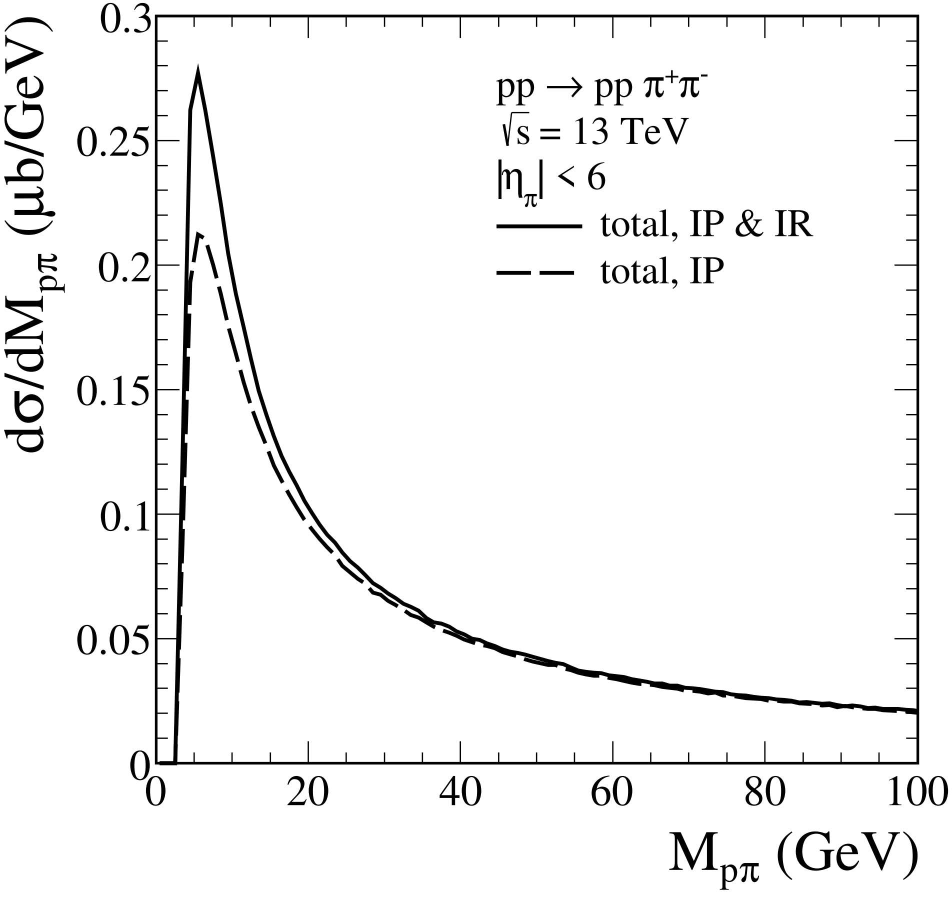


- Complete result (total) and the resonant and non-resonant contributions.
- $f_2(1270)$ production is more than 3 orders of magnitude smaller than $\rho(770)$.
  The production of higher-mass vector mesons decaying into $\pi^+\pi^-$ could easily be added here.
- Non-resonant DS contribution ($\gamma\gamma \to \pi^+\pi^-$) is negligibly small.

- $t_{1,2}$ distributions are strongly peaked at very small $|t_{1,2}|$. This is caused by the factors $1/t_{1,2}$ from the photon propagators.

Results





- Secondary reggeon exchanges contribute mainly at backward and forward pion pseudorapidity regions that correspond to low proton-pion invariant mass regions.

  The cut at $|\eta_\pi| < 2$ eliminates small proton-pion subenergies and we have $M_{p\pi} > 20$ GeV.

Summary

- We have proposed (Lebiedowicz, Nachtmann, Szczurek, arXiv: 2508.06334) a new model for the non-resonant (Drell-Söding, DS) contribution to the reactions:

$$\gamma^{(*)} + p \to \pi^+ + \pi^- + p\,,$$
$$p + p \to p + \pi^+ + \pi^- + p\,.$$

  Calculations have been done within the tensor-pomeron approach including the secondary reggeon exchanges and the odderon (assumed to couple to hadrons like a vector).

  This improves results presented in JHEP 01 (2015) 15 for real photoproduction of $\pi^+\pi^-$ pairs and in PRD 91 (2015) 074023 for the reaction $pp \to pp\pi^+\pi^-$ (revised DS model gives a larger cross section by a factor of 3.5). The interference effect (ρ(770) + DS) is more pronounced and leads to larger skewing of the observed spectral shape of ρ(770).

- Our findings should be important for the measurements of $pp \to pp\pi^+\pi^-$ reaction by the ALICE, ATLAS, CMS, and LHCb Collaborations at the LHC, even when the leading protons are not detected and only rapidity-gap conditions are checked, and for coherent $\pi^+\pi^-$ photoproduction in ultra-peripheral *p*A and AA collisions.

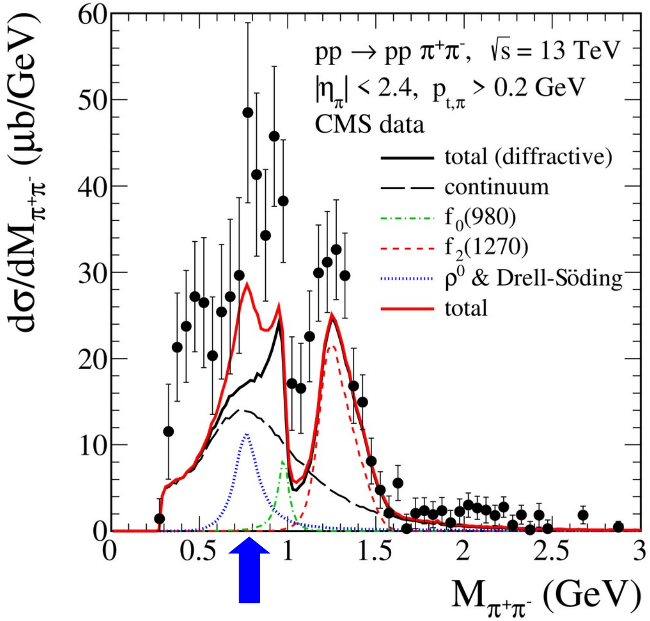
  Future measurements at the electron-ion colliders (EIC and LHeC) would be very helpful to improve our understanding of non-perturbative processes, in principle to check of the basic structures of the tensor-pomeron model.

Thank you for listening !

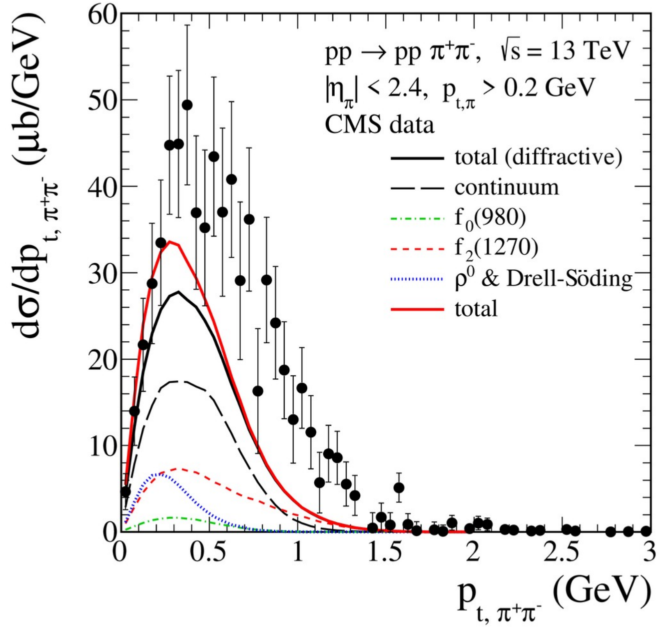
Thank you for listening !

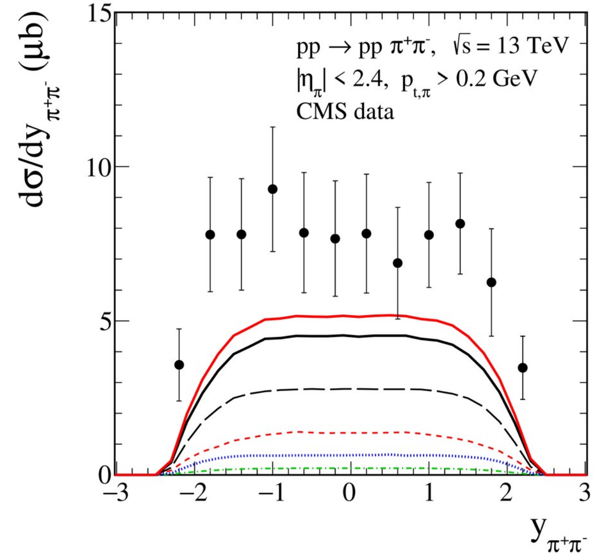
$pp \to pp\, \pi^+\pi^-$

- **Comparison to CMS data, Eur. Phys. J. C 80 (2020) 718**
  → rapidity gap method (gaps between the $\pi^+\pi^-$ system and the outgoing protons) – no proton tagging
  This measurement is not fully exclusive → the data contains contributions associated with one and both protons undergoing dissociation



photoproduction contribution

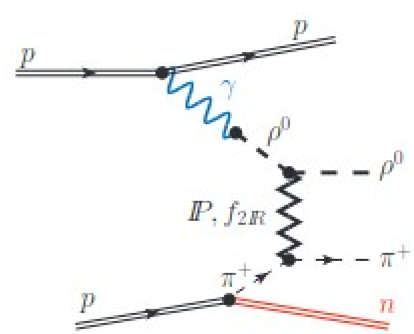


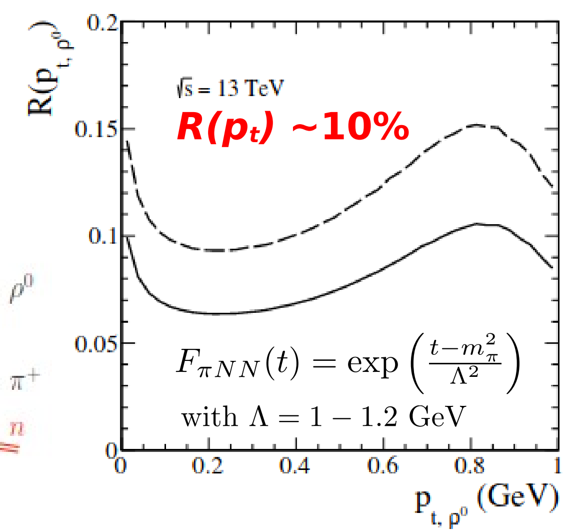


- PL, Nachtmann, Szczurek, PRD95 (2017) 034036
  Drell-Hiida-Deck type mechanism with centrally produced ρ^0 associated with a very forward/backward πN system

$$\text{Plotted is } R(p_{t,\rho^0}) = \frac{d\sigma_{pp \to pN\rho^0\pi}/dp_{t,\rho^0}}{d\sigma_{pp \to pp\rho^0}/dp_{t,\rho^0}}$$

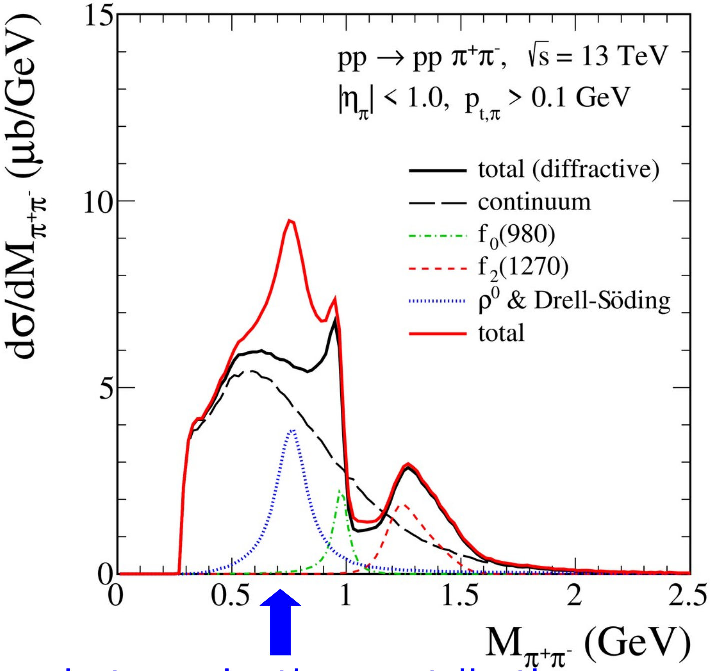
where $pN\rho^0\pi$ stands for $pn\rho^0\pi^+$ and $pp\rho^0\pi^0$





$pp \to pp\ \pi^+\pi^-$

PRELIMINARY RESULTS



photoproduction contribution