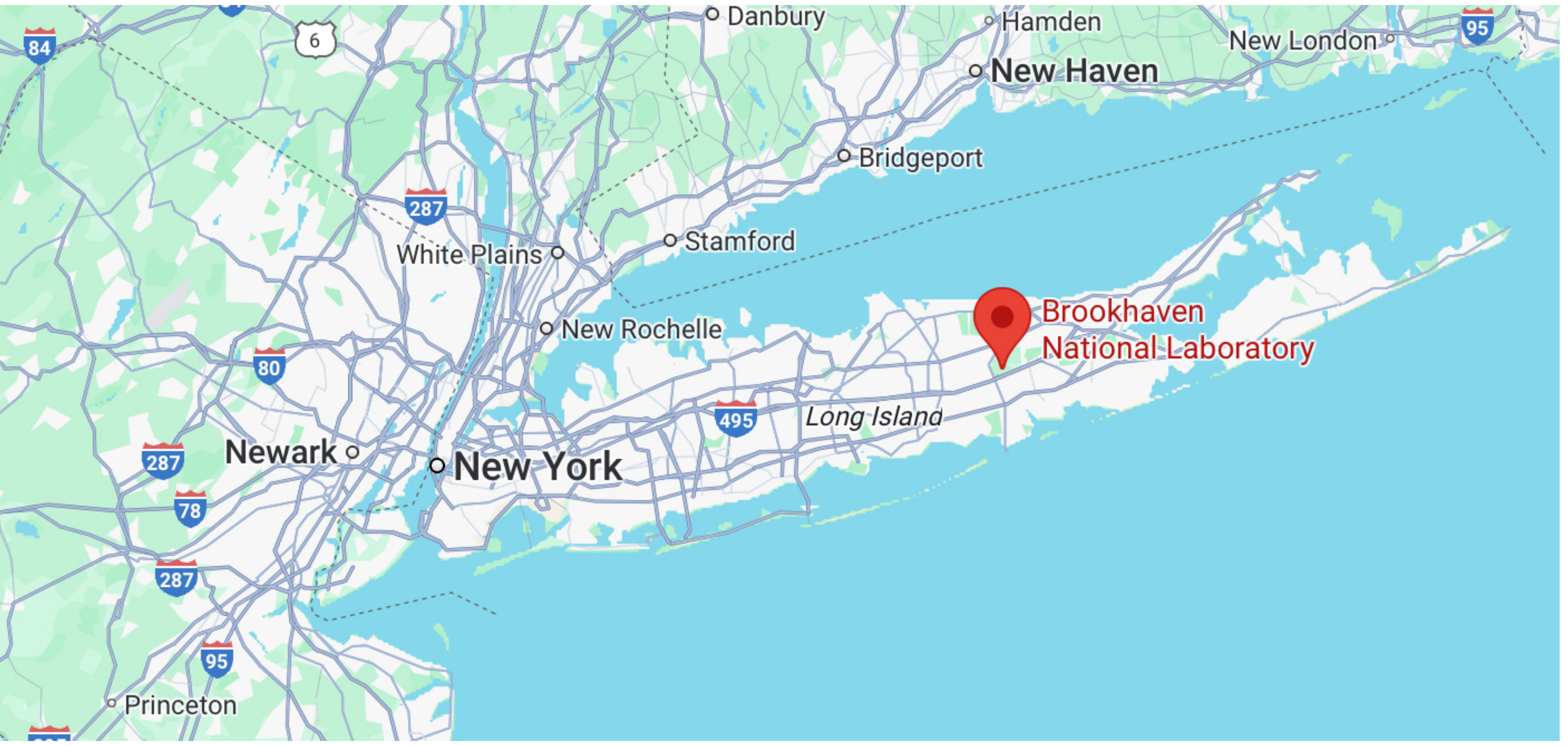
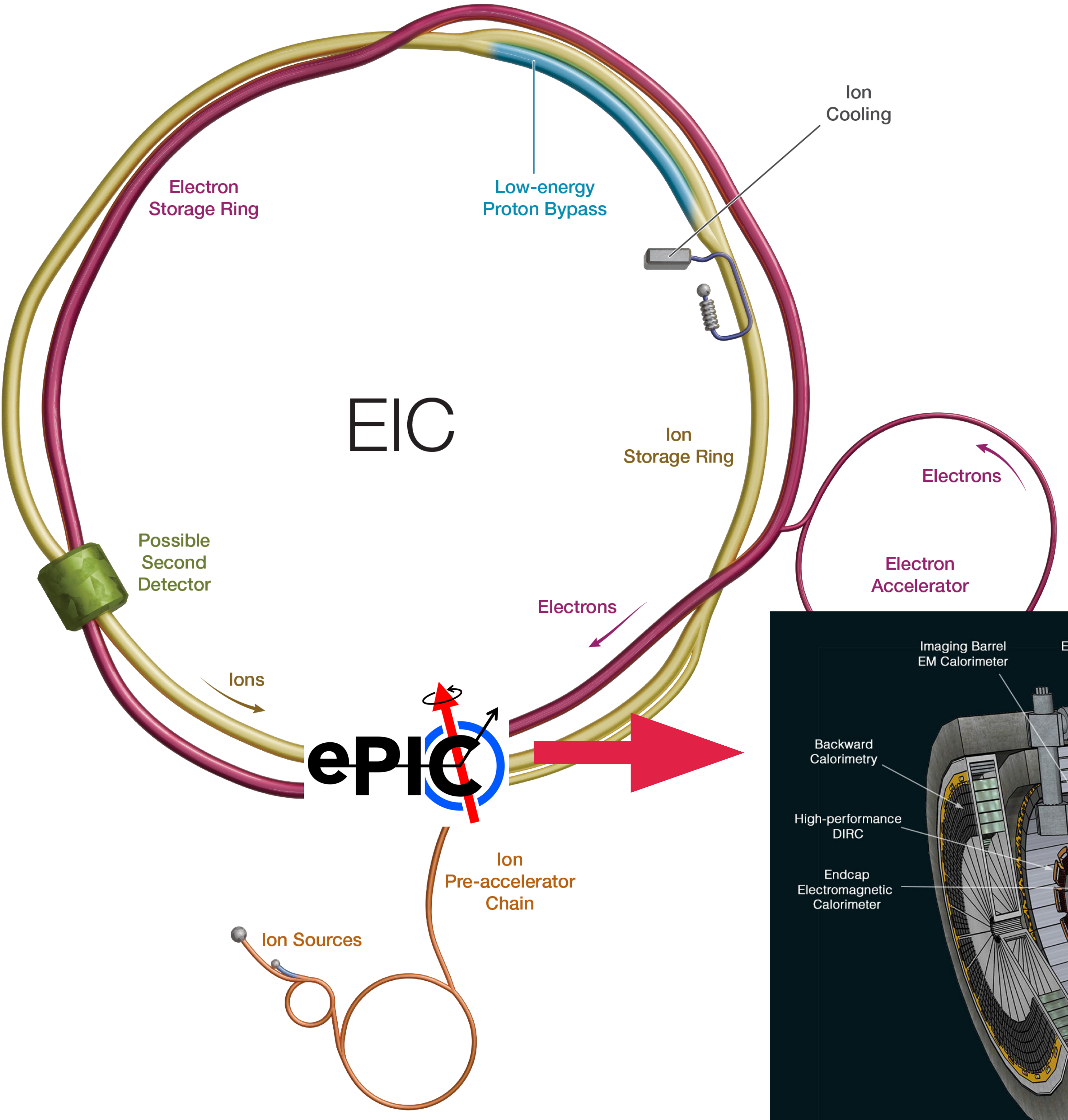
Extracting the gluon density at the Electron-Ion Collider from F_L measurements

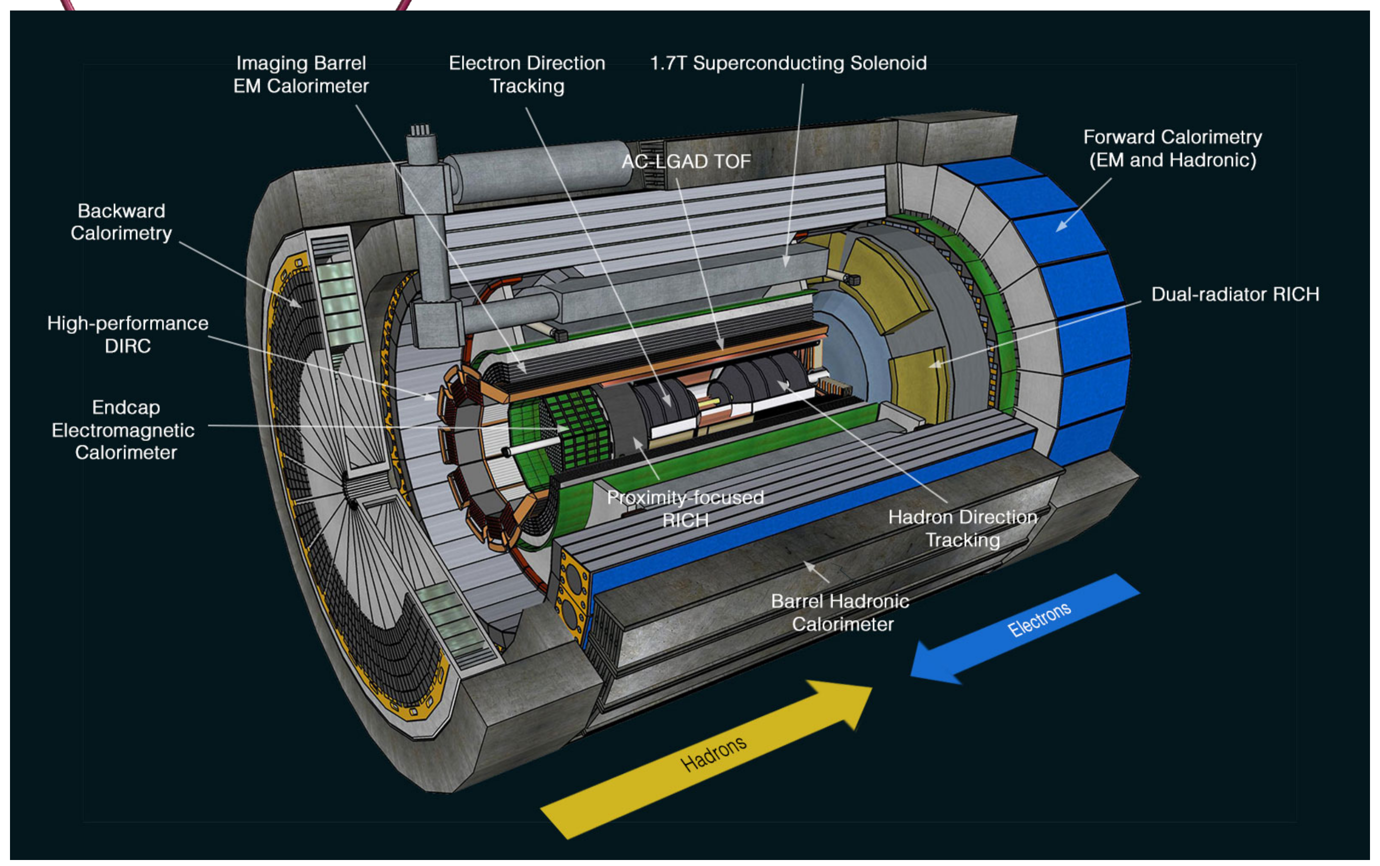
Javier Jiménez-López in collaboration with Katarzyna Wichmann and Paul R. Newman

Synergies between the EIC and the LHC
Krakow, Poland
September 22-24, 2025



The Electron-Ion Collider (EIC)





Hadron energy: 41 – 275 GeV

Electron energy: 5 – 18 GeV

$\rightarrow \sqrt{s} = 29 - 141$ GeV

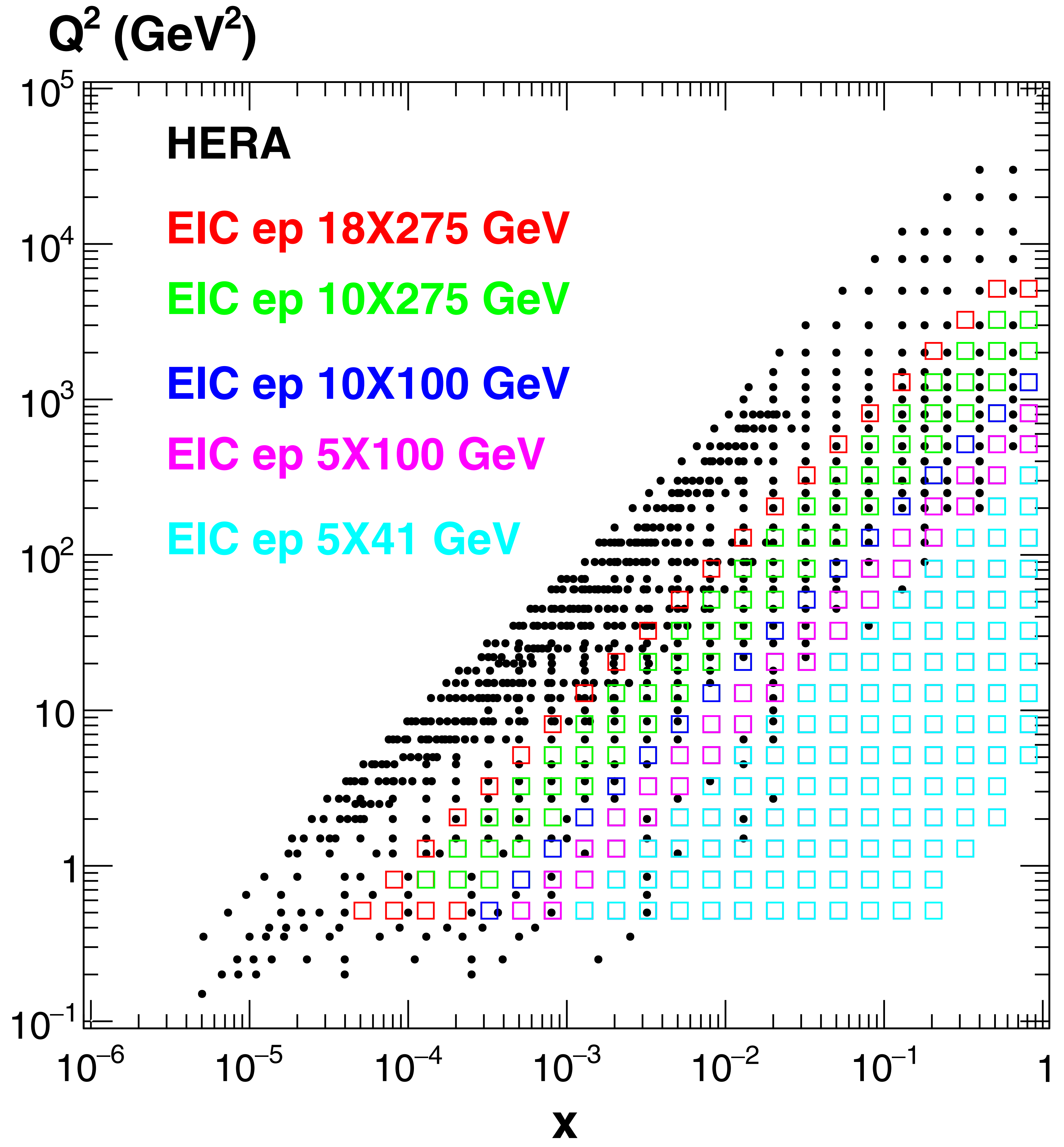
70 % polarisation for e^-, p and He

Heavier, unpolarised ions (up to U)

$\mathscr{L}_{\text{int}} = 10 - 100 \text{ fb}^{-1}/\text{year}$

The Electron-Ion Collider (EIC)

First $e^{-}p$ collider since HERA (1992 – 2007)



Extended kinematic phase space coverage at higher x than HERA with overlap at lower x

The Altarelli-Martinelli relation

Connects the unpolarised gluon PDF with the longitudinal structure function F_L.

- Formalised by Altarelli and Martinelli (*Altarelli et al., Physics Letters B* **76**, 89-94).
- Further simplified in various papers (*Zijlstra et al., Nuclear Physics B* **383**, 525-574; *Cooper-Sarkar et al., Zeitschrift für Physik C Particles and Fields* **39**, 281-290; *Borun et al., The European Physical Journal C* **72**, 2221).

In it's approximate form, the Altarelli-Martinelli relation is:

$$f_1^g(x, Q^2) \approx 1.77 \, \frac{3\pi}{2\alpha_s(Q^2)} \, F_L(x, Q^2)$$

Note: this equation is only valid for low values of Bjorken x.

Outcome of the DESY 2024 Summer Student Program

PHYSICAL REVIEW D **111**, 056014 (2025)

Prospects for measurements of the longitudinal proton structure function F_L at the Electron Ion Collider

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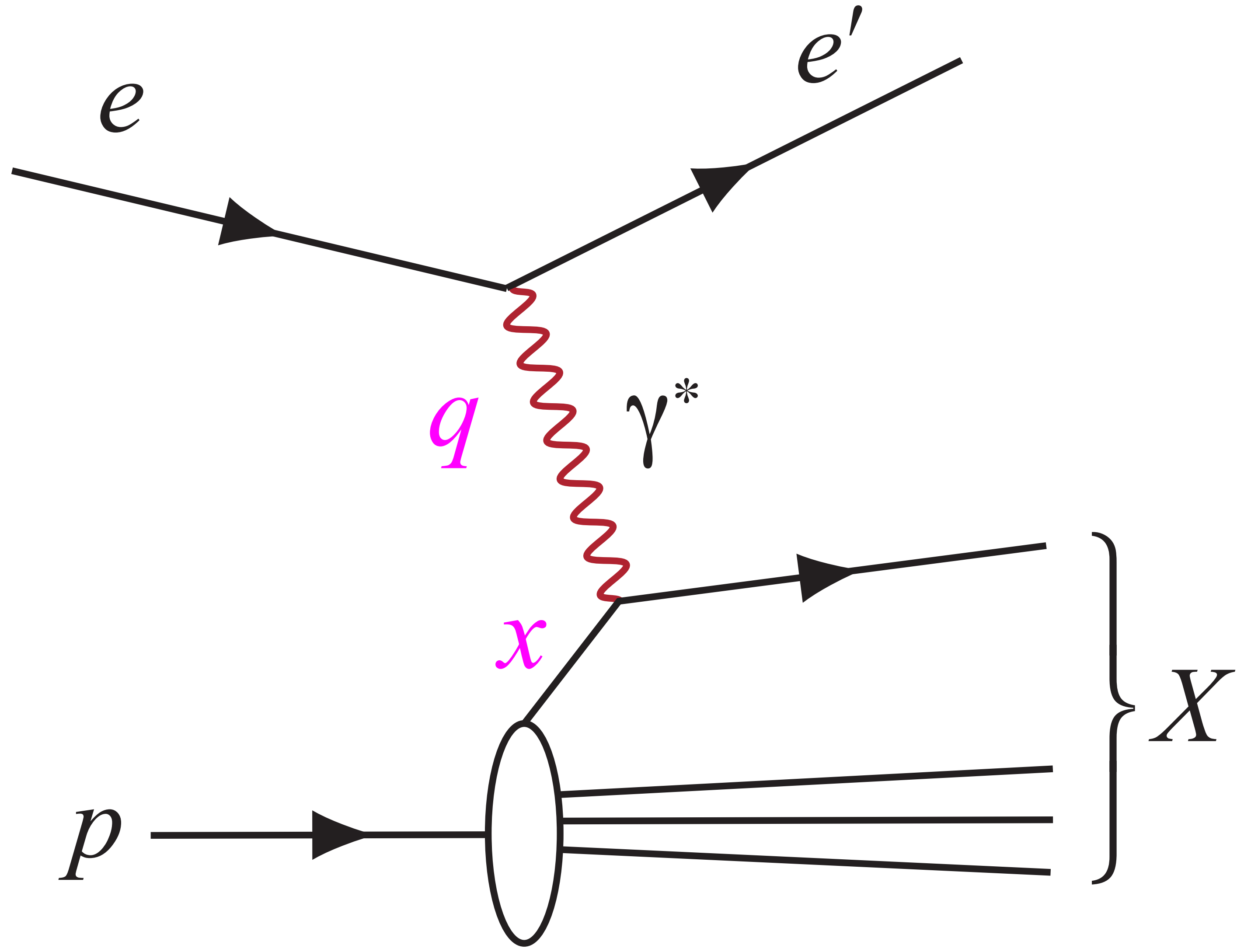
Extraction of F_L at the EIC

The reduced cross-section of the $e^- p$ NC DIS for $Q^2 \ll M_Z^2$ is:

$$\sigma_r(x, Q^2, y) = F_2(x, Q^2) - \frac{y^2}{Y_+} F_L(x, Q^2)$$

- x: Bjorken scaling variable
- y: inelasticity
- Q^2: virtuality
- $Y_+ = 1 + (1 - y)^2$



As x and Q^2 are known, F_2 and F_L can be extracted using a linear fit.

This is the well-known Rosenbluth-type separation method (*Rosenbluth., Phys. Rev.* **79**, 615).

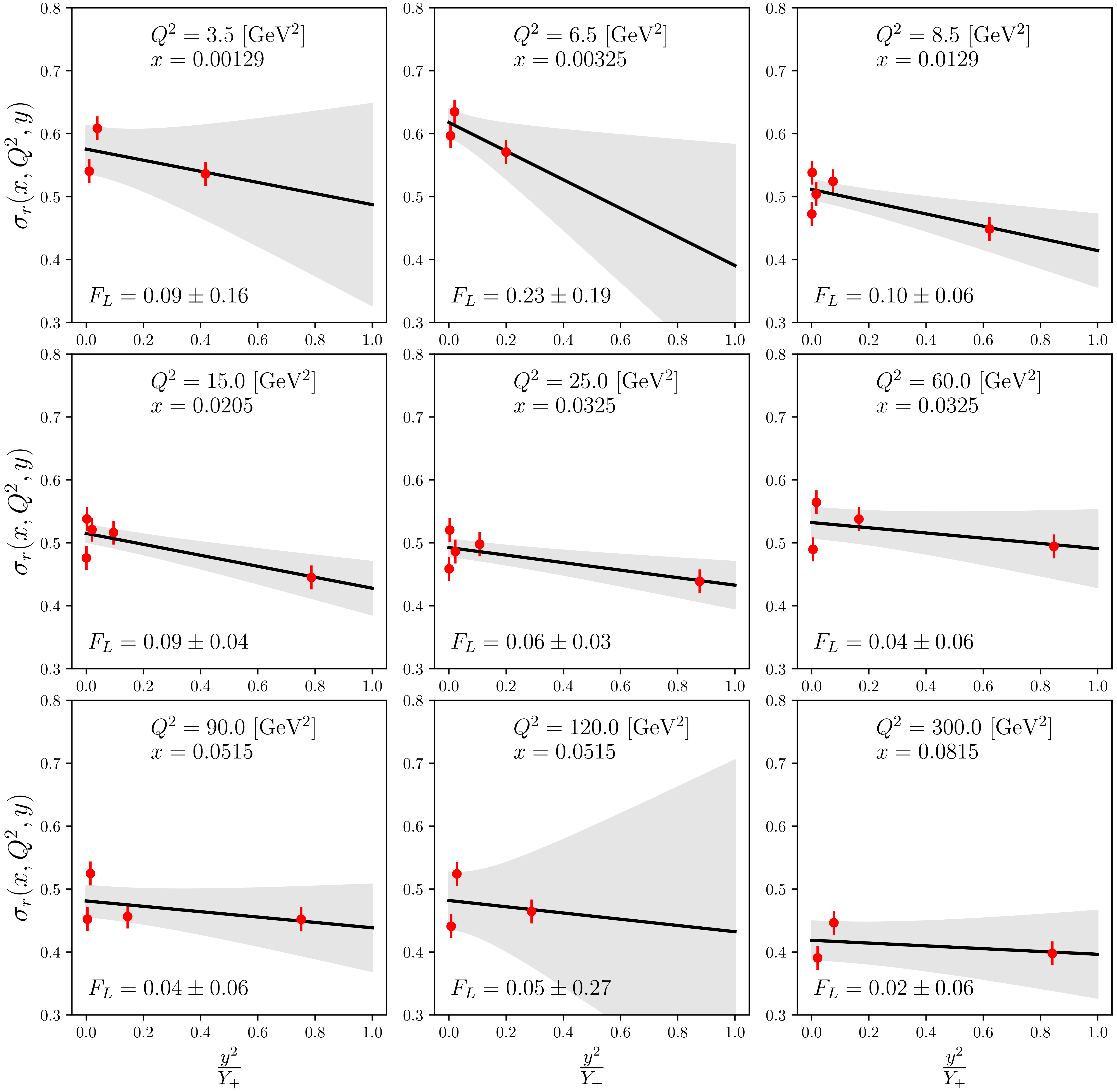
Pseudo-data simulation

e-beam energy (GeV)	p-beam energy (GeV)	$\sqrt{s}$ (GeV)	Integrated lumi (fb^{-1})
18	275	141	15.4
10	275	105	100
10	100	63	79.0
5	100	45	61.0
5	41	29	4.4

- Reduced cross-sections generated with HERAPDF2.0 NNLO.
- Smearing procedure for two different uncertainty scenarios:
  - Conservative: 1.9 % of uncorrelated systematics and 3.4 % of correlated systematics $\Longrightarrow$ total uncertainty of 3.9 % .
  - Optimistic: total uncertainty of 1 % .

Note that the uncertainty due to the normalisation between different beam energies is not considered.

Example fits



- Each point comes from a different c.o.m. energy.
- At least 3 points per (x, Q^2) bin are required to attempt the extraction of F_L.
- The point with the highest y^2/Y_+ comes from the lowest c.o.m. energy.

Averaging over MC replicas

In order to get a final measurement of F_L and it's uncertainties, we apply the following averaging procedure (*Armesto et al., Phys. Rev. D* **105**, 074006):

$$\bar{v} = \mathcal{S}_1/N \qquad (\Delta v)^2 = \frac{\mathcal{S}_2 - \mathcal{S}_1^2/N}{N-1}$$

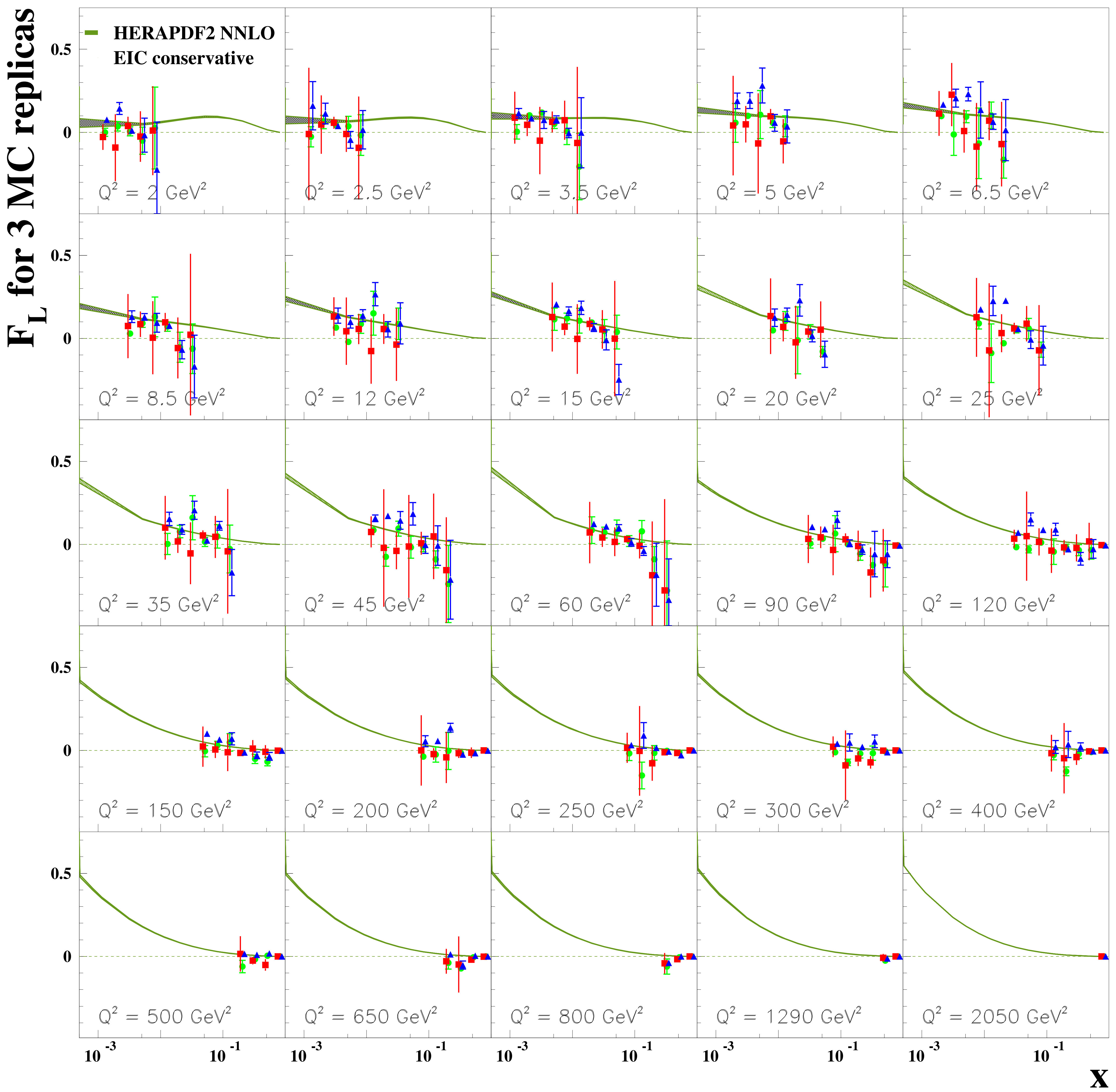
where $\mathcal{S}_n = \sum_{i=1}^{N} v_i^n$ and v_i stands for the extracted value of F_L in the i-th MC replica.

For each generated pseudo-data point:

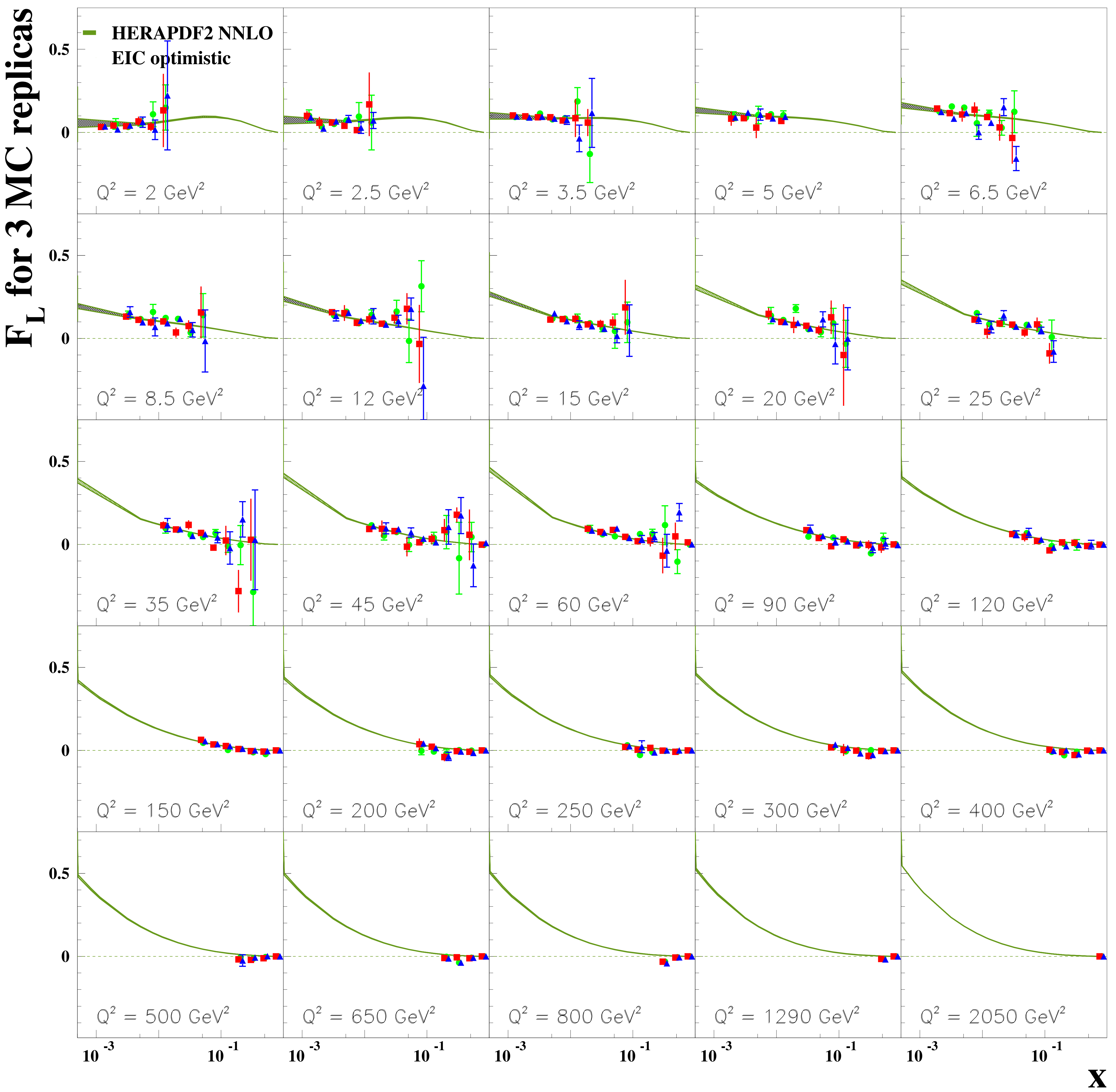
- Performed Gaussian smearing 1000 times.
- Bins whose absolute uncertainty were larger than 0.3 are not considered for the analysis:
    - This criterion removes around 30 % of the points for the conservative scenario and 20 % for the optimistic one.
    - These points might be recovered once real data is available.

Example replicas

Conservative scenario

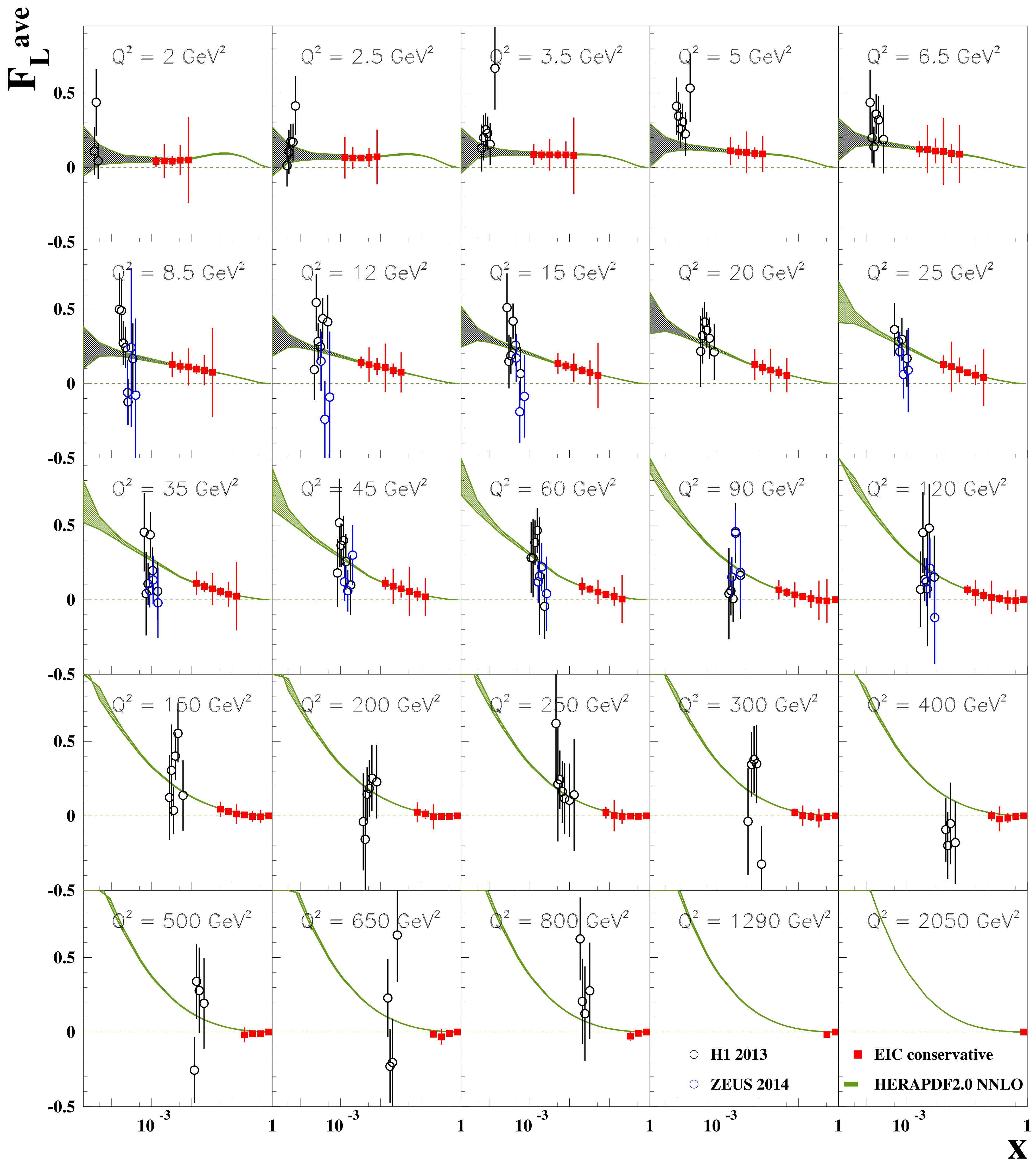


Optimistic scenario

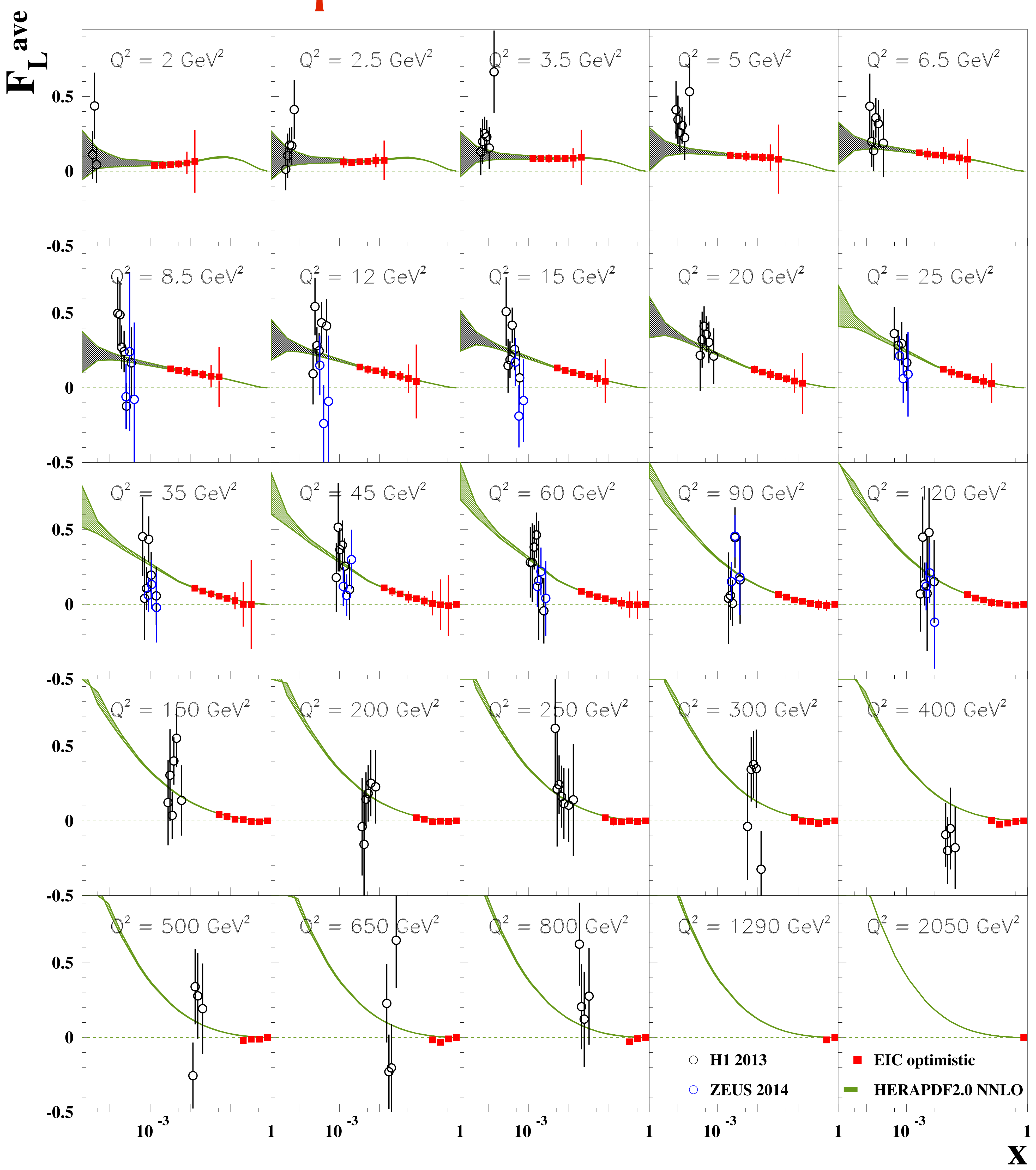


F_L averaged over 1000 MC replicas

Conservative scenario

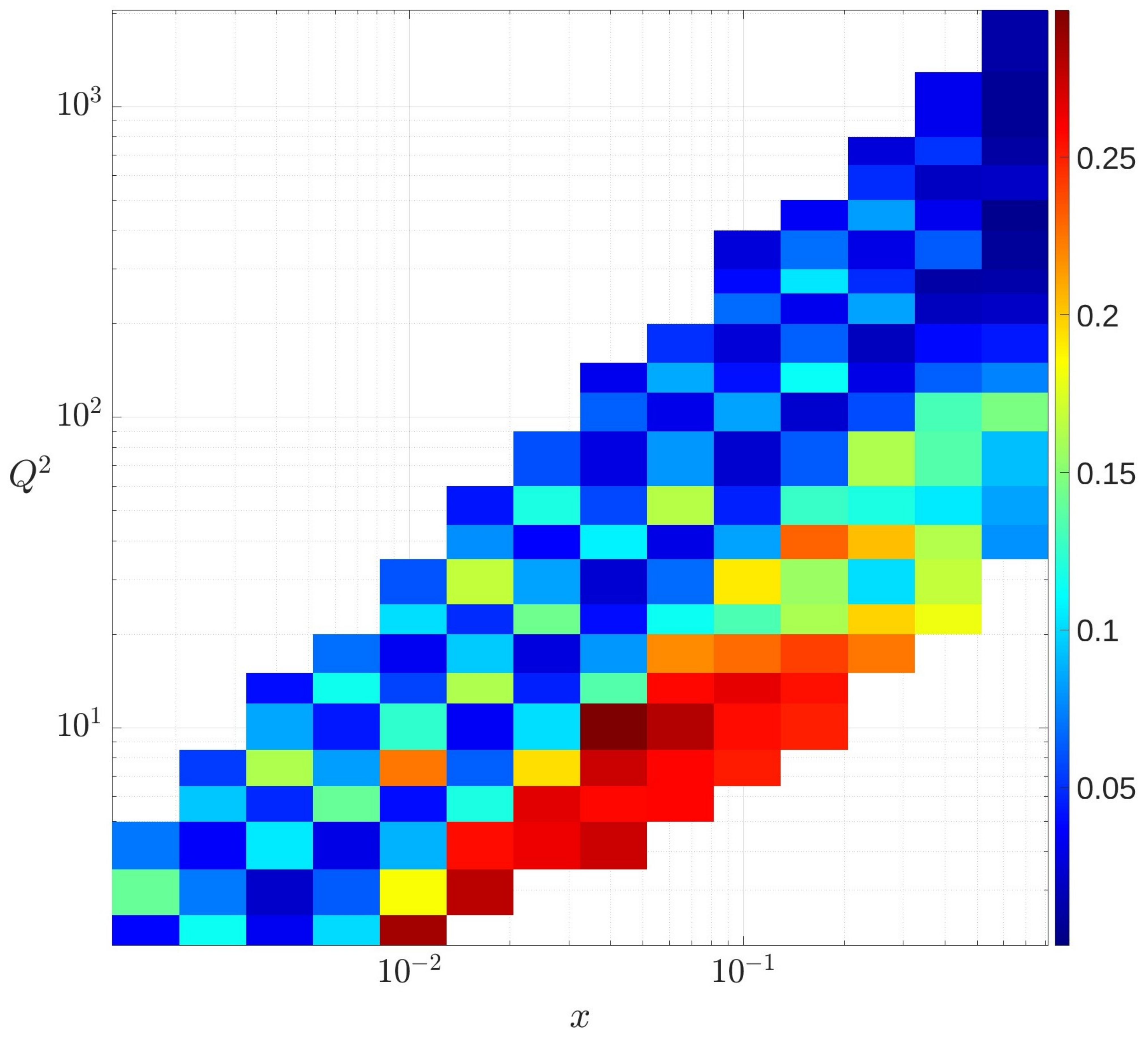


Optimistic scenario

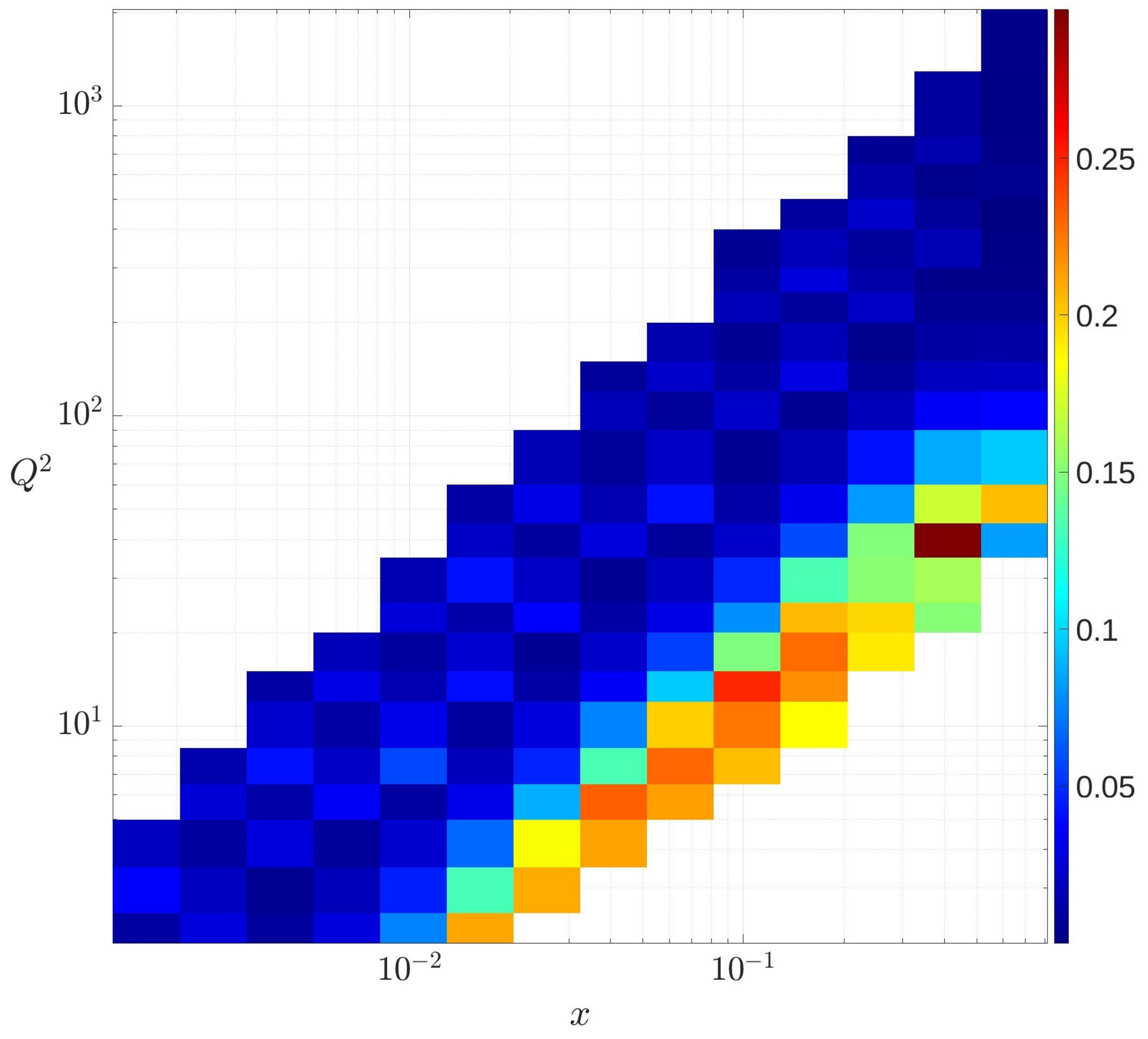


F_L absolute uncertainties with the MC replica method

Conservative scenario



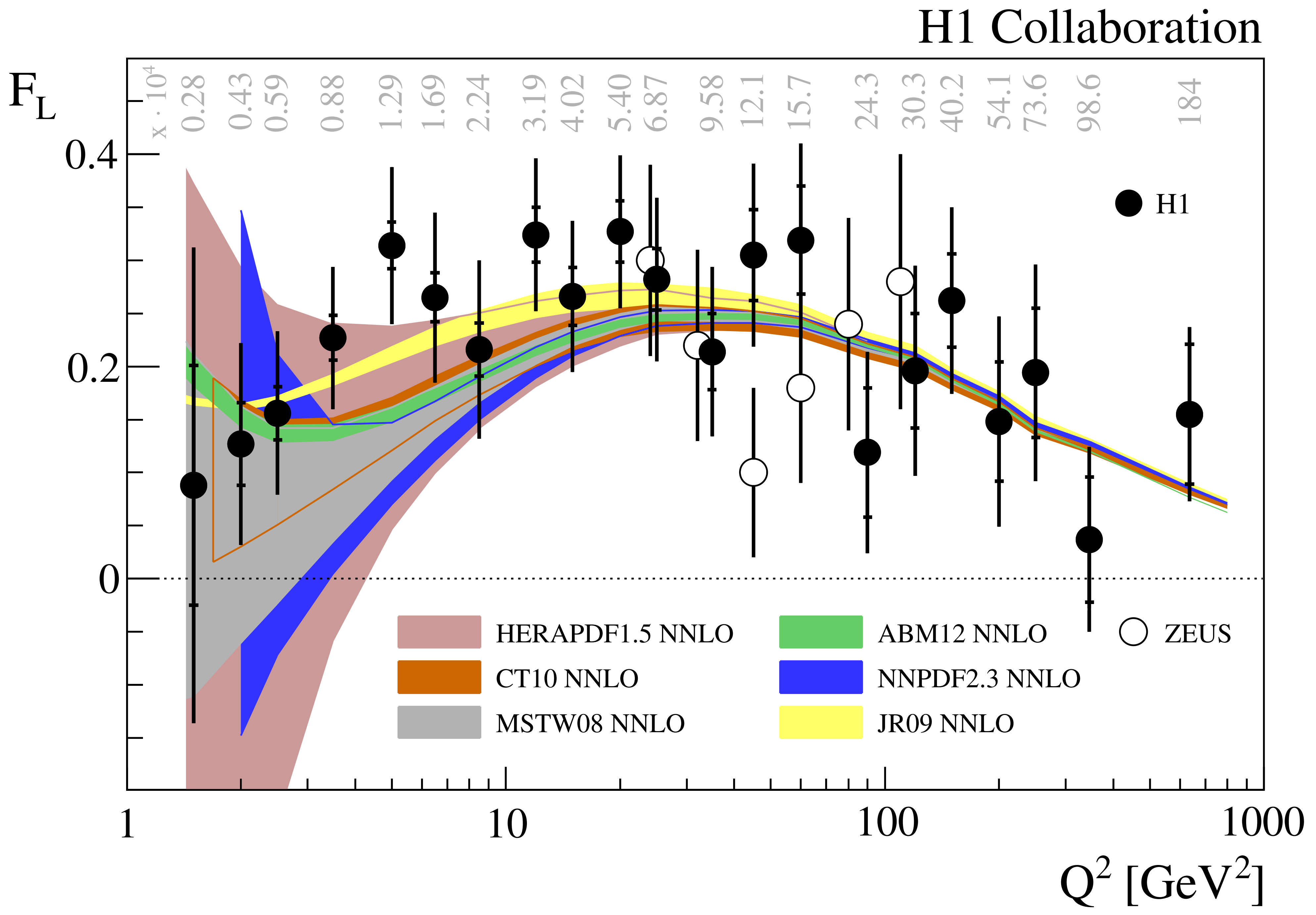
Optimistic scenario



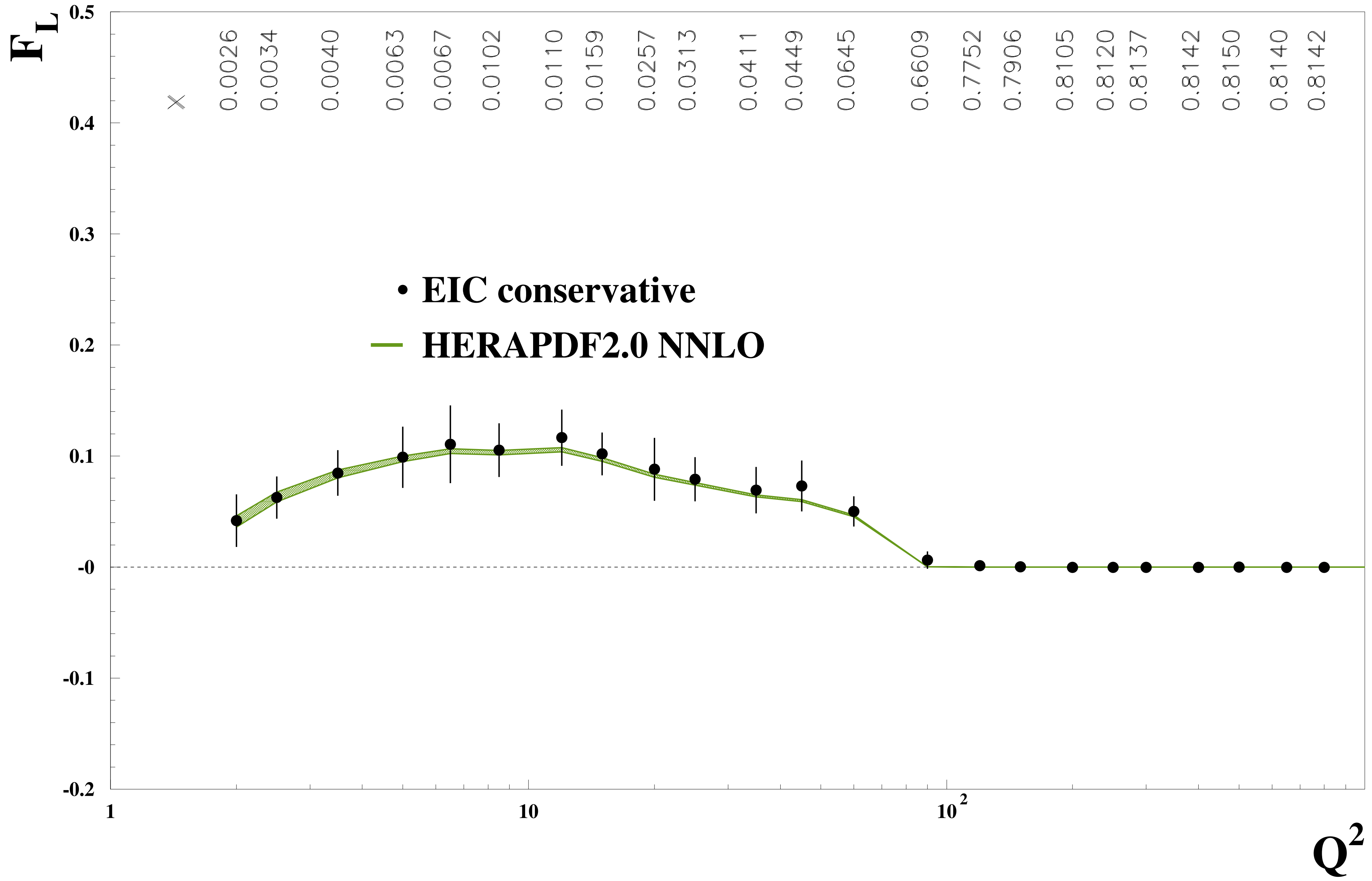
Great precision on F_L measurements, further improved in the optimistic scenario

$F_L(Q^2)$: comparison with HERA

HERA and EIC results cannot be directly compared as the phase space region they cover is different. However, the uncertainties in the measurements can be directly compared.



Andreev et al., (H1 collaboration), Eur. Phys. J. C **74**, 2814



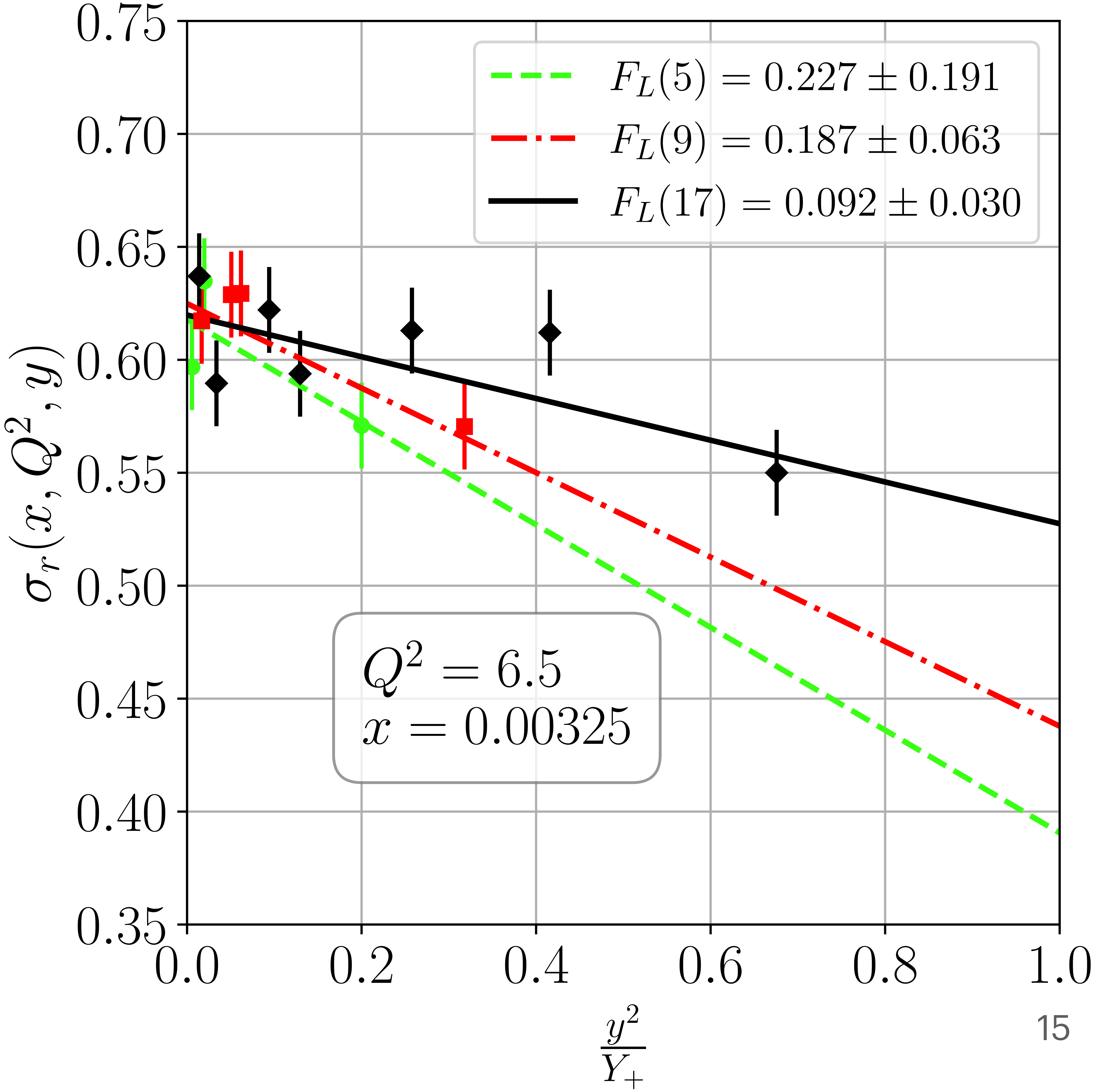
Unprecedented precision of F_L measurements at the EIC

Possible beam energy configurations at EIC

		E_p [GeV]					
		41	100	120	165	180	275
E_e [GeV]	5	29	45	49	57	60	74
	10	40	63	69	81	(85)	105
	18	54	85	93	109	114	141

- S-5 is the baseline configuration and is illustrated in green.
- S-9 is obtained by adding the red values.
- S-17 is obtained by adding the rest except the degenerated case $10 \times 180\ \mathrm{GeV}^2$ (marked in blue).

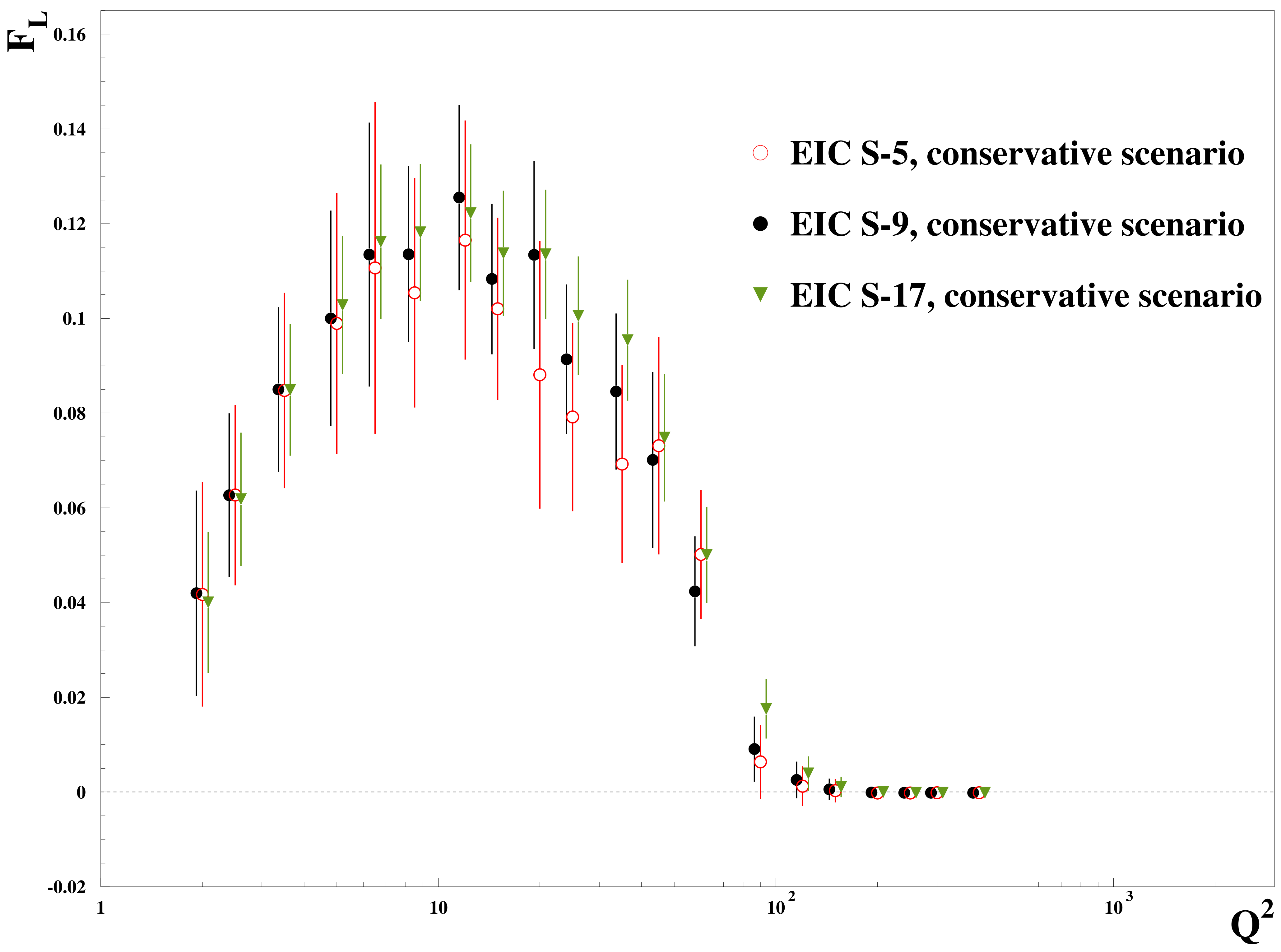
Effect of adding more beam energies

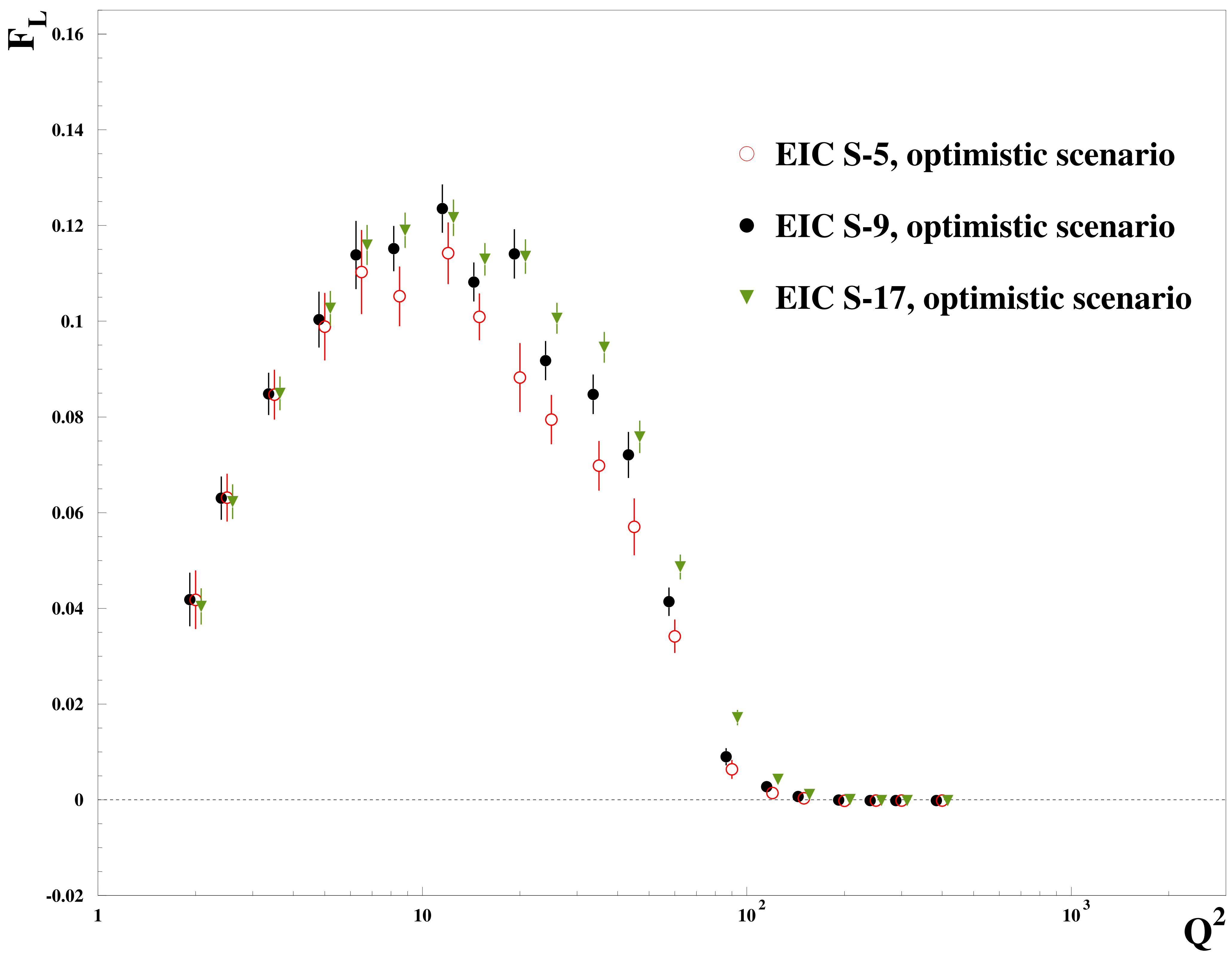


Adding more beam configurations increments the number of points available at high y^2/Y_+, significantly improving the quality of the fit

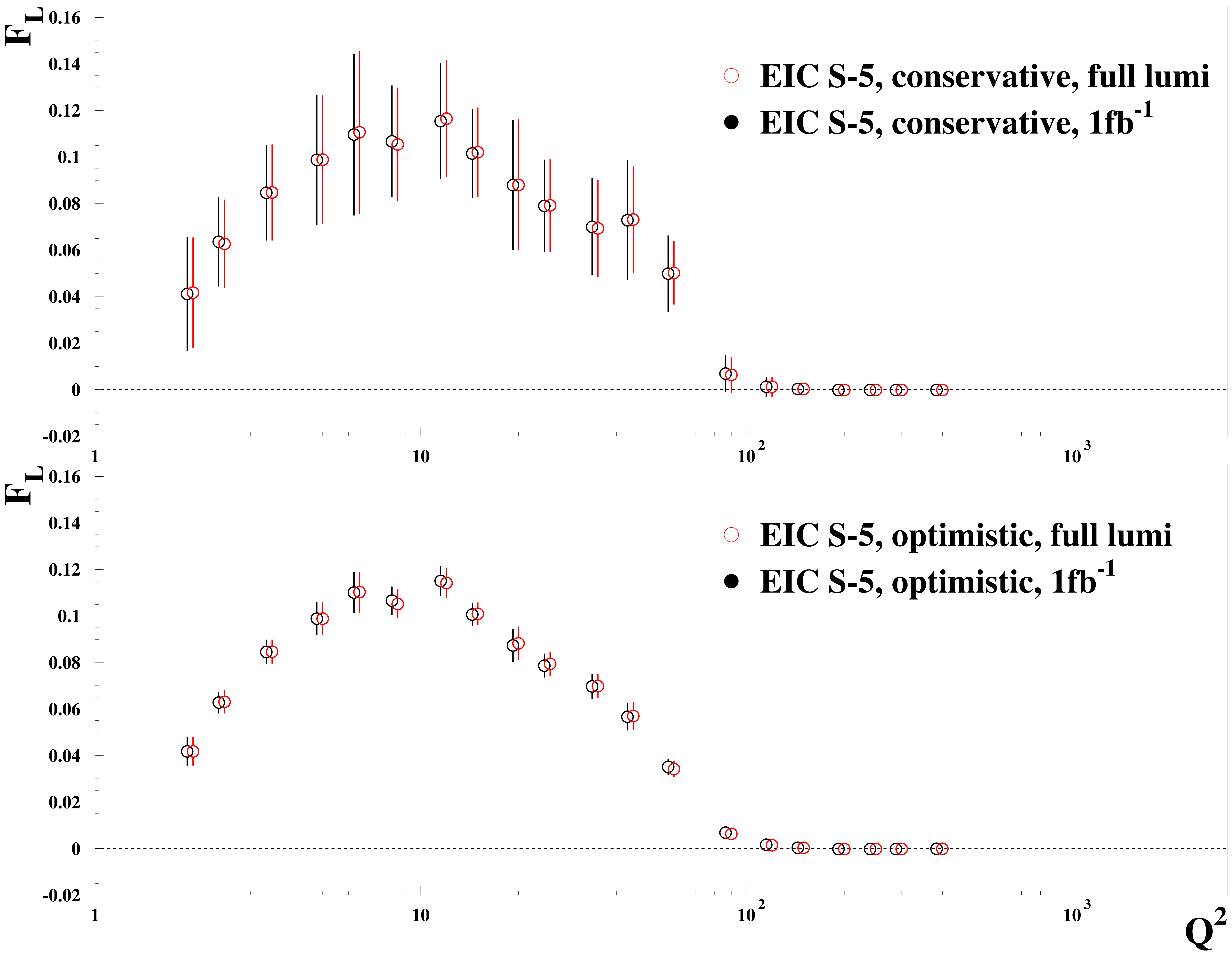
Possible improvements

- F_L measurements will significantly improve as more beam energies become available.
- However, the greatest improvement comes from reducing the uncertainties rather than adding additional beam energy configurations.



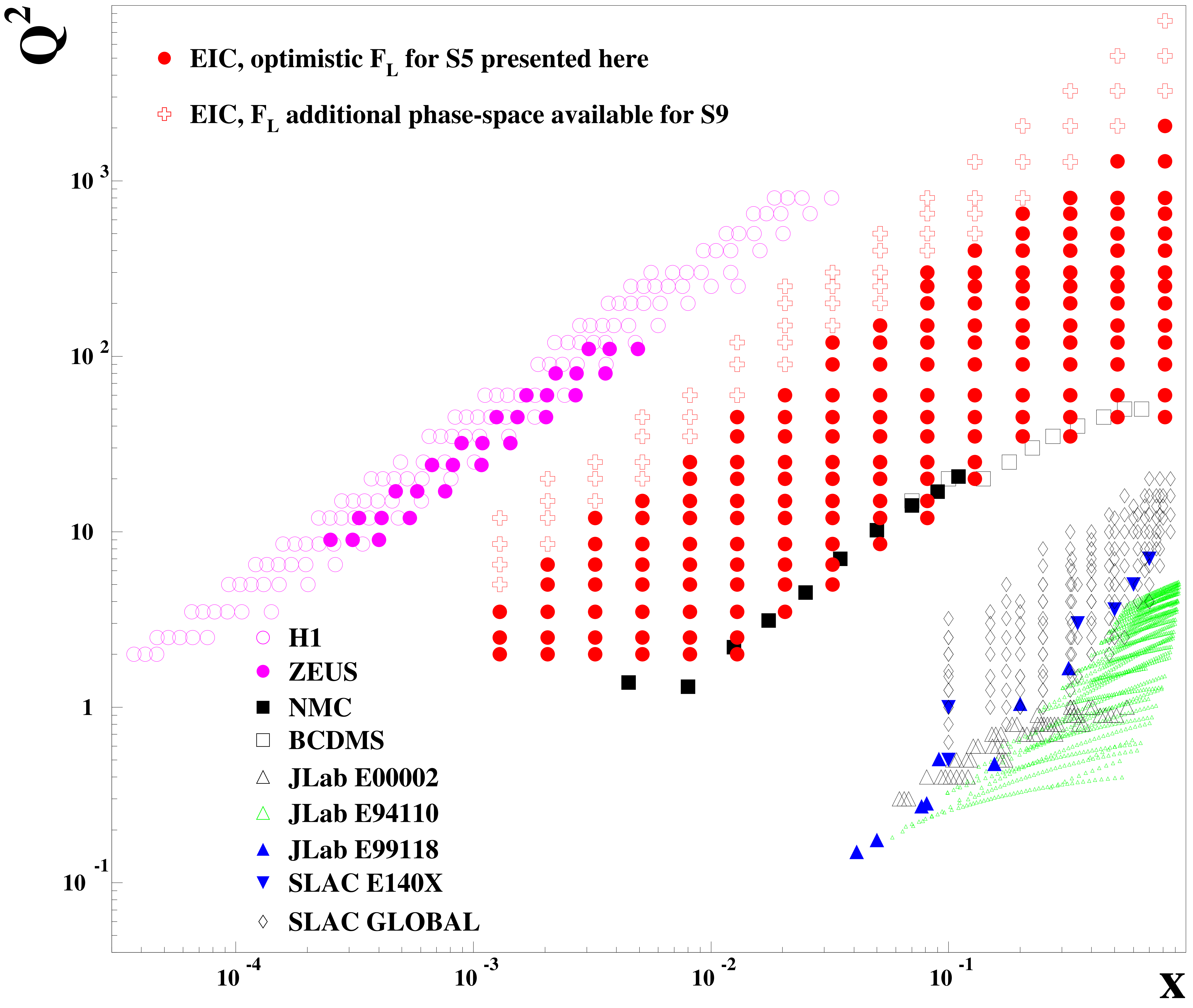


Extraction of F_L with $\mathscr{L}_{\text{int}} = 1\ \text{fb}^{-1}$



An integrated luminosity of $1\ \text{fb}^{-1}$ is enough to provide a precise F_L extraction

Kinematic coverage in phase space



EIC has great potential to measure the F_L structure function in an unexplored region of phase space

The strong coupling "constant"

The strong coupling constant is a function of the energy scale μ and in the $\overline{\text{MS}}$ scheme it's evolution with μ is given by:

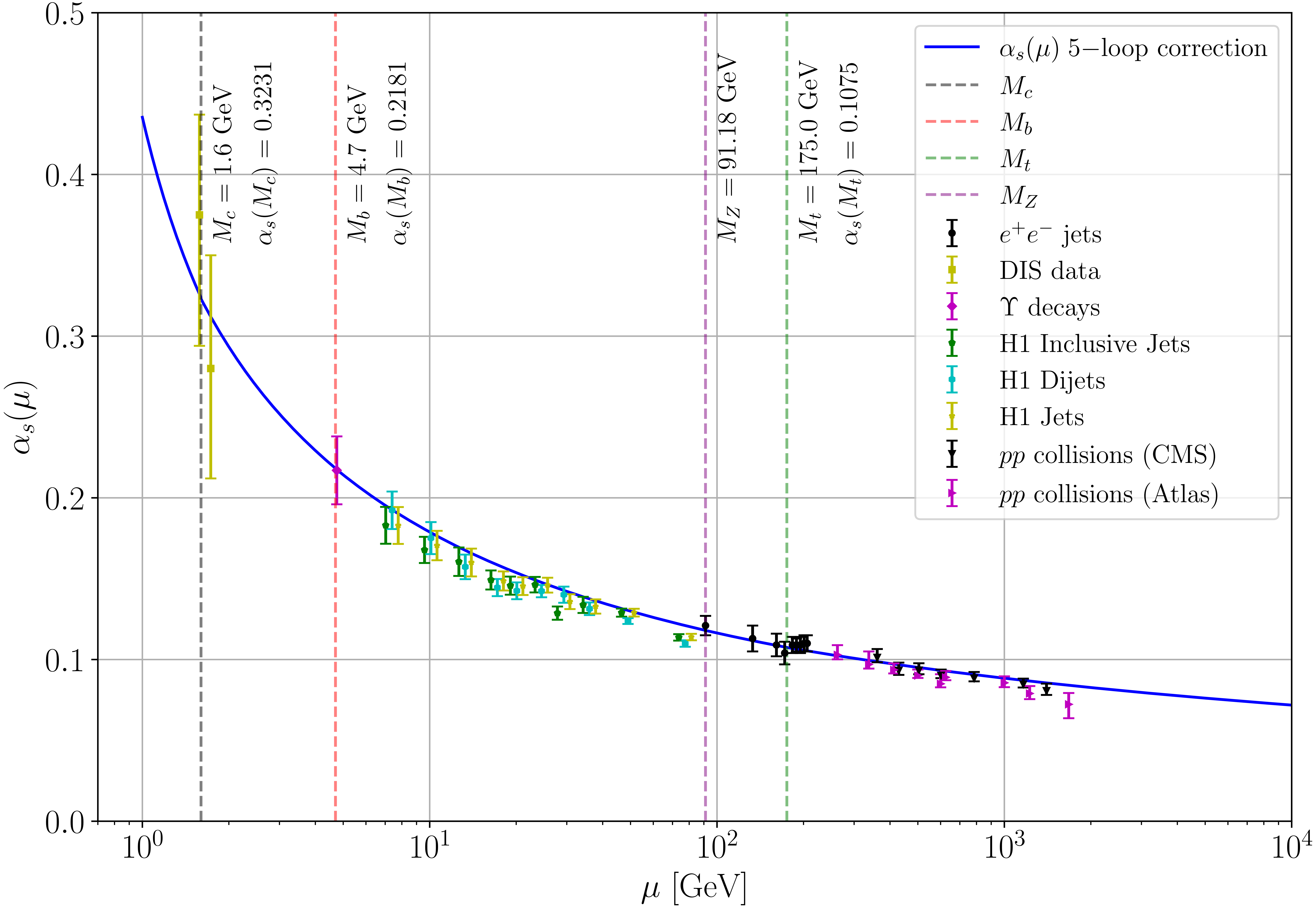
$$\mu^2 \frac{d}{d\mu^2} \alpha_s^{(n_f)}(\mu) = - \sum_{i \geq 0} \beta_i^{(n_f)} \left(\frac{\alpha_s^{(n_f)}(\mu)}{\pi} \right)^{i+2}$$

where n_f is the number of active flavours and the coefficients $\beta_i^{(n_f)}$ where taken from *Baikov et al., Phys. Rev. Lett.* **118**, 082002.

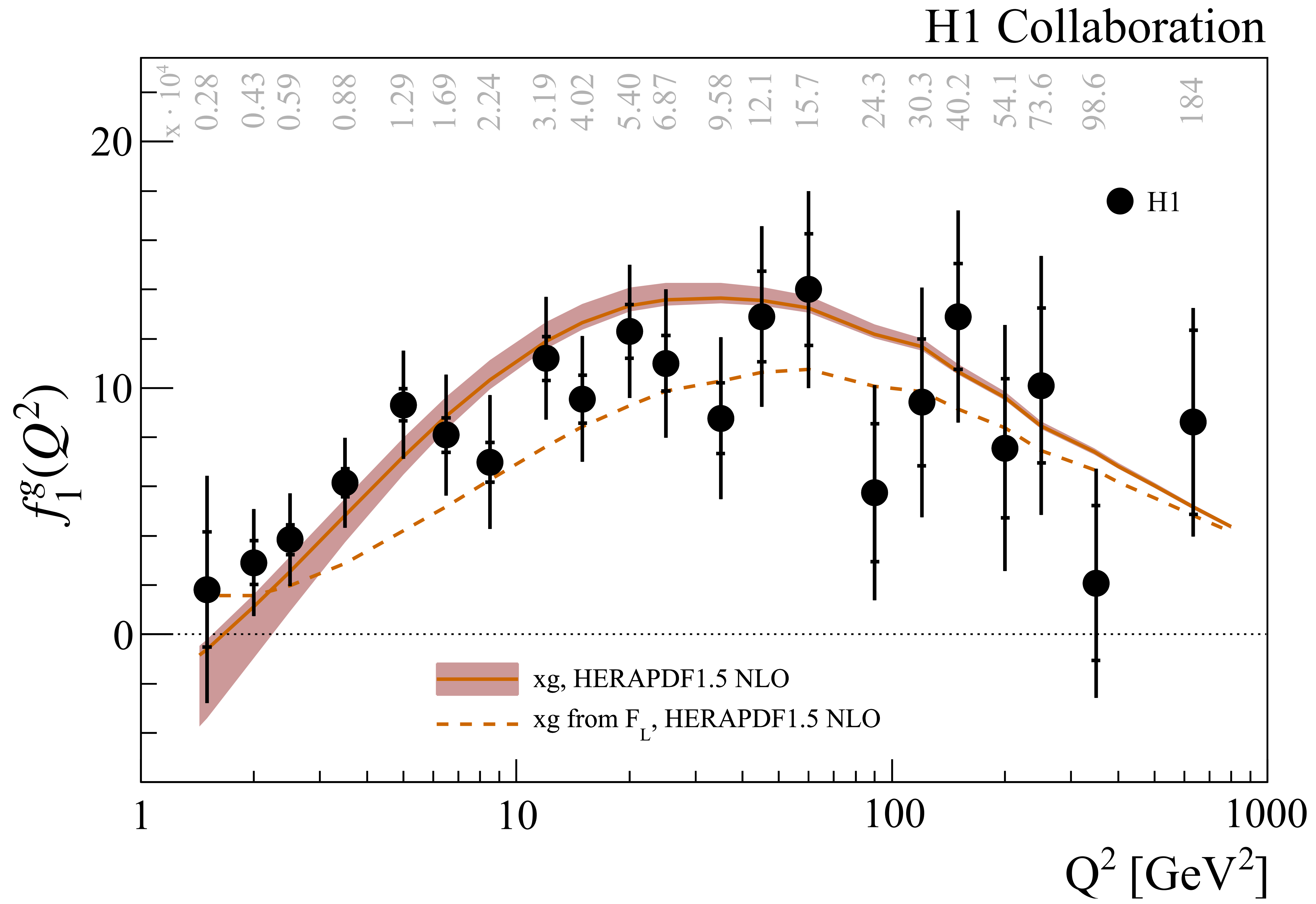
The initial condition for the running of α_s was chosen to be the world average value:

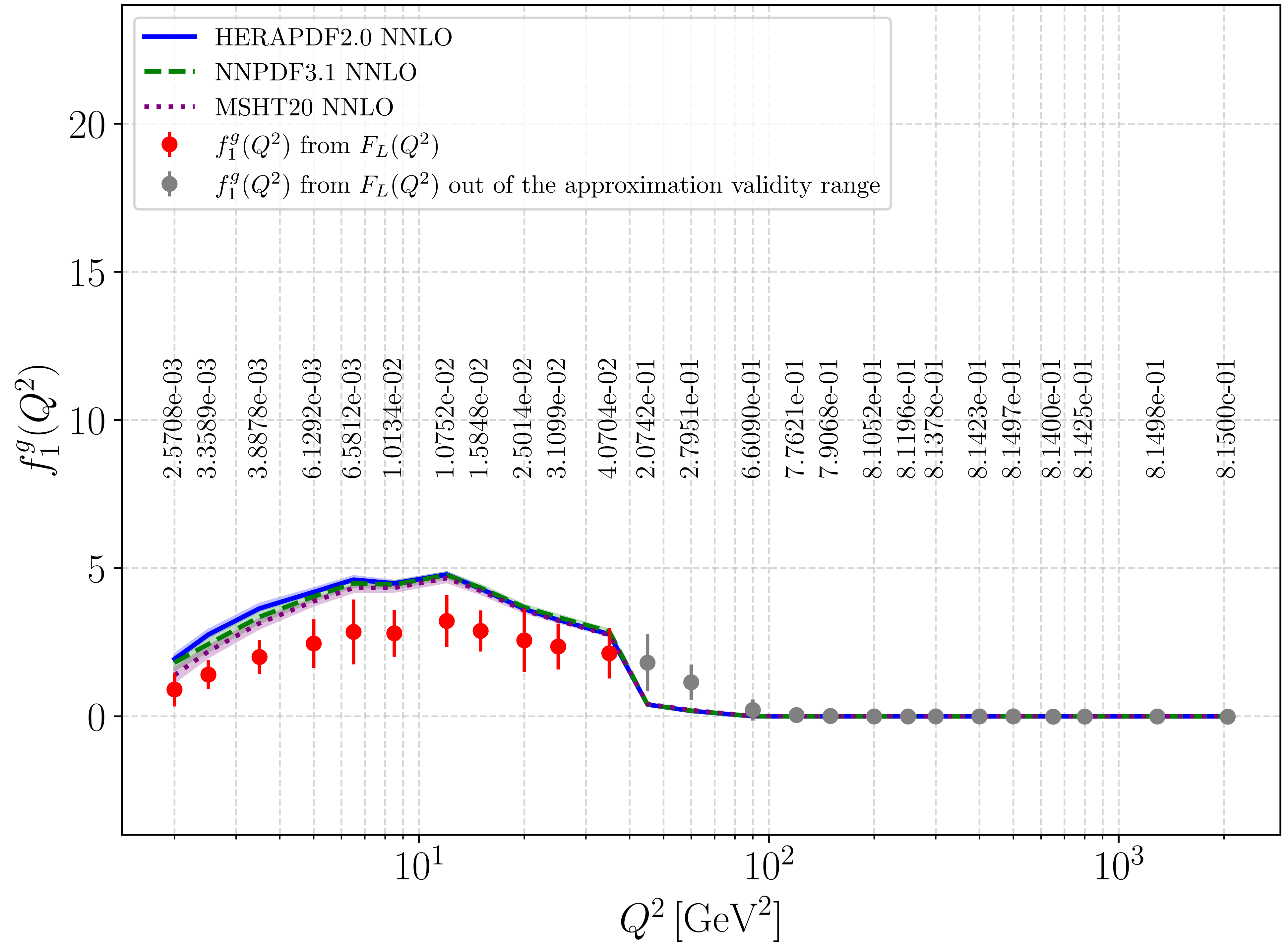
$$\alpha_s(\mu = M_Z) = 0.1180 \pm 0.0009$$

5-loop evolution for $\alpha_s(\mu)$



$f_1^g(Q^2)$: comparison with HERA

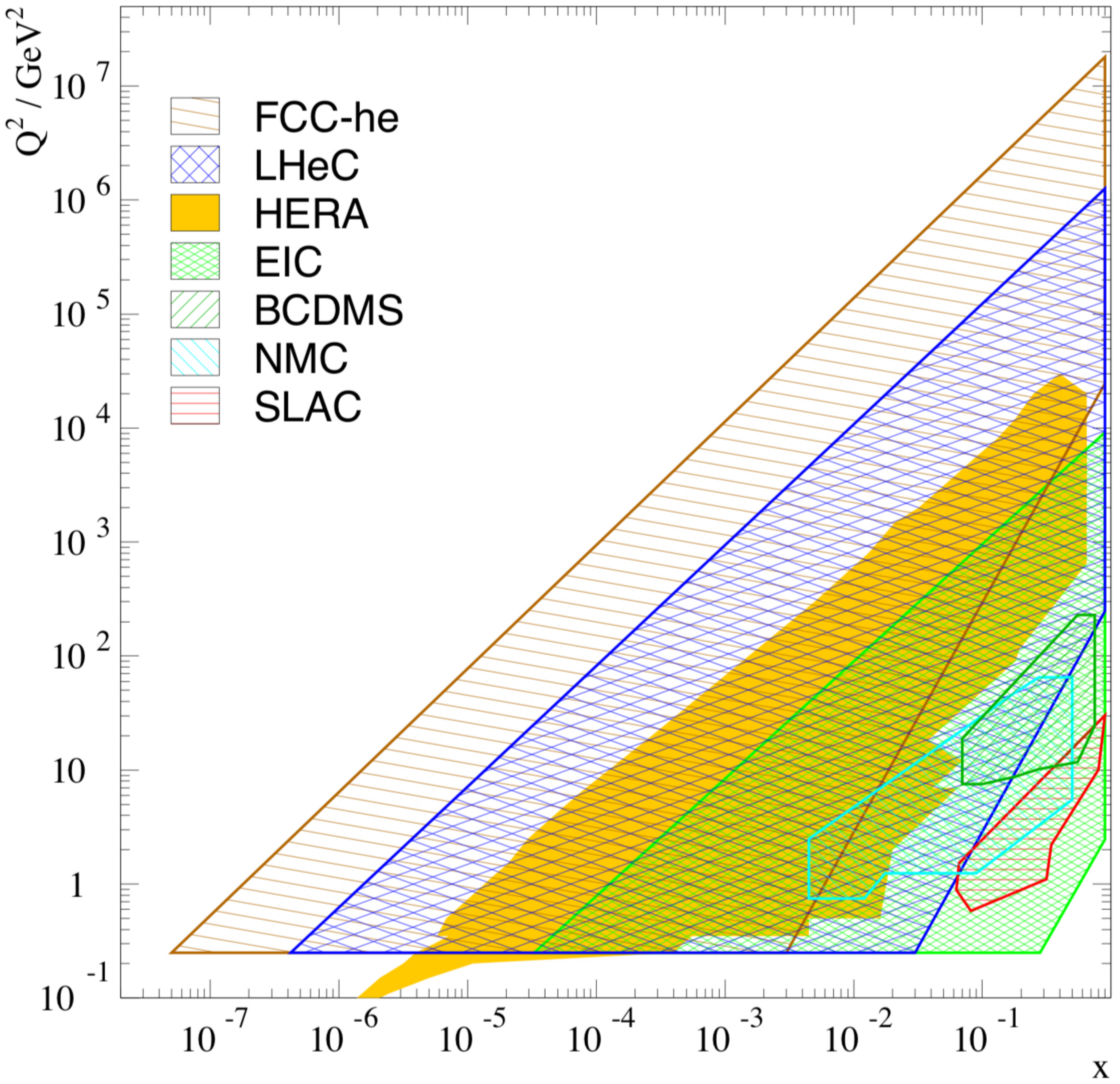




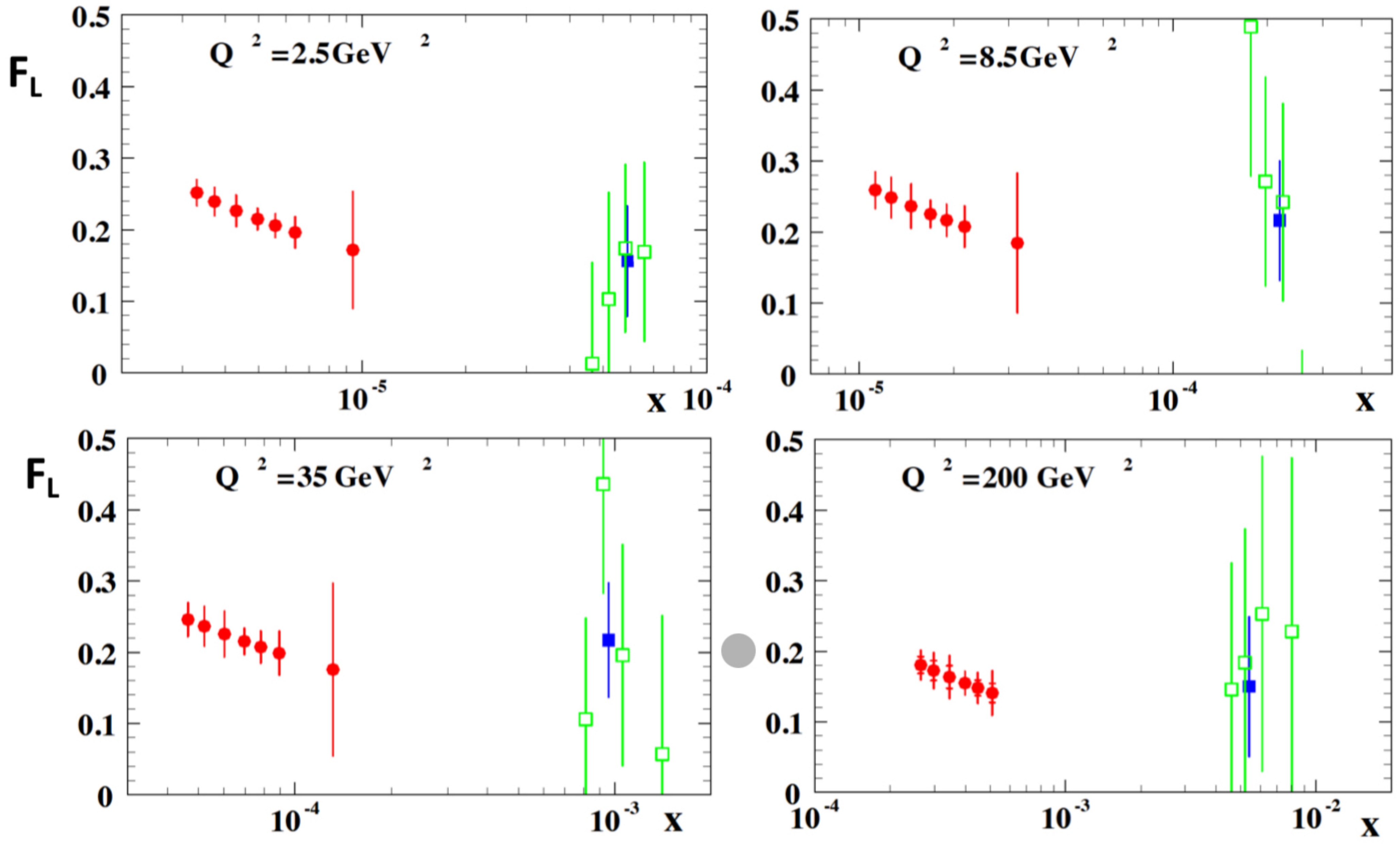
Andreev et al., (H1 collaboration), Eur. Phys. J. C **74**, 2814

$$\delta f_1^g(x,Q^2) = \frac{3}{2}\frac{1.77\pi}{\alpha_s\left(Q^2\right)}\sqrt{\alpha_s^2\left(Q^2\right)\left(\delta F_L\right)^2 + F_L^2\left(\delta\alpha_s\left(Q^2\right)\right)^2}$$

Future colliders impact on F_L



P. Agostini et al. 2021 J. Phys G: Nucl. Part. Phys. **48** 110501



P. Agostini et al. 2021 J. Phys G: Nucl. Part. Phys. **48** 110501

Future colliders will considerably expand the accessible kinematic region, allowing measurements in unexplored sections and reducing uncertainties

Conclusions and final remarks

The upcoming Electron-Ion Collider (EIC) will revolutionise our ability to probe the longitudinal structure function and, through it, the gluon content of the proton.

In particular:

- The longitudinal structure function F_L can be extracted with unprecedented precision across a vast, previously unexplored kinematic domain.
- F_L measurements will provide an almost direct determination of the gluon density $f_1^g(x, Q^2)$.
- Precise measurements of F_L are feasible even with $\mathscr{L}_{\text{int}} = 1\ \text{fb}^{-1}$ (total luminosity achieved at HERA), which will be reached in a short period of time at the EIC, meaning that an early extraction could be done in the first few years of operation.

Thank you for your time!

Backup

Obtaining F_L as a function of Q^2

The average value of F_L is calculated as:

$$\overline{F_L} = \frac{\sum_{i=1}^{N} \omega_i F_L^{(i)}}{\sum_{i=1}^{N} \omega_i} \qquad \text{with} \qquad \omega_i = \frac{1}{\left[\Delta F_L^{(i)}\right]^2}$$

The uncertainty on the averaged measurement is given by:

$$\delta_{\text{avg}} = \sqrt{\frac{1}{\sum_{i=1}^{N} \omega_i}}$$

The same procedure was applied to obtain the averaged value of x using the weights calculated for the average value of F_L.

Decoupling relation in the $\overline{\text{MS}}$ scheme

As the Appelquist-Carazzone decoupling theorem does not hold in the $\overline{\text{MS}}$ scheme, the decoupling must be done “by hand” every time the number of active flavours changes during the evolution of $\alpha_s(\mu)$.

$$\alpha_s^{(n_f-1)}(\mu_0) = \zeta_g^2 \alpha_s^{n_f}(\mu_0)$$

$$\left(\zeta_g^{\overline{\text{MS}}}\right)^2 = 1 + \frac{\alpha_s^{(n_f)}(\mu)}{\pi}\left(-\frac{1}{6}L\right) + \left(\frac{\alpha_s^{(n_f)}(\mu)}{\pi}\right)^2\left(\frac{11}{72} - \frac{11}{24}L + \frac{1}{36}L^2\right)$$
$$+\left(\frac{\alpha_s^{(n_f)}(\mu)}{\pi}\right)^3\left[\frac{564731}{124416} - \frac{82043}{27648}\zeta_3 - \frac{955}{576}L + \frac{53}{576}L^2\right.$$
$$\left.-\frac{1}{216}L^3 + (n_f-1)\left(-\frac{2633}{31104} + \frac{67}{576}L - \frac{1}{36}L^2\right)\right] \text{ with } L = \log\left(\frac{\mu_0^2}{m_h^2}\right)$$

$\beta_i^{(n_f)}$ coefficients

$$\beta_0^{(n_f)} = \frac{1}{4}\left[11 - \frac{2}{3}n_f\right]$$

$$\beta_1^{(n_f)} = \frac{1}{16}\left[102 - \frac{38}{3}n_f\right]$$

$$\beta_2^{(n_f)} = \frac{1}{64}\left[\frac{2857}{2} - \frac{5033}{18}n_f + \frac{325}{54}n_f^2\right]$$

$$\beta_3^{(n_f)} = \frac{1}{256}\left[\frac{149753}{6} + 3546\zeta_3 + \left(-\frac{1078361}{162} - \frac{6508}{27}\zeta_3\right)n_f + \left(\frac{50065}{162} + \frac{6472}{81}\zeta_3\right)n_f^2 + \frac{1093}{729}n_f^3\right]$$

$$\beta_4^{(n_f)} = \frac{1}{4^5}\left\{\frac{8157455}{16} + \frac{621885}{2}\zeta_3 - \frac{88209}{2}\zeta_4 - 288090\zeta_5\right.$$
$$+n_f\left[-\frac{336460813}{1944} - \frac{4811164}{81}\zeta_3 + \frac{33935}{6}\zeta_4 + \frac{1358995}{27}\zeta_5\right]$$
$$+n_f^2\left[\frac{25960913}{1944} + \frac{698531}{81}\zeta_3 - \frac{10526}{9}\zeta_4 - \frac{381760}{81}\zeta_5\right]$$
$$\left.+n_f^3\left[-\frac{630559}{5832} - \frac{48722}{243}\zeta_3 + \frac{1618}{27}\zeta_4 + \frac{460}{9}\zeta_5\right] + n_f^4\left[\frac{1205}{2916} - \frac{152}{81}\zeta_3\right]\right\}$$