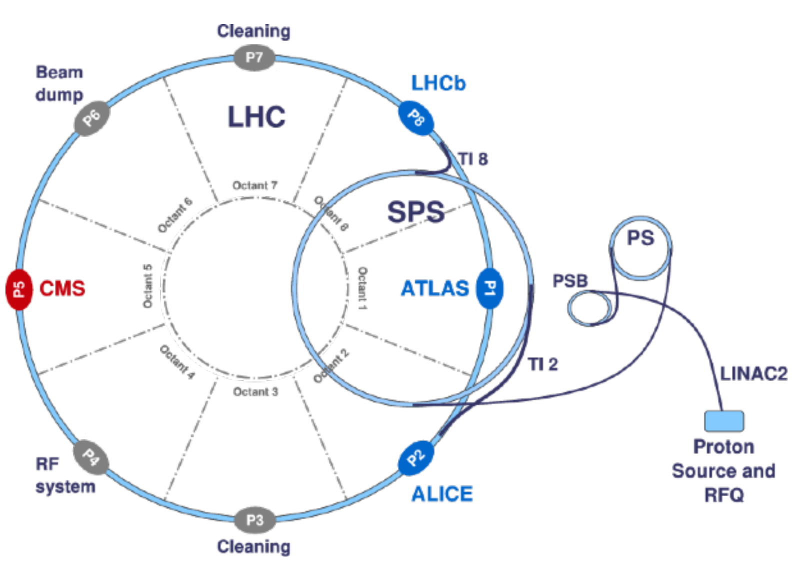
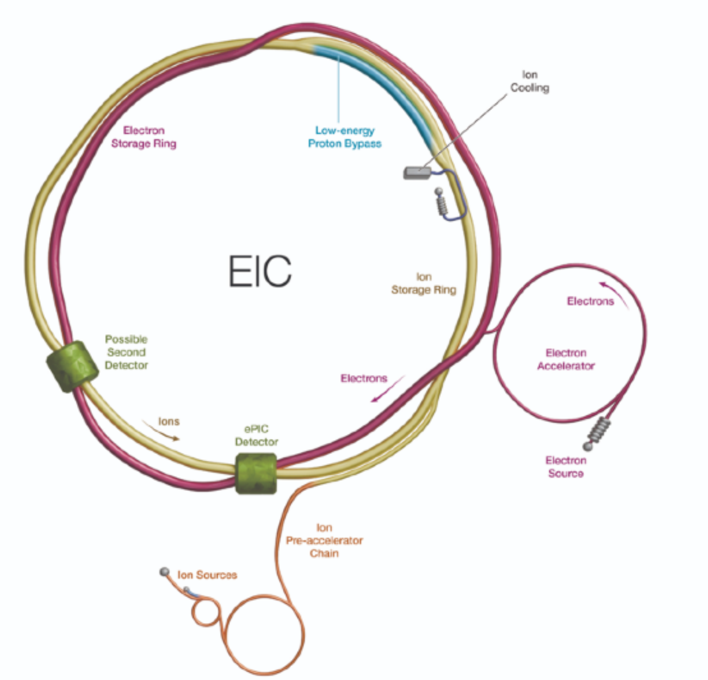
university of groningen / faculty of science and engineering / van swinderen institute for particle physics and gravity

EIC-LHC synergies for gluon distributions

Daniël Boer, Van Swinderen Institute for Particle Physics and Gravity
University of Groningen, The Netherlands





JENA Workshop "Synergies between the EIC and the LHC" @ Kraków, September 22, 2025

EoI website

https://indico.ph.tum.de/event/7004/

JENAA Expression of Interest: "Synergies between the Electron-Ion Collider and the Large Hadron Collider experiments"

Jun 20 – 21, 2022
Europe/Berlin timezone

Enter your search term

- Overview
- Endorse this Expression of Interest
- List of Endorsers
- Activities

Activities

Upcoming activities:

Third JENAA Workshop "Synergies between the EIC and the LHC" to be held September 22-24, 2025, at The Henryk Niewodniczański Institute of Nuclear Physics Polish Academy of Sciences, Kraków, Poland [organizers: Krzysztof Kutak, Leszek Motyka, Jacek Otwinowski, Antoni Szczurek, Sahil Upadhyaya, Jakub Wagner] Workshop poster

On this website you can find the EoI, endorse it if you like, find the future and past activities, and feel free to propose/organize an activity

Contact persons: Daniël Boer, Pasquale Di Nezza, Maria Vittoria Garzelli

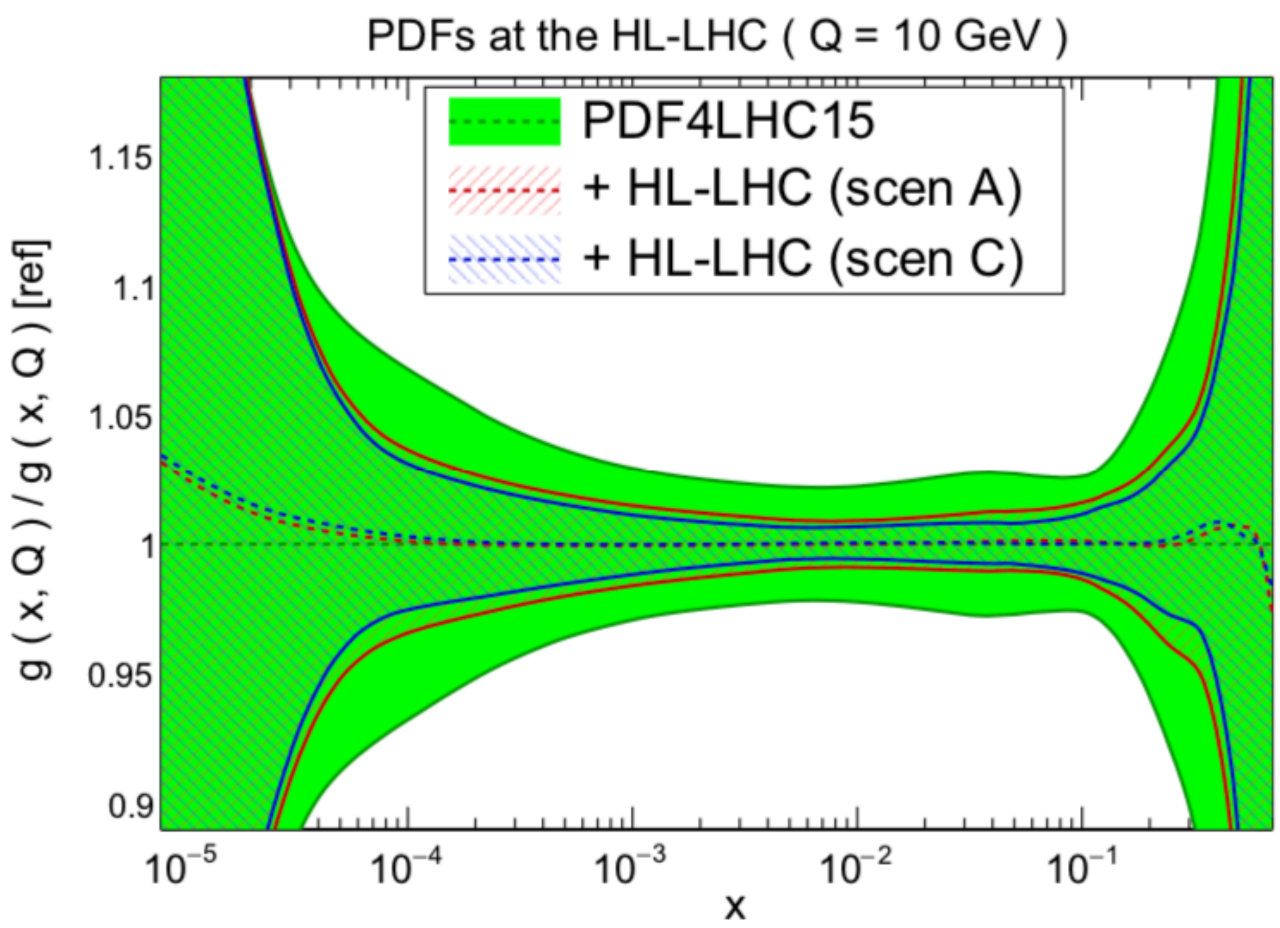
Outline

- Collinear gluon PDFs
- Gluon TMDs
- Gluon GPDs
- Gluon GTMDs
- Gluon double distributions
- Diffractive gluon distributions

Collinear gluon PDFs

PDFs at LHC and EIC

g



Khalek, Bailey, Gao, Harland-Lang, Rojo, EPJC 78 (2018) 962

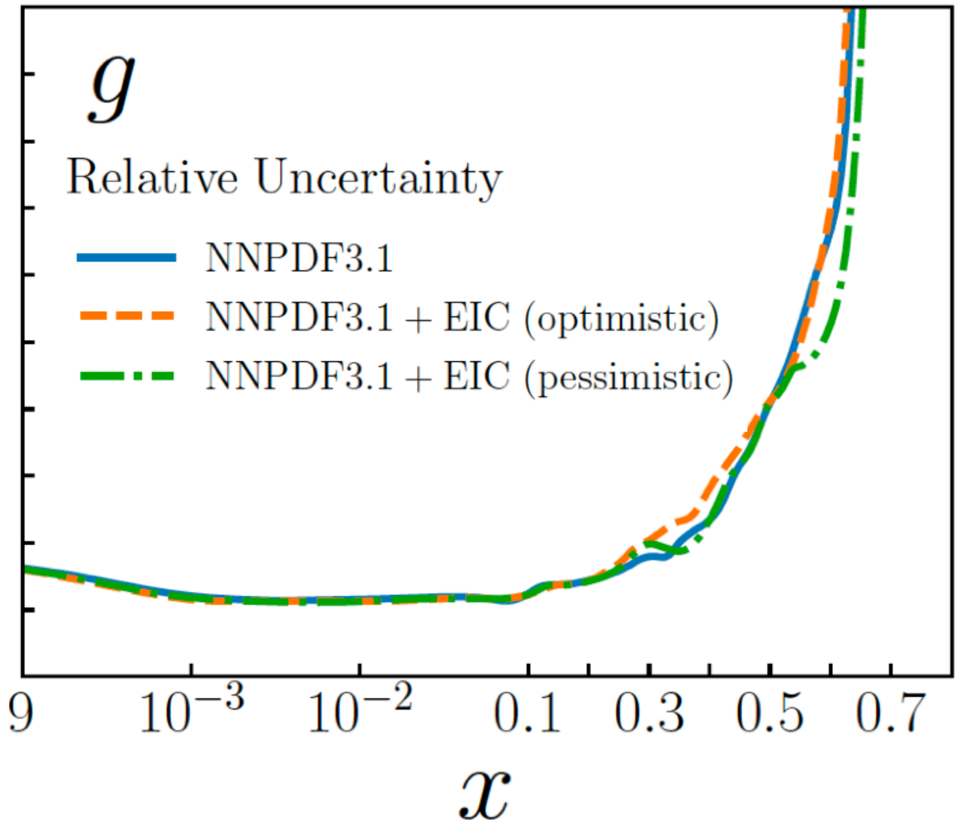
HL-LHC expected to reach % level accuracy of proton and Pb PDFs down to x~10^{-4} [for x <0.1 one is mostly dealing with gluons]

EIC will cover a similar region and can study PDFs for many other nuclei

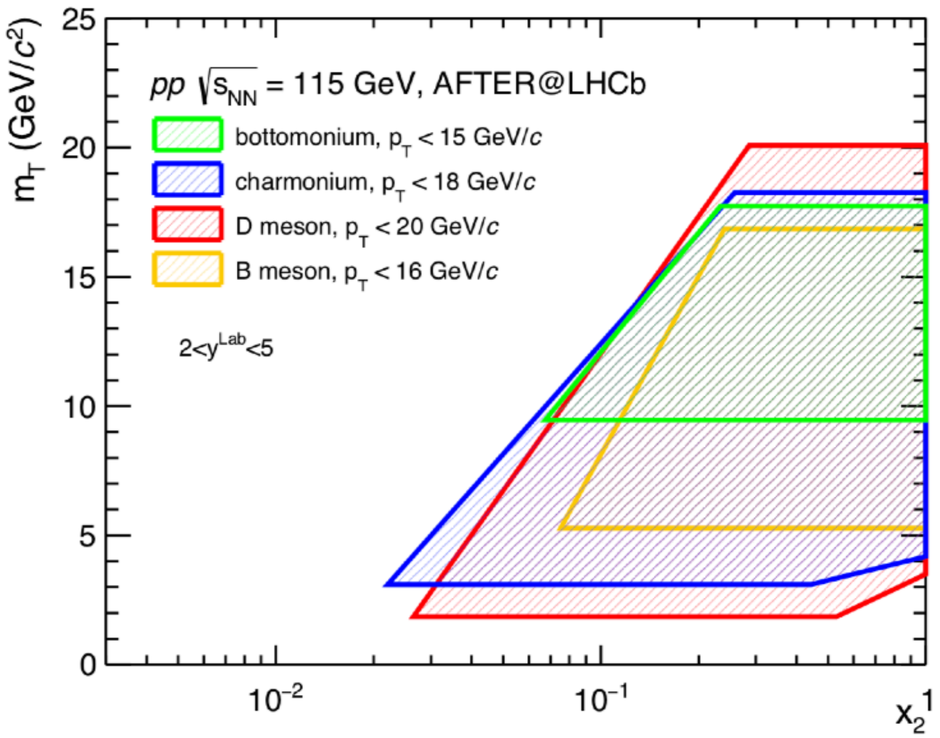
The parton distributions at large x are required for specific BSM physics studies and at small x for saturation physics studies

PDFs at LHC and EIC

EIC will improve PDFs mostly at large x, but mostly for u and s quarks



Abdul Khalek, Ethier, Nocera, Rojo, 2021



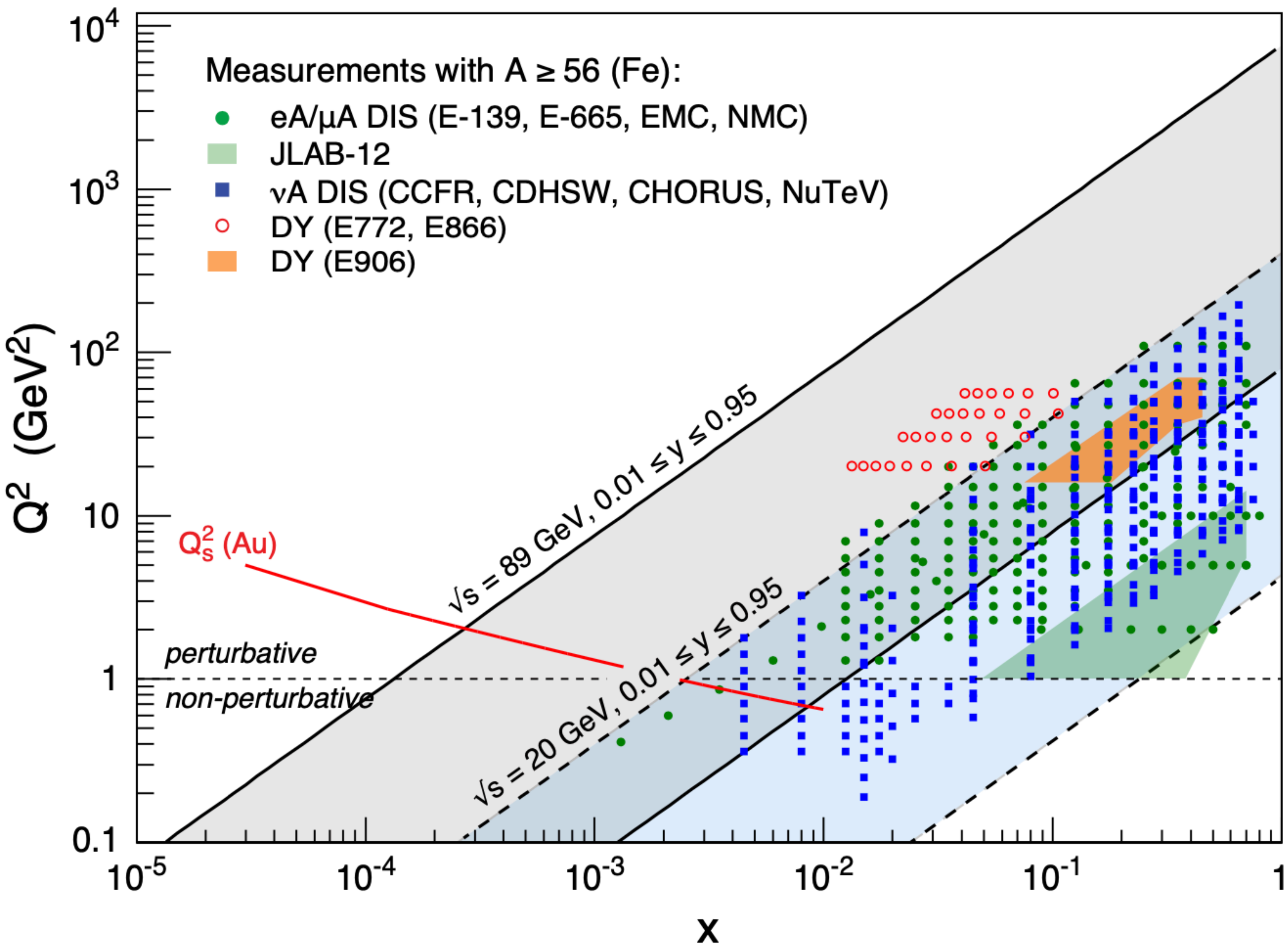
Hadjidakis, Kikola, Lansberg, Massacrier, Echevarria, *et al.*, 2021

Large-x gluon studies can be done in Fixed Target experiments at LHC, e.g. in Υ(1S) production

EIC will provide new information on gluons in various nuclei

Nuclear PDFs from EIC

EIC kinematic coverage in eA & existing FT data



EIC Yellow Report, arxiv:2103.05419

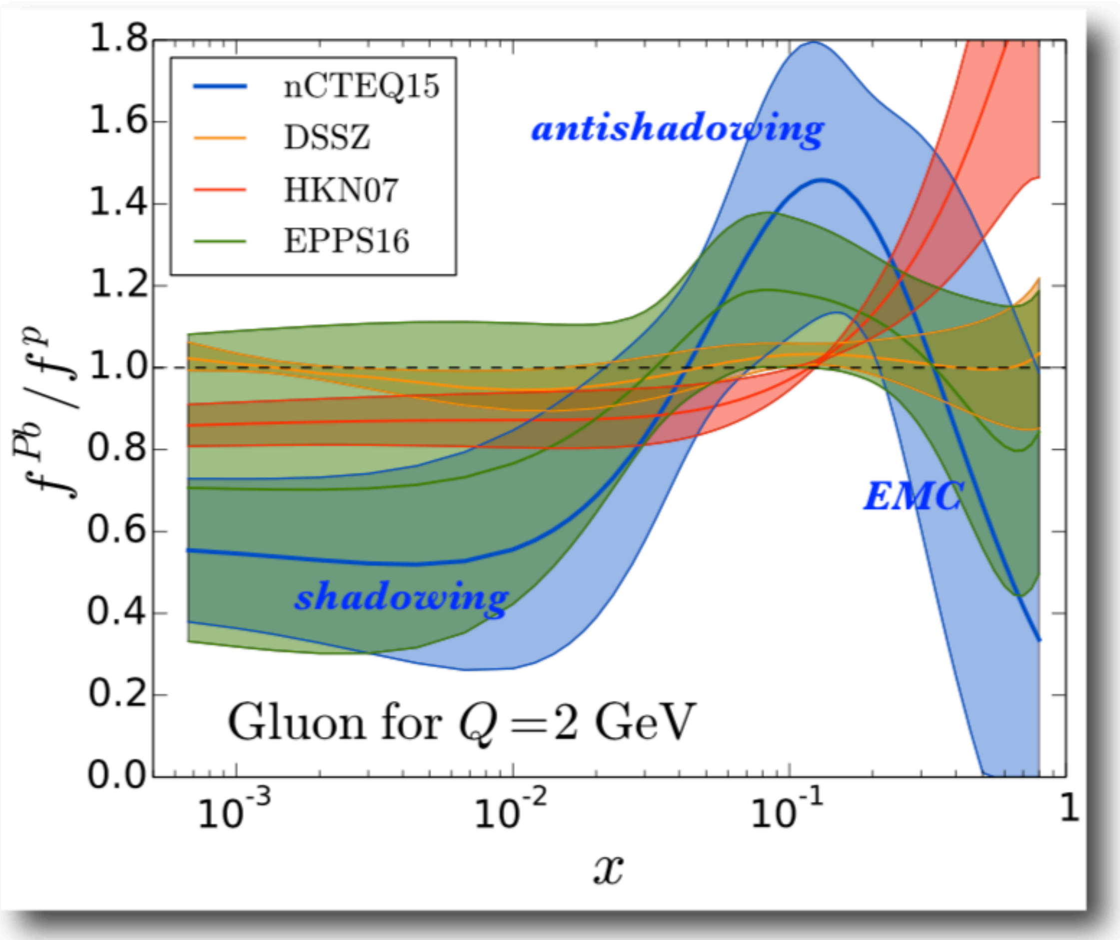
In the future this can be turned into a 3D plot: (Q^2, x, A)

EIC will mostly study p and Au, but can obtain data for nuclei up to U
Complements Pb measurements at LHC and neutrino DIS measurements

Partonic structure of nuclei

The parton distributions of nuclei vs nucleons remain to be understood still

gluon nuclear PDF (Pb)



From M. Echevarria, DIS2019 & 1807.00603

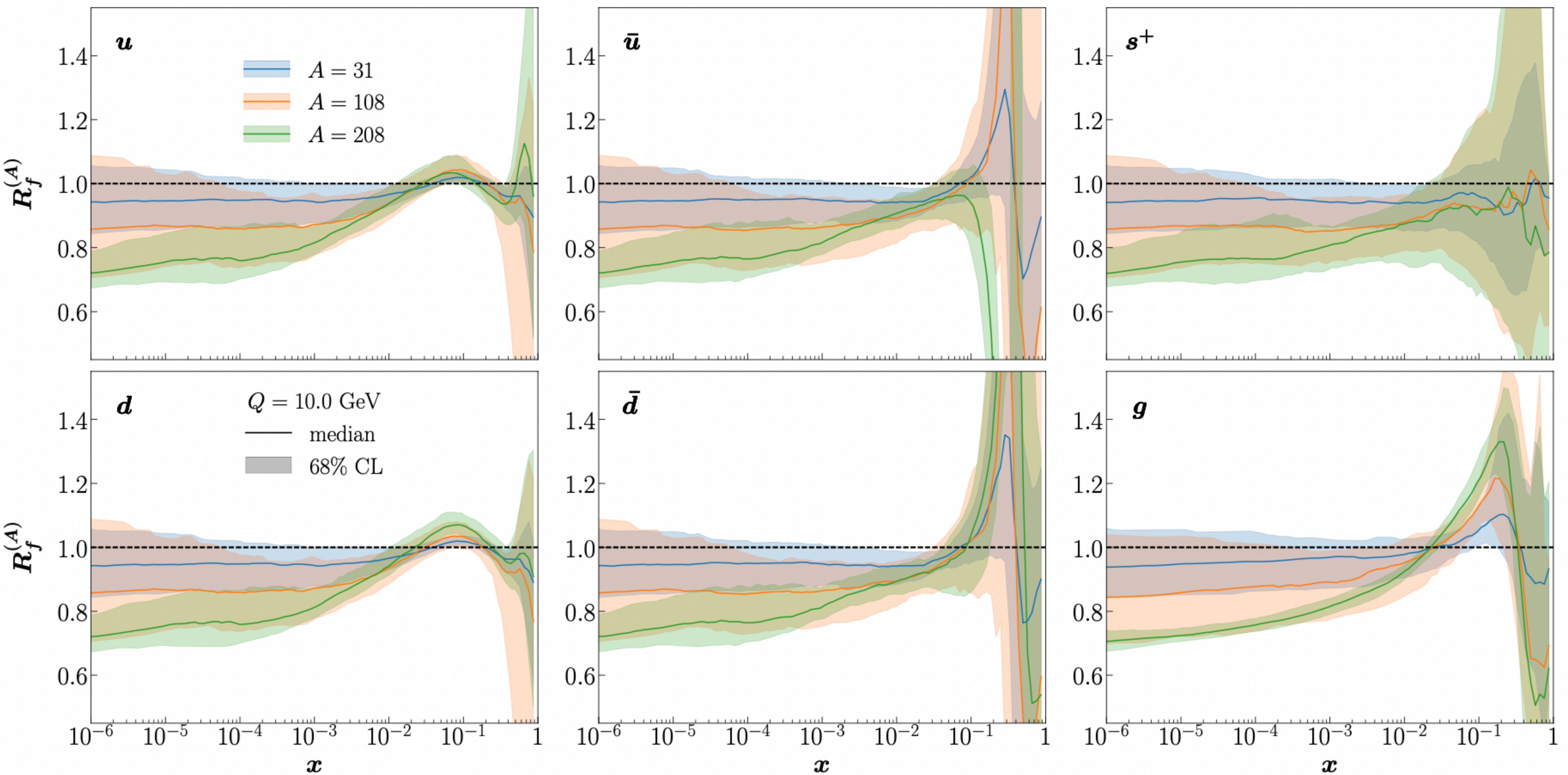
Nuclear PDFs are not well constrained over the entire x region, displays various effects: EMC effect & (anti)shadowing

To reach conclusions about the underlying physics effects, the uncertainties will need to be reduced first by LHC and EIC data

LHC data provides Pb PDFs from p-Pb and Pb-Pb collisions

PDFs as functions of A

nNNPDF3.0, 2022



$$f_i^A(x) = \frac{1}{A}\left(Z f_i^{p/A}(x) + (A-Z) f_i^{n/A}(x)\right)$$

$$\int_0^1 dx\, x\left(\Sigma^{(p/A)}(x,Q_0) + g^{(p/A)}(x,Q_0)\right) = 1\,, \quad \forall A$$

$$\int_0^1 dx\, \left(u^{(p/A)}(x,Q_0) - \bar{u}^{(p/A)}(x,Q_0)\right) = 2\,, \quad \forall A$$

$$\int_0^1 dx\, \left(d^{(p/A)}(x,Q_0) - \bar{d}^{(p/A)}(x,Q_0)\right) = 1\,, \quad \forall A$$

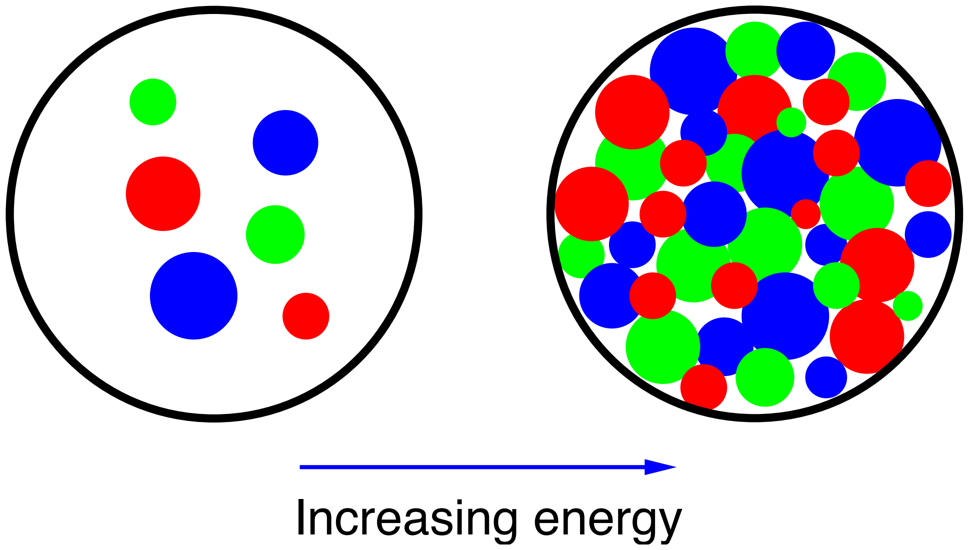
$$\int_0^1 dx\, \left(s^{(p/A)}(x,Q_0) - \bar{s}^{(p/A)}(x,Q_0)\right) = 0\,, \quad \forall A$$

Sum rules and agreement with A=1 are imposed

Small x gluons

The PDFs at small x tell us about nonlinear QCD effects → parton saturation

When x decreases, the density of gluons (n_g) increases



Scattering off a proton becomes scattering off multiple gluons simultaneously

Never directly observed in the gluon distribution yet

Dijet production in pp collisions suggested to study this
[Kutak, Sapeta, 2012; Van Hameren, Kakkad, Kotko, Kutak, Sapeta, 2023]

That process actually involves unintegrated gluon distributions or TMDs and their complications, like process dependence

EIC is important here: ep & eA provide the baseline for pp, pA & AA

Gluon TMDs

Gluons TMDs

Gluon TMD correlator: $\Gamma_g^{\mu\nu}(x, p_T) \propto \langle P | F^{+\nu}(0)\, \mathcal{U}\, F^{+\mu}(\xi^-, \xi_T)\, \mathcal{U}' | P \rangle$

transverse momentum dependent (TMD)

For unpolarized protons:

$$\Gamma_U^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{x}{2} \left\{ -g_T^{\mu\nu} f_1^g(x, \boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu} \frac{\boldsymbol{p}_T^2}{2M_p^2} \right) h_1^{\perp\, g}(x, \boldsymbol{p}_T^2) \right\}$$

unpolarized gluon TMD

linearly polarized gluon TMD

Gluons inside *unpolarized* protons can be polarized!

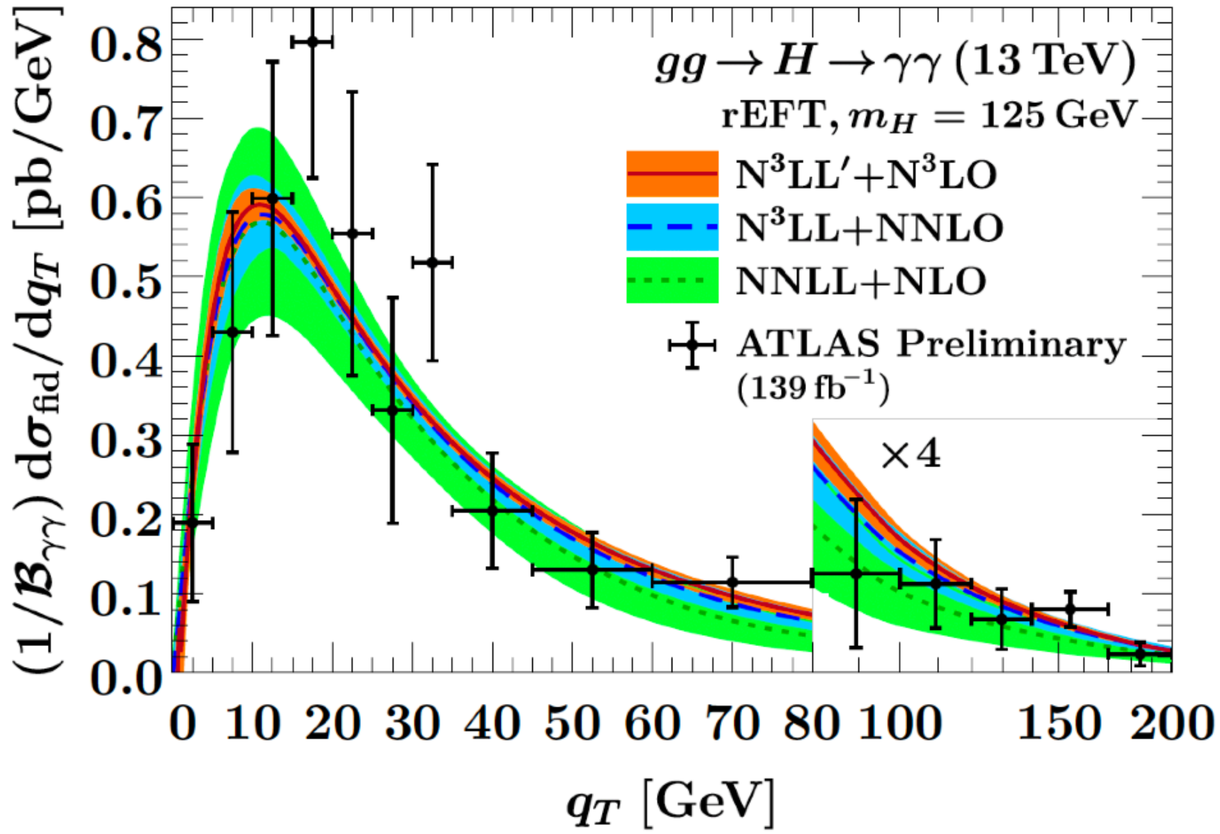
Mulders, Rodrigues '01

Allows spin studies with unpolarized hadrons, e.g. in Higgs production, quarkonium production, open heavy quark pair production

$h_1^{\perp g}$ shows up in the angle integrated cross sections, but also in cos 2ϕ and cos 4ϕ modulations, allows for many ways to study this distribution at LHC and EIC

Gluon TMDs in Higgs production

Higgs p_T distribution

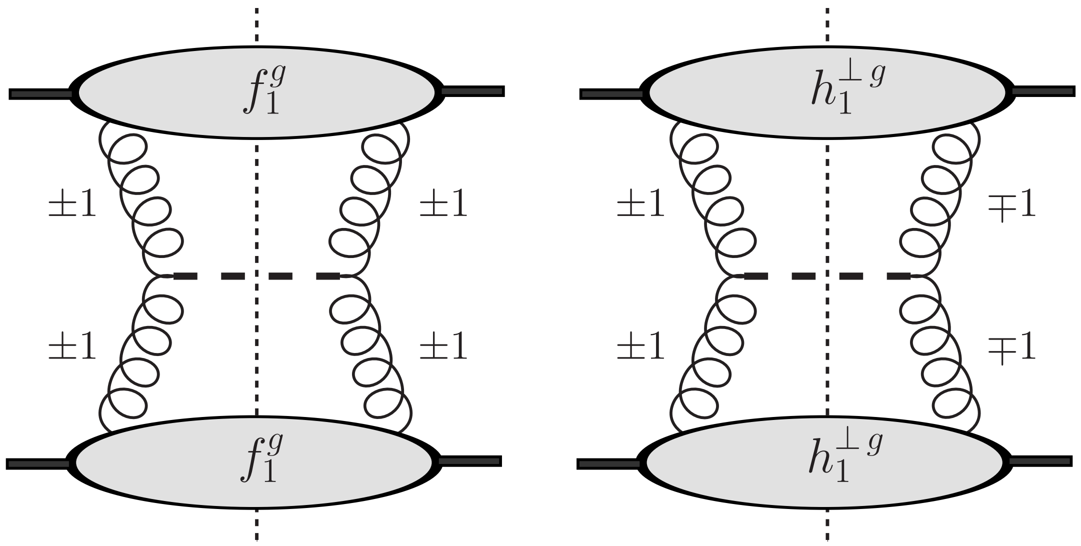


Billis, Dehnadi, Ebert, Michel, Tackmann, 2021

Probes mostly the unpolarized gluon distribution, perturbative description quite good (just 5-10% theoretical uncertainty at "low" p_T < 10 GeV)

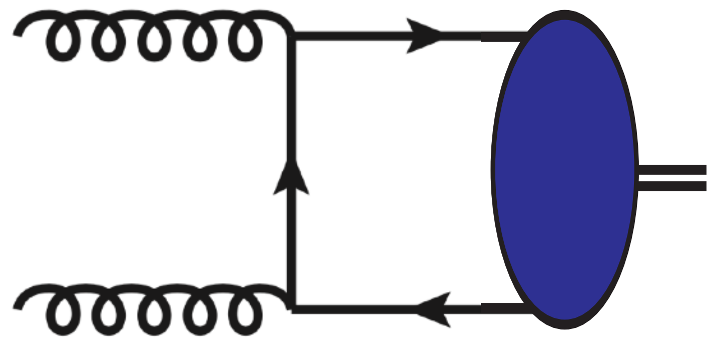
There is a (2-5%) contribution from linearly polarized gluons in Higgs production

Catani, Grazzini, 2010; Sun, Xiao, Yuan, 2011;
DB, Den Dunnen, Pisano, Schlegel, Vogelsang, 2012;
Echevarria, Kasemets, Mulders, Pisano, 2015



Charmonium & Bottomonium production

$pp \to \chi_Q X$



$\chi_{c0}\,,\,\chi_{b0}$

DB, Pisano, 2012

In LO NRQCD the differential cross sections in pp and pA are:

$$\frac{d\sigma(\eta_Q)}{dy\, d^2\boldsymbol{q}_T} = \frac{2}{9}\frac{\pi^3\alpha_s^2}{M^3\, s}\langle 0|\mathcal{O}_1^{\eta_Q}({}^1S_0)|0\rangle\, \mathcal{C}\,[f_1^g\, f_1^g]\;\left[1 - R(\boldsymbol{q}_T^2)\right]$$

$$\frac{d\sigma(\chi_{Q0})}{dy\, d^2\boldsymbol{q}_T} = \frac{8}{3}\frac{\pi^3\alpha_s^2}{M^5\, s}\langle 0|\mathcal{O}_1^{\chi_{Q0}}({}^3P_0)|0\rangle\, \mathcal{C}\,[f_1^g\, f_1^g]\;\left[1 + R(\boldsymbol{q}_T^2)\right]$$

$$\frac{d\sigma(\chi_{Q2})}{dy\, d^2\boldsymbol{q}_T} = \frac{32}{9}\frac{\pi^3\alpha_s^2}{M^5\, s}\langle 0|\mathcal{O}_1^{\chi_{Q2}}({}^3P_2)|0\rangle\, \mathcal{C}\,[f_1^g\, f_1^g]$$

$$R(\boldsymbol{q}_T^2) \equiv \frac{\mathcal{C}[w h_1^{\perp g} h_1^{\perp g}]}{\mathcal{C}[f_1^g f_1^g]}$$

For χ_{Q1} there is no contribution due to Landau-Yang theorem

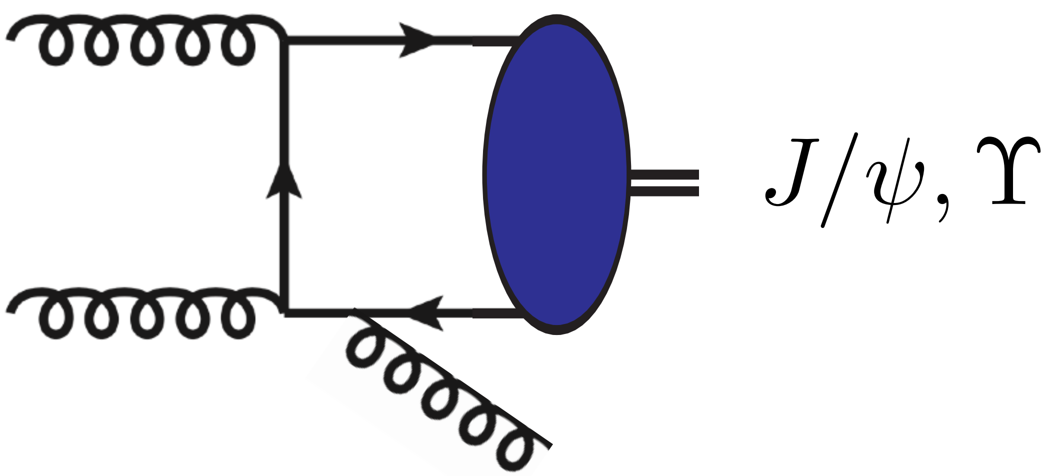
Comparing $pp \to H\, X$, where $H = \chi_{c0}$, χ_{b0} or Higgs allows to test TMD evolution

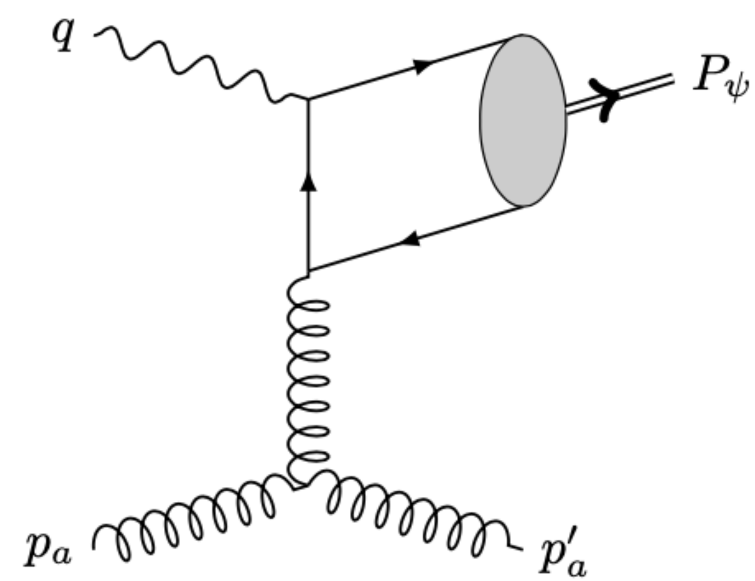
J/ψ production

J/ψ production often considered in NRQCD

Bodwin, Braaten, Lepage, 1995; …

J/ψ production in electron-proton and proton-proton collisions has been considered at large transverse momenta to ensure collinear factorization





Berger, Qiu, Wang, 2005;
Kang, Ma, Qiu, Sterman, 2014;
…

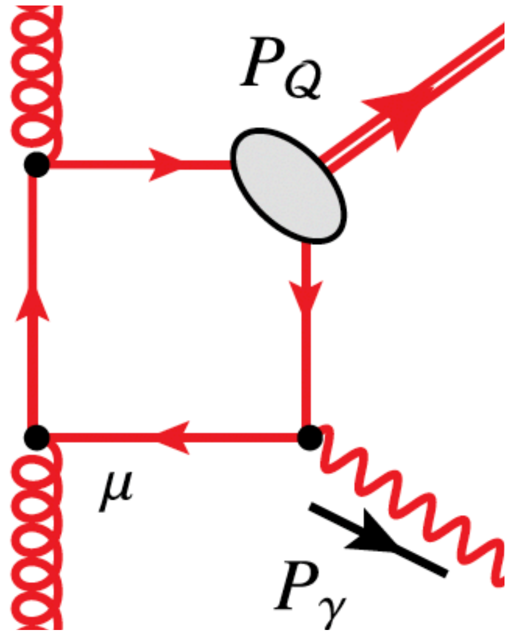
$$d\sigma_{\rm NRQCD}(\mathcal{Q}+X) = \sum_n d\sigma(Q\bar{Q}[n]+X) \times \langle \mathcal{O}^{\mathcal{Q}}[n] \rangle$$

LDMEs

It turns out to be hard to describe all HERA, Tevatron & LHC simultaneously

Low p_T data from LHC and EIC may help to clarify the situation

J/ψ production in TMD regime in pp

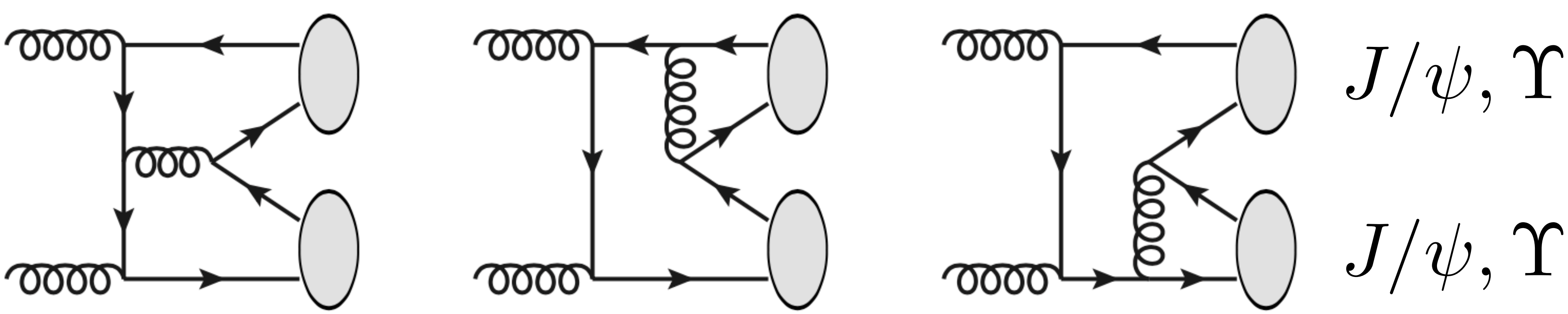


Associated production of J/ψ with a photon, both with large pT, but their sum needs to be small

A good probe of the gluon TMD

Den Dunnen, Lansberg, Pisano, Schlegel, 2014

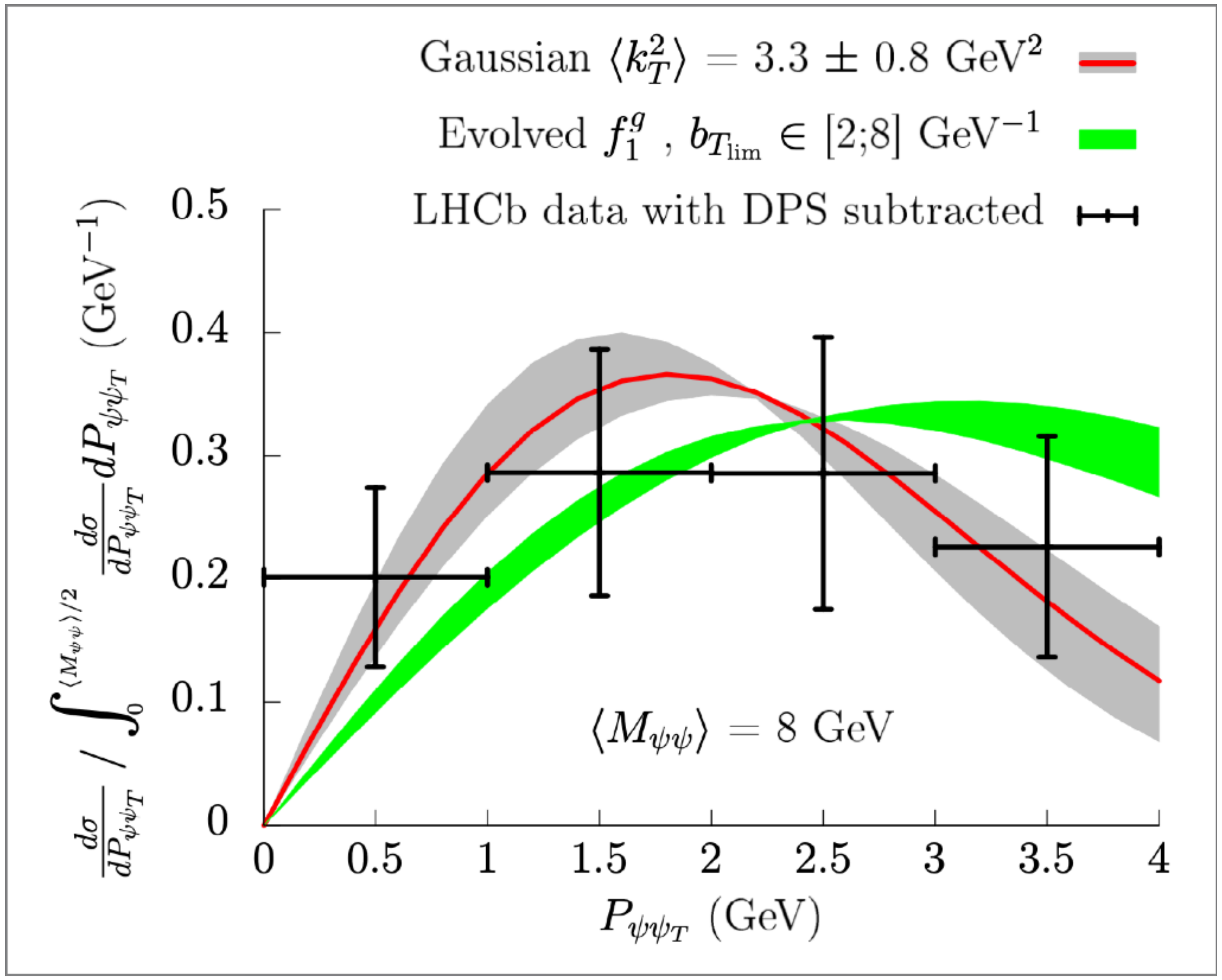
Works in double J/ψ production too:



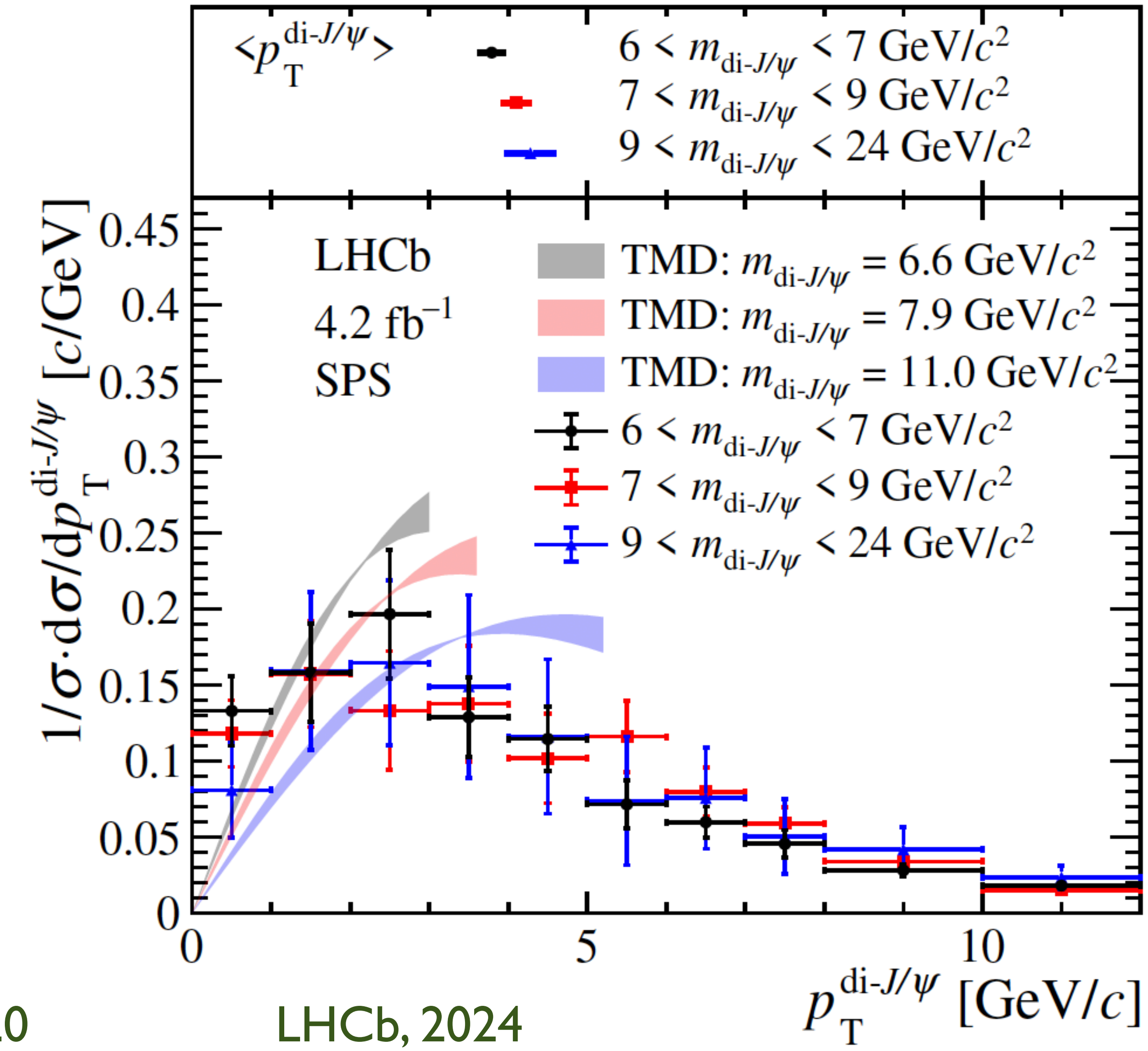
CS–CS ≫ CO–CO

Lansberg, Pisano, Scarpa, Schlegel, 2018; Scarpa, DB, Echevarria, Lansberg, Pisano, Schlegel, 2020

J/ψ pair production



Scarpa, DB, Echevarria, Lansberg, Pisano, Schlegel, 2020



LHCb, 2024

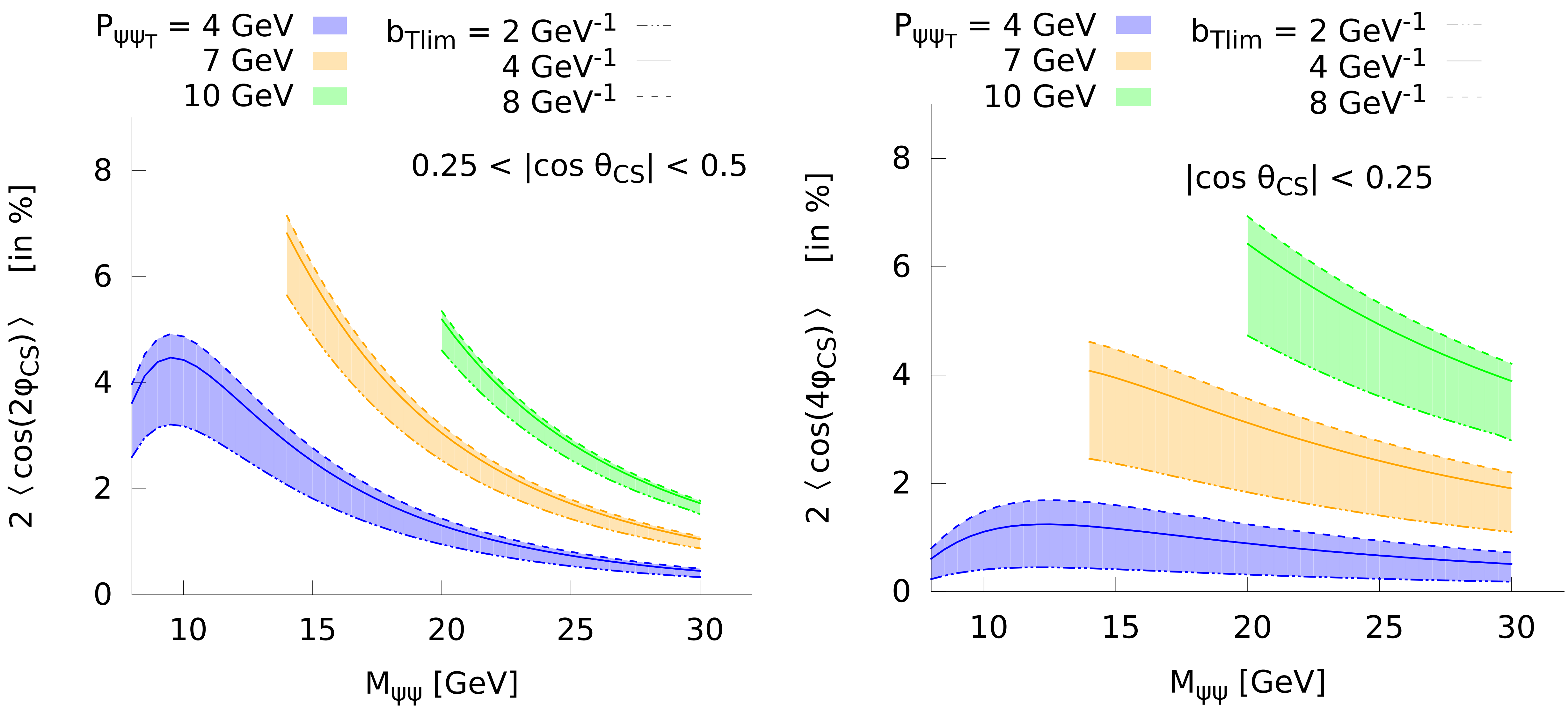
J/ψ pair invariant mass allows to study TMD evolution

The shape of this normalized & DPS subtracted cross section and its scale evolution is not fully described by the TMD description (also not within uncertainties from nonperturbative physics)

[cf. Jelle Bor, PhD thesis, 2025]

Linear gluon polarization in di-J/Ψ production

$h_1^{\perp g}$ can be probed through angular modulations in $p\,p \to J/\psi\, J/\psi\, X$

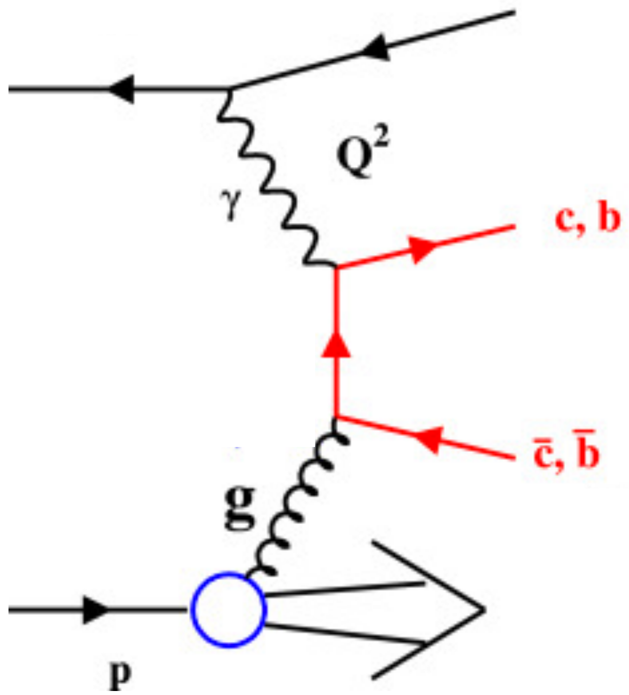


Estimated to lead to 1-5% level azimuthal modulations at LHC (incl. TMD evolution)

Scarpa, DB, Echevarria, Lansberg, Pisano, Schlegel, 2019; Bor, PhD thesis, 2025

Open heavy quark production at EIC

Unpolarized open heavy quark production at EIC probes $h_1^{\perp\, g}(x, p_T^2)$



The heavy quarks will not be exactly back-to-back in the transverse plane:

$$K_\perp = (K_{Q\perp} - K_{\bar{Q}\perp})/2$$

$$q_T = K_{Q\perp} + K_{\bar{Q}\perp}$$

$$|q_T| \ll |K_\perp|$$

$\phi_T, \phi_\perp$ are the angles of q_T, $K_\perp$

Linear gluon polarization shows up as a $\cos 2\phi_T$ or $\cos 2(\phi_T - \phi_\perp)$ distribution

DB, Brodsky, Mulders & Pisano, 2010

Up to 10% asymmetries at EIC

DB, Pisano, Mulders, Zhou, 2016

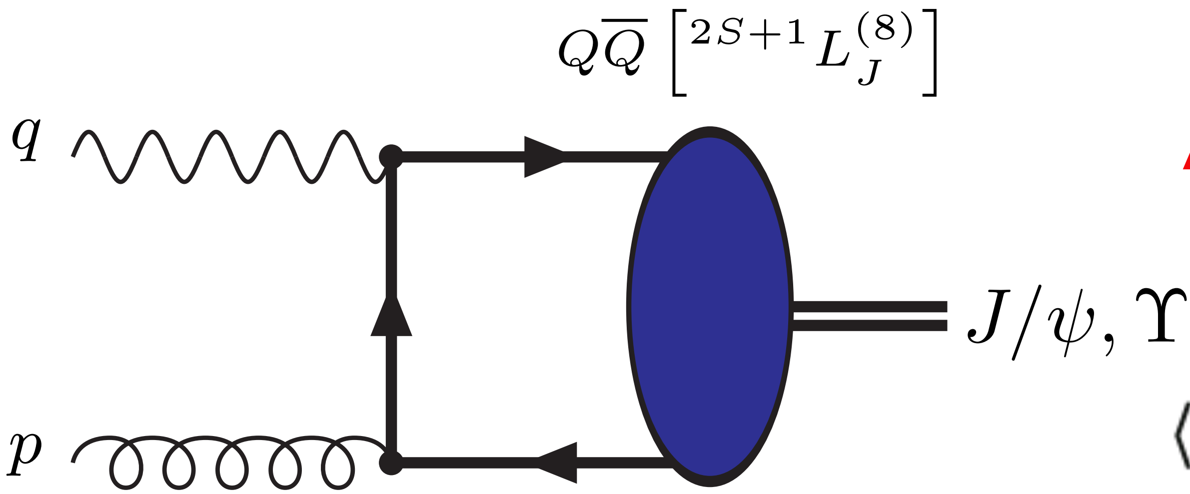
Similarly $h_1^{\perp g}$ is accessible in inclusive dijet production at EIC

Metz, Zhou 2011; Pisano, DB, Brodsky, Buffing, Mulders, 2013; Dumitru, Lappi, Skokov, 2015; Caucal, Salazar, Schenke, Stebel, Venugopalan, 2024

Quarkonium production in ep

$e\,p \to e'\,\mathcal{Q}\,X$ with $\mathcal{Q}$ either a J/ψ or a Υ meson

Mukherjee, Rajesh, 2017; Sun, Zhang, 2017; Bacchetta, DB, Pisano, Taels, 2018;
Kishore, Mukherjee, 2018; Kishore, Mukherjee, Siddiqah, 2021; …



A cos(2ϕ$_T$) asymmetry probes $h_1^{\perp\,g}$

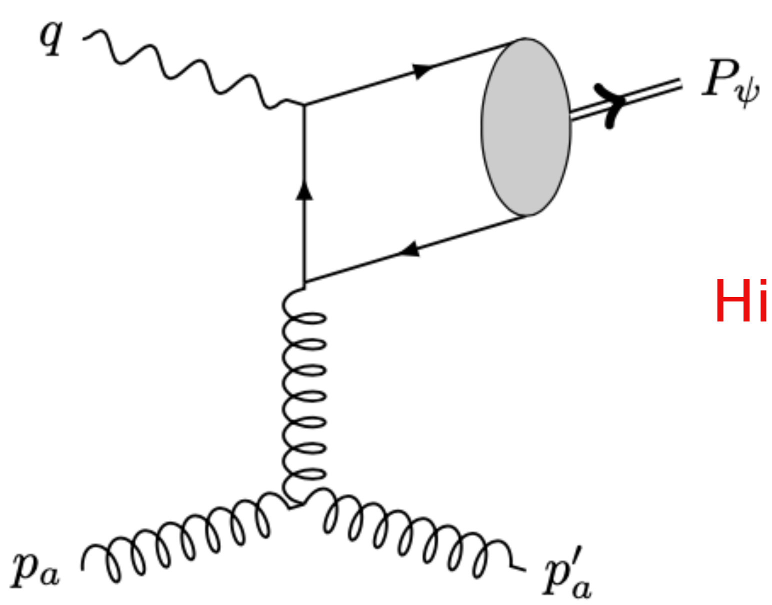
$$\langle\cos 2\phi_T\rangle = \frac{(1-y)\,\mathcal{B}_T^{\gamma^* g\to\mathcal{Q}}}{[1+(1-y)^2]\mathcal{A}_{U+L}^{\gamma^* g\to\mathcal{Q}} - y^2\mathcal{A}_L^{\gamma^* g\to\mathcal{Q}}} \times \frac{\boldsymbol{q}_T^2}{2M_p^2}\,\frac{h_1^{\perp\,g}(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}.$$

In LO NRQCD the prefactor of the asymmetry depends on kinematic variables and on two poorly known Color Octet (CO) Long Distance Matrix Elements (LDMEs)

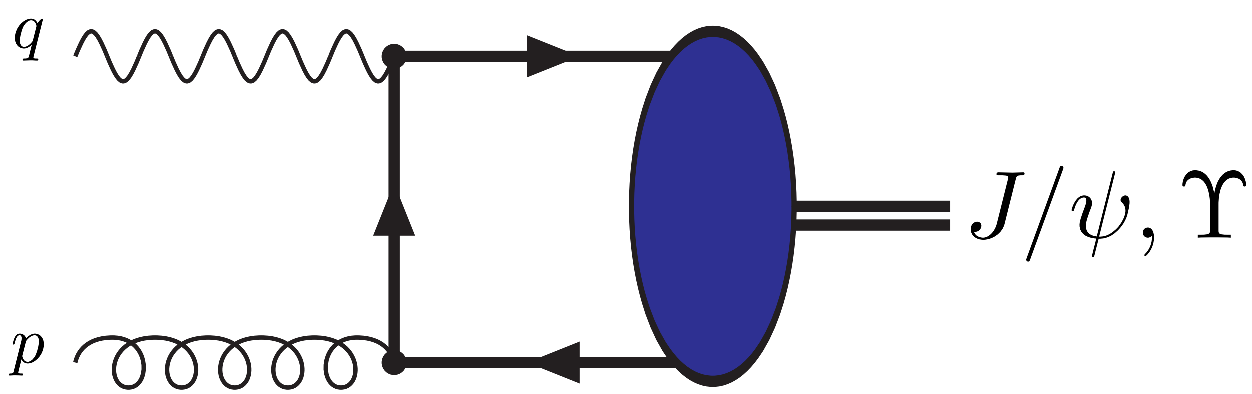
Despite the large uncertainties sizable cos2ϕ$_T$ asymmetries are possible at EIC

Bacchetta, DB, Pisano, Taels, 2018; Bor, DB, 2022; Kishore, Mukherjee, Pawar, Siddiqah, 2022

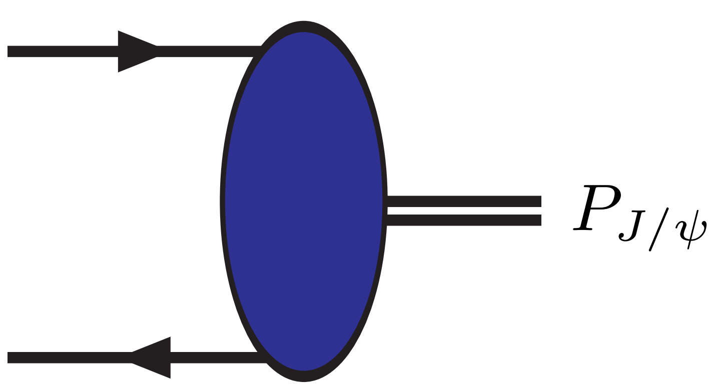
Matching high and low p_T data



High p_T production: collinear factorization



Low p_T production: TMD factorization

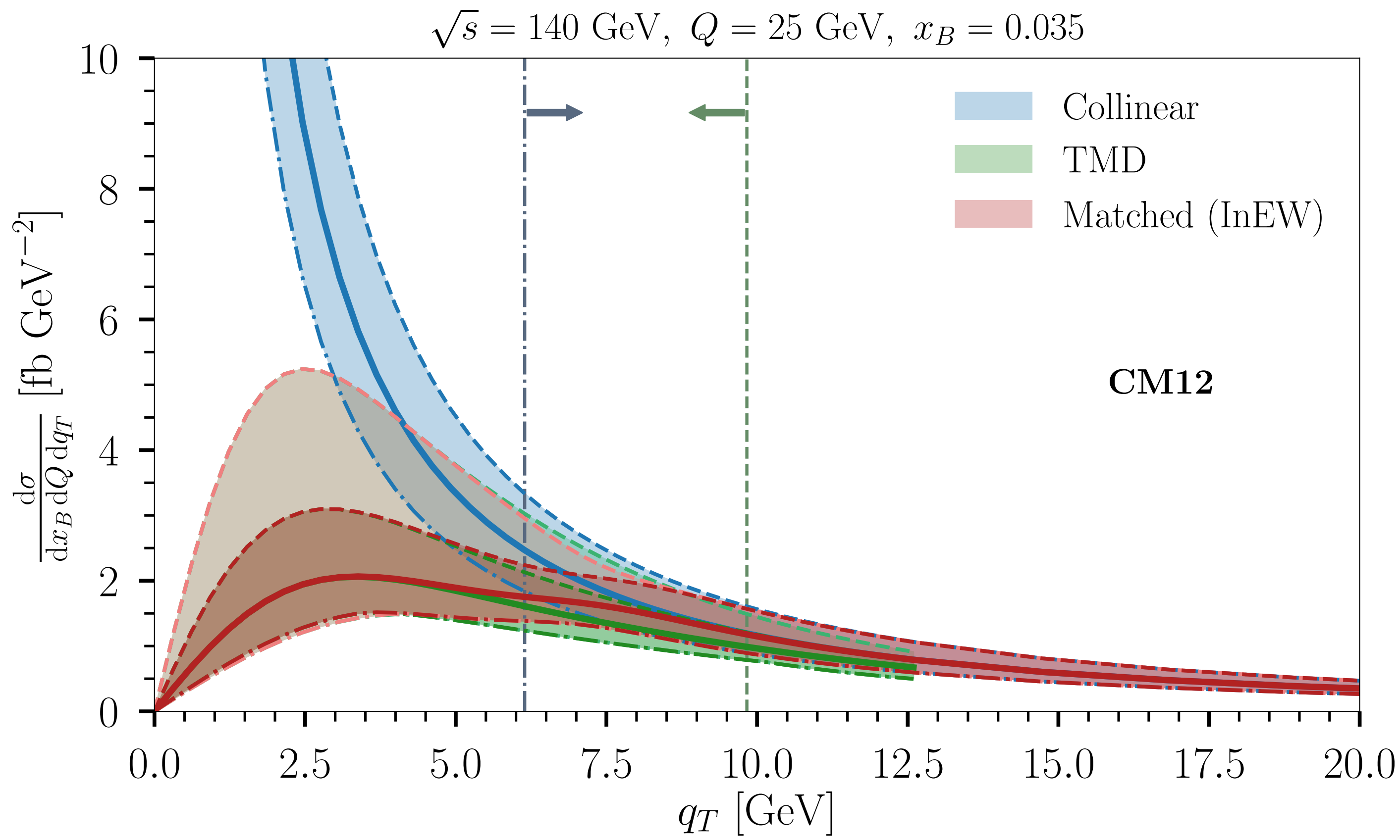


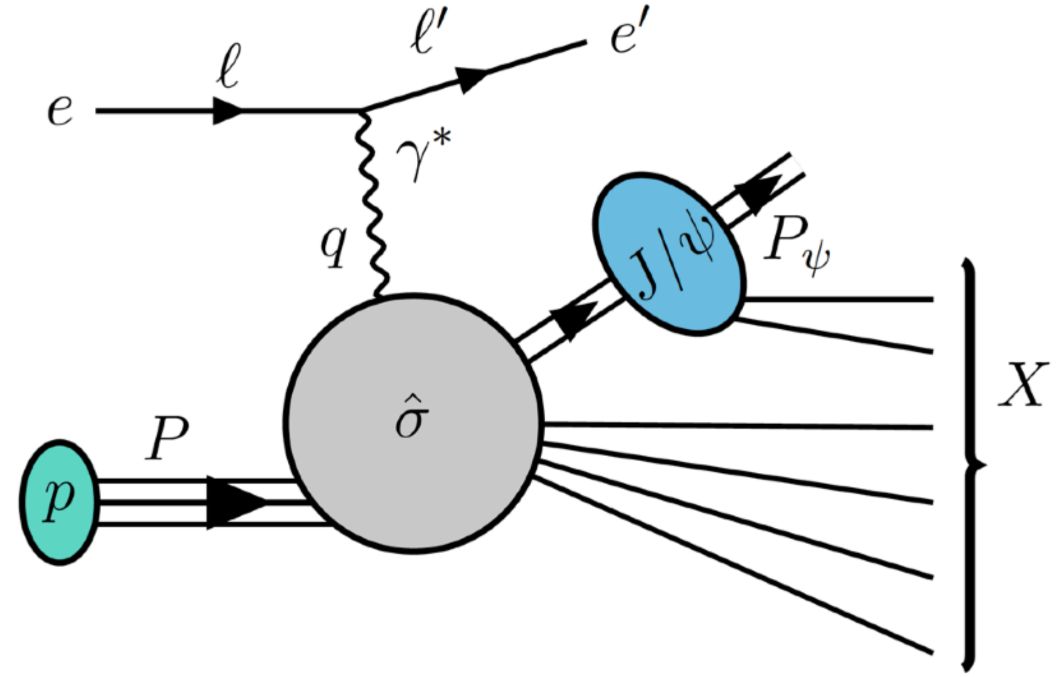
One becomes sensitive to the transverse momentum distribution of the gluon inside the proton *and* of the heavy quark pair inside the quarkonium

TMD factorization of quarkonium production will involve new shape functions

Echevarria, 2019; Fleming, Makris & Mehen, 2019; DB, D'Alesio, Murgia, Pisano, Taels, 2020; DB, Bor, Maxia, Pisano, Yuan, 2023

Matching





Matching the TMD and collinear (FO) expressions with uncertainties using Inverse Error Weighting (InEW)

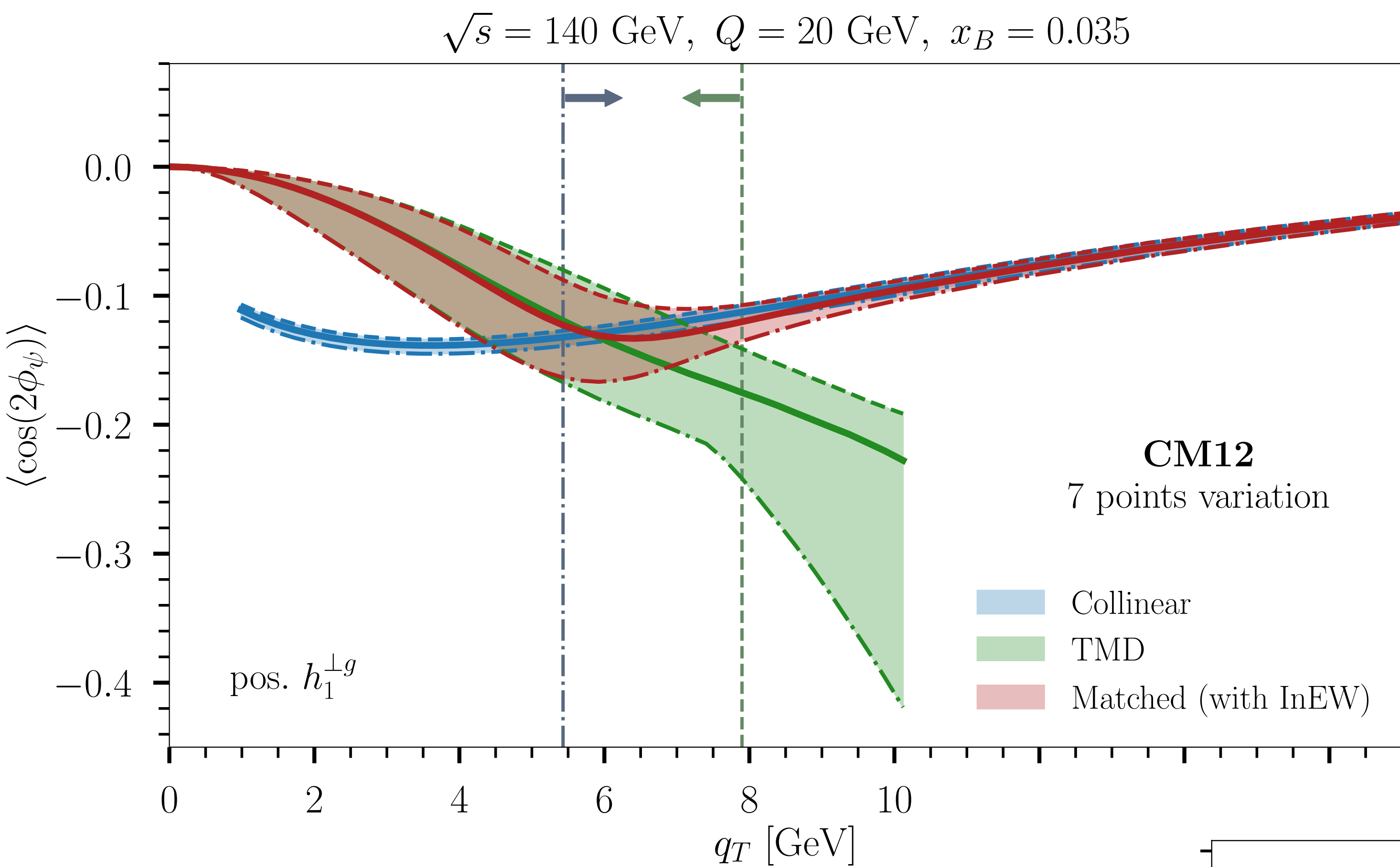
Maxia, DB, Bor, 2025

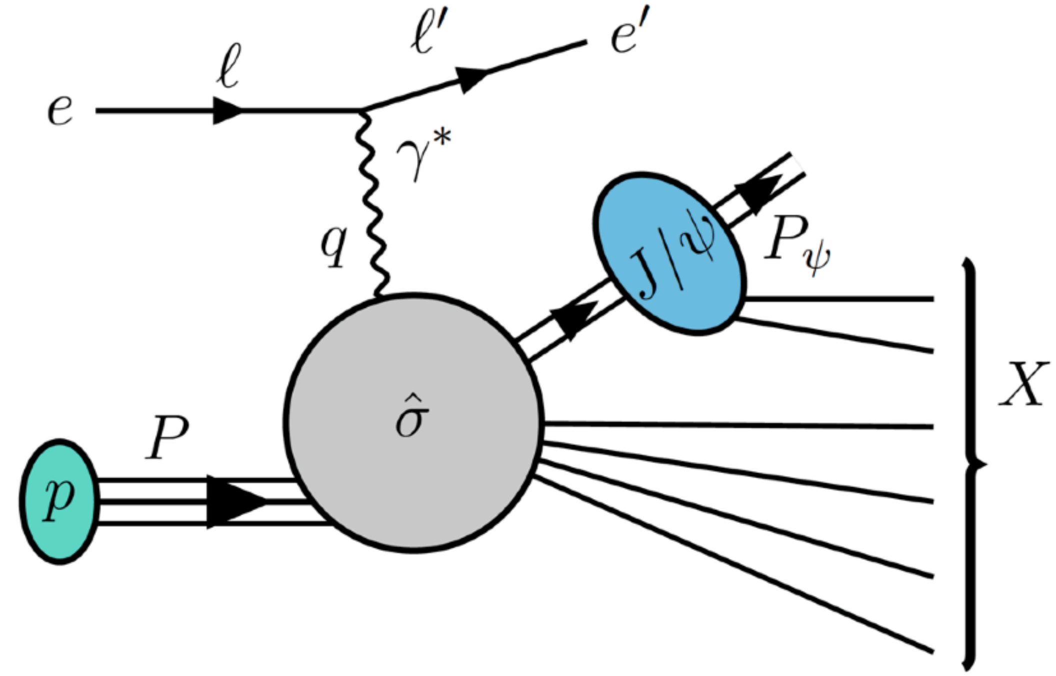
Uncertainty bands include LDME, scale & nonperturbative Sudakov uncertainties

EIC data will teach us about LDMEs, shape functions, and their process dependence

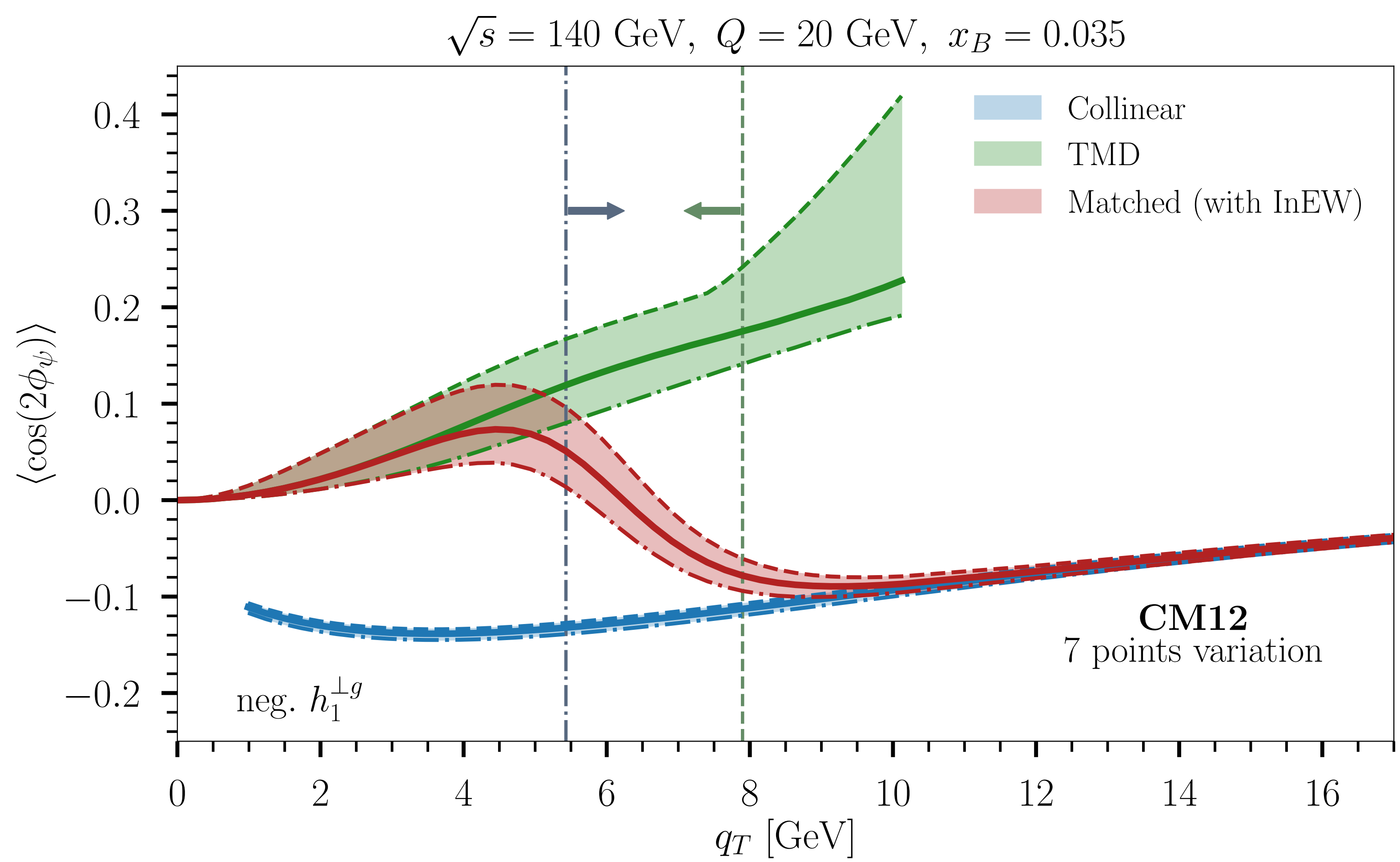
Comparing LHC data on J/ψ+γ and J/ψ pair production and EIC data on open heavy quark pairs and quarkonium production hopefully will shed light on why it is difficult to describe HERA, Tevatron & LHC simultaneously

Asymmetry





Sign of $h_1^{\perp g}$ not known



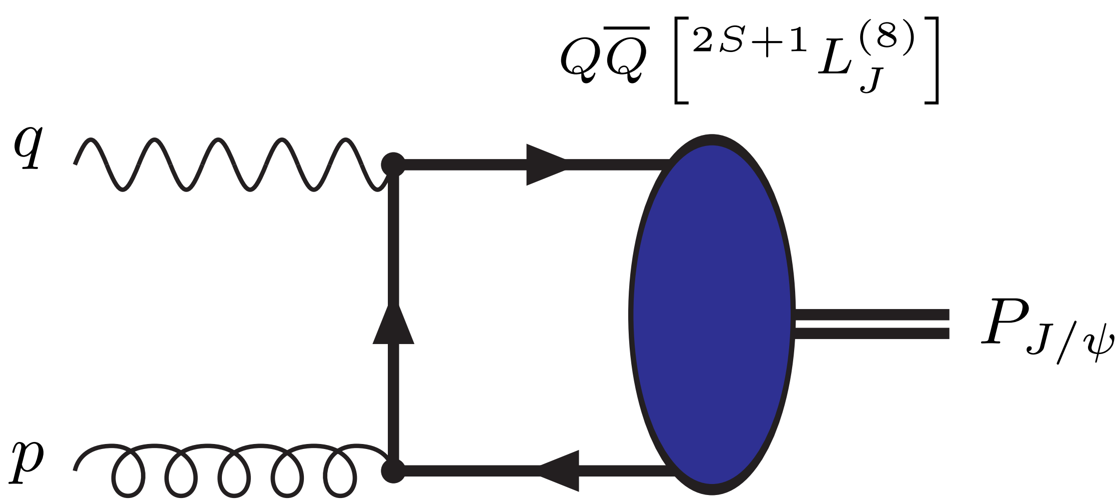
Modulations can be in the 10% range at EIC, a node is possible

Maxia, DB, Bor, 2025

Quarkonium production in ep↑

$e\,p^{\uparrow} \to e'\,\mathcal{Q}\,X$ with $\mathcal{Q}$ either a J/ψ or a Υ meson

Godbole, Misra, Mukherjee, Rawoot, 2012/3; Godbole, Kaushik, Misra, Rawoot, 2015; Mukherjee, Rajesh, 2017; Rajesh, Kishore, Mukherjee, 2018; …



Using LO NRQCD the Sivers asymmetry is:

$$A^{\sin(\phi_S-\phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p}\,\frac{f_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S+\phi_T)}} = \frac{\boldsymbol{q}_T^2}{M_p^2}\,\frac{h_1^{\perp\,g}(x,\boldsymbol{q}_T^2)}{h_1^g(x,\boldsymbol{q}_T^2)}$$

$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S-3\phi_T)}} = -\frac{1}{2}\,\frac{h_1^{\perp\,g}(x,\boldsymbol{q}_T^2)}{h_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}$$

$$\frac{A^{\sin(\phi_S-3\phi_T)}}{A^{\sin(\phi_S+\phi_T)}} = -\frac{\boldsymbol{q}_T^2}{2M_p^2}\,\frac{h_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}{h_1^g(x,\boldsymbol{q}_T^2)}$$

CO NRQCD LDMEs cancel out in ratios of asymmetries at LO

Bacchetta, DB, Pisano, Taels, 2018

Higher order corrections and shape functions will complicate this simple picture

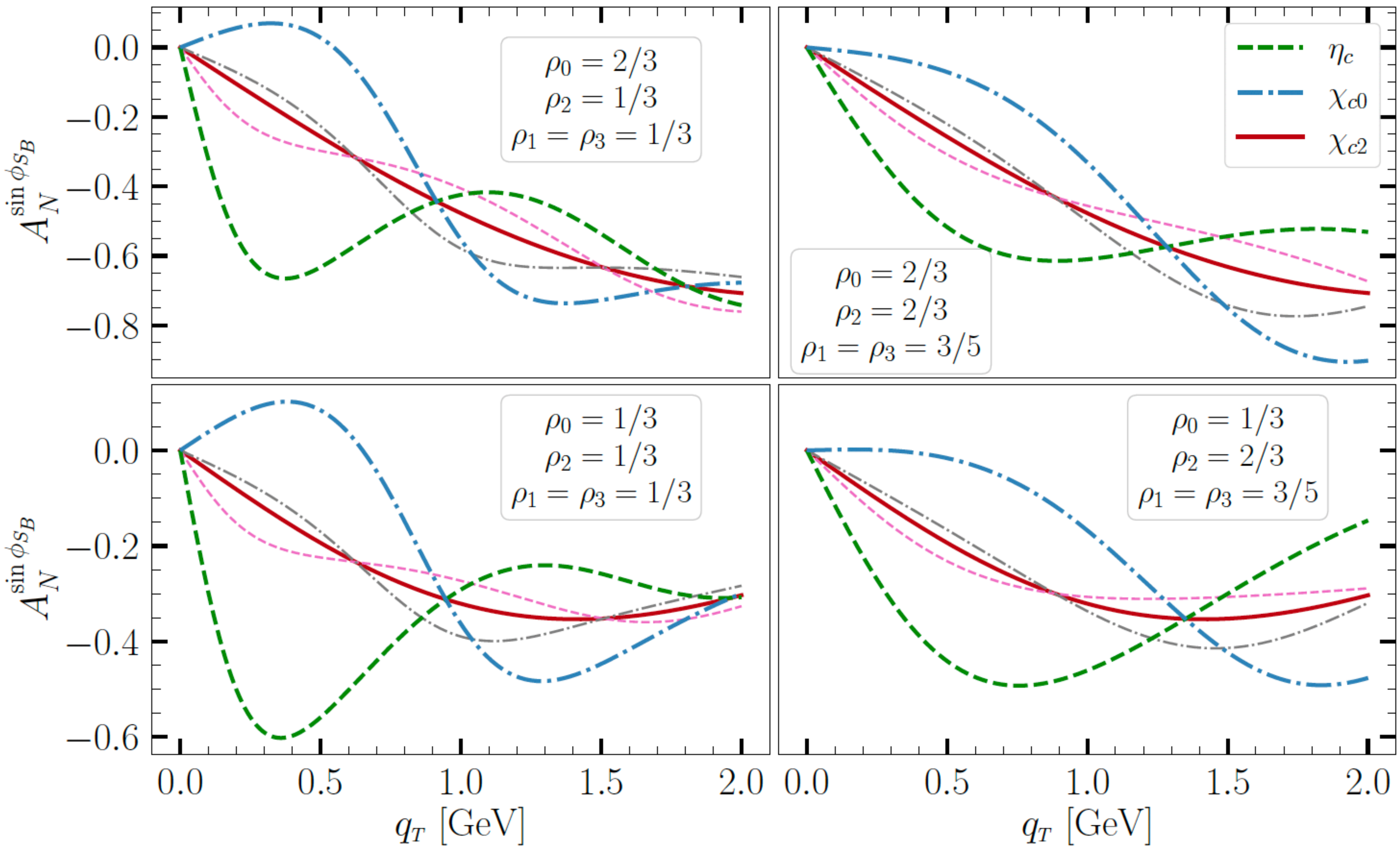
Synergies with LHCspin

$p^{\uparrow}p \to \eta_{c,b}\, X$

$$F_{TU}^{\eta_Q,\sin\phi_{S_A}} = H^{\eta_Q}\Big(\mathcal{C}\big[w_{TU}^{f}\, f_1^g f_{1T}^{\perp g}\big] - \mathcal{C}\big[w_{TU}^{h}\, h_1^{\perp g} h_1^g\big] + \mathcal{C}\big[w_{TU}^{h^\perp}\, h_1^{\perp g} h_{1T}^{\perp g}\big]\Big)\, \langle 0|\mathcal{O}_1^{\eta_Q}(^1S_0)|0\rangle\,,$$

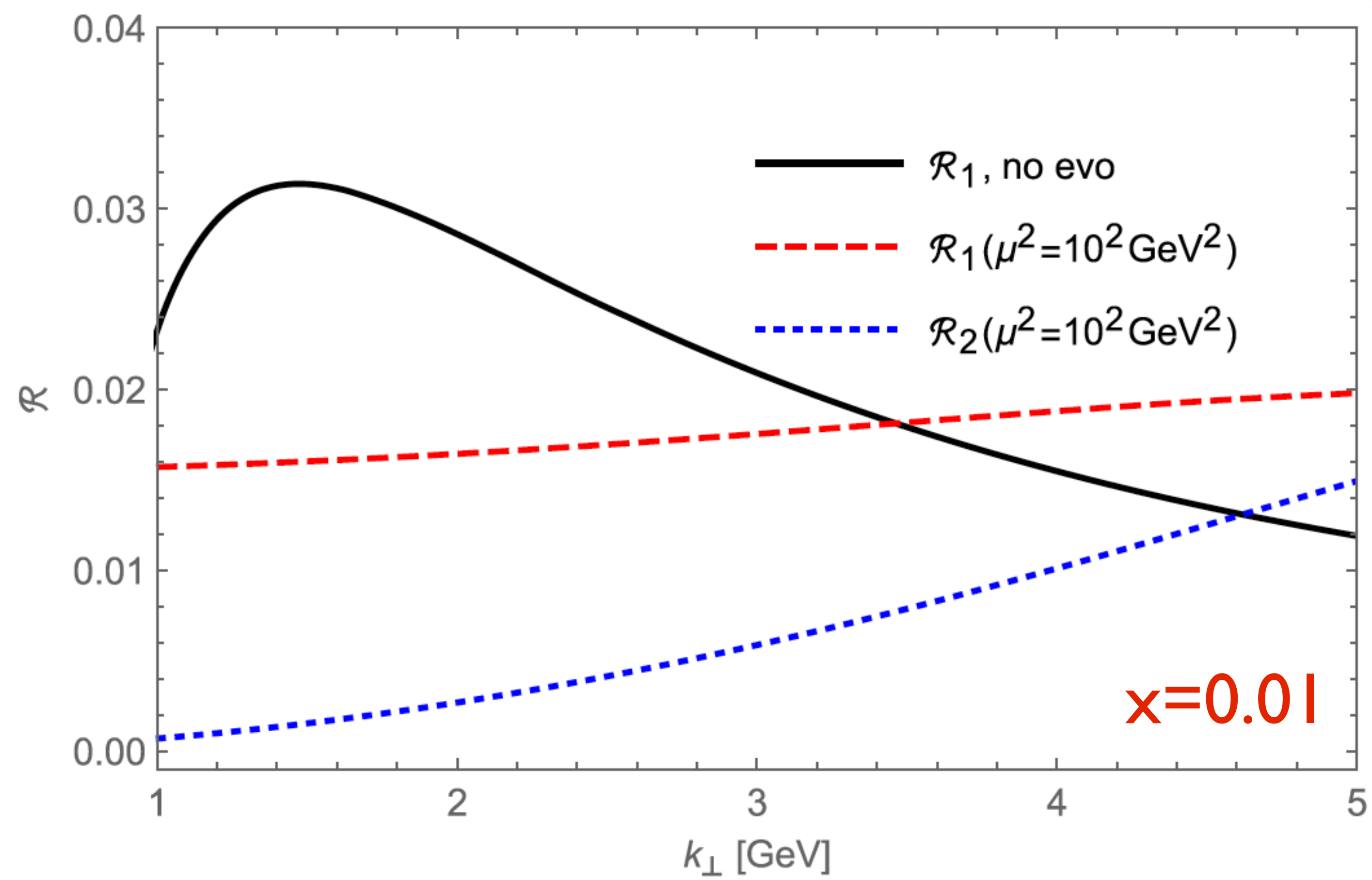
$$F_{TU}^{\chi_{Q0},\sin\phi_{S_A}} = H^{\chi_{Q0}}\Big(\mathcal{C}\big[w_{TU}^{f}\, f_1^g f_{1T}^{\perp g}\big] + \mathcal{C}\big[w_{TU}^{h}\, h_1^{\perp g} h_1^g\big] - \mathcal{C}\big[w_{TU}^{h^\perp}\, h_1^{\perp g} h_{1T}^{\perp g}\big]\Big)\, \langle 0|\mathcal{O}_1^{\chi_{Q0}}(^3P_0)|0\rangle\,,$$

$$F_{TU}^{\chi_{Q2},\sin\phi_{S_A}} = H^{\chi_{Q2}}\, \mathcal{C}\big[w_{TU}^{f}\, f_1^g f_{1T}^{\perp g}\big]\, \langle 0|\mathcal{O}_1^{\chi_{Q2}}(^3P_2)|0\rangle\,,$$



Kato, Maxia, Pisano, 2024

T-odd gluon TMDs at small x - scale evolution



$$\mathcal{R}_1(\mu^2) = \frac{x f_{1T}^{\perp g}(\mu^2)}{x f_1^g(\mu^2)}$$

$$\mathcal{R}_2(\mu^2) = \frac{\frac{\mathbf{k}_\perp^2}{2M^2} x h_{1T}^{\perp g}(\mu^2)}{x f_1^g(\mu^2)}$$

Initial scale 1.6 GeV

DB, Hagiwara, Jian Zhou & Ya-Jin Zhou, 2022

At small x the T-odd dipole gluon TMDs are equal but scale evolution does not preserve this equality and is the evolution of DP and WW distributions the same?

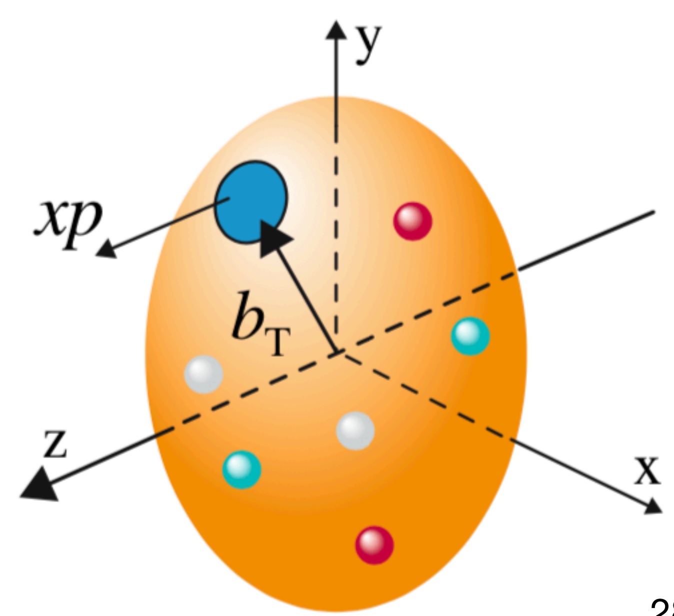
Comparison of $p^\uparrow p \rightarrow \eta_c X$ and $p^\uparrow p \rightarrow \eta_b X$ can test this (for WW gluons TMDs)

Conclusion: also for gluon distribution of polarized protons there are synergies between EIC and FT experiments at LHC

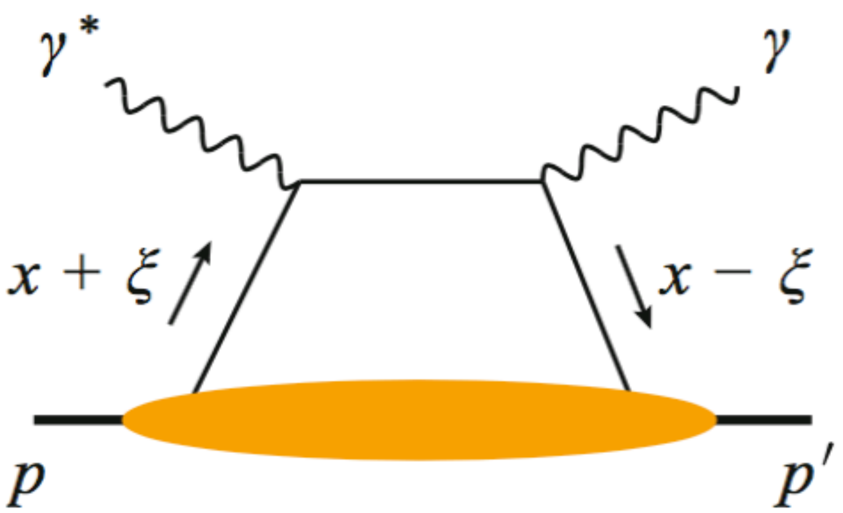
Gluon GPDs

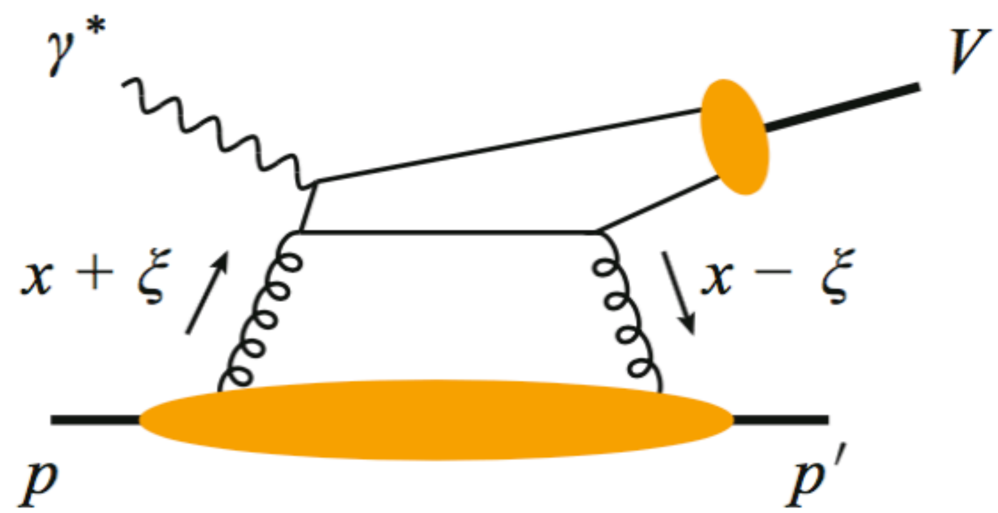
GPDs and MPI

- Exclusive and diffractive processes allow one to probe transverse spatial distributions, Generalized Parton Densities (GPDs)
- Exclusive reactions like DVCS or DVMP studied extensively by COMPASS, HERMES, JLab experiments
- GPDs provide a quantitative baseline expectation for the correlations between different partons. This is relevant for studies of multiparton interactions (MPI) in pp, pA and AA collisions



GPDs





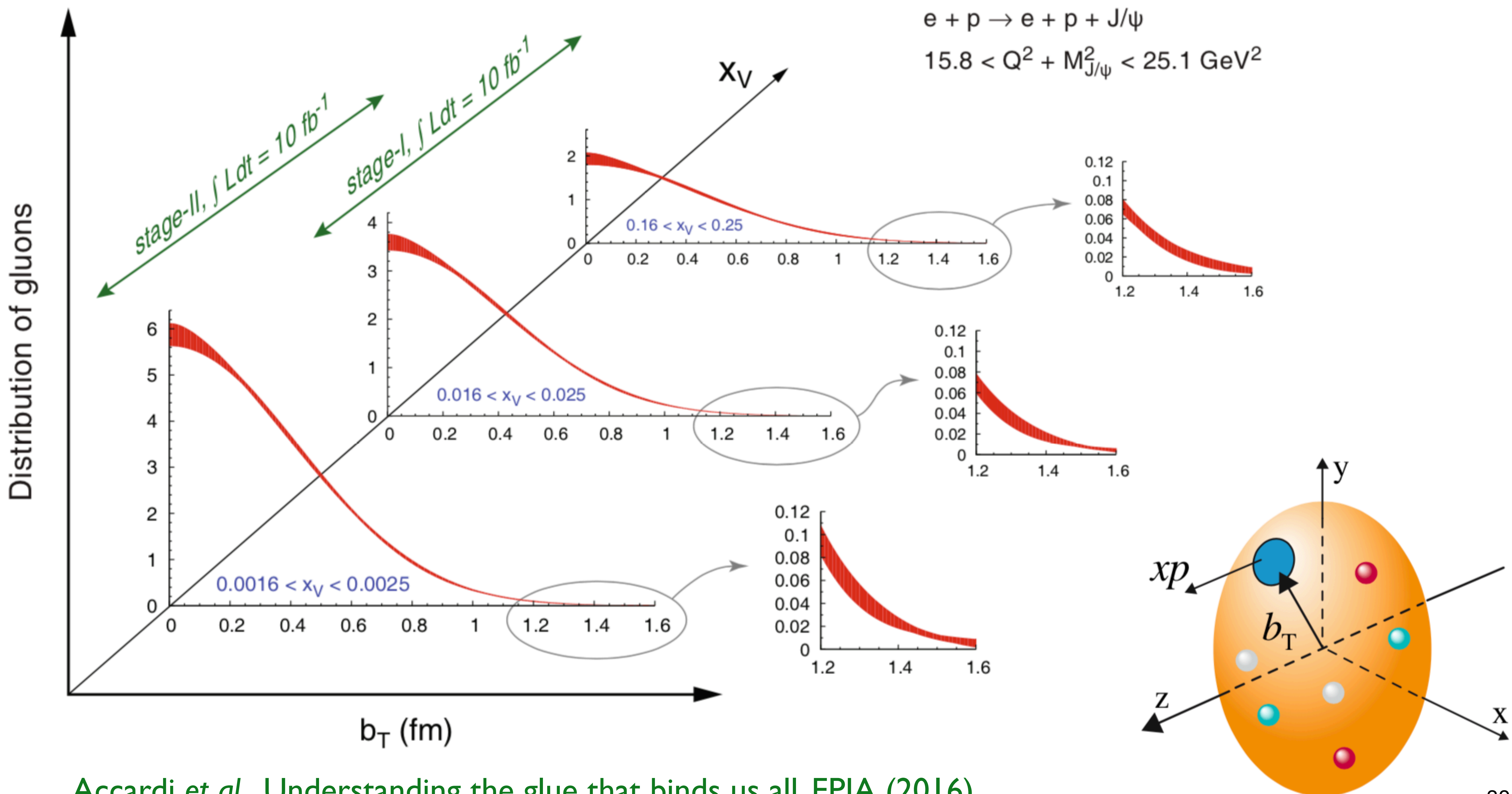
At EIC GPDs will be extracted in order to study Orbital Angular Momentum

$$J^q = \frac{1}{2} \int \mathrm{d}x \, x \, [H^q(x, \xi, t = 0) + E^q(x, \xi, t = 0)]$$

Idem for gluons, using vector mesons

Gluon GPD from exclusive J/ψ production

Projected precision of the transverse spatial distribution of gluons

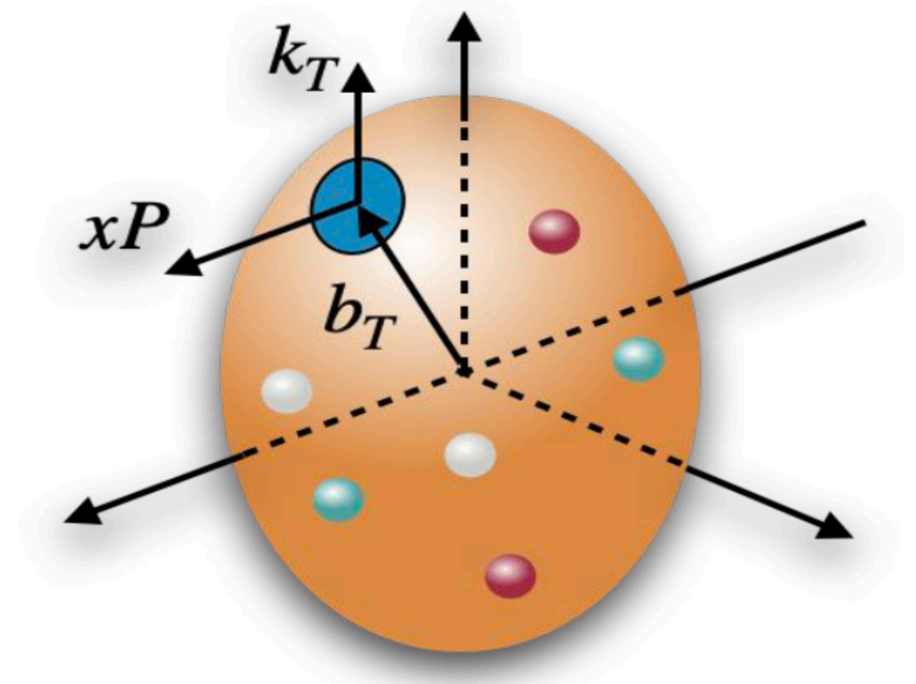


Accardi *et al.*, Understanding the glue that binds us all, EPJA (2016)

3D momentum and spatial distributions

TMDs - 3D momentum structure (x & k_T)
GPDs - 3D spatial structure (ξ & t or z & b_T)

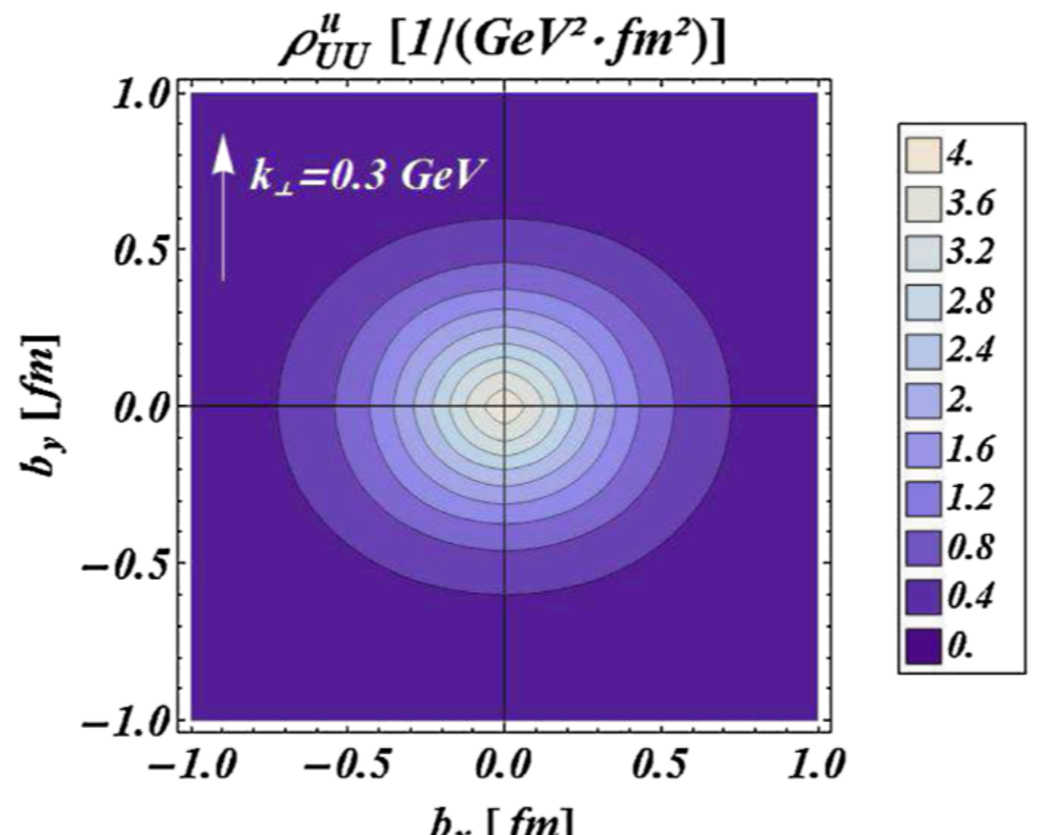
GTMDs - combined 5D (or 6D) structure



GTMD physics connects exclusive & diffractive reactions to semi-inclusive ones

Teaches us about orbital angular momentum

Lorce, Pasquini, 2011; Hatta, 2011; …



Gluon GTMDs

Gluon GTMDs for unpolarized protons

For unpolarized protons there are 2 (real valued) gluon TMDs:

$$\Gamma_U^{ij}(x,\boldsymbol{k}_T) \;=\; \frac{x}{2}\left[-\,g_T^{ij}\, f_1(x,\boldsymbol{k}_T^2) + \frac{k_T^{ij}}{M^2}\, h_1^{\perp}(x,\boldsymbol{k}_T^2)\right]$$

$$a_T^{ij} \equiv a_T^i a_T^j - \frac{1}{2} a_T^2 \delta_T^{ij}$$

Mulders, Rodrigues, 2001

For GTMDs one has one more vector so more anisotropic terms can arise

For unpolarized protons there are 4 (complex valued) gluon GTMDs

$$G^{[U,U']\,ij}(x,\boldsymbol{k}_T,\boldsymbol{\Delta}_T) = x\left(\delta_T^{ij}\,\mathcal{F}_1 + \frac{k_T^{ij}}{M^2}\,\mathcal{F}_2 + \frac{\Delta_T^{ij}}{M^2}\,\mathcal{F}_3 + \frac{k_T^{[i}\Delta_T^{j]}}{M^2}\,\mathcal{F}_4\right)$$

DB, van Daal, Mulders, Petreska, 2018

Lorcé, Pasquini, 2013; More, Mukherjee, Nair, 2018

Also for GTMDs gauge links *[U,U']* matter → WW and DP versions at small x

Dipole gluon GTMD

In the x→0 the dipole gluon GTMD becomes a correlator of a single Wilson loop:

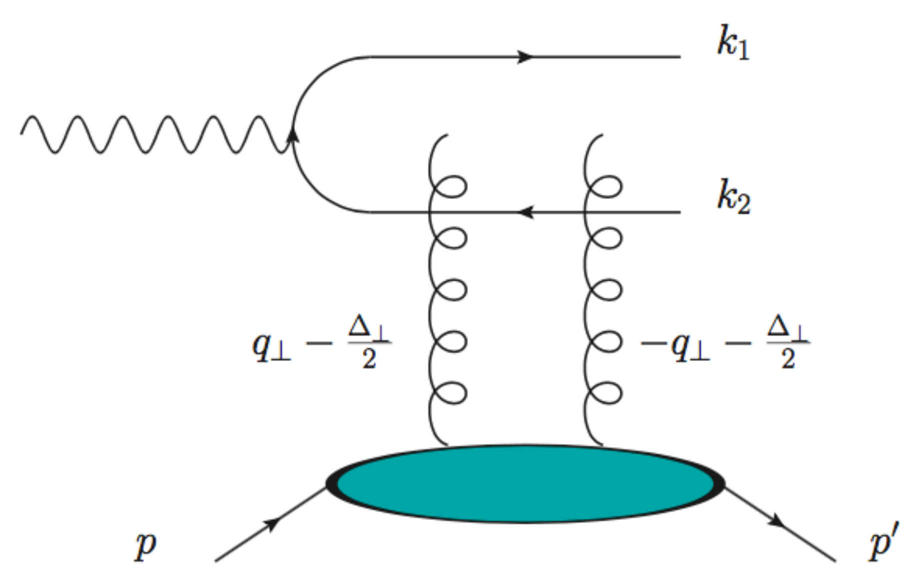
$$G^{[+,-]\,ij}(\boldsymbol{k},\boldsymbol{\Delta}) = \frac{2N_c}{\alpha_s}\left[\frac{1}{2}\left(\boldsymbol{k}^2 - \frac{\boldsymbol{\Delta}^2}{4}\right)\delta_T^{ij} + k_T^{ij} - \frac{\Delta_T^{ij}}{4} - \frac{k_T^{[i}\Delta_T^{j]}}{2}\right] G^{[\square]}(\boldsymbol{k},\boldsymbol{\Delta})$$

$$G^{[\square]}(\boldsymbol{k},\boldsymbol{\Delta}) \equiv \int \frac{d^2\boldsymbol{x}\, d^2\boldsymbol{y}}{(2\pi)^4}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})+i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}}\, \frac{\langle p'|\, S^{[\square]}(\boldsymbol{x},\boldsymbol{y})\, |p\rangle\big|_{\text{LF}}}{\langle P|P\rangle},$$

DB, van Daal, Mulders, Petreska, 2018

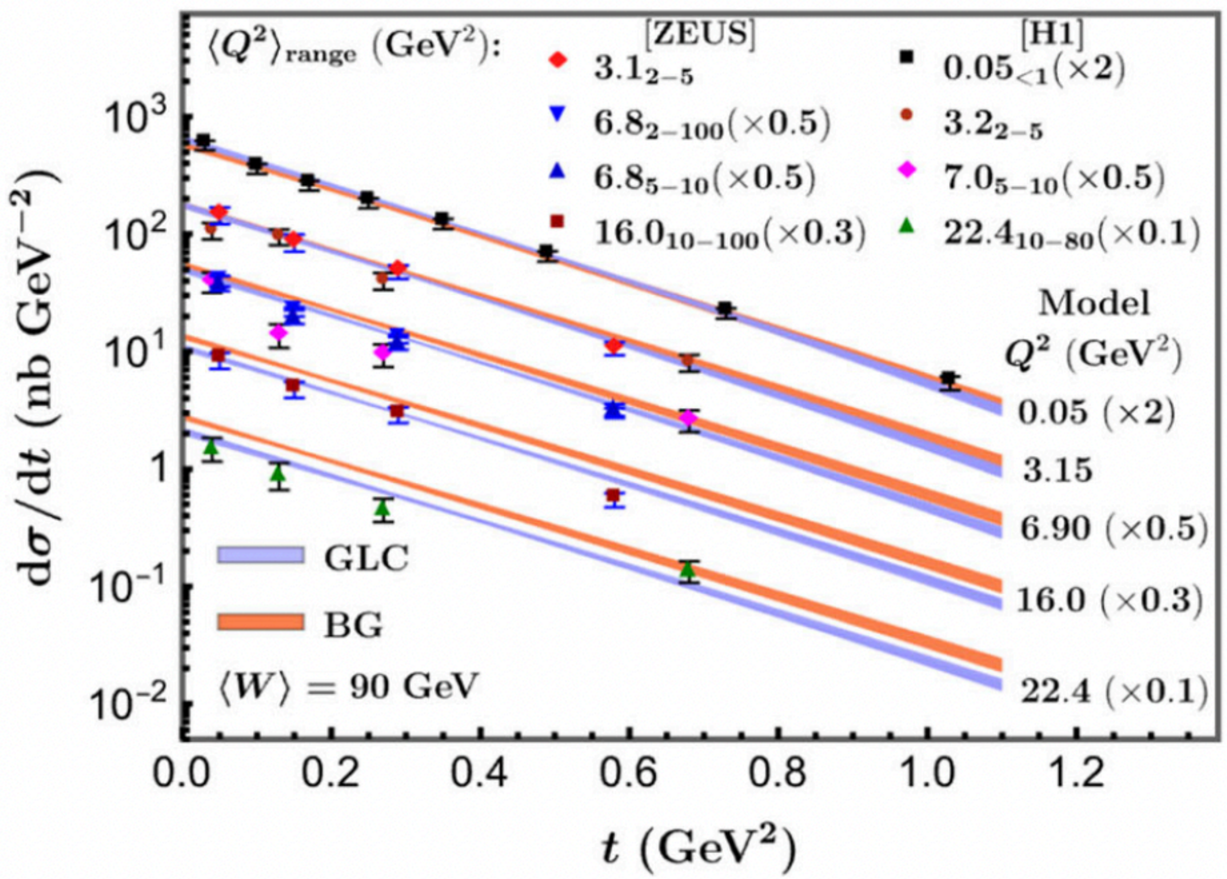
Probe gluon GTMDs via exclusive coherent diffractive dijet production in eA

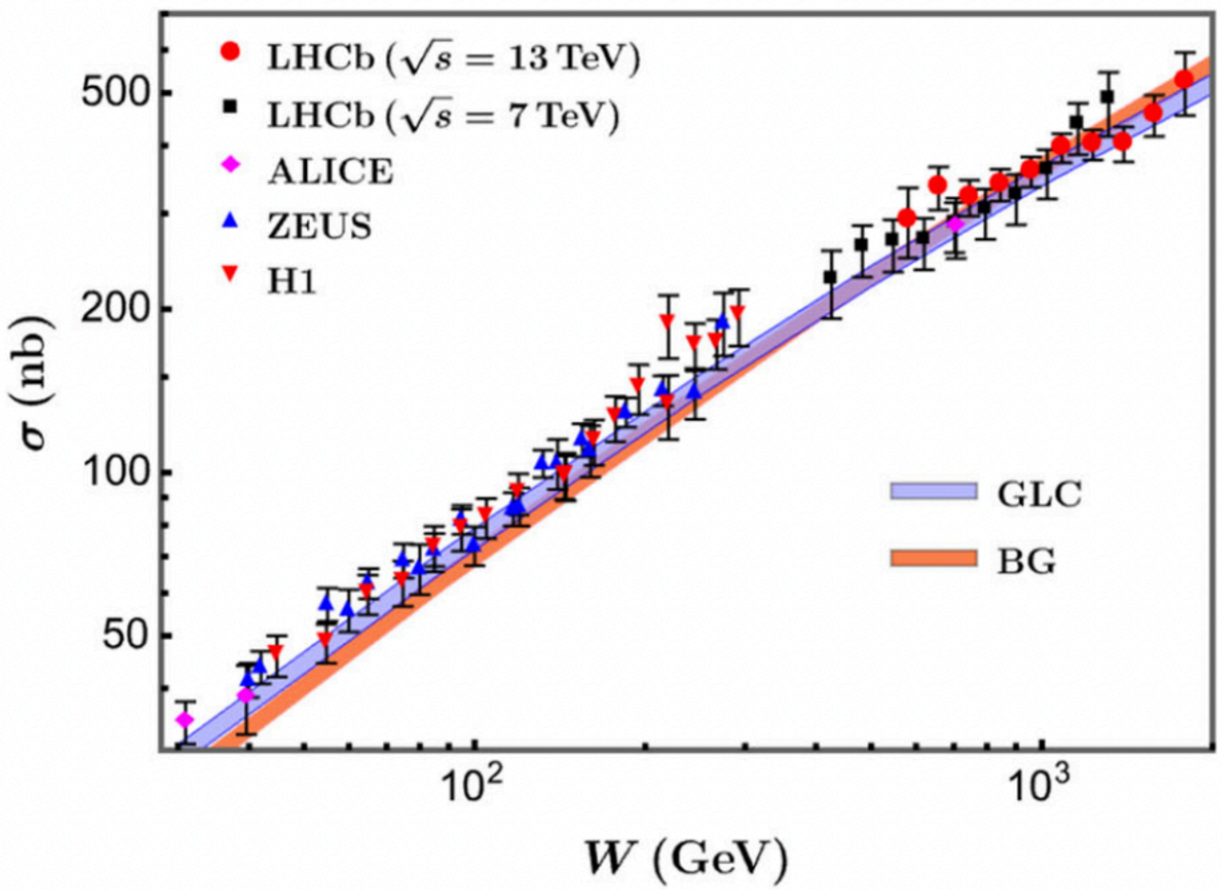
Altinoluk, Armesto, Beuf, Rezaeian, 2016; Hatta, Xiao, Yuan, 2016



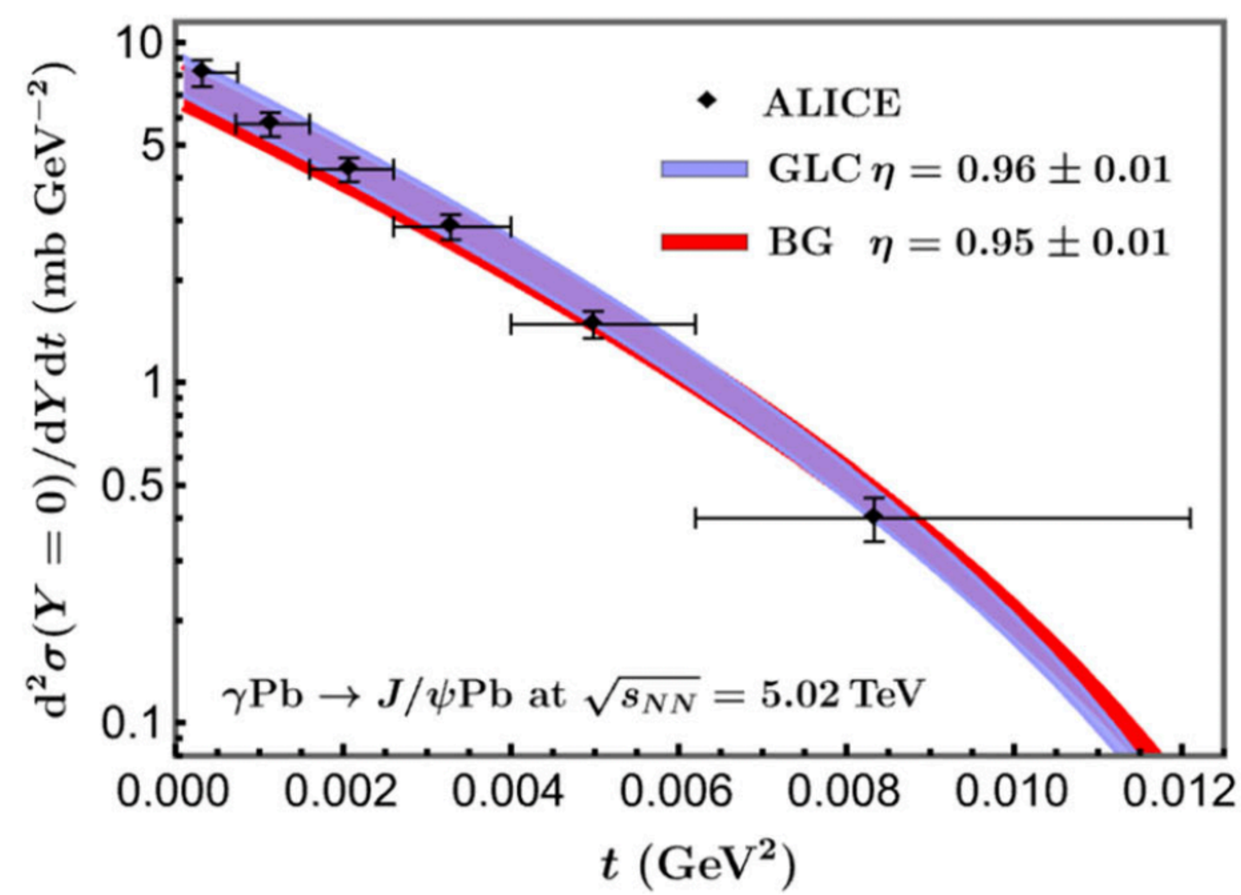
Also accessed in diffractive J/ψ production

The transverse momentum dependence of the GTMD is probed indirectly





Diffractive J/ψ production data of HERA (H1 & ZEUS) can be described reasonably by a small-x GTMD model

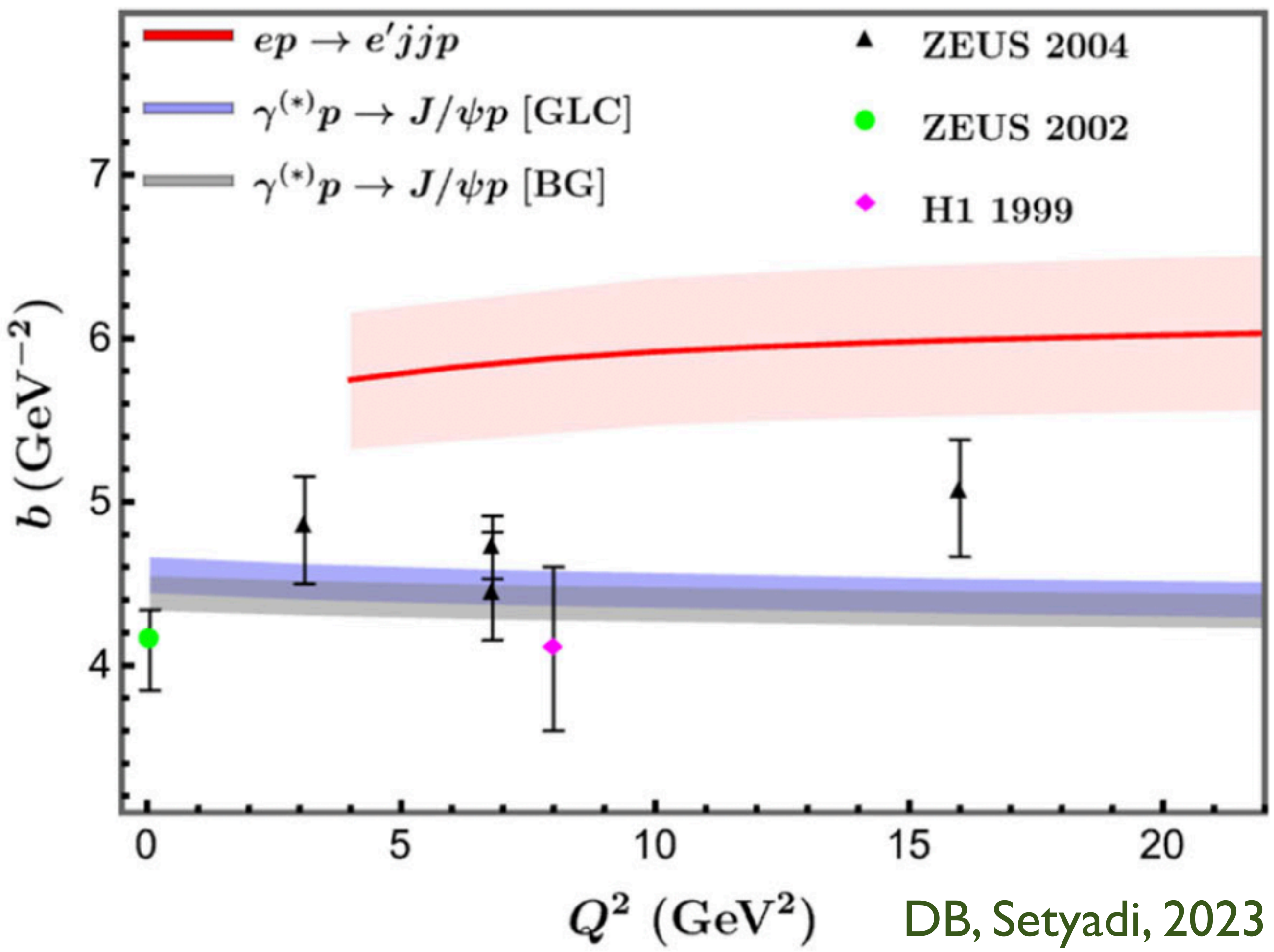


Description of ALICE UPC data qualitatively fine as well

DB, Setyadi, 2023

Tension between diffractive dijet and J/ψ production

There is tension between the dijet and J/ψ data regarding the steepness of the t-slope



DB, Setyadi, 2023

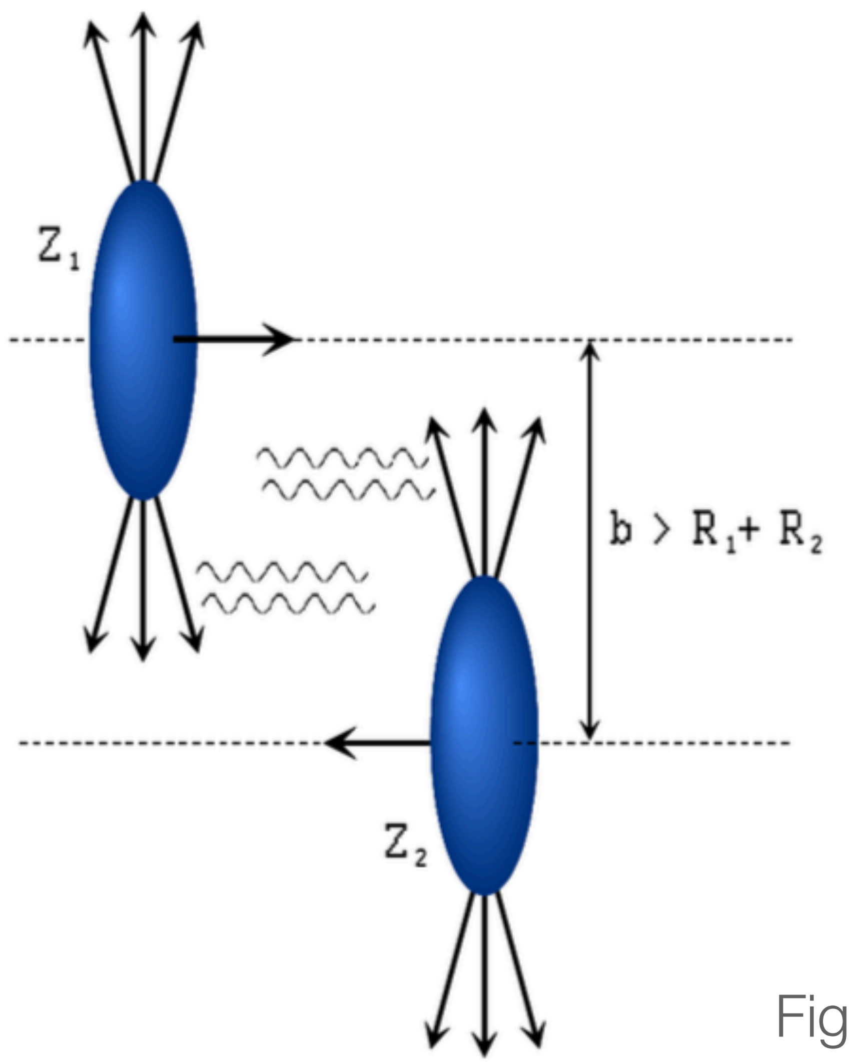
Applicability to H1 data (gluon dominance) is questioned by Linek, Luszczak, W. Schäfer, Szczurek, 2024

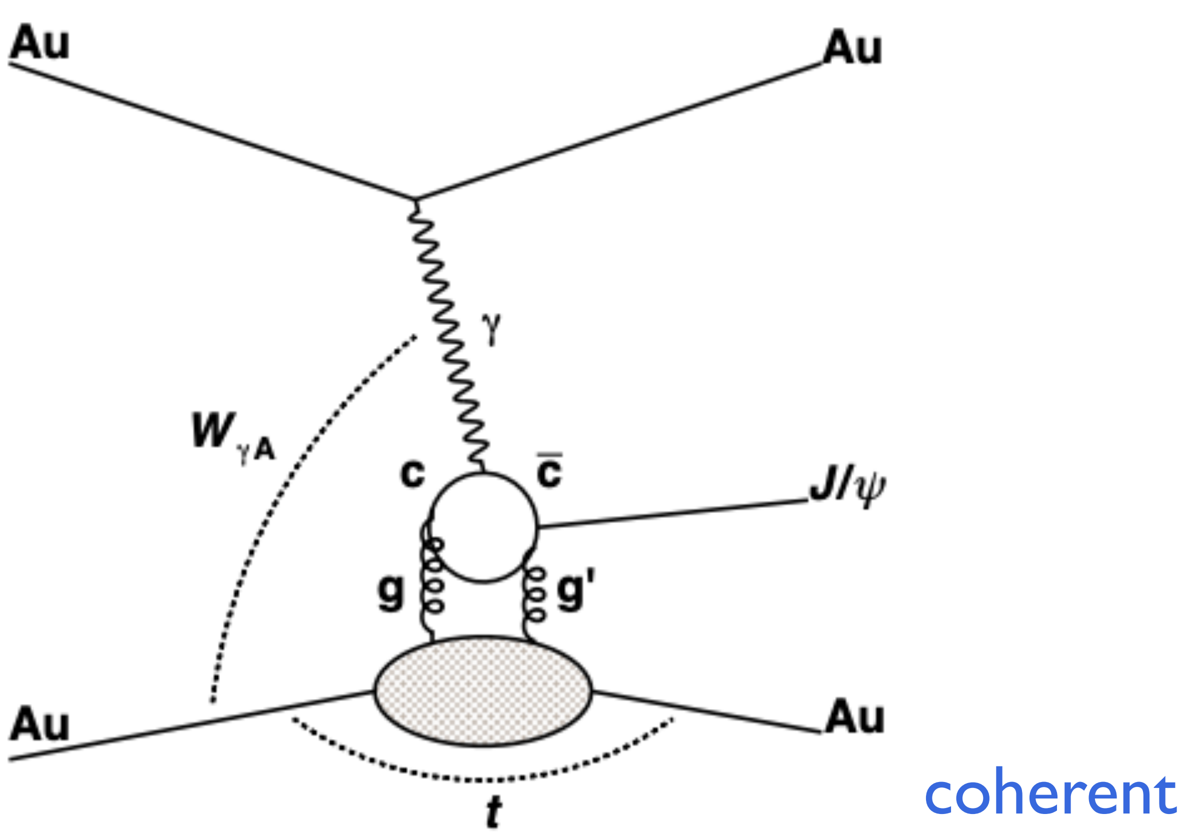
H1 dijet data may actually be in the ERBL region ($\xi > x$)

UPC data from RHIC and LHC and especially EIC data can shed further light on these issues, in order to check whether a common GTMD description is possible

UPCs: photonuclear production

UPCs at RHIC and LHC can be compared to photoproduction DIS data from HERA (proton) and EIC (proton & nuclei)





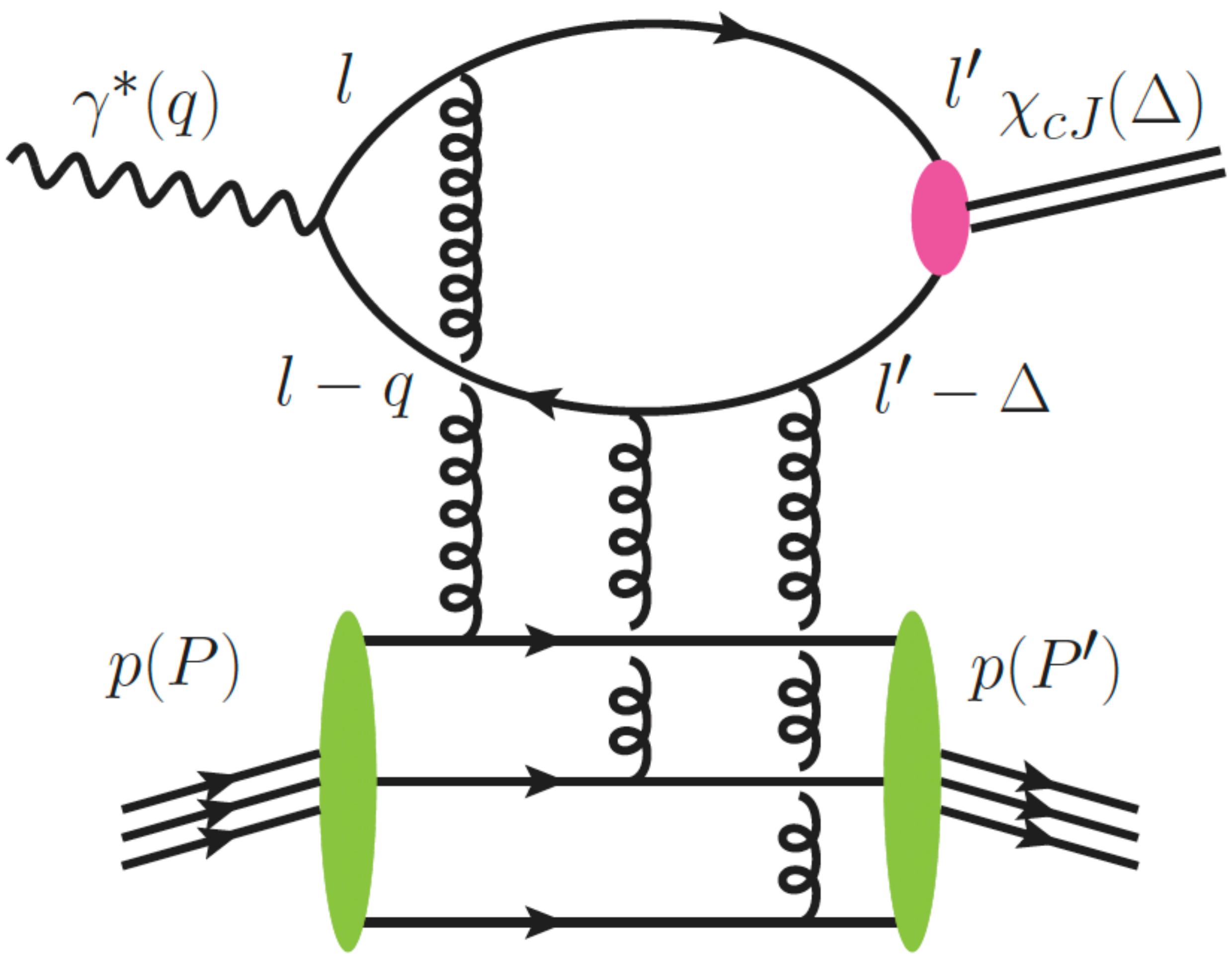
Figures taken from a presentation by Jaroslav Adam

Exclusive diffractive processes can be used to study diffractive PFDs, GPDs and GTMDs at EIC in photo- *and* electro-production

Exclusive χ_c production at EIC

This process probes the odderon:

$$\mathcal{O}(\boldsymbol{x}, \boldsymbol{y}) \equiv \frac{1}{2iN_c} \mathrm{Tr}\left(U^{[\square]} - U^{[\square]\dagger}\right)$$

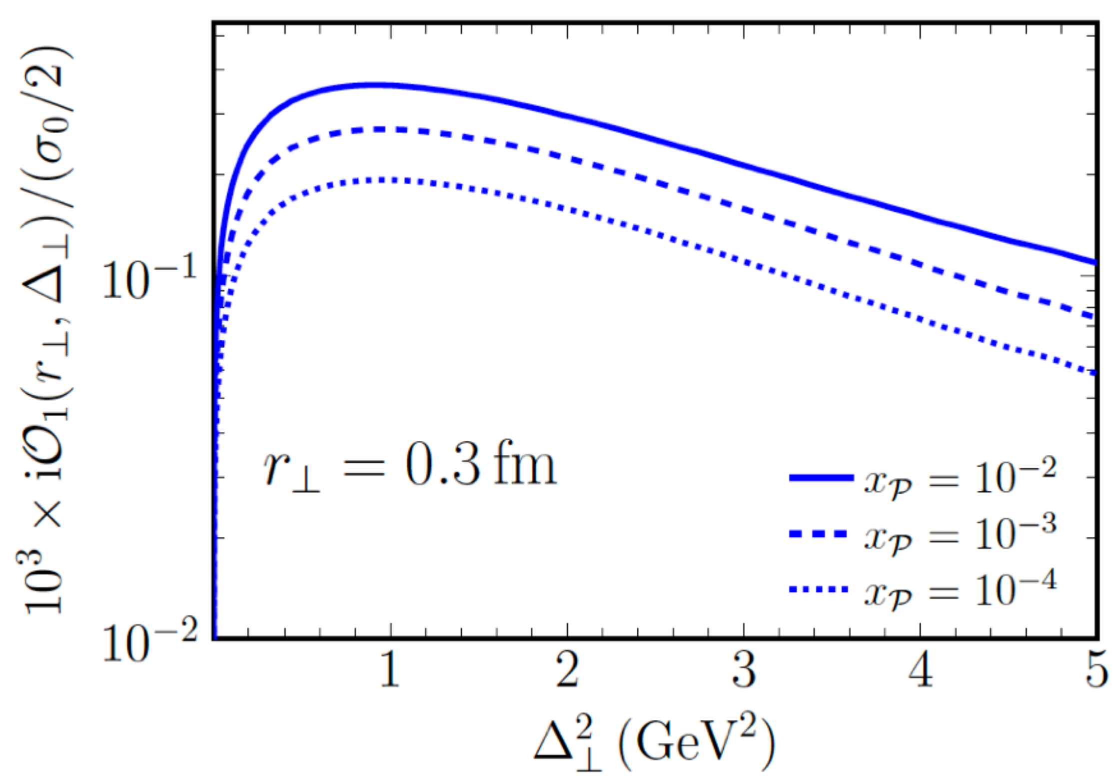


C-even final state requires
C-odd t-channel exchange

Constructive interference
with photon t-channel exchange
for $|t| \sim 1$ GeV^2

Benić, Dumitru, Kaushik, Motyka, Stebel, 2024

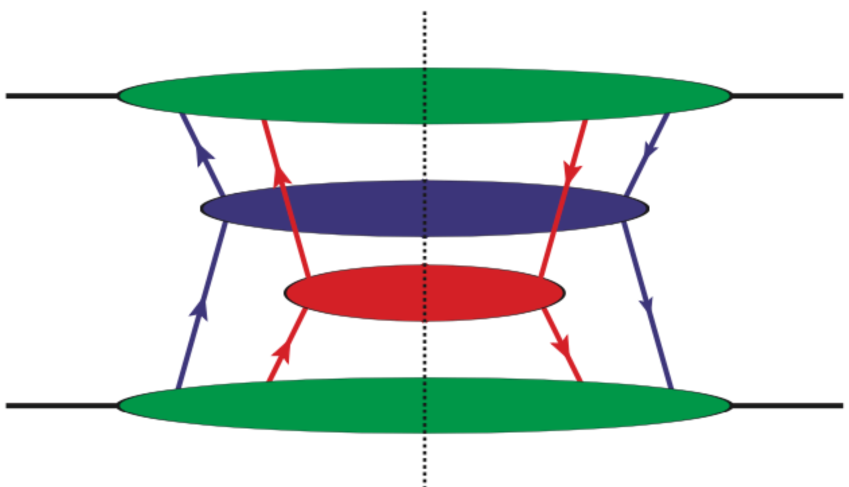
First azimuthal harmonic
found to be rather small

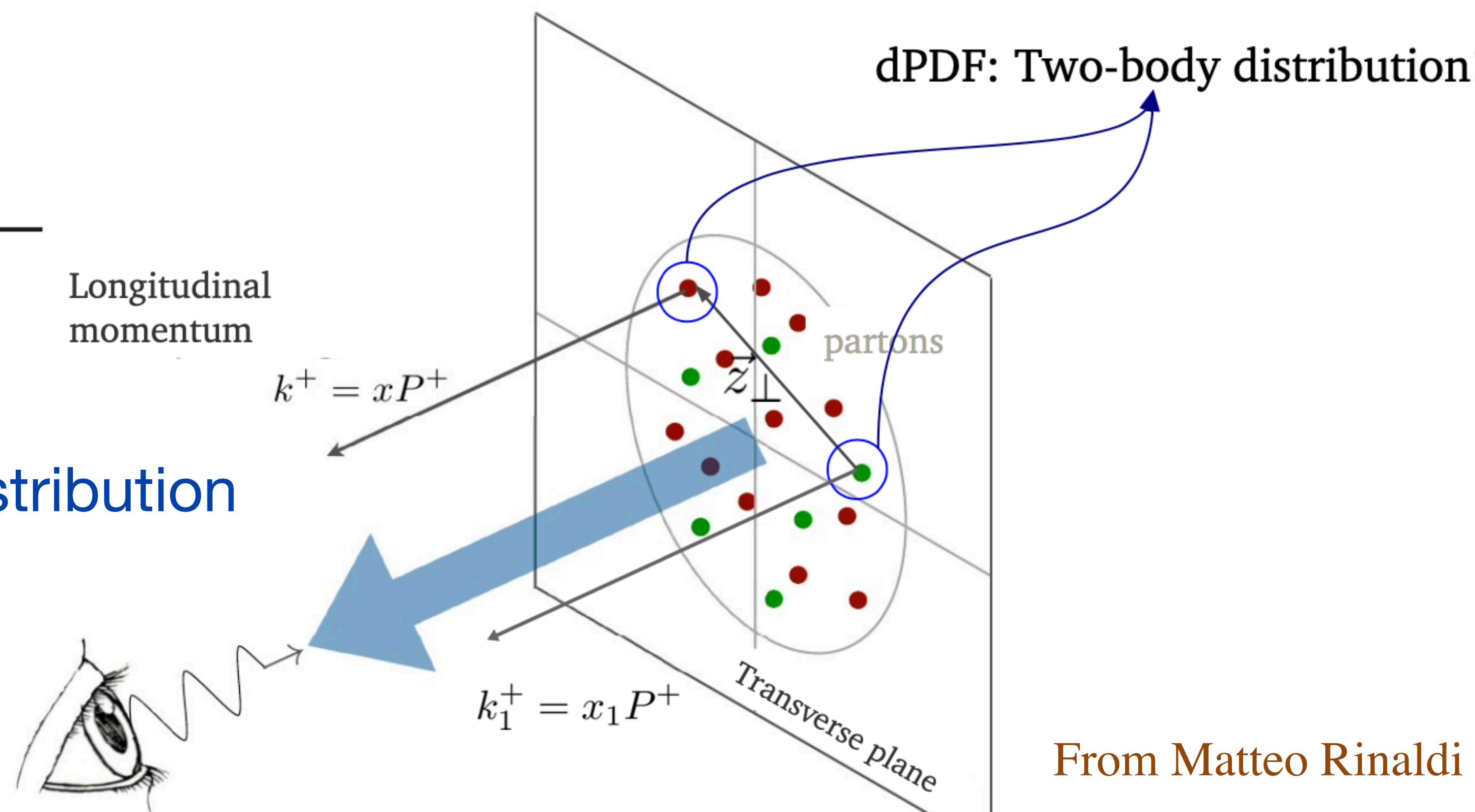


Gluon DPDs

Double Parton Scattering

DPS: 2 hard scatterings off partons in the same hadron simultaneously





DPD = Double Parton Distribution

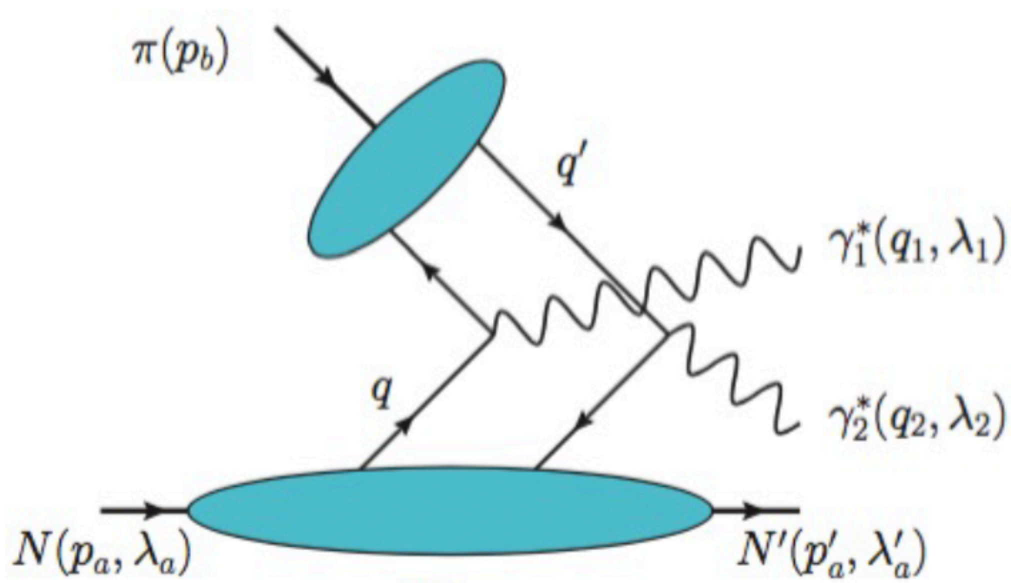
$$F(x, x'; z_\perp)$$

From Matteo Rinaldi at MPI@LHC 2015

DPDs capture the spin, color & flavor correlations between partons

GTMDs from exclusive DPS

If the hadron stays intact, then there is a connection to GTMDs:



DPD → GTMD²

Exclusive double Drell-Yan process probes quark GTMDs

Bhattacharya, Metz, Zhou, 2017
Echevarria, Gutierrez Garcia, Scimemi, 2022

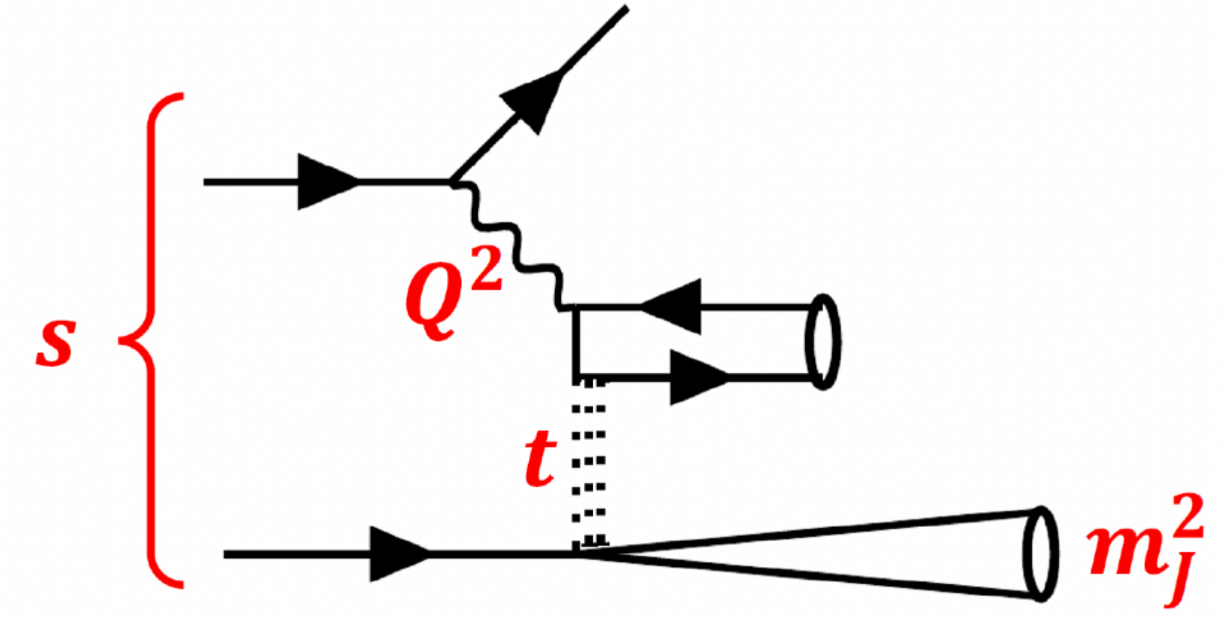
Likewise, exclusive double production of pseudoscalar quarkonia (η_c or η_b) in nucleon-nucleon collisions probes gluon GTMDs

Bhattacharya, Metz, Kumar Ojha, Tsai, Zhou, 2018

Exclusive coherent diffractive processes at EIC: σ ~ 2 DP gluon GTMDs
Exclusive double pseudoscalar quarkonia production at LHC: 4 WW gluon GTMDs

Diffractive distributions

Diffractive PDFs



Factorization from EFT

$$F_j^D(x, Q^2, \beta, t, m_J^2) = \mathcal{P}_j^{\mu\nu}\, S_{\mu\nu}(Q^2, \beta, t, \tau_{i\perp}) \otimes_\perp B(t, m_J^2, \tau_{i\perp})$$

Collins' hard scattering approach

$$F_{2/L}^D(x, Q^2, \beta, t) = \sum_i \int_\beta^1 \frac{d\zeta}{\zeta}\, H_{2/L}^{(i)}\left(\frac{\beta}{\zeta}, Q^2\right) f_i^D\left(\zeta, Q^2, \frac{x}{\beta}, t\right)$$

"Diffractive PDF" (dPDF)

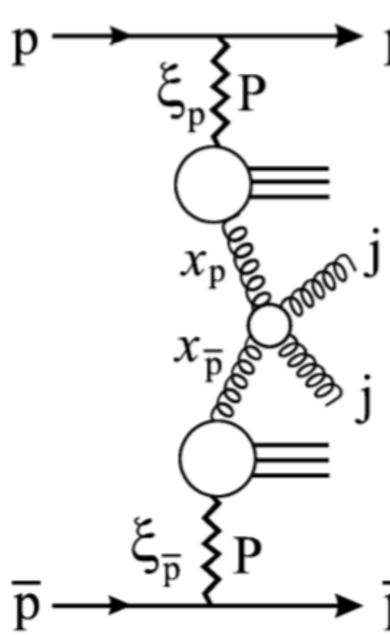
- Imposes *only* $\lambda_t = \sqrt{-t}/Q \ll 1$
- EFT also agrees with this for λ_t and $\lambda \ll 1$ $\quad \lambda = Q/\sqrt{s}$

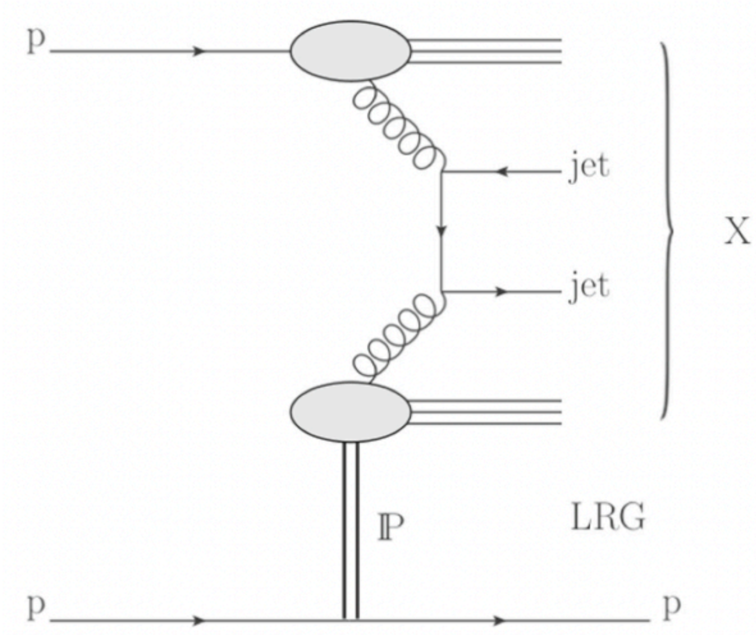
Berera/Soper, hep-ph/9509239. Collins, hep-ph/9709499. Frankfurt et al., 2203.12289.

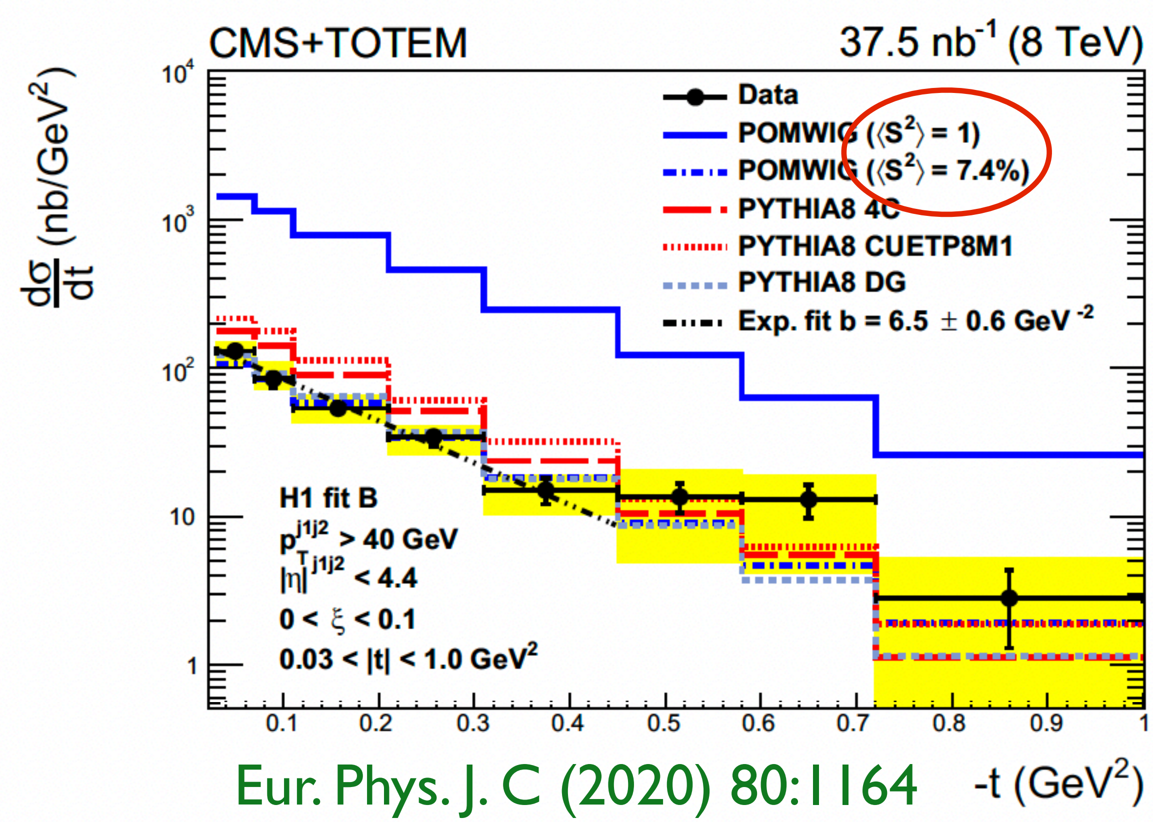
Lee, Schindler & Stewart, 2508.10231; Slide by Stella Schindler shown at QCD evolution 2024

Diffractive PDFs & Factorization Breaking

Diffractive dijet production indicates **non-factorization** in $p\bar{p}$ and pp collisions [$Sp\bar{p}S$, Tevatron, LHC] compared to ep [HERA]





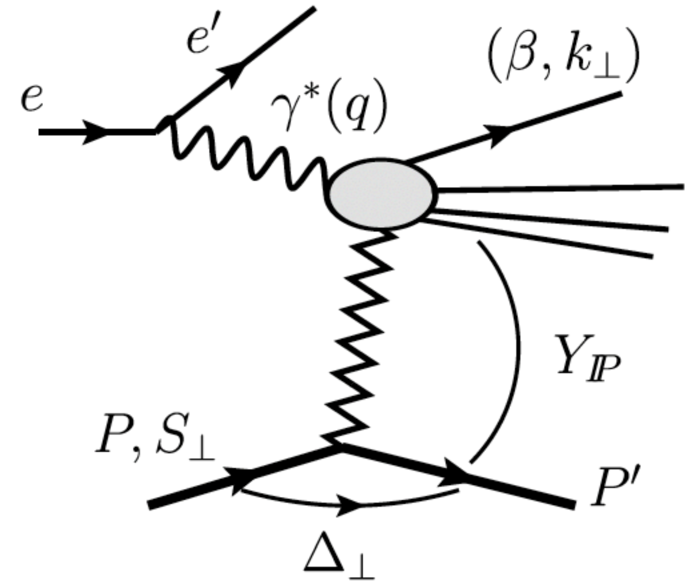


Eur. Phys. J. C (2020) 80:1164

Inclusive dijet observables in pp that probe TMDs (transverse momentum dependent PDFs) are also expected to be **non-factorizing**

Understanding of the origin and magnitude of the **non-factorization** is needed for **global analyses of multi-dimensional PDFs** and for that both ep and pp data are needed

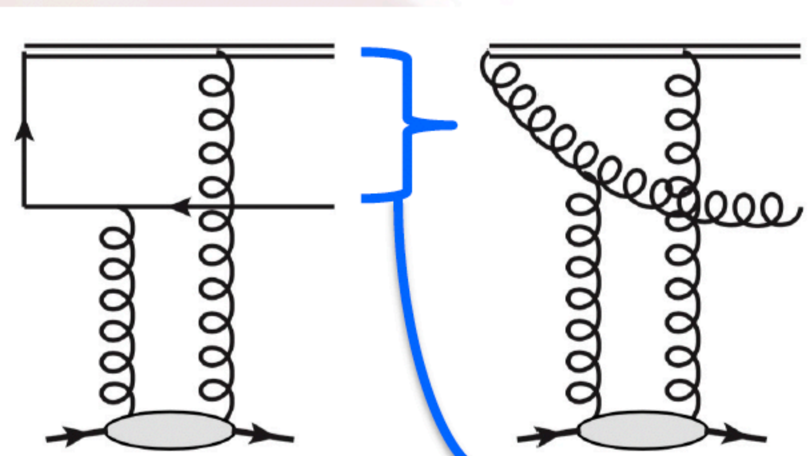
TMD Diffractive PDFs



Semi-inclusive coherent diffractive scattering

Compute the diffractive PDFs at small-x

- Definition is similar to TMDs for inclusive processes
- Requires large rapidity gap/color-singlet exchange



$$x\frac{d f_q^D(\beta,k_\perp;x_{I\!P})}{dY_{I\!P}\,dt\,d\phi_\Delta} = \int d^2k_{1\perp}\, d^2k_{2\perp}\, \mathcal{F}_{x_{I\!P}}(k_{1\perp},\Delta_\perp) \times \mathcal{F}_{x_{I\!P}}(k_{2\perp},\Delta_\perp)\frac{N_c\beta}{(2\pi)^2}\mathcal{T}_q(k_\perp,k_{1\perp},k_{2\perp})$$

5/27/24 5

Slide by Feng Yuan

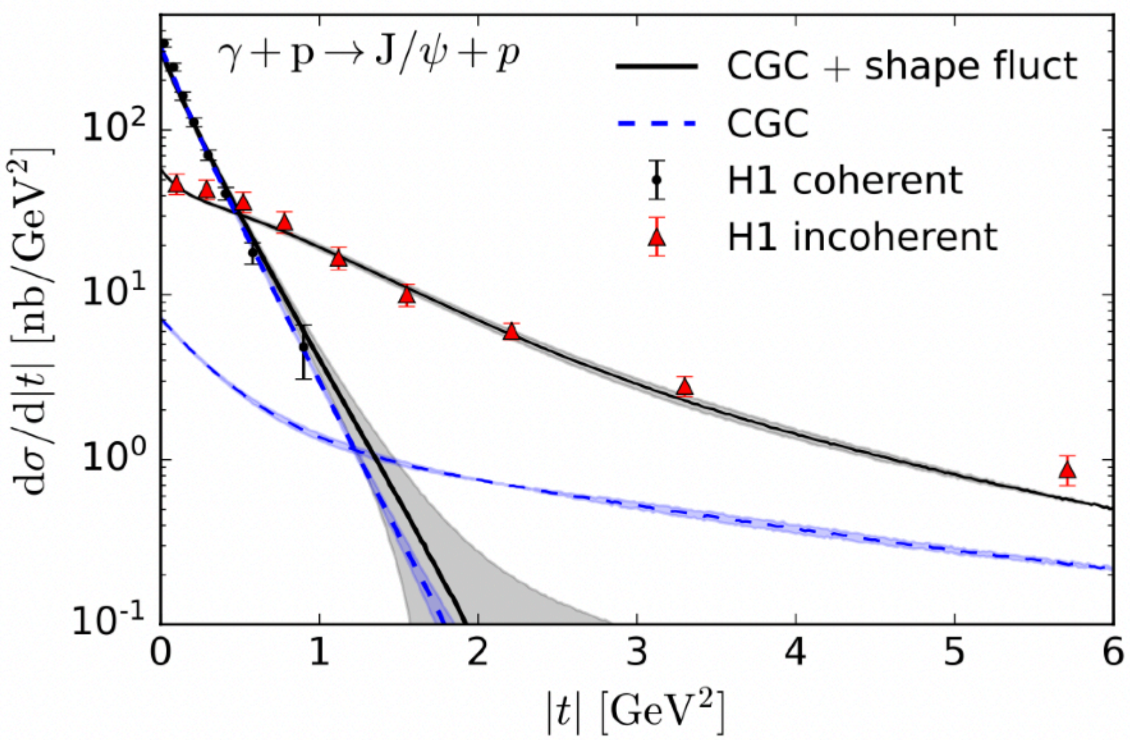
There is no linear polarization analogue!

Iancu, Mueller, Triantafyllopoulos, 2021; Hatta, Xiao, Yuan, 2022; Hatta, Yuan, 2024

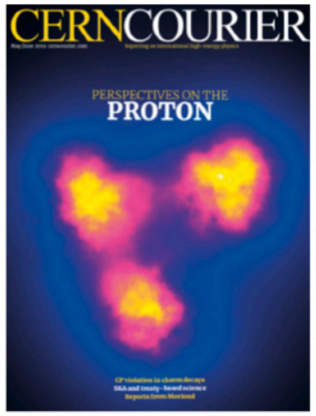
Incoherent diffractive J/ψ production

Large geometry fluctuations required by the HERA data ($x_{\mathbb{P}} \approx 0.001$)

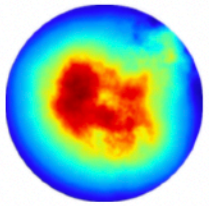
Study simultaneously coherent ($\sim$ average interaction) and incoherent ($\mathcal{A}$ variance)



CGC + shape fluct



CGC



- Parametrize e-b-e fluctuating geometry, fit parameters to data
- Incoherent σ: substructure geometry or color charge fluctuations depending on $|t|$

Orignal: H.M, B. Schenke, 1607.01711 (PRL), recent: 2202.01998 (HM, Schenke, Shen, Zhao), similar setup e.g.: Bendova, Cepila, Contreras; Cepila, Contreras, Krelina, Takaki; Traini, Blaizot; Kumar, Toll; Demirci, Lappi, Schlichting

Incoherent production is sensitive to the shape fluctuations in the proton

Hot spots mean that gluons are clustered (generated by the constituent quarks?)

Conclusions

Conclusions

- Lots of synergies between experiments at the LHC (HL, FT, possibly LHeC) and at EIC, consisting of different measurements of related quantities (including WW vs DP), similar/overlapping measurements, supporting measurements (providing baselines), as HERA versus Tevatron & LHC data comparisons already have demonstrated

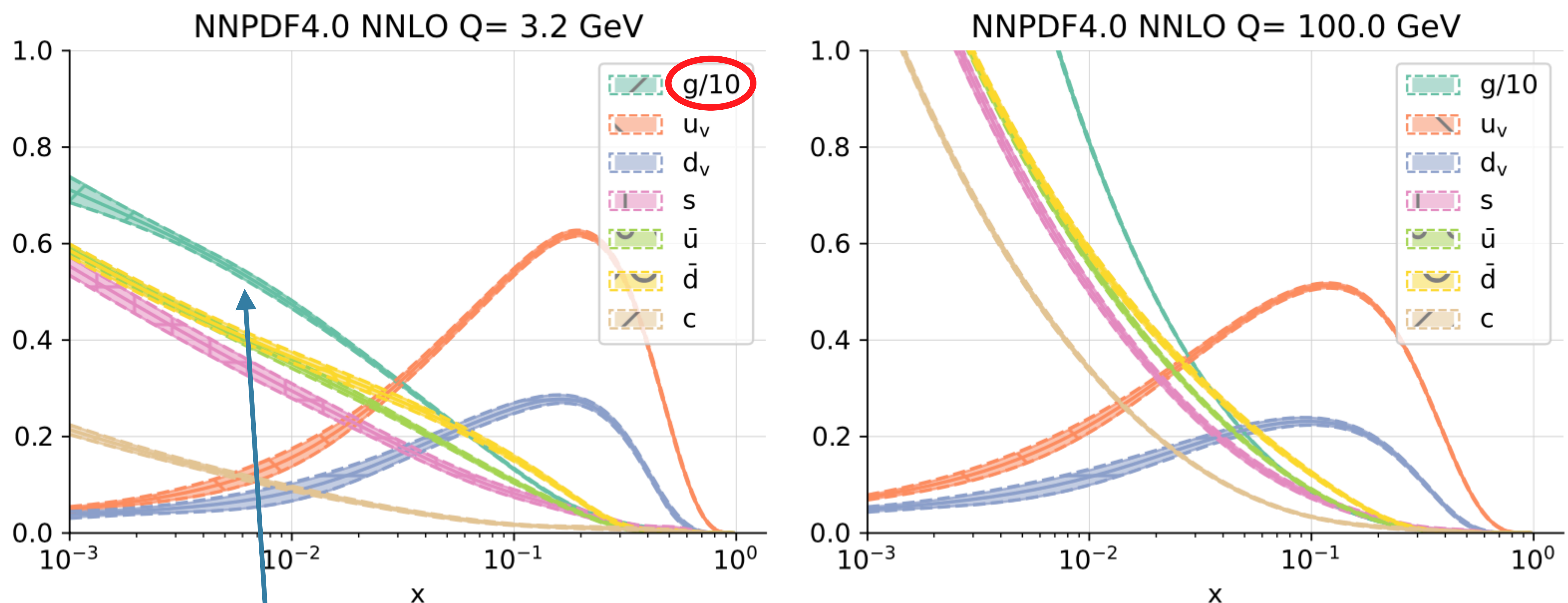
- Many QCD studies possible, involving lots of different distributions that are often linked directly or indirectly, but a complete picture is lacking still, especially regarding process dependence & factorization (breaking)

- There are many challenges for the theoretical descriptions of the various processes, concerning nuclear and saturation effects, the quarkonium production process, (non-)factorization, gauge links, evolution, MPI, etc

All this forms a rich, decades long program

Back-up slides

Parton densities (PDFs) of the proton at present



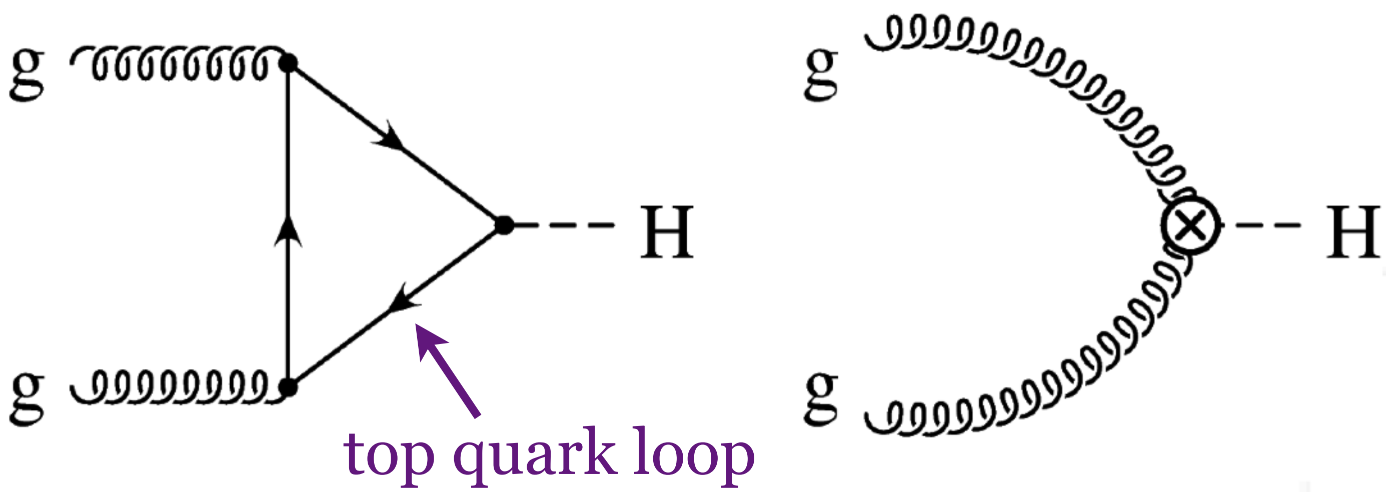
Fraction of overall proton momentum carried by parton

small momentum partons
(x<0.1) are mostly gluons

This picture is Q^2 dependent
but stays qualitatively the same

Large x gluons

The parton distributions at large x are required for specific BSM physics studies

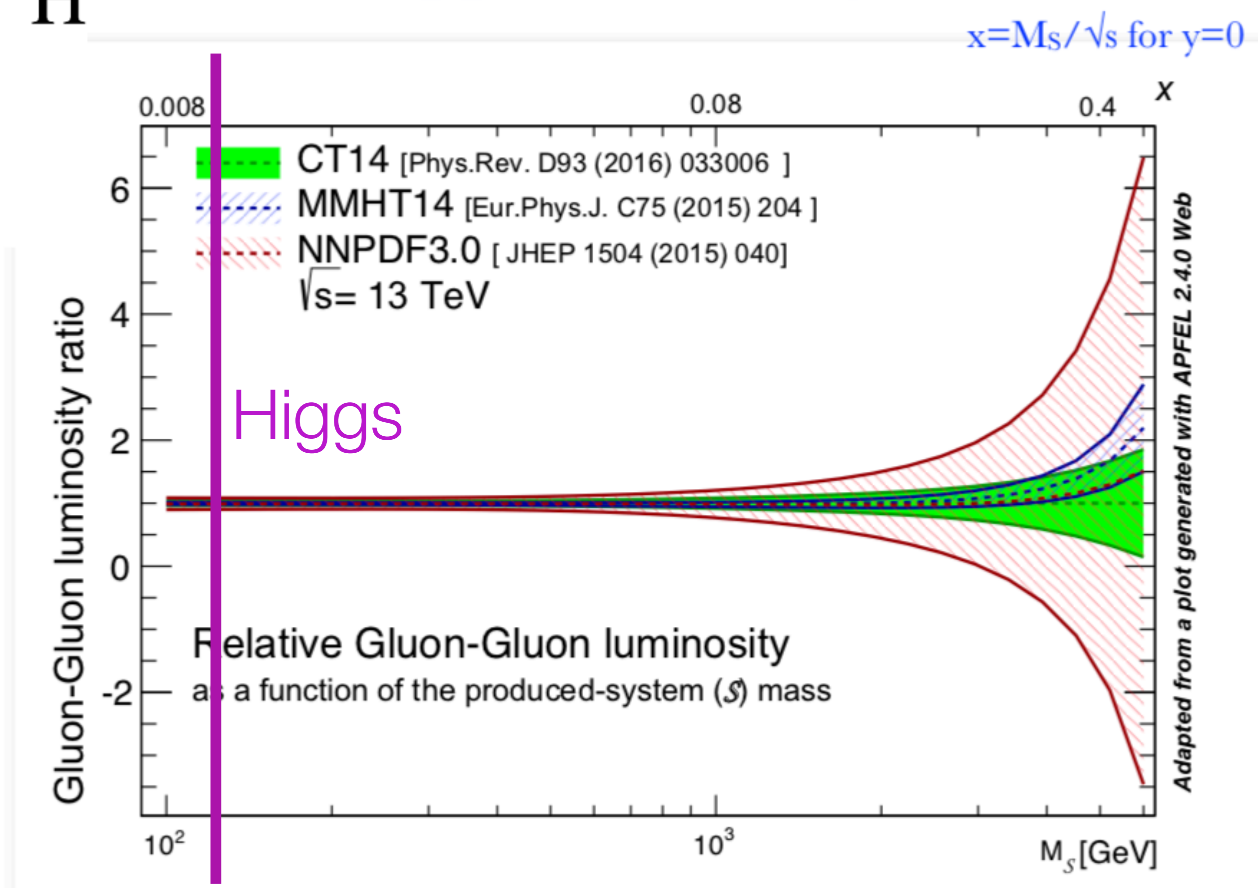


Gluons in Higgs production at LHC have x~0.01 (in well measured range)

Heavier states are produced from larger x partons

Will be improved by EIC, FT@LHC & HL-LHC experiments

gg luminosity

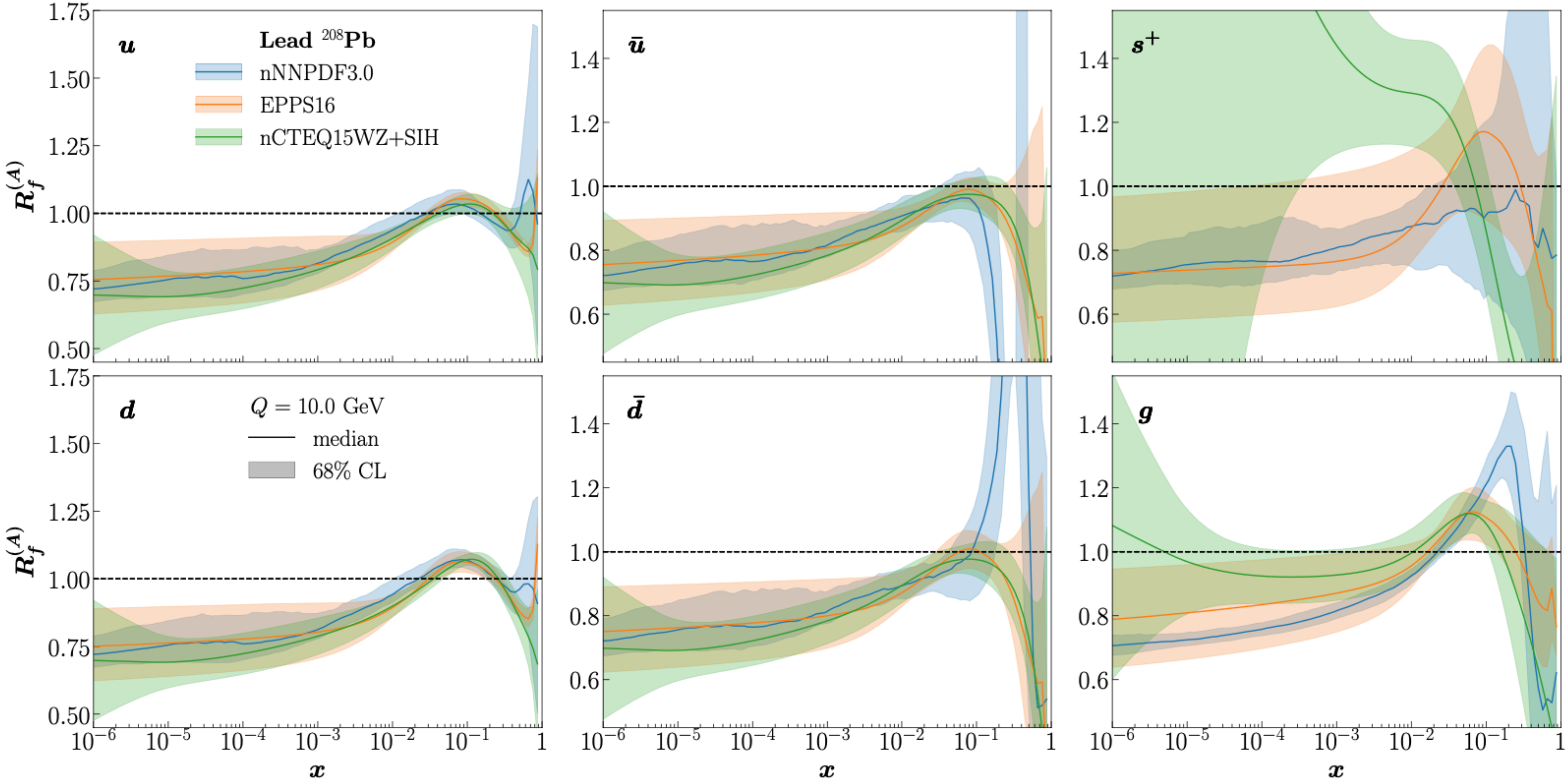


$$\frac{\partial \mathcal{L}_{ab}}{\partial \tau} = \frac{1}{s} \int_{\tau}^{1} \frac{dx}{x} f_a(x, M_S^2) f_b(\tau/x, M_S^2), \qquad \tau = M_S^2/s$$

From M. Echevarria, DIS2019 & 1807.00603

Nuclear modification factor

$$R_f^{(A)}(x,Q) \equiv \frac{f^{(N/A)}(x,Q)}{\frac{Z}{A}f^{(p)}(x,Q) + \frac{(A-Z)}{A}f^{(n)}(x,Q)}$$



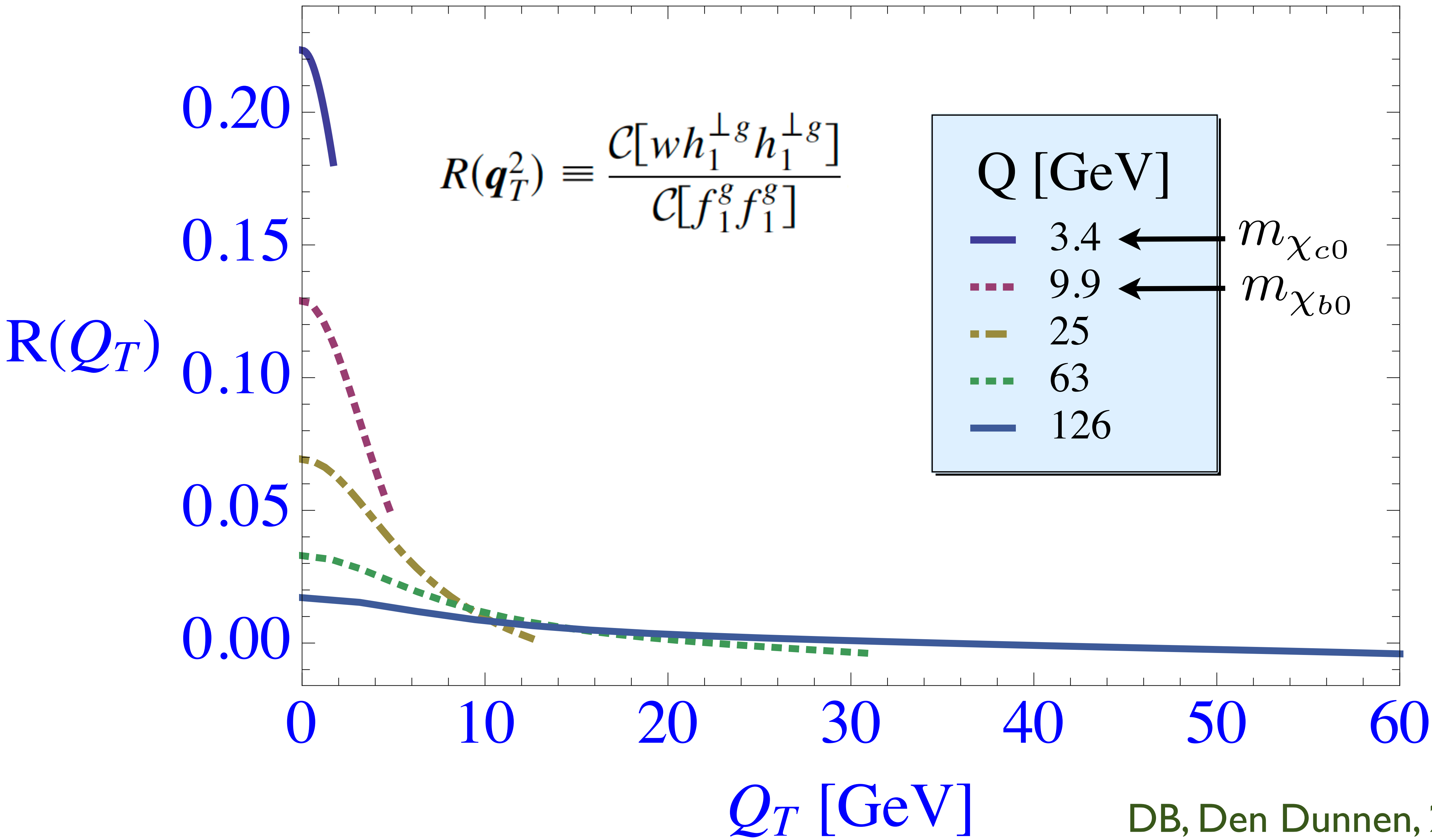
nNNPDF3.0, 2022

Comparison of NNPDF to parametrization fits shows large differences

Effect of TMD evolution

Comparing $pp \to H\,X$, where $H = \chi_{c0}$, χ_{b0} or Higgs allows to test TMD evolution

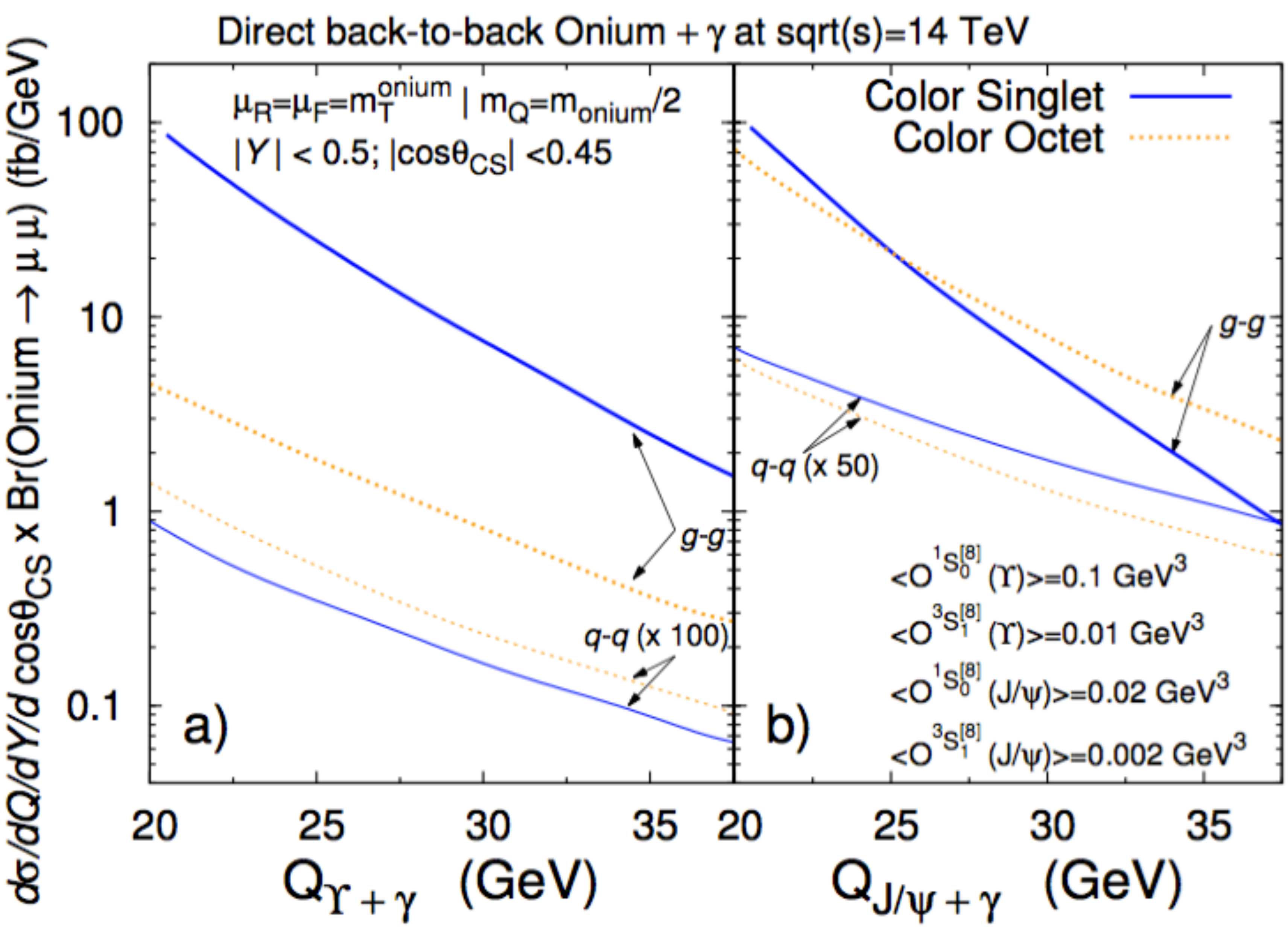
The relative contribution from linearly polarized gluons w.r.t. unpolarized gluons decreases with increasing mass of the produced state (which sets the hard scale):



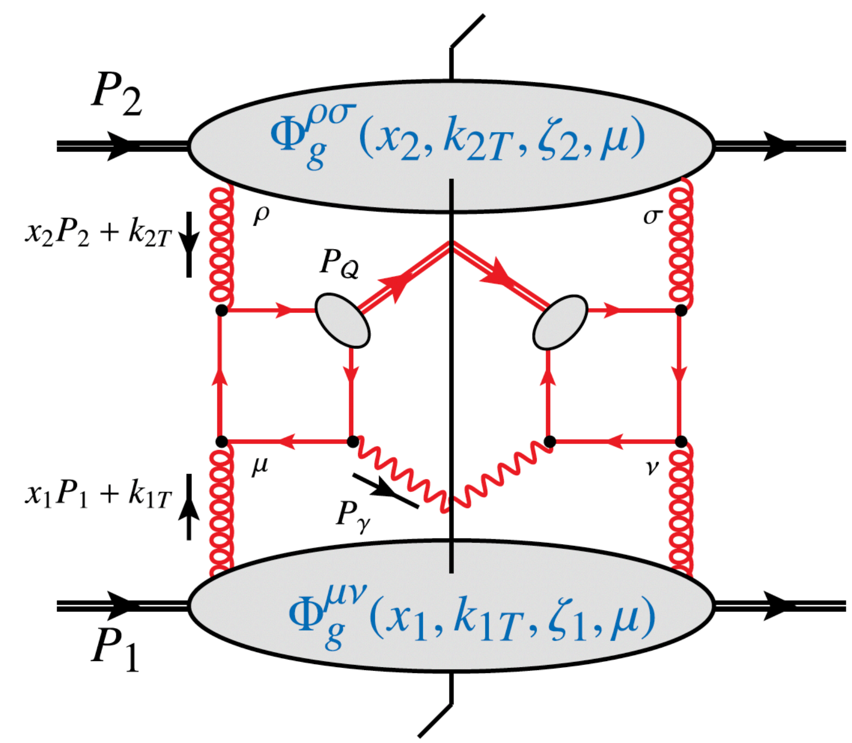
DB, Den Dunnen, 2014

Associated J/ψ production

$p\,p \to \mathcal{Q}\,\gamma\,X$ could be a good process to extract $f_1^g(x, p_T^2)$ at LHC

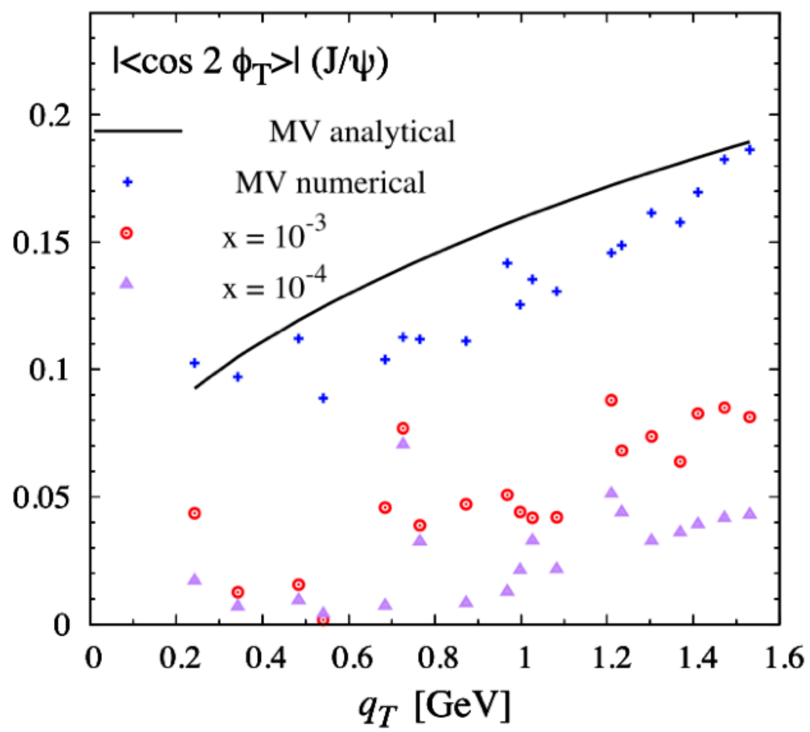


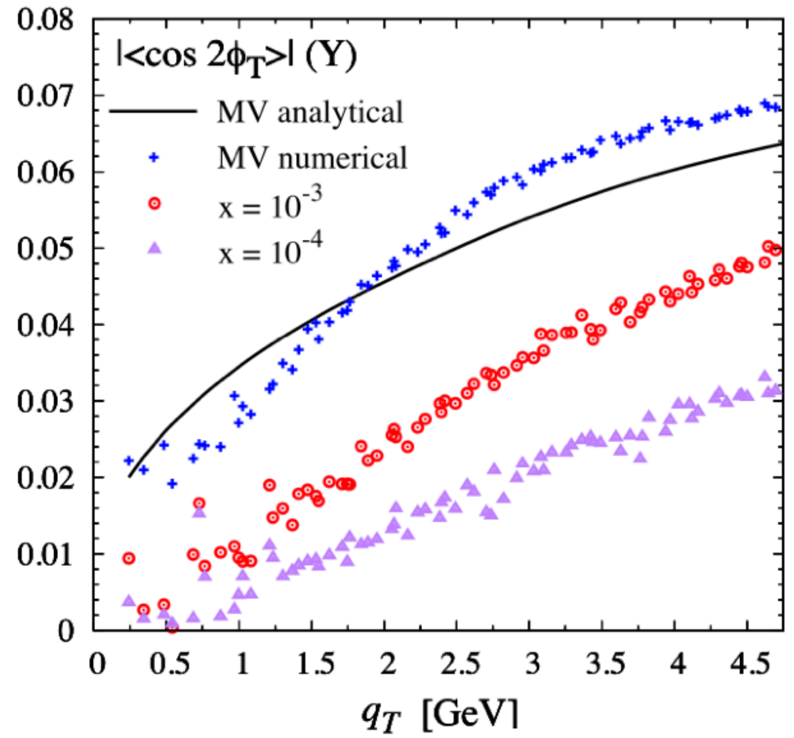
Den Dunnen, Lansberg, Pisano, Schlegel, 2014



The CS contribution dominates in Υ+γ production and for lower invariant mass of the pair also in J/ψ+γ production

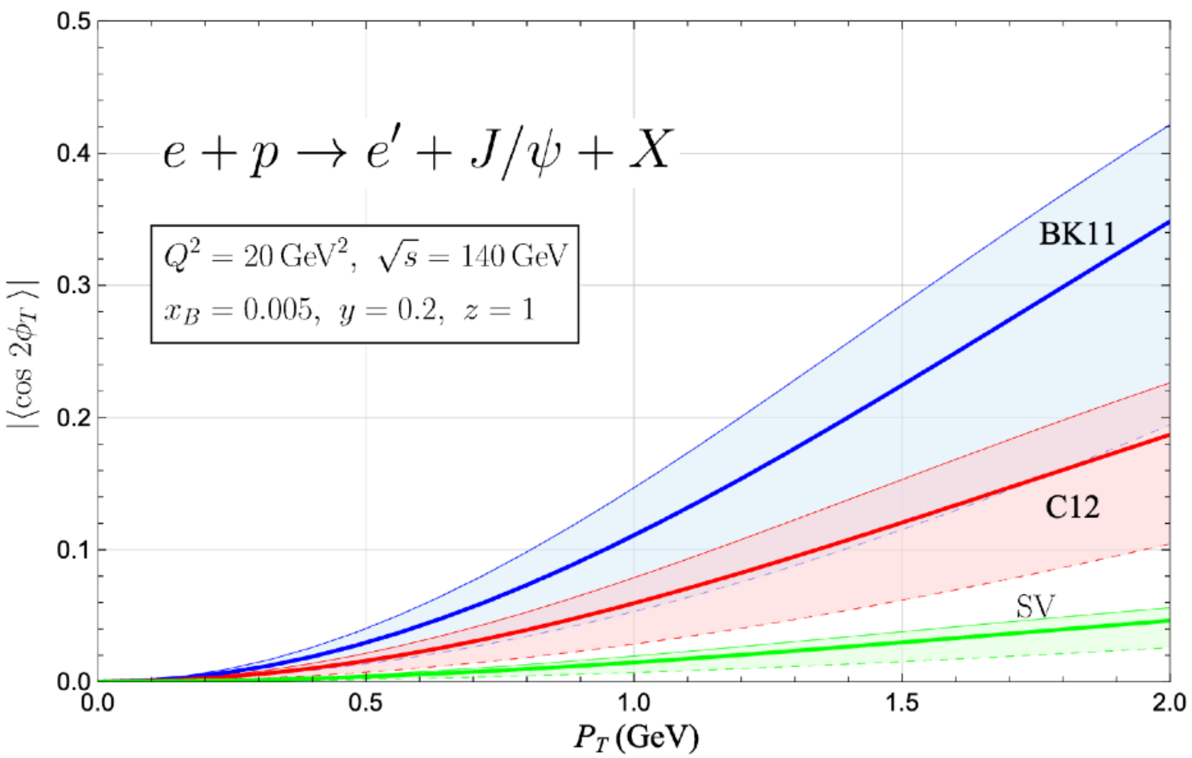
$\cos 2\phi_T$ asymmetry - predictions



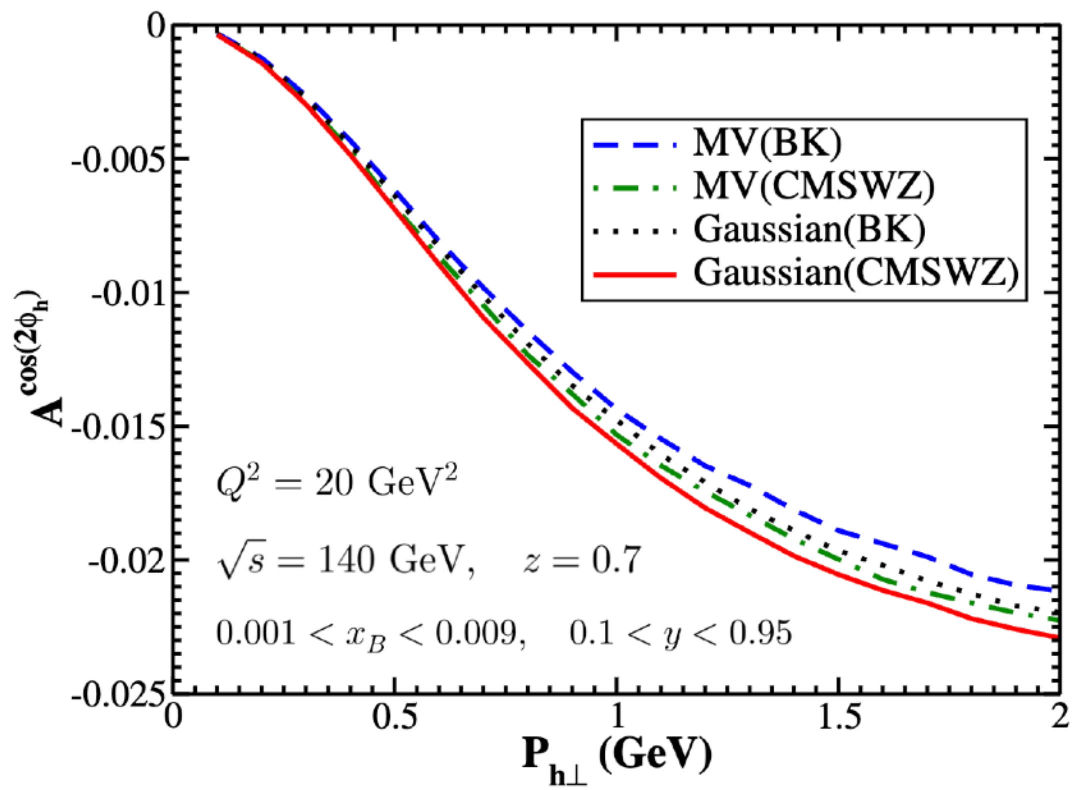


Asymmetries for $Q=M_Q, y=0.1$ in the small-x MV model, including nonlinear evolution

Bacchetta, DB, Pisano, Taels, 2018



Bor, DB, 2022

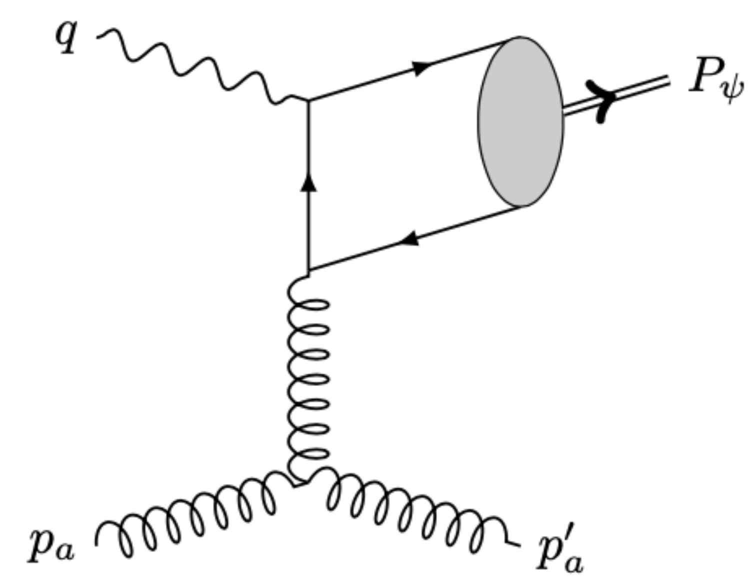


Kishore, Mukherjee, Pawar, Siddiqah, 2022

Despite the large uncertainties sizable $\cos 2\phi_T$ asymmetries are possible at EIC

Perturbative tail of the shape function

LO NRQCD: $\Delta^{[n]}(\boldsymbol{k}_T^2;\mu^2) = \langle 0|\mathcal{O}(n)\,|0\rangle\,\delta^2(\boldsymbol{k}_T)$



Taking into account non-analytic behavior of the hard scattering amplitude ($z \to 1$ and $q_T \to 0$ limits do not commute)

Perturbative tail:

$$\Delta^{[n]}(z,\boldsymbol{k}_T^2;\widetilde{Q}^2) = -\frac{\alpha_s}{2\pi^2 \boldsymbol{k}_T^2} C_A \left(1 + \log\frac{M_\psi^2}{M_\psi^2 + Q^2}\right) \langle \mathcal{O}[n]\rangle \delta(1-z)$$

$$\mu_H^2 \equiv \widetilde{Q}^2 = M_\psi^2 + Q^2$$

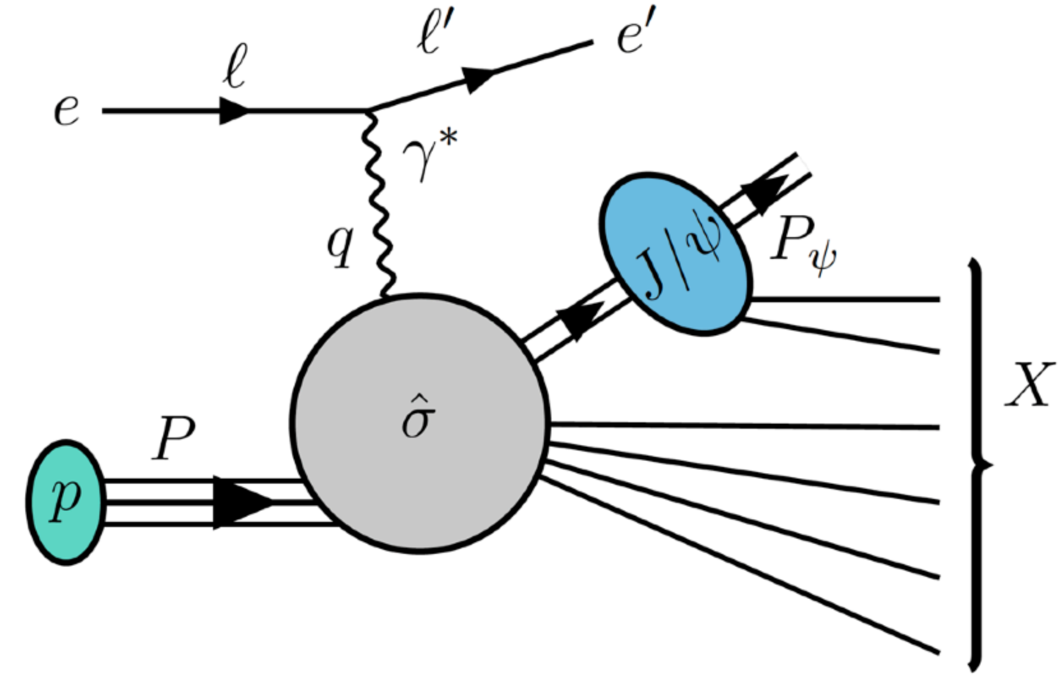
DB, Bor, Maxia, Pisano, Yuan, 2023

Even before taking the hard scale to be Q^2 dependent, the tail of the shape function is Q^2 dependent

Process dependence!

In agreement with open heavy quark pair expression by Zhu, Sun, Yuan, 2013

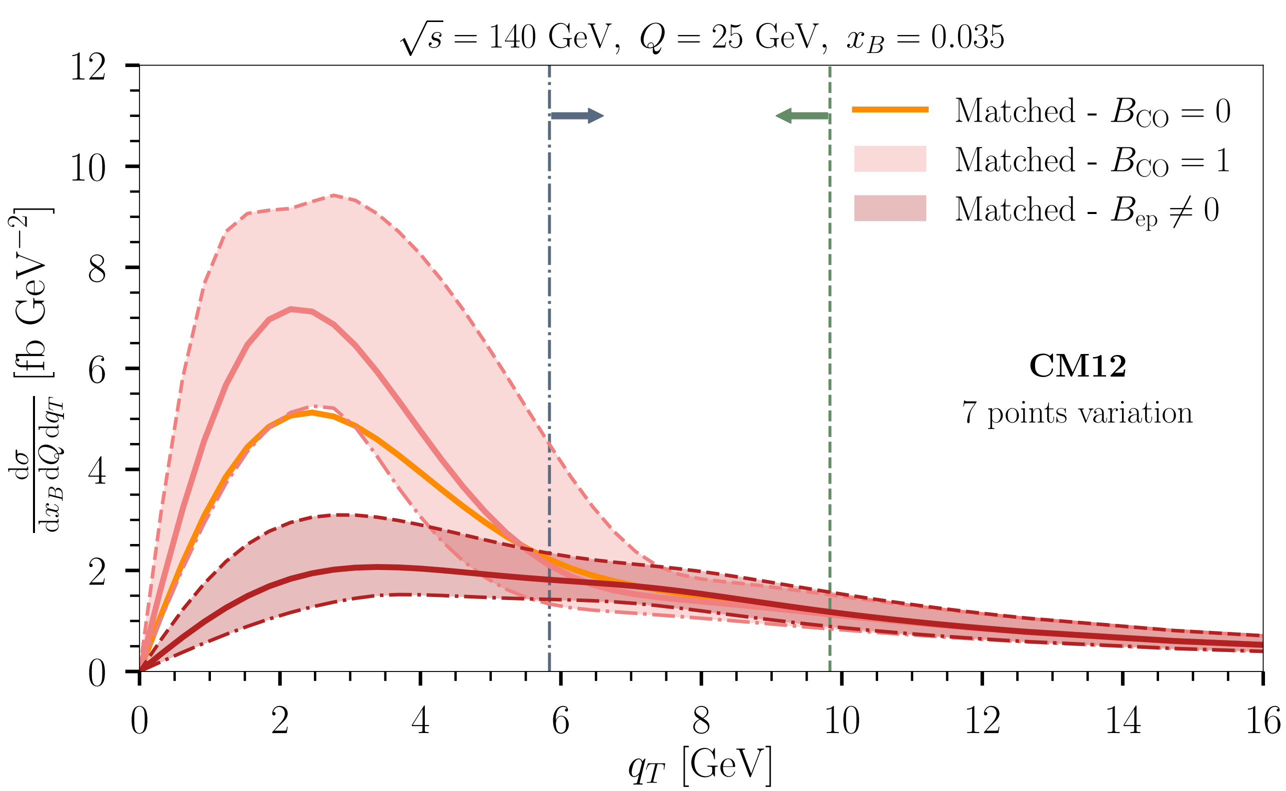
Matching

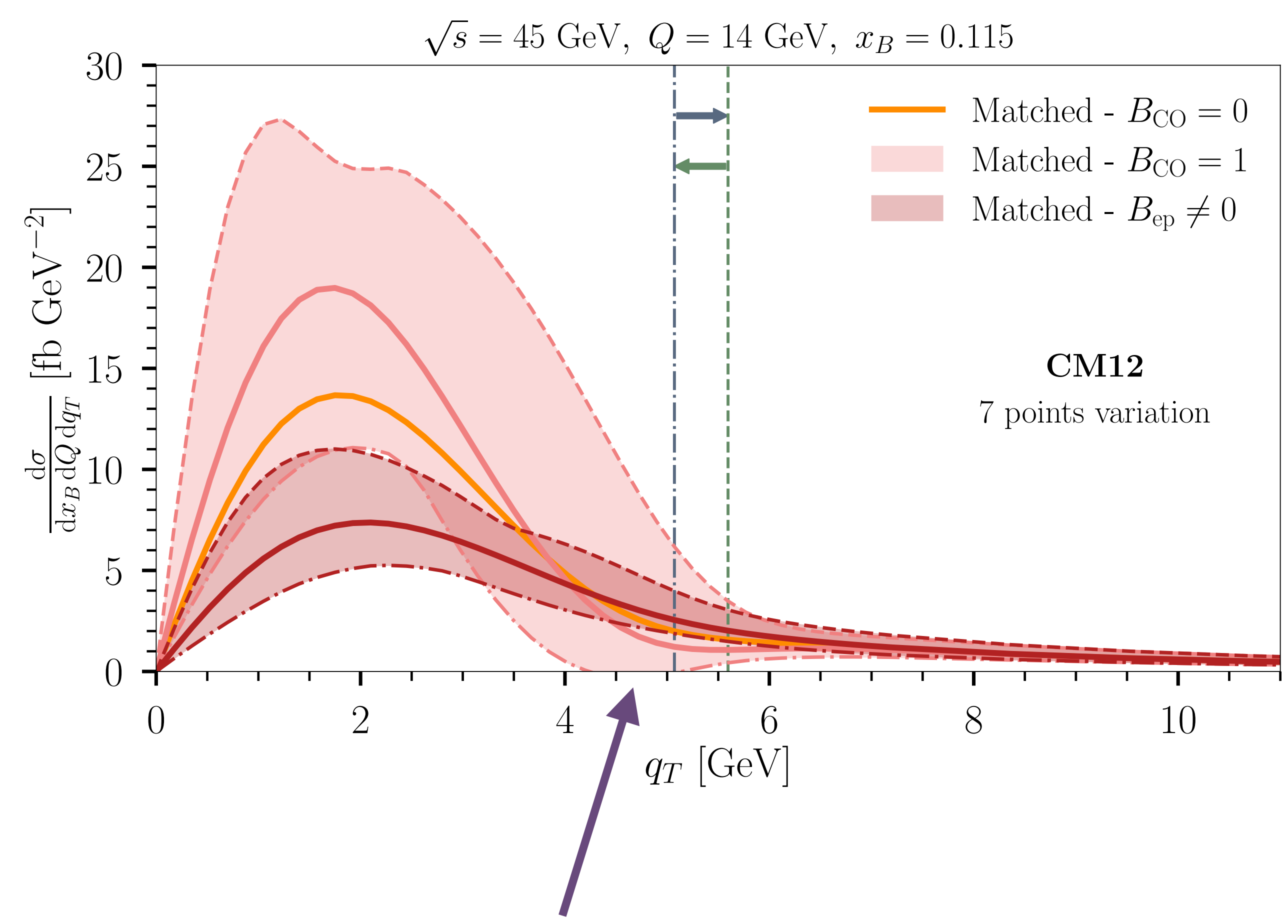


Uncertainty bands include LDME, scale & nonperturbative Sudakov uncertainties

$$S_{\text{pert}} = \int_{\mu_b^2}^{\mu^2} \frac{d\eta^2}{\eta^2} \left[A_g(\alpha_s(\eta)) \log \frac{\mu^2}{\eta^2} + B_g(\alpha_s(\eta)) \right] + \int_{\mu_b^2}^{\mu^2} \frac{d\eta^2}{\eta^2} B_{\text{CO}}(\alpha_s(\eta))$$

Maxia, DB, Bor, 2025

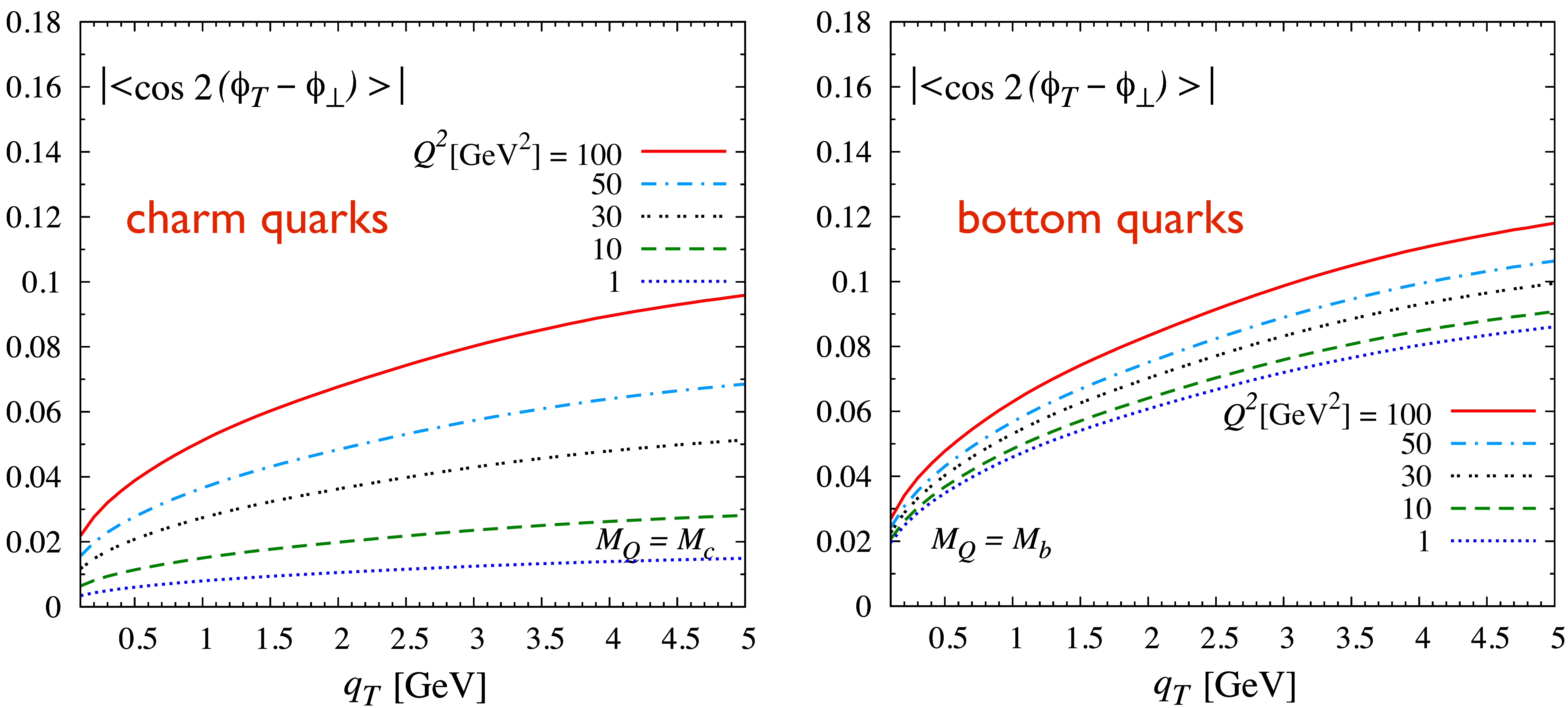




Process dependence matters

Negative cross sections can arise and asymmetries overshooting 1

Asymmetries in heavy quark pair production



small x
MV model

$|\boldsymbol{K}_\perp| = 10\,\mathrm{GeV}$

$z = 0.5$

$y = 0.3$

Up to 10% asymmetries at EIC

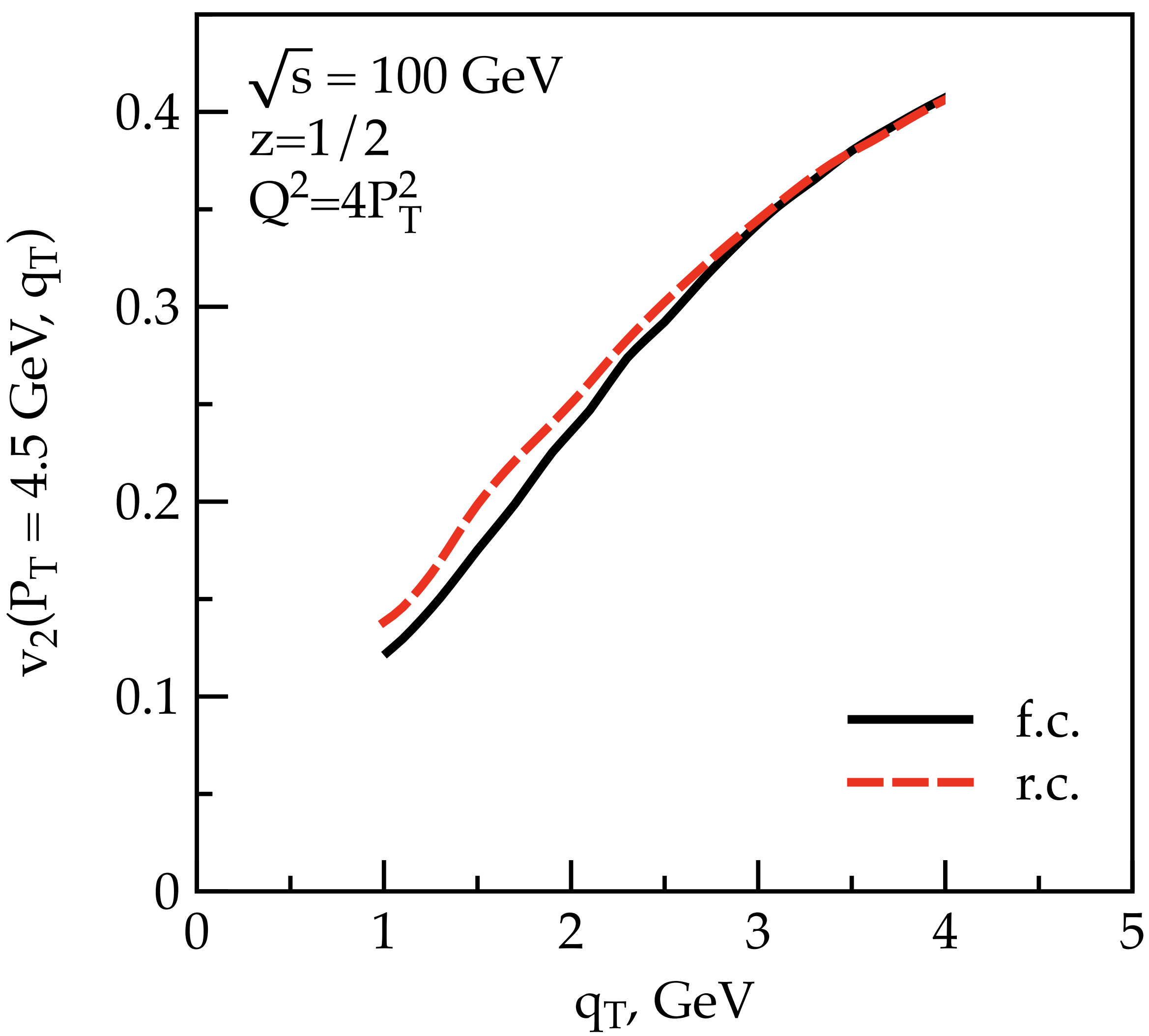
DB, Pisano, Mulders, Zhou, 2016

However, this does not include TMD or x-evolution yet

Inclusive dijet production at EIC

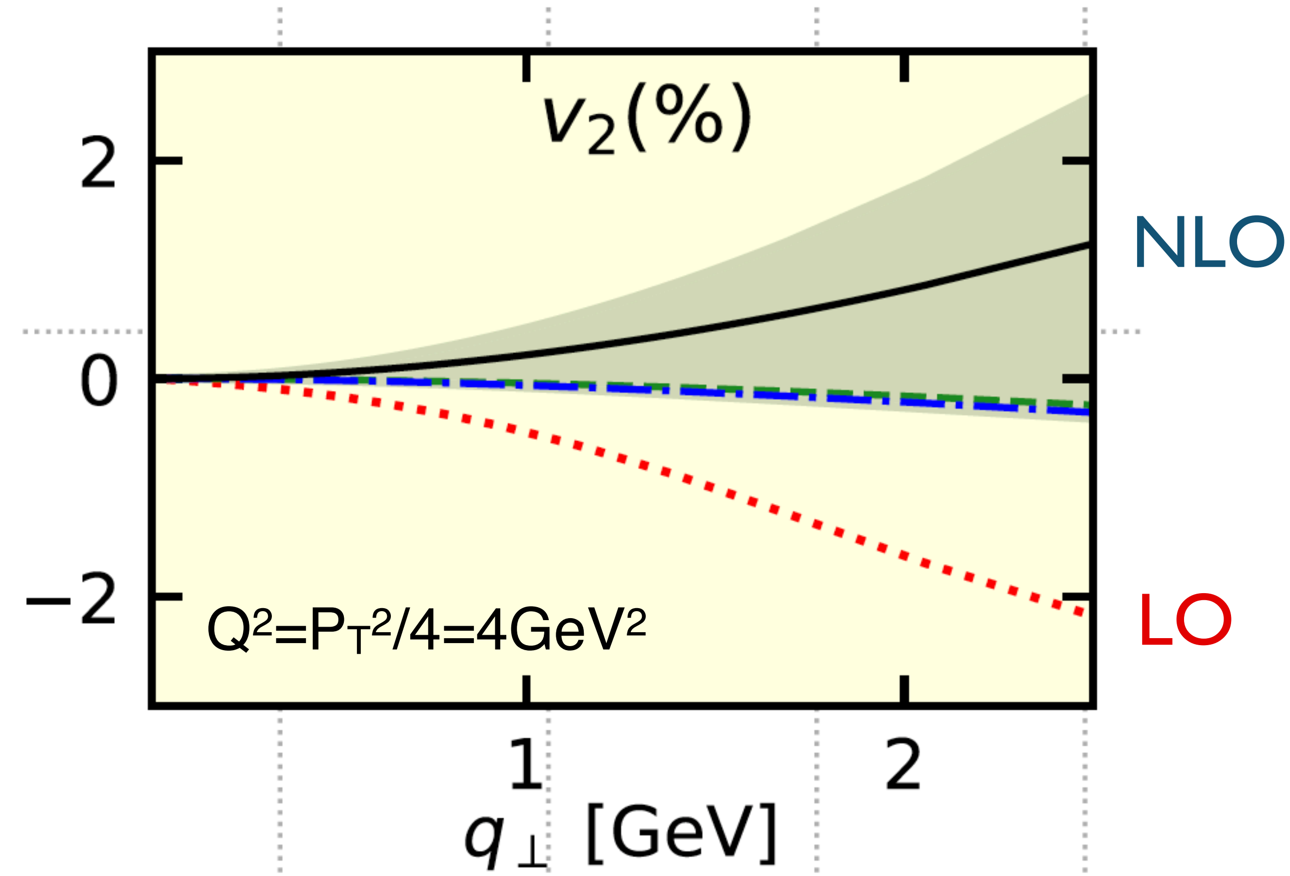
$\phi = \phi_T - \phi_\perp$

Linear gluon polarization shows itself through a $\cos 2\phi$ distribution ("v_2")



Large effects are found

Dumitru, Lappi, Skokov, 2015



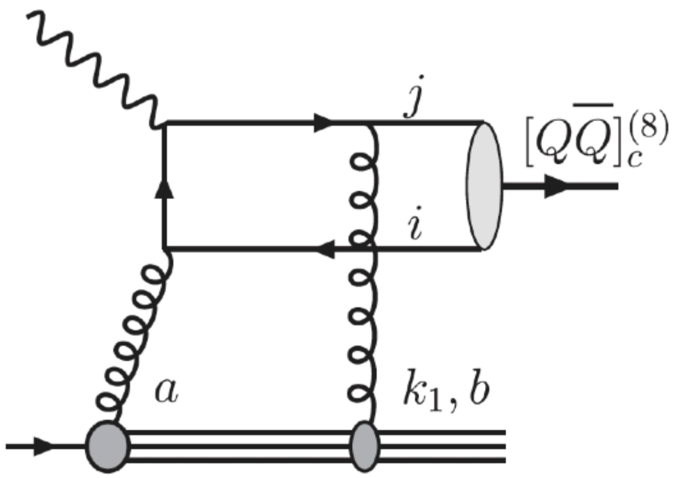
Effect of including NLO corrections

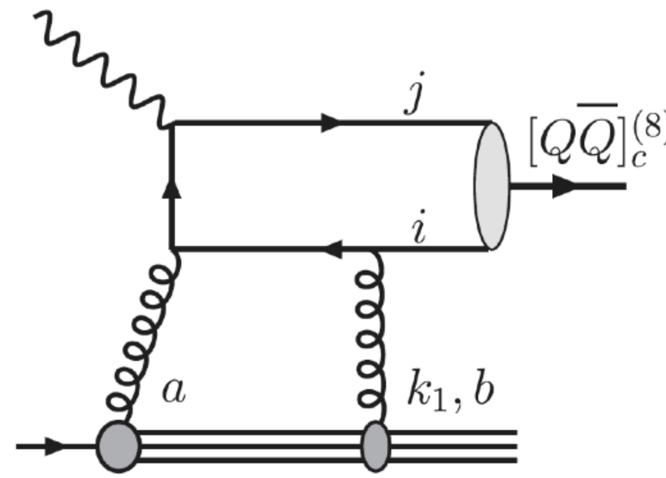
Caucal, Salazar, Schenke, Stebel, Venugopalan, 2024

Sign of v_2 is matter of definition and depends on sign of $h_1^\perp$, but it can apparently flip due to HO corrections; note that the probed (WW) distribution does not satisfy the same BK equation as f_1 (unlike the DP one)

Effects of ISI & FSI

SSA from gluon Sivers

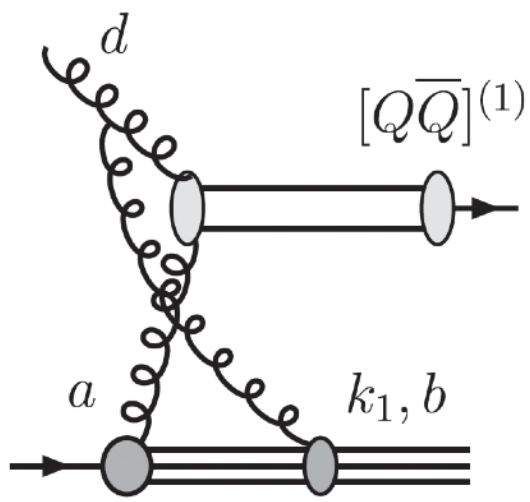




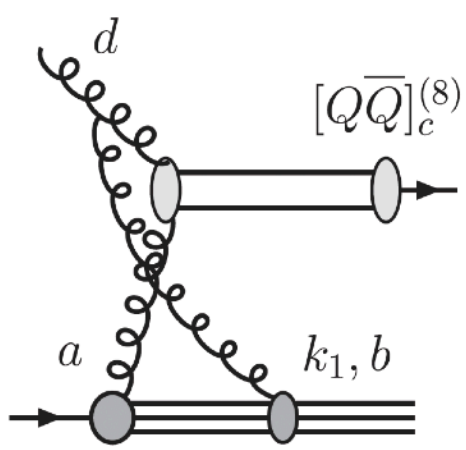
Nonzero SSA only for CO in ep

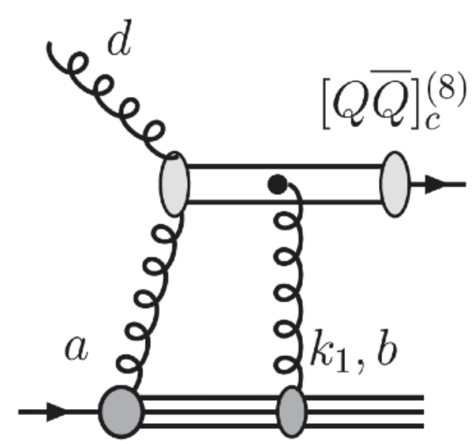
But not at twist-3 (large p_T)
cf. Shinsuke Yoshida's talk

F. Yuan, 2008



Nonzero SSA only for CS in pp



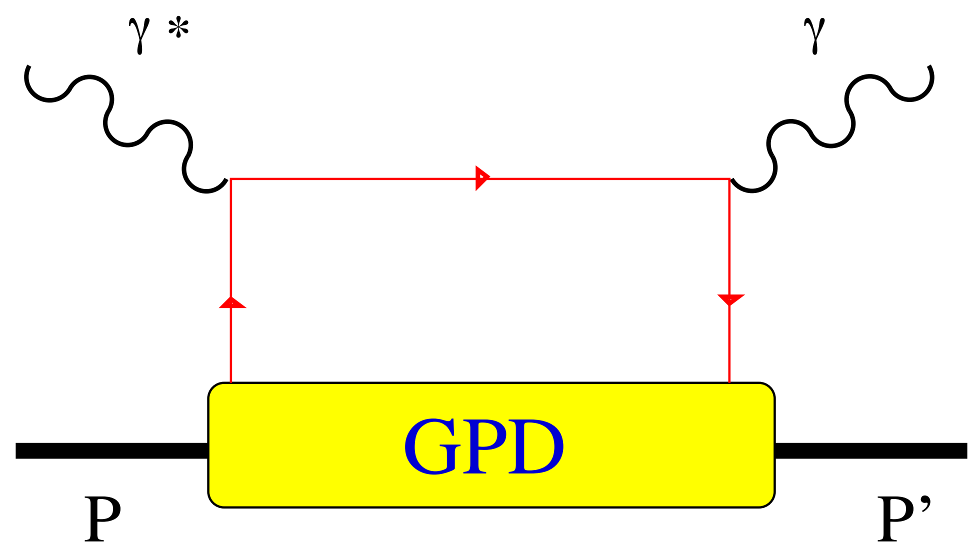


Zero SSA for CO in pp

Interesting to check to what extent this type of process dependence is correct: synergy with polarized FT experiments at LHC

GPDs

Deeply Virtual Compton Scattering (DVCS):



Theoretical description involves Generalized Parton Distributions (GPDs)

GPDs are off-forward matrix elements (P' ≠ P)

This describes the spatial distribution of quarks inside nucleons

b_T is *not* the Fourier conjugate of k_T

$b_\perp$ = transverse spatial distance w.r.t. the "center" of the proton

The transverse center of longitudinal momentum: $\mathbf{R}_\perp^{CM} \equiv \sum_i x_i \mathbf{r}_{\perp i}$

[Burkardt 2000; Soper 1977]

GTMDs - 5D parton distributions

Off-forward distributions, like GPDs, give access to the transverse spatial distributions; here the proton stays intact but gets a momentum kick

GTMDs can be seen as:

- off-forward TMDs
- transverse momentum dependent GPDs
- Fourier transforms of Wigner distributions

$$G(x, \boldsymbol{k}_T, \boldsymbol{\Delta}_T) \stackrel{FT}{\longleftrightarrow} W(x, \boldsymbol{k}_T, \boldsymbol{b}_T)$$

Meißner, Metz, Schlegel, 2009 — Ji, 2003; Belitsky, Ji & Yuan, 2004

GTMDs combine all properties of TMDs and GPDs, such as process dependence & nontrivial impact parameter dependence

Dipole gluon GTMD

In the x→0 the dipole gluon GTMD becomes a correlator of a single Wilson loop:

$$G^{[+,-]\,ij}(\boldsymbol{k},\boldsymbol{\Delta}) = \frac{2N_c}{\alpha_s}\left[\frac{1}{2}\left(\boldsymbol{k}^2 - \frac{\boldsymbol{\Delta}^2}{4}\right)\delta_T^{ij} + k_T^{ij} - \frac{\Delta_T^{ij}}{4} - \frac{k_T^{[i}\Delta_T^{j]}}{2}\right] G^{[\Box]}(\boldsymbol{k},\boldsymbol{\Delta})$$

$$G^{[\Box]}(\boldsymbol{k},\boldsymbol{\Delta}) \equiv \int \frac{d^2\boldsymbol{x}\, d^2\boldsymbol{y}}{(2\pi)^4}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})+i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}}\, \frac{\langle p'|\, S^{[\Box]}(\boldsymbol{x},\boldsymbol{y})\,|p\rangle\big|_{\mathrm{LF}}}{\langle P|P\rangle},$$

All gluon polarization states (linear & circular) become related:

$$\lim_{x,\xi\to 0} x\mathcal{F}_1 = \lim_{x,\xi\to 0} x\mathcal{F}_2^{(1)} = -4\lim_{x,\xi\to 0} x\mathcal{F}_3^{(1)} = -2\lim_{x,\xi\to 0} x\mathcal{F}_4^{(1)} = \mathcal{E}^{(1)}$$

$$\mathcal{F}_i^{(n)} \equiv [(\boldsymbol{k}^2 - \boldsymbol{\Delta}^2/4)/(2M^2)]^n\, \mathcal{F}_i$$

DB, van Daal, Mulders, Petreska, 2018

Not expected to hold for the WW GTMD, except at large $k_\perp$

Real part of $G^{[\Box]}(k,\Delta)$ only depends on k^2, Δ^2 and $(k \cdot \Delta)^2$

It means that also the δ_T^{ij} term can be anisotropic now

Elliptic Wigner distributions

$$xW(x,\boldsymbol{b},\boldsymbol{k}) = x\mathcal{W}_0(x,\boldsymbol{b}^2,\boldsymbol{k}^2) + 2\cos(\phi_b - \phi_k)\, x\mathcal{W}_1(x,\boldsymbol{b}^2,\boldsymbol{k}^2)$$
$$+ 2\cos 2(\phi_b - \phi_k)\, x\mathcal{W}_2(x,\boldsymbol{b}^2,\boldsymbol{k}^2) + \ldots$$

The cos 2(ϕ_b-ϕ_k) part is called “the” elliptic Wigner distribution

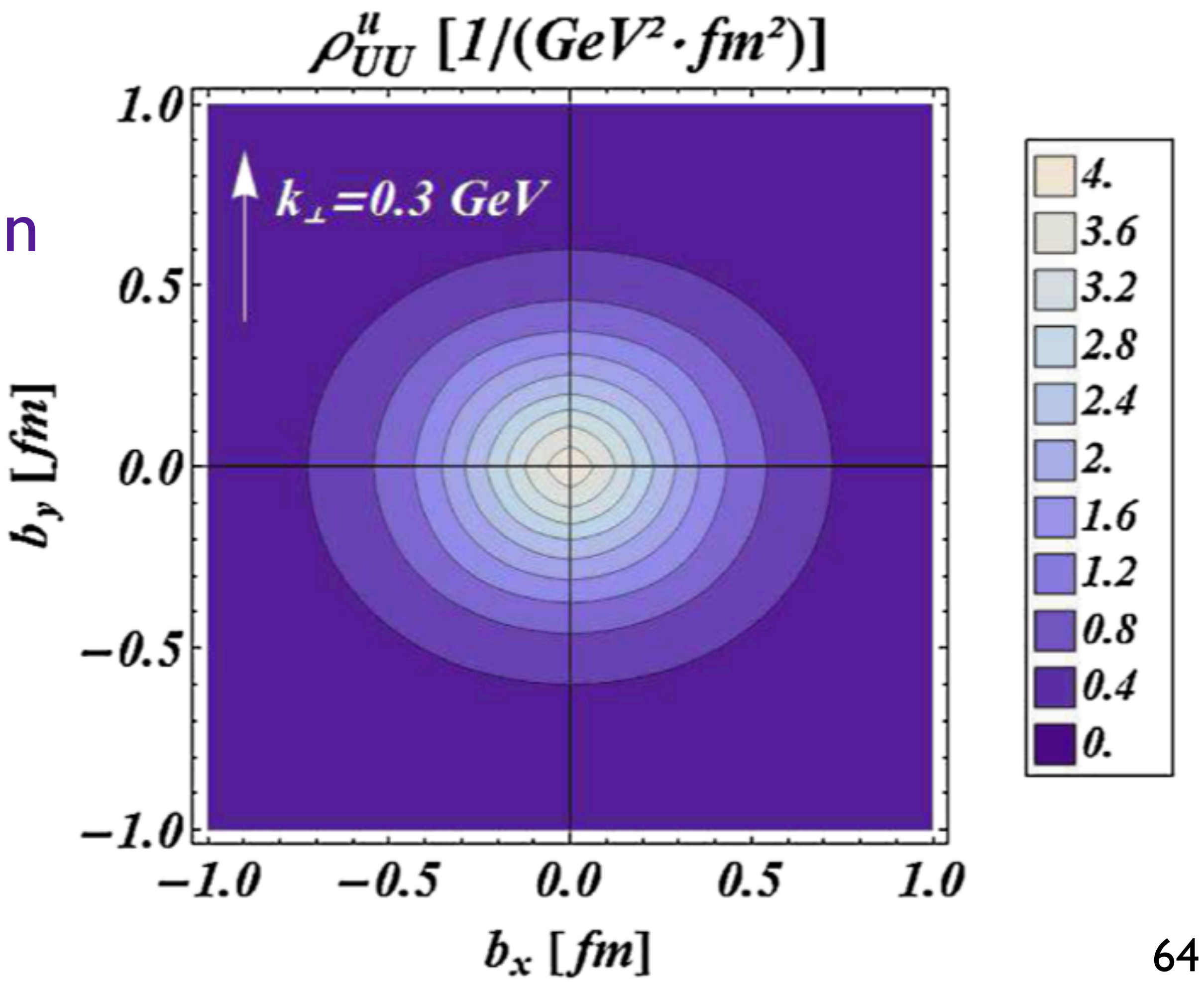
Hatta, Xiao, Yuan, 2016; J. Zhou, 2016; Mäntysaari, Mueller, Schenke, 2019; Salazar, Schenke, 2019

There can be such an elliptic piece in each Wigner distribution

Hence 4 different ones for unpolarized protons, reducing to 1 in the small-x limit

A nonzero elliptic quark Wigner distribution in the lightcone constituent quark model:

Lorcé, Pasquini, 2011



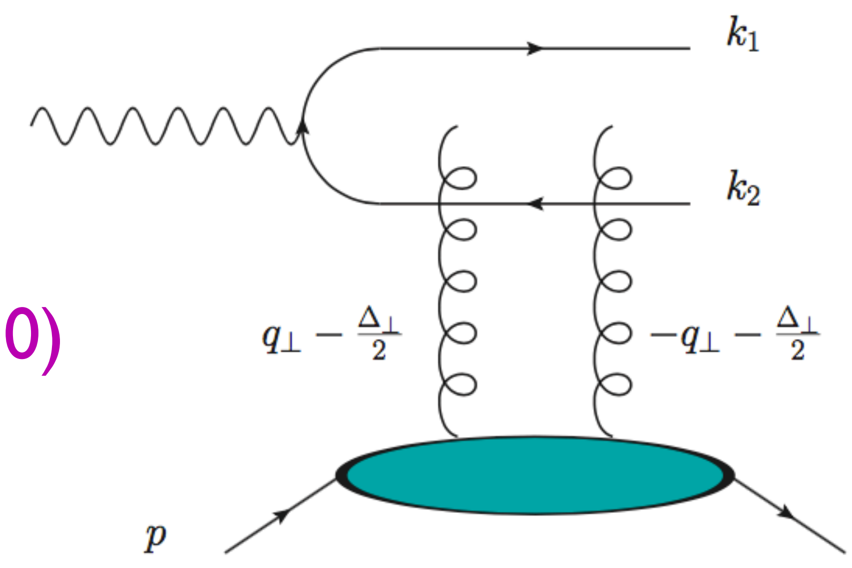
Due to quark orbital angular momentum

Lorcé, Pasquini, 2011; Hatta, 2011

Diffractive dijet production

Probe gluon GTMDs via hard diffractive dijet production in eA ($\Delta_\perp \neq 0, \xi = 0$)

Altinoluk, Armesto, Beuf, Rezaeian, 2016; Hatta, Xiao, Yuan, 2016



Earlier suggested to probe gluon GPDs ($\Delta_\perp = 0, \xi \neq 0$)

Braun, Ivanov, 2005

The DP GTMD correlator appears, as opposed to the inclusive case

$$\frac{d\sigma}{dy_1 dy_2 d^2\boldsymbol{k}_{1\perp} d^2\boldsymbol{k}_{2\perp}} \propto \int d^2\boldsymbol{q}_\perp d^2\boldsymbol{q}'_\perp \mathcal{F}^{[\Box]}(\boldsymbol{q}_\perp, \boldsymbol{\Delta}_\perp)\mathcal{F}^{[\Box]}(\boldsymbol{q}'_\perp, \boldsymbol{\Delta}_\perp)\mathcal{A}(\boldsymbol{K}_\perp, \boldsymbol{q}_\perp, \boldsymbol{q}'_\perp, \epsilon_f^2)$$

Altinoluk, Armesto, Beuf, Rezaeian, 2016; Hatta, Xiao, Yuan, 2016

$$\epsilon_f^2 = z(1-z)Q^2$$

$$G^{[\Box]} \to \mathcal{F}^{[\Box]} \qquad S^{[\Box]}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) \to 1 - S^{[\Box]}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)$$

The transverse momentum dependence of the GTMD is probed indirectly

Diffractive dijet production

The transverse momentum dependence of the GTMD is probed indirectly

$$\frac{d\sigma_T^{\gamma^* p\to jjp}}{dK_\perp d\Delta_\perp^2} = \frac{(2\pi)^4\alpha_{em}}{16N_c}\sum_f e_f^2 \int dz\, \left[z^2+(1-z)^2\right] \frac{\mathcal{A}_\mathrm{T}^2(K_\perp,\Delta_\perp,z,Q,y)}{K_\perp}$$

$$\mathcal{A}_\mathrm{T}(K_\perp,\Delta_\perp,z,Q,y) = \int \frac{d^2q_\perp}{(2\pi)^3}\left[\frac{K_\perp\cdot(K_\perp-q_\perp)}{z(1-z)Q^2+(K_\perp-q_\perp)^2}\right]\mathcal{F}_0^{[\Box]}(x,q_\perp,\Delta_\perp)\Big|_{x=s/(yQ^2)}$$

By varying Q^2 and $K_\perp$ one probes differently weighted integrals over the GTMD

Idem for the longitudinal photon polarization (which requires $Q^2 \neq 0$):

$$\frac{d\sigma_L^{\gamma^* p\to jjp}}{dK_\perp d\Delta_\perp^2} = \frac{(2\pi)^4\alpha_{em}}{4N_c}\sum_f e_f^2 \int dz\, z^2(1-z)^2 \frac{\mathcal{A}_\mathrm{L}^2(K_\perp,\Delta_\perp,z,Q,y)}{K_\perp}$$

$$\mathcal{A}_\mathrm{L}(K_\perp,\Delta_\perp,z_i,Q) = \int \frac{d^2q_\perp}{(2\pi)^3}\left[\frac{QK_\perp}{z(1-z)Q^2+(K_\perp-q_\perp)^2}\right]\mathcal{F}_0^{[\Box]}(x,q_\perp,\Delta_\perp)\Big|_{x=s/(yQ^2)}$$

DB, Setyadi, 2021

Diffractive J/ψ production

Again the transverse momentum dependence of the GTMD is probed indirectly

$$\mathcal{A}_{T,L} = \frac{\pi i}{2N_c} \int_0^1 dz \int d^2 r_\perp \left(\Psi_V^* \Psi_\gamma\right)_{T,L}(r_\perp, z) \int d^2 q_\perp J_0\left(|q_\perp + \delta_\perp| r_\perp\right) \mathcal{F}_0^{[\Box]}(x, q_\perp, \Delta_\perp)$$

$$\delta_\perp = (\frac{1}{2} - z)\Delta_\perp$$

Kowalski, Teaney, 2003; Kowalski, Motyka, Watt, 2006; many others

By varying *t* one probes (slightly) differently weighted integrals
Using different quarkonia also changes the weights ($r_\perp \sim 1/M_V$)

Often one considers this process to probe GPDs, which one (formally) recovers upon applying a collinear expansion: $r_\perp \sim 1/M_V$ means $q_\perp r_\perp \ll 1$

$$J_0\left(|q_\perp + \delta_\perp| r_\perp\right) \approx 1 - \frac{(q_\perp + \delta_\perp)^2 r_\perp^2}{4}$$

$$\mathcal{A}_{T,L} \approx \frac{\pi i}{8N_c} \int_0^1 dz \int d^2 r_\perp r_\perp^2 \left(\Psi_V^* \Psi_\gamma\right)_{T,L}(r_\perp, z) \int d^2 q_\perp q_\perp^2 \mathcal{F}_0^{[\Box]}(x, q_\perp, \Delta_\perp)$$

$$= \frac{\pi^3 i \alpha_s}{N_c} \int_0^1 dz \int d^2 r_\perp r_\perp^2 \left(\Psi_V^* \Psi_\gamma\right)_{T,L}(r_\perp, z)\, x H_g(x, \Delta_\perp)$$

See also Bertone, 2022

This yields an expression in terms of a GPD (requires regularization)

MV-like model

We consider the MV-like model:

$$\mathcal{F}^{[\Box]}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = 4N_c \int \frac{d^2\boldsymbol{r}_\perp d^2\boldsymbol{b}_\perp}{(2\pi)^2} e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} e^{i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} e^{-\epsilon_r r_\perp^2} \left[1 - \exp\left(-\frac{1}{4} r_\perp^2 \chi Q_s^2(b_\perp) \ln\left[\frac{1}{r_\perp^2 \Lambda^2} + e\right]\right)\right]$$

Similar to Hagiwara, Hatta, Pasechnik, Tasevsky & Teryaev, 2017; Salazar, Schenke, 2019

χ sets the normalization of Q_s and is x dependent (of GBW form)

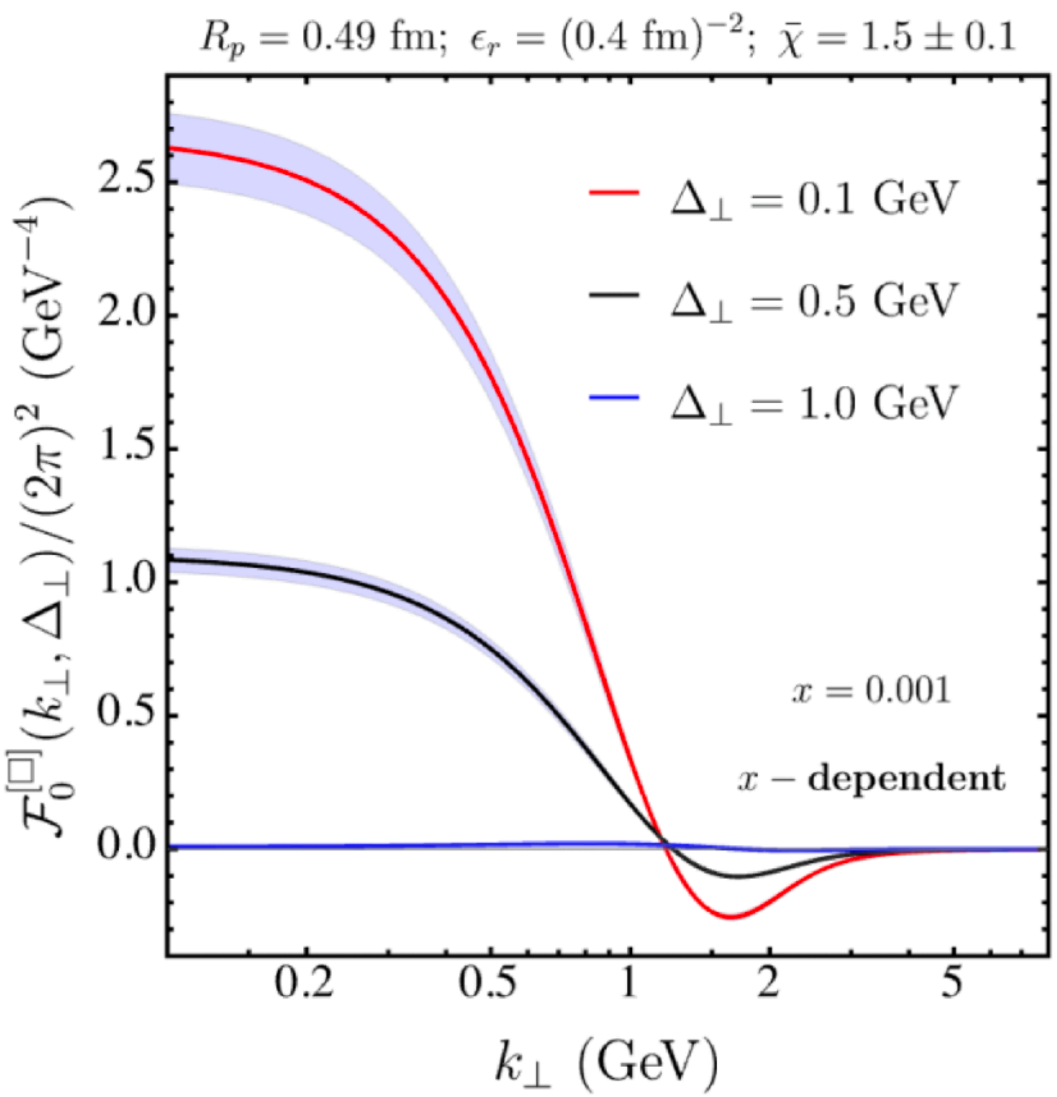
$$\chi(x) = \bar{\chi}\left(\frac{x_0}{x}\right)^\lambda \qquad x_0 = 3\times 10^{-4} \quad \lambda = 0.29$$

Q_s is proportional to the proton (Gaussian) or nuclear (Woods-Saxon) profile

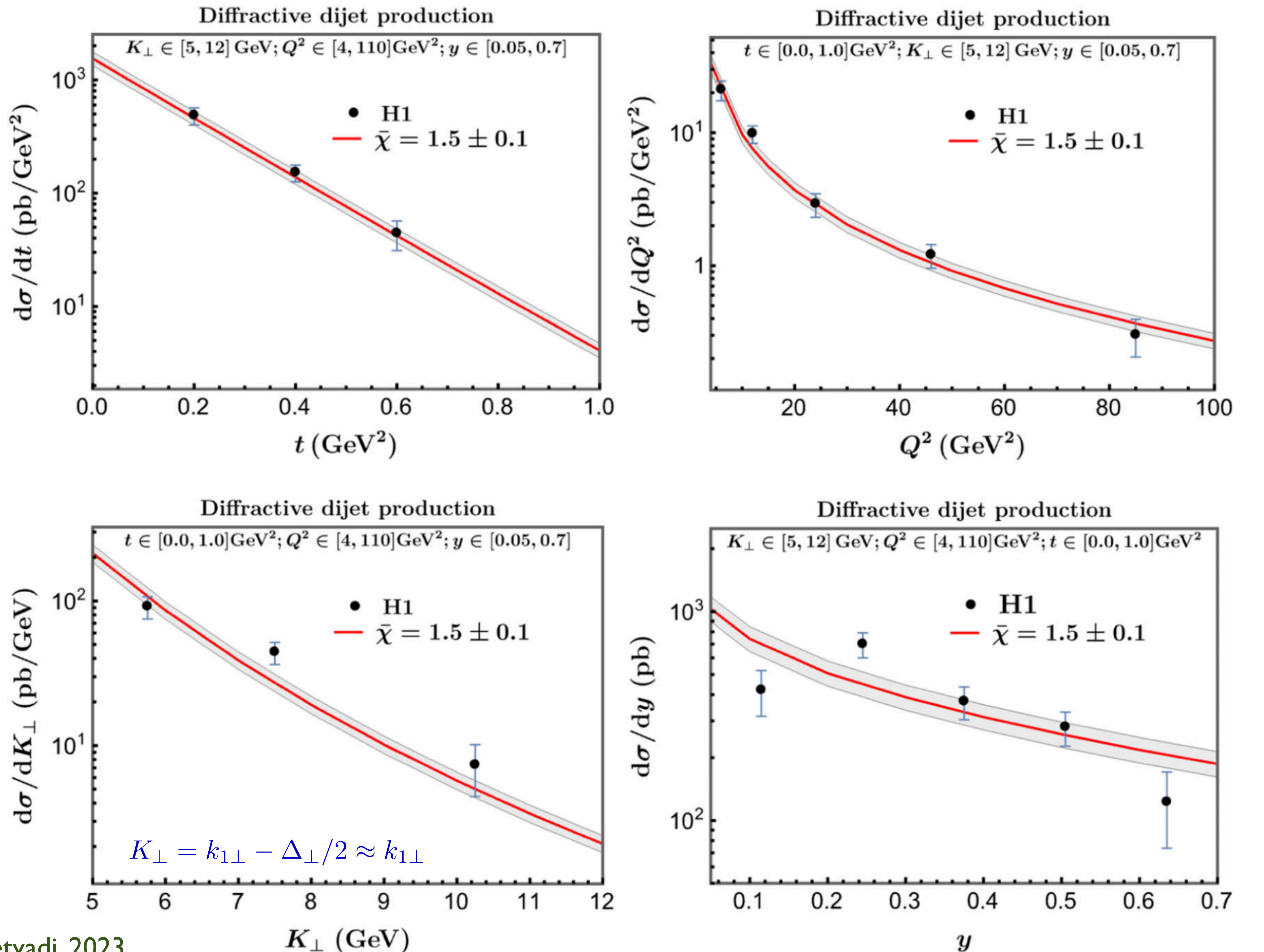
For details see DB, Setyadi, 2023

Dominant contribution from:

$$\Delta_\perp \ll K_\perp \text{ or } M_V$$



Best fit of H1 dijet data with Rp = 0.49 fm, λ = 0.29, and ϵ_r = (0.4 fm)$^{-2}$



DB, Setyadi, 2023

Odderon GTMDs

S[□] can also have an imaginary part:

$$S^{[\Box]}(\boldsymbol{x},\boldsymbol{y}) = \mathcal{P}(\boldsymbol{x},\boldsymbol{y}) + i\mathcal{O}(\boldsymbol{x},\boldsymbol{y})$$

$$\mathcal{P}(\boldsymbol{x},\boldsymbol{y}) \equiv \frac{1}{2N_c}\mathrm{Tr}\left(U^{[\Box]} + U^{[\Box]\dagger}\right) \qquad \mathcal{O}(\boldsymbol{x},\boldsymbol{y}) \equiv \frac{1}{2iN_c}\mathrm{Tr}\left(U^{[\Box]} - U^{[\Box]\dagger}\right)$$

This "odderon" operator is C-odd and T-odd

$$\begin{aligned} G_{(d)}^{(\text{T-odd})\,ij}(\boldsymbol{k},\boldsymbol{\Delta}) &\equiv \frac{1}{2}\left(G^{[+,-]\,ij}(\boldsymbol{k},\boldsymbol{\Delta}) - G^{[-,+]\,ij}(\boldsymbol{k},\boldsymbol{\Delta})\right) \\ &= \frac{N_c}{\alpha_s}\left[\frac{1}{2}\left(\boldsymbol{k}^2 - \frac{\boldsymbol{\Delta}^2}{4}\right)\delta_T^{ij} + k_T^{ij} - \frac{\Delta_T^{ij}}{4} - \frac{k_T^{[i}\Delta_T^{j]}}{2}\right] \\ &\quad \times \left(G^{[\Box]}(\boldsymbol{k},\boldsymbol{\Delta}) - G^{[\Box^\dagger]}(\boldsymbol{k},\boldsymbol{\Delta})\right) \end{aligned}$$

$$G^{[\Box]}(\boldsymbol{k},\boldsymbol{\Delta}) - G^{[\Box^\dagger]}(\boldsymbol{k},\boldsymbol{\Delta}) \propto \int \frac{d^2\boldsymbol{x}\, d^2\boldsymbol{y}}{(2\pi)^4}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y}) + i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}}\, \langle \mathcal{O}(\boldsymbol{x},\boldsymbol{y})\rangle$$

Odderon GTMDs

Hermiticity and PT constraints imply:

$$G^{[\Box]*}(\boldsymbol{k},\boldsymbol{\Delta}) = G^{[\Box]}(\boldsymbol{k},-\boldsymbol{\Delta}) \qquad G^{[\Box]*}(\boldsymbol{k},\boldsymbol{\Delta}) = G^{[\Box^\dagger]}(-\boldsymbol{k},-\boldsymbol{\Delta})$$

$$G^{[\Box]}(\boldsymbol{k},\boldsymbol{\Delta}) - G^{[\Box^\dagger]}(\boldsymbol{k},\boldsymbol{\Delta}) \text{ only depends on odd powers of } \boldsymbol{k}\cdot\boldsymbol{\Delta}$$

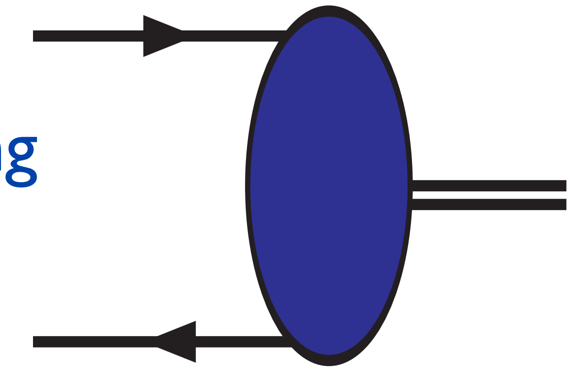
Odderon (for $\xi = 0$) involves only odd harmonics $\cos[(2n+1)(\phi_k - \phi_\Delta)]$

$$\begin{aligned} xW(x,\boldsymbol{b},\boldsymbol{k}) &= x\mathcal{W}_0(x,\boldsymbol{b}^2,\boldsymbol{k}^2) + 2\cos(\phi_b - \phi_k)\, x\mathcal{W}_1(x,\boldsymbol{b}^2,\boldsymbol{k}^2) \\ &\quad + 2\cos 2(\phi_b - \phi_k)\, x\mathcal{W}_2(x,\boldsymbol{b}^2,\boldsymbol{k}^2) + \ \ldots \end{aligned}$$

For $\xi \neq 0$ odd powers of $k \cdot \Delta$ can appear in the real parts as well

Future EIC data should specify the region (ERBL and/or DGLAP) probed and check for odd harmonics

Effect of smearing



In reality the process of $Q\bar{Q} \to J/\psi$ involves some k_T-smearing

TMD factorization requires inclusion of shape function Δ

Echevarria, 2019; Fleming, Makris & Mehen, 2019

Using LO NRQCD the unpolarized structure function becomes:

$$\mathcal{F}_{UU,T} = \frac{2\pi^2\alpha_s e_c^2}{M_\psi(M_\psi^2+Q^2)}\left[\langle 0|\mathcal{O}({}^1S_0^{[8]})|0\rangle + 4\,\frac{(7M_\psi^4+2M_\psi^2Q^2+3Q^4)}{M_\psi^2(M_\psi^2+Q^2)^2}\langle 0|\mathcal{O}({}^3P_0^{[8]})|0\rangle\right]$$

$$\times\, f_1^g(x,p_T^2)\Big|_{\boldsymbol{p}_T=\boldsymbol{q}_T}$$

introducing a shape function

$$\int d^2\boldsymbol{p}_T \int d^2\boldsymbol{k}_T\, \delta^2(\boldsymbol{q}_T-\boldsymbol{p}_T-\boldsymbol{k}_T)\, f_1^g(x,\boldsymbol{p}_T^2;\mu^2)\,\Delta^{[n]}(\boldsymbol{k}_T^2,\mu^2)$$

LO NRQCD: $\Delta^{[n]}(\boldsymbol{k}_T^2;\mu^2) = \langle 0|\mathcal{O}(n)\,|0\rangle\,\delta^2(\boldsymbol{k}_T)$

More generally, the shape function will be a smeared out delta function with a tail