Double Parton Scattering and Double PDFs

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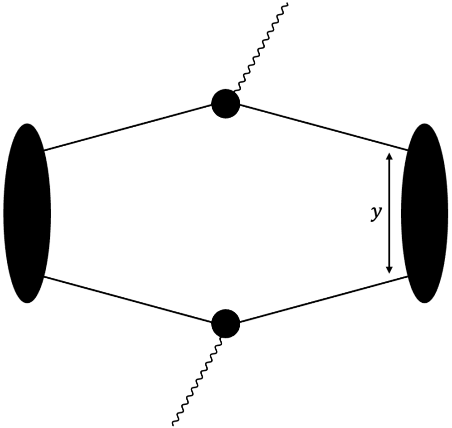
Outline

- Double Parton Scattering (DPS) and Double Parton Distributions (DPDs)
- DPS in same-sign WW
- What same-sign WW tells us about correlations in the proton

Double Parton Scattering

DPS: Two partons from each proton collide into two separate hard scatters.



$$\sigma_{DPS}^{(A,B)} = \int F_{ik}\left(x_1, x_2, \boldsymbol{y}\right) \otimes \hat{\sigma}_{ij}^{A} \hat{\sigma}_{kl}^{B} \otimes F_{jl}\left(x_1' x_2', \boldsymbol{y}\right) d^2\boldsymbol{y}$$

Factorisation now proven for double colour-singlet production in QCD.
(Diehl, Gaunt, Ostermeier, Plößl, Schafer, JHEP 1601 (2016) 076, Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089, Diehl, Nagar, JHEP 1904 (2019) 124, Vladimirov, JHEP 1804 (2018) 045)

Pocket Formula

This talk is about correlations, but for now lets ignore them

$$F^{ij}(x_1, x_2, \boldsymbol{y}) \to f^i(x_1)\, f^j(x_2)\, G(\boldsymbol{y}).$$

This gives the "DPS pocket formula":

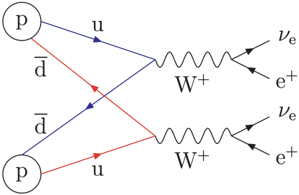
$$\sigma_D^{(A,B)} = \frac{\sigma_S^{(A)}\sigma_S^{(B)}}{\sigma_{\mathrm{eff}}}$$

with

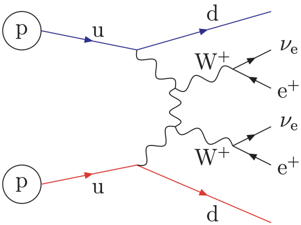
$$\sigma_{\mathsf{eff}} = \frac{1}{\int G(\boldsymbol{y}) d^2\boldsymbol{y}}$$

DPS in Same-Sign WW

Same-sign WW in single scattering suppressed by multiple coupling constants and must involve jets to conserve charge. This allows DPS to compete.



(a)



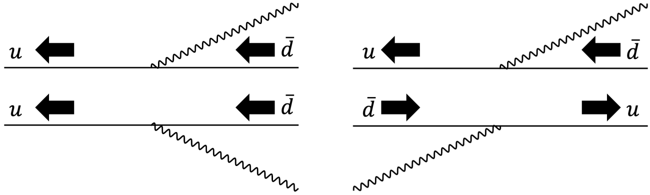
(b)

Formally DPS is power suppressed with respect to SPS

$$\sigma_{DPS} \sim \sigma_{SPS} \frac{\Lambda^2_{QCD}}{Q^2}$$

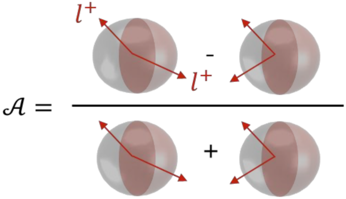
Correlations in DPS

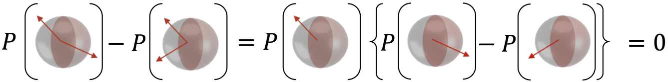
- A significant motivation to study DPS is to investigate correlations between partons in the proton.
- DPS offers insights into momentum, valence, spin and colour correlations.
- Same-sign WW offers a particularly nice insight into spin correlations due the W boson coupling only to LH particles/ RH antiparticles.



Observing Correlations

Asymmetry can be used to probe correlations.
Gaunt, Kom, Kulesza, Stirling, Eur.Phys.J. C69 (2010) 53





One must be careful with phase space cuts when using asymmetry to study correlations.

Momentum and Valence Number Conservation

- The requirement that the momentum and valence numbers of the proton be conserved leads to correlations.
- These correlations induce positive asymmetry as they reduce the likelihood of obtaining two high-x partons from one proton.
- dPDFs must obey the sum rules which are as follows:

Momentum sum rule encodes available momentum after “taking out” j_1:

$$\sum_{j_2} \int dx_2 x_2 D_{j_1 j_2}(x_1, x_2, Q) = (1 - x_1) f_{j_1}(x_1, Q)$$

Number sum rule encodes number of j_2 quarks - number of $\bar{j_2}$ quarks after “taking out” j_1:

$$\int dx_2 D_{j_1 j_{2v}}(x_1, x_2, Q) = \left(N_{j_{2v}} - \delta_{j_1 j_2} + \delta_{j_1 \bar{j_2}} \right) f_{j_1}(x_1, Q)$$

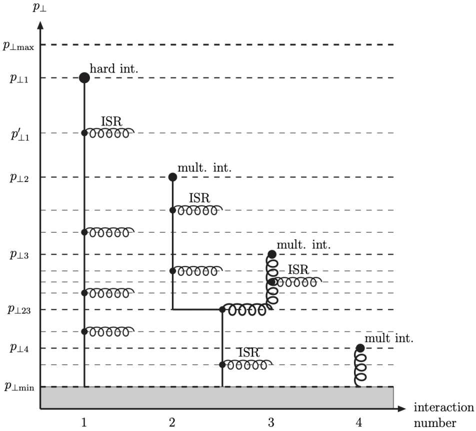
Gaunt, Stirling, 0910.4347
Blok, Dokshitzer, Frankfurt, Strikman, 1306.3763
Diehl, Plößl, Schäfer,1811.00289

Building dPDFs That Satisfy the Sum Rules

Unfortunately we do not have dPDFs from experimental fits, we must therefore build models which use information from single PDFs. Several different approaches:

- Pythia MPI model
- x-ordered modification to Pythia model
- GS09 dPDFs

Pythia MPI model



- Pythia uses single PDFs for hardest interaction and evolves backwards from there, interleaving ISR and MPI
- Dynamically modifies raw sPDFs to account for number and valence effects: $f_i^r(x, Q) \to f_i^{m \leftarrow j_1, x_1 \ldots j_{n-1}, x_{n-1}}(x, Q)$

Sjostrand, Skands, JHEP 03 (2004) 053, Eur.Phys.J.C 39 (2005) 129-154

Pythia MPI model

Momentum "Squeezing"

$f_i^{m\leftarrow j_1,x_1\dots j_{n-1},x_{n-1}}(x,Q) = \frac{1}{X} f_i^r\left(\frac{x}{X},Q\right)$ where $X = 1 - \sum_{i=1}^{n-1} x_i$

Valence Number Subtraction

$$f_{i_v}^{m\leftarrow j_1,x_1\dots j_{n-1},x_{n-1}}(x,Q) = \frac{N_{i_v n} n}{N_{i_v 0}} \frac{1}{X} f_{i_v}^r\left(\frac{x}{X},Q\right)$$

Companion Quark Addition

$$q_c(x,x_s) = C(x_s)\, P_{g\to q\bar{q}}\left(\frac{x_s}{x_s+x}\right)\frac{f_g(x_s+x)}{x_s+x}$$

Sea Quark and Gluon Rescaling

Steps 2 and 3 break the fact that step 1 guarantees:
$\sum_i \int dx\, x\, f_i^{m\leftarrow j_1,x_1\dots j_{n-1},x_{n-1}}(x,Q) = X$

Sjostrand, Skands, hep-ph/0402078,hep-ph/0408302

Pythia MPI model

The problem with this model is that it is based on hardness ordering and this is incompatible with equal scales processes such as same-sign WW. The dPDFs should be symmetric under $x_1 \leftrightarrow x_2, j_1 \leftrightarrow j_2$. This approach satisfies the sum rules to some extent, but can be improved

x_1	Momentum sum rule ($j_1 = u$). Should = 1.	$\bar{u}u$ number sum rule. Should = 3.
10^{-6}	0.979	2.961
10^{-3}	0.980	3.351
10^{-1}	1.014	3.491
0.2	1.047	3.580
0.4	1.133	3.858
0.8	1.679	7.048

Naively symmetrised Pythia dPDFs

Fedkevych, Gaunt, JHEP 02 (2023) 090

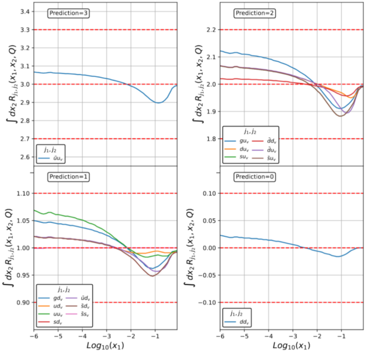
Improved Model

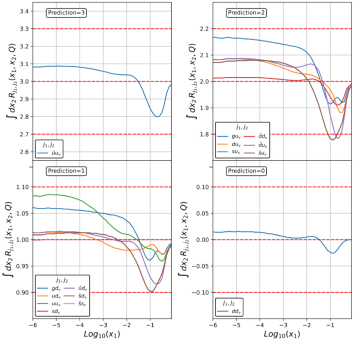
An improved model has been developed which satisfies these symmetries and is therefore more suited to equal scales processes. The modifications are:

- Order scatters in x rather than Q and has smooth transitions
- Improve "companion quark mechanism" so that it is symmetric and conforms better to picture of quark and companion arising from $g \to q\bar{q}$ splitting.

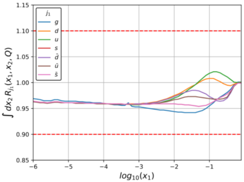
Fedkevych, Gaunt, Smith, in preparation

Improved Model Sum Rules

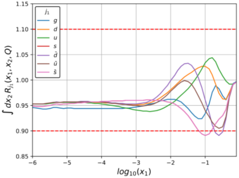




dPDFs constructed at $Q = M_Z$



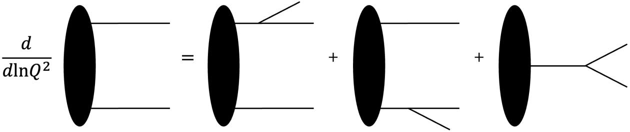
dPDFs constructed at $Q = 2$ GeV



Sum Rules well satisfied.

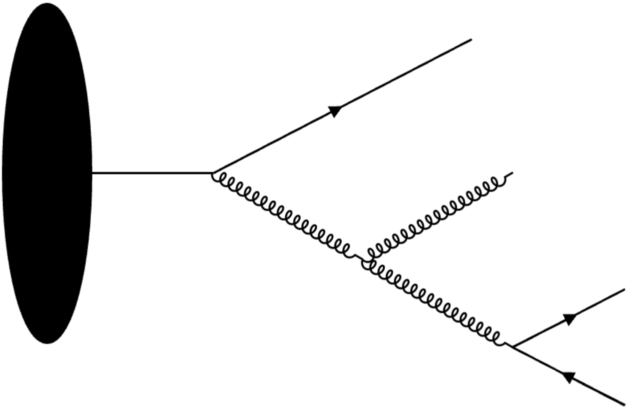
GS09

- dPDFs created to satisfy the sum rules.
- Modelled as product of single PDFs and a phase space suppression factor plus valence and companion adjustments at low scale and evolved using dDGLAP:



Perturbative Splitting Contribution

Partons can originate from the perturbative splitting of a single mother parton. This correlation again induces positive asymmetry as it adds a contribution which strongly prefers the high-x low-x configuration.



Spin Correlations

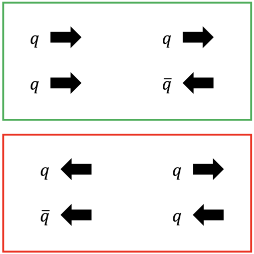
- Due to the coupling of the W boson only certain spin configurations will contribute to the cross section.
- Spin correlations encoded in polarised dPDFs which must satisfy the positivity bound:

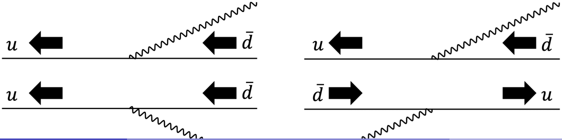
$$f_{a,b} = \uparrow\uparrow + \downarrow\downarrow + \downarrow\uparrow + \downarrow\uparrow$$

$$f_{\Delta a,\Delta b} = \uparrow\uparrow + \downarrow\downarrow - \downarrow\uparrow - \downarrow\uparrow$$

$$|f_{\Delta a,\Delta b}| \leq f_{a,b}$$

Mekhfi, Phys. Rev. D32 (1985) 2380
Diehl, Ostermeier and Schafer (JHEP 1203 (2012))
Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009





Spin Correlations

Build models to understand the maximum possible effects of spin correlations. Construct at $Q = 1\text{GeV}$ and evolve to W mass. 2 models proposed by Cotogno, Kasemets, Myska: Phys.Rev.D100 (2019) 1, 011503, JHEP 10 (2020) 214

Positive Polarisation

Saturate the positivity bound with positive polarised dPDFs:

$$f_{\Delta a \Delta b}(x_1, x_2, \boldsymbol{y}; Q_0) = f_{ab}(x_1, x_2, \boldsymbol{y}; Q_0) = f(x_1; Q_0)\, f(x_2; Q_0)\, G(\boldsymbol{y})$$

Mixed Polarisation

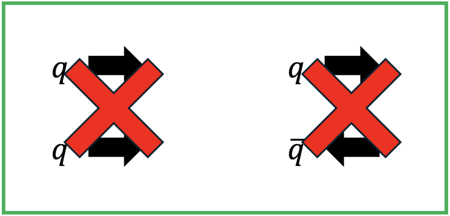
Saturate the positivity bound with flavour dependent polarised dPDFs:

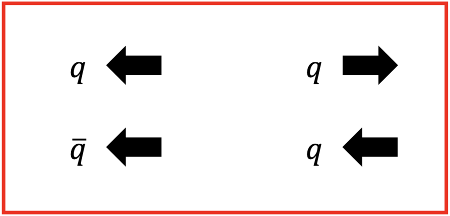
$$f_{\Delta a \Delta b}(x_1, x_2, \boldsymbol{y}; Q_0) = (-1)^n f_{ab}(x_1, x_2, \boldsymbol{y}; Q_0)$$

Where $n = 1$ if $ab = qq, \bar{q}\bar{q}$ and $n = 2$ otherwise.

A third model, negative polarisation is useful to compare to mixpol and is just -pospol.

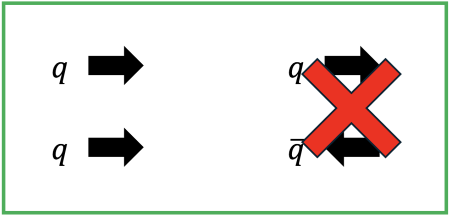
Mixed Polarisation

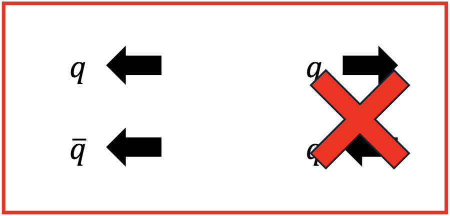


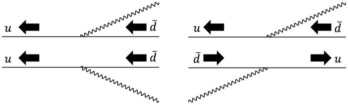


- Both configurations that contribute to W^+W^+ are killed.
- Cross section fully generated by evolution
- Evolution should primarily generate positive asymmetry since two high-x is unlikely.

Positive Polarisation

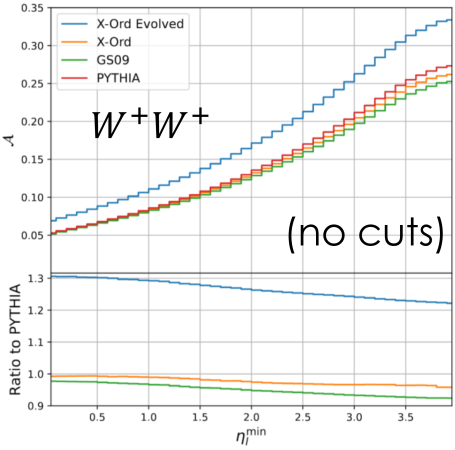






- Only align contribution allowed to contribute at low scale.
- If we assume generally $f_q > f_{\bar{q}}$, two quarks will push the Ws into the same hemisphere
- Can this effect withstand evolution?

Effects of Moment and Valence Number Conservation

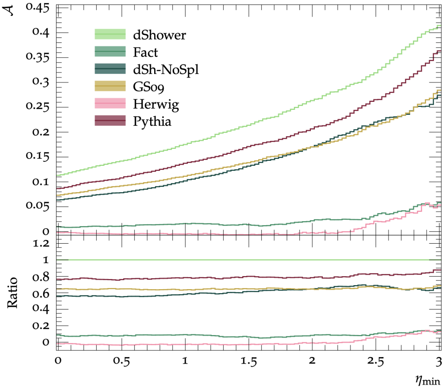


Fedkevych, Gaunt, Smith, in preparation

- Positive asymmetry, large increase with minimum rapidity cuts
- Very small differences between Pythia and x-ordered since ssWW doesn't probe companion quark
- X-order evolved and GS09 show large differences

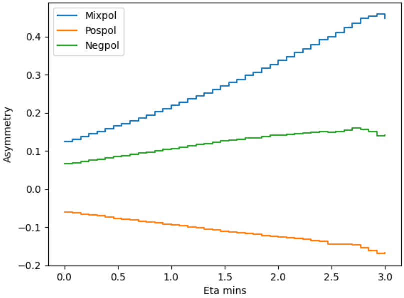
Effects of Splitting Contribution

- dShower is an MC that properly accounts for y dependence in splitting contribution
- These results are for $n_f = 3$, effects are partially washed out for $n_f = 5$
- GS09 captures about half the effect compared to dShower since it does not properly account for the **y** dependence



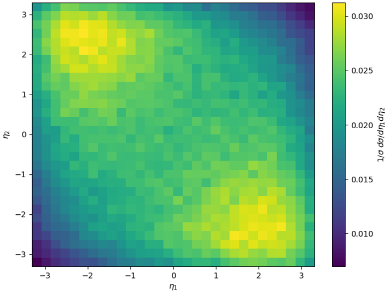
Cabouat, Gaunt, Ostrolenk, JHEP 11 (2019) 061

Effects of Spin Correlations



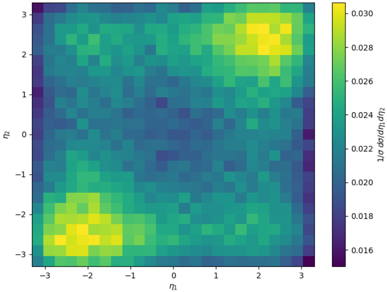
- Mixpol and pospol generate positive and negative asymmetry as expected
- At $|\eta| > 0$ approximately a 5-10% effect
- Mixpol especially good at generating high asymmetry when compared to negpol

Understanding Mixpol



- Mixpol behaves as expected producing strong bias towards opposite hemispheres
- Lower cross section since it must all be generated by evolution
- Very few events in same hemisphere corners, especially at high rapidities

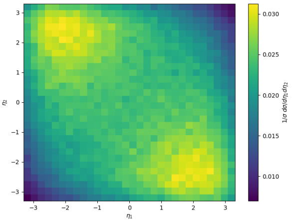
Understanding Pospol

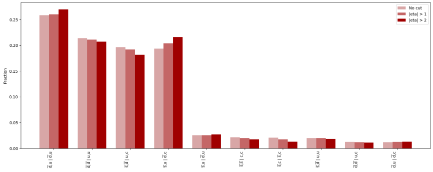


- Strong bias towards same hemisphere events
- Evolution still drives some opposite hemisphere events
- Could be problematic if it compensates the positive asymmetry from other sources

Why is Mixpol So Good?

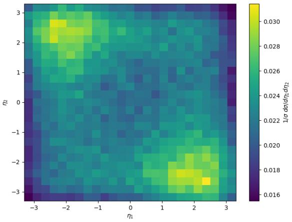
Mixpol

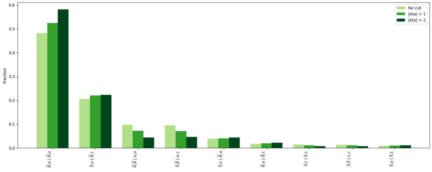




Evolution gives strong preference towards large-x small-x.

Negpol



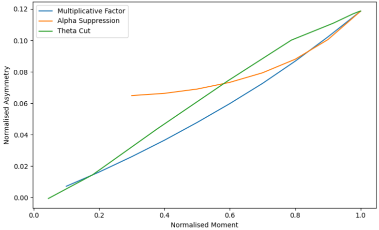


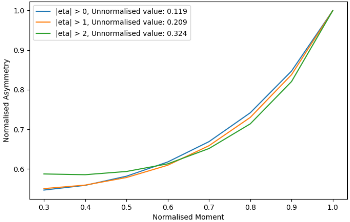
Valenceness of $\bar{d}$ pushes some events into same hemisphere

Placing Constraints on Spin Contributions

- Lattice-QCD should be able to constrain the moment of the polarised dPDFs
- We should be able to place a fairly strong constraint on the size of the spin contribution given this moment
- Different models of constraining asymmetry lead to similar predictions

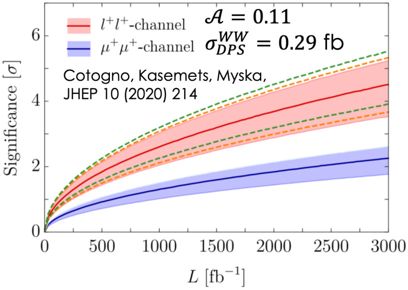
$$M_{ij} = \int_{x_{\min}}^{1} x_1 x_2 F_{ij}(x_1, x_2, Q^2) dx_1 dx_2.$$

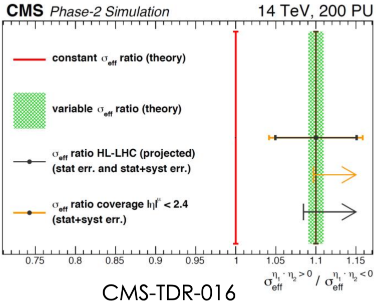




Prospects of Determining Correlations

- Should be able to distinguish correlations from the DPS pocket formula if the total effect of the correlations is $\mathcal{A} > 0.05$
- Spin effects can have negative contribution to $\mathcal{A}$ and potentially comparable to the effect from other correlations. Lattice info can help constrain spin effects and indicate if measurable $\mathcal{A} \neq 0$ remains





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Summary

- DPS is an important contribution to same-sign WW production, and studying it can reveal information on correlations between partons in the proton.
- The lepton pseudorapidity asymmetry $\mathcal{A}$ is an observable that probes such correlations. $\mathcal{A} \neq 0$ implies correlations!
- The main correlations that affect $\mathcal{A}$ are:
  - Momentum and number sum rule correlations
  - Correlations induced by perturbative splittings
  - Spin Correlations
- Each yields a few per cent impact on $\mathcal{A}$. First two effects give $\mathcal{A} > 0$, whilst spin correlations can give a positive or a negative contribution to $\mathcal{A}$.