First insight into TMD fragmentation physics at photon-photon colliders

Synergies between EIC & LHC - Krakow, 22-24 Sept 2025

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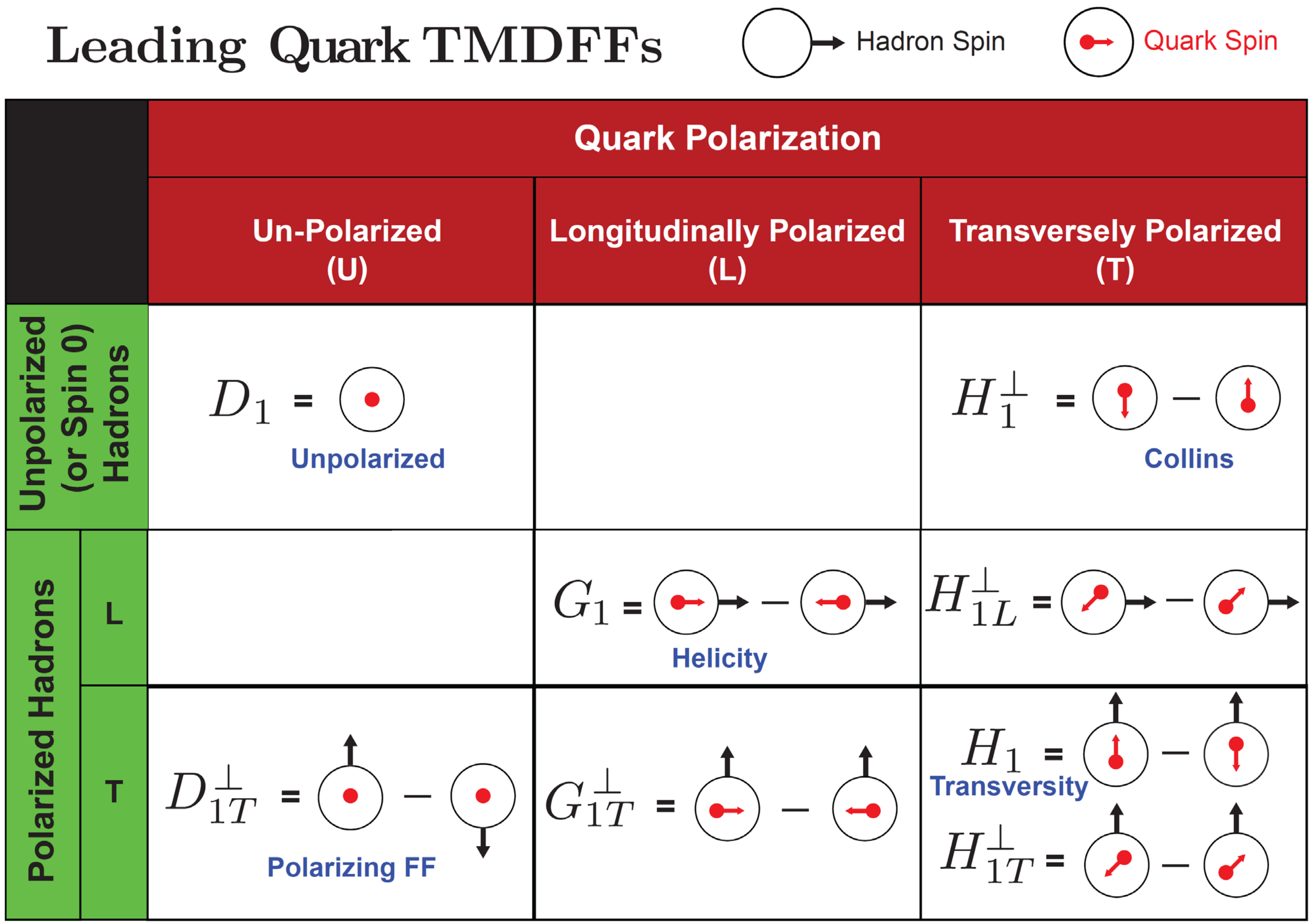
based on work with S. Anedda & C. Pisano: PRD 112, 014013 (2025) ArXiv: 2504.12802[hep-ph]

Outline

- Motivations and physical interest
- Complementarity with SIDIS (Hermes, Compass, EIC) and e^+e^- annihilations: flavor separation for quark TMD fragmentation functions (unpolarized, Collins, polarizing FF); UPCs at LHC, RHIC
- Theoretical scheme: TMD approach & helicity formalism at leading order and leading twist
- Illustrative case: two opposite-hemisphere hadron production in $\gamma^*\gamma$ collisions at lepton colliders
- Results: cross sections, azimuthal modulations and moments (no phenomenology yet)
- Conclusions and perspectives

Improving flavor separation for quark unpolarized and polarized LT TMD fragmentation functions

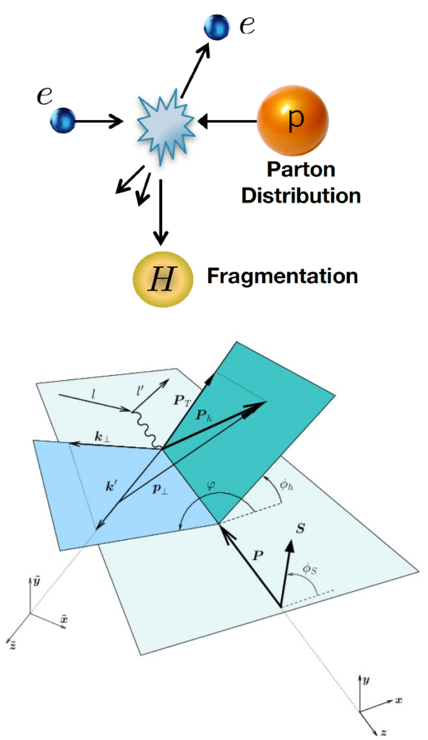


Spin 1/2 hadrons
Analogous table for gluons
[transv. pol. → linear. pol.

TMD Handbook
2304.03302 [hep-ph]

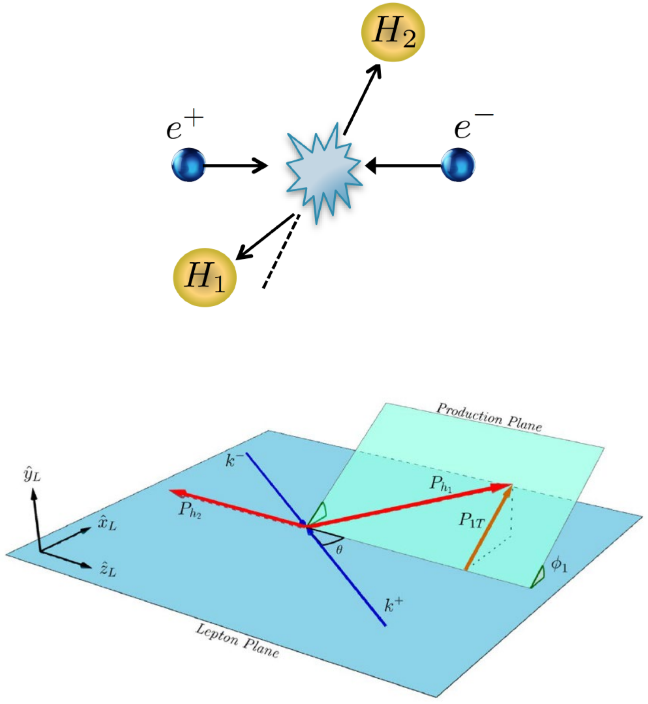
TMD fragmentation processes in photon-photon collisions
A useful complementary tool in combination with SIDIS and e^+e^- SIA processes

Semi-Inclusive DIS



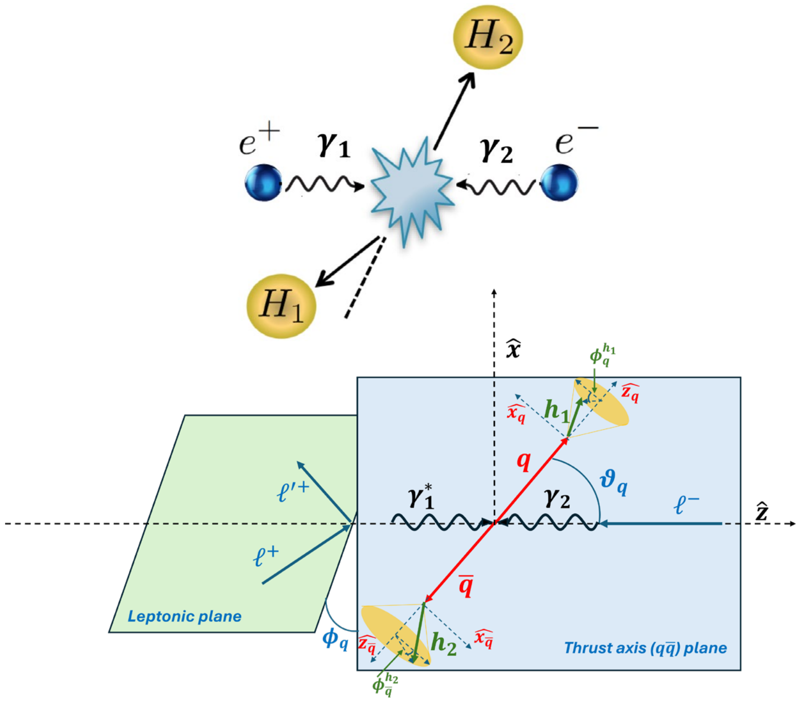
$$d\sigma \propto \sum_q e_q^2 \, f_{q/h}(x) \, D_q^h(z)$$

Dihadron in e+e-



$$d\sigma \propto \sum_q e_q^2 \, D_q^{h_1}(z_1) \, D_{\bar{q}}^{h_2}(z_2)$$

Dihadron in $\gamma\gamma$



$$d\sigma \propto \sum_q e_q^4 \, D_q^{h_1}(z_1) \, D_{\bar{q}}^{h_2}(z_2)$$

Motivations

- Different relative weight (16:1) among u-type and d-type quarks w.r.t. SIDIS and e^+e^- (4:1) [e_q^4 vs. e_q^2]
- Useful for quark flavor separation in the fragmentation sector – role of charm quark FFs (e.g. for hyperons)
- Possibility of probing different energy scales (TMD evolution) within the same experimental setup
- Several possible (present and future) experimental setups:
  Circular and linear lepton colliders (FCC-ee, CEPC, ILC, CLIC,…) with virtual/quasi-real photons
  Laser Compton back-scattering (beam polarization) in e^+e^- colliders
  UPCs at LHC, RHIC
- Possible strong synergy among EIC for SIDIS, UPCs at LHC, future circular and linear e^+e^- colliders
- Timely in view of the European Strategy for PP and the ongoing proposal/planning of future colliders and detectors for particle physics

Additional comments and open points

- **Feasible with sufficient precision? (luminosity, statistics, required detector performances)**
- **Different energy scales playing a role: photon virtuality, collider cm energy, photon-photon cm energy, large jet ($q, \bar{q}$) transverse momentum w.r.t. the colliding beams (t-channel process)**
- **Only analytical results presented here, phenomenological studies just starting, particularly for two quasi-real photon collisions [UPCs at LHC, lepton colliders]**
- **Comments, , suggestions welcome! (Both on theoretical and experimental aspects)**
- **A case study process as proxy to SIDIS:** $\ell^+\ell^- \to \ell'^+\gamma_1^*\gamma_2 \to \ell'^+ q\bar{q} \to \ell'^+ h_1 h_2 + X$

An illustrative case study process:

$$\ell^+\ell^- \to \ell'^+\boldsymbol{\gamma_1^*\gamma_2} \to \ell'^+\boldsymbol{q\bar{q}} \to \ell'^+\boldsymbol{h_1 h_2 + X}$$

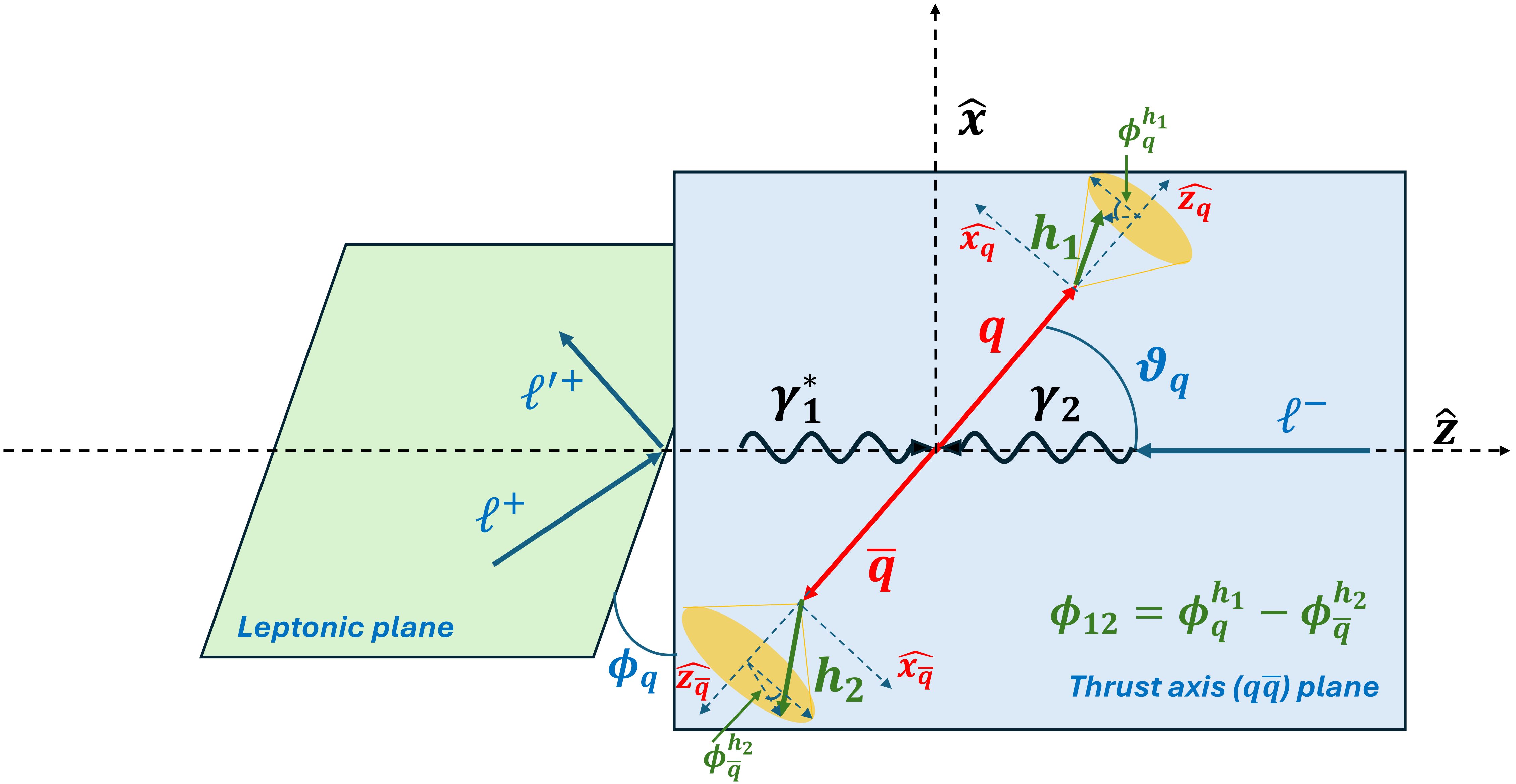
$$\ell^+(l_+)\ell^-(l_-) \to \ell'^+(l'_+)\gamma_1^*(q_1)\gamma_2(q_2) \to \ell'^+(l'_+)q(K_q)\bar{q}(K_{\bar{q}}) \to \ell'^+(l'_+)h_1(P_1)h_2(P_2) + X$$

- **TMD approach within helicity formalism, leading order and leading-twist**
- **Simple em initial state, TMD factorization should be guaranteed as for SIDIS and e^+e^- SIA processes**
- **Two energy scales – small scale: intrinsic parton motion in the fragmentation process – large one: transverse momentum of the $q\bar{q}$ jet w.r.t. the photon beams**
- **TMD fragmentation functions are universal and process independent** see e.g. Collins, Metz, PRL 93, 252001 (2004)
- **Jet thrust-axis configuration (adopted here) cleaner from the theoretical point of view**
- **Experimentally more difficult, hadron-plane configuration preferred (azimuthal distribution of one final hadron w.r.t. the second one (no thrust axis measurement required)**

U. D'Alesio, FM, M. Zaccheddu, JHEP 10 (2021) 078

- **Theoretical formalism within the hadron-plane scheme already available; can be easily implemented here**
- **CSS TMD evolution with energy scale not yet implemented; all required formalism already available, can be easily plugged-in** U. D'Alesio, L. Gamberg, FM, M. Zaccheddu, JHEP 12 (2022) 074
- **Possible competing contribution $\gamma^* g \to q\bar{q}$ should be suppressed; distinguishable experimentally due to additional hadronic production along the second lepton beam; can also be implemented**

Kinematics of the process $\ell^+\ell^- \to \ell'^+\gamma_1^*\gamma_2 \to \ell'^+q\bar{q} \to \ell'^+h_1h_2 + X$



$$\ell^+(l_+)\ell^-(l_-) \to \ell'^+(l'_+)\gamma_1^*(q_1)\gamma_2(q_2) \to \ell'^+(l'_+)q(K_q)\bar{q}(K_{\bar{q}}) \to \ell'^+(l'_+)h_1(P_1)h_2(P_2) + X$$

General expression of the differential cross section

Kinematical factors

$$d\sigma^{\ell^+\ell^-\to\ell'^+h_1h_2X} = \frac{1}{4(l_+\cdot l_-)}\frac{d^3\boldsymbol{l}'_+}{2(2\pi)^3 l'^0_+}\frac{d^3\boldsymbol{K}_q}{2(2\pi)^3K^0_q}\frac{d^3\boldsymbol{K}_{\bar{q}}}{2(2\pi)^3K^0_{\bar{q}}}(2\pi)^4\delta^{(4)}(q_1+q_2-K_q-K_{\bar{q}})$$
$$\times\sum_q\sum_{\{\lambda_i\}}\tilde{\rho}_{\lambda_1,\lambda'_1}(\gamma_1^*)\rho_{\lambda_2,\lambda'_2}(\gamma_2)f_{\gamma/\ell^-,\mathcal{P}^{\ell^-}_{\hat{z}_-}}(\xi)\frac{d\xi}{\xi}\hat{H}_{\lambda_q,\lambda_{\bar{q}};\lambda_1,\lambda_2}\hat{H}^*_{\lambda'_q,\lambda'_{\bar{q}};\lambda'_1,\lambda'_2}$$
$$\times\hat{D}^{h_1}_{\lambda_q,\lambda'_q}(z_1,\boldsymbol{p}_{\perp 1})dz_1d^2\boldsymbol{p}_{\perp 1}\hat{D}^{h_2}_{\lambda_{\bar{q}},\lambda'_{\bar{q}}}(z_2,\boldsymbol{p}_{\perp 2})dz_2d^2\boldsymbol{p}_{\perp 2}.$$

Dynamical kernel:
Initial polarization state
& hard scattering process

Dynamical kernel:
Soft TMD fragmentation process

Scale dependence of WW photon distribution and TMD FFs is implied and not shown

Kinematical factors – invariant variables

$$\frac{1}{4(l_+ \cdot l_-)} \frac{d^3 \boldsymbol{l}'_+}{2(2\pi)^3 l'^0_+} \frac{d^3 \boldsymbol{K}_q}{2(2\pi)^3 K^0_q} \frac{d^3 \boldsymbol{K}_{\bar{q}}}{2(2\pi)^3 K^0_{\bar{q}}} (2\pi)^4 \delta^{(4)}(q_1 + q_2 - K_q - K_{\bar{q}})$$

$$s = (l_+ + l_-)^2 = 2 l_+ \cdot l_-, \qquad Q^2 = -q_1^2 = -(l_+ - l'_+)^2 = 2 l_+ \cdot l'_+$$

$$x_B = \frac{Q^2}{2 l_- \cdot q_1}, \qquad y = \frac{l_- \cdot q_1}{l_- \cdot l_+}, \qquad Q^2 = x_B y s.$$

$$W^2 = (q_1 + l_-)^2 = (1 - x_B) y s = \frac{1 - x_B}{x_B} Q^2$$

$$\hat{s} = (q_1 + q_2)^2 = (\xi - x_B) y s = \frac{\xi - x_B}{x_B} Q^2$$

$$\hat{t} = -\zeta \xi y s, \qquad \hat{u} = -(1 - \zeta) \xi y s.$$

$$\zeta_q = \frac{K_q \cdot l_-}{q_1 \cdot l_-}, \qquad \zeta_{\bar{q}} = \frac{K_{\bar{q}} \cdot l_-}{q_1 \cdot l_-}$$

$$\zeta_{\bar{q}} \equiv \zeta = \frac{1}{e^{\eta_q - \eta_{\bar{q}}} + 1} = \frac{e^{-\hat{\eta}}}{2 \cosh \hat{\eta}} = \frac{K_T}{\sqrt{\hat{s}}} e^{-\hat{\eta}},$$

$$\zeta_q \equiv 1 - \zeta = \frac{1}{e^{\eta_{\bar{q}} - \eta_q} + 1} = \frac{e^{\hat{\eta}}}{2 \cosh \hat{\eta}} = \frac{K_T}{\sqrt{\hat{s}}} e^{\hat{\eta}}$$

Kinematical factors – evaluation

$$\frac{d^3 \boldsymbol{l}'_+}{2(2\pi)^3 l'^0_+} = \frac{1}{16\pi^2} sy dx_B dy$$

$$\delta^{(4)}(q_1 + q_2 - K_q - K_{\bar{q}}) = \delta(q_1^+ + q_2^+ - K_q^+ - K_{\bar{q}}^+) \times \delta(q_1^- + q_2^- - K_q^- - K_{\bar{q}}^-) \times \delta^{(2)}(-\boldsymbol{K}_{qT} - \boldsymbol{K}_{\bar{q}T}),$$

$$\delta(q_1^+ + q_2^+ - K_q^+ - K_{\bar{q}}^+)\delta(q_1^- + q_2^- - K_q^- - K_{\bar{q}}^-) = \frac{2}{ys}\delta(1 - \zeta_q - \zeta_{\bar{q}})\delta\left(\xi - x_B - \frac{\boldsymbol{K}_T^2}{\zeta_q \zeta_{\bar{q}} ys}\right),$$

$$\frac{d^3 \boldsymbol{K}_q}{2(2\pi)^3 K_q^0} \frac{d^3 \boldsymbol{K}_{\bar{q}}}{2(2\pi)^3 K_{\bar{q}}^0} \delta^{(2)}(-\boldsymbol{K}_{qT} - \boldsymbol{K}_{\bar{q}T}) = \frac{1}{4(2\pi)^6} \frac{dK_q^3}{K_q^0} d^2\boldsymbol{K}_T \frac{dK_{\bar{q}}^3}{K_{\bar{q}}^0}$$

$$\frac{dK_q^3}{K_q^0} = d\eta_q = \frac{d\zeta_q}{\zeta_q} \qquad d^2\boldsymbol{K}_T = K_T dK_T d\phi_q = (1/2) d\boldsymbol{K}_T^2 d\phi_q$$

$$\frac{d\sigma^{\ell^+\ell^- \to \ell'^+ h_1 h_2 X}}{dx_B dy d\zeta d\boldsymbol{K}_T^2 d\phi_q d\xi dz_1 d^2\boldsymbol{p}_{\perp 1} dz_2 d^2\boldsymbol{p}_{\perp 2}} = \frac{1}{2^9 \pi^4} \frac{1}{\zeta(1-\zeta)\xi s} \sum_q \sum_{\{\lambda_i\}} \tilde{\rho}_{\lambda_1,\lambda'_1}(\gamma_1^*) \rho_{\lambda_2,\lambda'_2}(\gamma_2) f_{\gamma/\ell^-,\mathcal{P}^{\ell^-}_{\hat{z}_-}}(\xi) \hat{H}_{\lambda_q,\lambda_{\bar{q}};\lambda_1,\lambda_2} \hat{H}^*_{\lambda'_q,\lambda'_{\bar{q}};\lambda'_1,\lambda'_2} \times \hat{D}^{h_1}_{\lambda_q,\lambda'_q}(z_1, \boldsymbol{p}_{\perp 1}) \hat{D}^{h_2}_{\lambda_{\bar{q}},\lambda'_{\bar{q}}}(z_2, \boldsymbol{p}_{\perp 2}) \delta\left(\xi - x_B - \frac{\boldsymbol{K}_T^2}{\zeta(1-\zeta) ys}\right),$$

Dynamical kernel – lepton and photon polarization states

Normalized helicity density matrix of virtual photon γ_1^* – for ratios as dN/N etc.

$$\rho(\gamma_1^*) = \frac{1}{2(2-y)^2} \times \begin{pmatrix} 1+(1-y)^2+\mathcal{P}^{\ell^+}_{\hat{z}_+} y(2-y) & -e^{-i\phi_\ell}\sqrt{2(1-y)}[(2-y)+\mathcal{P}^{\ell^+}_{\hat{z}_+} y] & -e^{-i2\phi_\ell}2(1-y) \\ -e^{i\phi_\ell}\sqrt{2(1-y)}[(2-y)+\mathcal{P}^{\ell^+}_{\hat{z}_+} y] & 4(1-y) & e^{-i\phi_\ell}\sqrt{2(1-y)}[(2-y)-\mathcal{P}^{\ell^+}_{\hat{z}_+} y] \\ -e^{i2\phi_\ell}2(1-y) & e^{i\phi_\ell}\sqrt{2(1-y)}[(2-y)-\mathcal{P}^{\ell^+}_{\hat{z}_+} y] & 1+(1-y)^2-\mathcal{P}^{\ell^+}_{\hat{z}_+} y(2-y) \end{pmatrix}$$

Normalization factor: reinsert it for cross sections etc.

$$\mathrm{Tr}[\tilde{\rho}] = \frac{2e^2(2-y)^2}{Q^2y^2} \equiv \frac{2e^2(2-y)^2}{x_B y^3 s}$$

Normalized helicity density matrix for quasi-real photon γ_2/ℓ^-

$$\rho(\gamma_2) = \frac{1}{2}\begin{pmatrix} 1+\mathcal{P}^{\gamma_2}_{\hat{z}_2} & 0 \\ 0 & 1-\mathcal{P}^{\gamma_2}_{\hat{z}_2} \end{pmatrix}$$

Quasi-real photon unpolarized and longitudinally polarized WW distributions

$$[\rho_{++}(\gamma_2)+\rho_{--}(\gamma_2)]f_{\gamma/\ell^-,\mathcal{P}^{\ell^-}_{\hat{z}_-}}(\xi) = f_{\gamma,+/\ell^-,\mathcal{P}^{\ell^-}_{\hat{z}_-}}(\xi) + f_{\gamma,-/\ell^-,\mathcal{P}^{\ell^-}_{\hat{z}_-}}(\xi) = f_{\gamma/\ell}(\xi),$$

$$[\rho_{++}(\gamma_2)-\rho_{--}(\gamma_2)]f_{\gamma/\ell^-,\mathcal{P}^{\ell^-}_{\hat{z}_-}}(\xi) = \mathcal{P}^{\gamma_2}_{\hat{z}_2} f_{\gamma/\ell^-,\mathcal{P}^{\ell^-}_{\hat{z}_-}}(\xi) = f_{\gamma,+/\ell^-,\mathcal{P}^{\ell^-}_{\hat{z}_-}}(\xi) - f_{\gamma,-/\ell^-,\mathcal{P}^{\ell^-}_{\hat{z}_-}}(\xi) = \mathcal{P}^{\ell^-}_{\hat{z}_-}\Delta_L f_{\gamma/\ell}(\xi)$$

For unpol. and longitudinally pol. WW photon disitributions, see e.g. D. De Florian, S. Frixione, PLB 457, 236 (1999)

Dynamical kernel – hard scattering $\gamma_1^*(q_1,\lambda_1)\gamma_2(q_2,\lambda_2) \to q(K_q,\lambda_q)\bar{q}(K_{\bar{q}},\lambda_{\bar{q}})$

Helicity amplitudes – massless quarks, parity conservation

$$\hat{H}_{+-;1,1} = -\hat{H}_{-+;-1,-1} = -2\sqrt{3}e^2e_q^2\frac{Q^2}{\hat{s}+Q^2}\sqrt{\frac{\hat{u}}{\hat{t}}} = -2\sqrt{3}e^2e_q^2\frac{x_B}{\xi}\sqrt{\frac{1-\zeta}{\zeta}},$$

$$\hat{H}_{+-;1,-1} = -\hat{H}^*_{-+;-1,1} = -2\sqrt{3}e^2e_q^2e^{i2\phi_q}\frac{\hat{s}}{\hat{s}+Q^2}\sqrt{\frac{\hat{u}}{\hat{t}}} = -2\sqrt{3}e^2e_q^2e^{i2\phi_q}\frac{\xi-x_B}{\xi}\sqrt{\frac{1-\zeta}{\zeta}},$$

$$\hat{H}_{+-;-1,1} = -\hat{H}^*_{-+;1,-1} = 2\sqrt{3}e^2e_q^2e^{-i2\phi_q}\frac{\hat{s}}{\hat{s}+Q^2}\sqrt{\frac{\hat{t}}{\hat{u}}} = \sqrt{3}e^2e_q^2e^{-i2\phi_q}\frac{\xi-x_B}{\xi}\sqrt{\frac{\zeta}{1-\zeta}},$$

$$\hat{H}_{+-;-1,-1} = -\hat{H}_{-+;1,1} = 2\sqrt{3}e^2e_q^2\frac{Q^2}{\hat{s}+Q^2}\sqrt{\frac{\hat{t}}{\hat{u}}} = 2\sqrt{3}e^2e_q^2\frac{x_B}{\xi}\sqrt{\frac{\zeta}{1-\zeta}},$$

$$\hat{H}_{+-;0,\pm1} = -\hat{H}_{-+;0,\pm1} = \pm2\sqrt{6}e^2e_q^2e^{\mp i\phi_q}\frac{\sqrt{\hat{s}}Q}{\hat{s}+Q^2} = \pm2\sqrt{6}e^2e_q^2e^{\mp i\phi_q}\frac{\sqrt{x_B(\xi-x_B)}}{\xi}.$$

Dynamical kernel – Fragmentation process and TMD FFs for spin zero/unpol. hadrons

TMD FFs for spin-zero and unpolarized hadrons – general definition in the helicity formalism

$$\hat{D}^{h/a}_{\lambda_a,\lambda'_a}(z,\boldsymbol{p}_\perp) = \sum_{\lambda_h} \sum\!\!\!\!\!\!\!\int_{X,\lambda_X} \hat{\mathcal{D}}_{\lambda_h,\lambda_X;\lambda_a}(z,\boldsymbol{p}_\perp)\hat{\mathcal{D}}^*_{\lambda_h,\lambda_X;\lambda'_a}(z,\boldsymbol{p}_\perp)$$

Only two independent LT FFs (for parity-conserving interactions) – Unpolarized and Collins FFs

$$\hat{D}^{h/a}_{++}(z,\boldsymbol{p}_\perp) = \hat{D}^{h/a}_{--}(z,\boldsymbol{p}_\perp) = D^h_a(z,p_\perp)$$

$$\hat{D}^{h/a}_{+-}(z,\boldsymbol{p}_\perp) = D^{h/a}_{+-}(z,p_\perp)e^{i\phi^h_a}$$

$$\hat{D}^{h/a}_{-+}(z,\boldsymbol{p}_\perp) = -[\hat{D}^{h/a}_{+-}(z,\boldsymbol{p}_\perp)]^* = -D^{h/a}_{+-}(z,p_\perp)e^{-i\phi^h_a}$$

$$\Delta^N D^h_{a^\uparrow}(z,p_\perp) = \frac{2p_\perp}{zm_h}H_1^{\perp,a}(z,p_\perp) = -i2D^{h/a}_{+-}(z,p_\perp)$$

Integrating over $p_\perp$ - collinear unpolarized FF and first $p_\perp$ moment of the Collins FF

$$\int \mathrm{d}^2\boldsymbol{p}_\perp D^h_a(z,p_\perp) = D^h_a(z)$$

$$\begin{aligned}\int \mathrm{d}^2\boldsymbol{p}_\perp \Delta^N D^h_{a^\uparrow}(z,p_\perp) &\equiv \int \mathrm{d}^2\boldsymbol{p}_\perp \frac{2p_\perp}{zm_h}H_1^{\perp,a}(z,p_\perp)\\ &= 2\pi\int \mathrm{d}p_\perp p_\perp \Delta^N D^h_{a^\uparrow}(z,p_\perp)\\ &= \Delta^N D^h_{a^\uparrow}(z) = 4H_1^{\perp(1/2)a}(z).\end{aligned}$$

Can be directly used in the jet thrust-axis configuration
The hadronic-plane conf. requires an explicit functional form of the transverse component of the FFs to perform $p_\perp$ integrations

Final expression of the differential cross section

$$\frac{d\sigma^{\ell^+\ell^-\to\ell'^+h_1h_2X}(\mathcal{P}_+,\mathcal{P}_-)}{dx_B\,dy\,d\zeta\,d\phi_q\,d\xi\,dz_1\,dz_2\,d\phi_{12}} = \frac{3\alpha^3}{8\pi^2}\frac{1}{x_B y^2\xi^3 s}\sum_q e_q^4\Big\{\Big[A_U + \mathcal{P}_+\mathcal{P}_- A_L + \Big(A_U^{\cos\phi_q} + \mathcal{P}_+\mathcal{P}_- A_L^{\cos\phi_q}\Big)\cos\phi_q$$
$$+ A_U^{\cos 2\phi_q}\cos 2\phi_q\Big] D_q^{h_1}(z_1) D_{\bar q}^{h_2}(z_2) + \Big[\Big(B_U^{\cos\phi_{12}} + \mathcal{P}_+\mathcal{P}_- B_L^{\cos\phi_{12}}\Big)\cos\phi_{12}$$
$$+ \Big(B_U^{\cos(\phi_q-\phi_{12})} + \mathcal{P}_+\mathcal{P}_- B_L^{\cos(\phi_q-\phi_{12})}\Big)\cos(\phi_q-\phi_{12})$$
$$+ \Big(B_U^{\cos(\phi_q+\phi_{12})} + \mathcal{P}_+\mathcal{P}_- B_L^{\cos(\phi_q+\phi_{12})}\Big)\cos(\phi_q+\phi_{12}) + B_U^{\cos(2\phi_q-\phi_{12})}\cos(2\phi_q-\phi_{12})$$
$$+ B_U^{\cos(2\phi_q+\phi_{12})}\cos(2\phi_q+\phi_{12})\Big]\Delta^N D_{q^\uparrow}^{h_1}(z_1)\Delta^N D_{\bar q^\uparrow}^{h_2}(z_2)\Big\}.$$

Azimuthal coefficients A_U, A_L for the unpolarized FF term

$$A_U = 2\left\{[1+(1-y)^2][x_B^2+(\xi-x_B)^2]\frac{1-2\zeta(1-\zeta)}{\zeta(1-\zeta)} + 16(1-y)x_B(\xi-x_B)\right\}f_{\gamma/\ell}(\xi)$$

$$A_U^{\cos\phi_q} = -8(2-y)\sqrt{1-y}(\xi-2x_B)\sqrt{x_B(\xi-x_B)}\frac{1-2\zeta}{\sqrt{\zeta(1-\zeta)}}f_{\gamma/\ell}(\xi),$$

$$A_U^{\cos 2\phi_q} = 16(1-y)x_B(\xi-x_B)f_{\gamma/\ell}(\xi),$$

$$A_L = -2y(2-y)\xi(\xi-2x_B)\frac{1-2\zeta(1-\zeta)}{\zeta(1-\zeta)}\Delta_L f_{\gamma/\ell}(\xi),$$

$$A_L^{\cos\phi_q} = 8y\sqrt{1-y}\xi\sqrt{x_B(\xi-x_B)}\frac{1-2\zeta}{\sqrt{\zeta(1-\zeta)}}\Delta_L f_{\gamma/\ell}(\xi).$$

Azimuthal coefficients B_U, B_L for the Collins FF term

$$B_U^{\cos\phi_{12}} = \{[1+(1-y)^2][x_B^2+(\xi-x_B)^2] - 8(1-y)x_B(\xi-x_B)\} f_{\gamma/\ell}(\xi)$$

$$B_U^{\cos(\phi_q-\phi_{12})} = -2(2-y)\sqrt{1-y}(\xi-2x_B)\sqrt{x_B(\xi-x_B)}\sqrt{\frac{\zeta}{1-\zeta}} f_{\gamma/\ell}(\xi),$$

$$B_U^{\cos(\phi_q+\phi_{12})} = 2(2-y)\sqrt{1-y}(\xi-2x_B)\sqrt{x_B(\xi-x_B)}\sqrt{\frac{1-\zeta}{\zeta}} f_{\gamma/\ell}(\xi),$$

$$B_U^{\cos(2\phi_q-\phi_{12})} = 2(1-y)x_B(\xi-x_B)\frac{\zeta}{1-\zeta} f_{\gamma/\ell}(\xi)$$

$$B_U^{\cos(2\phi_q+\phi_{12})} = 2(1-y)x_B(\xi-x_B)\frac{1-\zeta}{\zeta} f_{\gamma/\ell}(\xi),$$

$$B_L^{\cos\phi_{12}} = -y(2-y)\xi(\xi-2x_B)\Delta_L f_{\gamma/\ell}(\xi),$$

$$B_L^{\cos(\phi_q-\phi_{12})} = 2y\sqrt{1-y}\xi\sqrt{x_B(\xi-x_B)}\sqrt{\frac{\zeta}{1-\zeta}}\Delta_L f_{\gamma/\ell}(\xi),$$

$$B_L^{\cos(\phi_q+\phi_{12})} = -2y\sqrt{1-y}\xi\sqrt{x_B(\xi-x_B)}\sqrt{\frac{1-\zeta}{\zeta}}\Delta_L f_{\gamma/\ell}(\xi)$$

Double longitudinal spin asymmetry A_{LL}- azimuthal moments

$$\begin{aligned} d\sigma^{\text{unp}} &= \frac{1}{4}[d\sigma(1,1) + d\sigma(1,-1) + d\sigma(-1,1) + d\sigma(-1,-1)] \\ &= \frac{1}{2}[d\sigma(1,1) + d\sigma(1,-1)], \end{aligned}$$

$$\Delta_L \sigma = d\sigma(1,1) - d\sigma(1,-1) = d\sigma(-1,-1) - d\sigma(-1,+1).$$

$$A_{LL} = \frac{d\sigma(1,1) - d\sigma(1,-1)}{d\sigma(1,1) + d\sigma(1,-1)} = \frac{\Delta_L \sigma}{2 d\sigma^{\text{unp}}}$$

$$\langle d\sigma^{\text{unp}} | n_q; m_{12} \rangle = 2 \frac{\int d\phi_q d\phi_{12} d\sigma^{\text{unp}}(\phi_q, \phi_{12}) \cos[n_q \phi_q + m_{12}\phi_{12}]}{\int d\phi_q d\phi_{12} d\sigma^{\text{unp}}(\phi_q, \phi_{12})}$$

$$\langle A_{LL} | n_q; m_{12} \rangle = 2 \frac{\int d\phi_q d\phi_{12} A_{LL} d\sigma^{\text{unp}}(\phi_q, \phi_{12}) \cos[n_q \phi_q + m_{12}\phi_{12}]}{\int d\phi_q d\phi_{12} d\sigma^{\text{unp}}(\phi_q, \phi_{12})}$$

TABLE I. Summary of the relevant azimuthal moments of the unpolarized cross section $d\sigma^{\rm unp}$ and of the longitudinal azimuthal asymmetry A_{LL} according to Eqs. (32) and (33).

n_q	m_{12}	$\langle d\sigma^{\rm unp} \vert n_q; m_{12}\rangle$	$\langle A_{LL} \vert n_q; m_{12}\rangle$
0	0		$\frac{A_L}{A_U}$
± 1	0	$\frac{A_U^{\cos\phi_q}}{A_U}$	$\frac{A_L^{\cos\phi_q}}{A_U}$
± 2	0	$\frac{A_U^{\cos 2\phi_q}}{A_U}$	0
0	± 1	$\frac{B_U^{\cos\phi_{12}}}{A_U} \frac{\sum_q e_q^4 \Delta^N D_{q^\uparrow}^{h_1} \Delta^N D_{\bar{q}^\uparrow}^{h_2}}{\sum_q e_q^4 D_q^{h_1} D_{\bar{q}}^{h_2}}$	$\frac{B_L^{\cos\phi_{12}}}{A_U} \frac{\sum_q e_q^4 \Delta^N D_{q^\uparrow}^{h_1} \Delta^N D_{\bar{q}^\uparrow}^{h_2}}{\sum_q e_q^4 D_q^{h_1} D_{\bar{q}}^{h_2}}$
1	± 1	$\frac{B_U^{\cos(\phi_q \pm \phi_{12})}}{A_U} \frac{\sum_q e_q^4 \Delta^N D_{q^\uparrow}^{h_1} \Delta^N D_{\bar{q}^\uparrow}^{h_2}}{\sum_q e_q^4 D_q^{h_1} D_{\bar{q}}^{h_2}}$	$\frac{B_L^{\cos(\phi_q \pm \phi_{12})}}{A_U} \frac{\sum_q e_q^4 \Delta^N D_{q^\uparrow}^{h_1} \Delta^N D_{\bar{q}^\uparrow}^{h_2}}{\sum_q e_q^4 D_q^{h_1} D_{\bar{q}}^{h_2}}$
-1	∓ 1	"	"
2	± 1	$\frac{B_U^{\cos(2\phi_q \pm \phi_{12})}}{A_U} \frac{\sum_q e_q^4 \Delta^N D_{q^\uparrow}^{h_1} \Delta^N D_{\bar{q}^\uparrow}^{h_2}}{\sum_q e_q^4 D_q^{h_1} D_{\bar{q}}^{h_2}}$	0
-2	∓ 1	"	"

Kinematical constraints

To collect statistics, we can further integrate the (un)polarized cross section and (separately) the numerator and denominator of A_{LL} and the corresponding azimuthal moments over some of the remaining variables: x_B, y, ζ, ξ and, for the fragmentation sector, z_1, z_2 .

To safely remain in the region of validity of our TMD factorization approach and DIS regime for the virtual photon some kinematical constraints need to be fulfilled:

$$Q^2,\ \ K_T^2 = \zeta(1-\zeta)\hat{s} \ \geq\ Q_0^2 \quad \Rightarrow \quad \hat{s} \geq 4\,Q_0^2$$

$$Q_0^2 \leq Q^2 \leq s - 4Q_0^2,$$

$$\frac{Q^2}{s} \leq x_B \leq \frac{Q^2}{Q^2 + 4Q_0^2},$$

$$\frac{Q^2 + 4Q_0^2}{s} \leq y, \xi \leq 1.$$

$$\frac{1}{2}\left\{1 - \sqrt{1 - \frac{4Q_0^2}{\hat{s}}}\right\} \leq \zeta \leq \frac{1}{2}\left\{1 + \sqrt{1 - \frac{4Q_0^2}{\hat{s}}}\right\}$$

Within these constraints, for phenomenology it is important to identify kinematical regions that maximize the sensitivity to the asymmetries and the azimuthal moments . They can be different from case to case. This requires a dedicated study (currently in progress)

Conclusions and outlook

- Two almost back-to-back hadron production in (virtual and quasi-real) photon-photon collisions as complementary tool to SIDIS and e^+e^- SIA annihilations aiming at improving quark flavor separation for TMD (unpolarized and Collins) fragmentation functions

S. Anedda, FM, C. Pisano, PRD 112, 014013 (2025)

- Cross section and azimuthal distributions (moments) evaluated in a leading order and leading twist scheme within helicity approach in the jet thrust-axis kinematics

- TMD CSS evolution with scale and hadron-plane configuration (no jet thrust-axis required) already available in the same scheme from e^+e^- annihilations; can be easily implemented

- Extension to more general situations (spin zero + spin 1/2 hadrons, two spin 1/2 hadrons, polarized hadrons) also already available from e^+e^- annihilations

- Phenomenological studies starting now, in progress: two quasi-real photons at lepton colliders, UPCs at LHC first; studies for future circular (pA, AA, ee) and lepton linear colliders, laser Compton back-scattering

- Spontaneous Lambda polarization in pseudoscalar meson + Lambda hyperon production in e^+e^- collisions at Belle: several open points concerning isospin symmetry violation in the TMD polarizing Lambda FF and the role of charm quark contribution; SIDIS at EIC and this process in UPCs at LHC may help in clarifying these aspects

Belle Collab., PRL 122, 042001 (2019) - see e.g. U. D'Alesio, L. Gamberg, FM, M. Zaccheddu, PRD 108, 094004 (2023)

Thank you for your attention!