Exploring exclusive factorization beyond the leading twist regime within saturation physics

Lech Szymanowski

National Centre for Nuclear Research, Warsaw

in collaboration with

Renaud Boussarie, Michael Fucilla and Samuel Wallon

based on

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Content

Exploring exclusive factorization beyond the leading twist regime within saturation physics

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iiii. **Coordinate** and **momentum space** representations

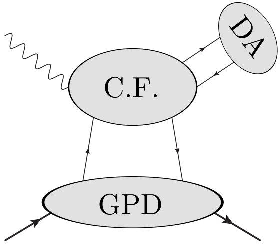
iiiii. Linearization **[Caron-Huot (2013)]** $\Longrightarrow$ **BFKL results**

Deeply virtual meson production (DVMP)

- Exclusive ρ-meson leptoproduction

  $\gamma^{(*)}(p_\gamma) + P(p_0) \to \rho(p_\rho) + P(p_0')$

- Extensively studied at HERA



- NLO corrections to the production of a longitudinally polarized ρ-meson at small-x

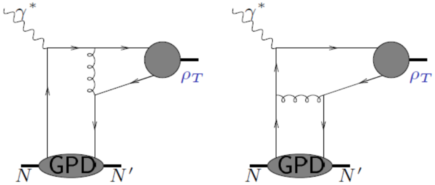
[Ivanov, Kotsky, Papa (2004)]

[Boussarie, Grabovsky, Ivanov, LS, Wallon (2017)]

[Mäntysaari, Pentalla (2022)]

Transversally polarized vector meson production starts at **twist-3**

- the dominant DA of ρ_T is of twist 2 and chiral-odd ($[\gamma^\mu, \gamma^\nu]$ coupling)
- *unfortunately* $\gamma^* \, N \to \rho_T \, N' = 0$
  - This cancellation is true at any order : such a process would require a helicity transfer of 2 from a photon.
  - lowest order diagrammatic argument:



$$\gamma^\alpha [\gamma^\mu, \gamma^\nu] \gamma_\alpha \to 0$$

[Diehl, Gousset, Pire (1999)], [Collins, Diehl (2000)]

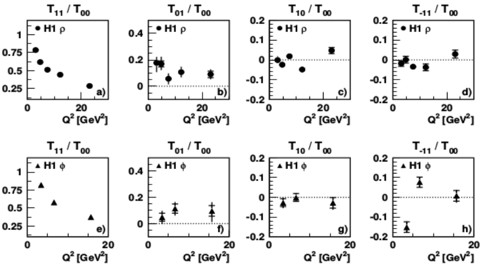
Collinear treatment at twist-3 leads to **end point singularities**

[Mankiewicz, Piller (2000)] **[Anikin, Teryaev (2002)]**

Transversely polarized vector meson production

- HERA data for the ρ and ϕ meson **[F.D. Aaron et al. (2010)]**

$$\gamma^*(\lambda_\gamma)p \to V(\lambda_V)p \qquad \lambda_\gamma = 0, 1, -1 \quad \text{and} \quad \lambda_V = 0, 1, -1$$



Transversely polarized ρ-meson production

- Momentum space impact factor for the exclusive ρ-meson production at the twist-3 in the dilute limit (BFKL scheme) and forward case

  [Anikin, Ivanov, Pire, LS, Wallon (2009)]

- Phenomenological studies at small-x

  [Besse, LS, Wallon (2013)]

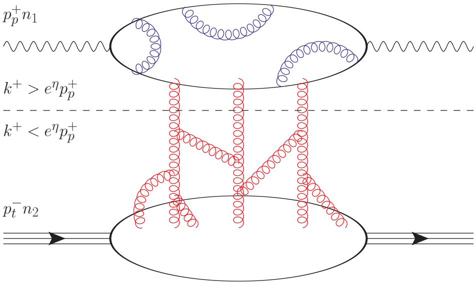
  [Bolognino, Celiberto, Ivanov, Papa (2018)]

  [Bolognino, Szczurek, Schäfer (2019)]

  [Mäntysaari, Pentalla (2022)]

Shockwave approach

- High-energy approximation $s = (p_p + p_t)^2 \gg \{Q^2\}$



$$p_p = p_p^+ n_1 - \frac{Q^2}{2p_p^+} n_2$$

$$p_t = \frac{m_t^2}{2p_t^-} n_1 + p_t^- n_2$$

$$p_p^+ \sim p_t^- \sim \sqrt{\frac{s}{2}}$$

$$n_1^2 = n_2^2 = 0 \quad n_1 \cdot n_2 = 1$$

- Separation of the gluonic field into "fast" (quantum) part and "slow" (classical) part through a rapidity parameter $\eta < 0$

[I. Balitsky (1996-2001)]

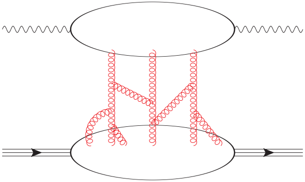
$$\mathcal{A}^\mu(k^+, k^-, \vec{k}) = A^\mu(k^+ > e^\eta p_p^+, k^-, \vec{k}) + b^\mu(k^+ < e^\eta p_p^+, k^-, \vec{k})$$

$e^\eta \ll 1$

Shockwave approach

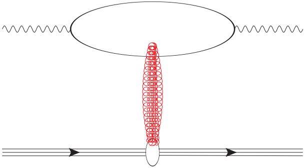
- Large longitudinal boost: $\Lambda = \sqrt{\frac{1+\beta}{1-\beta}} \sim \frac{\sqrt{s}}{m_t}$

$$\begin{cases} b^+(x^+, x^-, \vec{x}) & = \Lambda^{-1} b_0^+(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \\ b^-(x^+, x^-, \vec{x}) & = \Lambda b_0^-(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \\ b^i(x^+, x^-, \vec{x}) & = b_0^i(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \end{cases}$$



$b_0^\mu(x)$

boost $\longrightarrow$



$b^\mu(x^+, x^-, \vec{x}) = \delta(x^+) \mathbf{B}(\vec{x}) n_2^\mu + \mathcal{O}\left(\Lambda^{-1}\right)$

Shockwave approximation

- Light-cone gauge $A \cdot n_2 = 0$

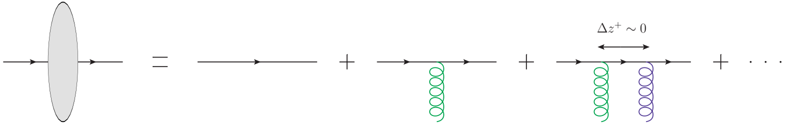
$A \cdot b = 0 \implies$ *Simple effective Lagrangian*

Shockwave approach

- Interactions with the simple shockwave field

  i. Independence from x^- $\Longrightarrow$ conservation of p^+ (**eikonal** approx.)
  ii. $\delta(x^+)$ $\Longrightarrow$ interactions at a **single transverse coordinate**.

- Quark line through the shockwave



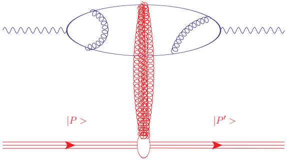
$$V_{\vec{z}_i} = 1 + ig \int_{-\infty}^{+\infty} dz_i^+ b_\eta^- \left(z_i^+, \vec{z}_i\right) + (ig)^2 \int_{-\infty}^{+\infty} dz_i^+ dz_j^+ b_\eta^- \left(z_i^+, \vec{z}_i\right) b_\eta^- \left(z_j^+, \vec{z}_i\right) \theta\left(z_{ij}^+\right) + \cdots$$

- Multiple interactions with the target → ***path-ordered Wilson lines***

$$V_{\vec{z}}^\eta = \mathcal{P} \exp\left[ig \int_{-\infty}^{+\infty} dz_i^+ b_\eta^- \left(z_i^+, \vec{z}\right)\right]$$

Shockwave approach

- Factorization in the shockwave approximation



$$\mathcal{M}^\eta = N_c \int d^d \boldsymbol{z}_1 d^d \boldsymbol{z}_2 \, \Phi^\eta(\boldsymbol{z}_1, \boldsymbol{z}_2) \langle P' \left| \mathcal{U}^\eta_{12}(\boldsymbol{z}_1, \boldsymbol{z}_2) \right| P \rangle$$

- **Dipole operator** $\mathcal{U}^\eta_{ij} = 1 - \frac{1}{N_c} \mathrm{Tr}\left(V^\eta_{\vec{z}_i} V^{\eta\dagger}_{\vec{z}_j} \right)$
- Evolution equations
  - **Balitsky-JIMWLK** evolution equations
    [Balitsky (1995)]
    [Jalilian-Marian, Iancu, McLerran, Weigert, Kovner, Leonidov]
  - Large $N_c \rightarrow$ **Balitky-Kovchegov (BK)** non-linear equation
    [Balitsky (1995)] [Kovchegov (1999)]
  - Evolution at the NLO
    [Balitsky, Chirilli (2007)] [Kovner, Lublinsky, Mulian (2013)]

Theoretical framework

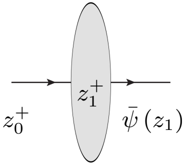
- **Effective background field operator** formalism of small-x physics

$$[\psi_{\text{eff}}\ (z_0)]_{z_0^+<0} = \psi\,(z_0) - \int \mathrm{d}^D z_2 G_0\,(z_{02})\left(V^\dagger_{\boldsymbol{z}_2} - 1\right)\gamma^+\psi\,(z_2)\,\delta(z_2^+)$$

$$\left[\bar{\psi}_{\text{eff}}\ (z_0)\right]_{z_0^+<0} = \bar{\psi}\,(z_0) + \int \mathrm{d}^D z_1 \bar{\psi}\,(z_1)\,\gamma^+\,(V_{\boldsymbol{z}_1} - 1)\,G_0\,(z_{10})\,\delta(z_1^+)$$

$$\left[A^{\mu a}_{\text{eff}}\ (z_0)\right]_{z_0^+<0} = A^{\mu a}\,(z_0) + 2i\int \mathrm{d}^D z_3 \delta(z_3^+) F^b_{-\sigma}\,(z_3)\,G^{\mu\sigma_\perp}\,(z_{30})\left(U^{ab}_{\boldsymbol{z}_3} - \delta^{ab}\right)$$

e.g. of antiquark effective operator:



free quark propagator: $G_0(z) = \int \frac{d^D l}{(2\pi)^D} e^{-il\cdot z} \frac{i\not{k}}{k^2+i0}$

- A fermionic line starts at the light-cone time $z_0^+ < 0$
- freely propagates to z_1^+
- it interacts eikonally at z_1^+ with the background shockwave field.

Effective background field operators cntd

- Such operators serve to construct amplitudes involving non-perturbative matrix elements of general off light-cone correlators, i.e. **without any reference to the twist-expansion**
- Proof by induction
- Shockwave effective Feynman rules are reproduced

$$\left[v_{\alpha}^{ij}\left(p_{\bar{q}}, z_0\right)\right]_{z_0^+<0} \equiv \left[\psi_{\mathrm{eff},\alpha}^{j}\left(z_0\right)\right]_{z_0^+<0} \left|i, p_{\bar{q}}\right\rangle = -\frac{(-i)^{d/2}}{2(2\pi)^{d/2}} \left(\frac{p_{\bar{q}}^+}{-z_0^+}\right)^{d/2} \theta(p_{\bar{q}}^+)\theta(-z_0^+)$$

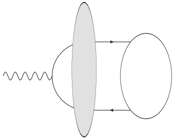
$$\times \int d^d z_2 V_{\vec{z}_2}^{ij\dagger} \frac{-z_0^+\gamma^- + \hat{z}_{20\perp}}{-z_0^+} \gamma^+ \frac{v(p_{\bar{q}})}{\sqrt{2p_{\bar{q}}^+}} \exp\left\{ i p_{\bar{q}}^+ \left(z_0^- - \frac{\vec{z}_{20}^{\,2}}{2z_0^+} + i0 \right) - i\vec{p}_{\bar{q}} \cdot \vec{z}_{20} \right\}$$

$$G_{ij}\left(z_2, z_0\right)\Big|_{z_2^+>0>z_0^+} \equiv \overbrace{\psi_i(z_2)\left[\overline{\psi}\right.}_{\mathrm{eff},j}\left.\left(z_0\right)\right]_{z_0^+<0}$$

$$= \frac{i\Gamma(d+1)}{4(2\pi)^{d+1}} \int d^d \vec{z}_1 V_{ij}\left(\vec{z}_1\right) \frac{\left(z_2^+\gamma^- + \hat{z}_{21\perp}\right)\gamma^+\left(-z_0^+\gamma^- + \hat{z}_{10\perp}\right)}{\left(-z_0^+ z_2^+\right)^{\frac{D}{2}} \left(-z_{20}^- + \frac{\vec{z}_{21}^2}{2z_2^+} - \frac{\vec{z}_{10}^2}{2z_0^+} + i\varepsilon\right)^{d+1}} \theta(z_2^+)\theta(-z_0^+)$$

ρ-meson production: diagrams

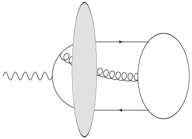
- **Two-body contribution**



i. Dependence of the leading Fock state wave function – with a minimal number of (valence) partons – on **transverse momentum**

$$\mathcal{A}_2 = -ie_f \int \mathrm{d}^D z_0 \theta(-z_0^+) \left\langle P\left(p'\right) M\left(p_M\right) \left| \overline{\psi}_{\mathrm{eff}}\left(z_0\right) \hat{\varepsilon}_q \mathrm{e}^{-i(q\cdot z_0)} \psi_{\mathrm{eff}}\left(z_0\right) \right| P\left(p\right) \right\rangle$$

- **Three-body contribution**

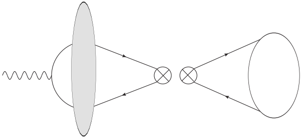


i. Distribution with a **non-minimal parton configuration**

$$\mathcal{A}_{3,q} = \left(-ie_q\right)\left(ig\right) \int \mathrm{d}^D z_4 \mathrm{d}^D z_0 \theta(-z_4^+)\theta(-z_0^+)$$
$$\times \left\langle P\left(p'\right) M\left(p_M\right) \left| \overline{\psi}_{\mathrm{eff}}\left(z_4\right) \gamma_\mu A^{\mu a}_{\mathrm{eff}}(z_4) t^a G(z_{40}) \hat{\varepsilon}_q \mathrm{e}^{-i(q\cdot z_0)} \psi_{\mathrm{eff}}\left(z_0\right) \right| P\left(p\right) \right\rangle$$

ρ-meson production: factorization

- **Two-body contribution**



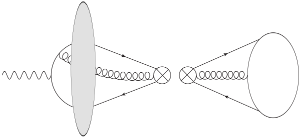
$$\mathcal{A}_2 = ie_f \int \mathrm{d}^D z_0 \int \mathrm{d}^D z_1 \int \mathrm{d}^D z_2 \theta(-z_0^+)\delta(z_1^+)\delta(z_2^+) \left\langle M\left(p_M\right) \left| \overline{\psi}\left(z_1\right) \Gamma^\lambda \psi\left(z_2\right) \right| 0 \right\rangle$$

$$\times \left\langle P\left(p'\right) \left| 1 - \frac{1}{N_c} \mathrm{tr}\left(V_{\boldsymbol{z}_1} V_{\boldsymbol{z}_2}^\dagger\right) \right| P\left(p\right) \right\rangle \frac{1}{4} \mathrm{tr}_D \left[\gamma^+ G_0\left(z_{10}\right) \hat{\varepsilon}_q \mathrm{e}^{-i(q \cdot z_0)} G_0\left(z_{02}\right) \gamma^+ \Gamma_\lambda\right]$$

hard part

ρ-meson production: factorization

- **Three-body contribution**



$$\mathcal{A}_{q3} = -ie_q \int \mathrm{d}^D z_4 \mathrm{d}^D z_3 \mathrm{d}^D z_2 \mathrm{d}^D z_1 \mathrm{d}^D z_0 \theta(-z_4^+)\delta(z_3^+)\delta(z_2^+)\delta(z_1^+)\theta\left(-z_0^+\right) \mathrm{e}^{-i(q\cdot z_0)}$$
$$\times \left\langle P\left(p'\right) \left| \mathrm{tr}\left(V_{\boldsymbol{z}_1} t^a V^\dagger_{\boldsymbol{z}_2} t^b U^{ab}_{\boldsymbol{z}_3}\right) \right| P\left(p\right) \right\rangle \left\langle M\left(p_M\right) \left| \overline{\psi}\left(z_1\right) \Gamma^\lambda g F_{-\sigma}\left(z_3\right) \psi\left(z_2\right) \right| 0 \right\rangle$$
$$\times \frac{1}{N_c^2-1} \mathrm{tr}_D \left[\gamma^+ G_0\left(z_{14}\right) \gamma_\mu G^{\mu\sigma_\perp}\left(z_{34}\right) G_0\left(z_{40}\right) \hat{\varepsilon}_q G_0\left(z_{02}\right) \gamma^+ \Gamma_\lambda\right] - \text{n.i.}$$

hard part

Expression of $U^{ab}_{\boldsymbol{z_3}}$ in adjoint through fundamental representation:

$$\mathrm{tr}\left(V_{\boldsymbol{z}_1} t^a U^{ab}_{\boldsymbol{z}_3} V^\dagger_{\boldsymbol{z}_2} t^b\right) = \frac{1}{2}\left[\mathrm{tr}\left(V_{\boldsymbol{z}_1} V^\dagger_{\boldsymbol{z}_3}\right) \mathrm{tr}\left(V_{\boldsymbol{z}_3} V^\dagger_{\boldsymbol{z}_2}\right) - \frac{1}{N_c}\mathrm{tr}\left(V_{\boldsymbol{z}_1} V^\dagger_{\boldsymbol{z}_2}\right)\right].$$

Results: two-body contribution

- **Dipole amplitude**

$$\mathcal{A}_2 = \int_0^1 \mathrm{d}x \int \mathrm{d}^2\boldsymbol{r}\Psi_2\left(x,\boldsymbol{r}\right) \int \mathrm{d}^d\boldsymbol{b}\ \mathrm{e}^{i(\boldsymbol{q}-\boldsymbol{p}_M)\cdot\boldsymbol{b}} \left\langle P\left(p'\right) \middle| 1 - \frac{1}{N_c}\mathrm{tr}\left(V_{\boldsymbol{b}+\bar{x}\boldsymbol{r}} V^\dagger_{\boldsymbol{b}-x\boldsymbol{r}}\right) \middle| P\left(p\right) \right\rangle$$

- **Coordinate-space impact factor**

$$\Psi_2\left(x,\boldsymbol{r}\right) = e_q \delta\left(1 - \frac{p_M^+}{q^+}\right)\left(\varepsilon_{q\mu} - \frac{\varepsilon_q^+}{q^+} q_\mu\right)$$

$$\times\left[\phi_{\gamma^+}(x,\boldsymbol{r})\left(2x\bar{x}q^\mu - i(x-\bar{x})\frac{\partial}{\partial r_{\perp\mu}}\right) + \epsilon^{\mu\nu+-}\phi_{\gamma^+\gamma^5}(x,\boldsymbol{r})\frac{\partial}{\partial r_\perp^\nu}\right] K_0\left(\sqrt{x\bar{x}Q^2\boldsymbol{r}^2}\right)$$

- Two-body vacuum to meson matrix elements

$$\phi_{\gamma^+}(x,\boldsymbol{r}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dr^- e^{ixp_M^+ r^-} \left\langle M\left(p_M\right) \middle| \overline{\psi}\left(r\right)\gamma^+\psi\left(0\right) \middle| 0 \right\rangle_{r^+=0}$$

$$\phi_{\gamma^+\gamma^5}(x,\boldsymbol{r}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dr^- e^{ixp_M^+ r^-} \left\langle M\left(p_M\right) \middle| \overline{\psi}\left(r\right)\gamma^+\gamma^5\psi\left(0\right) \middle| 0 \right\rangle_{r^+=0}$$

at this stage, r^2 is arbitrary, in principle off the light-cone.

Results: three-body contribution

$$\mathcal{U}^{\eta}_{ij} = 1 - N_c^{-1}\,\mathrm{Tr}\left(V^{\eta}_{\vec{z}_i} V^{\eta\dagger}_{\vec{z}_j}\right)$$

- Three-body amplitude: involves dipole and double dipole contributions

$$\mathcal{A}_3 = \left(\prod_{i=1}^{3}\int dx_i\theta(x_i)\right)\delta(1-x_1-x_2-x_3)\int d^2\boldsymbol{z}_1 d^2\boldsymbol{z}_2 d^2\boldsymbol{z}_3 e^{i\boldsymbol{q}(x_1\boldsymbol{z}_1+x_2\boldsymbol{z}_2+x_3\boldsymbol{z}_3)}$$

$$\times\Psi_3(x_1,x_2,x_3,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{z}_3)\left\langle P(p')\left|\mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_3}\mathcal{U}_{\boldsymbol{z}_3\boldsymbol{z}_2} - \mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_3} - \mathcal{U}_{\boldsymbol{z}_3\boldsymbol{z}_2} + \frac{1}{N_c^2}\mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_2}\right|P(p)\right\rangle$$

- **Coordinate-space impact factor** (with $Z = \sqrt{x_1x_2\boldsymbol{z}_{12}^2 + x_1x_3\boldsymbol{z}_{13}^2 + x_2x_3\boldsymbol{z}_{23}^2}$)

$$\Psi_3(x_1,x_2,x_3,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{z}_3) = \frac{e_q q^+}{2(4\pi)}\frac{N_c^2}{N_c^2-1}\left(\varepsilon_{q\rho} - \frac{\varepsilon_q^+}{q^+}q_\rho\right)$$

$$\times\left\{\chi_{\gamma^+\sigma}\left[\left(4ig_{\perp\perp}^{\rho\sigma}\frac{x_1x_2}{1-x_2}\frac{Q}{Z}K_1(QZ) + T_1^{\sigma\rho\nu}(x_1,x_2,x_3)\frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2}K_0(QZ)\right) - (1\leftrightarrow 2)\right]\right.$$

$$\left.-\chi_{\gamma^+\gamma^5\sigma}\left[\left(4\epsilon^{\sigma\rho+-}\frac{x_1x_2}{1-x_2}\frac{Q}{Z}K_1(QZ) + T_2^{\sigma\rho\nu}(x_1,x_2,x_3)\frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2}K_0(QZ)\right) + (1\leftrightarrow 2)\right]\right\}$$

- Three-body vacuum to meson non-perturbative matrix elements

$$\chi_{\Gamma^\lambda,\sigma} \equiv \chi_{\Gamma^\lambda,\sigma}(x_1,x_2,x_3,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{z}_3) =$$

$$\int_{-\infty}^{\infty}\frac{\mathrm{d}z_1^-}{2\pi}\frac{\mathrm{d}z_2^-}{2\pi}\frac{\mathrm{d}z_3^-}{2\pi}\mathrm{e}^{-ix_1q^+z_1^- - ix_2q^+z_2^- - ix_3q^+z_3^-}\left\langle M(p_M)\left|\overline{\psi}(z_1)\Gamma^\lambda gF_{-\sigma}(z_3)\psi(z_2)\right|0\right\rangle_{z^+_{1,2,3}=0}$$

Again, at this stage, z_1^2, z_2^2, z_3^2 are arbitrary, in principle off the light-cone.

Light-cone collinear factorization

$$T_1^{\sigma\rho\nu}(\{x_i\}) = 4\left[2\frac{x_1(\bar{x}_1+x_2)}{x_3}g_{\perp\perp}^{\sigma\nu}q^\rho + \left(\frac{(\bar{x}_1+x_2)(\bar{x}_1-x_1)}{\bar{x}_1 x_3}g_{\perp\perp}^{\sigma\nu}g_{\perp\perp}^{\rho\mu} - \frac{1}{\bar{x}_1}\left(g_{\perp\perp}^{\nu\rho}g_{\perp\perp}^{\sigma\mu} - g_{\perp\perp}^{\rho\sigma}g_{\perp\perp}^{\nu\mu}\right)\right) i\partial_{z_{1\perp}\mu}\right]$$

$$T_2^{\sigma\rho\nu}(\{x_i\}) = \frac{4i}{\bar{x}_1}\left[2x_1\bar{x}_1 q^\rho \epsilon^{\nu\sigma+-} - \left(\left(1+\frac{2x_2}{x_3}\right)\left(g_{\perp\perp}^{\sigma\mu}\epsilon^{\nu\rho+-} - g_{\perp\perp}^{\rho\sigma}\epsilon^{\nu\mu+-}\right) + (x_1-\bar{x}_1)g_{\perp\perp}^{\rho\mu}\epsilon^{\nu\sigma+-}\right) i\partial_{z_{1\perp}\mu}\right]$$

Light-cone collinear factorization

- *Light-cone collinear factorization*

  [Ellis, Furmanski, Petronzio (1982)] [Anikin, Teryaev (2002)]
- Factorization around the dominant **light-cone direction** is naturally implemented in **momentum space**
- **Overcomplete set of distributions** must be reduced exploiting QCD equations of motion

$$\langle i(\hat{D}(0)\psi(0))_\alpha \bar{\psi}_\beta(z)\rangle = 0 \qquad \langle i\psi_\alpha(0)(\bar{\psi}(z)\overleftarrow{\hat{D}}(z))_\beta\rangle = 0$$

- Invariance of the amplitude under **rotation on the light-cone**

  [Anikin, Ivanov, Pire, LS, Wallon (2009)]

  i. Independence of the amplitude from the choice of n

  ii. Given a “natural” choice n_0, we can define

$$n^\mu = \alpha p^\mu + \beta n_0^\mu + n_\perp^\mu$$

  iii. Imposing $p \cdot n = 1$ and $n^2 = 0 \longrightarrow \beta = 1\, , \alpha = -n_\perp^2/2$

  iiii. The freedom is parametrized in terms of the transverse component

$$\frac{\partial \mathcal{A}}{\partial n_\perp^\mu} = 0$$

Covariant collinear factorization = non-local OPE (1)

- expansion in powers of the hard scale
  = expansion of **string operators** in powers of deviation from the light-cone
  [Balitsky, Braun (1989)]

  e.g. **up to twist 3:**

  - *2-body:* expansion in powers of r^2 $(r^2 \to 0)$ of

$$\left\langle M(p_M) \left| \overline{\psi}(r)\, \gamma^+ \psi(0) \right| 0 \right\rangle_{r^+=0} \quad \text{and} \quad \left\langle M(p_M) \left| \overline{\psi}(r)\, \gamma^+ \gamma^5 \psi(0) \right| 0 \right\rangle_{r^+=0}$$

  - *3-body:* expansion in powers of $(z_3 - z_1)^2, (z_2 - z_3)^2$
    $((z_3 - z_1)^2, (z_2 - z_3)^2 \to 0)$ of

$$\left\langle M(p_M) \left| \overline{\psi}(z_1)\, \Gamma^\lambda g F_{-\sigma}(z_3)\, \psi(z_2) \right| 0 \right\rangle_{z^+_{1,2,3}=0}$$

- each coefficient of this OPE expansion
  = finite sum of on-light-cone **non-local** correlators
- for each term in this Taylor expansion:
  vacuum-to-meson matrix elements contribute to **different kinematic twist:**
  - matrix element = linear combination of $p_{M\mu}$, r_μ, $\varepsilon^*_{M\mu}$ **(now** $r^2 = 0$**)**
  - coefficients depend on the available Lorentz inv. $p_M \cdot r$, $\varepsilon_M \cdot r$, m_M^2
  - these quantities have **different scaling** in the $Q \to \infty$ limit

Covariant collinear factorization = non-local OPE (2)

- example: *parametrization of the 2-body vector matrix element* (up to twist 3)

$$\left\langle M(p_M)\left|\overline{\psi}(r)\,\gamma^\mu[r,0]\psi(0)\right|0\right\rangle|_{r^2=0}$$

$$\sim f_M m_M \int_0^1 \mathrm{d}x e^{ix(p_M\cdot r)}\left[p_M^\mu \frac{(\varepsilon_M^*\cdot r)}{(p_M\cdot r)}\phi(x) + \varepsilon_{M,T}^{*\mu}\, g_\perp^{(v)}(x)\right]$$

$\sim Q$ twist 2 — ~ 1 twist 3

- **up to twist 3:**
  only the first term in the Taylor expansion of the off-light-cone matrix elements survives
  the next one is twist 4, i.e. z^2 suppressed
  we keep twist 2 and twist 3 terms:
  2-body: $\rightarrow$ twist-2 and twist-3 (kinematic) (see above example)
  3-body: $\rightarrow$ twist-3 (genuine)

Covariant collinear factorization

- **Covariant collinear factorization**

[Braun, Filyanov (1990)]

[Ball, Braun, Koike, Tanaka (1998)]

  i. Minimal basis of *independent distributions*

  ii. *Minimal numbers of parameters*

  iii. Easy to perform the calculation directly into coordinate space

- 2 and 3-body operators in gauge invariant form, on the light-cone $z^2 = 0$

$$\langle M(p_M)|\overline{\psi}(z)\Gamma_\lambda \, [z,0] \, \psi(0)|0\rangle$$
$$\langle M(p_M)|\overline{\psi}(z)\gamma_\lambda \, [z,tz] \, gF^{\mu\nu}(tz) \, [tz,0] \, \psi(0)|0\rangle$$
$$\langle M(p_M)|\overline{\psi}(z)\gamma_\lambda \, [z,tz] \, g\tilde{F}^{\mu\nu}(tz) \, [tz,0] \, \psi(0)|0\rangle$$

where

$$[z,0] = \mathcal{P}_{\exp}\left[ig\int_0^1 dt A^\mu \,(tz)\, z_\mu\right]$$

Results: twist 3 expansion

A subtlety: making contact with covariant collinear factorization

- before twist expansion, our result does do not contain gauge links between fields
- this should be taken into account, through:

$$\mathcal{P}\exp\left[ig\int_0^1 dt A^\mu(tz)z_\mu\right] = 1 + ig\int_0^1 dt A^\mu(tz)\, z_\mu + \dots\,,$$

- it does not affect the 3-body twist-3 result
- it does contribute to the 2-body twist-3 result

Results

Reminder:

- **Dipole amplitude**

$$\mathcal{A}_2 = \int_0^1 \mathrm{d}x \int \mathrm{d}^2\boldsymbol{r}\, \Psi_2(x, \boldsymbol{r}) \int \mathrm{d}^d\boldsymbol{b}\, \mathrm{e}^{i(\boldsymbol{q}-\boldsymbol{p}_M)\cdot\boldsymbol{b}} \left\langle P(p') \left| 1 - \frac{1}{N_c}\mathrm{tr}\left(V_{\boldsymbol{b}+\bar{x}\boldsymbol{r}} V^\dagger_{\boldsymbol{b}-x\boldsymbol{r}}\right) \right| P(p) \right\rangle$$

- Three-body amplitude: with **dipole** and **double dipole** contributions

$$\mathcal{A}_3 = \left(\prod_{i=1}^{3} \int dx_i \theta(x_i)\right) \delta(1 - x_1 - x_2 - x_3) \int d^2\boldsymbol{z}_1 d^2\boldsymbol{z}_2 d^2\boldsymbol{z}_3 e^{i\boldsymbol{q}(x_1\boldsymbol{z}_1 + x_2\boldsymbol{z}_2 + x_3\boldsymbol{z}_3)}$$

$$\times \Psi_3(x_1, x_2, x_3, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{z}_3) \left\langle P(p') \left| \mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_3}\mathcal{U}_{\boldsymbol{z}_3\boldsymbol{z}_2} - \mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_3} - \mathcal{U}_{\boldsymbol{z}_3\boldsymbol{z}_2} + \frac{1}{N_c^2}\mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_2} \right| P(p) \right\rangle$$

Results: twist 3 expansion

2-body twist-3 expanded result

coordinate space

$$\Psi_2(x,\boldsymbol{r}) = e_q m_M f_M \delta \left(1-\frac{p_M^+}{q^+}\right)\left(\varepsilon_{q\mu}-\frac{\varepsilon_q^+}{q^+}q_\mu\right)\left(\varepsilon^*_{M\alpha}-\frac{\varepsilon_M^{*+}}{p_M^+}p_{M\alpha}\right)$$

$$\times\left[-ir_\perp^\alpha\left(h(x)-\tilde{h}(x)\right)\left(2x\bar{x}q^\mu+(x-\bar{x})\frac{-i\partial}{\partial r_{\perp\mu}}\right)\right.$$

$$\left.+\epsilon^{\mu\nu+-}\epsilon^{+\alpha-\delta}r_{\perp\delta}\left(\frac{g_\perp^{(a)}(x)-\tilde{g}_\perp^{(a)}(x)}{4}\right)\frac{\partial}{\partial r_\perp^\nu}\right]K_0\left(\sqrt{x\bar{x}Q^2\boldsymbol{r}^2}\right),$$

with

$$h(x)=\int_0^x \mathrm{d}u\left(\phi(u)-g_\perp^{(v)}(u)\right),$$

$$\widetilde{h}(x)=\frac{f_{3M}^V}{f_M}\int_0^x \mathrm{d}x_q\int_0^{1-x}\mathrm{d}x_{\overline{q}}\,\frac{V\left(x_q,x_{\overline{q}}\right)}{\left(1-x_q-x_{\overline{q}}\right)^2},$$

$$\widetilde{g}_\perp^{(a)}(x)=4\frac{f_{3M}^A}{f_M}\int_0^x \mathrm{d}x_q\int_0^{1-x}\mathrm{d}x_{\overline{q}}\,\frac{A\left(x_q,x_{\overline{q}}\right)}{(1-x_q-x_{\overline{q}}+i\epsilon)^2}.$$

Results: twist 3 expansion

3-body twist-3 expanded result

coordinate space

$$\Psi_3\left(x_1, x_2, x_3, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{z}_3\right)$$

$$= \frac{e_q m_M c_f}{8\pi} \delta \left(1 - \frac{p_M^+}{q^+}\right) \left(\varepsilon_{q\rho} - \frac{\varepsilon_q^+}{q^+} q_\rho\right) \left(\varepsilon_M^{*\mu} - \frac{p_M^\mu}{p_M^+} \varepsilon_M^{*+}\right) \left(\prod_{j=1}^{3} \theta(x_j)\theta(1-x_j) e^{-i x_j \boldsymbol{p}_M \boldsymbol{z}_j}\right)$$

$$\times \Big\{ - i f_{3M}^V g_{\sigma\mu} V(x_1, x_2) \left[\left(4 i g_{\perp\perp}^{\rho\sigma} \frac{x_1 x_2}{1-x_2} \frac{Q}{Z} K_1(QZ) + T_1^{\sigma\rho\nu}(\{x_i\}) \frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2} K_0(QZ)\right) - (1 \leftrightarrow 2)\right]$$

$$- \epsilon_{-+\sigma\beta} f_{3M}^A g_{\perp\perp\mu}^{\beta} A(x_1, x_2) \left[\left(4 \epsilon^{\sigma\rho+-} \frac{x_1 x_2}{1-x_2} \frac{Q}{Z} K_1(QZ) + T_2^{\sigma\rho\nu}(\{x_i\}) \frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2} K_0(QZ)\right) + (1 \leftrightarrow 2)\right]$$

$V(x_1, x_2)$ = genuine twist-3 vector DAs

$A(x_1, x_2)$ = genuine twist-3 axial DAs

f_M^V and f_M^A = normalization constants

3-body twist-3 expanded result

momentum space

- Fourier transform

$$\Phi_3\left(\{x\},\{\boldsymbol{p}\}\right) = \left(\prod_{j=1}^{3}\int d^2\boldsymbol{z}_j e^{-i\boldsymbol{z}_j\boldsymbol{p}_j}\right)\Psi_3\left(\{x\},\{\boldsymbol{z}\}\right)$$

- **Momentum space impact factor** (after twist expansion)

$$\Phi_3\left(\{x\},\{\boldsymbol{p}\}\right) = \frac{e_q m_M}{4}c_f\left(\varepsilon_{q\rho} - \frac{\varepsilon_q^+}{q^+}q_\rho\right)\left(\varepsilon_M^{*\beta} - \frac{p_M^\beta}{p_M^+}\varepsilon_M^{*+}\right)\delta\left(1-\frac{p_M^+}{q^+}\right)$$
$$\times\left(\prod_{j=1}^{3}\frac{\theta(1-x_j)\theta(x_j)}{x_j}\right)\frac{(2\pi)^3\delta^{(2)}\left(\sum_{i=1}^3\boldsymbol{p}_i + x_i\boldsymbol{p}_M\right)}{\left[Q^2+\sum_{i=1}^3(\boldsymbol{p}_i+x_i\boldsymbol{p}_M)^2/x_i\right]}\Bigg\{g_{\beta\sigma}f_{3M}^V V(x_1,x_2)\left(4g_{\perp\perp}^{\rho\sigma}\frac{x_1x_2}{1-x_2}\right.$$
$$\left.+\tilde{T}_1^{\sigma\rho\nu}(\{x\})\big|_{\boldsymbol{k}_i=-x_i\boldsymbol{p}_M}\frac{x_1x_2(p_3+x_3p_M)_{\perp\nu} - x_1x_3(p_2+x_2p_M)_{\perp\nu}}{(\boldsymbol{p}_1+x_1\boldsymbol{p}_M)^2+x_1(1-x_1)Q^2}\right) - \epsilon_{-+\sigma\beta}f_{3M}^A A(x_1,x_2)$$
$$\times\left(4\frac{x_1x_2}{1-x_2}\epsilon^{\sigma\rho+-} + i\tilde{T}_2^{\sigma\rho\nu}(\{x\})\big|_{\boldsymbol{k}_i=-x_i\boldsymbol{p}_M}\frac{x_1x_2(p_3+x_3p_M)_{\perp\nu} - x_1x_3(p_2+x_2p_M)_{\perp\nu}}{(\boldsymbol{p}_1+x_1\boldsymbol{p}_M)^2+x_1(1-x_1)Q^2}\right)\Bigg\}$$
$$+(1\leftrightarrow 2)$$

3-body twist-3 expanded result cntd

momentum space

where

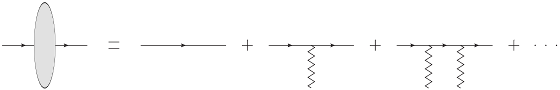
$$\tilde{T}_1^{\sigma\rho\nu}(\{x\}) = 4\left[2\frac{x_1(\bar{x}_1+x_2)}{x_3}g_\perp^{\sigma\nu}q^\rho - (k_1-p_1)_{\perp\mu}\right.$$
$$\left.\times\left(\frac{(\bar{x}_1+x_2)(\bar{x}_1-x_1)}{\bar{x}_1x_3}g_\perp^{\sigma\nu}g_\perp^{\rho\mu} - \frac{1}{\bar{x}_1}\left(g_\perp^{\nu\rho}g_\perp^{\sigma\mu} - g_\perp^{\rho\sigma}g_\perp^{\nu\mu}\right)\right)\right]$$

$$\tilde{T}_2^{\sigma\rho\nu}(\{x\}) = \frac{4i}{\bar{x}_1}\left[2x_1\bar{x}_1q^\rho\epsilon^{\nu\sigma+-} + (k_1-p_1)_{\perp\mu}\right.$$
$$\left.\times\left(\left(1+\frac{2x_2}{x_3}\right)\left(g_\perp^{\sigma\mu}\epsilon^{\nu\rho+-} - g_\perp^{\rho\sigma}\epsilon^{\nu\mu+-}\right) + (x_1-\bar{x}_1)g_\perp^{\rho\mu}\epsilon^{\nu\sigma+-}\right)\right],$$

Dilute regime: two-body contribution

- **Reggeon** definition **[Caron-Huot (2013)]** $\quad R^a(\boldsymbol{z}) \equiv \dfrac{f^{abc}}{gC_A} \ln\left(U^{bc}_{\boldsymbol{z}}\right)$
- Expansion of the *Wilson line* in Reggeized gluons

$$V_{\boldsymbol{z}_1} = 1 + ig\boldsymbol{t}^a R^a(\boldsymbol{z}_1) - \frac{1}{2} g^2 \boldsymbol{t}^a \boldsymbol{t}^b R^a(\boldsymbol{z}_1) R^b(\boldsymbol{z}_1) + O\left(g^3\right)$$



- **BFKL k_T-factorization**

$$\mathcal{A}_2^{\text{dilute}} = \frac{g^2}{4N_c} (2\pi)^d \delta^d(\boldsymbol{q} - \boldsymbol{p}_M - \boldsymbol{\Delta}) \int \frac{\mathrm{d}^d \ell}{(2\pi)^d} \mathcal{U}(\ell) \int_0^1 \mathrm{d}x$$

$$\times \underbrace{\left[\Phi_2\left(x, \boldsymbol{\ell} - \frac{x - \bar{x}}{2}\boldsymbol{\Delta}\right) + \Phi_2\left(x, -\boldsymbol{\ell} - \frac{x - \bar{x}}{2}\boldsymbol{\Delta}\right) - \Phi_2(x, \bar{x}\boldsymbol{\Delta}) - \Phi_2(x, -x\boldsymbol{\Delta})\right]}_{\Phi_{2,\text{BFKL}}(x, \boldsymbol{l}, \boldsymbol{\Delta})}$$

- $\mathcal{U}(\boldsymbol{l}) \rightarrow k_T$-unintegrated gluon density (UGD) in the BFKL sense

$$\mathcal{U}(\boldsymbol{\ell}) \equiv \int \mathrm{d}^d \boldsymbol{v} \mathrm{e}^{-i(\boldsymbol{\ell}\cdot\boldsymbol{v})} \left\langle P\left(p'\right) \middle| R^a\left(\frac{\boldsymbol{v}}{2}\right) R^a\left(-\frac{\boldsymbol{v}}{2}\right) \middle| P(p) \right\rangle ,$$

- Φ_2 is the Fourier transform of Ψ_2

Explicit two-body term in the dilute and $\Delta = 0$ limit

- **BK impact factor**

$$\Phi_{2,\boldsymbol{\Delta}=0}(x,\boldsymbol{l}) = 2\pi m_M f_M e_q \delta(1 - p_M^+/q^+)$$
$$\times \left[\frac{2\boldsymbol{l}^2}{\left[\boldsymbol{l}^2 + x\bar{x}Q^2\right]^2} T_{\text{f.}}\, \phi_{2,\text{f.}}(x) - \frac{x\bar{x}Q^2}{\left[\boldsymbol{l}^2 + x\bar{x}Q^2\right]^2} T_{\text{n.f.}}\, \phi_{2,\text{n.f.}}(x) \right]$$

- Helicity (flip and non-flip) structures and DAs combinations

$$T_{\text{n.f.}} = \boldsymbol{\varepsilon}_q \cdot \boldsymbol{\varepsilon}_M^* \qquad \phi_{2,\text{n.f.}}(x) = (2x-1)(h(x) - \tilde{h}(x)) + \frac{g_\perp^{(a)}(x) - \tilde{g}_\perp^{(a)}(x)}{4}$$

$$T_{\text{f.}} = \frac{(\boldsymbol{\varepsilon}_q \cdot \boldsymbol{l})(\boldsymbol{\varepsilon}_M^* \cdot \boldsymbol{l})}{\boldsymbol{l}^2} - \frac{\boldsymbol{\varepsilon}_q \cdot \boldsymbol{\varepsilon}_M^*}{2} \qquad \phi_{2,\text{f.}}(x) = (2x-1)(h(x) - \tilde{h}(x)) - \frac{g_\perp^{(a)}(x) - \tilde{g}_\perp^{(a)}(x)}{4}$$

- **Forward limit matching**

$$\Phi_{2,\boldsymbol{\Delta}=0}^{\text{BFKL}}(x,\boldsymbol{l}) = 2\left(\Phi_{2,\boldsymbol{\Delta}=0}(x,\boldsymbol{l}) - \Phi_{2,\boldsymbol{\Delta}=0}(x,\boldsymbol{0})\right)$$

- **BFKL impact factor**

$$\Phi_{2,\boldsymbol{\Delta}=0}^{\text{BFKL}}(x,\boldsymbol{l}) = 4\pi m_M f_M e_q \delta(1 - p_M^+/q^+)$$
$$\times \left[\frac{2\boldsymbol{l}^2}{\left[\boldsymbol{l}^2 + x\bar{x}Q^2\right]^2} T_{\text{f.}}\, \phi_{\text{f.}}(x) + \frac{\boldsymbol{l}^2(\boldsymbol{l}^2 + 2x\bar{x}Q^2)}{x\bar{x}Q^2\left[\boldsymbol{l}^2 + x\bar{x}Q^2\right]^2} T_{\text{n.f.}}\, \phi_{\text{n.f.}}(x) \right]$$

Explicit three-body term in the dilute and $\Delta = 0$ limit

- The 3-body BFKL impact factor is a combination of 12 BK impact factors

$$\Phi_3\left(\{x\},\{\boldsymbol{p}\}\right) = \left(\prod_{j=1}^{3}\int d^2\boldsymbol{z}_j e^{-i\boldsymbol{z}_j\boldsymbol{p}_j}\right)\Psi_3\left(\{x\},\{\boldsymbol{z}\}\right)$$

- Transverse to transverse transition in the **forward** and **dilute** limit

$$c_f = N_c^2/(N_c^2-1)$$

$$\mathcal{A}^{\text{dilute}}_{3T,\boldsymbol{\Delta}=\boldsymbol{0}} = e_q m_M \frac{g^2}{N_c}(2\pi)\delta\left(1-\frac{p_M^+}{q^+}\right)(2\pi)^2\delta^2\left(\boldsymbol{q}-\boldsymbol{p}_M\right)\int\frac{\mathrm{d}^d\boldsymbol{\ell}}{(2\pi)^d}\mathcal{U}(\boldsymbol{\ell})$$
$$\times\left(\prod_{i=1}^{3}\int_0^1\frac{dx_i}{x_i}\right)\frac{\delta(1-x_1-x_2-x_3)}{x_3}\frac{\boldsymbol{\ell}^2}{Q^2}\left\{T_{\text{f.}}\left[f_{3M}^V V\left(x_1,x_2\right)-f_{3M}^A A\left(x_1,x_2\right)\right]\right.$$
$$\times 2x_1\left(\frac{x_3c_f}{\boldsymbol{\ell}^2+\frac{x_2x_3}{x_2+x_3}Q^2}+\frac{x_3c_f}{\boldsymbol{\ell}^2+\frac{x_1x_3}{x_1+x_3}Q^2}-\frac{\bar{x}_3\left(1-c_f\right)}{\boldsymbol{\ell}^2+\frac{x_1x_2}{x_1+x_2}Q^2}+\frac{x_2-\bar{x}_1c_f}{\boldsymbol{\ell}^2+x_1\bar{x}_1Q^2}+\frac{x_1-\bar{x}_2c_f}{\boldsymbol{\ell}^2+x_2\bar{x}_2Q^2}\right)$$
$$-T_{\text{n.f.}}\left[f_{3M}^V V\left(x_1,x_2\right)+f_{3M}^A A\left(x_1,x_2\right)\right]$$
$$\left.\times\left(\frac{\left(1-c_f\right)x_1\bar{x}_3}{\bar{x}_3\boldsymbol{\ell}^2+x_1x_2Q^2}-\frac{c_fx_3^2}{\bar{x}_1\boldsymbol{\ell}^2+x_2x_3Q^2}-\frac{\left(x_2-\bar{x}_1c_f\right)x_1x_2}{\bar{x}_1\left(\boldsymbol{\ell}^2+x_1\bar{x}_1Q^2\right)}-\frac{\left(x_1-\bar{x}_2c_f\right)\bar{x}_2}{\left(\boldsymbol{\ell}^2+x_2\bar{x}_2Q^2\right)}\right)\right\}$$

- **The forward and dilute limit matches our previous result**

[Anikin, Ivanov, Pire, LS, Wallon (2009)]

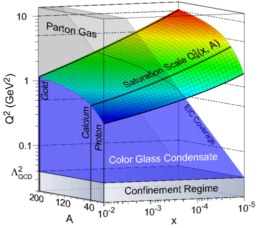
BFKL approach + twist-expansion via light-cone collinear factorization

Summary

- **Transversally polarized light vector meson production**
- DVMP in the **non-linear** regime in the transversely polarized case
- Both **forward** and **non-forward** results and **s-channel non-conserving helicity amplitudes**
- **Coordinate** and **momentum space** representations
- Reggeized gluon expansion **[Caron-Huot (2013)]** $\Longrightarrow$ **BFKL results**
- To be used for a complete description of HERA and future EIC data
- Higher-twist corrections are essential to describe medium energy data of exclusive processes:

  data for $ep \to e\pi^0 p$ need a twist 3 π^0 DA **[M. Defurne et al. (2016)]**

- Method to deal with twist corrections at small-x including saturation
- what's next? what about the NLO frontier?
  - Wandzura-Wilczek approximation: no genuine twist-3, i.e. no $q\bar{q}g$ 3-body in principle, "straightforward"
  - Full NLO? Out of reach for the moment without a full automatization of the calculations...



THANK YOU FOR ATTENTION !!