Probing the photon Wigner distribution with dilepton production at small q_T in UPCs

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Joint ECFA-NuPECC-APPEC Workshop

"Synergies between the EIC and the LHC"

Krakow, 24 September 2025

ECFA European Committee for Future Accelerators

Based on: *JHEP* 01 (2025) 076

In collaboration with: D. Boer, C. Pisano

Outline

(*Generalised transverse-momentum dependent distributions)

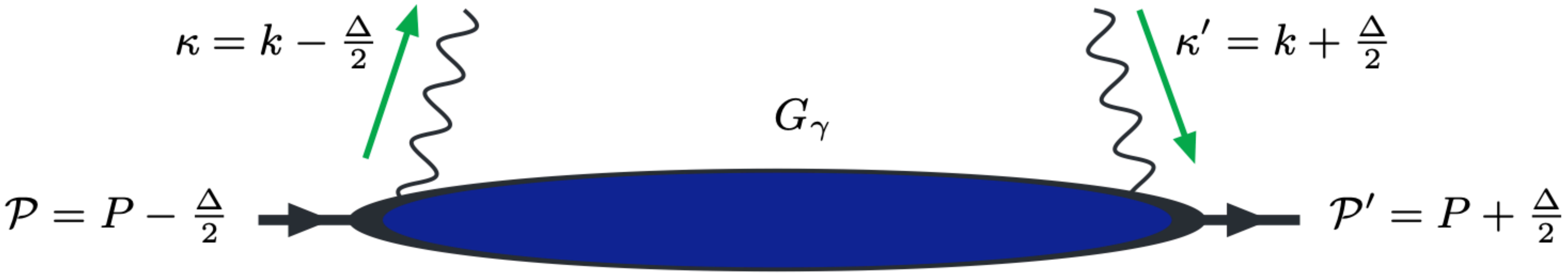
- **Part I: constructing the photon GTMDs***

- **Part II:** UPC **factorization** and **formalism**
  - Counter-intuitive features: *cut diagrams, feed-in,* $\sin\phi$ *asymmetries*

(**depending on the time)

- **Part III: Numerical** results and **mass dependence****
  - Cross section
  - Asymmetries

The GTMD correlator



$$G_\gamma^{\mu\nu}(x, \boldsymbol{k}_T, \xi, \boldsymbol{\Delta}_T) = \frac{1}{2x}\Big\{ -g_T^{\mu\nu}\, \underset{\text{(unp.)}}{\mathcal{F}_1^\gamma}(x, \boldsymbol{k}_T^2, \xi, \boldsymbol{\Delta}_T^2, \boldsymbol{k}_T\cdot\boldsymbol{\Delta}_T) + \frac{k_T^{\mu\nu}}{M_N^2}\, \underset{\text{(lin.)}}{\mathcal{F}_2^\gamma}(x, \boldsymbol{k}_T^2, \xi, \boldsymbol{\Delta}_T^2, \boldsymbol{k}_T\cdot\boldsymbol{\Delta}_T)$$

$$+ \frac{\Delta_T^{\mu\nu}}{M_N^2}\, \underset{\text{(lin.)}}{\mathcal{F}_3^\gamma}(x, \boldsymbol{k}_T^2, \xi, \boldsymbol{\Delta}_T^2, \boldsymbol{k}_T\cdot\boldsymbol{\Delta}_T) + \frac{k_T^{[\mu}\Delta_T^{\nu]}}{M_N^2}\, \underset{\text{(circ.)}}{\mathcal{F}_4^\gamma}(x, \boldsymbol{k}_T^2, \xi, \boldsymbol{\Delta}_T^2, \boldsymbol{k}_T\cdot\boldsymbol{\Delta}_T) \Big\}$$

$\boldsymbol{\xi \approx 0}$ (Suppressed as the “-” light components)

For photon GTMD in UPCs:

$$\mathcal{F}_1^\gamma = \frac{k_T^2 - \frac{1}{4}\Delta_T^2}{2M_N^2}\mathcal{F}_2^\gamma = -2\frac{k_T^2 - \frac{1}{4}\Delta_T^2}{M_N^2}\mathcal{F}_3^\gamma = \frac{k_T^2 - \frac{1}{4}\Delta_T^2}{M_N^2}\mathcal{F}_4^\gamma = f_{\gamma/A}$$

Nucleus form factor dependence

Fermi, *Z.Phys. 29* (1924) Weizsäcker, *Z.Phys. 88* (1934) Williams, *Phys.Rev. 45* (1934)

According to the **Equivalent photon approximation** (**EPA**) photon GTMDs are related to the nucleus **Form Factor** (**FF**)

$$x f_{\gamma/A}(x, \kappa_T, \kappa_T') \propto Z^2 \alpha \mathcal{N}(\kappa_T, \kappa_T') F_A\left(\kappa_T + x^2 M_N^2\right) F_A^*\left(\kappa_T' + x^2 M_N^2\right)$$

Longitudinally, the nucleus momentum is shared between the nucleons: $x = \omega_\gamma / E_N$

Wood, Saxon, *Phys.Rev. 95* (1954)

For $b \lesssim R$: **Wood-Saxon**-like potential $F_A(k) = \frac{4\pi\rho_0}{A} \int_0^\infty \mathrm{d}r \, \frac{r^2}{1 + \exp\left[(r-R)/d\right]} \frac{\sin(kr)}{kr}$

Vidovic, Greiner, Best, Soff, *Phys.Rev.C 47* (1993)

For $b \gg R$: reduction to point-like $F_A(k) = 1 \Rightarrow f_\gamma(x, b) \approx \frac{Z^2 \alpha}{2\pi} x m^2 \left[K_1(xmb)\right]^2$

for $x \ll 1$ spin effects are negligible

Budnev, Ginzburg, Meledin, Serbo, *Phys.Rept. 15* (1975)

$K_1(b) \sim \sqrt{\frac{1}{b}}$ at high b

The photon GTMDs

$$x f_{\gamma/A}(x, \kappa_T, \kappa_T') \propto Z^2 \alpha \mathcal{N}(\kappa_T, \kappa_T') F_A\left(\kappa_T + x^2 M_N^2\right) F_A^*\left(\kappa_T' + x^2 M_N^2\right)$$

↓

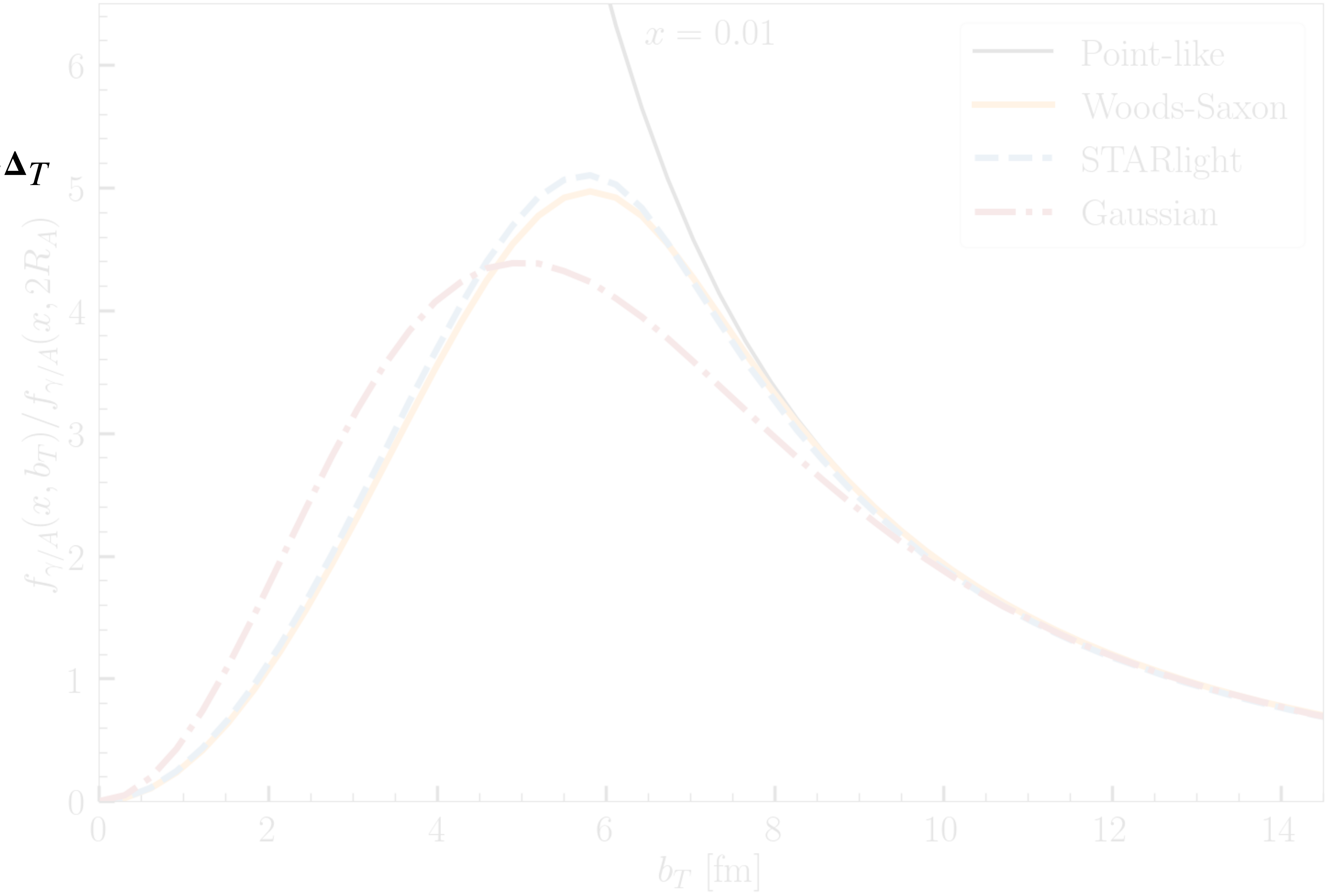
$$f_{\gamma/A}(x, b_T) \propto \int \mathrm{d}^2\boldsymbol{k}_T\, \mathrm{d}^2\boldsymbol{\Delta}_T\, f_{\gamma/A}(x, \kappa_T, \kappa_T')\, \mathrm{e}^{-i \boldsymbol{b}_T \cdot \boldsymbol{\Delta}_T}$$

From a GTMD to the b_T distribution

emitted γ absorbed γ

$$\kappa_T = k_T + \frac{\Delta_T}{2} \qquad \kappa_T' = k_T - \frac{\Delta_T}{2}$$

Symmetric definition



The photon GTMDs

$$x f_{\gamma/A}(x, \kappa_T, \kappa'_T) \propto Z^2 \alpha \mathcal{N}(\kappa_T, \kappa'_T)\, F_A\left(\kappa_T + x^2 M_N^2\right) F_A^*\left(\kappa'_T + x^2 M_N^2\right)$$

$$f_{\gamma/A}(x, b_T) \propto \int \mathrm{d}^2\boldsymbol{k}_T\, \mathrm{d}^2\boldsymbol{\Delta}_T\, f_{\gamma/A}(x, \kappa_T, \kappa'_T)\, \mathrm{e}^{-i \boldsymbol{b}_T \cdot \boldsymbol{\Delta}_T}$$

From a GTMD to the b_T distribution

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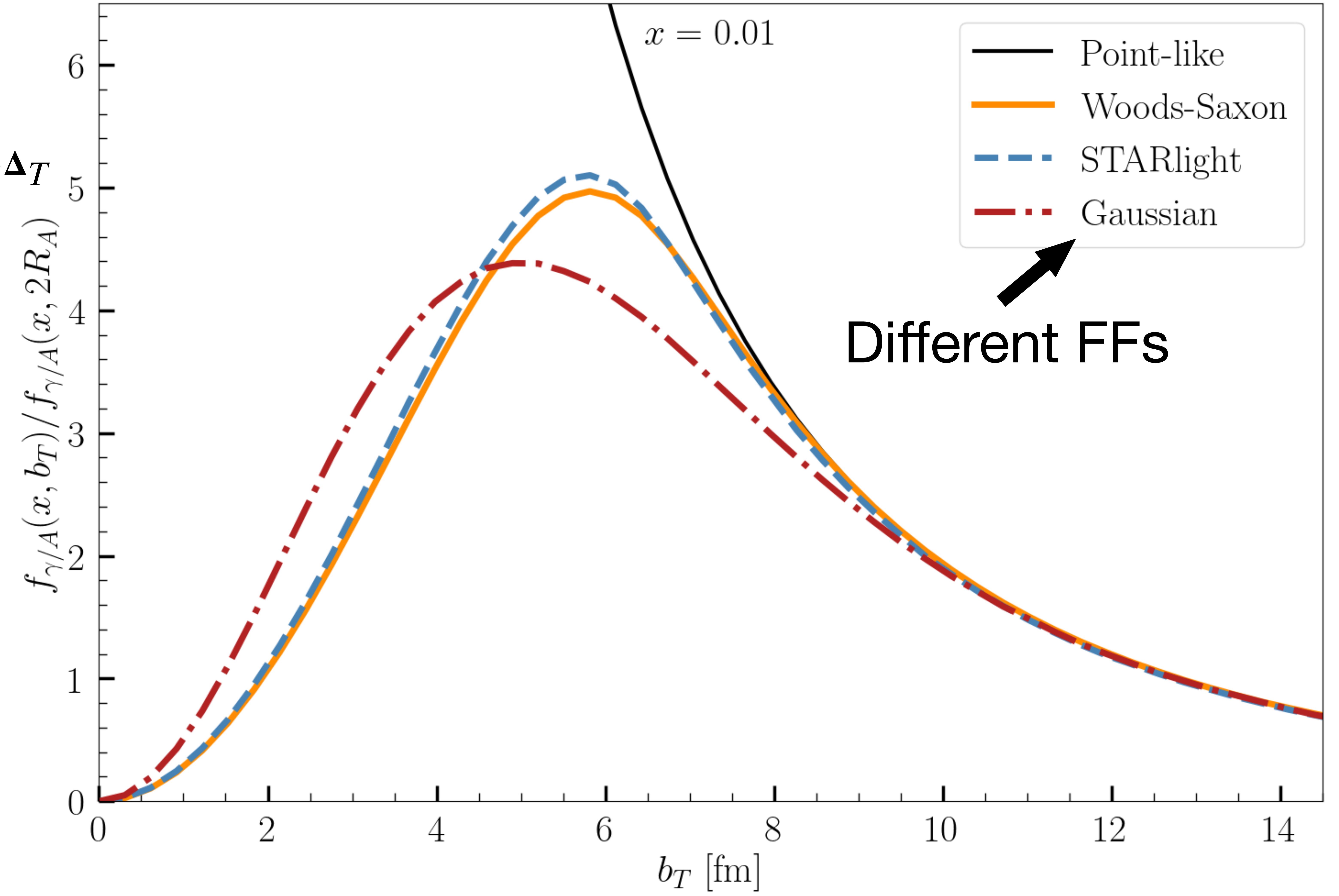
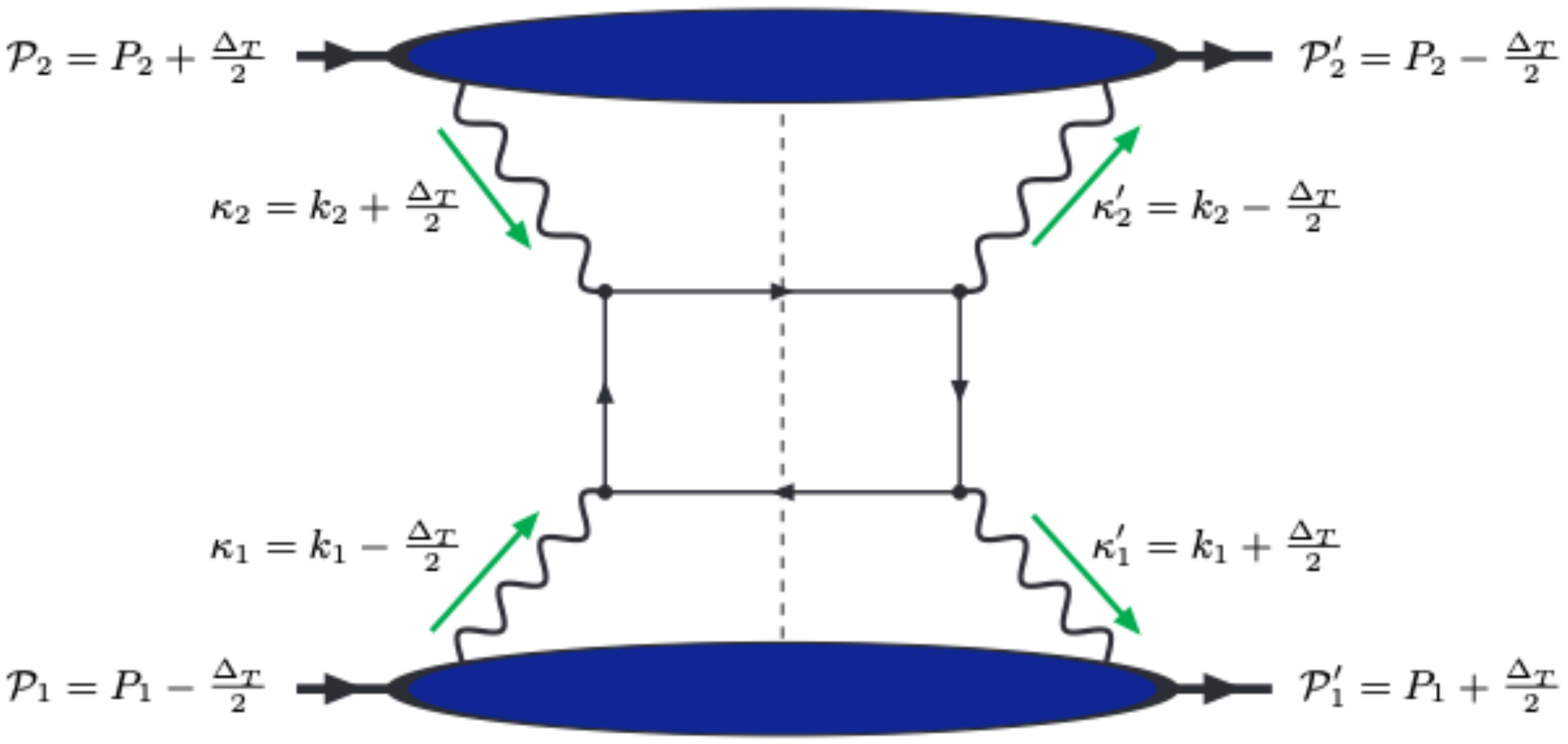


Different FFs

Representative cut(?) diagram

The LO diagram involving the photon GTMD correlator

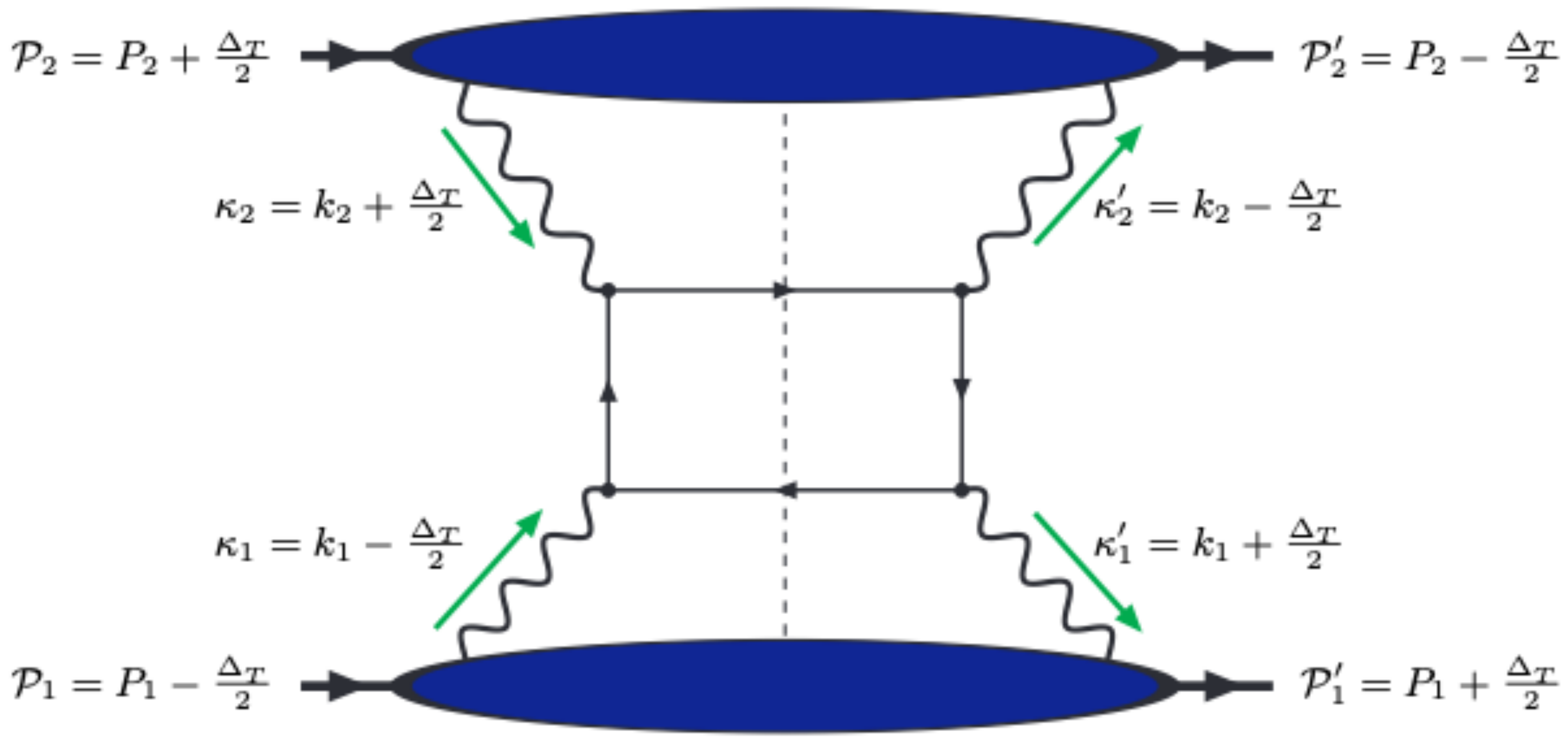


Usually the particle is emitted and absorbed before the cut

Is the *cut* a real cut?

Representative cut(!) diagram

The LO diagram involving the photon GTMD correlator



Usually the particle is emitted and absorbed before the cut

Is the *cut* a real cut?

Actually yes!

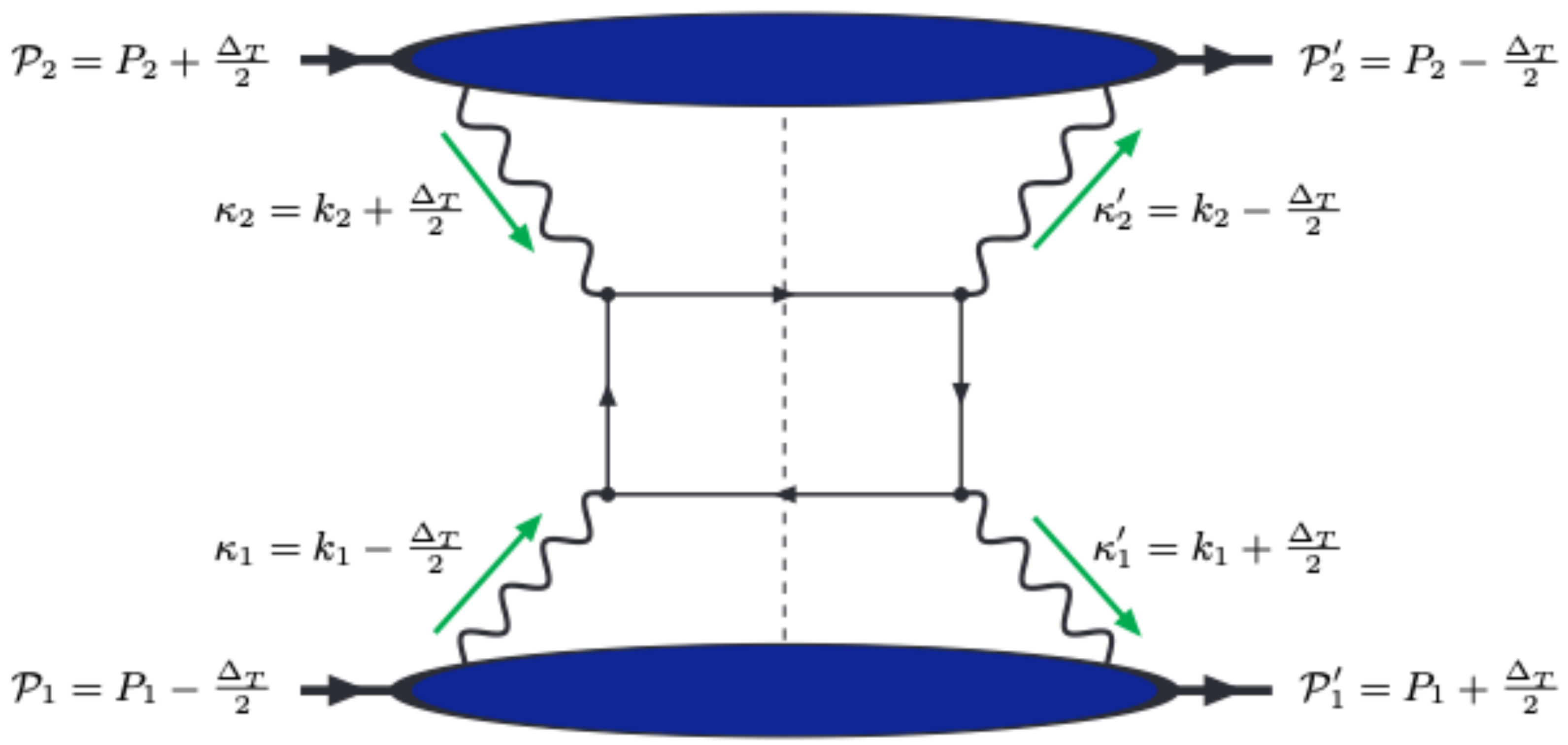
Final state is fixed: $\kappa'_1 + \kappa'_2 = \kappa_1 + \kappa_2 = K_1 + K_2$ $\Delta_{2T} = -\Delta_{1T}$

(final state) ensure positive cross-sections

When measuring b_T the initial state *is not* in a momentum eigenstate (as GPDs) Dhiel, *EPJC 25* (2002)

From Δ_T to b_T space

Boer, LM, Pisano, *JHEP 01* (2025)



From diagram differential cross section in Δ_T space

$$\frac{d\sigma}{dPS\, d^2\boldsymbol{\Delta}_T}$$ decomposed into structure functions

$$F^1 \qquad F^{\cos n\phi_{ab}} \qquad F^{\sin n\phi_{ab}} \qquad \phi_{ab} = \phi_a - \phi_b$$

$$dPS = dy_1\, dy_2\, d^2\boldsymbol{K}_T\, d^2\boldsymbol{q}_T$$

y_i final particle rapidity

$\boldsymbol{K}_{1T} \approx -\boldsymbol{K}_{2T} \approx \boldsymbol{K}_T$ (hard)

$\boldsymbol{q}_T = \boldsymbol{K}_{1T} + \boldsymbol{K}_{2T}$ (soft)

The Δ_T is not experimentally accessible while b_T (impact parameter) is

$$\frac{d\sigma}{dPS\, d^2\boldsymbol{b}_T} = \int \frac{d^2\boldsymbol{\Delta}_T}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{\Delta}_T} \frac{d\sigma}{dPS\, d^2\boldsymbol{\Delta}_T} \longrightarrow$$

Two questions to address:

1. How $\Delta_T \cdot k_T \neq 0$ affects the FT?
2. $\sin\phi$ asymmetries in b_T space?

Q1: $\Delta_T \cdot k_T$ and feed-in contributions

Boer, LM, Pisano, *JHEP 01* (2025)

GTMDs depend on two vectors that belong to the transverse plane

$$\mathcal{F}_i^{\gamma}(x, \boldsymbol{k}_T^2, \boldsymbol{\Delta}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{\Delta}_T) = \sum_{m=0}^{\infty} F_i^{\gamma\,(2m)} \cos^{2m}\phi_{k\Delta}$$

→ even powers of $(\Delta_T \cdot k_T)^{2m}$

→ no parity-violating terms ($\sin(\phi)$)

Contributions from higher m cause modulations in $2n\phi_\Delta$ to **"feed-into"** $2n'\phi_b$

Q1: $\Delta_T \cdot k_T$ and feed-in contributions

GTMDs depend on two vectors that belong to the transverse plane

$$\mathscr{F}_i^{\gamma}(x, \boldsymbol{k}_T^2, \boldsymbol{\Delta}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{\Delta}_T) = \sum_{m=0}^{\infty} F_i^{\gamma\,(2m)} \cos^{2m}\phi_{k\Delta}$$

Contributions from higher m cause modulations in $2n\phi_\Delta$ to **"feed-into"** $2n'\phi_b$

Example for $m = 1$

from convolution expansion

$$\int_0^{2\pi} \mathrm{d}\phi_\Delta\, \mathrm{e}^{i\boldsymbol{b}_T\cdot\boldsymbol{\Delta}_T} \cos(2n\phi_{\Delta a}) \cos^2(\phi_{q\Delta}) = (-1)^n \frac{\pi}{2} \Big[2\, J_{2n}(b_T\,\Delta_T) \cos(2n\phi_{ba}) - J_{2n+2}(b_T\,\Delta_T) \cos 2(n\phi_{ba} + \phi_{bq}) - J_{2n-2}(b_T\,\Delta_T) \cos 2(n\phi_{ba} - \phi_{bq}) \Big]$$

Note that number of ϕ_a is fixed between left- and right-hand sides

Q2: $\sin\phi$ asymmetries

Pisano, Boer, Brodsky, Buffing, Mulders, *JHEP 10* (2013)

In the forward limit ($\Delta_T \to 0$) there is no $\sin\phi$ asymmetries

In UPC the cross section in Δ_T-space has $\sin\phi$ asymmetries but:

- always involve ϕ_Δ
- always associated with additional $\sin\phi$ terms in the GTMD convolution $\mathscr{C}[w\,\mathscr{F}_i\,\mathscr{F}_j]$

→ $\sin\phi$ terms can only *feed-into* other ***even*** terms

Example for $m = 1$ (sine contributes only via the feed-in effect)

$$\int \mathrm{d}\phi_{k_1}\, \mathrm{e}^{i\,\boldsymbol{q}_T\cdot\boldsymbol{k}_{1T}}\, \mathrm{d}\phi_\Delta\, \mathrm{e}^{i\,\boldsymbol{b}_T\cdot\boldsymbol{\Delta}_T} \sin(2\phi_{q1})\, \sin(4\phi_{\Delta q})\, \cos^2(\phi_{1\Delta}) \propto \cos(4\phi_{qK})$$

from GTMD convolution

from GTMD expansion

The cross section in b_T space - recap

$$\frac{\mathrm{d}\sigma}{\mathrm{dPS}\,\mathrm{d}^2\boldsymbol{b}_T} \propto \int \frac{\mathrm{d}\boldsymbol{\Delta}_T^2}{4\pi} \Big\{ J_0(b_T\,\Delta_T)\Big[F^0 + F^{\cos 2\phi_{qK}} \cos 2\phi_{qK} + F^{\cos 4\phi_{qK}} \cos 4\phi_{qK}\Big]$$

$$-J_2(b_T\,\Delta_T)\Big[F^{\cos 2\phi_{\Delta K}} \cos 2\phi_{bK} + F^{\cos 2\phi_{q\Delta}} \cos 2\phi_{qb} + F^{\cos 2(\phi_{qK}+\phi_{\Delta K})} \cos 2(\phi_{qK}+\phi_{bK})\Big]$$

$$+J_4(b_T\,\Delta_T)\, F^{\cos 4\phi_{\Delta K}} \cos 4\phi_{bK} \Big\} + \dots$$

Dots stands for additional terms driven by feed-in

Since ***feed-in*** affects solely $\phi_{q\Delta}$ (hence ϕ_{qb}) we cannot have asymmetries with more than $\mathbf{4\,\phi_K}$

n° of $\phi_K = 0$ ➡ Includes the isotropic term

n° of $\phi_K = 2$ ➡ Only for “massive” final state (muon, tau, heavy quarks, ...)

n° of $\phi_K = 4$

Details of the calculation

Additional points to address:

- b_T integration should include **survival probability** (of nuclei and/or rapidity gap)
  - ➡ Instead, b_T is integrated in the strict region $\left[2R_A, \infty\right)$

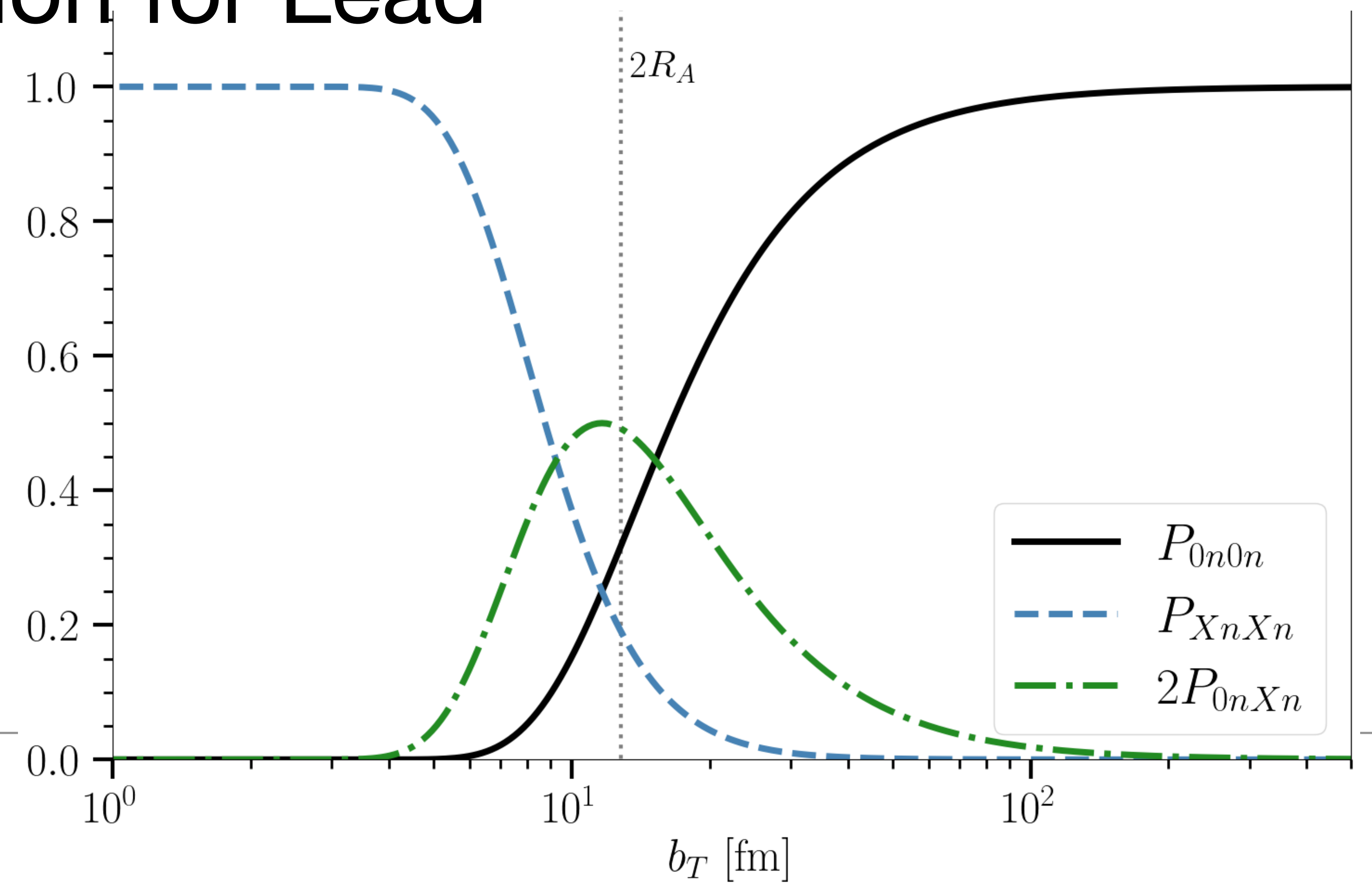
$$\frac{\mathrm{d}\sigma}{\mathrm{dPS}} = \int_{b_{\min}}^{\infty}\int_0^{2\pi} \mathrm{d}^2\boldsymbol{b}_T \frac{\mathrm{d}\sigma}{\mathrm{dPS}\,\mathrm{d}^2\boldsymbol{b}_T} = \int \mathrm{d}^2\boldsymbol{\Delta}_T \left[\delta^{(2)}(\boldsymbol{\Delta}_T) - \frac{b_{\min}}{2\pi\Delta_T} J_1(b_{\min}\,\Delta_T)\right] \frac{\mathrm{d}\sigma}{\mathrm{dPS}\,\mathrm{d}^2\boldsymbol{\Delta}_T}$$

- neutron tagging (NT) to separate $0n$ and Xn channels
  - ➡ Gamma-UPC has the overall best NT implementation for Lead
  - ➡ Instead, based on a simple modelization from

CMS PAS HIN-21-015 (2024)

Baur, Hencken, Trautmann, *J. Phys. G 24* (1998)

$$P_{0n} = \mathrm{e}^{-94\,\mathrm{fm}^2/b_T^2}$$



Normalized cross section - STAR

$$x f_{\gamma/A}(x, k_T, \Delta_T) = \frac{Z^2\alpha}{\pi^2} \frac{k_T^2 - \Delta_T^2/4}{N(x,\kappa_T,\epsilon)\, N(x,\kappa_T',\epsilon)} F_A(\kappa_T^2 + x^2M_N^2)\, F_A^*(\kappa_T'^2 + x^2M_N^2)$$

→ STARlight Form Factor

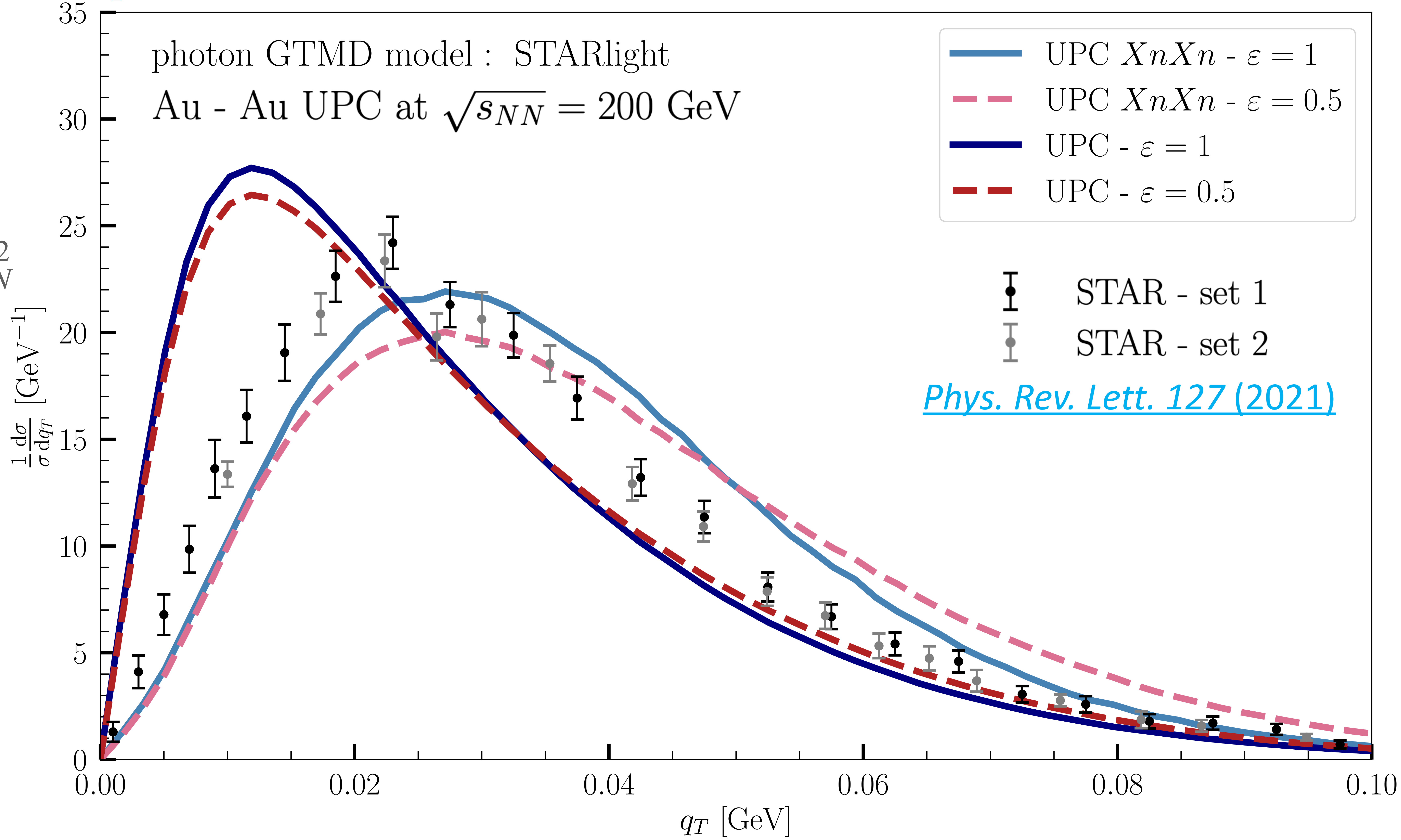
$$\kappa_T = k_T + \frac{\Delta_T}{2} \qquad \kappa_T' = k_T - \frac{\Delta_T}{2}$$

$$N(x,\kappa_T,\varepsilon) = \kappa_T^2 + (1-\varepsilon)\, k_T \cdot \Delta_T + x^2 M_N^2$$

$$0 \le \varepsilon \le 1$$

suppress feed-in

Data slightly favour unsuppressed feed-in



Asymmetries in e^+e^- production

Boer, LM, Pisano, *JHEP 01* (2025)

For $q_T \to 0$

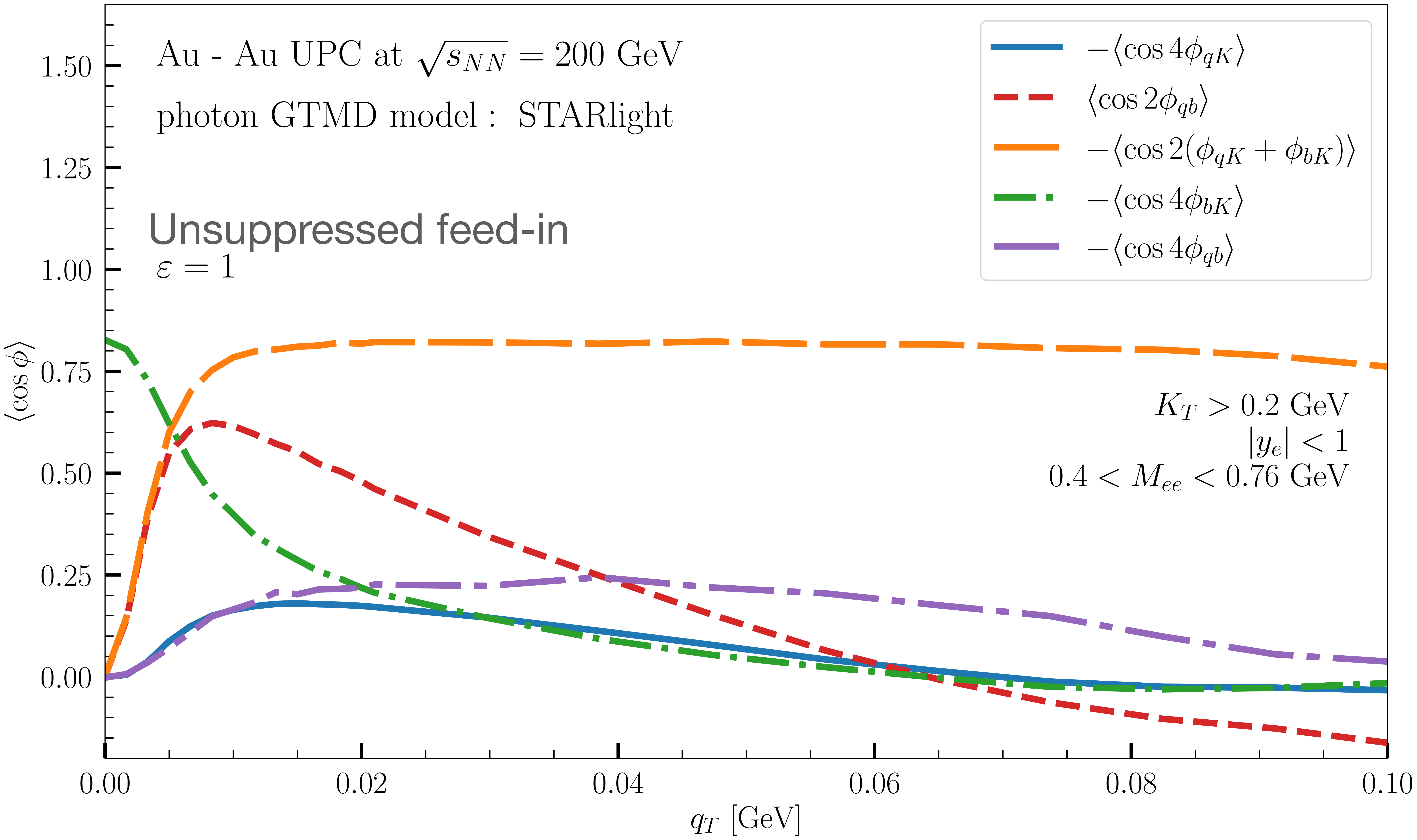
only $\langle \cos 4\phi_{bK} \rangle \neq 0$

$\langle \cos 2(\phi_{qK} + \phi_{bK}) \rangle$

largest

$\langle \cos 4\phi_{qb} \rangle \neq 0$

only from feed-in



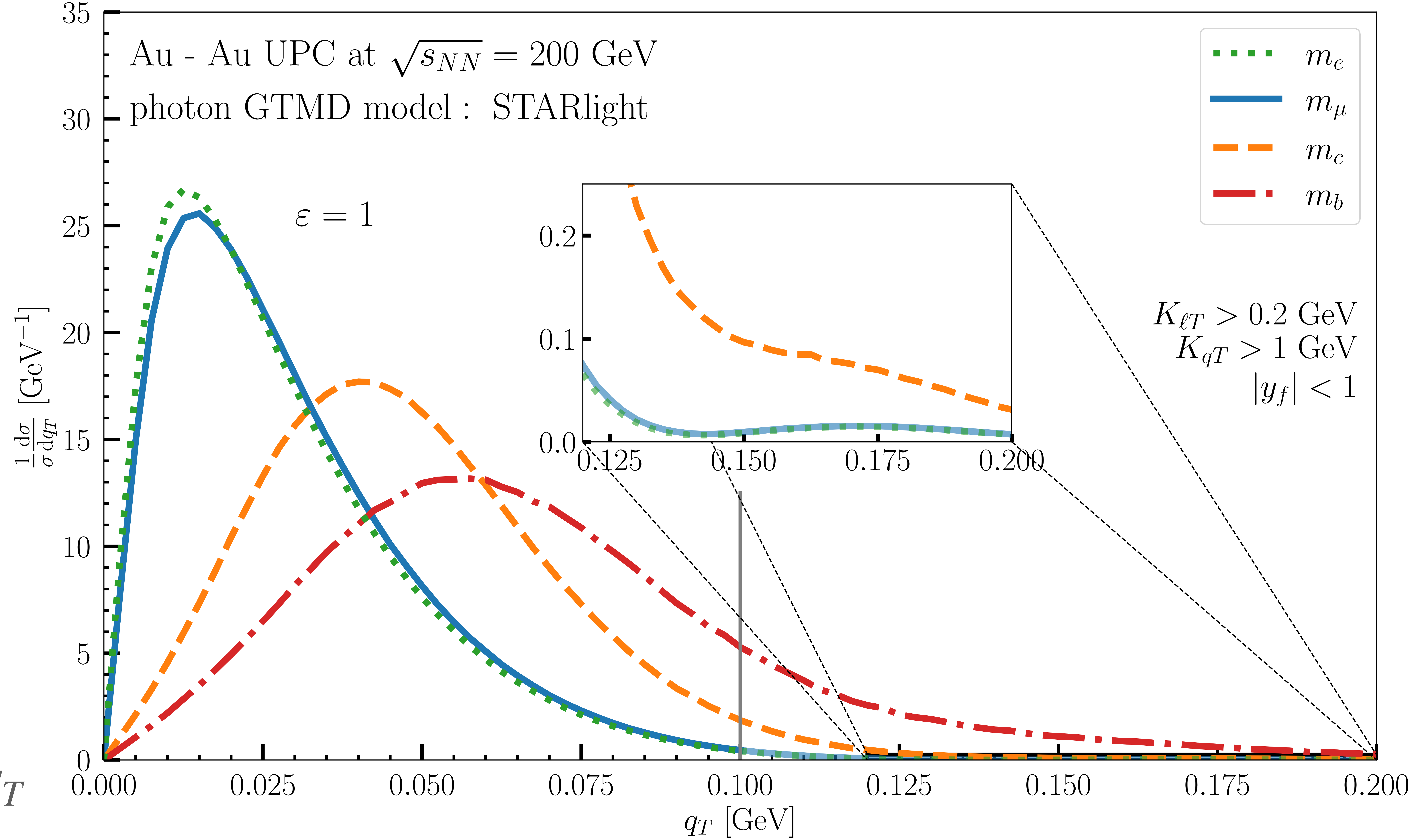
Normalized cross section - mass dependence

We take $K_{qT} > K_{\ell T}$

Validity of the framework is limited for light leptons

Cross section is **broader** for heavy quarks

Oscillations might be **smeared out** by corrections at "high" q_T

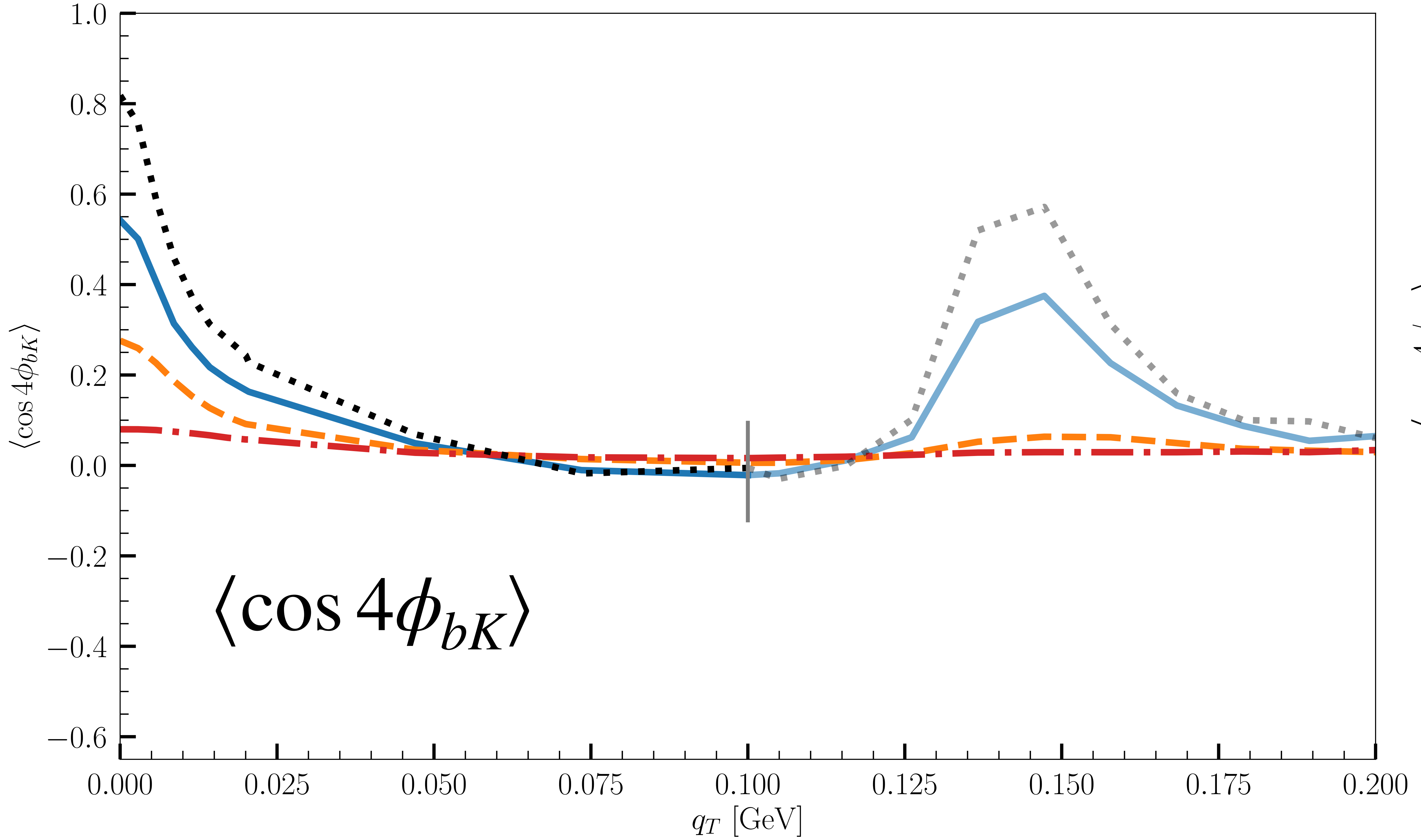


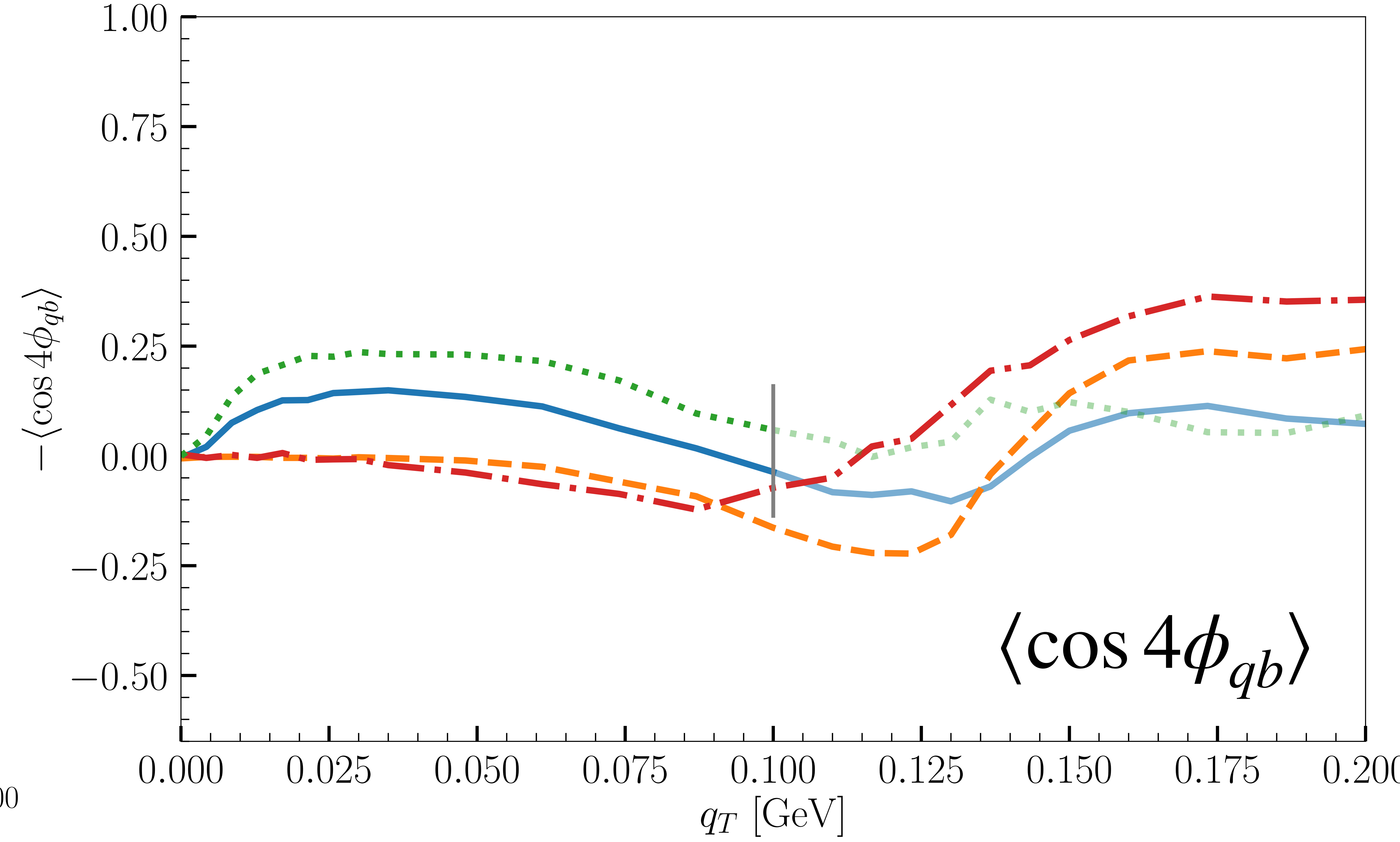
Asymmetries - mass dependence ($0\phi_b$)

Vertical line corresponding to the limit of factorization reliability

For **heavier states** the dominant ϕ_{qK} dependence is $\cos 2\phi_{qK}$

Asymmetries - mass dependence ($4\phi_b$)





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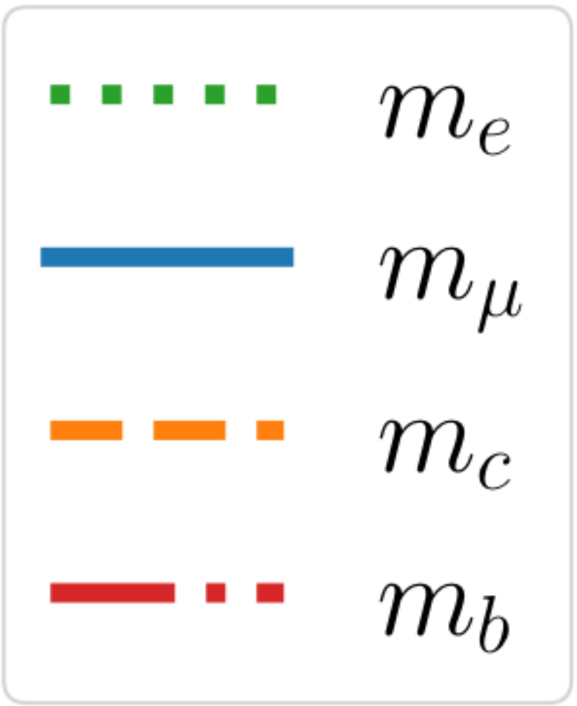


Observation of **oscillating behaviour** to test higher-order effects

Summary of the seminar

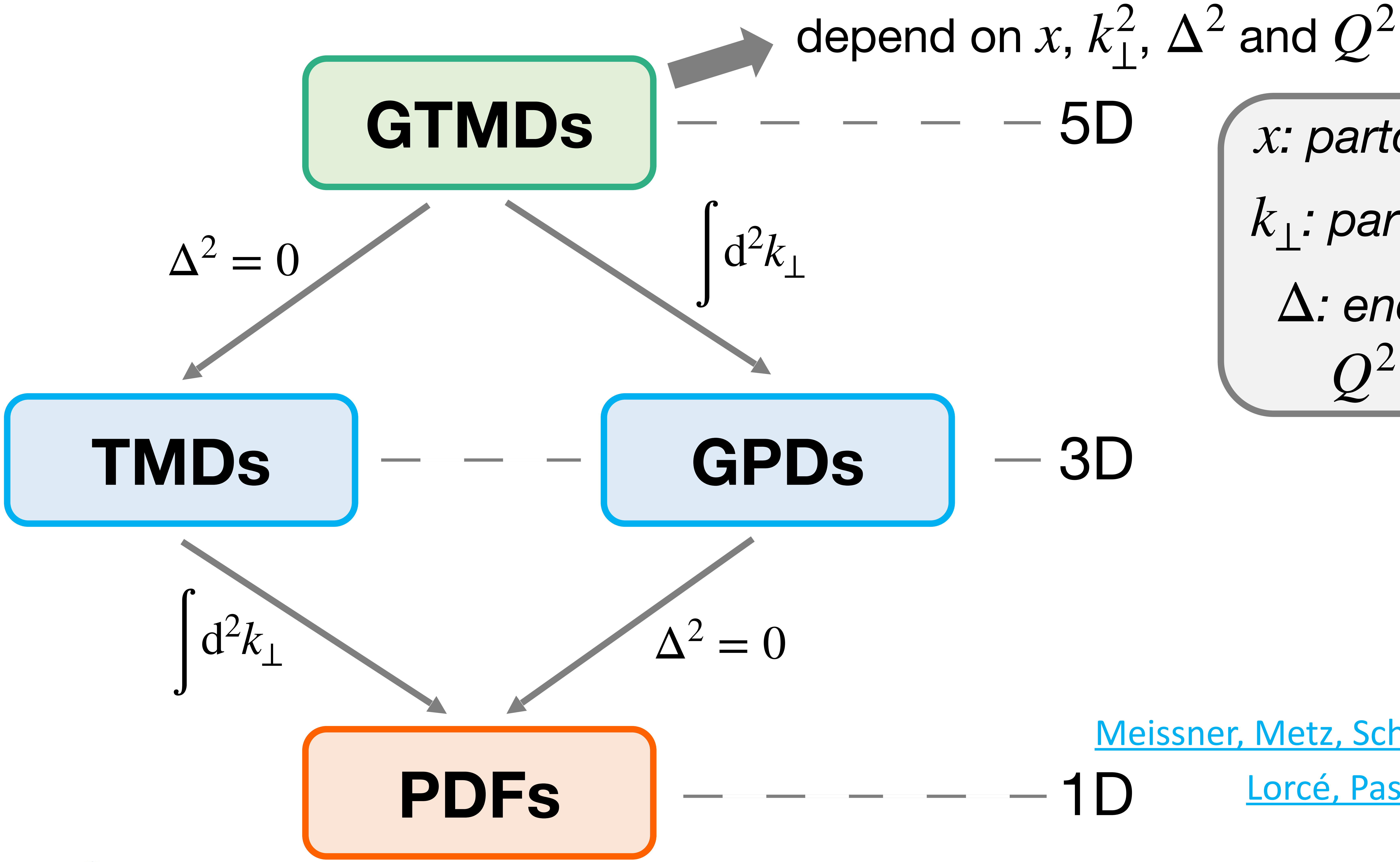
- Highly-accelerated heavy ions act like a photon emitter
  - ➡ Distribution of photons described in terms of **GTMDs**
- The GTMD depends on two transverse vectors
  - ➡ Anisotropies contributions lead to a ***feed-in* effect**
- According to QED there is ***no*** $\sin\phi$ ***asymmetry*** expected in b_T space
- Experimental data can clarify more regarding:
  - ➡ Unsuppressed feed-in contributions
  - ➡ Oscillating behaviour and high-order effects

Probing the photon Wigner distribution with dilepton production at small q_T in UPCs

Back-up slides

The photon GTMDs

Analogous to the quark/gluon GTMDs



x: parton mom. fraction

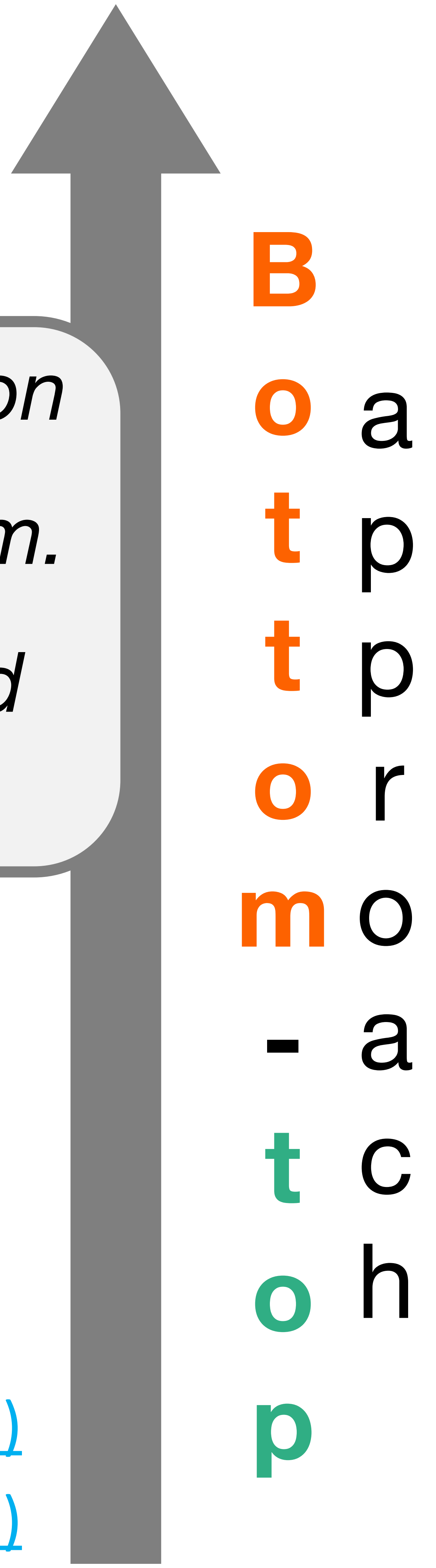
$k_\perp$: parton transv. mom.

Δ: energy transferred

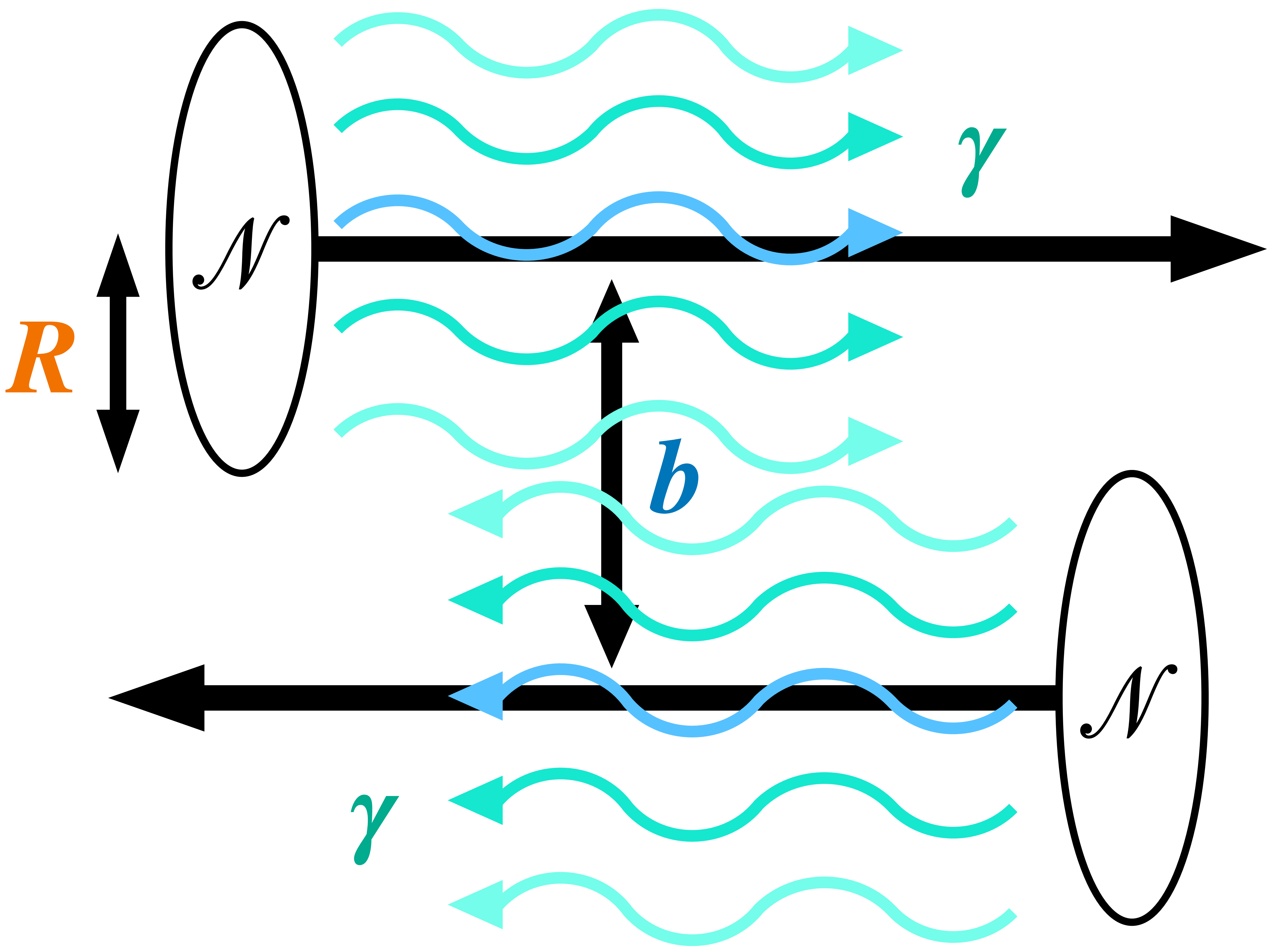
Q^2 energy scale

Meissner, Metz, Schlegel, *JHEP 08* (2009)

Lorcé, Pasquini, *JHEP 09* (2013)



Ultra-peripheral collisions



The impact parameter b measures the distance between the two nuclei $\mathcal{N}$/nucleons N

$b = 0$: central collisions

$60\,\% \lesssim b \lesssim 80\,\%$: peripheral

$b \gtrsim 90\,\%$: ultra peripheral

Here we have strictly considered $b > 2R$ (100%)

S-matrix non-diagonal components

The cross section can be seen as an integral over the probability of transition in function of $\boldsymbol{b}$

$$\sigma = \int \mathrm{d}^2\boldsymbol{b}\ P(\boldsymbol{b}) \qquad\qquad P(\boldsymbol{b}) = |\,{}_{out}\langle \phi_1 \dots \phi_n \,|\, \mathcal{T} \,|\, \phi_A \phi_B \rangle_{in}|^2$$

$\mathcal{T}$ imposes momentum conservation and introduces the amplitude $\mathscr{A}(k_A, k_B \to p_f)$

$|\,\phi_1 \dots \phi_n\rangle_{out}$ introduces the final state phase space integration Π_f

$|\,\phi_A \phi_B\rangle_{in}$ are states localised in different space-points with a momentum imbalance

$$P(\boldsymbol{b}) \sim \Pi_f \int \mathrm{d}^3\boldsymbol{k}_T\ \phi_A(\boldsymbol{k}_A)\,\phi_B(\boldsymbol{k}_B) \int \mathrm{d}^3\bar{\boldsymbol{k}}_T\ \phi_A(\bar{\boldsymbol{k}}_A)\,\phi_B(\bar{\boldsymbol{k}}_B)\,\mathrm{e}^{-i\boldsymbol{b}\cdot(\boldsymbol{k}_T - \bar{\boldsymbol{k}}_T)}\ \mathscr{A}(k_A, k_B \to \{p_f\})\,\mathscr{A}^*(k_A, k_B \to \{p_f\})$$

Expression in Δ_T space

$$\frac{d\sigma}{dPS\, d^2\boldsymbol{\Delta}_T} \propto F^0 + F^{\cos 2\phi_{q\Delta}} \cos 2\phi_{q\Delta} + F^{\sin 2\phi_{q\Delta}} \sin 2\phi_{q\Delta} + F^{\cos 2\phi_{qK}} \cos 2\phi_{qK} + F^{\sin 2\phi_{qK}} \sin 2\phi_{qK} + F^{\cos 2\phi_{\Delta K}} \cos 2\phi_{\Delta K}$$

$$+ F^{\cos 4\phi_{qK}} \cos 4\phi_{qK} + F^{\sin 4\phi_{qK}} \sin 4\phi_{qK} + F^{\cos 4\phi_{\Delta K}} \cos 4\phi_{\Delta K}$$

$$+ F^{\cos 2(\phi_{qK}+\phi_{\Delta K})} \cos 2(\phi_{qK} + \phi_{\Delta K}) + F^{\sin 2(\phi_{qK}+\phi_{\Delta K})} \sin 2(\phi_{qK} + \phi_{\Delta K})$$

$$\phi_{ab} = \phi_a - \phi_b$$

$$dPS = dy_1\, dy_2\, d^2\boldsymbol{K}_T\, d^2\boldsymbol{q}_T$$

y_i final particle rapidity

$\boldsymbol{K}_{1T} \approx -\boldsymbol{K}_{2T} \approx \boldsymbol{K}_T$ (hard)

$\boldsymbol{q}_T = \boldsymbol{K}_{1T} + \boldsymbol{K}_{2T}$ (soft)

Each structure function depends on the convolution of two GTMDs

$$\mathscr{C}[w\,\mathscr{F}_i\,\mathscr{F}_j](\boldsymbol{q}_T^2, \boldsymbol{\Delta}_T^2, \boldsymbol{\Delta}_T \cdot \boldsymbol{q}_T) \equiv \int d^2k_{1T}\, d^2k_{2T}\, \delta^2(\boldsymbol{k}_{1T} + \boldsymbol{k}_{2T} - \boldsymbol{q}_T)\, w(\boldsymbol{k}_{1T}, \boldsymbol{k}_{2T}, \boldsymbol{\Delta}_T)\, \mathscr{F}_i(x_1, \boldsymbol{k}_{1T}^2, \boldsymbol{\Delta}_T^2, \boldsymbol{k}_{1T} \cdot \boldsymbol{\Delta}_T)\, \mathscr{F}_j(x_2, \boldsymbol{k}_{2T}^2, \boldsymbol{\Delta}_T^2, -\boldsymbol{k}_{2T} \cdot \boldsymbol{\Delta}_T)$$

$$x_{(1,2)} \approx \frac{M_T}{\sqrt{s_{NN}}} \left(e^{\pm y_1} + e^{\pm y_2}\right)$$

TMD cross section

Pisano, Boer, Brodsky, Buffing, Mulders, *JHEP 10* (2013)

In the forward limit ($\Delta_T \to 0$) there is <u>no</u> $\sin\phi$ asymmetries

$$\lim_{\Delta_T \to 0} \mathscr{F}_1^{\gamma} = f_1^{\gamma} \qquad \lim_{\Delta_T \to 0} \mathscr{F}_2^{\gamma} = h_1^{\perp\gamma}$$

$$z = \frac{K_1 \cdot \kappa_1}{\kappa_1 \cdot \kappa_2} \qquad M_T = \sqrt{M^2 + K_T^2}$$

$$F^0_{\Delta_T \to 0} = 2\left[z^2 + (1-z)^2 + 4\,z\,(1-z)\left(1 - \frac{M^2}{M_T^2}\right)\frac{M^2}{M_T^2}\right]\mathscr{C}[f_1^{\gamma}\, f_1^{\gamma}] - z\,(1-z)\,\frac{M^4}{M_T^4}\,\mathscr{C}[w_0^{22}\, h_1^{\perp\gamma}\, h_1^{\perp\gamma}]$$

$$F^{\cos 2\phi_{qK}}_{\Delta \to 0} = 4\,z\,(1-z)\,\frac{M^2}{M_T^2}\left(1 - \frac{M^2}{M_T^2}\right)\mathscr{C}[w_{c2}^{12}\, f_1^{\gamma}\, h_1^{\perp\gamma} + w_{c2}^{21}\, h_1^{\perp\gamma}\, f_1^{\gamma}]$$

$$F^{\cos 4\phi_{qK}}_{\Delta \to 0} = -\,z\,(1-z)\left(1 - \frac{M^2}{M_T^2}\right)^2 \mathscr{C}[w_{c4}^{22}\, h_1^{\perp\gamma}\, h_1^{\perp\gamma}]$$

Two forms of the hard function: $H_0 = z^2 + (1-z)^2 \qquad H_1 = z\,(1-z)$

Absence of $\sin\phi$ asymmetries - an example

We consider the $\phi_{qK} = \phi_q - \phi_K$ dependence of the cross section for e^+e^- production

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\phi_{qK}} \propto \left(\widetilde{F}^{0} + \widetilde{F}^{\cos 4\phi_{qK}} \cos 4\phi_{qK} + \widetilde{F}^{\sin 4\phi_{qK}} \sin 4\phi_{qK} \right)$$

$$\widetilde{F}^{0} = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_T}{8\pi} \frac{\mathrm{d}^2\boldsymbol{b}_T}{2\pi} \mathrm{d}\overline{\mathrm{PS}}\, \mathrm{e}^{-i\boldsymbol{b}_T\cdot\boldsymbol{\Delta}_T} \left[F^{0} + F^{\cos 2\phi_{q\Delta}} \cos 2\phi_{q\Delta} + F^{\sin 2\phi_{q\Delta}} \sin 2\phi_{q\Delta} \right]$$

$$\widetilde{F}^{\cos 4\phi_{qK}} = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_T}{8\pi} \frac{\mathrm{d}^2\boldsymbol{b}_T}{2\pi} \mathrm{d}\overline{\mathrm{PS}}\, \mathrm{e}^{-i\boldsymbol{b}_T\cdot\boldsymbol{\Delta}_T} \left[F^{\cos 4\phi_{qK}} + F^{\cos 4\phi_{q\Delta}} \cos 4\phi_{q\Delta} + F^{\cos 2(\phi_{qK}+\phi_{\Delta K})} \cos 2\phi_{q\Delta} - F^{\sin 2(\phi_{qK}+\phi_{\Delta K})} \sin 2\phi_{q\Delta} \right]$$

$$\widetilde{F}^{\sin 4\phi_{qK}} = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_T}{8\pi} \frac{\mathrm{d}^2\boldsymbol{b}_T}{2\pi} \mathrm{d}\overline{\mathrm{PS}}\, \mathrm{e}^{-i\boldsymbol{b}_T\cdot\boldsymbol{\Delta}_T} \left[F^{\sin 4\phi_{qK}} + F^{\cos 4\phi_{q\Delta}} \sin 4\phi_{q\Delta} + F^{\cos 2(\phi_{qK}+\phi_{\Delta K})} \sin 2\phi_{q\Delta} + F^{\sin 2(\phi_{qK}+\phi_{\Delta K})} \cos 2\phi_{q\Delta} \right] = 0$$

The last term is *zero* due to the presence of ϕ_q in $\mathrm{d}\overline{\mathrm{PS}}$

$$\int_0^{2\pi} \mathrm{d}\phi_q \left[F^{\sin 4\phi_{qK}} + F^{\cos 4\phi_{q\Delta}} \sin 4\phi_{q\Delta} + F^{\cos 2(\phi_{qK}+\phi_{\Delta K})} \sin 2\phi_{q\Delta} + F^{\sin 2(\phi_{qK}+\phi_{\Delta K})} \cos 2\phi_{q\Delta} \right] = 0$$

Explicit b_T integration with $b_{\min}$

To obtain the expressions in UPC we take $b_{\min} = 2R_A$

forward limit!

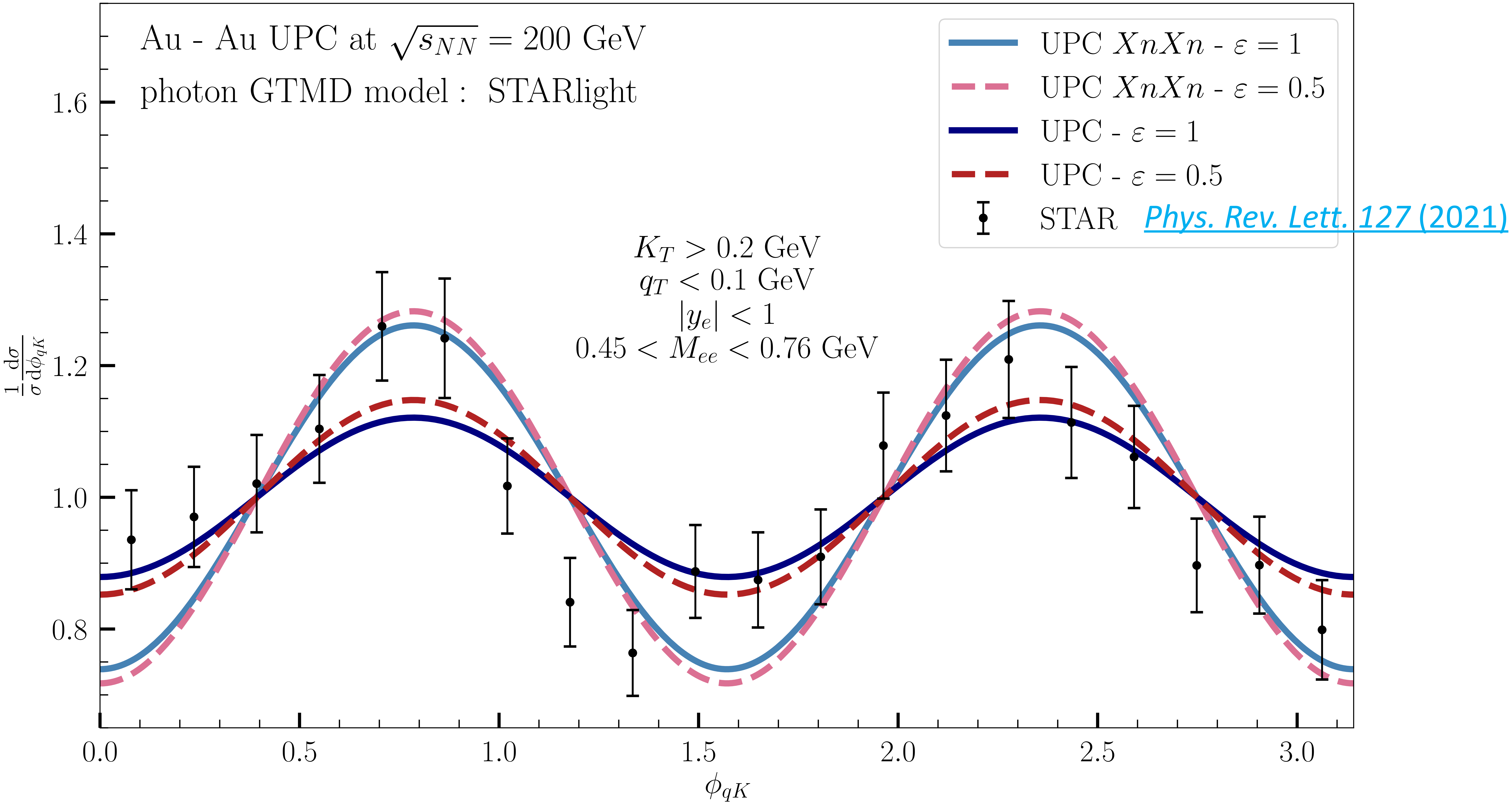
$$\frac{d\sigma}{dPS} = \int d^2\boldsymbol{b}_T \frac{d\sigma}{dPS\, d^2\boldsymbol{b}_T} = \int d^2\boldsymbol{\Delta}_T \left[\delta^{(2)}(\boldsymbol{\Delta}_T) - \frac{b_{\min}}{2\pi\Delta_T} J_1(b_{\min}\,\Delta_T)\right] \frac{d\sigma}{dPS\, d^2\boldsymbol{\Delta}_T}$$

$$\int d^2\boldsymbol{b}_T \frac{d\sigma}{dPS\, d^2\boldsymbol{b}_T} \cos 2\phi_{ba} = -\int \frac{d^2\boldsymbol{\Delta}_T}{2\pi\Delta_T} \left[b_{\min} J_1(b_{\min}\,\Delta_T) + \frac{2}{\Delta_T} J_0(b_{\min}\,\Delta_T)\right] \frac{d\sigma}{dPS\, d^2\boldsymbol{\Delta}_T} \cos 2\phi_{\Delta a}$$

$$\int d^2\boldsymbol{b}_T \frac{d\sigma}{dPS\, d^2\boldsymbol{b}_T} \cos 4\phi_{ba} = \int \frac{d^2\boldsymbol{\Delta}_T}{2\pi\Delta_T} \left[\frac{4}{\Delta_T}\Big(3\,J_2(b_{\min}\,\Delta_T) + J_0(b_{\min}\,\Delta_T)\Big) - b_{\min} J_1(b_{\min}\,\Delta_T)\right] \frac{d\sigma}{dPS\, d^2\boldsymbol{\Delta}_T} \cos 4\phi_{\Delta a}$$

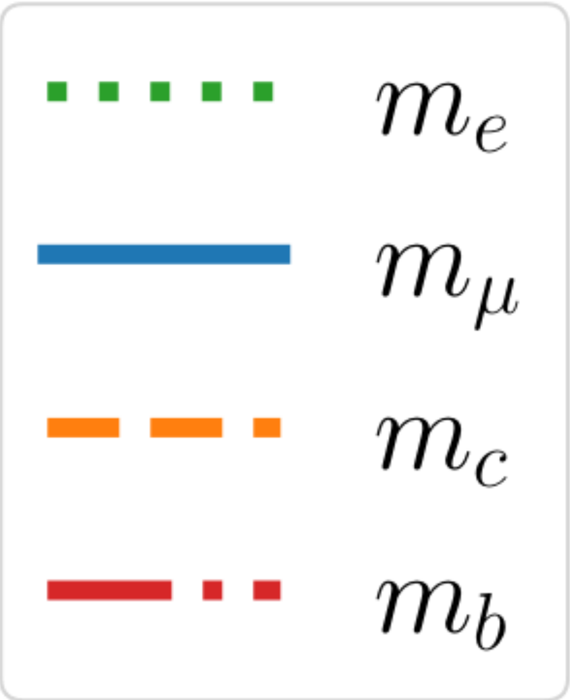
Note that this one-to-one correspondence is only for the *weight*

The $\cos 4\phi_{ee}$ asymmetry - STAR



Asymmetries - mass dependence ($2\phi_b$)

Vertical line corresponding to the limit of factorization reliability



Major **differences** in **sign** and **significance**
between leptons and quarks might be experimentally accessible

Observation of **oscillating behaviour** to test higher-order effects