Heavy-quark mass effects in off-light-cone distributions

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[V. Bertone, M. Fucilla, C. Mezrag. EPJC 85 (2025) 8, 889]

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THE ULAM PROGRAMME

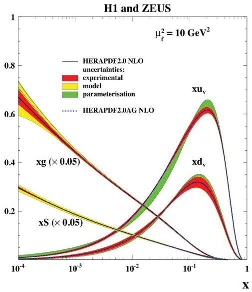
Parton Distribution Functions (PDFs)

- PDFs encode the **momentum distribution** of quarks and gluons (partons) inside a hadron
- Essential ingredient in high-energy processes involving hadrons (e.g. Drell–Yan, Deep Inelastic Scattering, etc...)
- Defined within the framework of **QCD factorization**:

Hard process $\otimes$ **PDFs** $\otimes$ **Fragmentation functions**

- **Collinear parton distribution functions** $f(x, \mu^2)$
- *Universal* (process-independent) and **non-perturbative**

  $\Rightarrow$ extracted from experiment



- **Possible to compute PDFs from first principles using QCD?**
- Can be crucial for more **complex distributions** (e.g. GPDs)

Parton distribution functions from Lattice QCD

- PDFs have non-perturbative nature $\longrightarrow$ **Lattice QCD?**
- Parton distributions are given in terms of **non-local light-cone correlators** (intrinsically Minkowskian); problem for the lattice!

$$f(x) = \frac{1}{2\pi} \int dz^- e^{-ixp^+ z^-} \langle p|\bar{\psi}(0)\Gamma^\lambda[0,z]\psi(z)|p\rangle \big|_{z^+, z_\perp = 0}$$

  - $[0,z] \longrightarrow$ gauge link
  - $\Gamma^\lambda \longrightarrow$ Dirac structure

- Breakthrough: **Quasi-PDF approach** **[X. Ji (2013)]**

  $\Longrightarrow$ **Large Momentum Effective Theory (LaMET)**

  Min-Huan's talk

- General idea: Compute a *quasi distribution*, $\mathcal{Q}(x, P^3)$, which is purely spatial (**equal-time correlator**) and uses nucleons with finite momentum P_3
- At LO, differs from the light-cone PDFs by terms $\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_3^2}, \frac{m_N^2}{P_3^2}\right)$

$$\mathcal{Q}(x, P^3) \xrightarrow{P^3 \to \infty} f(x)$$

- Alternative: **Short-distance factorization** **[Radyushkin (2017)]**

Short-distance factorization

- Matrix element $M^\alpha(z,p) = \langle p|\bar{\psi}(0)\gamma^\alpha[0,z]\psi(z)|p\rangle$ with $z = (0,0,0,z_3)$

$$M^\alpha(z,p) = p^\alpha \mathcal{M}(\nu, z^2) + z^\alpha \mathcal{M}_z(\nu, z^2)$$

$\nu = -pz$ is the **Ioffe time** $\qquad$ $\alpha = 0$ avoids contamination on the lattice

- $\mathcal{M}(\nu, z^2)$ is the off-light-cone leading-twist **Ioffe time distribution**
- The Fourier transform of this object gives a **pseudoPDF**

$$M^0(z,p) = 2p^0 \mathcal{M}(\nu, z^2) = 2p^0 \int_{-1}^{1} dx\, \mathcal{P}(x, z_3^2)\, e^{ix\nu}$$

- $\mathcal{P}(x, z_3^2)$ is a natural generalization of the light-cone PDF
- The *light-cone* Ioffe-time distribution and PDF can be obtained by the standard parametrization $z^\mu = z^- n_2^\mu$

$$\mathcal{I}(-p^+ z^-, 0) = \mathcal{I}(\nu) = \int_{-1}^{1} dx\, f(x)\, e^{ix\nu}$$

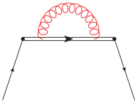
- The matching must be done order by order in perturbation theory

$\Longrightarrow$ Study **loop corrections** to the Ioffe-time distribution at small-z_3^2 (perturbative regime) **[Radyushkin (2018)]**

One-loop corrections: Wilson line correction

- Correction at the operator level $\mathcal{O}^\alpha$

  In the spirit of **[Balitsky, Braun (1989)]**



$$\mathcal{O}^\alpha_{\text{Wils.}} = (ig)^2 \frac{C_F}{2} \int_0^1 dt_1 \int_0^1 dt_2 z^\mu z^\nu D_{\mu\nu}(z(t_1 - t_2))\bar{\psi}(0)\gamma^\alpha\psi(z) \equiv \Gamma_{\text{Wils.}}(z)\bar{\psi}(0)\gamma^\alpha\psi(z)$$

$$\Gamma_{\text{Wils.}}(z) = -\frac{g^2 C_F}{8\pi^{D/2}} \Gamma\left(\frac{D}{2} - 1\right) \, (-z^2) \int_0^1 dt_1 \int_0^1 dt_2 \frac{1}{[-z^2(t_1 - t_2)^2]^{D/2-1}}$$

- Polyakov regularization: $1/(-z^2(t_1 - t_2)^2) \to 1/(-z^2(t_1 - t_2)^2 + a^2)$

  [Polyakov (1980)]

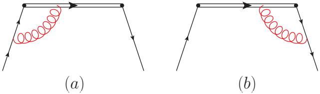
$$\Gamma_{\text{Wils.}}(z)|_{a\to 0} = -C_F \frac{\alpha_s}{2\pi} \left[\frac{\pi\,|z_3|}{a} - 2 - \ln \frac{z_3^2}{a^2} + \mathcal{O}\left(a^2/z_3^2\right) \right]$$

- Dimensional regularization $(D = 4 - 2\epsilon)$

$$\Gamma_{\text{Wils.}}(z) = -\frac{g^2 C_F}{(4\pi)^{D/2}} \frac{4\,\Gamma\left(\frac{D}{2} - 1\right)}{(D-3)(D-4)} \left(\frac{-z^2}{4}\right)^{2-D/2}$$

One-loop corrections: vertex diagram

- Vertex contribution



$$\mathcal{O}^{\alpha}_{\text{Vertex,b}} = g^2 C_F \int_0^1 dt \int d^D z_1 D_{\mu\nu}(z_1 - zt)\bar{\psi}(z_1)\gamma^{\mu} S_F(z_1)\gamma^{\alpha}\psi(z) z^{\nu}$$

- F.T. of $\bar{\psi}(z_1)$ and introducing the Schwinger parametrization for propagators

$$O^{\alpha}_b(z) = i^{3-D/2}\frac{2g^2 C_F}{(4\pi)^{D/2}}\int_0^1 dt \int_0^{\infty} d\sigma\, \sigma^{D/2-2} e^{-i\sigma t^2 z^2/4}$$
$$\int_0^1 d\beta \int d^4k e^{i\beta t(kz)}\bar{\psi}(k)\left[\frac{(1-\beta)(kz)}{\sigma} + \frac{tz^2}{4}\right]\gamma^{\alpha}\psi(z)\, e^{ik^2/\sigma}$$

- First term: isolate the UV-divergent contribution at small-t by the replacement $\bar{\psi}(\beta t z) = \bar{\psi}(0) + [\bar{\psi}(\beta t z) - \bar{\psi}(0)]$

$$O^{\alpha}_{\text{UV}}(z) = -\frac{g^2 C_F}{(4\pi)^{D/2}}\frac{2\Gamma\left(\frac{D}{2}-1\right)}{(D/2-2)}\left(\frac{-z^2}{4}\right)^{2-D/2}\bar{\psi}(0)\gamma^{\alpha}\psi(z)$$

One-loop corrections: vertex diagram

- Reminder of the subtraction → UV-finite contribution

$$O^{\alpha}_{\mathrm{UV,Fin}}(z) = -\frac{g^2 C_F}{8\pi^2}\int_0^1 du \int_0^{1-u} \mathrm{d}v \left(\delta(u)\left[\frac{\bar{v}}{v}\right]_+ + \delta(v)\left[\frac{\bar{u}}{u}\right]_+\right)\bar{\psi}(uz)\gamma^{\alpha}\psi(\bar{v}z)$$

- If the virtuality of the external quark leg is neglected, the second term becomes IR-divergent

$$O^{\alpha}_{\mathrm{IR}}(z) = -\frac{g^2 C_F}{8\pi^2}\int_0^1 du \int_0^{1-u} \mathrm{d}v \left(\ln\left(\frac{-z^2\mu^2}{4e^{-2\gamma_E-1}}\right) + \frac{1}{\epsilon_{\mathrm{IR}}}\right) \times \left(\delta(u)\left[\frac{\bar{v}}{v}\right]_+ + \delta(v)\left[\frac{\bar{u}}{u}\right]_+\right)\bar{\psi}(uz)\gamma^{\alpha}\psi(\bar{v}z)$$

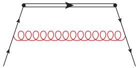
- The logarithm of $-z^2$ drives the **small-z^2 evolution**

- Additional IR-finite contribution

$$O^{\alpha}_{\mathrm{IR,Fin}}(z) = -\frac{g^2 C_F}{4\pi^2}\int_0^1 du \int_0^{1-u} \mathrm{d}v \left(\delta(u)\left[\frac{\ln v}{v}\right]_+ + \delta(v)\left[\frac{\ln u}{u}\right]_+\right)\bar{\psi}(uz)\gamma^{\alpha}\psi(\bar{v}z)$$

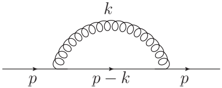
One-loop corrections: box and quark self-energy diagrams

- Box contribution



$$O^0_{\text{Box}}\ (z_3) = -\frac{\bar{g}^2 C_F}{8\pi^2}\int_0^1 du \int_0^{1-u}\ \mathrm{d}v \left\{ \ln\left(\frac{-z^2\mu^2}{4e^{-2\gamma_E-1}}\right) + \frac{1}{\epsilon_{\text{IR}}}\right\} \bar{\psi}\,(uz)\,\gamma^0\psi\,(\bar{v}z)$$

- Quark self-energy correction



$$\Gamma_{\text{Self.}} = \left.\frac{d\Sigma(p)}{d\not{p}}\right|_{\not{p}=m}$$

$$\Gamma_{\text{Self.}} = \frac{\bar{g}^2 C_F}{(4\pi)^{D/2}}\left[\frac{1}{\left(\frac{D}{2}-2\right)} + \frac{1}{\epsilon_{IR}}\right]$$

Full one-loop corrections

- One-loop corrections in the operator form ($\Gamma^0 = \{\gamma^0, \gamma^0\gamma^5\}$)

[Radyushkin (2018,2019)]

$$\delta\mathcal{O}^0(z_3) = -\frac{\alpha_s}{2\pi} C_F \int_0^1 du \int_0^{1-u} dv \bar{\psi}(uz)\Gamma^0\psi(\bar{v}z)$$
$$\times \left\{ \left(\delta(v)\left[\frac{\bar{u}}{u}\right]_+ + \delta(u)\left[\frac{\bar{v}}{v}\right]_+ + 1 \right) \left(\ln\left(\frac{-z^2\mu^2}{4e^{-2\gamma_E-1}}\right) + \frac{1}{\epsilon_{\mathrm{IR}}} \right) \right.$$
$$\left. +2\left(\delta(v)\left[\frac{\ln u}{u}\right]_+ + \delta(u)\left[\frac{\ln v}{v}\right]_+ - 1 \right) + Z'\left(z^2\right)\delta(u)\delta(v) \right\}$$

$\downarrow$ What does this **single equation** contain?

- Its logarithmic dependence on z^2 reflects the UV-behavior of the operator
- **Renormalization group equations** of QCD distributions (**ERBL**, **DGLAP**, and **GPD evolution** equations) are determined by deriving with respect to $\ln(-z^2)$ **[Balitsky, Braun (1987)] [Radyushkin (2018,2019)]**
- One can get the **matching kernel** for quark **PDFs**, $\langle p|\mathcal{O}^0(z)|p\rangle$, **DAs**, $\langle M|\mathcal{O}^0(z)|0\rangle$, **GPDs**, $\langle p'|\mathcal{O}^0(z)|p\rangle$
- $Z'(z^2)$ collects the UV-singular part and the contributions of the quark self-energy diagram $\propto \delta(u)\,\delta(v)$

Reduced Ioffe-time distribution

- The *off-light cone Ioffe-time distribution* is obtained by computing

$$\mathcal{M}^{1-\text{loop}}(\nu, z^2) = \langle p|\mathcal{O}^0(z_3)|p\rangle = \mathcal{M}^0(\nu) + \frac{\bar{g}^2 C_F}{8\pi^2}\Bigg\{\tilde{Z}(z^2)\mathcal{M}^0(\nu)$$

$$-\int_0^1 d\beta \left[\frac{1+\beta^2}{1-\beta}\left(\ln\left(\frac{-z^2\mu^2}{4e^{-2\gamma_E-1}}\right)+\frac{1}{\epsilon_{\text{IR}}}\right)+\frac{4\ln(1-\beta)}{1-\beta}-2(1-\beta)\right]_+ \mathcal{M}^0(\beta\nu)\Bigg\}$$

$$\tilde{Z}(z^2) = \frac{3}{2}\frac{1}{\epsilon_{\text{UV}}} + \frac{3}{2}\ln\left(\frac{-z^2\mu^2}{4e^{-2\gamma_E}}\right) + \frac{5}{2} \equiv \frac{3}{2}\frac{1}{\epsilon_{\text{UV}}} + \tilde{Z}_R(z^2)$$

- The *light-cone Ioffe-time distribution* at one-loop reads

$$\mathcal{I}^{1-\text{loop}}(\nu, \mu^2) = \mathcal{M}^0(\nu) + \frac{g^2 C_F}{8\pi^2}\int_0^1 d\beta \left[\frac{1+\beta^2}{1-\beta}\right]_+ \left(\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}}\right)\mathcal{M}^0(\beta\nu)$$

- Once the distributions are $\overline{\text{MS}}$ renormalized, their difference gives the matching kernel

$$\mathcal{I}(\nu, \mu^2) = \mathcal{M}(\nu, z^2) + \frac{\bar{g}^2 C_F}{8\pi^2}\int_0^1 d\beta \Bigg\{\Bigg[\frac{1+\beta^2}{1-\beta}\ln\left(\frac{-z^2\mu^2}{4e^{-2\gamma_E-1}}\right)$$

$$+\frac{4\ln(1-\beta)}{1-\beta} - 2(1-\beta)\Bigg]_+ - \tilde{Z}_R(z^2)\delta(1-\beta)\Bigg\}\mathcal{M}^0(\beta\nu)$$

- **Reduced Ioffe-time distribution**: $\mathfrak{M}(v, -z^2) \equiv \mathcal{M}(v, -z^2)/\mathcal{M}(0, -z^2)$

Parton distribution function of the charm

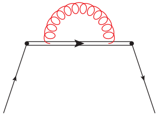
- Parton distributions of **heavy quarks** are key quantities for understanding the structure of the proton.
- The NNPDF collaboration has provided evidence for the existence of an **intrinsic charm** component in the proton

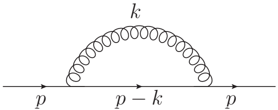
  [NNPDF Collaboration (2022)] [NNPDF Collaboration (2023)]
- An exciting prospect would be the extraction of the charm PDF directly from **Lattice QCD**.
- The short-distance factorization regime, $z_3^2 \Lambda_{\mathrm{QCD}}^2 \ll 1$, allows for **leading-twist matching**

- For **heavy quarks**, an additional perturbative scale m_Q enters the problem, introducing power corrections to the matching kernel of the form $z_3^2 m_Q^2$.
- However, the regime $z_3^2 m_Q^2 \ll 1$ is **not accessible** at current lattice spacings
- Very small z_3^2 values also imply a **limited Ioffe-time window** $\nu = P_3 z_3$, reducing sensitivity to the shape of the PDF in x
- Idea: treat m_Q as a perturbative scale, and incorporate its dependence explicitly in the **perturbative matching kernel**
- Collinear factorization with heavy quarks is achieved by neglecting higher twist terms $\sim \Lambda_{\mathrm{QCD}}/Q$, **independently** of the value of m_Q with respect to Q (i.e. for $Q \geq m_Q$) **[J. Collins (1998)]**

$$F = \hat{F}(m_Q^2) \otimes f + \text{remainder}$$

Massive case: Wilson line and quark self-energy





- The Wilson line self-energy is not affected

$$\mathcal{M}_{\text{Wils.}}(\nu, z^2) = -\frac{g^2 C_F}{(4\pi)^{D/2}} \frac{2\,\Gamma\left(\frac{D}{2}-1\right)}{(D-3)\left(\frac{D}{2}-2\right)} \left(\frac{-z^2}{4}\right)^{2-D/2} \mathcal{M}^0(\nu)$$

- The One-loop corrections the quark external legs is

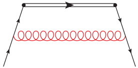
$$\mathcal{M}_{\text{Self.}}(\nu, z^2, m^2) = -\frac{2g^2 C_F}{(4\pi)^{D/2}} \left[(m^2)^{D/2-2} \frac{\Gamma\left(3-\frac{D}{2}\right)}{\epsilon_{IR}} + 2 \right] \mathcal{M}^0(\nu)$$

$$+ \frac{g^2 C_F}{(4\pi)^{D/2}} \left[\frac{1}{\left(\frac{D}{2}-2\right)} \left(\frac{-z^2}{4e^{-2\gamma_E}}\right)^{2-D/2} + \ln\left(\frac{-z^2 m^2}{4e^{-2\gamma_E}}\right) \right] \mathcal{M}^0(\nu)$$

- The new term contains a infrared singularity associated with the *massless gluon dynamics* → regularized in dimReg with $D = 4 - 2\epsilon_{\text{IR}}$ ($\epsilon_{IR} < 0$)

Massive case: box contribution

- Box contribution

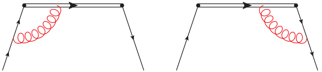


$$\mathcal{M}_{\text{Box}}(\nu, z^2, m^2) = \frac{2g^2 C_F}{(4\pi)^{D/2}} \Bigg\{ \Bigg[(m^2)^{D/2-2}\Gamma\left(3 - \frac{D}{2}\right)\left(\frac{1}{\epsilon_{\text{IR}}} + 2\right)$$
$$+2\left(\frac{1 - \sqrt{-z^2 m^2} K_1(\sqrt{-z^2 m^2})}{-z^2 m^2}\right)\Bigg] \mathcal{M}^0(\nu)$$
$$+ \int_0^1 d\beta \left[2(1-\beta) K_0\left(\sqrt{-z^2(1-\beta)^2 m^2}\right)\right]_+ \mathcal{M}^0(\beta\nu)$$
$$+\frac{1}{2}\int_0^1 d\beta \left[\frac{-4\beta}{1-\beta}\right]_+ \mathcal{M}^0(\beta\nu)\sqrt{-z^2(1-\beta)^2 m^2} K_1\left(\sqrt{-z^2(1-\beta)^2 m^2}\right) \Bigg\}$$

- The red term containing the infrared singularity cancel that in the quark self-energy
- $K_0\left(\sqrt{-z^2(1-\beta)^2 m^2}\right)$ generalizes the logarithm of the massless case and drives the **small-z^2 behaviour**
- The second term appears when enforcing the plus prescription structure in the term containing the Macdonald function K_0

Massive case: vertex contribution

- Vertex contribution



(a) (b)

$$\mathcal{M}_{\text{Vertex}}(\nu, z^2, m^2) = \frac{g^2 C_F}{(4\pi)^{D/2}} \Bigg\{ -\frac{2\Gamma\left(\frac{D}{2}-1\right)}{\left(\frac{D}{2}-2\right)} \left(\frac{-z^2}{4}\right)^{2-D/2} \mathcal{M}^0(\nu)$$

$$-8\mathcal{M}^0(\nu) R\big(\sqrt{-z^2 m^2}\big) + 2\int_0^1 d\beta \left[\frac{4\beta}{1-\beta} K_0\left(\sqrt{-z^2(1-\beta)^2 m^2}\right)\right]_+ \mathcal{M}^0(\beta\nu)$$

$$-2\int_0^1 d\beta \left[4\Phi(1-\beta, \sqrt{-z^2m^2})\mathcal{M}^0(\beta\nu) \; -4\left(\frac{\ln(1-\beta)+\beta}{1-\beta}\right) \mathcal{M}^0(\nu)\right]$$

$$+\int_0^1 d\beta \left[\frac{4\beta}{1-\beta}\right]_+ \mathcal{M}^0(\beta\nu)\sqrt{-z^2(1-\beta)^2m^2} K_1\left(\sqrt{-z^2(1-\beta)^2m^2}\right) \Bigg\}$$

- The Φ-function satisfies

$$\Phi(1-\beta, \sqrt{-z^2m^2}) = \frac{\beta + \ln(1-\beta)}{1-\beta} + \mathcal{O}((1-\beta)^0)$$

- Expansion for $z^2m^2 \to 0$

$$\Phi(1-\beta, \sqrt{-z^2m^2}) = \frac{\beta + \ln(1-\beta)}{1-\beta} + \mathcal{O}(z^2m^2) \qquad R\big(\sqrt{-z^2m^2}\big) = \mathcal{O}(z^2m^2)$$

Massive matching kernel in the $z^2m^2 \to 0$

- Full one-loop distribution in the $z^2m^2 \to 0$

$$\mathcal{M}^{1-\text{loop}}(\nu, z^2, m^2)\big|_{z^2m^2\to 0} = \frac{\bar{g}^2 C_F}{8\pi^2}\Bigg\{ -\int_0^1 d\beta \left[\frac{4\ln(1-\beta)}{1-\beta} - 2(1-\beta)\right]_+ \mathcal{M}^0(\beta\nu)$$
$$+\tilde{Z}(z^2)\mathcal{M}^0(\nu) - \int_0^1 d\beta \left[\frac{1+\beta^2}{1-\beta}\left(\ln\left(\frac{-z^2 m^2}{4e^{-2\gamma_E-1}}\right) + 2\ln(1-\beta)+1\right)\right]_+ \mathcal{M}^0(\beta\nu)\Bigg\}$$

$$\tilde{Z}(z^2) = Z(z^2)\big|_{z^2m^2\to 0} = \frac{3}{2}\left(\frac{1}{\epsilon_{\text{UV}}} + \ln\left(\frac{-z^2\mu^2}{4e^{-2\gamma_E}}\right)\right) + \frac{5}{2}$$

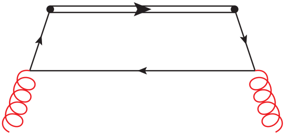
- Must be matched onto a PDF computed in the same scheme

$$\mathcal{I}^{1-\text{loop}}(\nu, \mu^2, m^2) = \frac{\bar{g}^2 C_F}{8\pi^2}\int_0^1 d\beta\, \mathcal{M}^0(\beta\nu)\left[\frac{1+\beta^2}{1-\beta}\left(\frac{1}{\epsilon_{\text{UV}}} \underbrace{-\ln\left(\frac{m^2}{\mu^2}\right) - 2\ln(1-\beta) - 1}_{\substack{\text{Heavy}-\text{quark threshold matching} \\ \text{for PDF evolution in a variable}- \\ \text{flavour}-\text{number scheme}}}\right)\right]_+$$

e.g. **[R. D. Ball et. al (2016)]**

Massive case: gluon-quark mixing

- Quark and **gluon distributions** become coupled at one-loop order



$$\mathcal{M}_{\text{gluon-mix}}(\nu, z^2, m^2) = \frac{\bar{g}^2}{8\pi^2} 2T_R \int_0^1 d\beta\; 2p^0 f_g^{(0)}(\beta\nu)$$

$$\left\{ [\beta^2 + (1-\beta)^2] \underbrace{K_0(\sqrt{-z^2 m^2})} + \beta(1-\beta) \underbrace{\sqrt{-z^2m^2} K_1(\sqrt{-z^2m^2})} \right\}$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

Massless : $z^2 m^2 \sim 0 \implies \quad -\frac{1}{2} \ln\left(\frac{-z^2m^2}{4e^{-2\gamma_E}}\right) \qquad\qquad 1$

- The result of this computation coincides with that of the gluon contribution to **heavy-quark TMDs** at one-loop order

[P. M. Nadolsky, N. Kidonakis, F. I. Olness, C. P. Yuan (2003)]

[R. von Kuk, J. K. L. Michel, and Z. Sun (2023)]

- The *gauge-link structure* does **not** play any role in this diagram.

Massive matching kernel

- **Matching relation**

$$\underbrace{f_Q\left(x, z^2, \mu^2\right)}_{\text{PseudoPDF}} = \sum_{i=Q,g} \int_x^1 \frac{dy}{y} \mathcal{C}_{Qi}\left(y, z^2\mu^2, z^2m^2, g\right) \underbrace{f_i\left(\frac{x}{y}, 0, \mu^2\right)}_{\text{PDF (LHAPDF)}}$$

[V. Bertone, M. F., C. Mezrag (2025)]

- **Diagonal contribution**

$$\mathcal{C}_{QQ}\left(y, z^2\mu^2, z^2m^2, g\right) = \delta(1-y) + \frac{\bar{g}^2 C_F}{8\pi^2}\Bigg\{ Z_{\mathrm{R}}(z^2)\delta(1-y)$$
$$+ \left[\frac{1+y^2}{1-y}\left(2K_0\left(\sqrt{-z^2m^2(1-y)^2}\right) + \ln\left(\frac{m^2}{\mu^2}\right) + 2\ln(1-y) + 1\right)\right.$$
$$\left. -4\frac{\ln(1-y)+y}{1-y}\right]_+ - 4\left(\Phi\left(1-y, \sqrt{-z^2m^2}\right) - \frac{\ln(1-y)+y}{1-y}\right)\Bigg\}$$

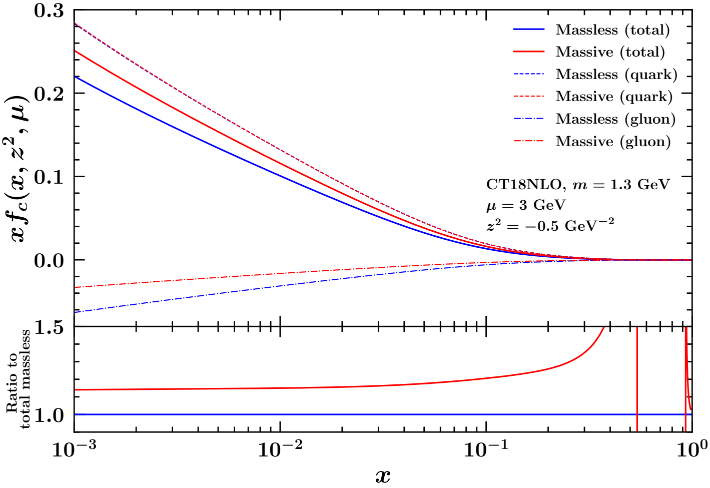
- **Off-diagonal contribution**

$$\mathcal{C}_{Qg}\left(y, z^2\mu^2, z^2m^2, g\right) = \frac{\bar{g}}{8\pi^2} 2T_R \Bigg\{ [y^2 + (1-y)^2]\left(K_0(\sqrt{-z^2m^2}) + \frac{1}{2}\ln\left(\frac{m^2}{\mu^2}\right)\right)$$
$$+ y(1-y)\sqrt{-z^2m^2}K_1(\sqrt{-z^2m^2})\Bigg\}$$

Heavy-quark mass effects

- Mass effects amount to **15** % in the relevant x region ($x \lesssim 0.1$)

[V. Bertone, M. Fucilla, C. Mezrag (2025)]



- The effect is entirely due to the **gluon channel**.

Thank you for the attention!

Backup

Factorization in presence of heavy-quarks

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Hard-scattering factorization with heavy quarks: A general treatment

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A detailed proof of hard-scattering factorization is given with the inclusion of heavy quark masses. Although the proof is explicitly given for deep-inelastic scattering, the methods apply more generally. The power-suppressed corrections to the factorization formula are uniformly suppressed by a power of Λ/Q, independently of the size of heavy quark masses, M, relative to Q. [S0556-2821(98)03819-3]

$$F = \hat{F} \otimes f + \text{remainder},$$

with the following properties:

(1) The coefficient function $\hat{F}(x/\xi, Q^2, M^2)$ is infra-red safe: it is dominated by virtualities of order Q^2.

(2) The parton density f is a renormalized matrix element of a light-cone operator.

(3) The remainder is suppressed by a power of Λ/Q.

(4) This suppression is uniform over the whole range $Q \gtrsim M$