

On the PB Sudakov:

NNLL coefficient, CS kernel and intrinsic-kt

- **Ola Lelek** on behalf of the Parton Branching & Cascade team



Soft gluon resummation

Different methods developed to deal with soft gluons

- In Monte Carlos: **Parton Showers (PS)**
- **Transverse Momentum Dependent (TMD)** factorization theorems
baseline: low q_\perp Collins-Soper-Sterman (**CSS**)
- SCET-based factorization
- small- x
- more recent **TMD Parton Branching (PB)**

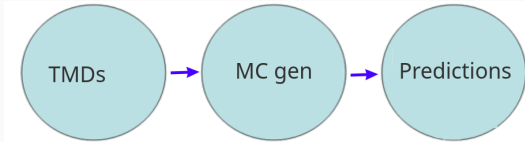
Different approaches have different origin, assumptions, motivations, application, mathematical formalism, successes and failures etc, ...

Connections and differences between them have to be understood

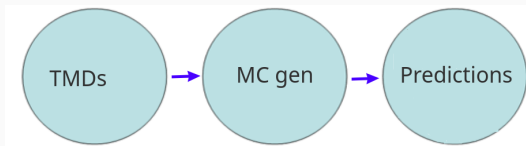
This talk:

TMD PB, especially **PB Sudakov** form factor and its relation to **Sudakov of CSS**

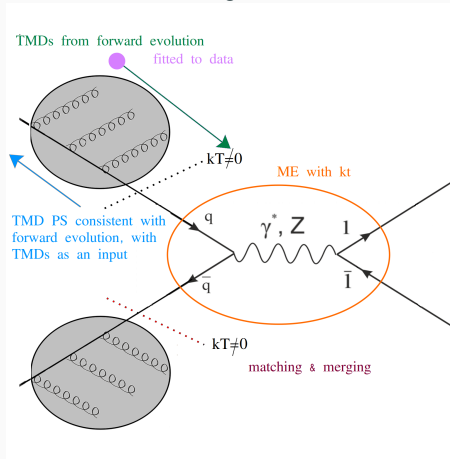
What is the TMD Parton Branching method?



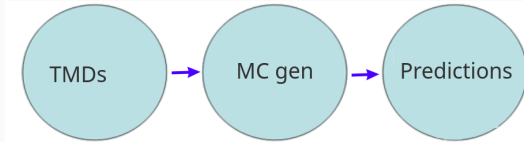
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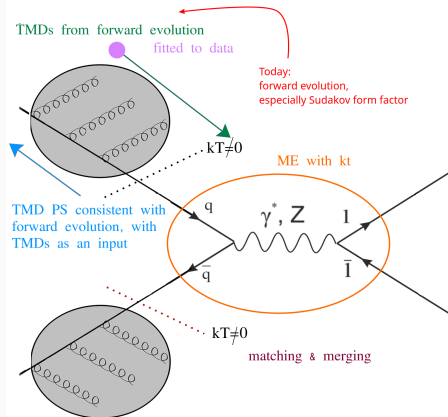
Building blocks:



What is the TMD Parton Branching method?



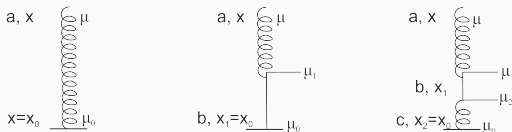
Building blocks:



TMD evolution equation

Evolution in the TMD PB method

Hautmann, Jung, Lelek, Radescu, Zlebicki, Phys.Lett.B 772 (2017) 446 & JHEP 01 (2018) 070



$$\begin{aligned} \tilde{A}_a(x, k_{\perp}^2, \mu^2) &= \Delta_a(\mu^2, \mu_0^2) \tilde{A}_a(x, k_{\perp}^2, \mu_0^2) + \sum_b \int \frac{d^2 \mu_{\perp 1}}{\pi \mu_{\perp 1}^2} \Theta(\mu_{\perp 1}^2 - \mu_0^2) \Theta(\mu^2 - \mu_{\perp 1}^2) \\ &\times \Delta_a(\mu^2, \mu_{\perp 1}^2) \int_x^{z_M} dz P_{ab}^R(z, \mu_{\perp 1}^2) \tilde{A}_b\left(\frac{x}{z}, |k_{\perp 1}|^2, \mu_{\perp 1}^2\right) \Delta_b(\mu_{\perp 1}^2, \mu_{\perp 0}^2) + \dots \end{aligned}$$

Intuitive **probabilistic interpretation** \iff easy to **solve by Monte Carlo (MC)** :

- Sudakov form factor

$$\begin{aligned} \Delta_a(\mu^2, \mu_0^2) &= \exp \left(- \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz z P_{ba}^R(z, \mu'^2) \right) \\ &\approx \exp \left(- \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \left(\int_0^{z_M} k_a(\alpha_s) \frac{1}{1-z} dz - d_a(\alpha_s) \right) \right) \end{aligned}$$

probability of an evolution without resolvable branchings between μ_0^2 and μ^2

- Splitting function $P_{ab}^R(z, \mu^2)$ - probability of $b \rightarrow a$

P_{qq}^R & P_{gg}^R - **divergent** for $z \rightarrow 1 \iff$ **soft gluons**: z_M defines **resolvable and non-resolvable** branchings

$\tilde{A} = xA$, z - splitting variable, $x = zx_1$, $z \in (0, 1)$

Transverse momentum in PB

- starting distribution at μ_0^2 :

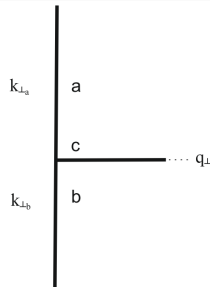
$$\tilde{A}_{a,0}(x, k_{\perp 0}^2, \mu_0^2) = \tilde{f}_{a,0}(x, \mu_0^2) \frac{1}{\pi q_s^2} \exp\left(\frac{-k_{\perp 0}^2}{q_s^2}\right)$$

- Initial distribution $\tilde{f}_{a,0}(x, \mu_0^2)$ obtained from **fits to inclusive DIS data**
- Intrinsic transverse momentum $k_{\perp 0}$ constraint from DY data
- transverse momentum **k calculated at each branching**

$$\mathbf{k}_a = \mathbf{k}_b - \mathbf{q}_c,$$

\mathbf{k} of the propagating parton is a sum of intrinsic transverse momentum and all emitted transverse momenta

$$\mathbf{k} = \mathbf{k}_0 - \sum_i \mathbf{q}_i \rightarrow \text{b TMD from parton branching}$$



How to relate q_{\perp} and the evolution scale μ' ?

→ Ordering condition: **Angular Ordering** (AO) of S. Catani, G. Marchesini, B. Webber (CMW)
scale associated with the rescaled transverse momentum

$$q_{\perp} = (1 - z)\mu'$$

AO assures PB TMDs do not have IR singularities

Moreover:

$$\alpha_s(q_{\perp})$$

AO picture

$$\Delta_a(\mu^2, \mu_0^2) \approx \exp \left(- \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \left(\int_0^{z_M} k_a(\alpha_s) \frac{1}{1-z} dz - d_a(\alpha_s) \right) \right)$$

AO of Catani-Marchesini-Webber (CMW): $q_\perp = (1-z)\mu'$

If we assume minimum $q_0 \rightarrow z_M = z_{\text{dyn}} = 1 - q_0/\mu'$

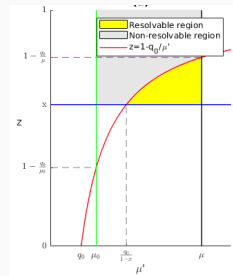
Let's use z_{dyn} as an intermediate scale, to divide the phase space:

- Perturbative: $z < z_{\text{dyn}}$, where $|q_\perp| > q_0$ (resolvable)
- NP: $z_{\text{dyn}} < z < z_M$ ($z_M = 1 - \epsilon$ with $0 \simeq \epsilon \ll 1$), where $|q_\perp| < q_0$ (non-resolvable)

$$\Delta_a(\mu^2, \mu_0^2) = \exp \left(- \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \left[\int_0^{z_{\text{dyn}}(\mu')} dz \frac{k_q(\alpha_s)}{1-z} - d_q(\alpha_s) \right] \right) \\ \times \exp \left(- \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_{z_{\text{dyn}}(\mu')}^{z_M \approx 1} dz \frac{k_q(\alpha_s)}{1-z} \right).$$

i.e.:

$$\Delta_a(\mu^2, \mu_0^2) = \Delta_a^{(P)}(\mu^2, \mu_0^2, q_0) \cdot \Delta_a^{(NP)}(\mu^2, \mu_0^2, \epsilon, q_0^2).$$

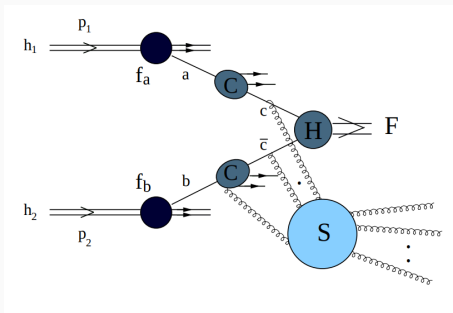


Nucl.Phys.B 949 (2019) 114795

Using dynamical $z_M = 1 - \frac{q_0}{\mu'}$ i.e. skipping the non-perturbative Sudakov in the evolution has interesting consequences

CSS Sudakov form factor

Collins-Soper-Sterman (CSS)



$$\frac{d\sigma}{dq_{\perp}} \sim \int d^2b \exp(i\mathbf{b} \cdot \mathbf{q}_{\perp}) \int dz_1 dz_2 H(Q^2) F_1(z_1, b, \text{scales}) F_2(z_2, b, \text{scales}) + Y$$

where the TMD: $F = f \otimes C \otimes \sqrt{\Delta}$

and the Sudakov Δ divided in perturbative and non-perturbative parts:

$$\Delta = \Delta^{(P)} \Delta^{(NP)}$$

$$\Delta_a^{\text{CSS1}}(Q, Q_0, b, x_a, x_{\tilde{a}}, b_{\text{max}}, C_1, C_2) =$$

$$\exp \left\{ - \int_{\mu_{b*}^2}^{\mu_Q^2} \frac{d\mu'^2}{\mu'^2} \left(A_a(\alpha_s) \ln \left(\frac{\mu_Q^2}{\mu'^2} \right) + B_a(\alpha_s) \right) \right\}$$

$$\times \exp \left\{ -g_{a/A}(x_a, b, b_{\text{max}}) - g_{\tilde{a}/B}(x_{\tilde{a}}, b, b_{\text{max}}) - g_{K,a}(b, b_{\text{max}}) \ln \frac{Q^2}{Q_0^2} \right\}$$

$A_a(\alpha_s)$ and $B_a(\alpha_s)$ have series expansions: $\mathcal{R}_a = \sum_n (\alpha_s/2\pi)^n \mathcal{R}_a^{(n)}$.

$\Delta^{(P)}$: Perturbative resummation

LL: $A_a^{(1)}$

NLL: $A_a^{(2)}$ and $B_a^{(1)}$,

NNLL by $A_a^{(3)}$ and $B_a^{(2)}$ etc.

Perturbative Sudakov

Perturbative Sudakov

After change of integration variables ($\mu' \rightarrow q_\perp = (1-z)\mu'_\perp$)

$$\Delta_a^{(P)}(\mu^2, q_0^2) = \exp \left(- \int_{q_0^2}^{\mu^2} \frac{dq_\perp^2}{q_\perp^2} \left[\frac{1}{2} k_a(\alpha_s) \ln \left(\frac{\mu^2}{q_\perp^2} \right) - d_a(\alpha_s) \right] \right)$$

the perturbative PB Sudakov coincides, in its overall structure, with the perturbative CSS1 Sudakov form factor

$$\Delta_a^{\text{CSS1 (P)}}(\mu^2, \mu_{b*}^2) = \exp \left(- \int_{\mu_{b*}^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \left[A_a(\alpha_s) \ln \left(\frac{\mu^2}{\mu'^2} \right) + B_a(\alpha_s) \right] \right)$$

One can **try to compare the exact forms of the PB coefficients with that of CSS1 to determine the logarithmic accuracy** achieved by the PB Sudakov form factor.

Perturbative Sudakov

After change of integration variables ($\mu' \rightarrow q_\perp = (1-z)\mu'_\perp$)

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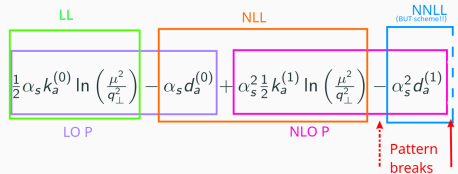
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One can **try to compare the exact forms of the PB coefficients with that of CSS1 to determine the logarithmic accuracy** achieved by the PB Sudakov form factor.

- LO P and LL: $k_a^{(0)} = A_a^{(1)} = \frac{1}{2} \gamma_{k,a}^{(1)}$
- LO P and NLL: $d_a^{(0)} = -\frac{1}{2} B^{(1)} = \frac{1}{2} \gamma_a^{(1)}$
- NLO P and NLL: $k_a^{(1)} = A_a^{(2)} = \frac{1}{2} \gamma_{k,a}^{(2)}$

Here however this simple pattern breaks !



Renormalization group mix the B , C , and H of the CSS formalism

$$d_a^{(1)} = -\frac{1}{2} B^{(2)} \bar{M}^S$$

Difference between PB and CSS literature:

$$B_q^{(2)\text{DY}} - (-2) \cdot d_q^{(1)} = 16 C_F \pi \beta_0 (\zeta_2 - 1) \text{ and } B_g^{(2)\text{H}} - (-2) \cdot d_g^{(1)} = 16 C_A \pi \beta_0 (\zeta_2 + \frac{11}{24})$$

Collinear anomaly: Becher & Neubert, Eur.Phys.J.C 71 (2011) 1665

the NNLL resummation coefficient $A_a^{(3)}$ differs from the NNLO DGLAP coefficient $k_a^{(2)}$.

The difference is:

$$A_a^{(3)} - k_a^{(2)} = C_a \pi \beta_0 \left[C_A \left(\frac{808}{27} - 28\zeta_3 \right) - \frac{112}{27} N_f \right]$$

One obtains this result by differentiating the CS kernel with respect to $\ln b_\star^2$:

$$A_a - k_a = - \frac{d\tilde{K}_a(b_\star, \mu_{b_\star})}{d \ln b_\star^2}$$

The **NNLL accuracy can be achieved by the usage of physical soft-gluon coupling**

Catani et al., Eur.Phys.J.C 79 (2019) 8, 685

Banfi et al., JHEP 01 (2019) 083

$$\alpha_s^{\text{NNLL}} = \alpha_s \left(1 + \mathcal{K}^{(2)} \left(\frac{\alpha_s}{2\pi} \right)^2 \right)$$

I. e. We modify

$$\Delta_a^{(P)}(\mu^2, q_0^2) = \exp \left(- \int_{q_0^2}^{\mu^2} \frac{dq_\perp^2}{q_\perp^2} \left[\frac{1}{2} k_a(\alpha_s^{\text{NNLL}}) \ln \left(\frac{\mu^2}{q_\perp^2} \right) - d_a(\alpha_s^{\text{NNLL}}) \right] \right)$$

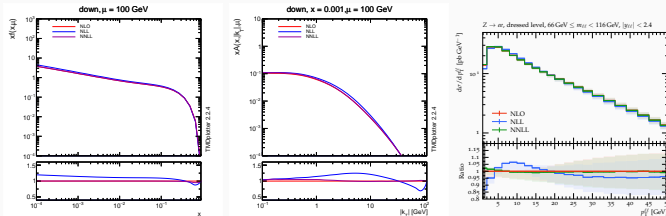
With NLO P

$$\begin{aligned}
 \ln(\Delta_a^{(P)}(\mu^2, q_0^2)) &= - \int_{q_0^2}^{\mu^2} \frac{dq_\perp^2}{q_\perp^2} \frac{\alpha_s^{\text{phys}}}{2\pi} \left(\ln \frac{\mu^2}{q_\perp^2} \left(k_a^{(0)} + \frac{\alpha_s^{\text{phys}}}{2\pi} k_a^{(1)} \right) - d_a^{(0)} - \frac{\alpha_s^{\text{phys}}}{2\pi} d_a^{(1)} \right) \\
 &= - \int_{q_0^2}^{\mu^2} \frac{dq_\perp^2}{q_\perp^2} \frac{\alpha_s}{2\pi} \left(\ln \frac{\mu^2}{q_\perp^2} \left(k_a^{(0)} + \frac{\alpha_s}{2\pi} k_a^{(1)} \right) - d_a^{(0)} - \frac{\alpha_s}{2\pi} d_a^{(1)} \right) \\
 &\quad + \frac{\alpha_s^2}{(2\pi)^2} \mathcal{K}^{(2)} k_a^{(0)} \frac{1}{2} \ln \frac{\mu^2}{q_\perp^2} + \dots
 \end{aligned}$$

where $\mathcal{K}^{(2)} \cdot k_a^{(0)} = A_a^3$.

All Sudakov coefficients at NNLL, i.e. $A_a^{(1)}$, $A_a^{(2)}$, $A_a^{(3)}$, $B_a^{(1)}$ and $B_a^{(2, \overline{MS})}$, are included in the PB

The middle row: standard NLO PB evolution



Non-perturbative Sudakov

$$\begin{aligned} \Delta_a^{\text{CSS2}}(Q, Q_0, b, x_a, x_{\tilde{a}}, b_{\text{max}}, C_1, C_2) = \\ \exp \left\{ - \int_{\mu_{b*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left(\gamma_{k,a}(\alpha_s) \ln \left(\frac{Q^2}{\mu'^2} \right) - 2\gamma_a(\alpha_s) \right) \right\} \\ \times \exp \left(-g_{a/A}(x_a, b, b_{\text{max}}) - g_{\tilde{a}/B}(x_{\tilde{a}}, b, b_{\text{max}}) - g_{K,a}(b, b_{\text{max}}) \ln \frac{Q^2}{Q_0^2} \right) \\ \times \exp \left(\tilde{K}_a(b_*, \mu_{b*}) \ln \frac{Q^2}{\mu_{b*}^2} \right) \end{aligned}$$

\tilde{K}_a perturbative CS kernel

$g_{K,a}$ the non-perturbative part of the CS kernel

$g_{a/A}$, $g_{\tilde{a}/B}$ and $g_{K,a}$ the same as in the CSS1

A_a and B_a and $\gamma_{k,a}$ and γ_a do not coincide at all orders but they are related with each other.

$$\begin{aligned}
 \Delta_a^{\text{CSS2}}(Q, Q_0, b, x_a, x_{\tilde{a}}, b_{\text{max}}, C_1, C_2) = & \\
 \exp \left\{ - \int_{\mu_{b*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left(\gamma_{k,a}(\alpha_s) \ln \left(\frac{Q^2}{\mu'^2} \right) - 2\gamma_a(\alpha_s) \right) \right\} & \\
 \times \exp \left(-g_{a/A}(x_a, b, b_{\text{max}}) - g_{\tilde{a}/B}(x_{\tilde{a}}, b, b_{\text{max}}) - g_{K,a}(b, b_{\text{max}}) \ln \frac{Q^2}{Q_0^2} \right) & \\
 \times \exp \left(\tilde{K}_a(b_*, \mu_{b*}) \ln \frac{Q^2}{\mu_{b*}^2} \right) & \leftarrow \text{CS kernel}
 \end{aligned}$$

\tilde{K}_a perturbative CS kernel

$g_{K,a}$ the non-perturbative part of the CS kernel

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\tilde{K}_a perturbative CS kernel

$g_{K,a}$ the non-perturbative part of the CSS2 kernel
 $g_{a/A}$, $g_{\tilde{a}/B}$ and $g_{K,a}$ the same as in the CSS1 formalism
 A_a and B_a and $\gamma_{k,a}$ and γ_a do not coincide with each other.

In CSS formalism:

$$\frac{\partial \ln f_f/H(x, b_t, \zeta, \mu)}{\partial \ln \sqrt{\zeta}} = \mathcal{K}(b_t, \mu)$$

CS kernel:

- governs the rapidity evolution
- contains non-perturbative information
- can be extracted from measurements
- is the only QCD function which is largely unknown

Non-perturbative PB Sudakov

Non-resolvable region:

$z_{\text{dyn}} < z < z_M$ ($z_M = 1 - \epsilon$ with $0 \simeq \epsilon \ll 1$), for which $|q_{\perp}| < q_0$

In PB, $\alpha_s = \alpha_s(q_{\perp}) \rightarrow$ **freeze at $\alpha_s(q_{\text{cut}})$**

$$\Delta_a^{(\text{NP})}(\mu^2, \mu_0^2, \epsilon, q_0) = \exp \left(- \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_{1-q_0\mu'}^{1-\epsilon} dz \frac{k_a(\alpha_s)}{1-z} \right) = \exp \left(- \frac{k_a(\alpha_s)}{2} \ln \left(\frac{\mu^2}{\mu_0^2} \right) \ln \left(\frac{q_0^2}{\epsilon^2 \mu_0 \mu} \right) \right)$$

$\ln \mu^2 / \mu_0^2$ resembles the structure of the NP CS kernel $g_{K,a}(b, b_{\text{max}}) \ln \frac{Q^2}{Q_0^2} \rightarrow$ **rapidity evolution**

Remarks:

In the CSS literature: NP CS kernel modelled and fitted to data

In PB: Modelling of the NP Sudakov probes the AO picture (i.e. z_{dyn}), & depends on α_s modelling (freezing)

b and μ are related to each other:

- b is Fourier transform of k_{\perp}
- k_{\perp} contains the whole evolution history, ($k_{\perp} = k_{\perp 0} - \sum_i q_{\perp,i}$)
- $q_{\perp,i}$ is related to the branching scales by the AO condition, i.e. $q_{\perp,i} = (1 - z_i) \mu'_{\perp,i}$

Next slides: **extract the CS kernel from the PB approach**

Models for numerical studies

We study 5 PB models which **differ in the amount of (soft) radiation**.

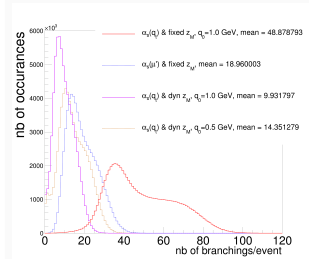
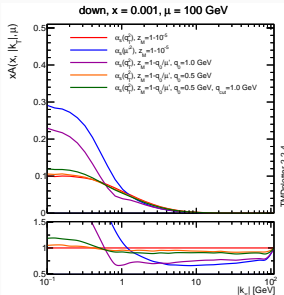
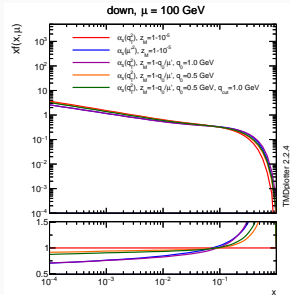
Amount of radiation **modelled in terms of α_s and z_M**

Models with fixed $z_M \approx 1$:

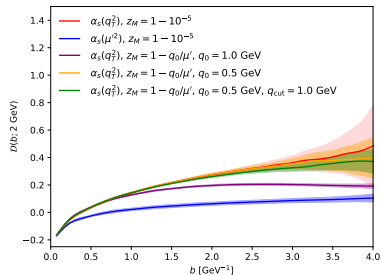
- $\alpha_s(q_\perp^2)$, $\alpha_s = \alpha_s(\max(q_0^2, q_\perp^2))$, $q_0 = 1.0$ GeV (red)
- $\alpha_s(\mu'^2)$ (blue)

Models with $\alpha_s(q_\perp^2)$ and dynamical $z_M = 1 - q_0/\mu'$ (i.e. no non-perturbative Sudakov):

- $q_0 = 1.0$ GeV (purple)
- $q_0 = 0.5$ GeV (orange)
- $q_0 = 0.5$ GeV and $q_{cut} = 1$ GeV (green)



CS kernels extracted from PB DY predictions



$$\mathcal{D}(b, \mu_0) = \frac{\ln(\Sigma_1(b)/\Sigma_2(b)) - \ln Z(Q_1, Q_2) - 2\Delta_R(Q_1, Q_2; \mu_0)}{4 \ln(Q_2/Q_1)} - 1$$

Σ_1 and Σ_2 - Hankel transformed DY cross sections

$$\Delta_R(Q_1, Q_2; \mu_0) = \int_{Q_2}^{Q_1} \frac{d\mu}{\mu} \gamma_F(\mu, Q_1) - 2 \ln \frac{Q_1}{Q_2} \int_{\mu_0}^{Q_2} \frac{d\mu}{\mu} \gamma_k(\mu)$$

$$Z(Q_1, Q_2) = \frac{\alpha_{\text{em}}^2(Q_1) |C_V(Q_1, \mu_{Q_1})|^2}{\alpha_{\text{em}}^2(Q_2) |C_V(Q_2, \mu_{Q_2})|^2}$$

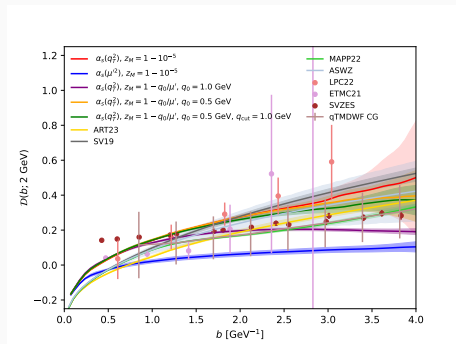
where C_V is the hard coefficient function.

All terms except Σ_1/Σ_2 are perturbative and known up to to $N^3\text{LO}$

The method of A. Bermudez Martinez and A. Vladimirov, Phys.Rev.D 106 (2022) 9, L091501

- different modelling of radiation can lead to a very different kernel behaviour, including different slopes.
- the results probe the AO picture, through α_s and resolution scale z_M
- the curves with $\alpha_s(q_\perp)$ are close to one another at small b
- instead, the curve with $\alpha_s(\mu')$ is already very different at small b
- note flattening behavior at large b in curve with $q_0 = 1 \text{ GeV}$

CS kernels extracted from PB DY predictions



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- instead, the curve with $\alpha_s(\mu')$ is already very different at small b
- note flattening behavior at large b in curve with $q_0 = 1$ GeV
- The curves spread over a wide range, covering extractions from other groups

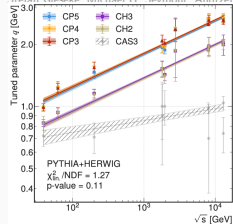
Interplay of non-perturbative Sudakov and Intrinsic-kt

Intrinsic kt vs center-of-mass energy & DY mass

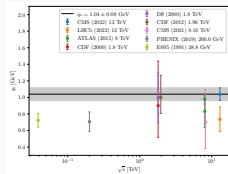
Pythia, Herwig: the intrinsic k_{\perp} is center-of-mass dependent

T. Sjostrand, Peter Z. Skands, JHEP 03 (2004) 053

Stefan Gieseke, Michael H. Seymour, Andrzej Siodmok, JHEP 06 (2008) 001



Phys.Rev.D 111 (2025) 7, 072003

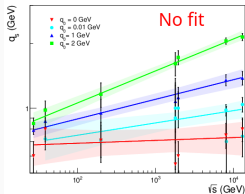


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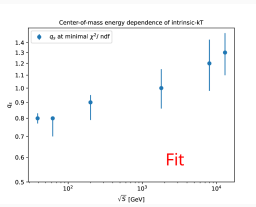
Method:

- replicas of PB-NLO-HERAI+II-2018-set2 created with q_s scanned between $q_s = 0.1$ and $q_s = 2.0$ GeV with a step of 0.1 GeV;
- prediction for each DY measurement obtained with each replica;
- for each measurement, the q_s providing the best χ^2 was extracted.

In PB, the \sqrt{s} dependence of intrinsic-kt much weaker than in other MCs



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The non-perturbative Sudakov ($z_M \rightarrow 1$) & $\alpha_s(q_{\perp})$ crucial for intrinsic kt independent of \sqrt{s}

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We discussed PB Sudakov form factor

- AO-driven division of the phase space allows to get the Perturbative and model the Non-Perturbative Sudakov
- P: logarithmic resummation (new: NNLL $A^{(3)}$ included, in addition to single-log NNLL $B^{(2)}$)
- NP: rapidity evolution (new: extractions of CS kernel from TMD PB, both with fixed and dynamical z_{max})

PB approach contains the Sudakov form factor (both perturbative and non-perturbative) exactly corresponding to the Sudakov of CSS formulation.

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Thank you