Lattice QCD Calculation of TMDs and GPDs Through LaMET

Min-Huan Chu

Adam Mickiewicz University

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IM. ADAMA MICKIEWICZA
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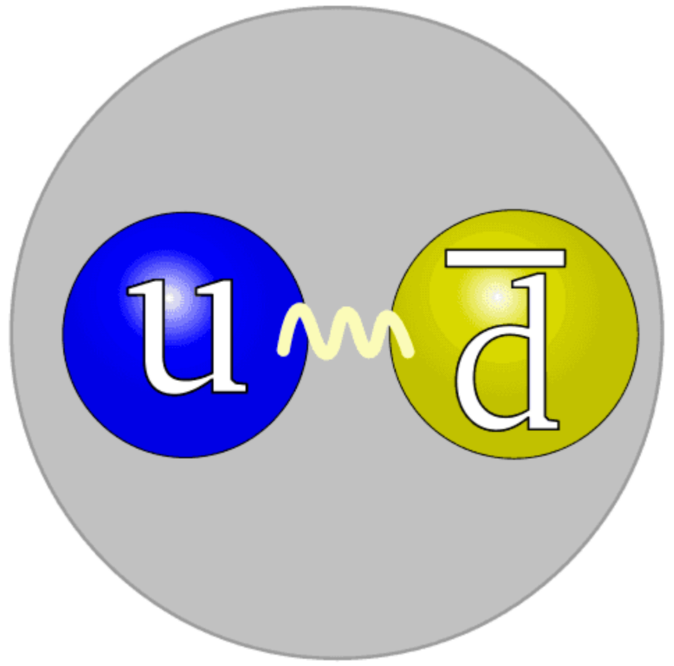
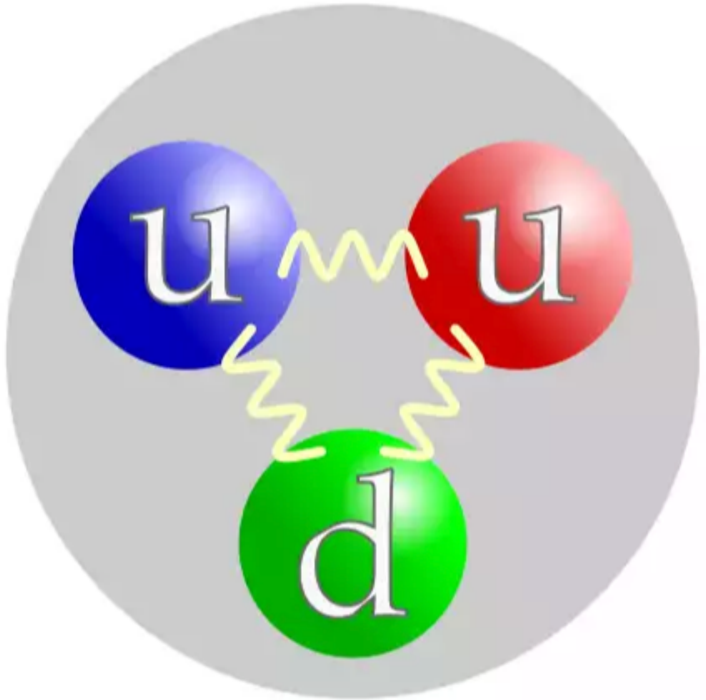
Outline

- Motivation
- Lattice QCD
- Large Momentum Effective Theory
- Framework and Results
- Summary





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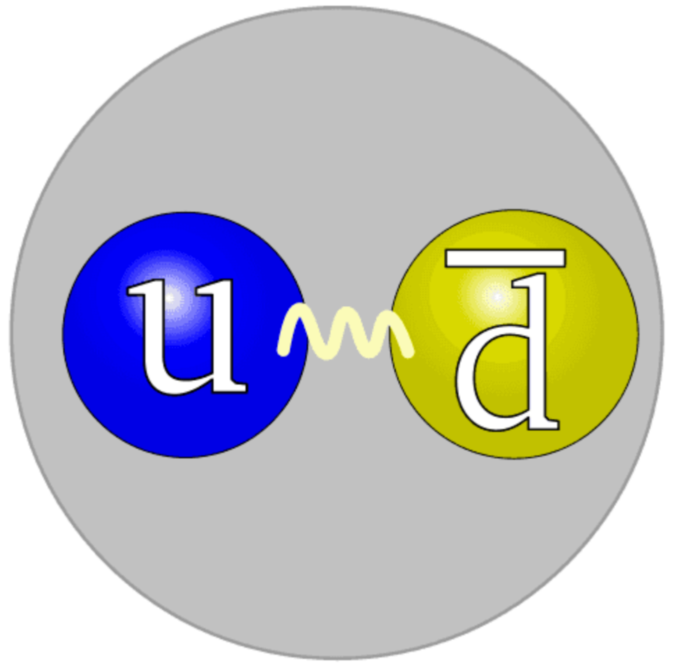
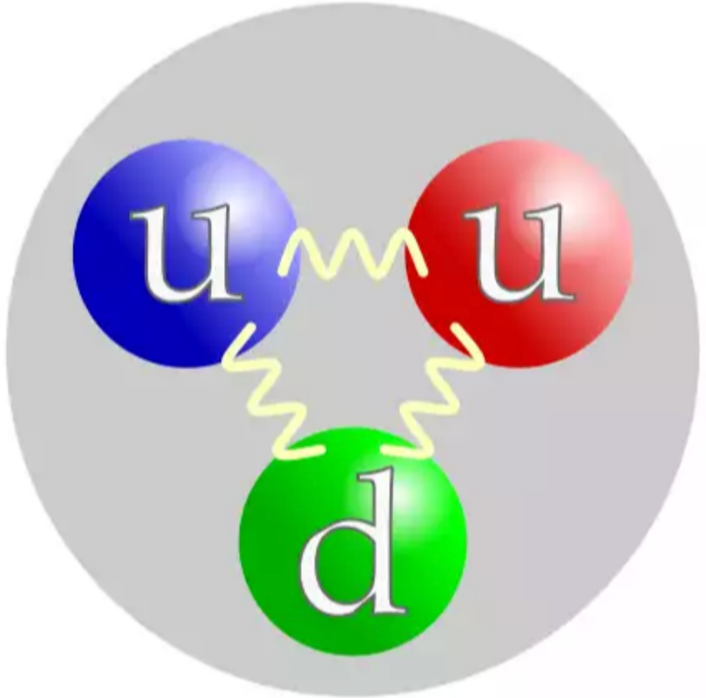
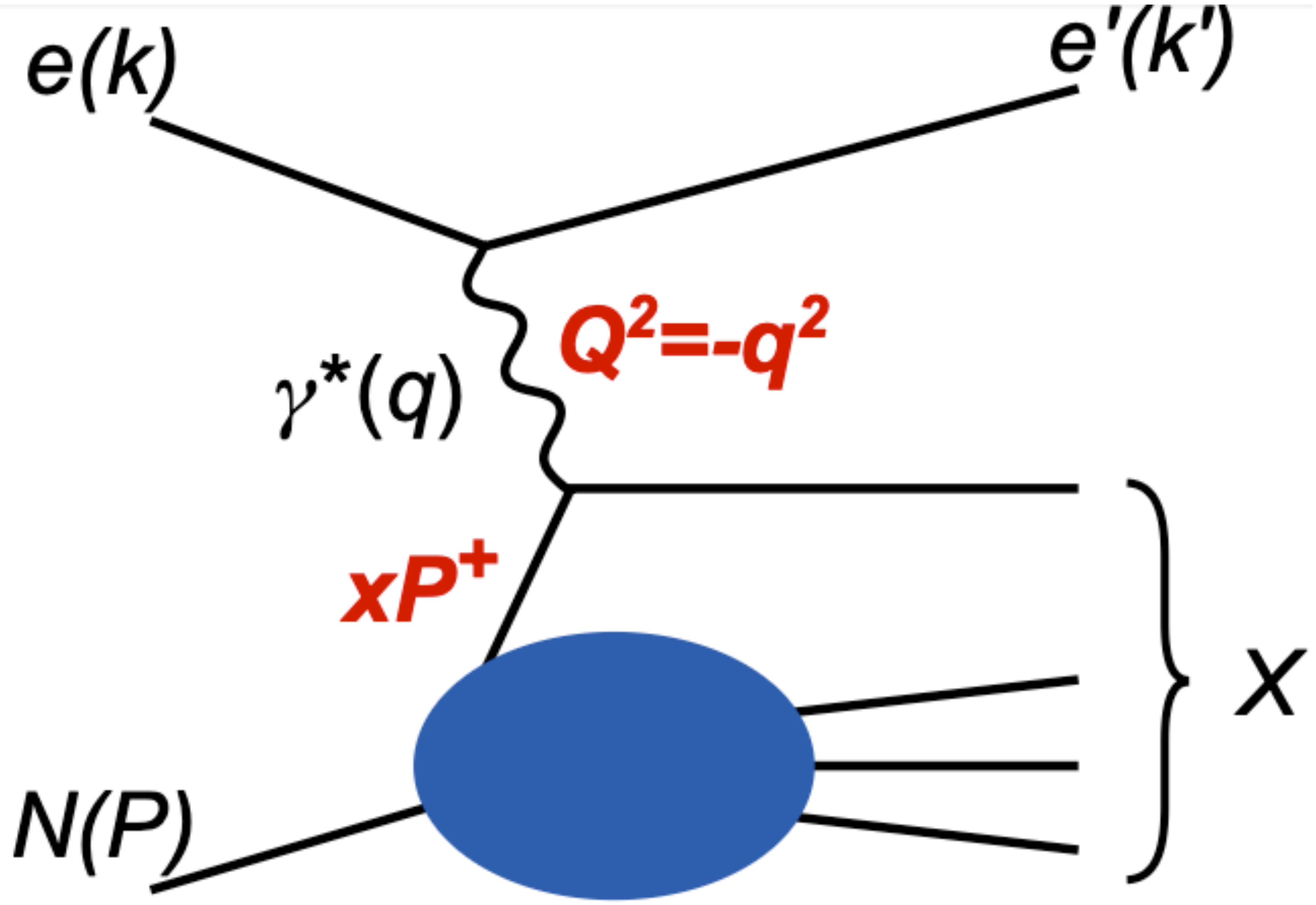




Motivation

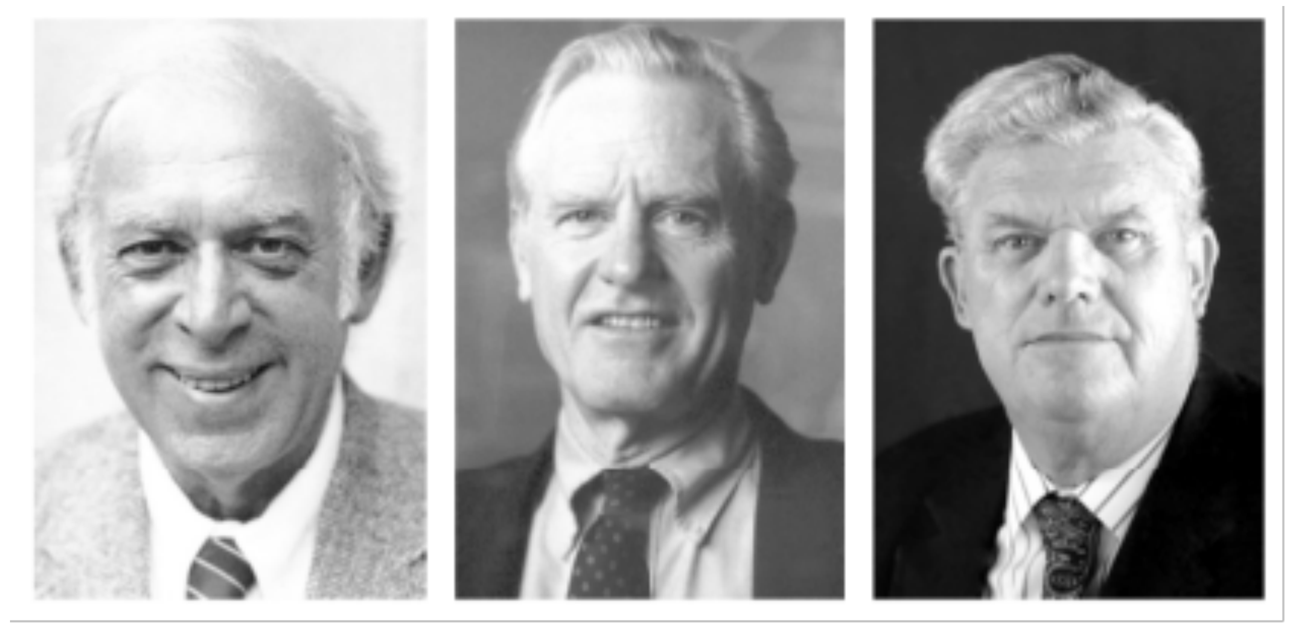
Deep Inelastic Scattering Process



hadronic part of cross section

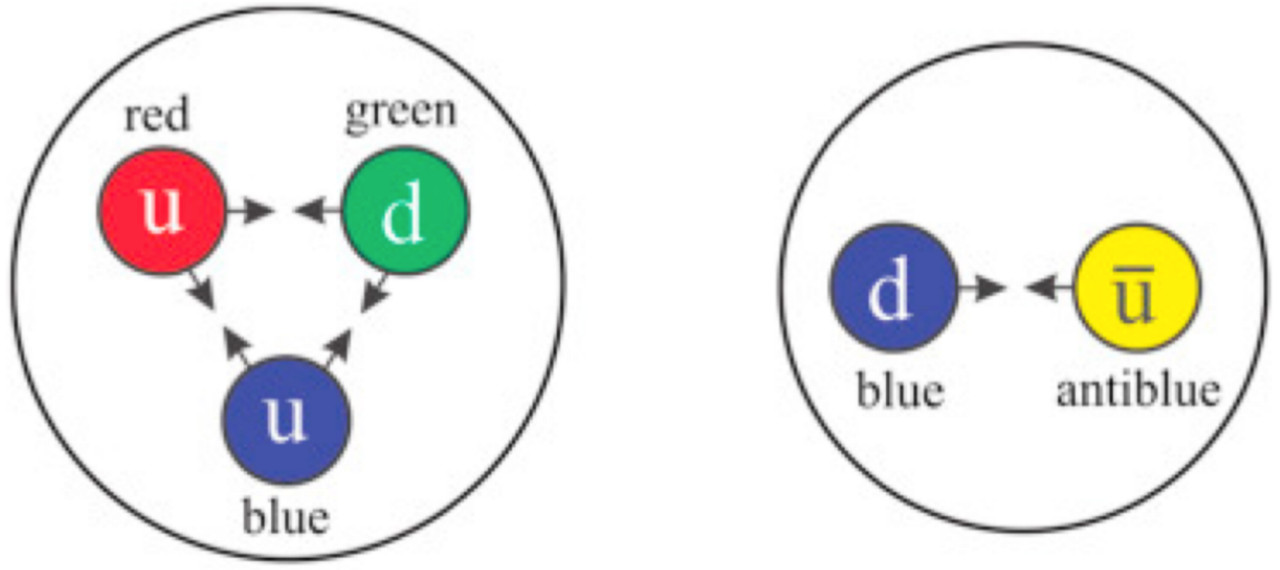
$$\frac{d\sigma}{d\Omega} \propto q(x)$$

Quarks in hadrons



J. Friedman
H. Kendall
R. Taylor
Nobel prize in 1990

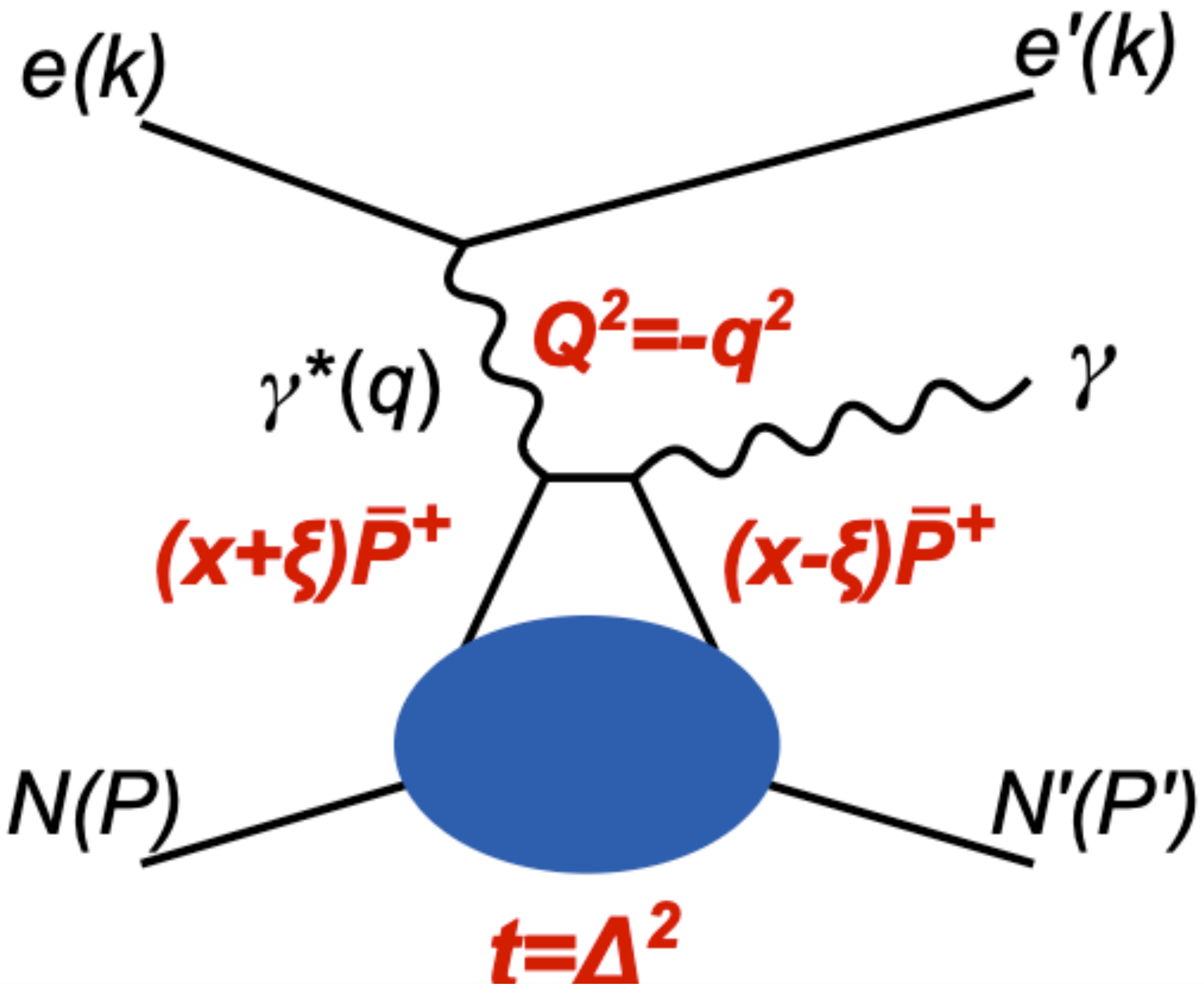
pictures



Baryon (proton, p⁺) Meson (negative pion, π⁻)

Motivation

Deep Virtual Compton Scattering



angular momentum
Ji's sum rule

nucleon tomographic figures

hadronic part of cross section related to $q(x, \xi, t)$

$$q(x, \xi, t) = \bar{u}(P') \left[\gamma^+ H^q(x, \xi, t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M} E^q(x, \xi, t) \right] u(P)$$

$H(x, \xi, t)$ $E(x, \xi, t)$

forward limit $\quad \xi \to 0,\ t \to 0$

$q(x)$

GPDs are important inputs!

Motivation

- Experimental analysis

global fitting

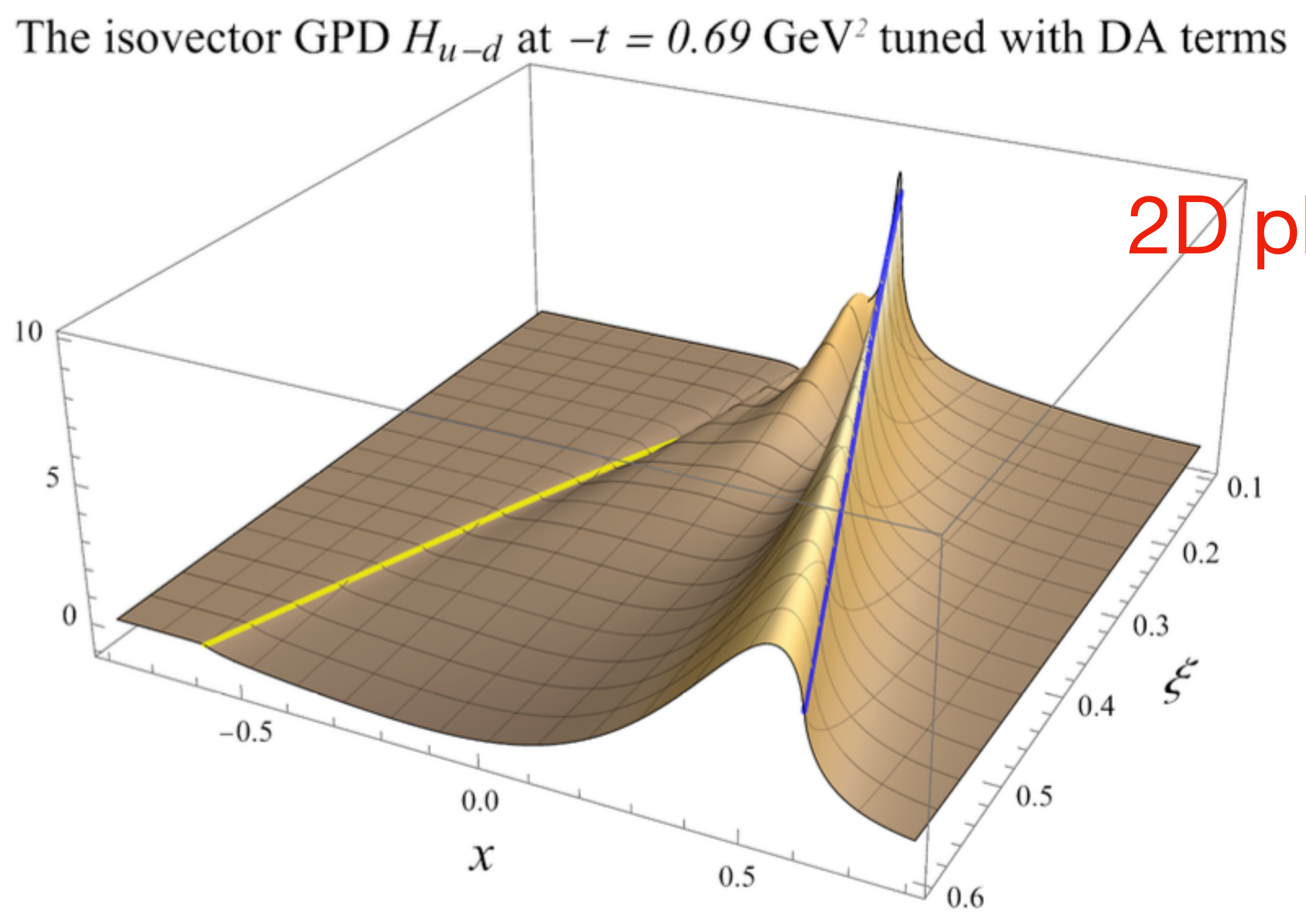
1. Kumerički et al., EPJ Web Conf. 112 (2016) 01012
2. Guo et al., JHEP 05 (2023) 150
3. Burkert et al., Nature 557 (2018) 7705, 396-399

…

- Phenomenological results

1. Moutarde et al., Eur.Phys.J.C 78 (2018) 11, 890
2. Hannaford-Gunn et al., Phys.Rev.D 110 (2024) 1, 014509
3. Dupre et al., Phys.Rev.D 95 (2017) 1, 011501

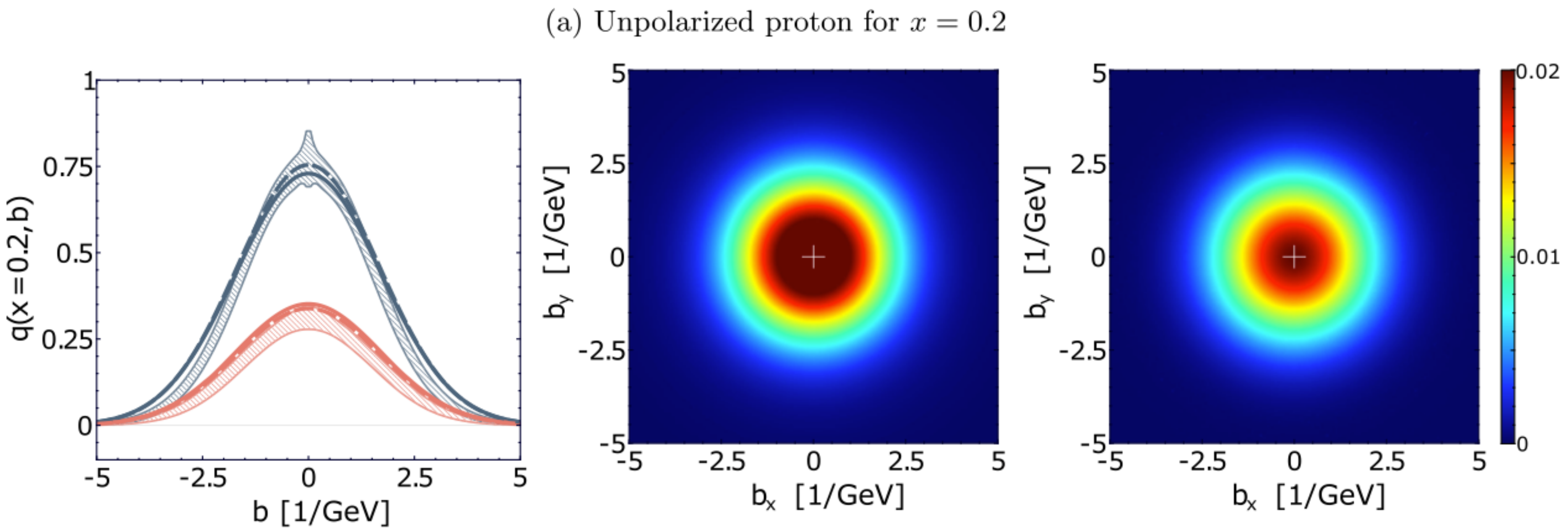
…



2D plots for $H_{u-d}(x,\xi)$

Y. Guo et al., JHEP 05 (2023) 150

tomographic figures



K. Cichy et al., Phys.Rev.D 110 (2024) 11, 114025

Motivation

- Lattice results of GPDs

1. unpolarized+helicity+one value of ξ+symmetric frame:
   C. Alexandrou et al., Phys.Rev.Lett. 125 (2020) 26, 262001

2. transversely polarized :
   C. Alexandrou et al., Phys.Rev.D 105 (2022) 3, 034501

3. unpolarized+asymmetric frame:
   S. Bhattacharya et al., Phys.Rev.D 106 (2022) 11, 114512

5. twist 3 axial+asymmetric frame:
   S. Bhattacharya et al., Phys.Rev.D 109 (2024) 3, 034508

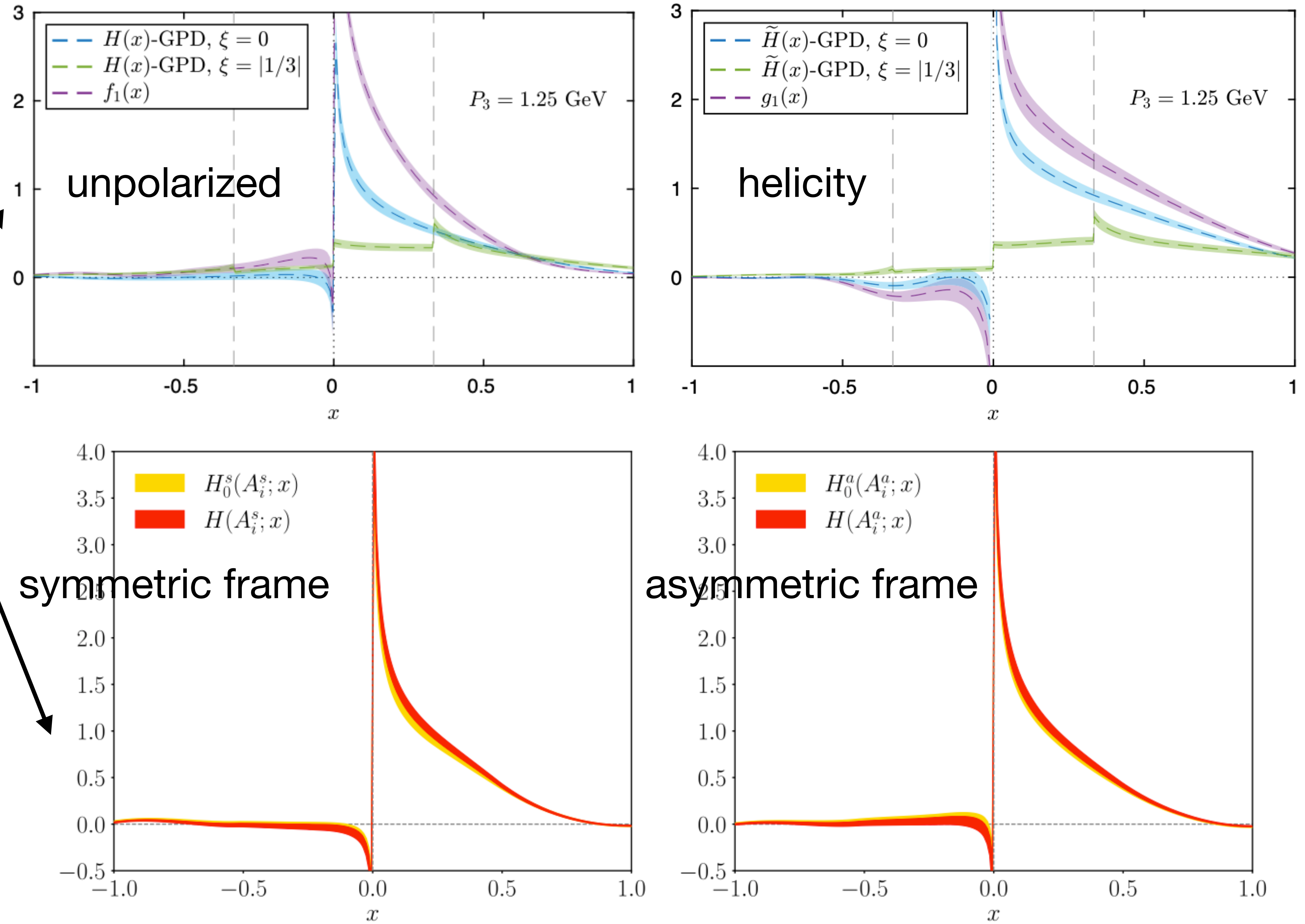
7. unpolarized+ξ dependence+asymmetric frame:
   M. Chu et al., arxiv: 2508.17998 (related to this talk)

…

asymmetric frame: save much computation in multi kinematic setups

- Progress

GPD features	unpolarized→polarized(helicity, transversely…)
	$\xi = 0 \rightarrow \xi \neq 0$
	symmetric frame→asymmetric frame
lattice	continuum limit, hybrid renormalization

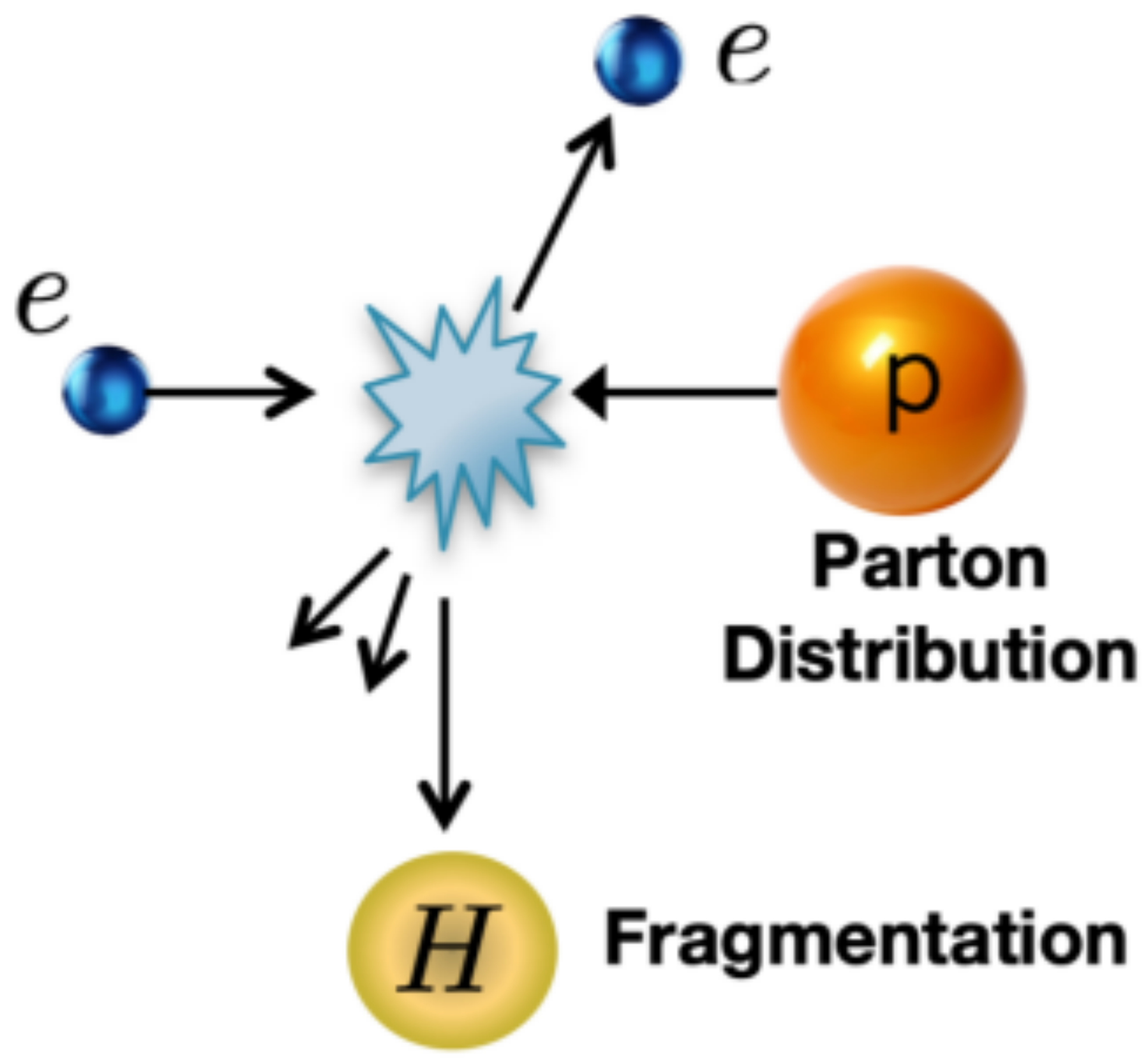


Motivation

TMD process

TMDs are important inputs!

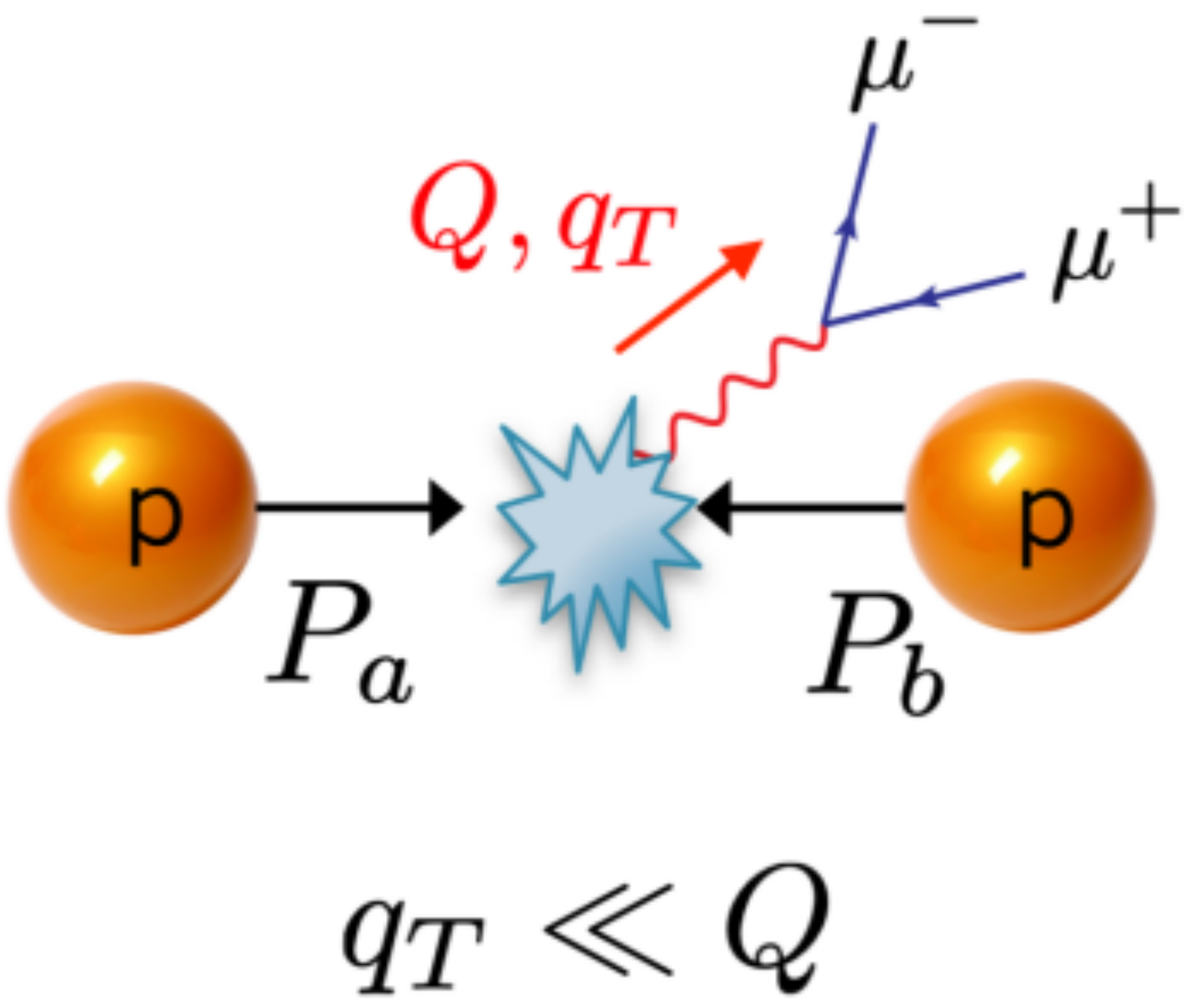
Semi-Inclusive DIS



LHC, FermiLab, RHIC, …

$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$

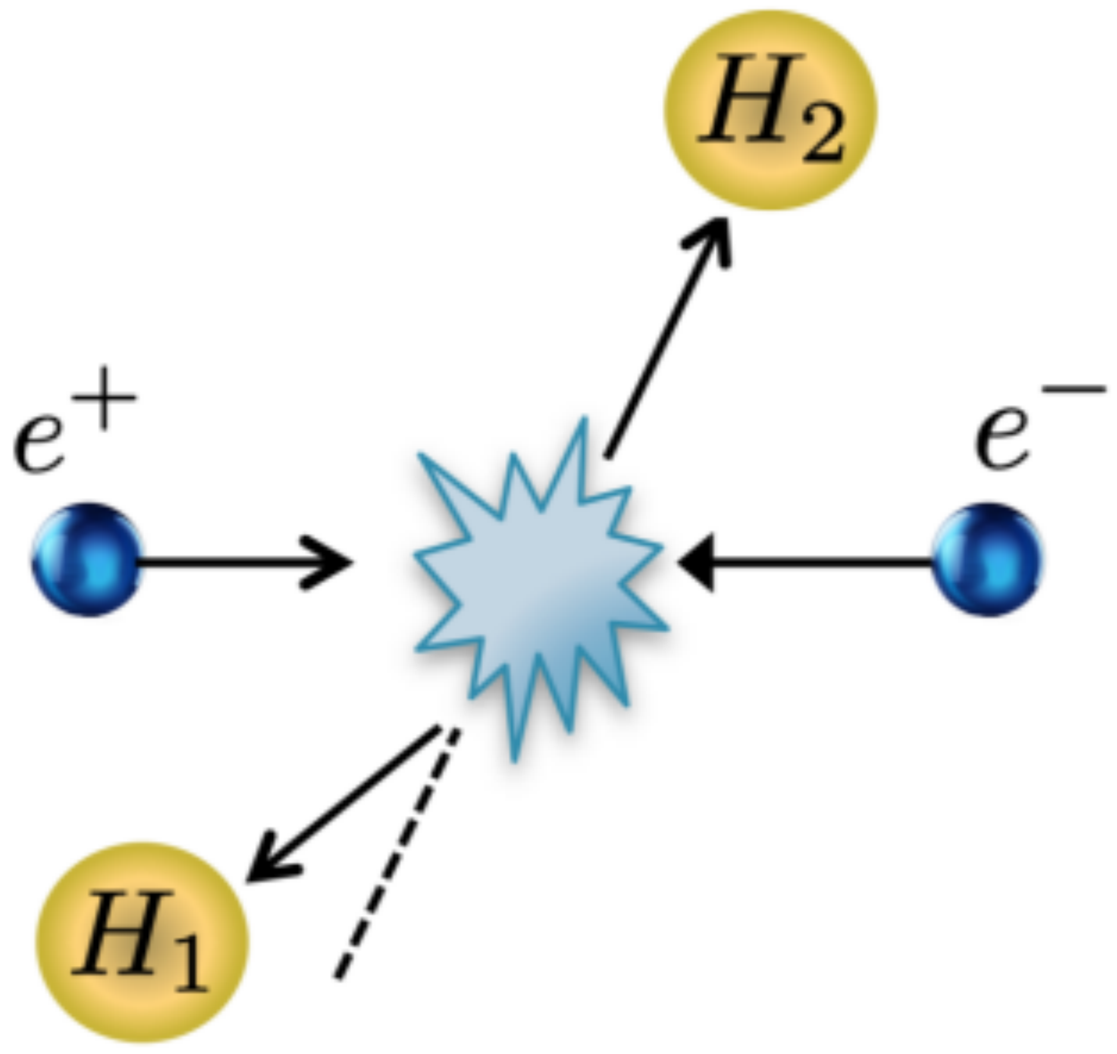
Drell-Yan



HERMES, COMPASS, JLab, EIC, …

$$\sigma \sim f_{q/P}(x, k_T) D_{h/P}(x, k_T)$$

Dihadron in e+e-



BESIII, Babar, Belle, …

$$\sigma \sim D_{h_1/P}(x, k_T) D_{h_2/P}(x, k_T)$$

Motivation

- Theoretical analysis

TMD factorization, evolution and resummation

1. Boussarie et al. TMD handbook, arxiv: 2304.03302

2. Collins, Foundations of perturbative QCD, Camb.Monogr.Part.Phys.Nucl.Phys.Cosmol. 32 (2011) 1-624

…

- Phenomenological results

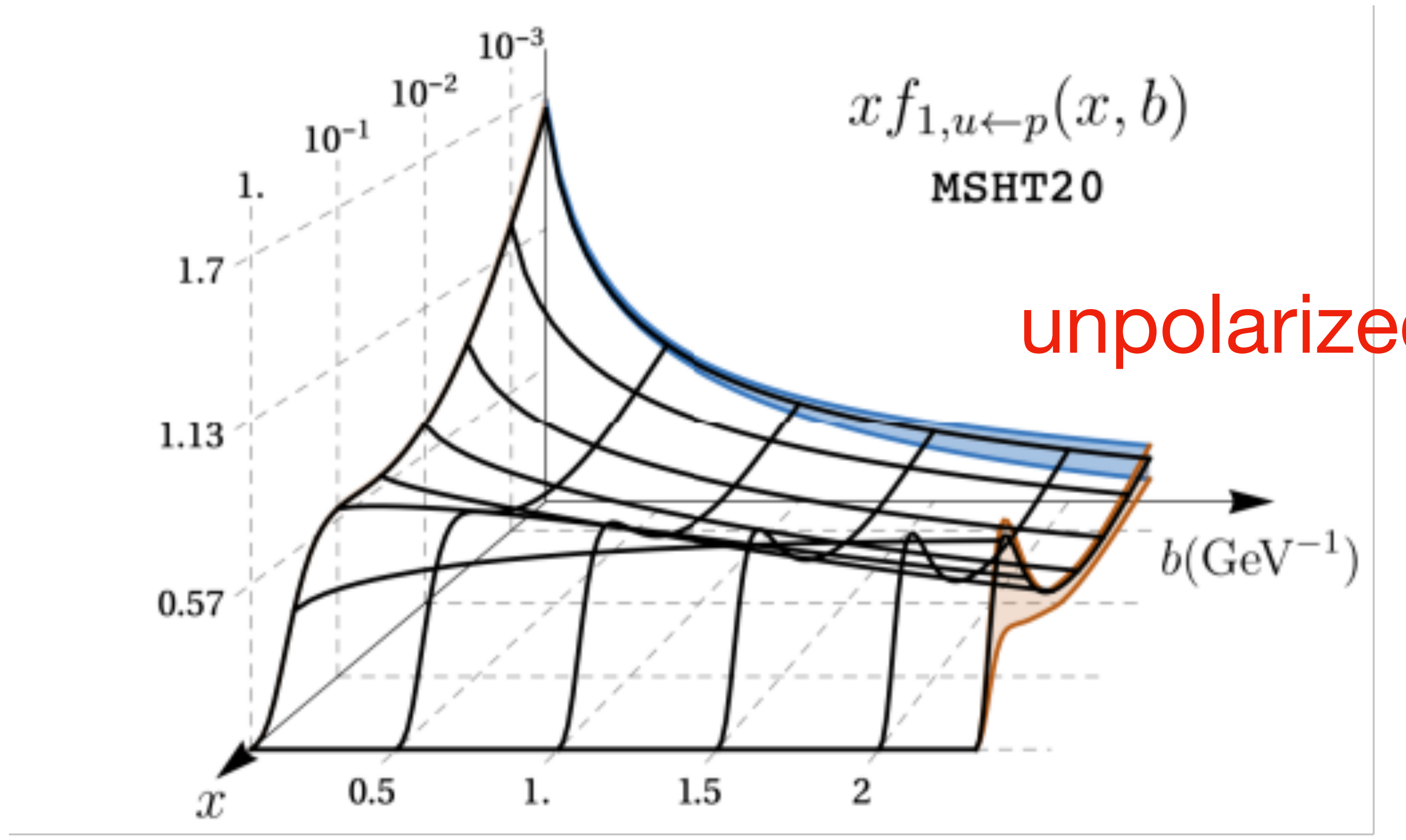
1. Bacchetta et al., JHEP 10 (2022) 127

2. Bury et al., JHEP 10 (2022) 118

3. Scimemi et al., JHEP 06 (2020) 137

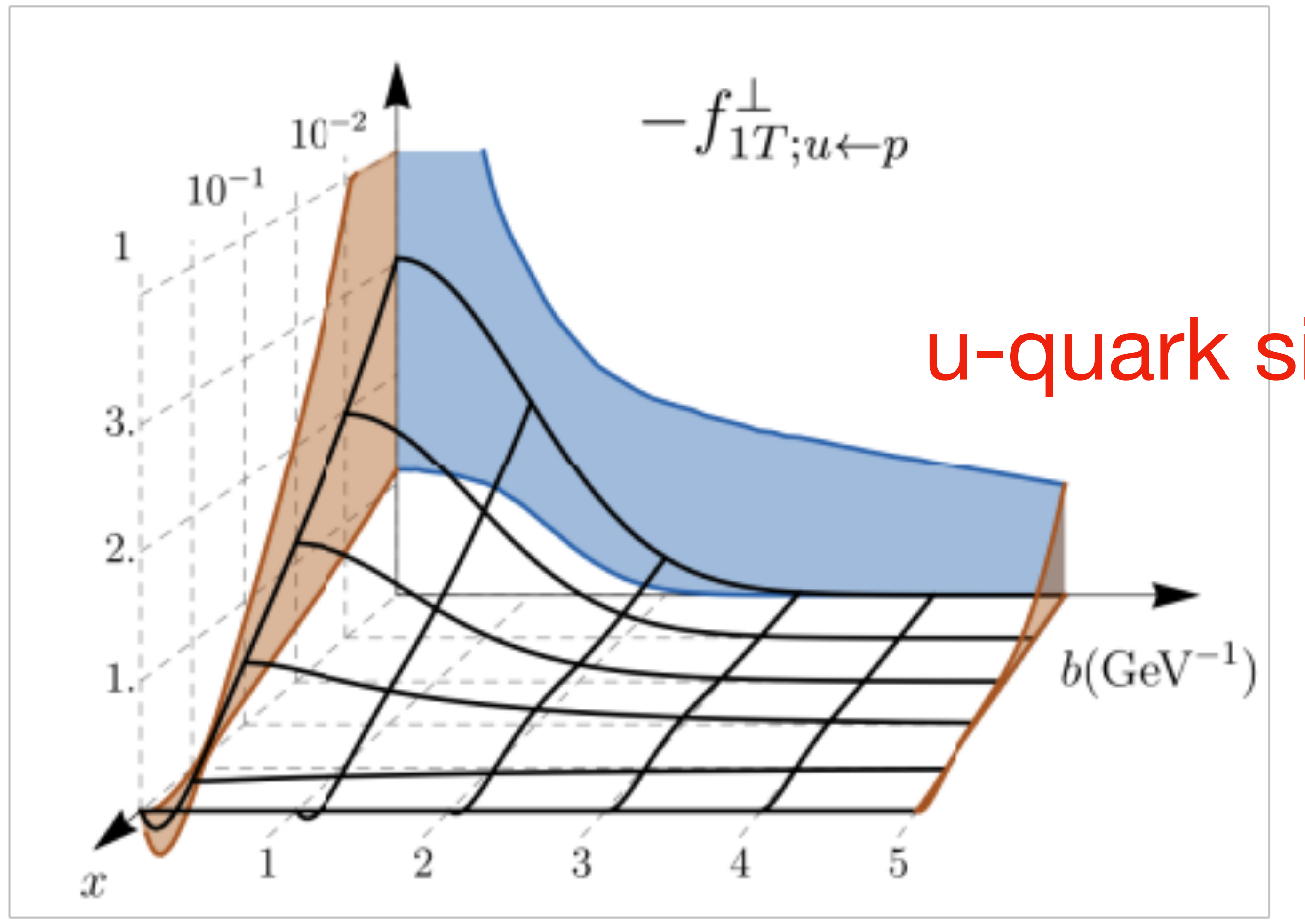
4. Bacchetta et al., JHEP 06 (2019) 051

…



unpolarized TMDPDF

M. Bury et al., JHEP 10 (2022) 118



u-quark sivers function

M. Bury et al., Phys.Rev.Lett. 126 (2021)

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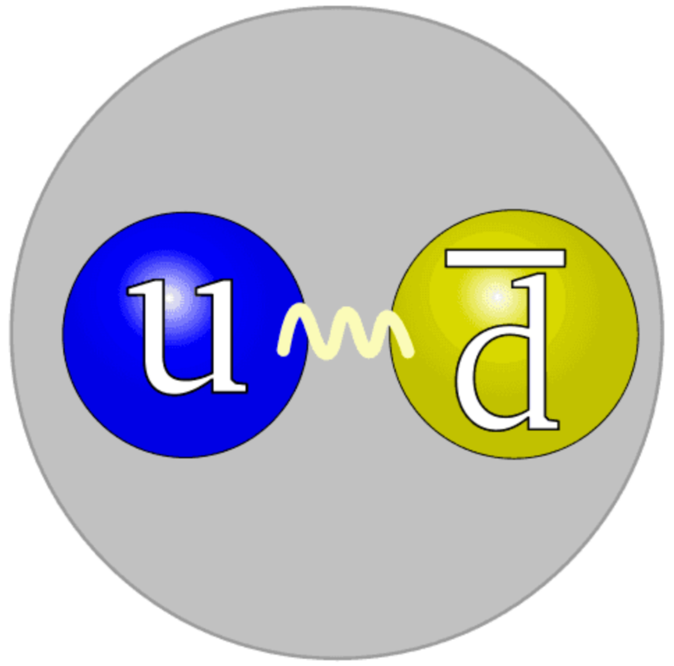
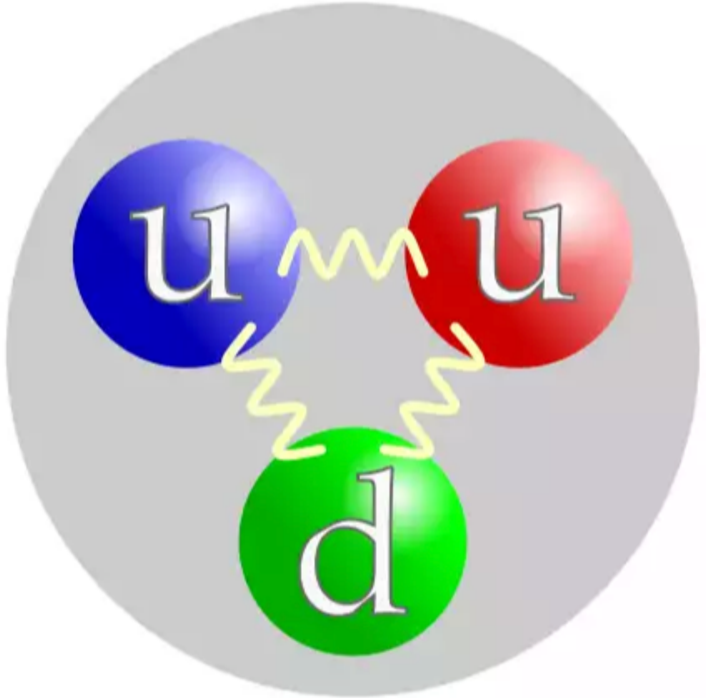




Lattice QCD

Discretized 4D Euclidean space-time

Lattice QCD action:

$$S_E^{\text{latt}} = -\sum_{\square} \frac{6}{g^2} \mathrm{Re\,tr}_N \left(U_{\square,\mu\nu} \right) - \sum_q \bar{q} \left(D_\mu^{\text{lat}} \gamma_\mu + a m_q \right) q$$

gauge action   fermion action

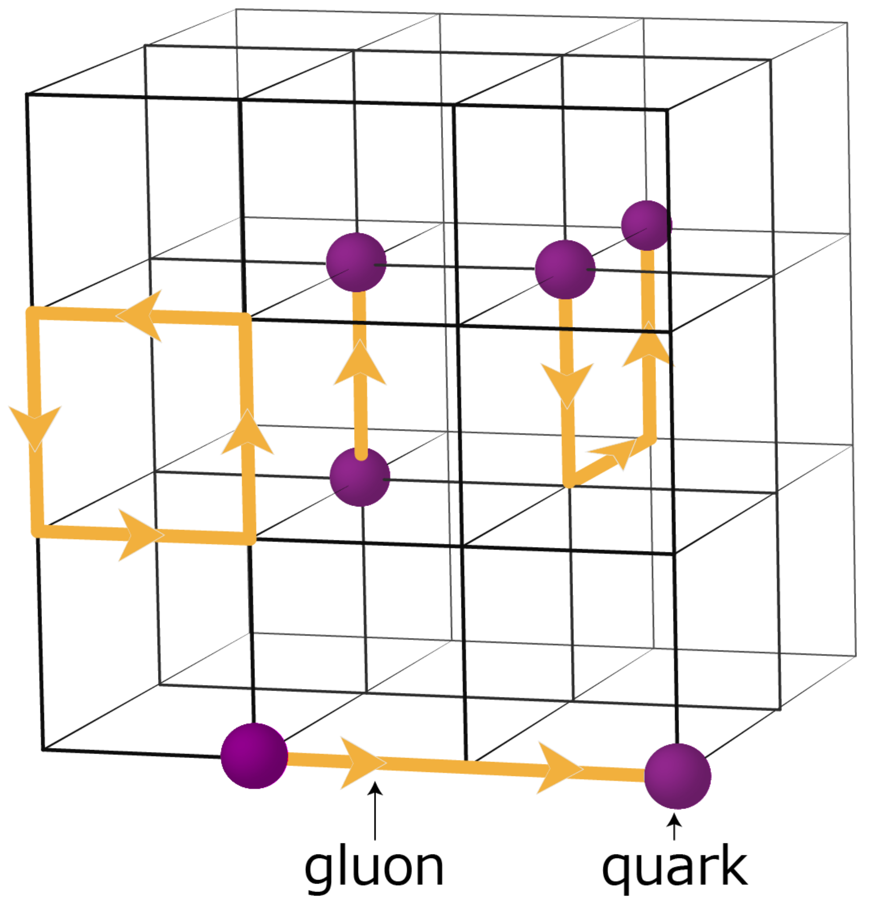
Quantum observables in path integral:

$$\langle 0 | \hat{O} | 0 \rangle = \frac{\int [d\bar{\psi}][d\psi][dA] \hat{O} e^{-S(\bar{\psi},\psi,A)}}{\int [d\bar{\psi}][d\psi][dA] e^{-S(\bar{\psi},\psi,A)}}$$

Similar to partiton functions in thermal physics

↓

The fields distributed with the probability density $e^{-S[\psi,\bar{\psi},A]}$



From continuum to lattice

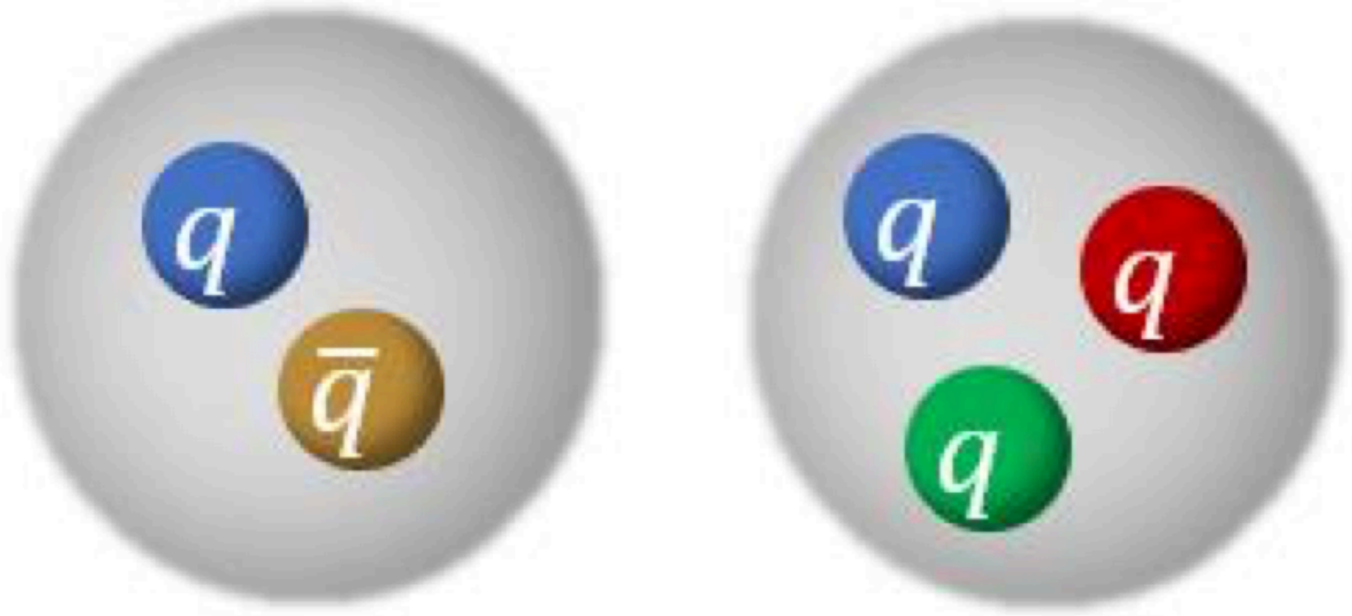
Continuum	Lattice
real time t	Euclidean time $\tau_E = -it$
Gauge field $A_\mu(x)$	Gauge link $U_\mu(n)$
Quark field Ψ	det of Dirac matrix

Observables: hadron spectrum

hadron operators

mesons: $\hat{O}_M = \bar{q}\Gamma q$

baryons: $\hat{O}_B = \epsilon_{abc} P_{\pm} q_{1a} (q_{2b}^T \Gamma^B q_{3c})$



meson baryon

Observables: hadron spectrum

two point correlation functions

$$\langle \hat{O}(t)\hat{\bar{O}}(0)\rangle = \langle 0 | e^{-iHt} e^{iHt} \hat{O}(t) e^{-iHt} e^{iHt} | H \rangle \langle H | \hat{\bar{O}}(0) | 0 \rangle$$
$$= e^{iE_H t} \langle 0 | \hat{O}(0) | H \rangle \langle H | \hat{\bar{O}}(0) | 0 \rangle$$

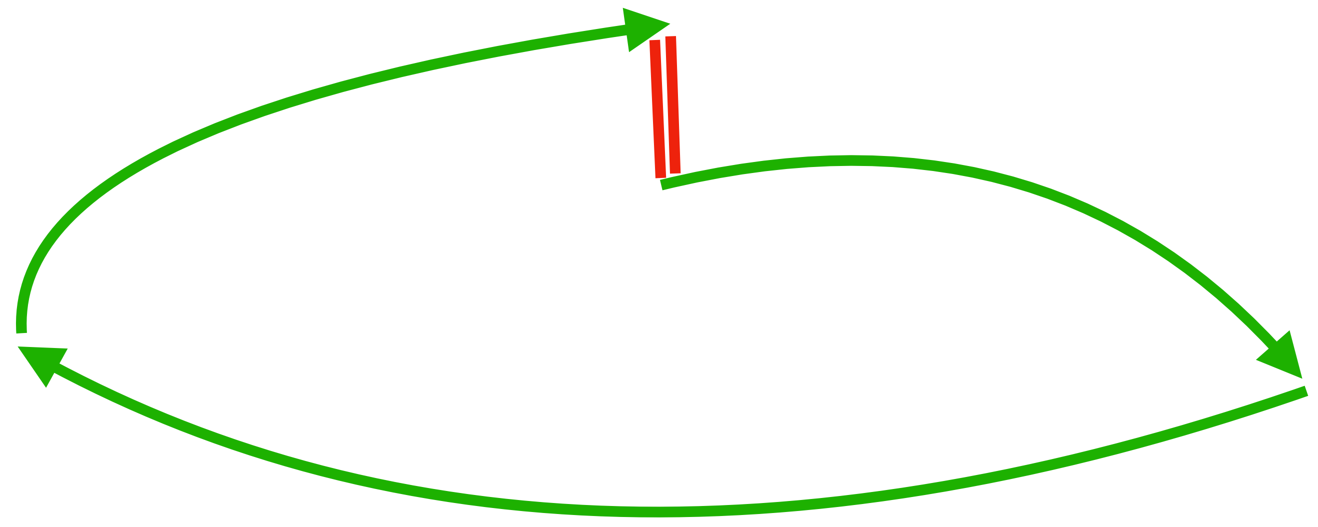
from continuum to lattice: $e^{iE_H t} \rightarrow e^{-E_H t}$

lattice calculation (take pion as e.g.)

$$\langle \hat{O}_\pi(n) \hat{\bar{O}}_\pi(m) \rangle = \langle \bar{d}(n)\gamma_5 u(n) \bar{u}(m) \gamma_5 d(m) \rangle$$
$$= -\,\mathrm{Tr}[\gamma_5 D_u^{-1}(n|m) \gamma_5 D_d^{-1}(m|n)]$$

quark propagators

Observables: hadron structure



reduction formula: from nonlocal 3pt to PDF

$$C_3(z,t,t_{sep}) = \int d^3x e^{-i\vec{p}\vec{x}} \int d^3y \langle 0 | \hat{O}_H(\vec{x},t_{sep}) \hat{O}_H(\vec{y},t;z) \hat{O}_H^{\dagger}(0,0) | 0 \rangle$$

$$= \langle 0 | \hat{O}_H(0,t_{sep}) | \pi(p) \rangle \langle \pi(p) | \hat{O}_H(0,t;z) | \pi(p) \rangle \langle \pi(p) | \hat{O}_H^{\dagger}(0,0) | 0 \rangle$$

cancelation

local 2pt: $C_2(0,t_{sep}) = \int d^3x e^{-i\vec{p}\vec{x}} \int d^3y \langle 0 | \hat{O}_H(\vec{x},t_{sep}) \hat{O}_H^{\dagger}(0,0) | 0 \rangle = \langle 0 | \hat{O}_H(0,t_{sep}) | \pi(p) \rangle \langle \pi(p) | \hat{O}_H^{\dagger}(0,0) | 0 \rangle$

parton distribution function of pion: $f(z,P^z) = \langle \pi(P^z) | \bar{\psi}(x) \gamma^t W(z,0) \psi(0) | \pi(P^z) \rangle$

Excited states: $\dfrac{C_3(z,t,t_{sep})}{C_2(0,t_{sep})} = f(z,P^z)(1 + c_1 e^{-i\Delta E t} + c_2 e^{-i\Delta E (t_{sep}-t)} + c_3 e^{-i\Delta E t_{sep}})$

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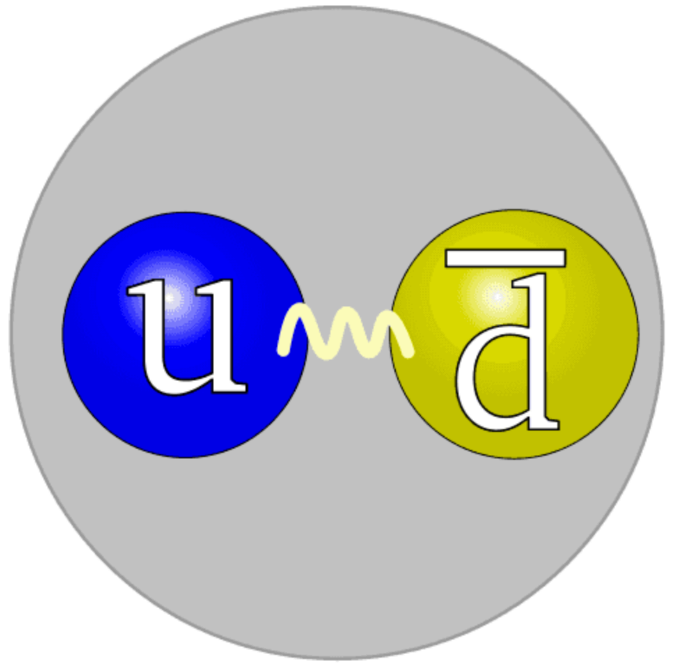
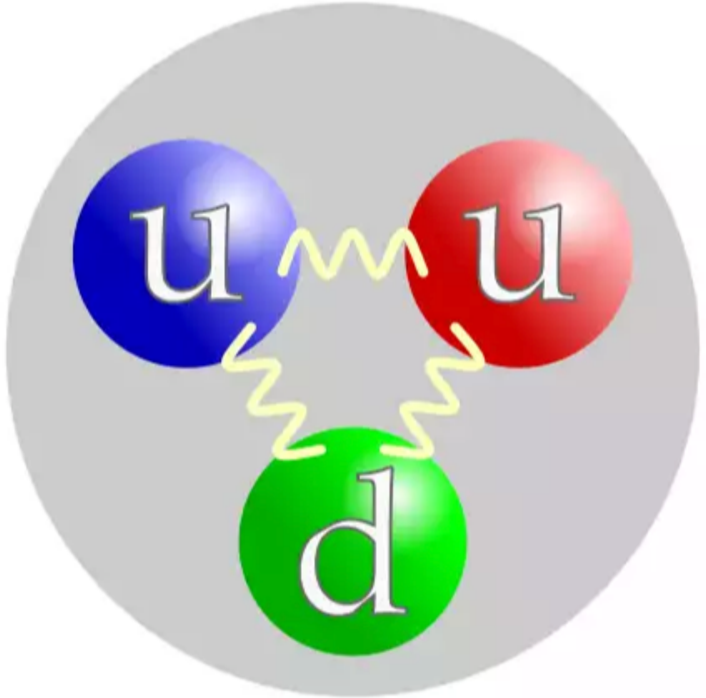




Large Momentum Effective Theory

main idea

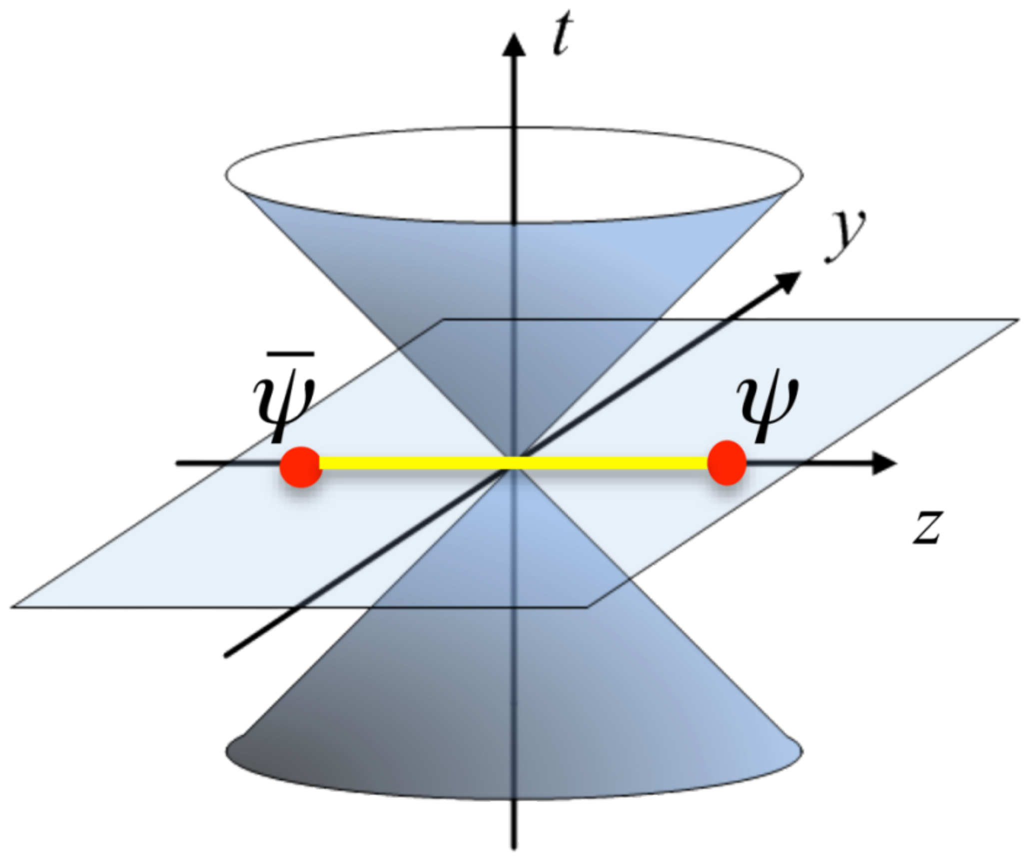
Equal time correlation

$$\tilde{\phi}(z) \sim \langle P^z | \bar{\psi}(\frac{z}{2})\Gamma U(\frac{z}{2}, -\frac{z}{2})\psi(-\frac{z}{2}) | P^z \rangle$$

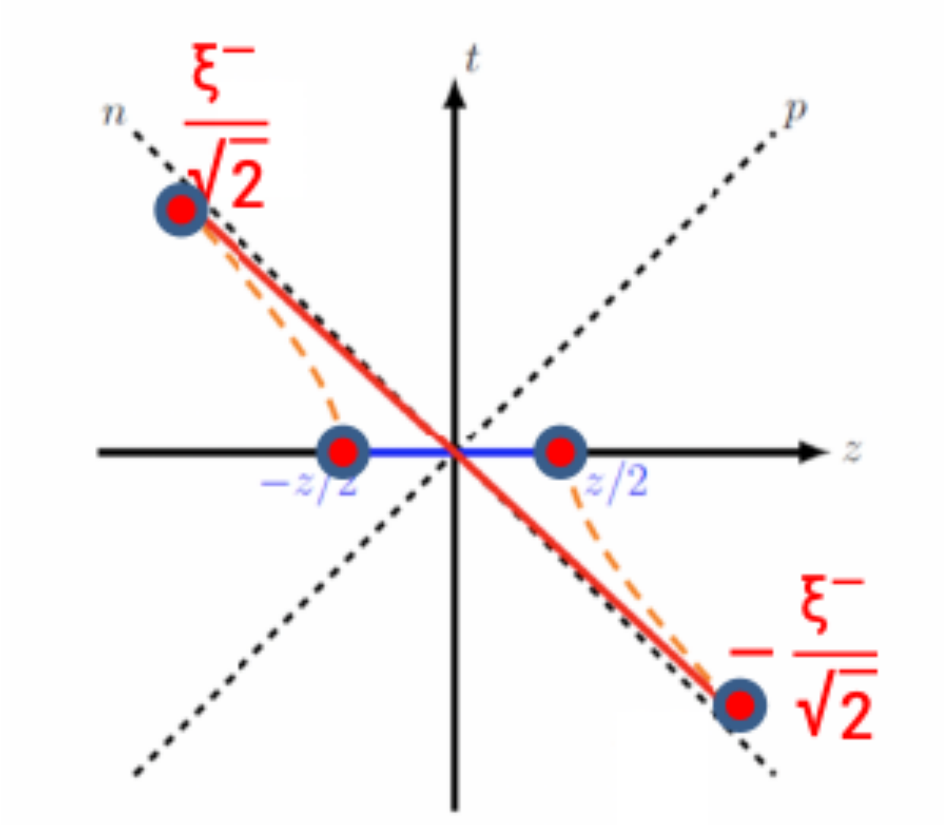
main idea

Light-cone correlation:

$$\phi(x) \sim \langle P^+ | \bar{\psi}(\frac{\xi^+}{\sqrt{2}})\Gamma U(\frac{\xi^+}{\sqrt{2}}, -\frac{\xi^+}{\sqrt{2}})\psi(-\frac{\xi^+}{\sqrt{2}}) | P^+ \rangle$$

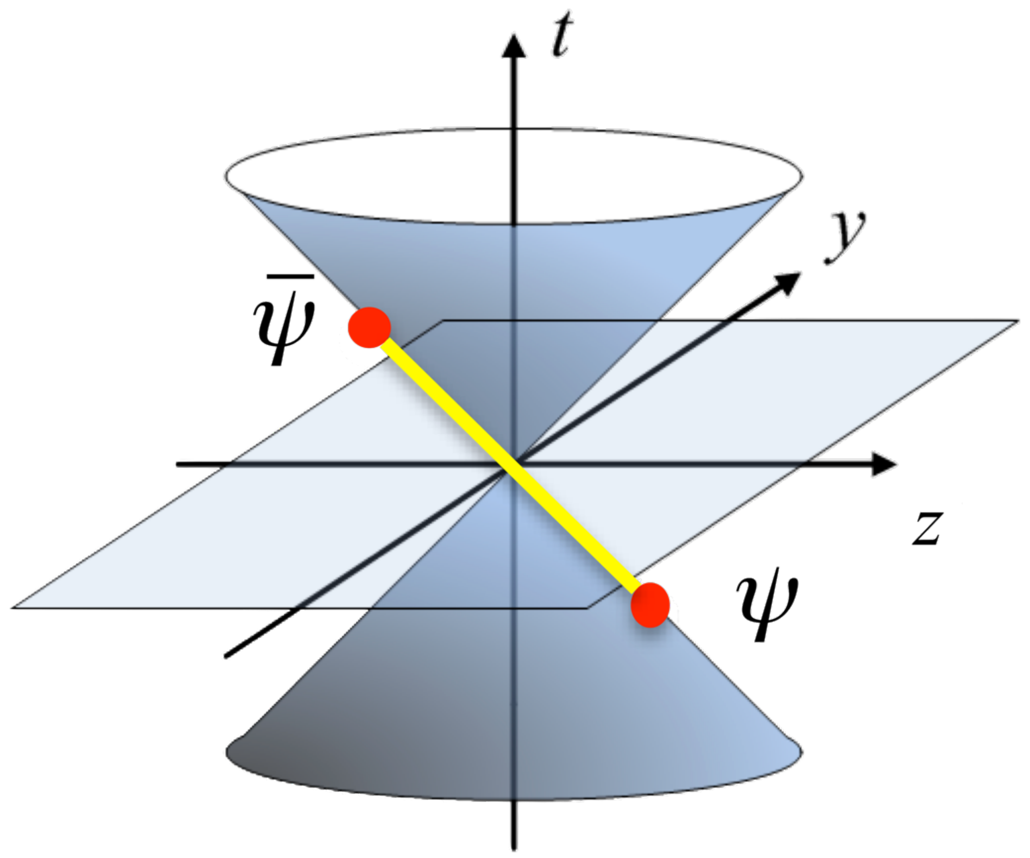


Lorentz transformation



X. Ji, Phys.Rev.Lett. 110 (2013) 262002

matching



Due to the IR structure are only based on states, then the difference between $\phi(x)$ and $\tilde{\phi}(x)$ is only UV structure, which can be perturbatively determined.

$$\tilde{\phi}(y, P^z, \mu) = \int_{-1}^{1} \frac{dx}{|x|} C(\frac{y}{x}, \frac{\mu}{xP^z})\phi(x, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{(yP^z)^2}, \frac{\Lambda^2_{QCD}}{((1-y)P^z)^2}\right)$$

renormalization

renormalization of non-local operators

$$\underbrace{\hat{O}^B_\Gamma(z,\Lambda)}_{\text{renormalized operator}} = Z_\Gamma(z,\Lambda,\mu)\underbrace{\hat{O}^R_\Gamma(z)}_{\text{bare operator}}$$

renormalized operator　　　bare operator

RI/MOM renormalization

C. Sturm, et,al Phys.Rev.D 80 (2009) 014501

J. Chen, et,al Phys.Rev.D 97 (2018) 1, 014505

$$\tilde{h}_R(z,P^z,p^z_R,\mu_R) = \lim_{a\to 0} Z^{-1}_{\hat{O}_\Gamma}(z,p^z_R,\mu_R,a)\tilde{h}_B(z,P^z,a)$$

$$\langle q|\hat{O}_B|q\rangle = \mathrm{Tr}[\Lambda^\Gamma_0(z,a,p)\hat{P}],\ \ Z_q Z^{-1}_{\hat{O}_\Gamma}\Lambda^\Gamma_0(z,a,p)|_{p=p_R} = \Lambda^\Gamma_{\text{tree}}(z,a,p)$$

Hybrid renormalization

X. Ji, et,al Nucl.Phys.B 964 (2021) 115311

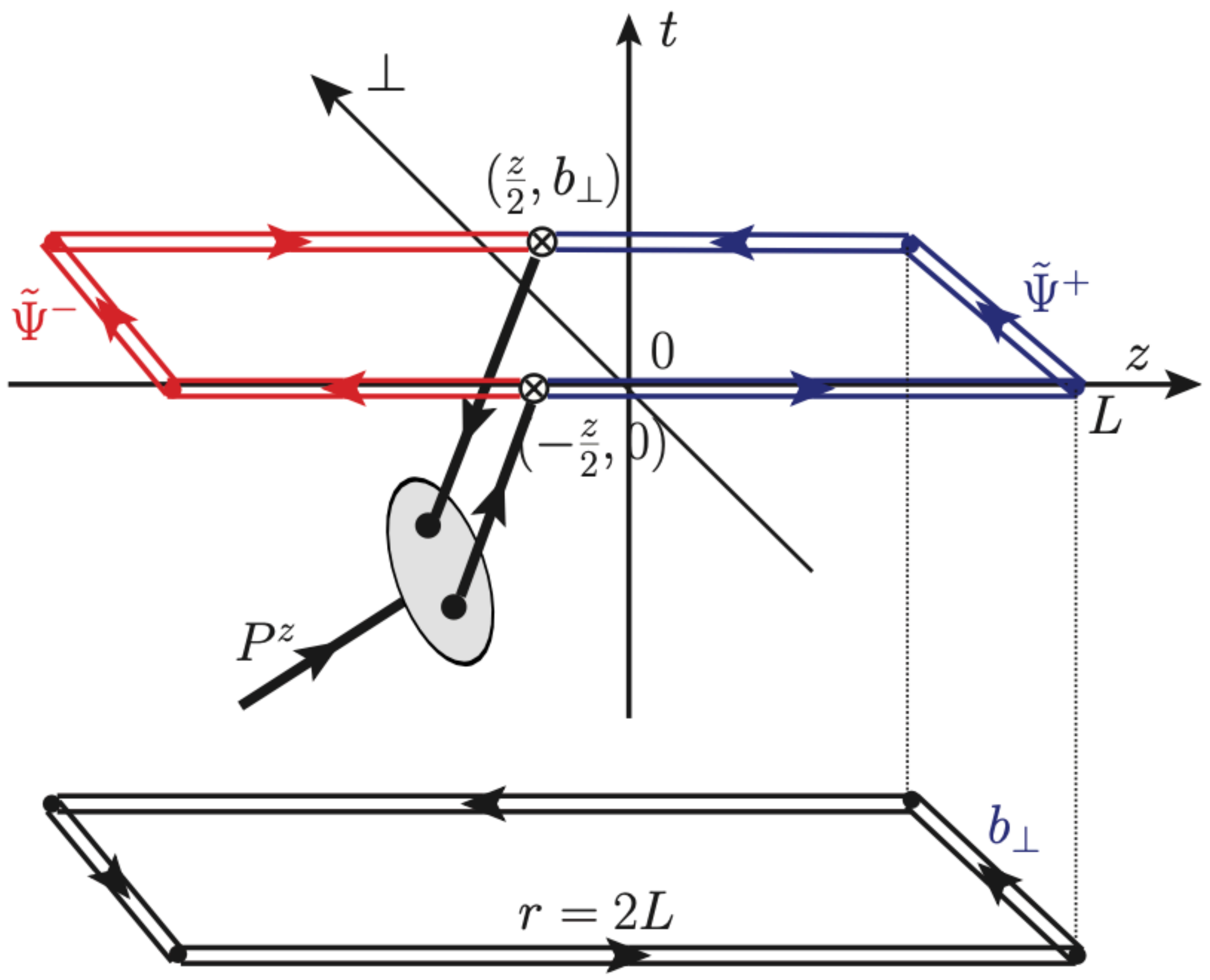
$$\tilde{h}^R(z,z_s;P_z) = \begin{cases} \dfrac{\tilde{h}^B(z,P_z)}{\tilde{h}^B(z,0)}, & |z|\le|z_s|, \\ \dfrac{\tilde{h}^B(z,P_z)}{\tilde{h}^B(z_s,0)}e^{(\delta m+\bar{m}_0)|z-z_s|}, & |z|>|z_s|. \end{cases}$$

renormalization

Wilson loop renormalization for TMD operators

$$\hat{O}_R(L,b_\perp,z) = \frac{\hat{O}_B(L,b_\perp,z)}{\sqrt{Z_E(2L+z,b_\perp)}}$$

gauge invariant



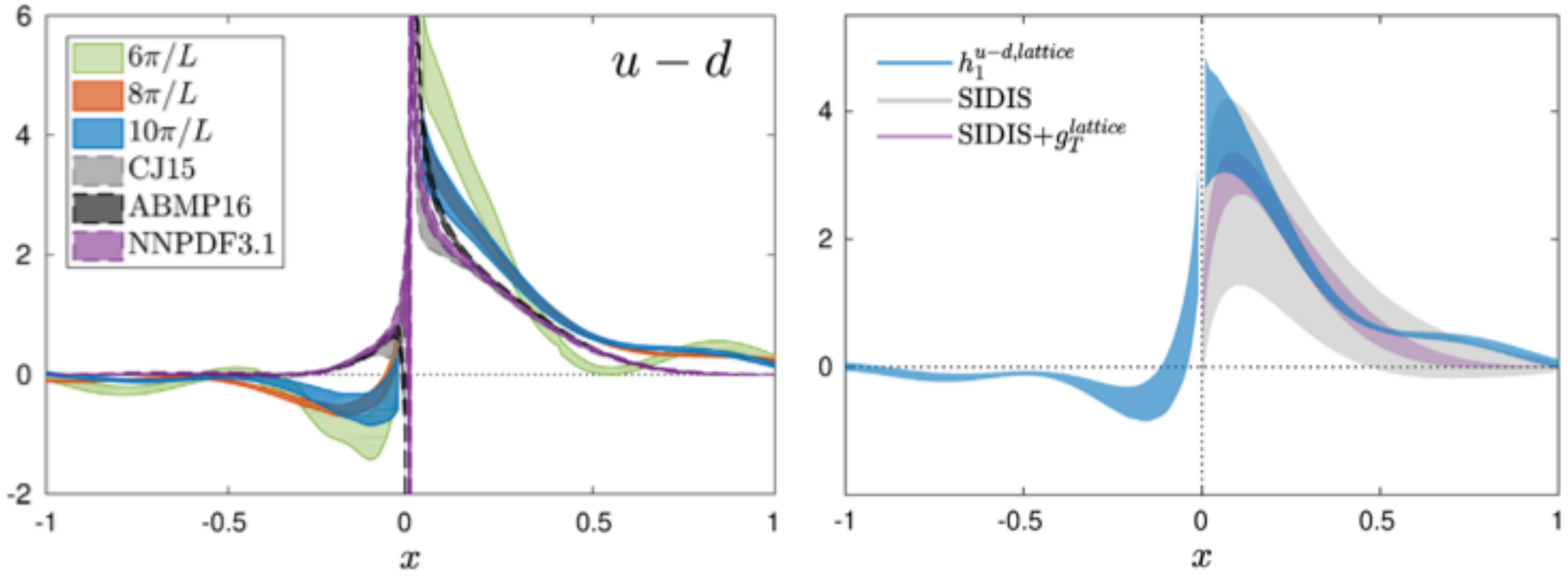
$\hat{O}_B$ and $\sqrt{Z_E}$ have the same linear divergence, which is eliminated at large L by the ratio of $\hat{O}_B$ and $\sqrt{Z_E}$.

Large Momentum Effective Theory

achievements

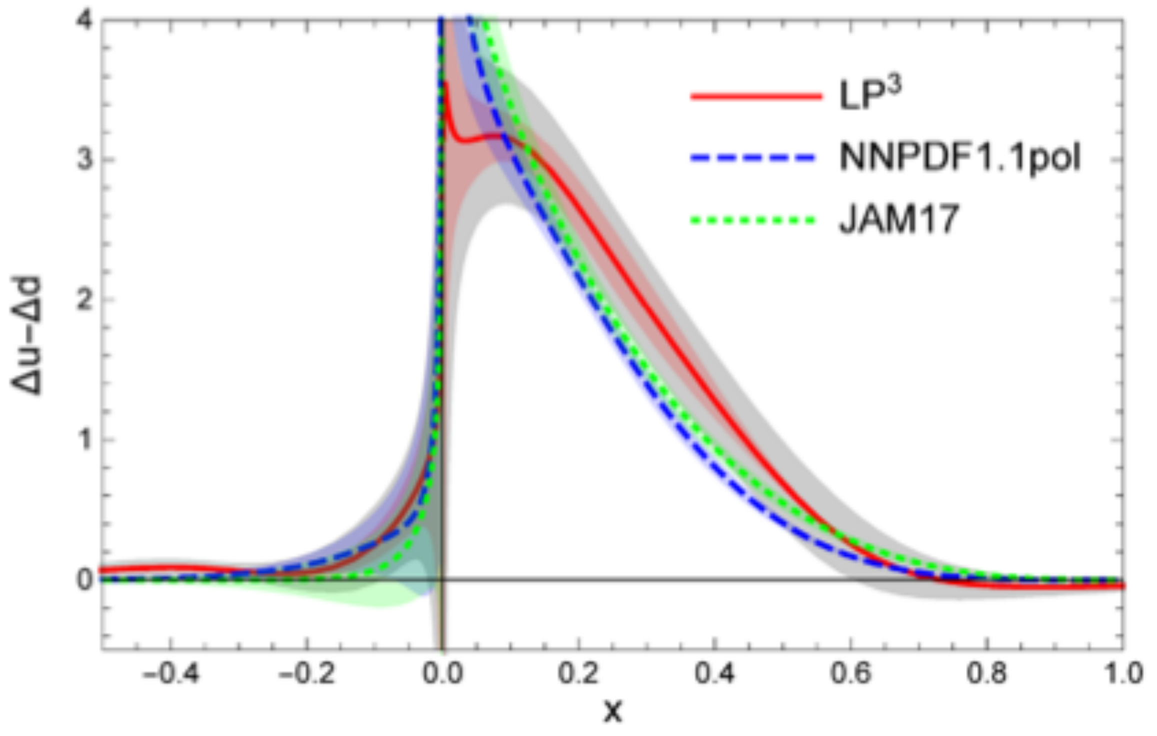
Proton unpolarized quark PDF

C. Alexandrou, et,al., Phys.Rev.D. 98 (2018) 9, 091503



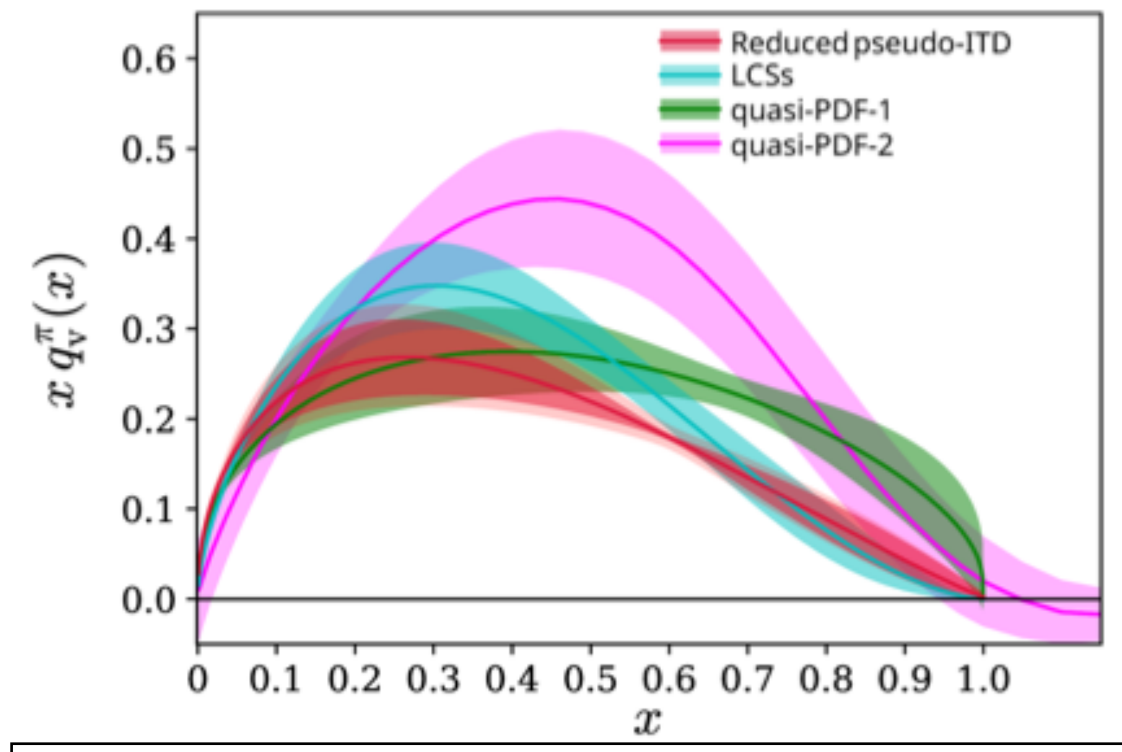
C. Alexandrou, et,al., Phys.Rev.Lett. 121 (2018) 11, 112001

Proton helicity quark PDF



H. Lin, et,al., Phys.Rev.Lett. 121 (2018) 24, 242003

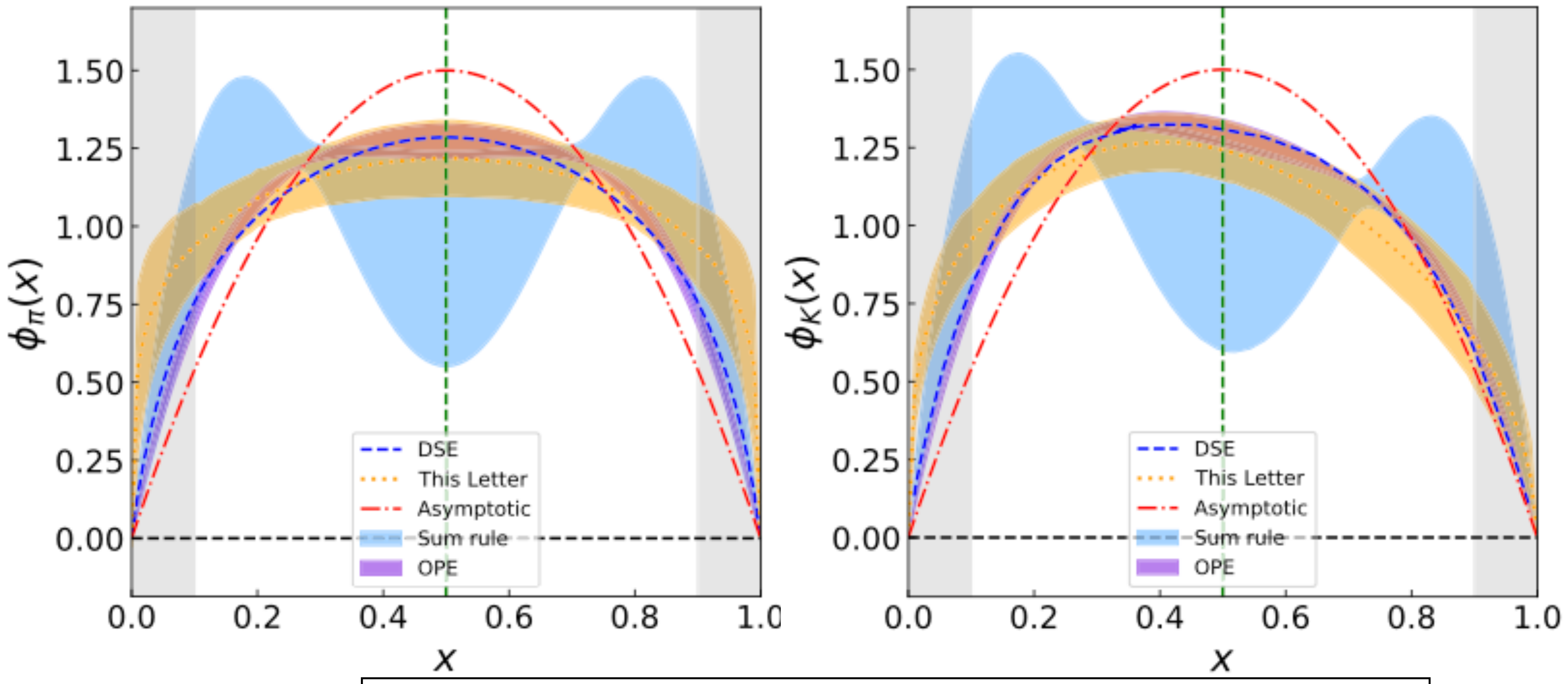
Pion valence quark PDF



B. Joo, et,al., Phys.Rev.D 100 (2019) 11, 114512

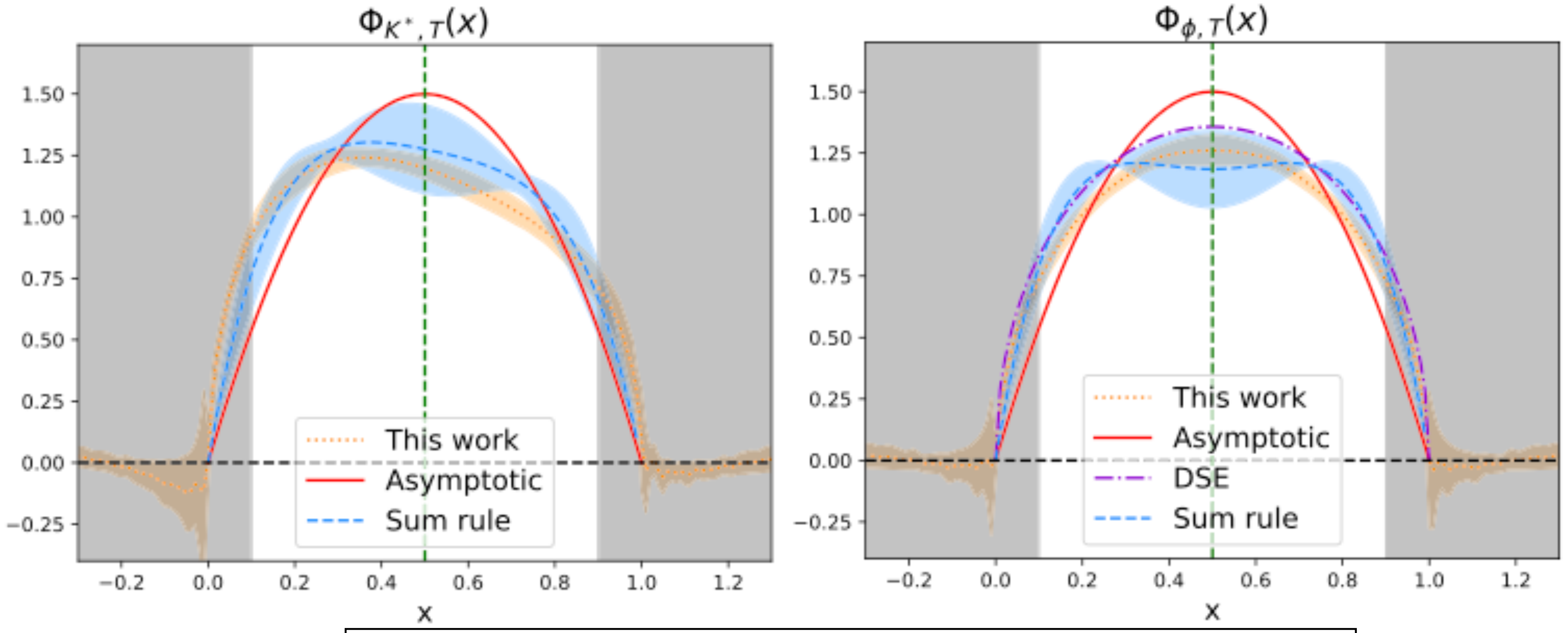
achievements

Pion and Kaon distribution amplitude



J. Hua, M. Chu et,al., Phys.Rev.Lett. 129 (2022) 13, 132001

Vector meson distribution amplitude

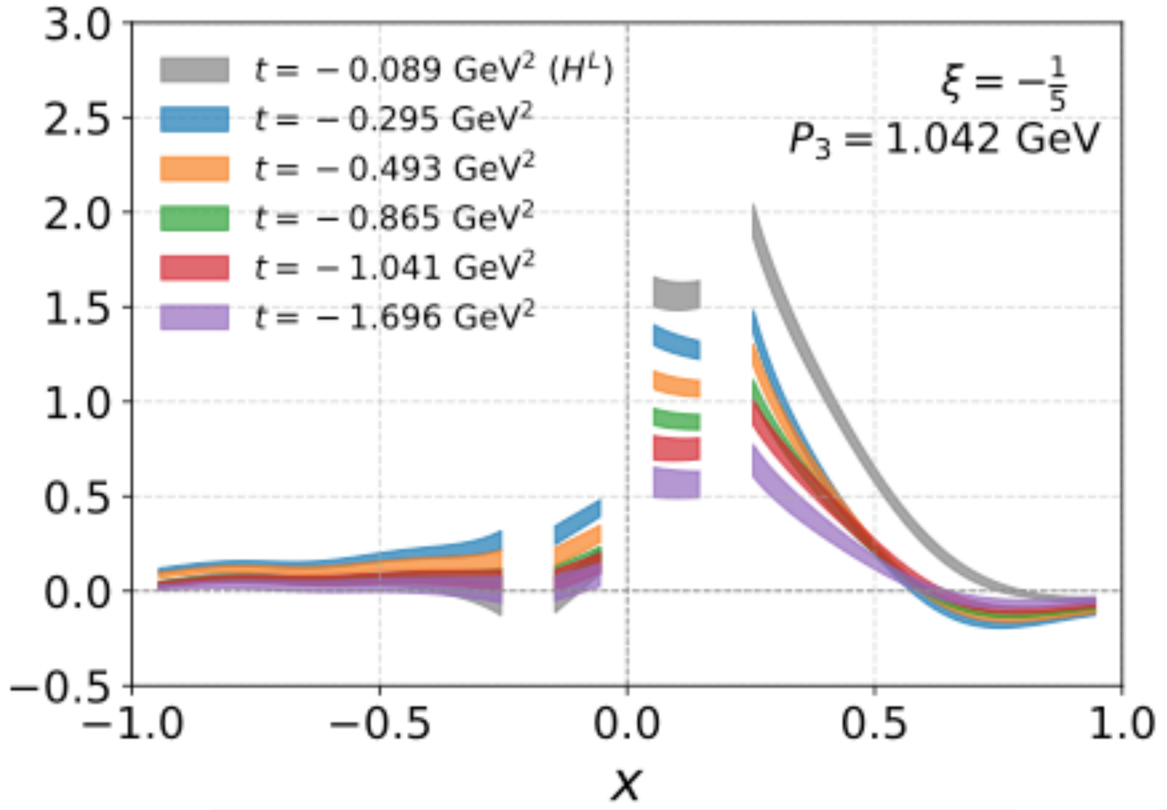


J. Hua, M. Chu et,al., Phys.Rev.Lett. 127 (2021) 6, 062002

Large Momentum Effective Theory

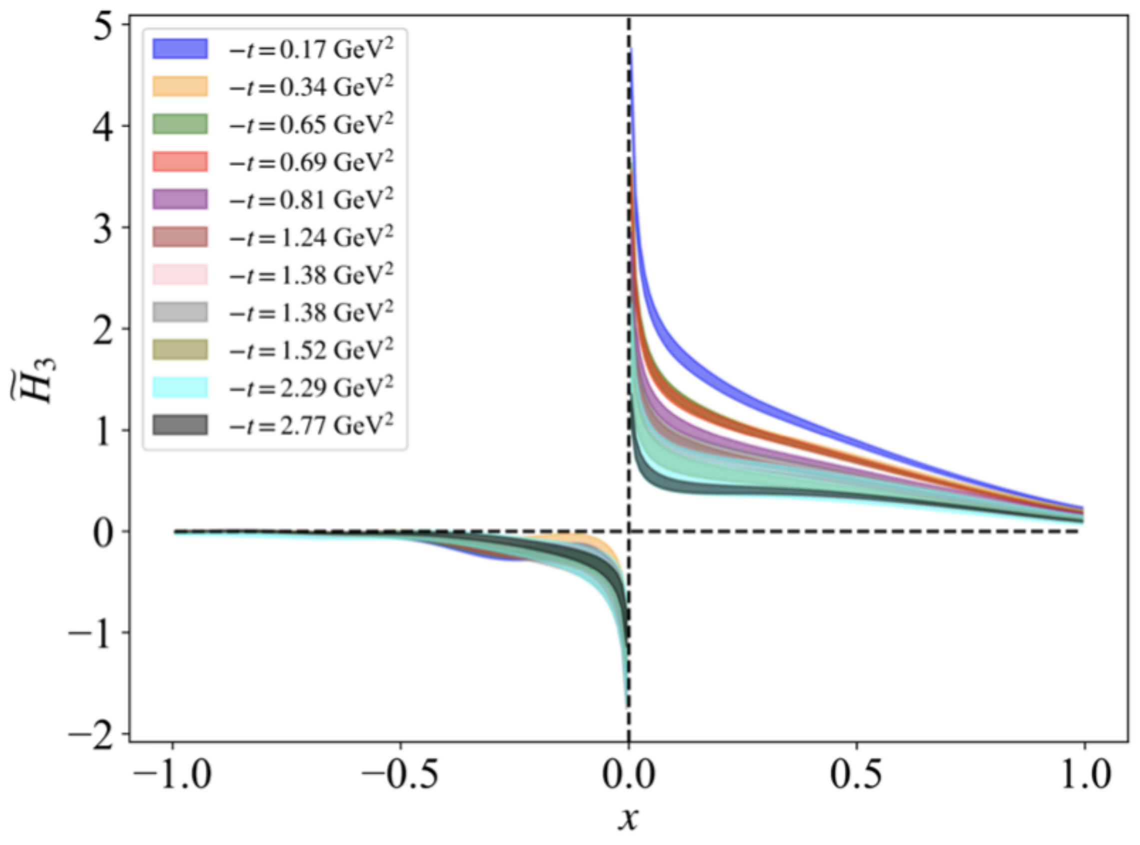
achievements

Proton unpolarized GPD related to this talk



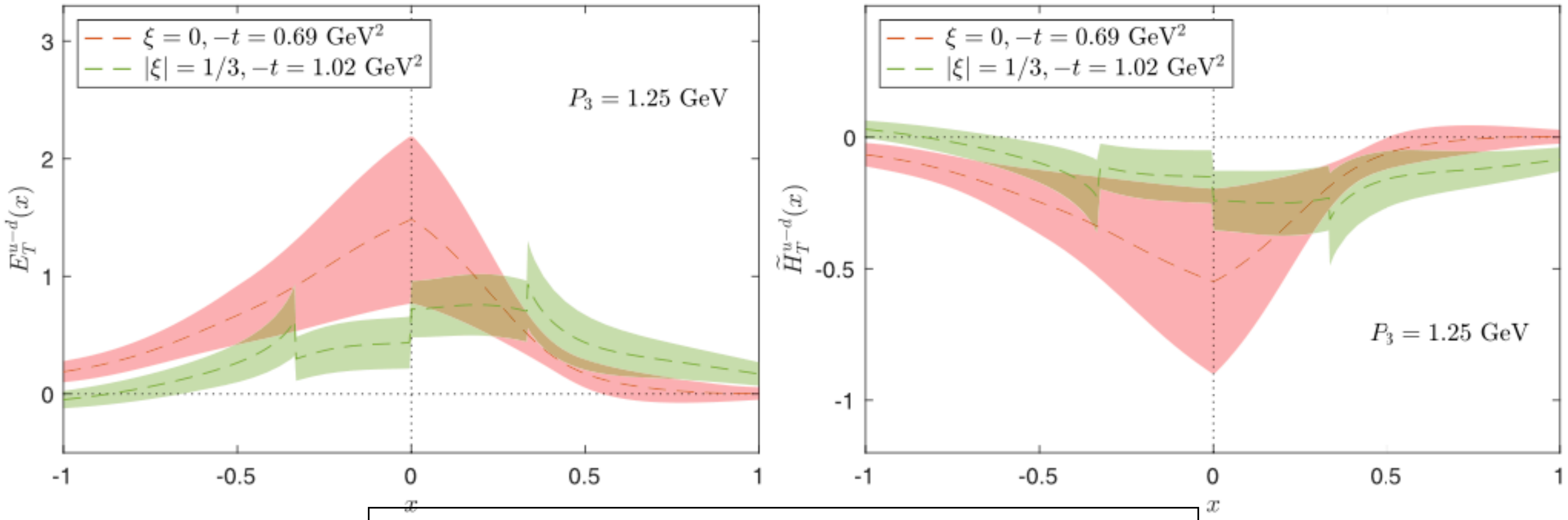
M. Chu, et,al., arxiv: 2508.17998

Proton axial vector GPD



S. Bhattacharya et al., Phys.Rev.D 109 (2024) 3, 034508

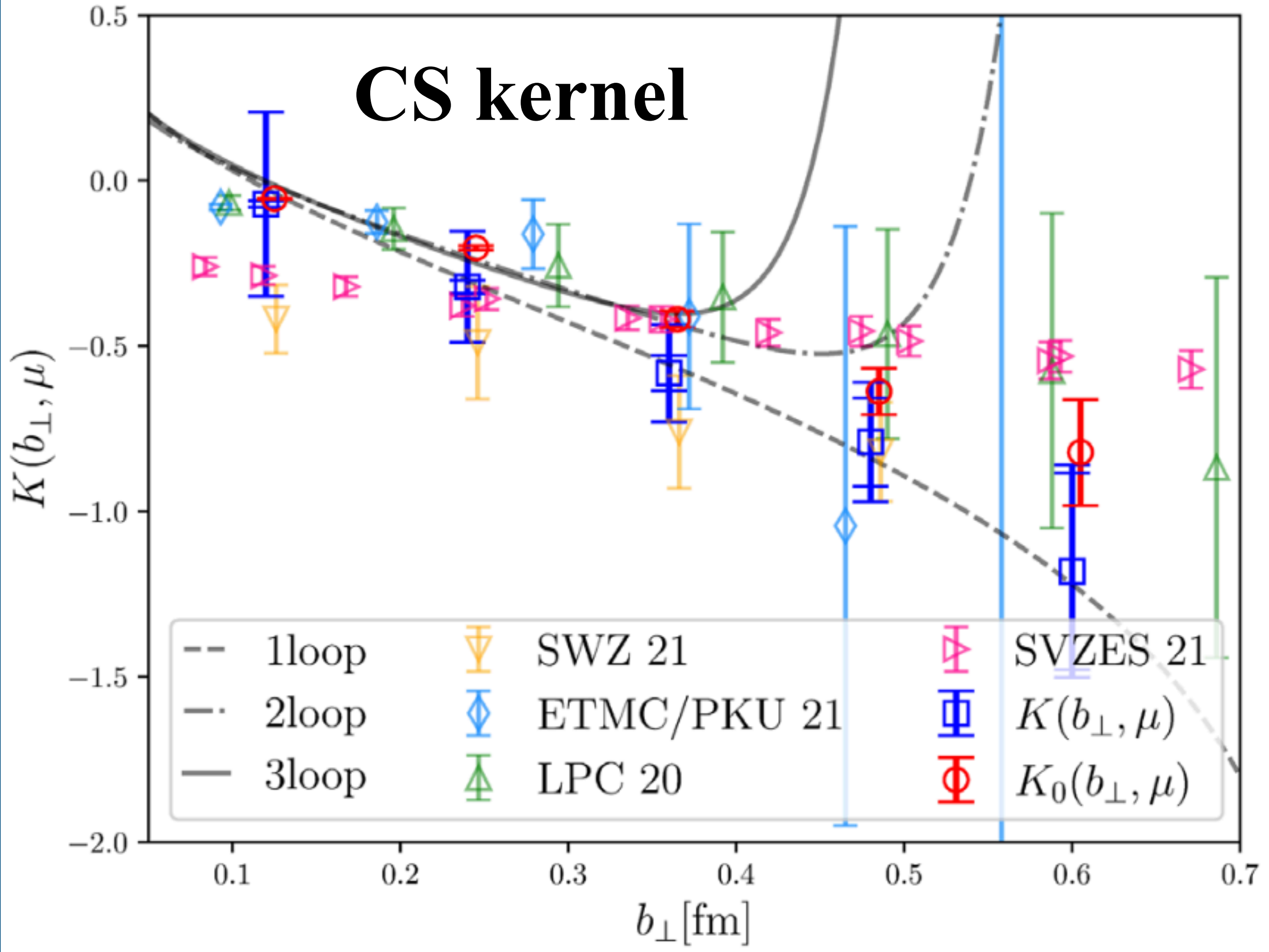
Proton transversity quark GPD



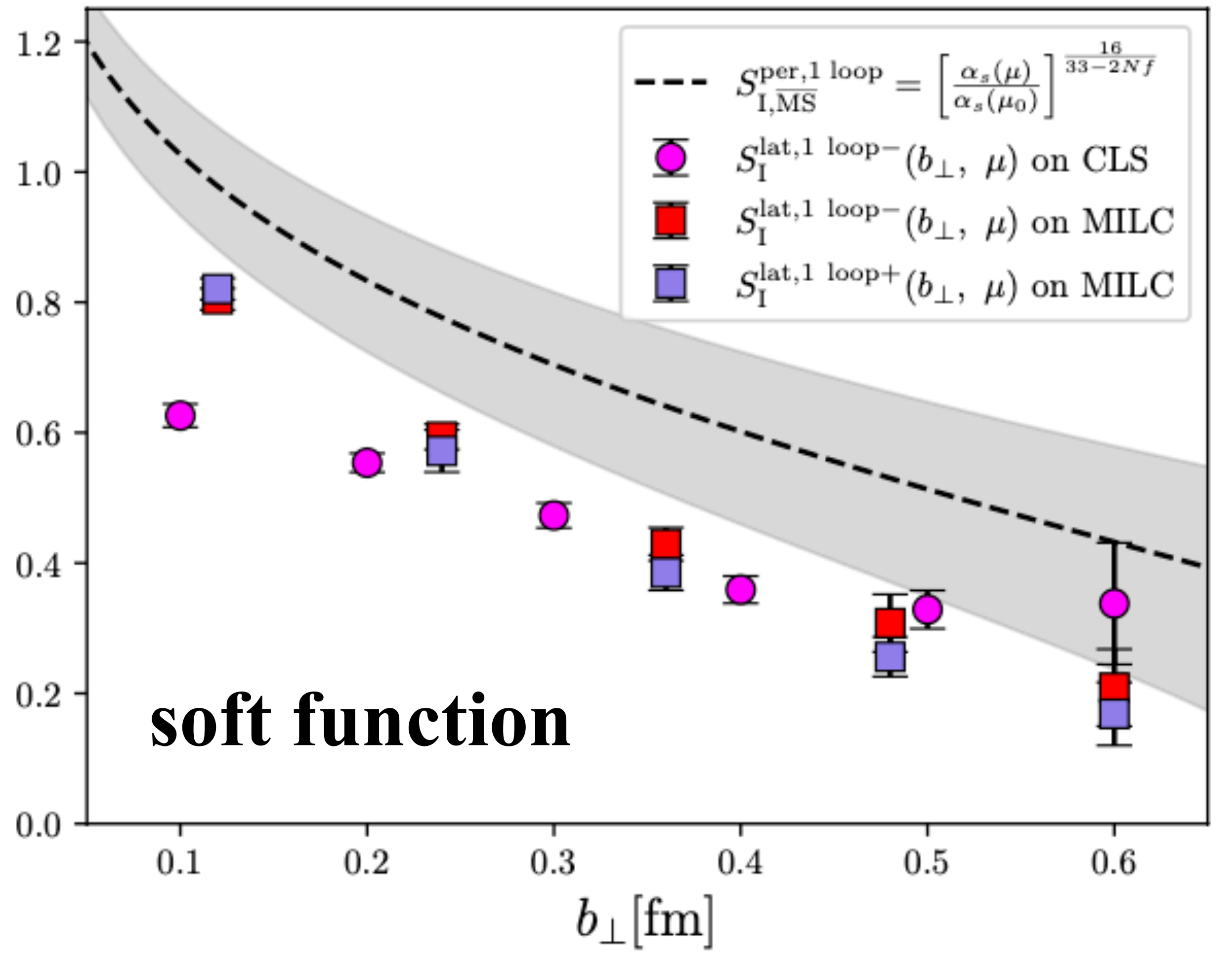
C. Alexandrou, et,al., Phys.Rev.D 105 (2022) 3, 034501

achievements

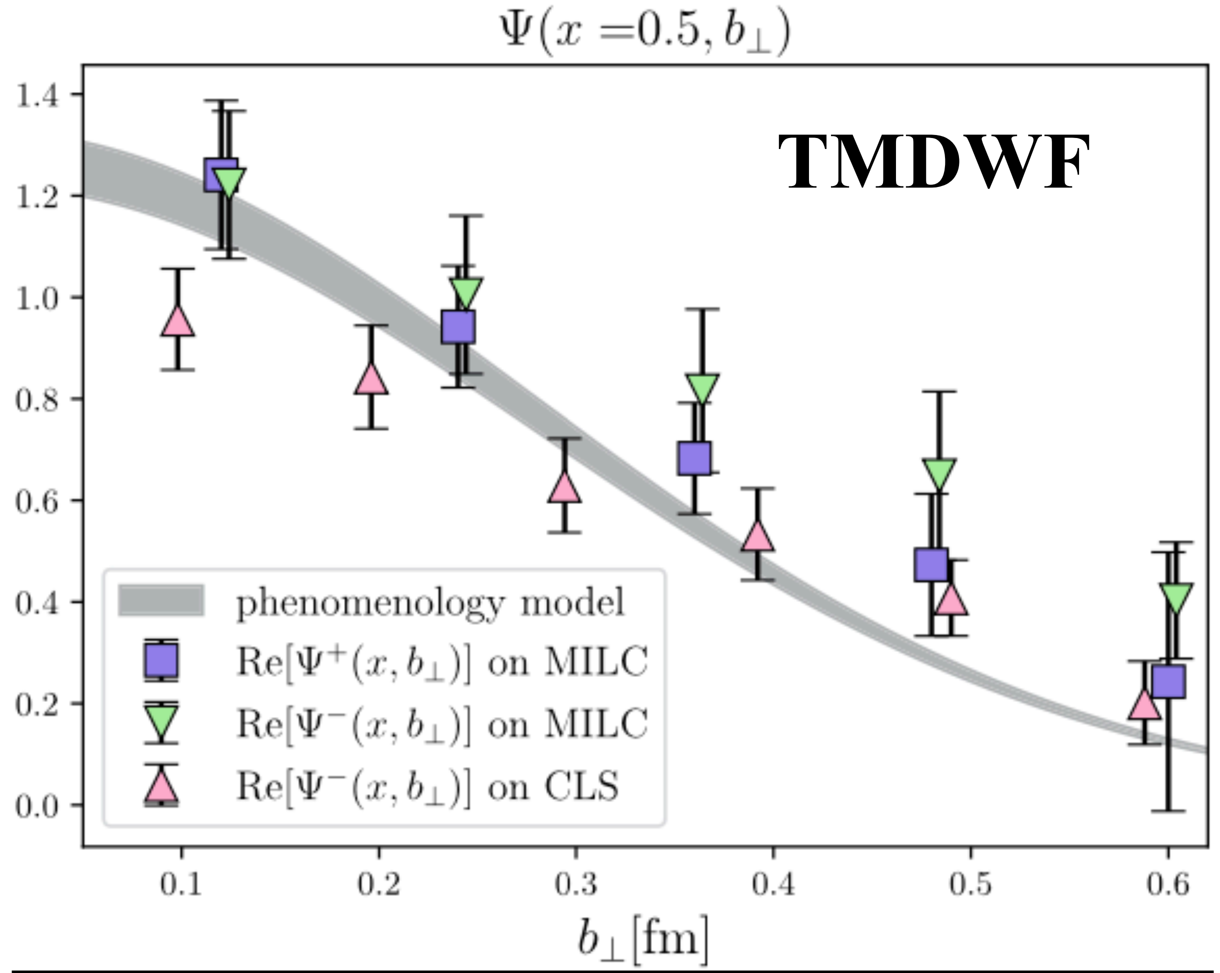
TMDs related to this talk



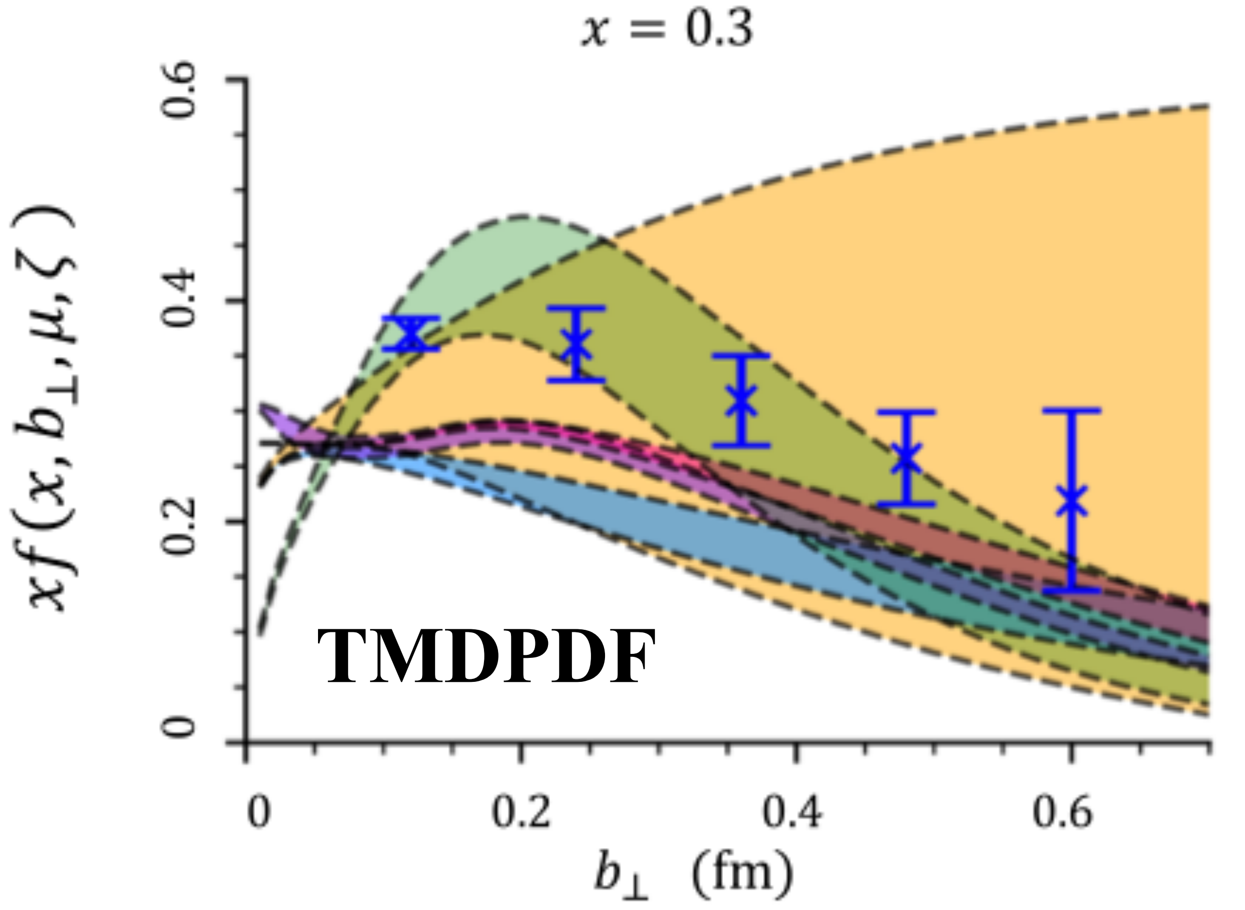
M. Chu, et,al., Phys.Rev.D 106 (2022) 3, 034509



M. Chu, et,al., JHEP 08 (2023) 172



M. Chu, et,al., Phys.Rev.D 109 (2024) 9, L091503



J. He, et,al., Phys.Rev.D 109 (2024) 11, 114513

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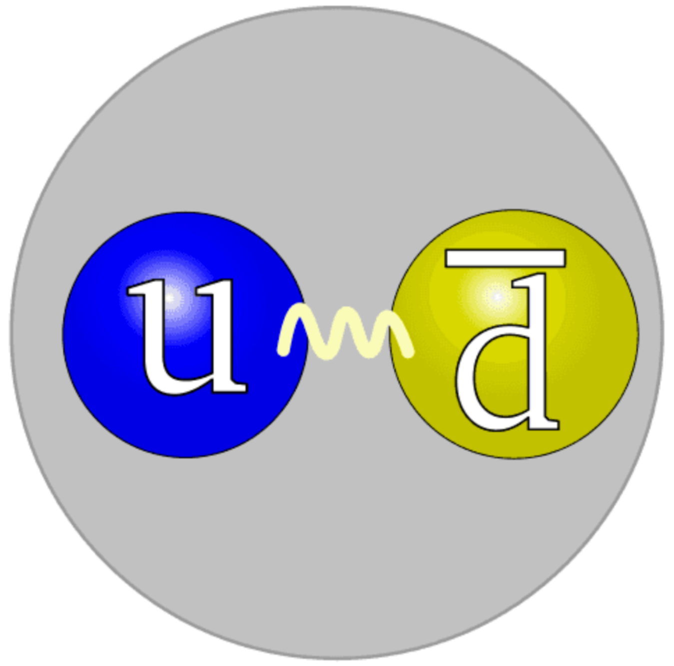
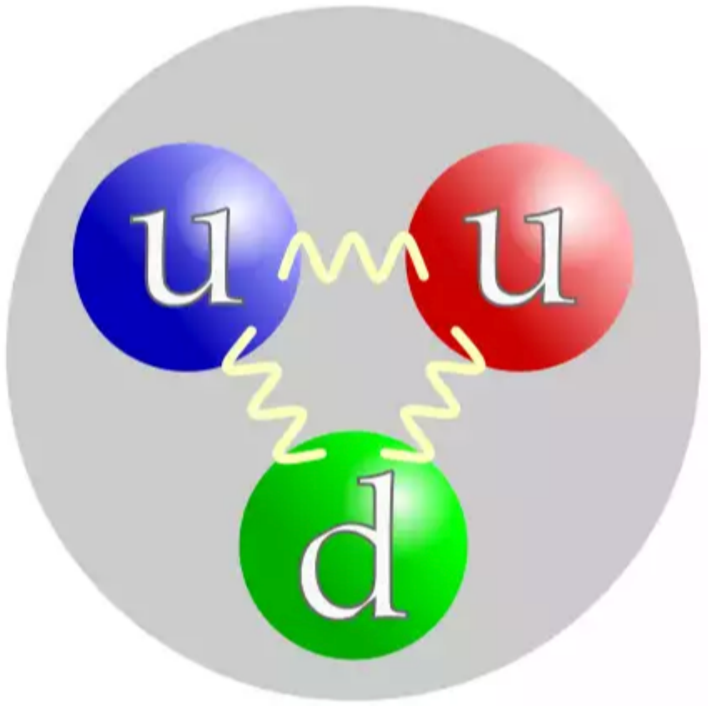




Framework and Results: GPD

definition of GPDs

$$F^\mu(z,P,\Delta) = \langle P_f | \bar{q}(-\frac{z}{2})\gamma^\mu W(-\frac{z}{2},\frac{z}{2}) q(\frac{z}{2}) | P_i \rangle$$

$$= \bar{u}(P_f,\lambda')\left[\frac{P^\mu}{m}A_1 + mz^\mu A_2 + \frac{\Delta^\mu}{m}A_3 + im\sigma^{\mu z}A_4 + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{m}A_6 + mz^\mu i\sigma^{z\Delta}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}A_8\right]u(P_i,\lambda)$$

In $\Delta_\perp = 0$ case:

$$F^{\mu,L}(z,P,\Delta_L) = \bar{u}(P_f,\lambda')\left[\frac{P^\mu}{m}A_1^L + mz^\mu A_2^L + im\sigma^{\mu z}A_4^L\right]u(P_i,\lambda)$$

definition of GPDs

Projecting to 16 components:

amplitudes

$$\Pi_\mu(\Gamma_\nu) = K\,\mathrm{Tr}\left[\Gamma_\nu\left(\frac{-i\not{P}_f + m}{2m}\right)\tilde{F}^\mu\left(\frac{-i\not{P}_i + m}{2m}\right)\right]$$

$$\Pi_\mu(\Gamma_\nu) = \sum_{i=0}^{8} C_i A_i$$

decomposition

H and E GPDs

$$H = A_1 - 2\xi A_3$$

$$E = -A_1 + 2\xi A_3 + 2A_5 + 2P_3 z A_6 - 4\xi P_3 z A_8$$

RI/MOM renormalization

FT and matching

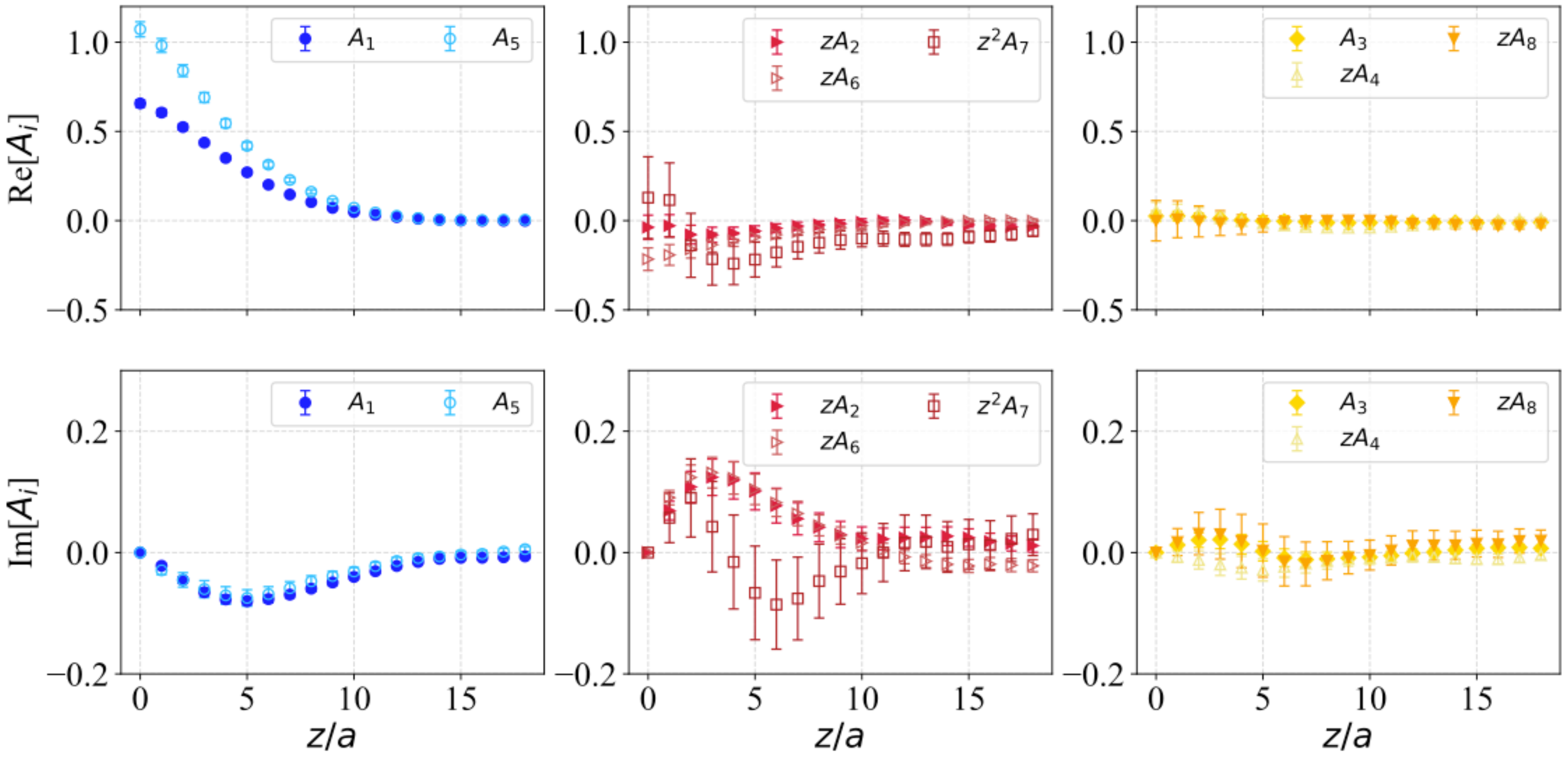
$$H(x,\xi,t),\quad E(x,\xi,t)$$

light-cone H and E GPDs

Framework and Results: GPD

numerical results: amplitudes

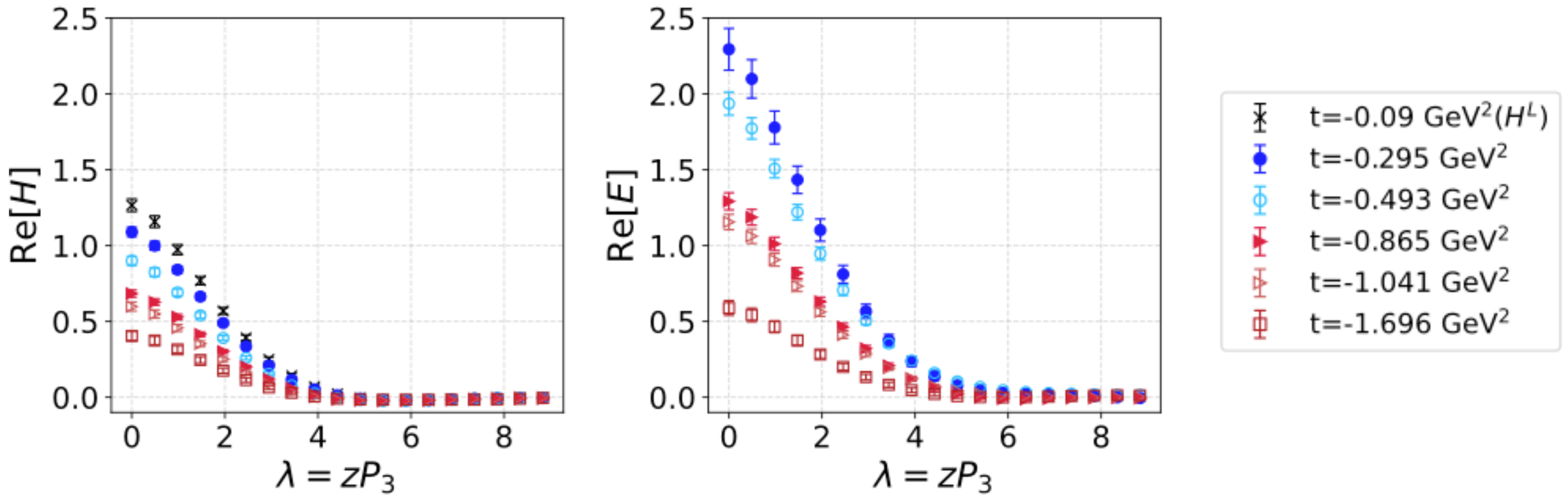
8 amplitudes



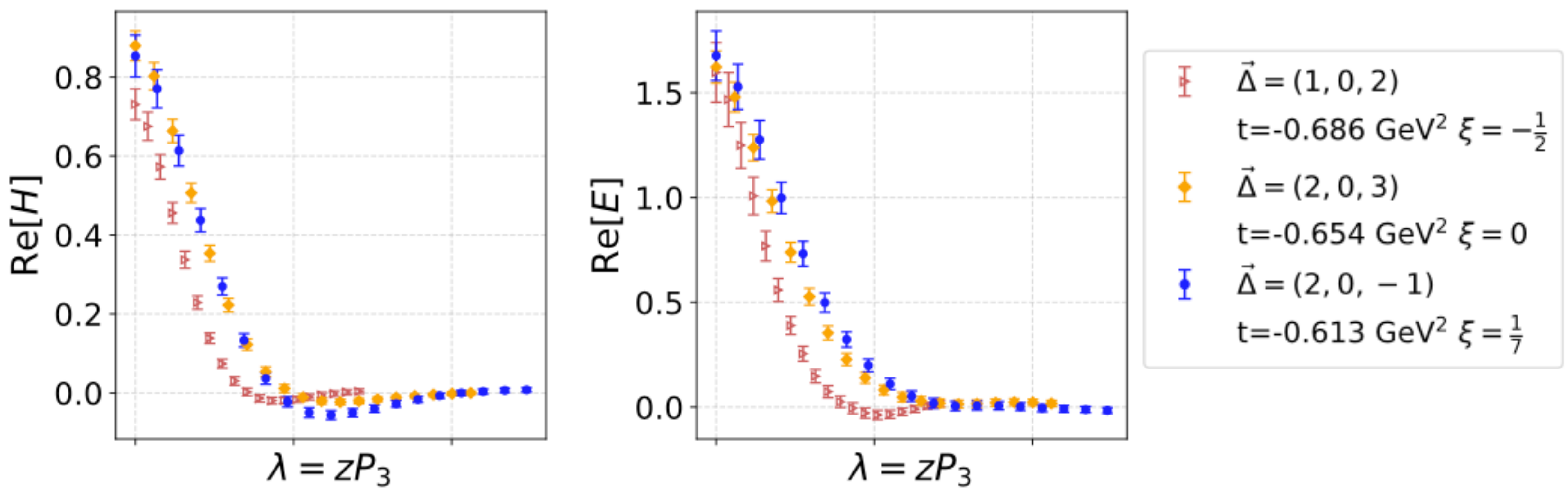
same as $\xi = 0$ case: A_1 and A_5 dominate

different from $\xi = 0$ case: A_2, A_6 and A_7 may have signal

numerical results: H and E GPDs



same as $\xi = 0$: H/E primarily decay with $-t$

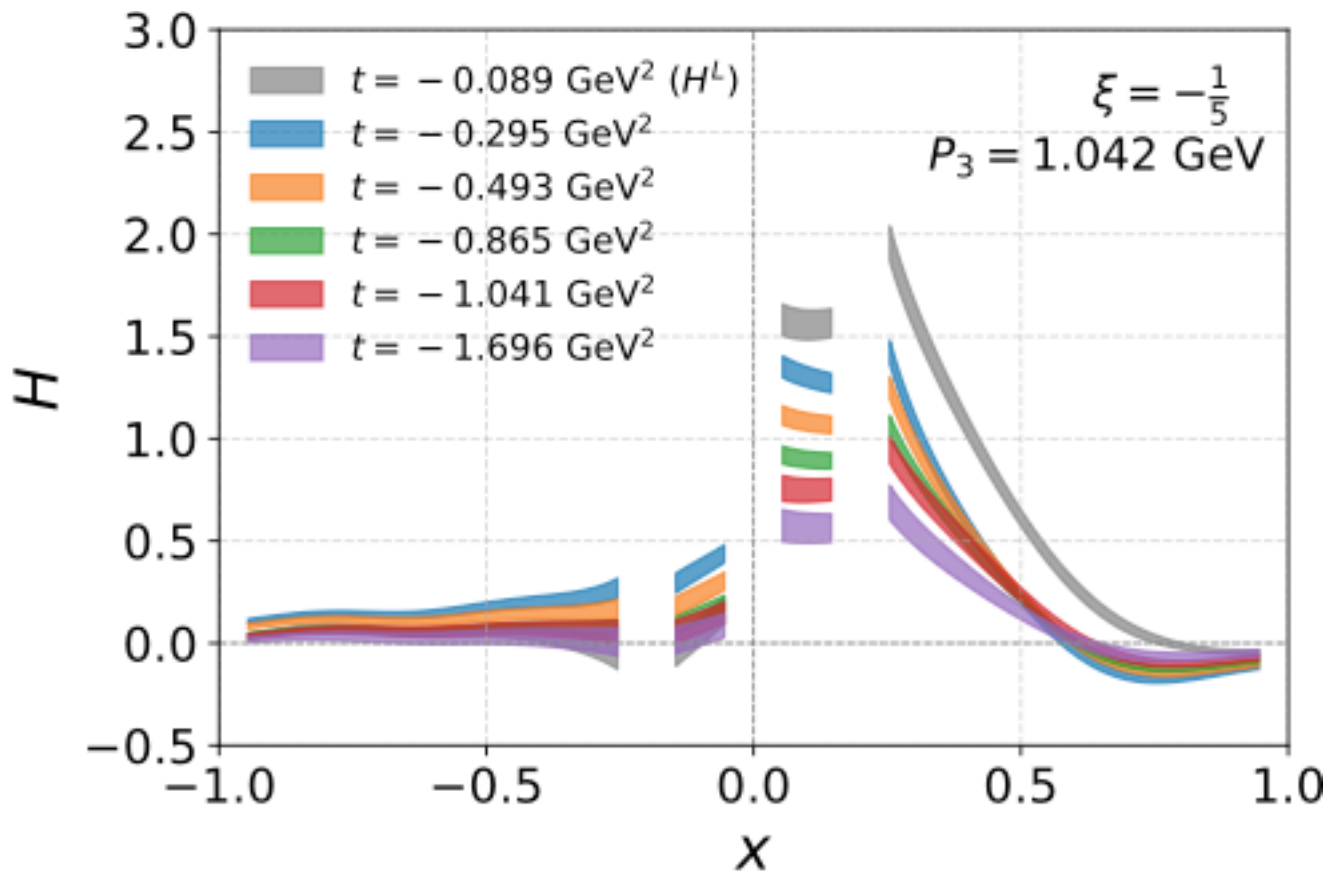


different from $\xi = 0$: ξ accelerates the decay

Framework and Results: GPD

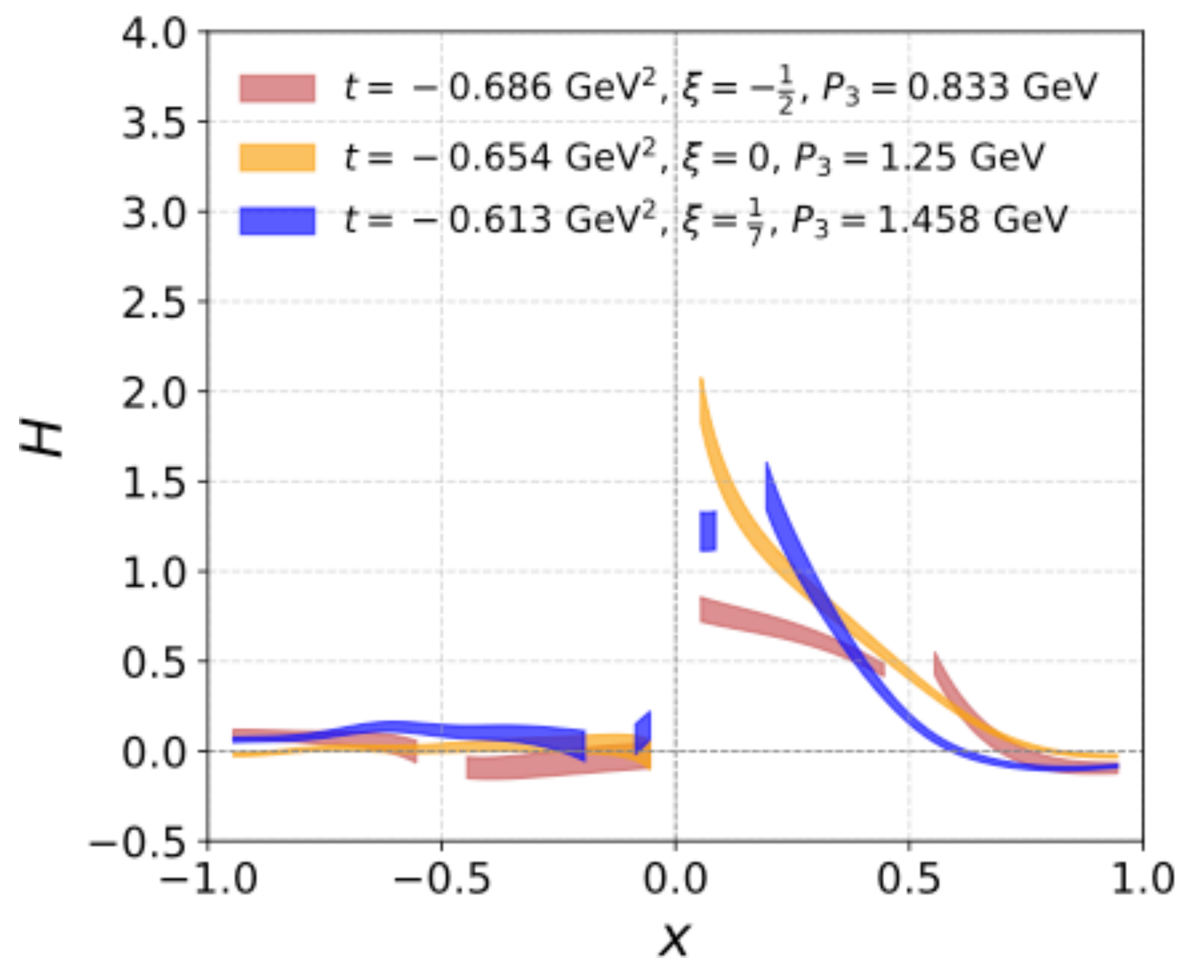
numerical results: light-cone GPDs

unreliable region: power correction at $x \to \pm|\xi|$ and $x \to \pm 1$, BG at $x \to 0$ (limitation of light-cone results)



H/E decay with $-t$

difficult to obtain information in the ERBL region



H/E suppress in ERBL but grows with ξ increase

discontinuity in unreliable region causes underwhelming results

solutions for GPD limitations

1. power correction: larger hadron momentum

2. BG : moments from SDE (in progress), model dependent reconstruction, neuron network reconstruction (in progress)

3. lattice artifacts: hybrid renormalization (in progress), continuum limit (in progress)

$H(x, \xi, t), \quad E(x, \xi, t)$

angular momentum

elastic form factor

nucleon tomographic

Framework and Results: GPD

Short-Distance Expansion approach

double ratio renormalization

$$F(z, \nu = zP_z) = F(z, P_z) = \frac{F^0(z, P_z)}{f(z,0)} \frac{f(0,0)}{f(z, P_z)}$$

matching in coordinate space

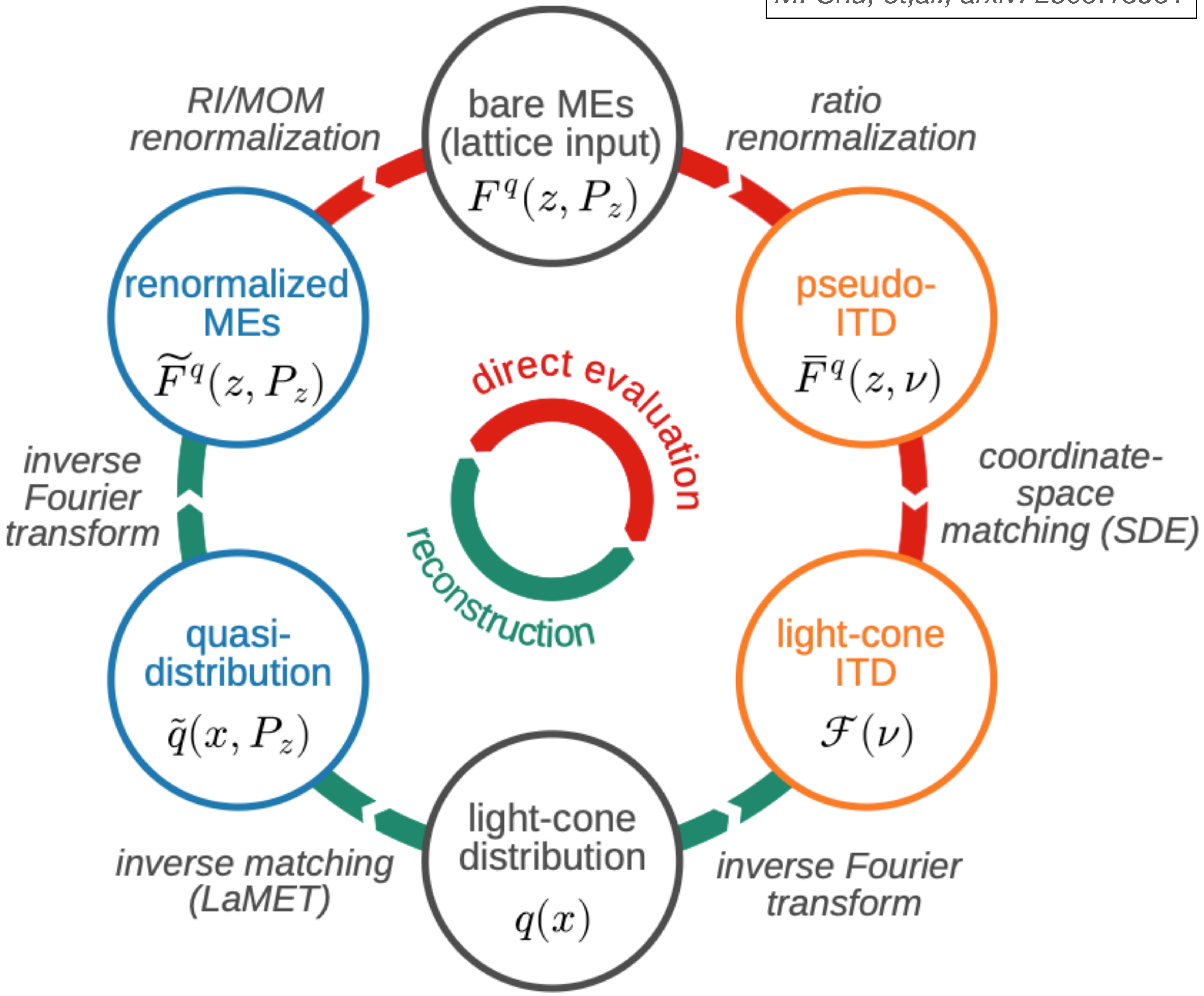
$$F(\nu, t, z^2) = \bar{F}(\nu, t, z^2) + \frac{\alpha_s C_F}{2\pi} \int_0^1 \left[\ln \frac{z^2 \mu^2 e^{2\gamma_E + 1}}{4} B(u) + L(u) \right] \bar{F}(\nu, \xi, t, z^2)$$

power correction of SDE: $\mathcal{O}(z^2)$ not related to x

power correction of LaMET: $\mathcal{O}\left(1/x^2 P_z^2\right)$,$\mathcal{O}\left(1/(1-x)^2 P_z^2\right)$

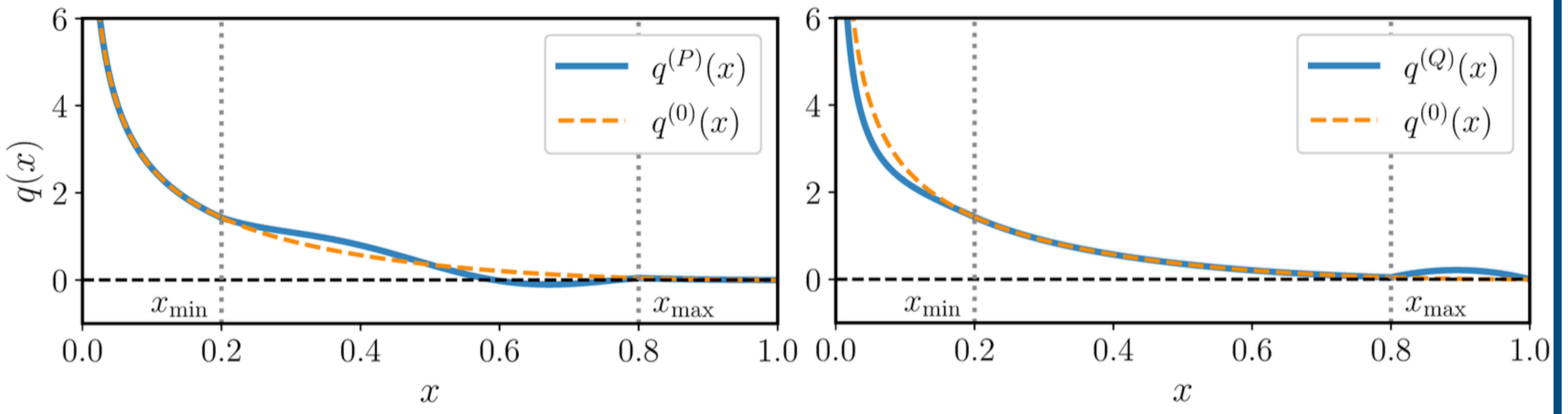
unified ANN reconstruction

M. Chu, et,al., arxiv: 2509.15931



Framework and Results: GPD

unified ANN reconstruction



0.2 0.8

SDE LaMET SDE x

$$q^{(Q)}(x) = \begin{cases} q^{(0)}(x) + q^{(1)}(x)\,, & 0 < x < x_{\min}\,, \\ q^{(0)}(x) + q^{(2)}(x)\,, & x_{\max} < x < 1\,, \\ q^{(0)}(x)\,, & \text{otherwise}\,, \end{cases}$$

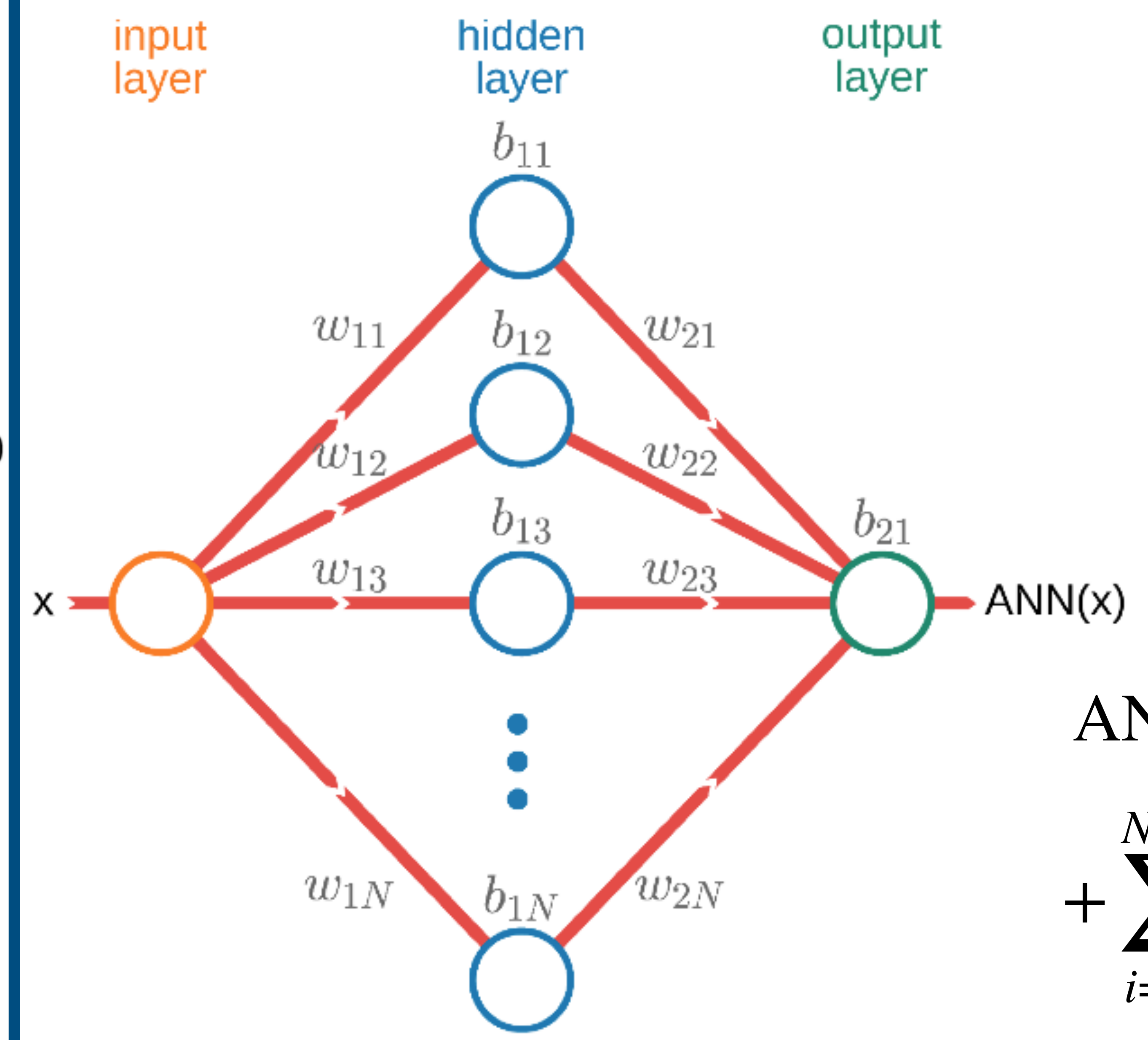
$$q^{(P)}(x) = \begin{cases} q^{(0)}(x) + q^{(3)}(x)\,, & x_{\min} < x < x_{\max}\,, \\ q^{(0)}(x)\,, & \text{otherwise}\,, \end{cases}$$

parametrizations of light-cone PDFs

M. Chu, et,al., arxiv: 2509.15931

$$q^{(0)}(x) = x^{\delta^{(0)}}(1-x)^{\rho^{(0)}}\,\mathrm{ANN}^{(0)}(x)$$

unified ANN reconstruction

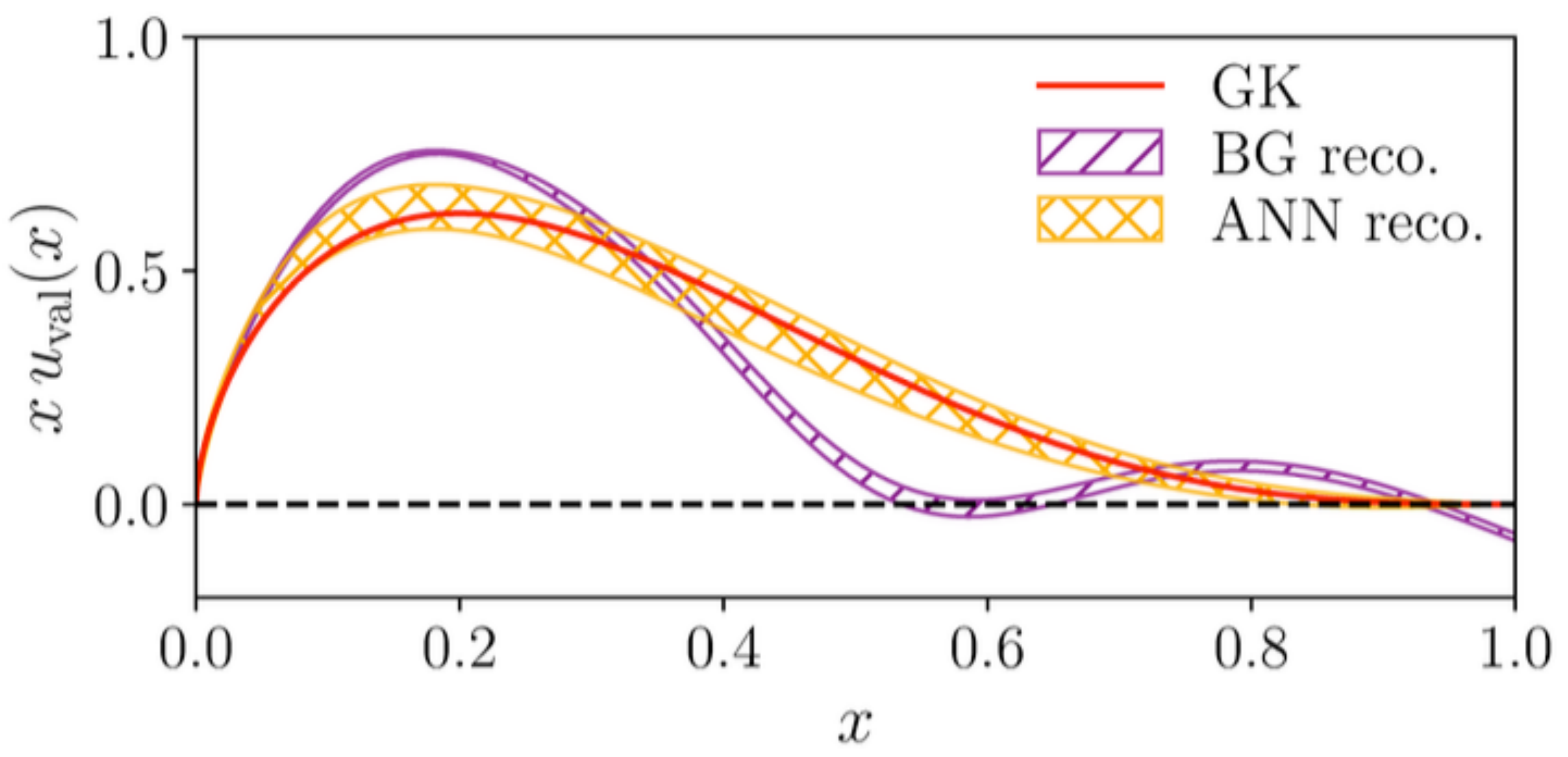


M. Chu, et,al., arxiv: 2509.15931

neuron network

$$\mathrm{ANN}^{(0)}(x) = b_{21}^{(0)} + \sum_{i=1}^{N^{(0)}} \left[w_{2i}^{(0)} \ln\left(1 + \exp(b_{1i}^{(0)} + w_{1i}^{(0)} x)\right)\right]$$

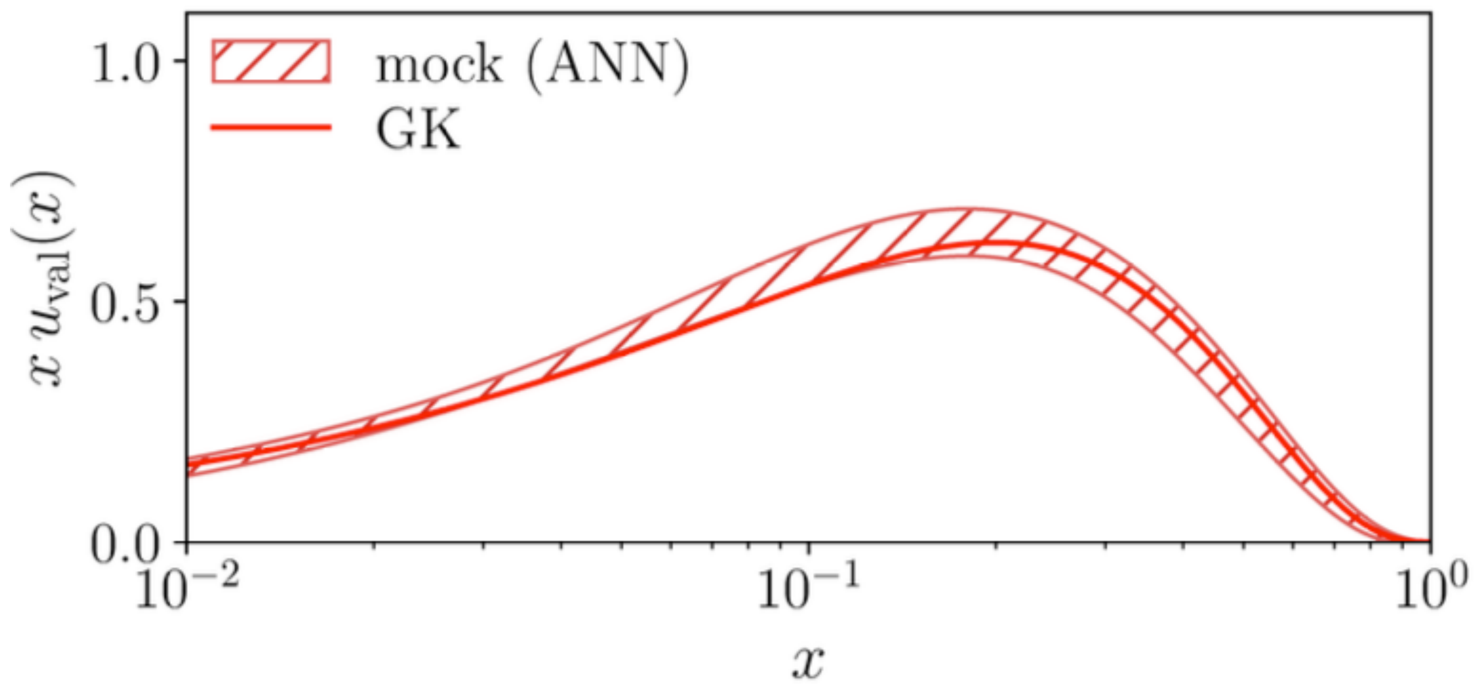
ANN v.s. BG



Framework and Results: GPD

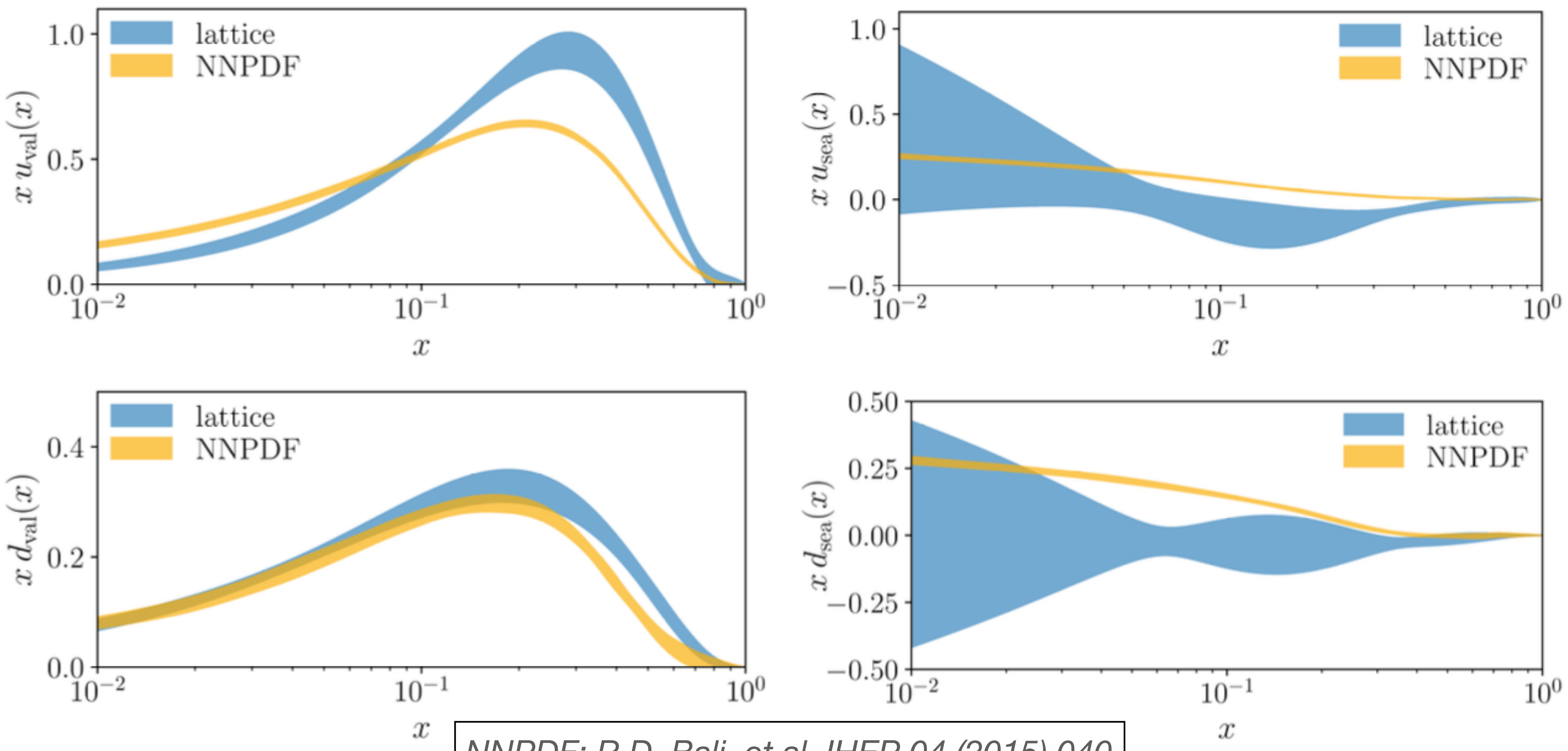
unified ANN reconstruction

mock data test



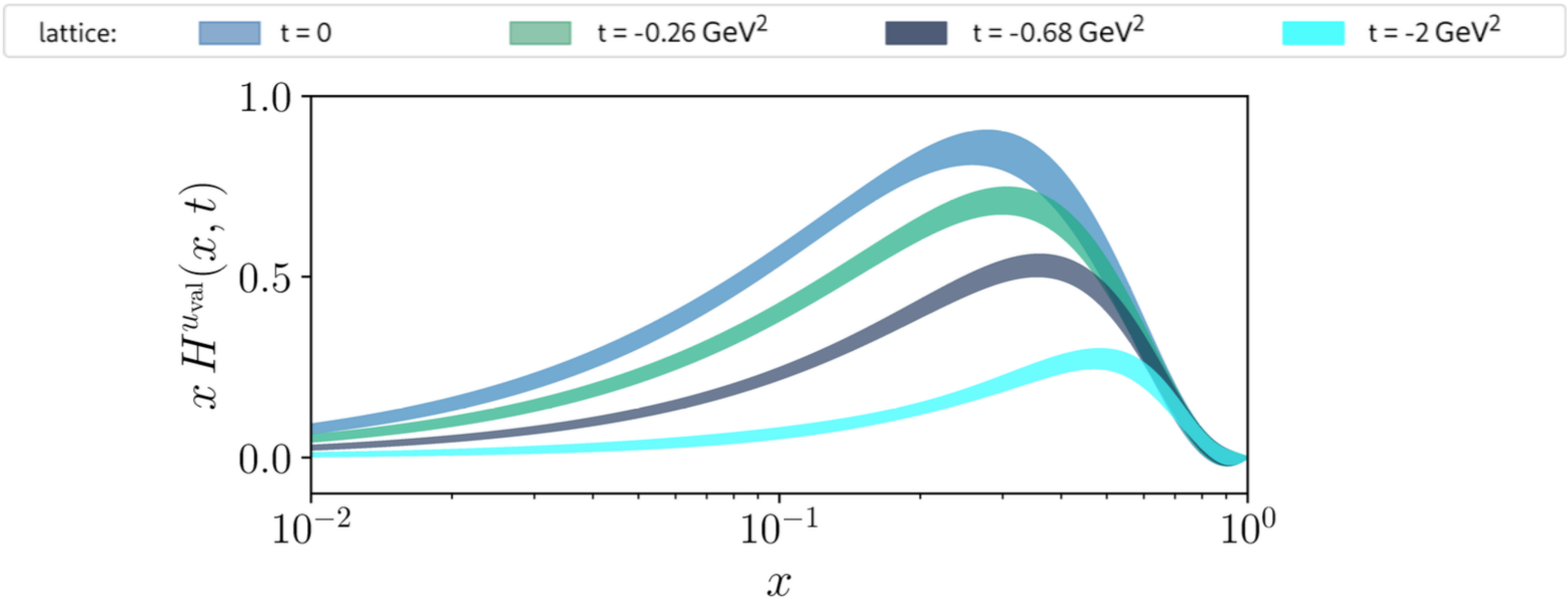
PDF results

M. Chu, et,al., arxiv: 2509.15931



NNPDF: R.D. Bali, et,al JHEP 04 (2015) 040

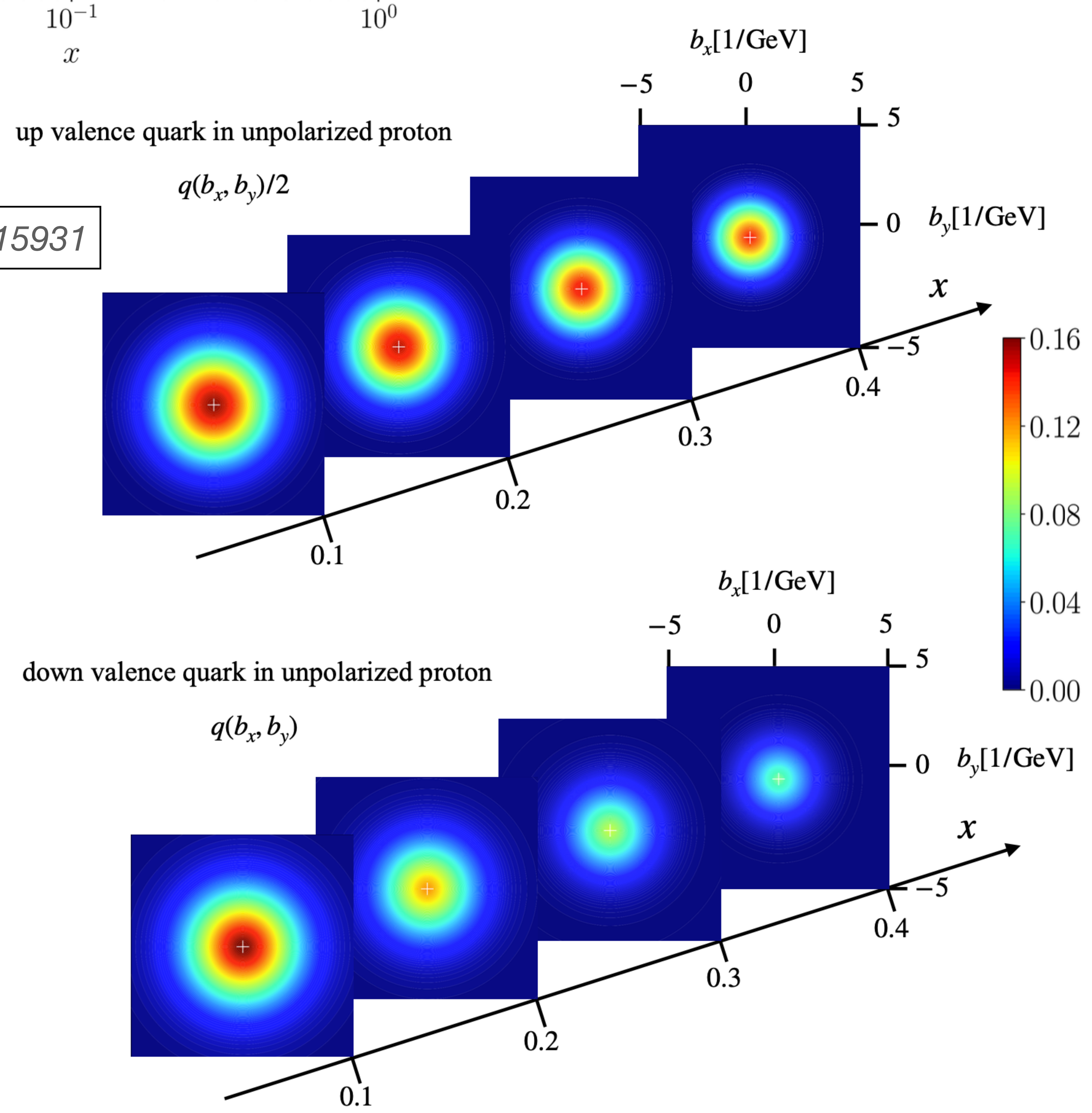
unified ANN reconstruction



GPD results

M. Chu, et,al., arxiv: 2509.15931

nucleon tomography

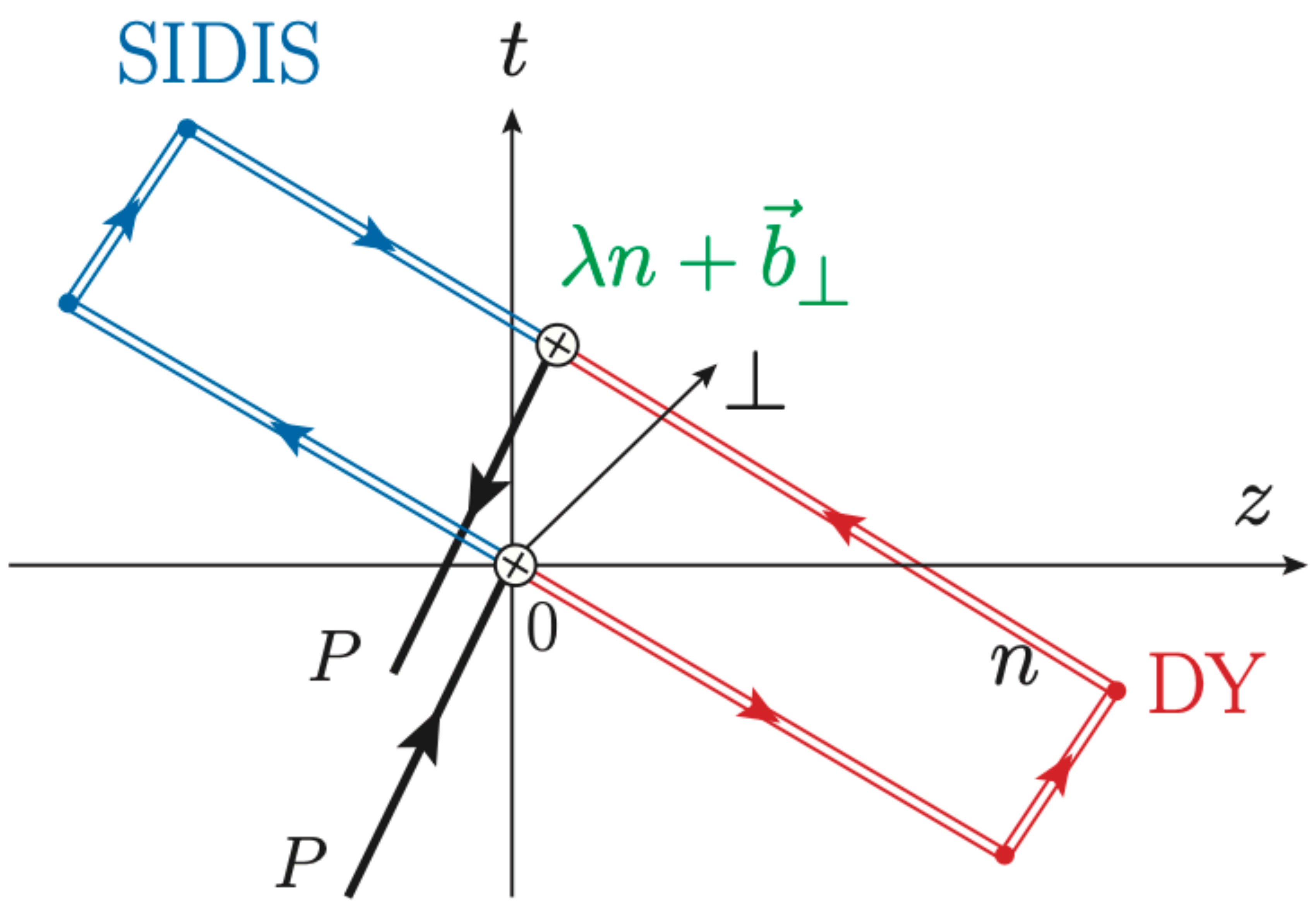


Framework and Results: TMDs

definition of TMDPDF/WF

$$f^{\pm}(x,\vec{k}_\perp) = \int \frac{d\lambda}{2\pi}\frac{db_\perp^2}{(2\pi)^2} e^{-i\lambda x + i\vec{k}_\perp \vec{b}_\perp}$$
$$\times \langle P | \bar{q}(\lambda n + \vec{b}_\perp)\Gamma W^{\pm}(\lambda n/2 + \vec{b}_\perp) q(0) | P \rangle$$

vacuum state for WF



TMD factorization

Both for TMDPDF/WF

X. Ji et al. Rev.Mod.Phys. 93 (2021) 3, 035005

Z. Deng et al. JHEP 09 (2022) 046

$$\tilde{f}^{\pm}\left(x, b_\perp, \mu, \zeta^z\right) S_I^{\frac{1}{2}}\left(b_\perp, \mu\right)$$
$$= H^{\pm}\left(x, \zeta^z, \mu\right) e^{\left[\frac{1}{2}K(b_\perp,\mu)\ln\frac{\mp\zeta^z + i\epsilon}{\zeta}\right]} f^{\pm}\left(x, b_\perp, \mu, \zeta\right)$$

TMDs have additional rapidity divergence!

Soft gluon radiation

rapidity dependent part: $e^{\frac{1}{2}K(b_\perp,\mu)\frac{\mp\zeta^z + i\epsilon}{\zeta}}$ — Collins-Soper kernel

rapidity independent part: $S_I^{\frac{1}{2}}(b_\perp, \mu)$ — intrinsic soft function

Framework and Results: TMDs

Collins-Soper kernel

from TMD factorization

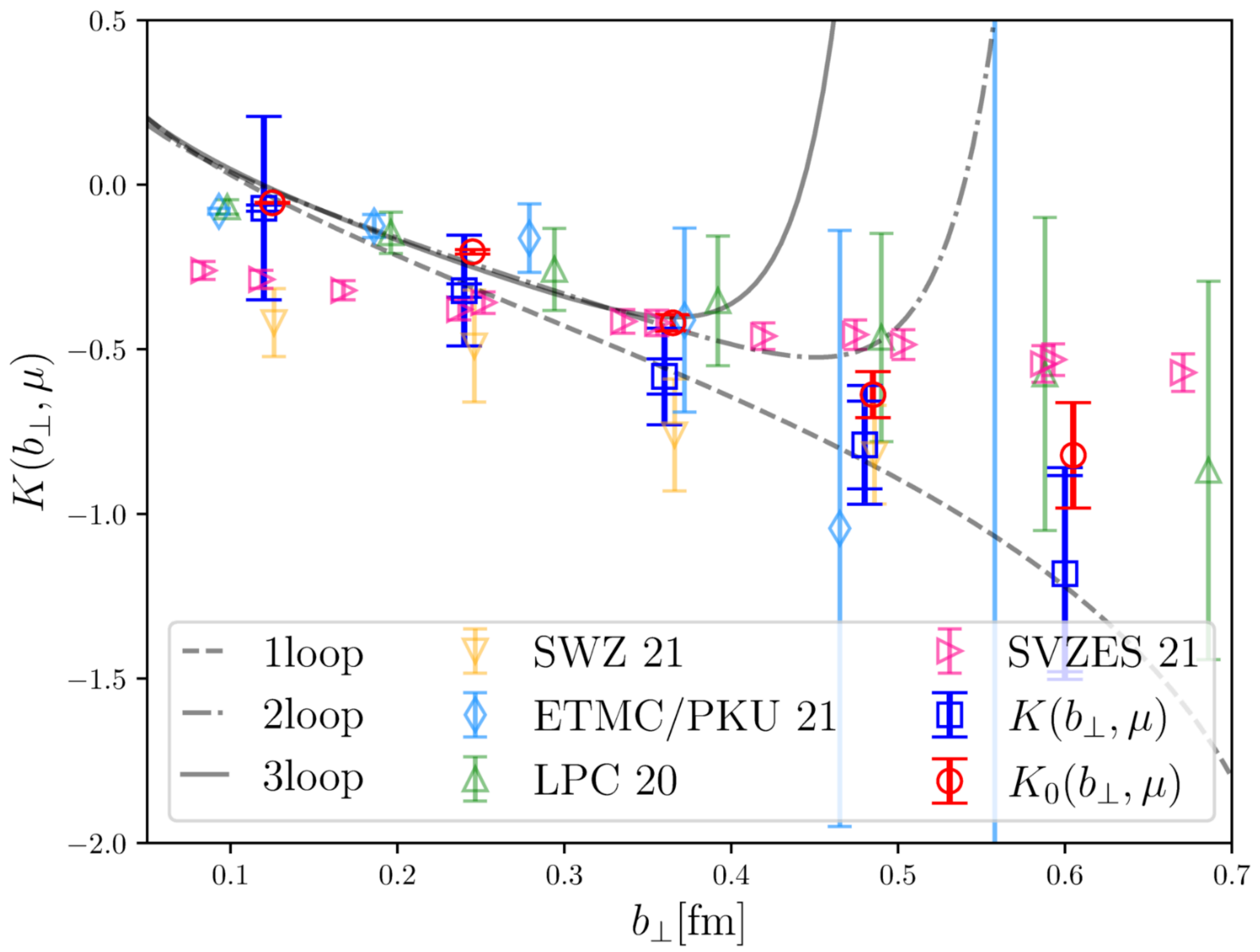
$$\tilde{f}^{\pm}\left(x, b_\perp, \mu, \zeta^z\right) S_I^{\frac{1}{2}}\left(b_\perp, \mu\right)$$

$$= H^{\pm}\left(x, \zeta^z, \mu\right) e^{\left[\frac{1}{2} K\left(b_\perp, \mu\right) \ln \frac{\mp \zeta^z + i\epsilon}{\zeta}\right]} f^{\pm}\left(x, b_\perp, \mu, \zeta\right)$$

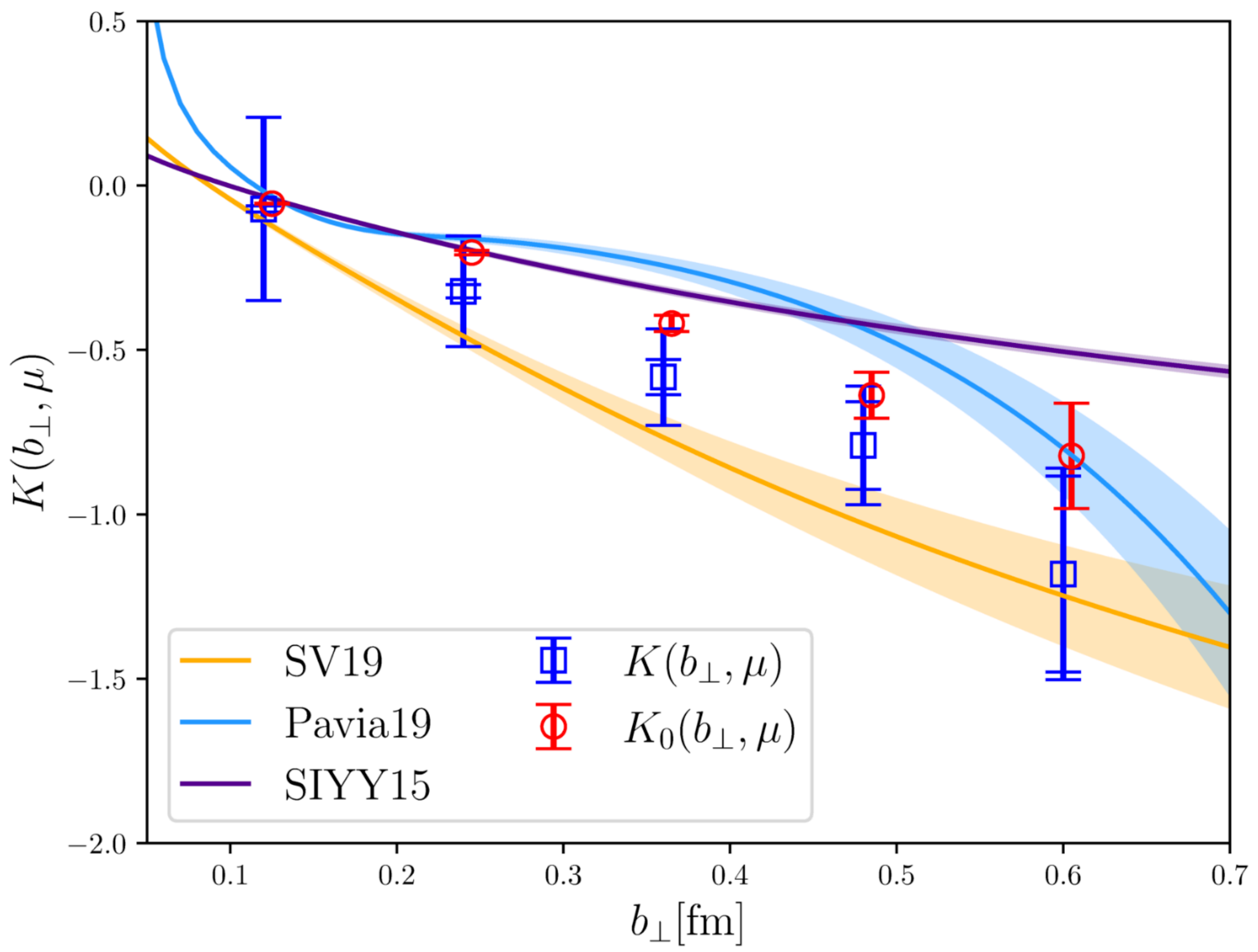
choosing different $\zeta^z = (2xP^z)^2$

$$K(b_\perp, \mu) = \frac{1}{\ln(P_2^z/P_1^z)} \ln \left[\frac{H^{\pm}(x, \zeta_1^z, \mu)\tilde{f}^{\pm}(x, b_\perp, \mu, \zeta_2^z)}{H^{\pm}(x, \zeta_2^z, \mu)\tilde{f}^{\pm}(x, b_\perp, \mu, \zeta_1^z)}\right]$$

Compared with lattice



Compared with pheno



Intrinsic soft function

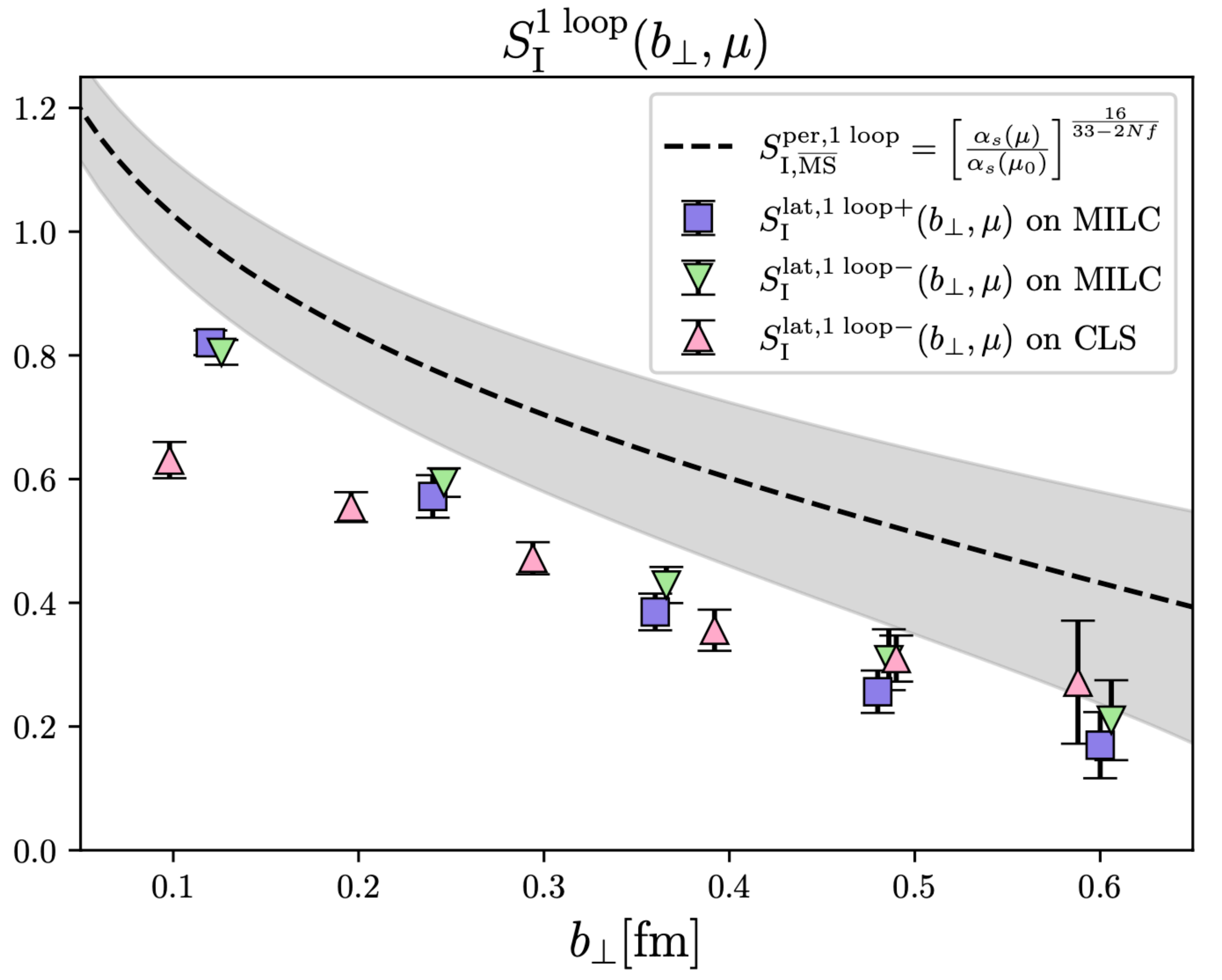
$$S_I\left(b_\perp, \mu\right) = \frac{F\left(b_\perp, P_1, P_2, \Gamma, \mu\right)}{\int dx_1 dx_2 H\left(x_1, x_2, \Gamma\right) \times \tilde{\Psi}^{\pm *}\left(x_2, b_\perp, \zeta^z\right) \tilde{\Psi}^{\pm}\left(x_1, b_\perp, \zeta^z\right)}$$

$$F\left(b_\perp, P_1, P_2, \Gamma, \mu\right) = \left\langle P_2 \left| \bar{q}(b_\perp)\Gamma q(b_\perp)\bar{q}(0)\Gamma' q(0) \right| P_1 \right\rangle$$

one loop correction

proper normalization

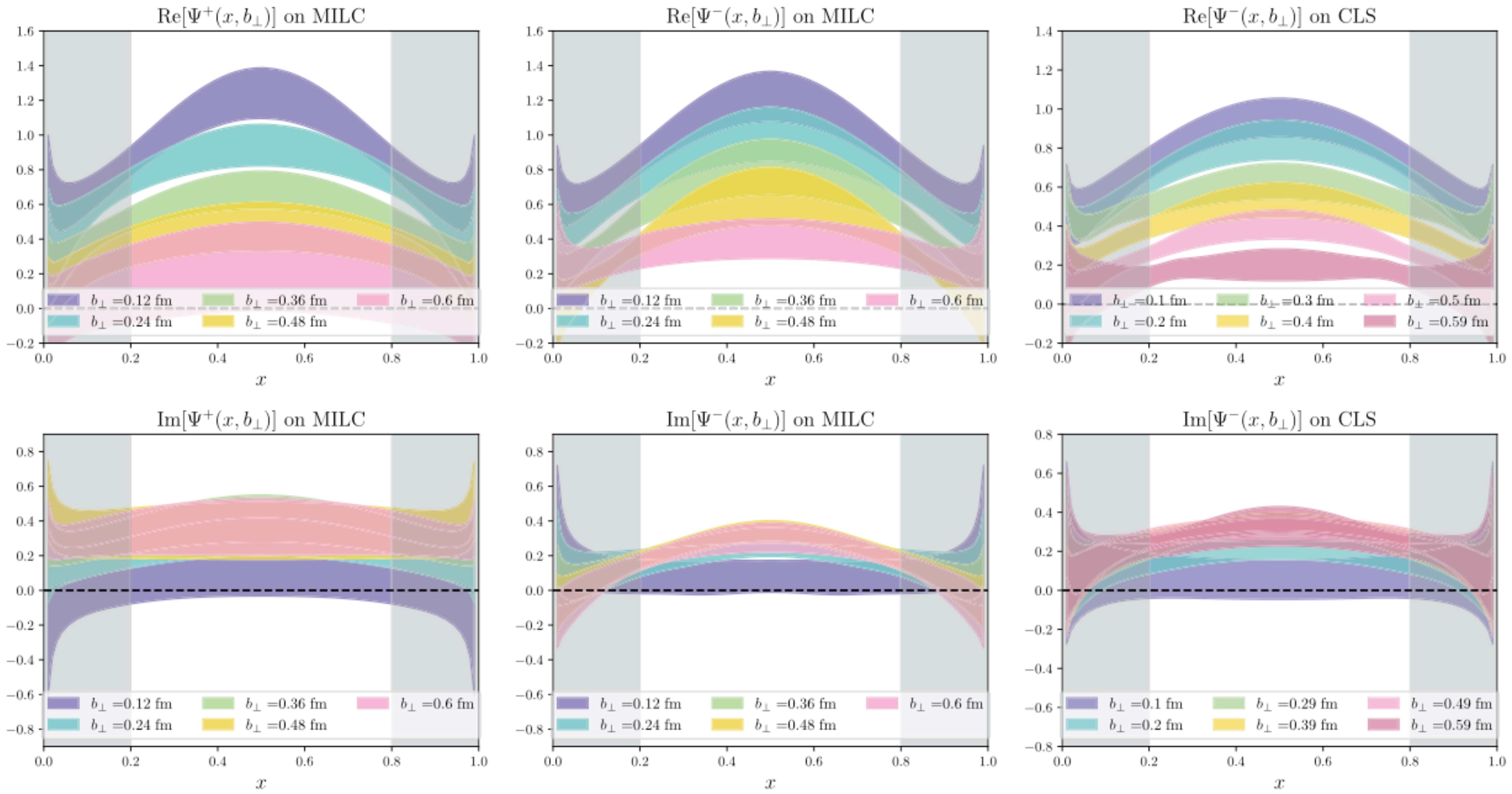
proper Dirac structure

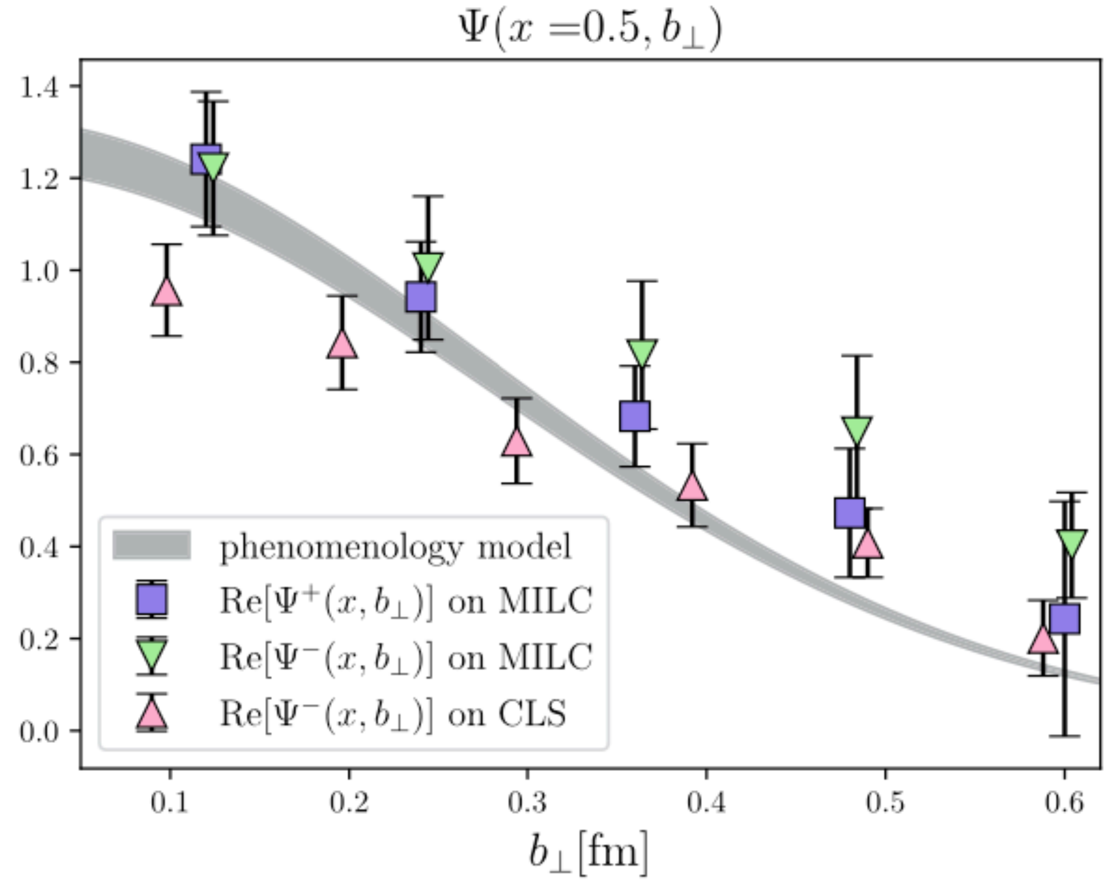


Framework and Results: TMDs

M. Chu et al., Phys.Rev.D 109 (2024) 9, L091503

TMDWF

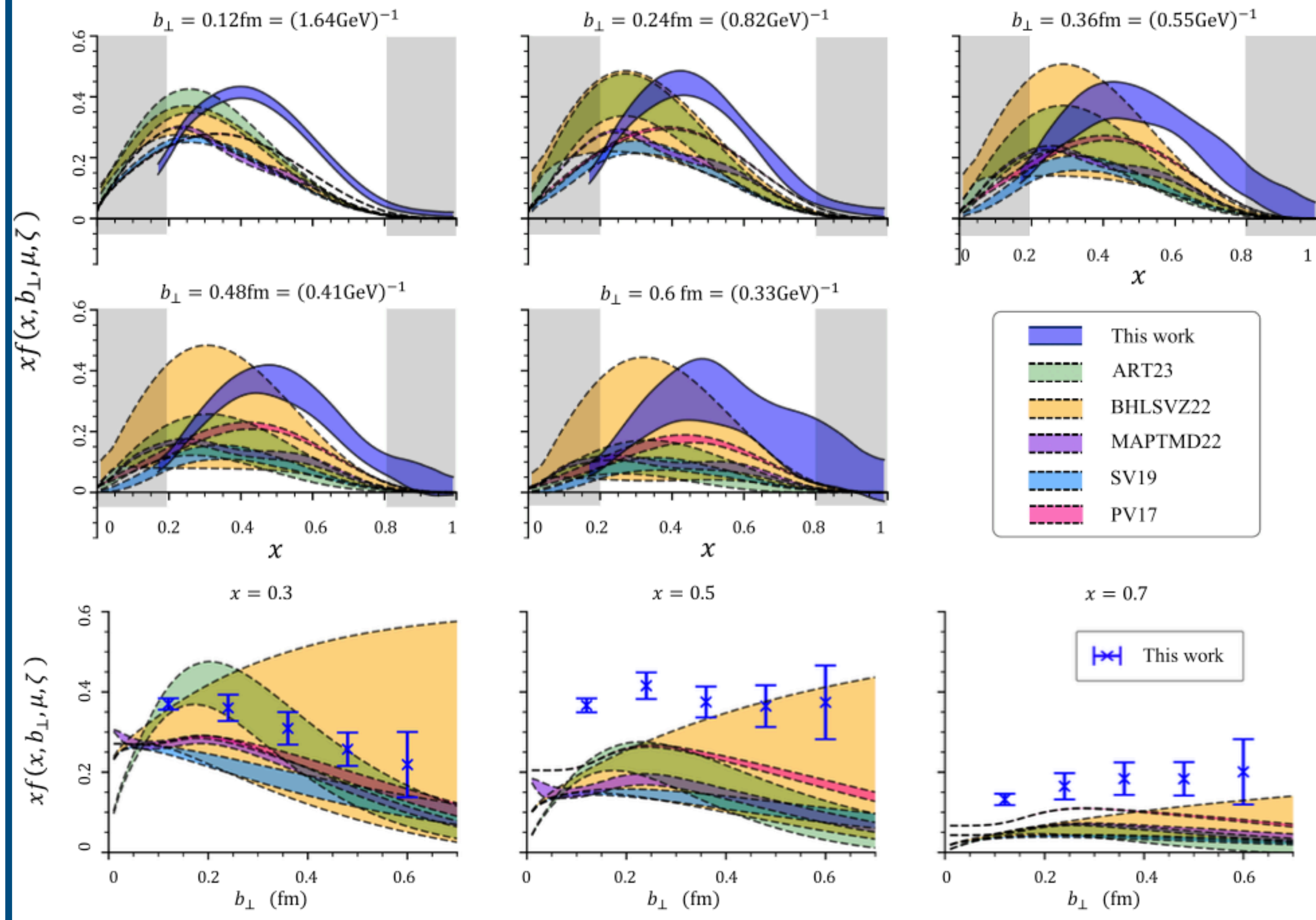




- non-negligible discrete effects
- control of systematic

J. He et al., Phys.Rev.D 109 (2024) 11, 114513

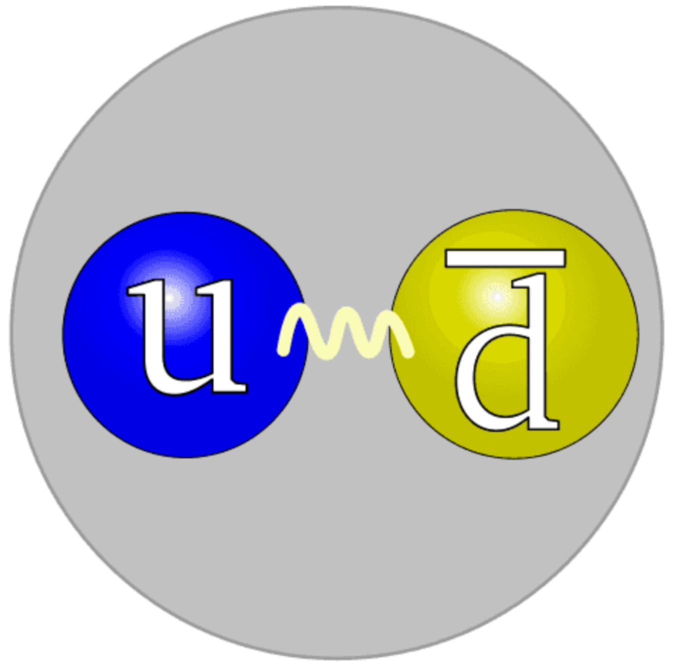
TMDPDF

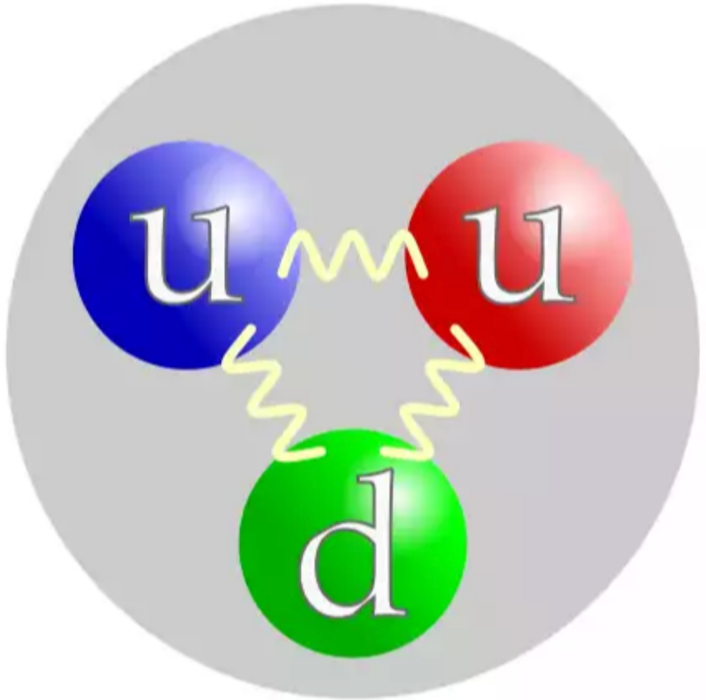


- inconsistency with pheno
- more certificates in different ensembles

Outline

- TMD physics
- Lattice QCD
- Large Momentum Effective Theory
- Framework and Results
- Summary





Summary

- GPDs and TMDs are crucial for understanding the structure of hadrons. LaMET based on lattice QCD provides first-principle approaches for these calculations.

- The GPD work presents calculations of unpolarized GPDs with non-zero skewness in asymmetric frame, highlighting the limitations of traditional lattice approaches. In the future ANN reconstruction based on unified structure of LaMET and SDE could be applied.

- The TMD works employed one-loop matching to derive results for Collins-Soper kernels and intrinsic soft function, facilitating preliminary analyses of physical TMDs.

Thank you!

Backups

Gluon on lattice

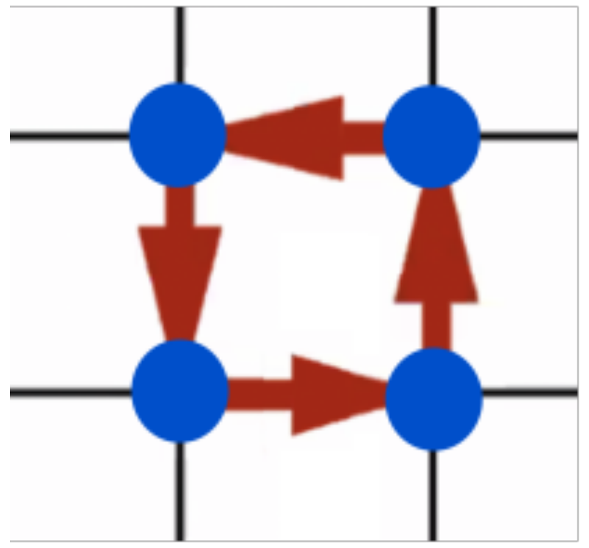
Gauge link/Wilson line

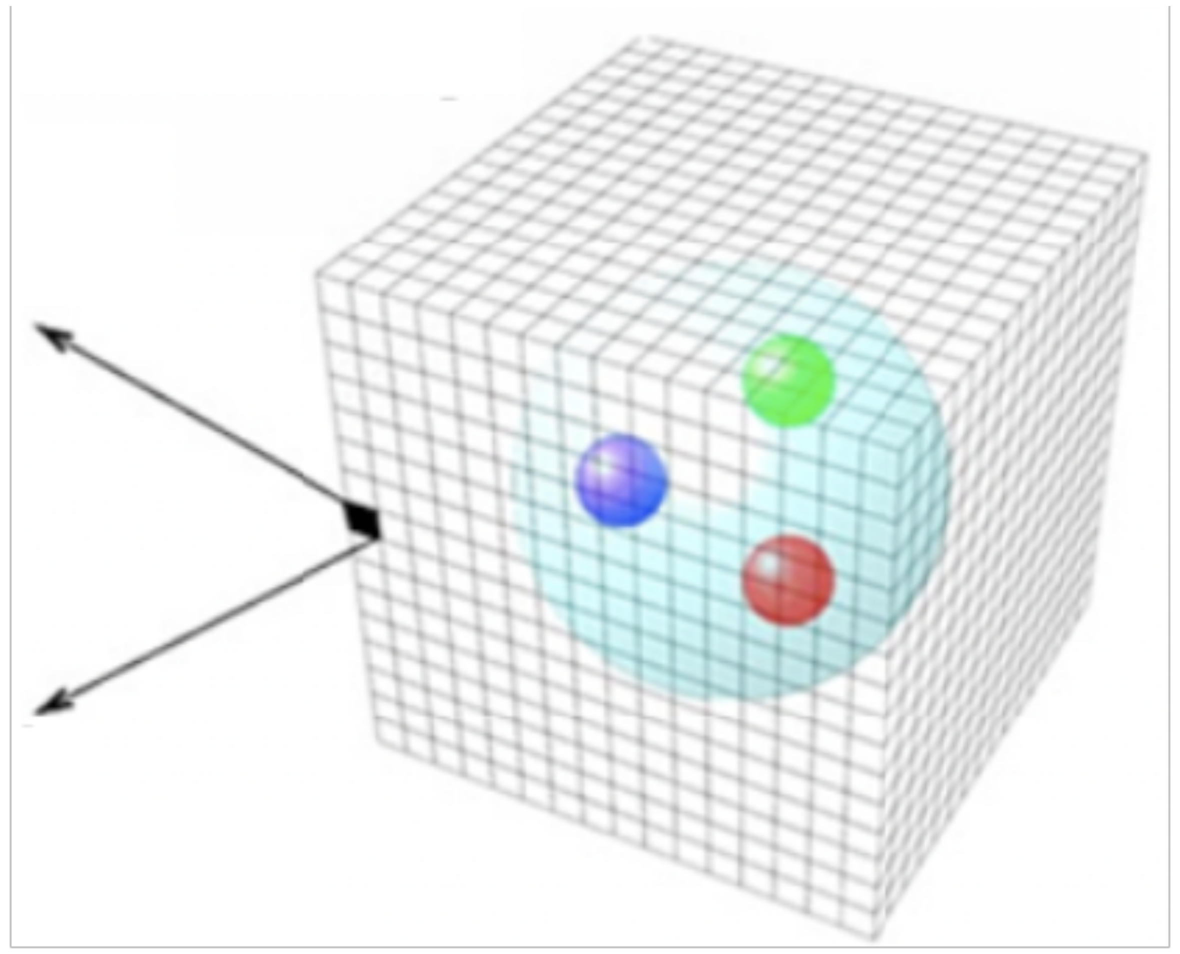
$$U_\mu(n) = \exp(iaA_\mu(n)) = 1 + iaA_\mu(n) + \mathcal{O}(a^2)$$

gluon action

$$S_G[U] = \frac{2}{g^2} \sum_{x\in\Lambda} \sum_{\mu<\nu} \mathrm{Re}[\mathrm{Tr}(1 - U_{\mu\nu}(x))]$$

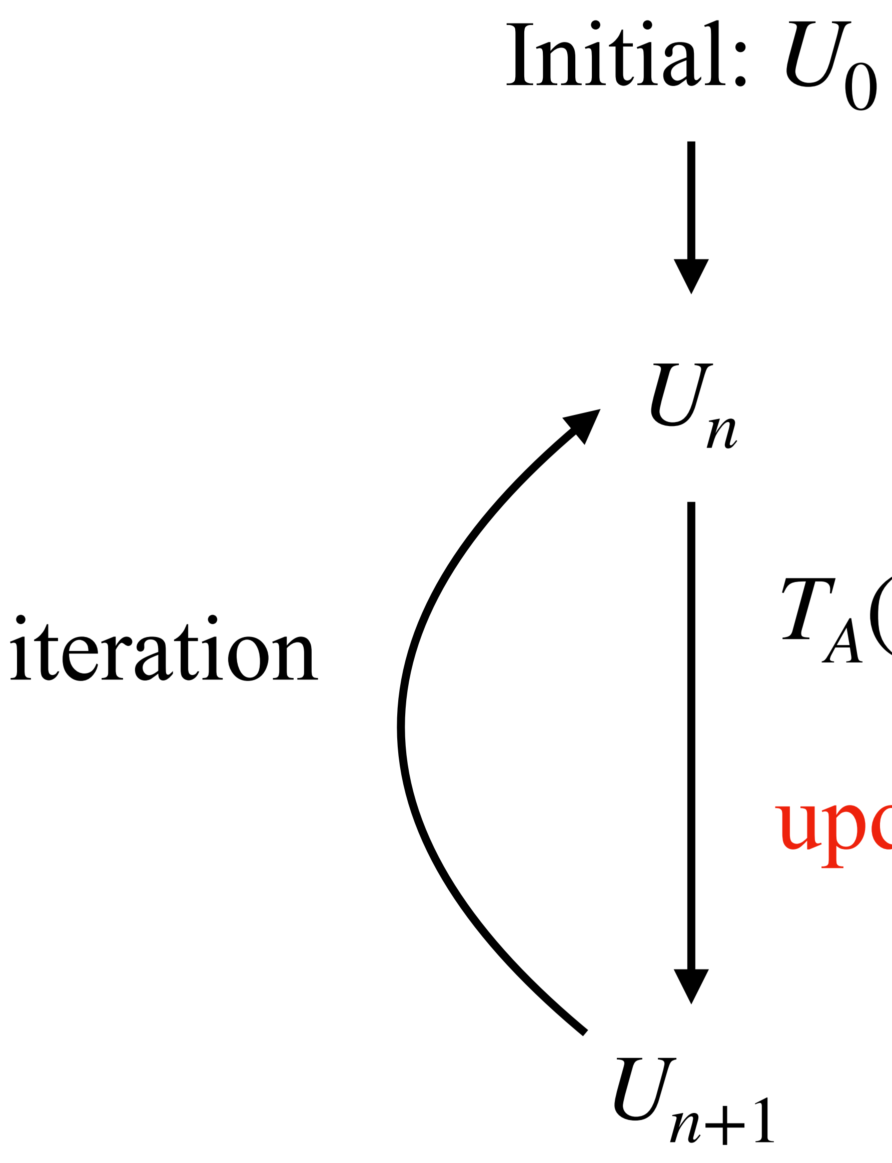
plaquette: 1×1 Wilson loop





Gluon on lattice

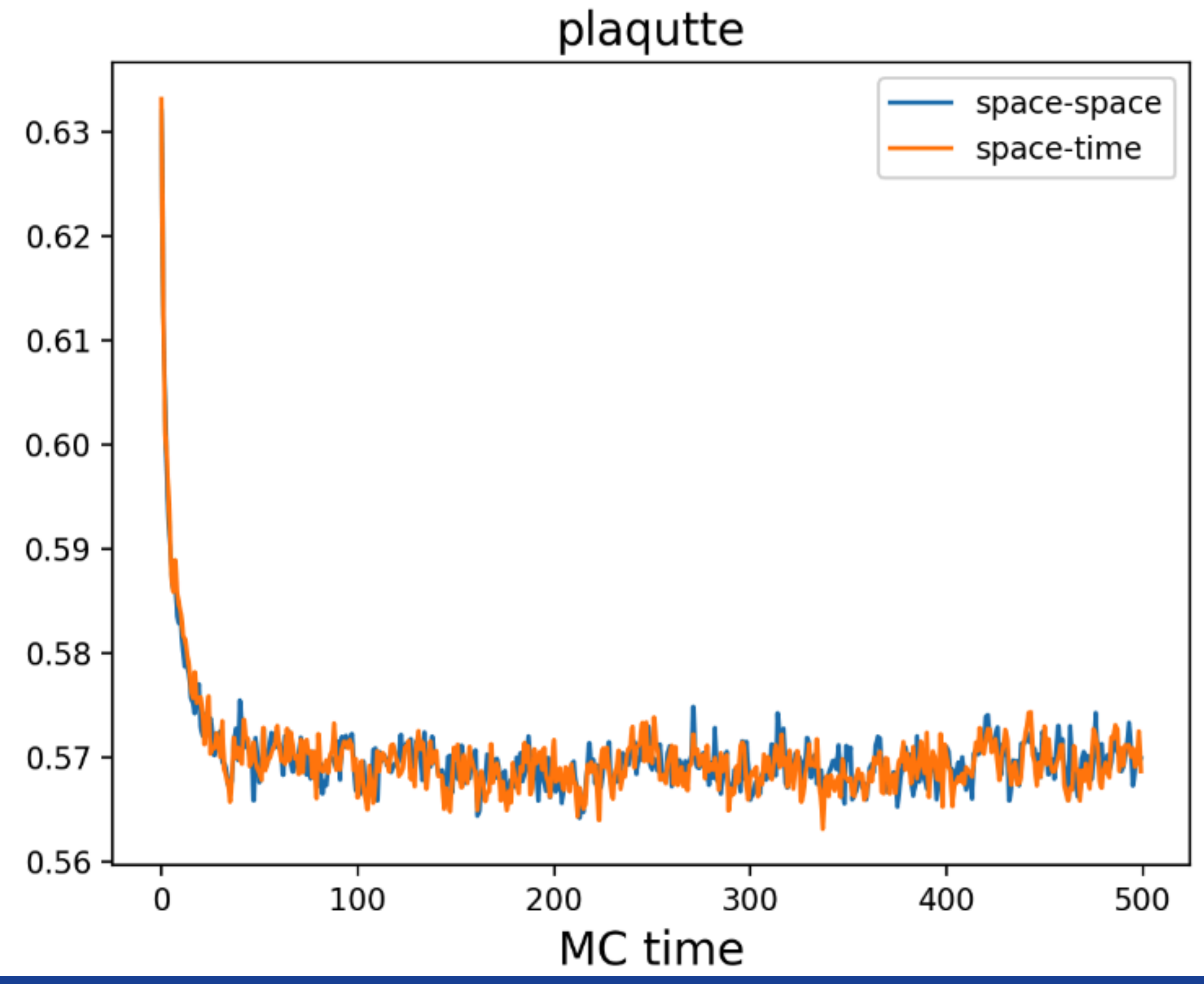
Configurations for pure gluon: Monte Carlo+Markov chain



$$T_A(U_{n+1}|U_n) = \min\left\{1, \frac{e^{-S[U_{n+1}]}}{e^{-S[U_n]}}\right\}$$

updating: correlation and efficiency

Observe the balance progress



Fermion on lattice

Fermion action

$$S_F[\psi, \bar{\psi}, U] = a^4 \sum_{m,n \in \Lambda} \sum_{a,b,\alpha,\beta} \bar{\psi}(n)_{\alpha,a} D(n\,|\,m)^{\alpha,\beta}_{a,b} \psi(m)_{\beta,b}$$

Grassmann number

Salam formula: $\int d\eta_N d\bar{\eta}_N \ldots d\eta_1 d\bar{\eta}_1 \exp\left(\sum_{i,j=1}^{N} \bar{\eta}_i M_{ij} \eta_j\right) = \det[M]$

$$Z[\psi, \bar{\psi}] = \int [d\bar{\psi}][d\psi] e^{-S_F}$$

$$= \int [d\bar{\psi}][d\psi] \exp\left[-a^4 \sum_{m,n,a,b,\alpha,\beta} \bar{\psi}(n)_{\alpha,a} D(n\,|\,m)^{\alpha,\beta}_{a,b} \psi(m)_{\beta,b}\right]$$

$$= \det[M]$$

Fermion in lattice

full QCD observables

$$\langle \hat{O}[U] \rangle_{F+G} = \frac{\int dU d\psi d\bar{\psi} \hat{O}[U] e^{-S_F} e^{-S_G}}{\int dU d\psi d\bar{\psi} e^{-S_F} e^{-S_G}} = \frac{\int dU \hat{O}[U] \det[M] e^{-S_G}}{\int dU \det[M] e^{-S_G}}$$

it breaks the form of partition function

Two choices

- probability density $e^{-S_G + \ln[\det[M]]}$ — widely used
- take $\det[M]$ as an observable

$$\langle \hat{O}[U] \rangle_{F+G} = \frac{\int dU \hat{O}[U] \det[M] e^{-S_G}}{\int dU e^{-S_G}} \times \frac{\int dU e^{-S_G}}{\int dU \det[M] e^{-S_G}}$$

Fermion on lattice

feature of Grassmann integration

$$\langle \eta_{i_1}\bar{\eta}_{j_1}\dots\eta_{i_n}\bar{\eta}_{j_n}\rangle_F = \frac{1}{Z_F}\int \Big(\prod_{i=1}^{N} d\eta_i d\bar{\eta}_i\Big)\eta_{i_1}\bar{\eta}_{j_1}\dots\eta_{i_n}\bar{\eta}_{j_n}\exp\left(\sum_{l,m=1}^{N}\bar{\eta}_l M_{lm}\eta_m\right)$$

$$= (-1)^n \sum_{P(1,2,\dots n)} sign(P)(M^{-1})_{i_1 j_{P1}}(M^{-1})_{i_2 j_{P2}}(M^{-1})_{i_1 j_{P1}}\dots(M^{-1})_{i_n j_{Pn}}$$

quark propagators

$$\langle \psi(n)^{\alpha}_{a}\bar{\psi}^{\beta}_{b}(m)\rangle_{F+G} = \frac{\langle a^{-4}D^{-1}(n\,|\,m)^{\alpha,\beta}_{a,b}\det[-a^4 D]\rangle_G}{\langle \det[-a^4 D]\rangle_G}$$

dimensions of matrix D: $N = n_s^3 \times n_t \times 4 \times 3$

determination of $\det[D]$: perturbative iteration/CG algorithm

Fermion in lattice

doubling problem in fermion

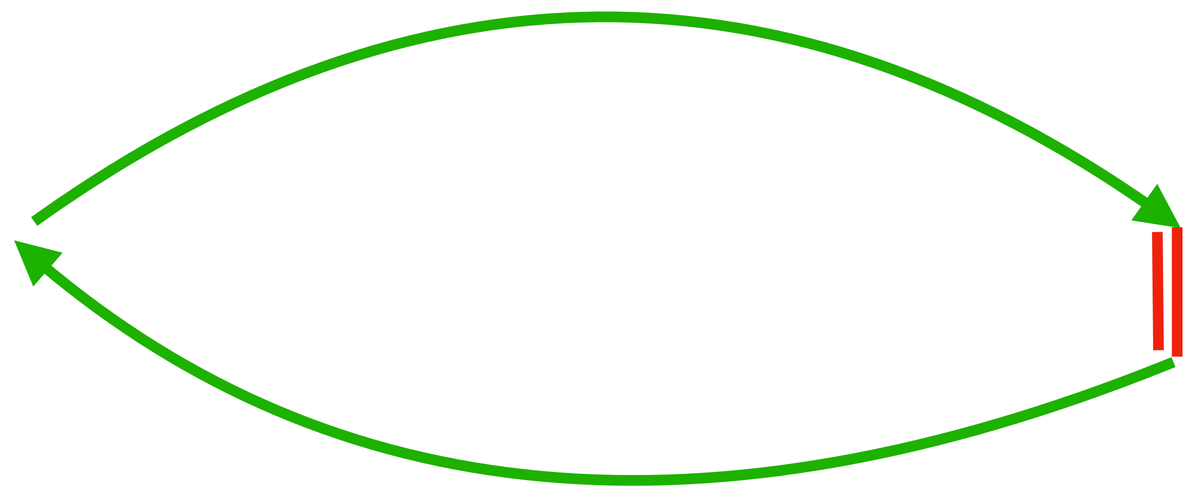
$$\tilde{D}^{-1}(p) = \frac{-ia^{-1}\sum_\mu \sin(p_\mu a)}{a^{-2}\sum_\mu \sin^2(p_\mu a)} \to (a \to 0) \to \frac{-i\sum_\mu \gamma_\mu p_\mu}{p^2}$$

fermion actions

Wilson, staggered, twist-mass, domain wall, ….

Gattringer and Lang's book: *Lect.Notes Phys.* 788 (2010) 1-343

Observables: hadron structure



reduction formula: from nonlocal 2pt to DA

$$
\begin{aligned}
C_2(z,t) &= \int d^3x e^{-i\vec{p}\vec{x}} \langle 0 | \hat{O}_H(\vec{x},t;z) \hat{O}_H^\dagger(0,0) | 0 \rangle \\
&= \int d^3x e^{-i\vec{p}\vec{x}} \langle 0 | \hat{O}_H(\vec{x},t;z) \sum_H \int \frac{d^3q}{(2\pi)^3 2E} | \pi(q) \rangle \langle \pi(q) | \hat{O}_H^\dagger(0,0) | 0 \rangle \\
&= \int \frac{d^3q_0}{(2\pi)^3 2E_0} \int d^3x e^{-i(\vec{p}-\vec{q}_0)\vec{x}} \langle 0 | \hat{O}_H(0,t;z) | \pi(q_0) \rangle \langle \pi(q_0) | \hat{O}_H^\dagger(0,0) | 0 \rangle \\
&= \langle 0 | \hat{O}_H(0,t;z) | \pi(p) \rangle \langle \pi(p) | \hat{O}_H^\dagger(0,0) | 0 \rangle \\
&= e^{-iE_0 t} \phi(z,P^z) \langle \pi(q_0) | \hat{O}_H^\dagger(0,0) | 0 \rangle
\end{aligned}
$$

insert the hadron state

Spatial Translation

integration

Pion distribution amplitude: $\phi(z,P^z) = \langle 0 | \bar{\psi}(z) \gamma^\mu \gamma_5 W(z,0) \psi(0) | \pi(P^z) \rangle$

Excited states: $\frac{C_2(z,t)}{C_2(0,t)} = \phi(z,P^z) \frac{1 + c_0 e^{-\Delta E t}}{1 + c_1 e^{-\Delta E t}}$