

Quantum process tomography at particle colliders

Michał Eckstein

Institute of Theoretical Physics, Jagiellonian University, Kraków, Poland



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in collaboration with

Clelia Altomonte (King's College, London), *Alan Barr* (Merton College, Oxford),
Paweł Horodecki (ICTQT, Gdańsk Univ.), *Kazuki Sakurai* (Warsaw Univ.)

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1 Motivation: *beyond* quantum theory

[M.E., P. Horodecki, *Proc. R. Soc. A.* **478**:20210806 (2022), arXiv:2103.12000]

- *Why* venturing beyond quantum theory?
- *What* might be out there?
- **How** to seek beyond-quantum effects?

2 Implementation: *quantum process tomography* in colliders

[C. Altomonte, A. Barr, M.E., P. Horodecki, K. Sakurai, arXiv:2412.01892]

- Quantum Field Theory and Standard Model *predictions*
- Procedure for an empirical *test*

3 Example: polarised $e^+e^- \rightarrow t\bar{t}$ process

4 Summary and prospects

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Standard Model \subset QFT = Quantum Mechanics + Special Relativity

Routes towards New Physics:

- 1 Beyond Standard Model, but still in QFT
 - SUSY, composite Higgs, dark sector, inflation, ...
- 2 Beyond Special Relativity, but assuming QM
 - QFT in curved spacetimes – 'semi-classical' (Unruh effect, ...)
 - quantum gravity (strings, loop etc.)
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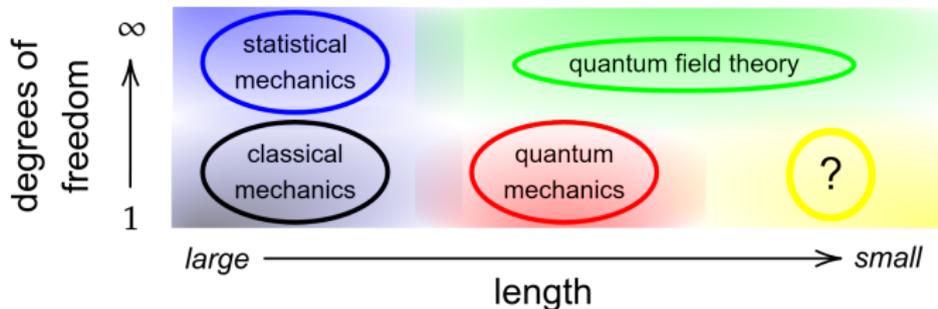
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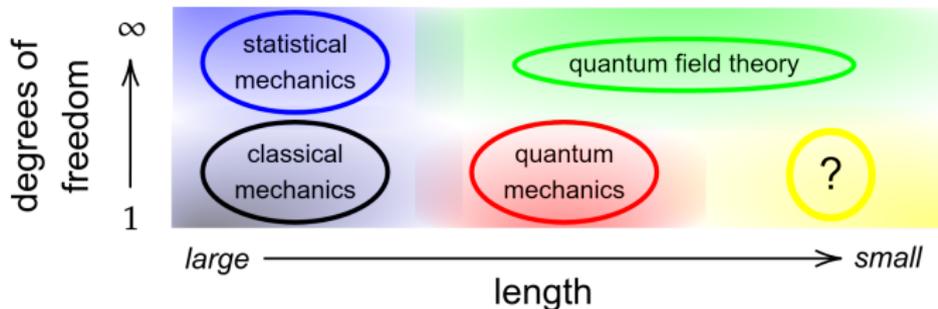
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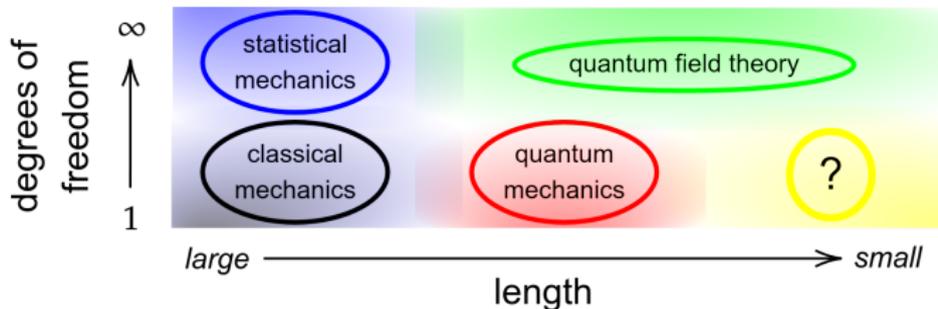
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- How to seek possible deviations from QM (*and* classicality)?

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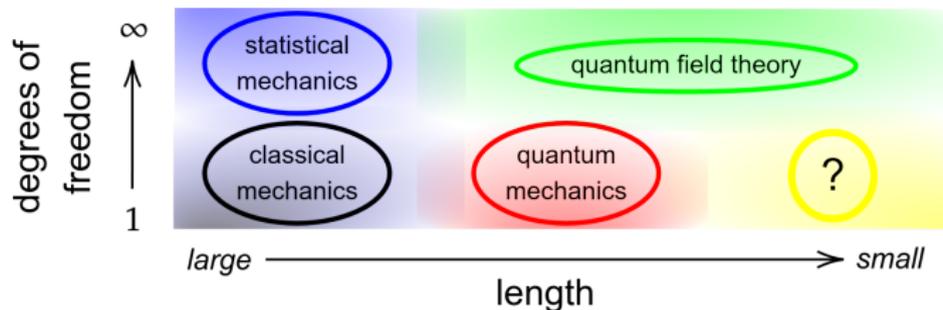
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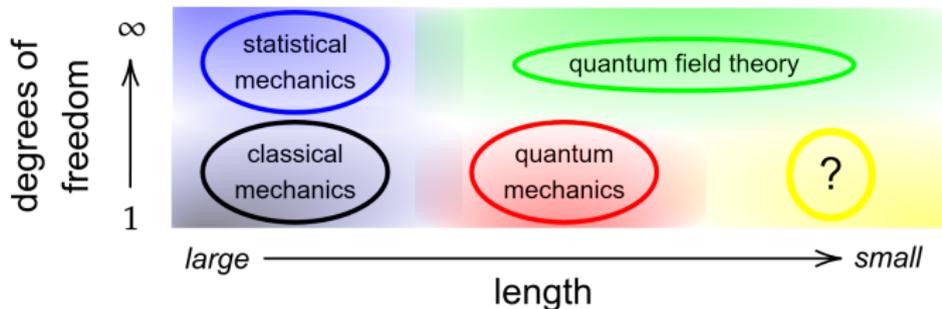
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The “theory independent” **black box** methodology

- Physical systems are treated as information-processing devices (“**black boxes**”), which can be probed by free agents.
- The conclusions are drawn from the **output–input correlations**.

$$P(\text{outputs} | \text{inputs})$$

Bell test: 2 agents (Alice and Bob) — 2 inputs (x, y) — 2 outputs (a, b)

The *experimental* (frequency) correlation function:

$$C_e(x, y) = P(a = b | x, y) - P(a \neq b | x, y)$$

The key assumption of *freedom of choice* (“measurement independence”):

$$P(x, y | \lambda) = P(x) \cdot P(y)$$

- No pre-correlations between the inputs (x, y) and the box (λ)

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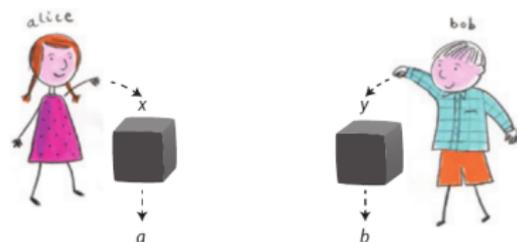
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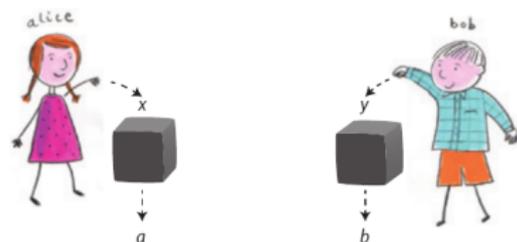
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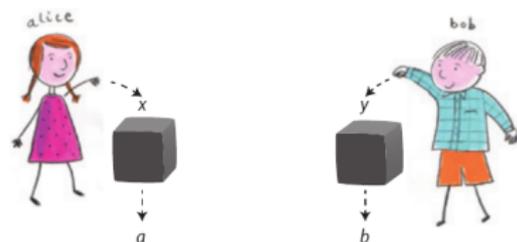
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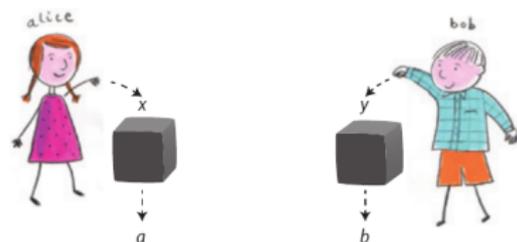
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Nonlocal correlations beyond quantum mechanics

Bell-CHSH inequality: 2 parties – 2 inputs – 2 outcomes

$$S := C_{\text{LHV}}(x, y) + C_{\text{LHV}}(x, y') + C_{\text{LHV}}(x', y) - C_{\text{LHV}}(x', y') \leq 2 < 2\sqrt{2}$$

No-signalling boxes [Popescu, Rohrlich (1994)]

$$P(a, b | x, y) = \begin{cases} \frac{1}{2}, & \text{if } a \oplus b = xy, \\ 0, & \text{otherwise,} \end{cases} \quad S_{\text{PR}} = 4.$$

- No-signalling principle admits correlations that are **stronger than entanglement**.

Violation of the Tsirelson bound would refute all (local) quantum models!

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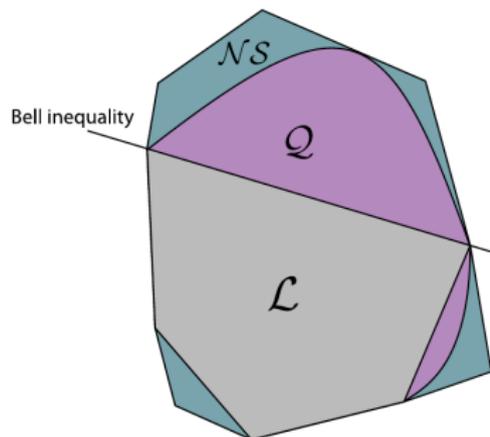
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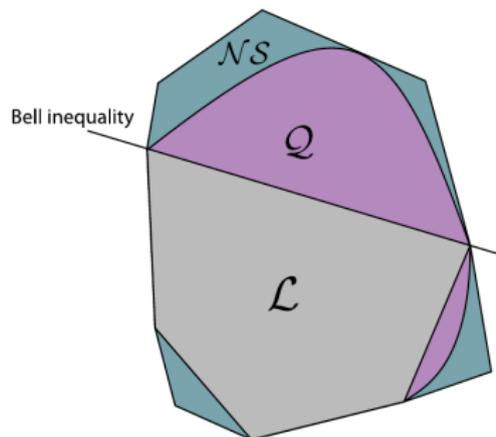
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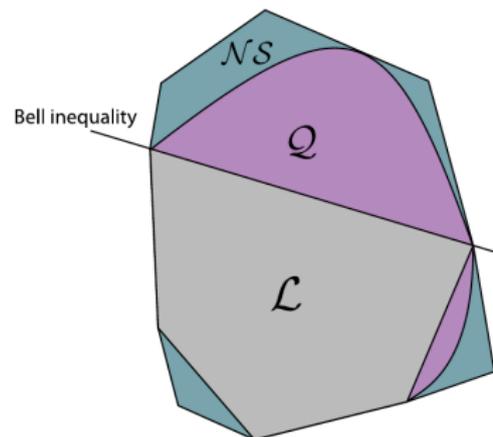
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Beyond linear quantum theory

Quantum mechanics is a **linear** theory.

- The Schrödinger equation is a *linear* PDE.
- The observables are *linear* operators and the states are *linear* functionals.

1 Wave function collapse models

[A. Bassi, K. Lochan, S. Satin, T.P. Singh, H. Ulbricht, *Rev. Mod. Phys.* **85**, 471 (2013)]

- Mostly aimed at explaining the 'quantum-to-classical' transition.
- Spontaneous collapse and mixing [K. Simonov, *PRA* **102** 022226 (2020)]

2 Nonlinear Schrödinger equation

- Schrödinger–Newton equation, aka the *Diosi – Penrose model*
- DeBroglie (1960), Białynicki-Birula–Mycielski (1976), Weinberg (1989), Polchinski (1991), Czachor (1998/2002), Rembieliński–Caban (2019-21)
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How to seek beyond-quantum effects?

1 Motivation: *beyond* quantum theory

[M.E., P. Horodecki, *Proc. R. Soc. A.* **478**:20210806 (2022), arXiv:2103.12000]

- *Why* venturing beyond quantum theory?
- *What* might be out there?
- **How** to seek beyond-quantum effects?

2 Implementation: *quantum process tomography* in colliders

[C. Altomonte, A. Barr, M.E., P. Horodecki, K. Sakurai, arXiv:2412.01892]

- Quantum Field Theory and Standard Model *predictions*
- Procedure for *experimental verification*

3 Example: polarised $e^+e^- \rightarrow t\bar{t}$ process

4 Summary and prospects

Quantum-data black box methodology

- We treat physical systems as **Q-data boxes**, i.e. *quantum-information* processing devices.
- A Q-data box is probed *locally* with quantum information.



- x are classical parameters (e.g. scattering kinematics)
- The *input state* is **prepared**.
- The *output state* is reconstructed from **quantum state tomography**.
- We assume that validity of QM *outside* the box, but not *inside* it.

Quantum-data black box methodology

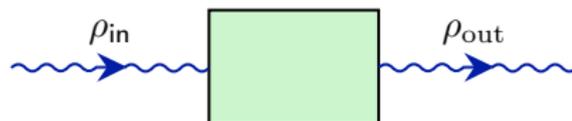
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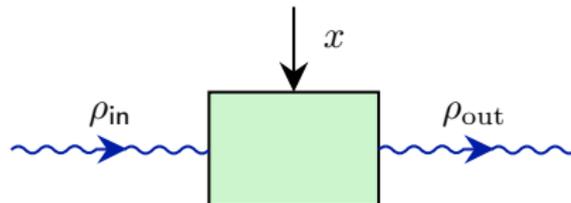
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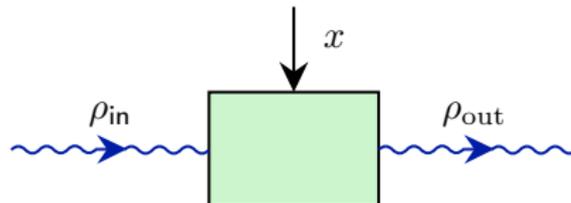
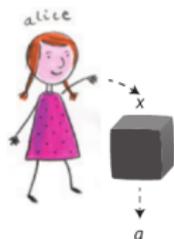
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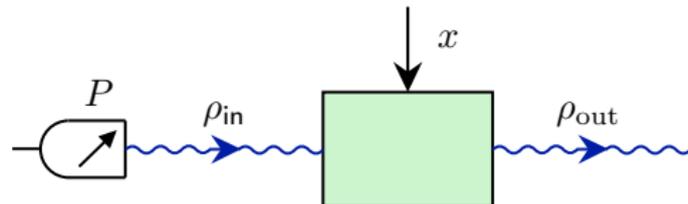
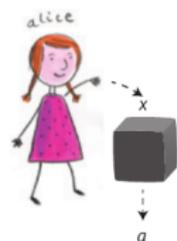


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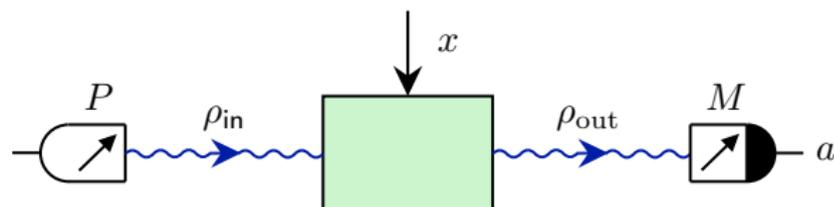
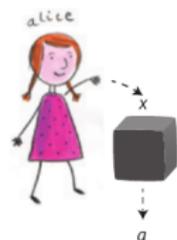


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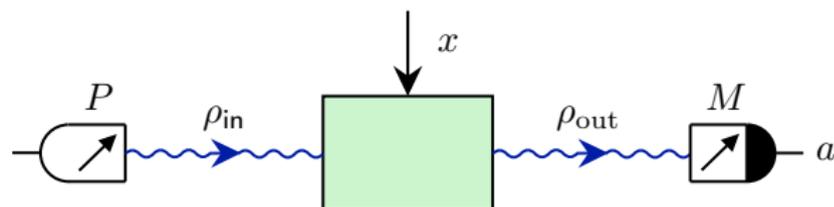
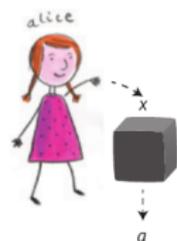


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Quantum state tomography:

- A (mixed) quantum state $\rho_{\text{out}} \in S(\mathcal{H})$ is an $n \times n$ matrix, $n = \dim \mathcal{H}$.
- Take a complete set of projectors $\{M_i\}_{i=1}^{n^2-1}$ (e.g. $\{\sigma_x, \sigma_y, \sigma_z\}$).
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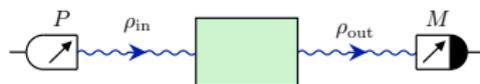
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$$\mathcal{E} : S(\mathcal{H}_{\text{in}}) \rightarrow S(\mathcal{H}_{\text{out}}), \quad \rho_{\text{in}} \mapsto \rho_{\text{out}} = \mathcal{E}(\rho_{\text{in}}), \quad \text{which is}$$

- linear, $\mathcal{E}(\sum_k \lambda_k \rho_{\text{in}}^k) = \sum_k \lambda_k \rho_{\text{out}}^k$;
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- A *selection* of the final states $\{\rho_{\text{out}}^{(k,x)}\}$ should yield a CP linear map

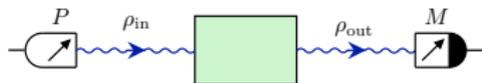
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Choi–Jamiołkowski isomorphism (aka “channel–state duality”)

A map \mathcal{I}_x is completely positive if and only if its *Choi matrix*

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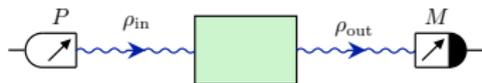
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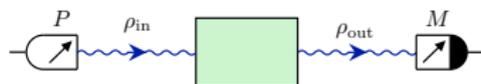
- linear**, $\mathcal{E}(\sum_k \lambda_k \rho_{\text{in}}^k) = \sum_k \lambda_k \rho_{\text{out}}^k$,
- completely positive**, i.e. $\mathcal{E} \otimes \mathbb{1}_N$ is positive for all N .
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Choi–Jamiołkowski isomorphism (aka “channel–state duality”)

A map \mathcal{I}_x is completely positive if and only if its *Choi matrix*

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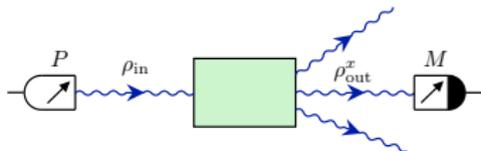
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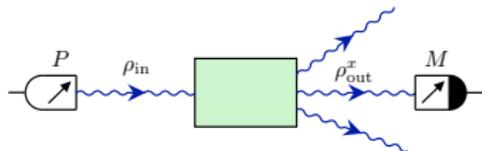
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Quantum process tomography as a foundational test

By performing the quantum tomography of a physical process we can reconstruct its Choi matrix and:

- compare it with theoretical predictions
↪ window for BSM physics
- reconstruct an *unknown* quantum dynamics
↪ low-energy QCD, gravity, ...
- study its properties
 - check its positivity (memory effects, non-Markovianity, ...)
 - some channels can 'simulate' beyond quantum correlations
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Quantum process tomography at colliders

1 Motivation: *beyond* quantum theory

[M.E., P. Horodecki, *Proc. R. Soc. A.* **478**:20210806 (2022), arXiv:2103.12000]

- *Why venturing beyond quantum theory?*
- *What might be out there?*
- *How to seek beyond-quantum effects?*

2 Implementation: *quantum process tomography* at colliders

[C. Altomonte, A. Barr, M.E., P. Horodecki, K. Sakurai, arXiv:2412.01892]

- Quantum Field Theory prediction
- Procedure for *experimental verification*

3 Example: polarised $e^+e^- \rightarrow t\bar{t}$ process

4 Summary and prospects

Quantum process tomography in particle colliders

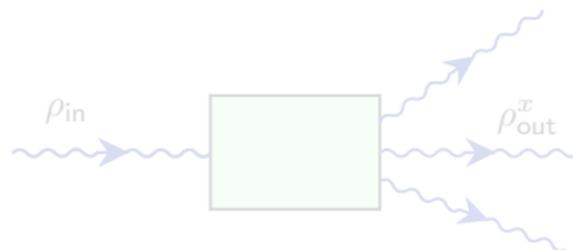
- We consider $2 \rightarrow 2$ scattering process: $\alpha\beta \rightarrow \gamma\delta$.
- We consider finite-dimensional Hilbert spaces

$$\mathcal{H}_{\text{in}} = \mathcal{H}_{\alpha} \otimes \mathcal{H}_{\beta},$$

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- 1 State preparation
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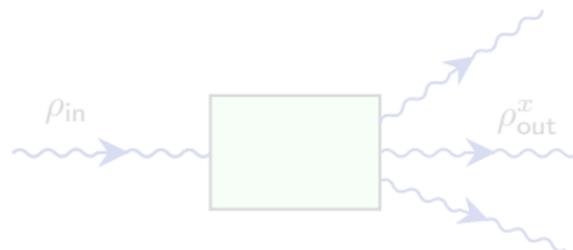
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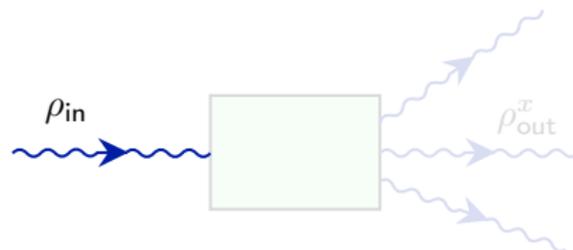
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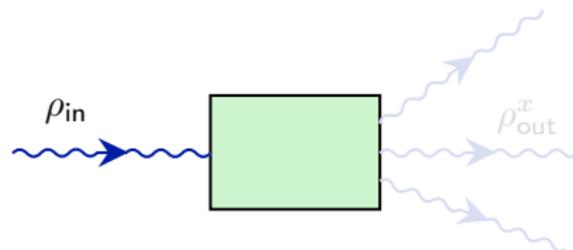
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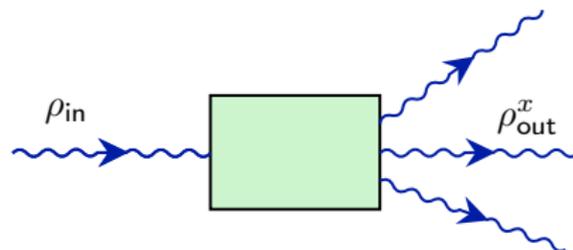
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Initial state preparation and evolution

- The two beams of particles are initially uncorrelated $\rho_{\text{in}} = \rho_{\text{in}}^{\alpha} \otimes \rho_{\text{in}}^{\beta}$.
- A beam can be polarised in a direction \mathbf{n} to a degree $q \cdot 100\%$

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- The projection operator, \mathcal{P}_x , implementing our selective measurement, is given by

$$\mathcal{P}_x = \sum_{A,B} \int_x d\Pi_{\gamma\delta} |p_f; A, B\rangle \langle p_f; A, B|,$$

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is the inclusive cross-section for unpolarised scattering.

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$$\mathcal{I}_x : \rho_{\text{in}} \mapsto \varrho_x,$$

for any choice x of final state momenta given by

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Quantum process tomography at colliders

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[M.E., P. Horodecki, *Proc. R. Soc. A.* **478**:20210806 (2022), arXiv:2103.12000]

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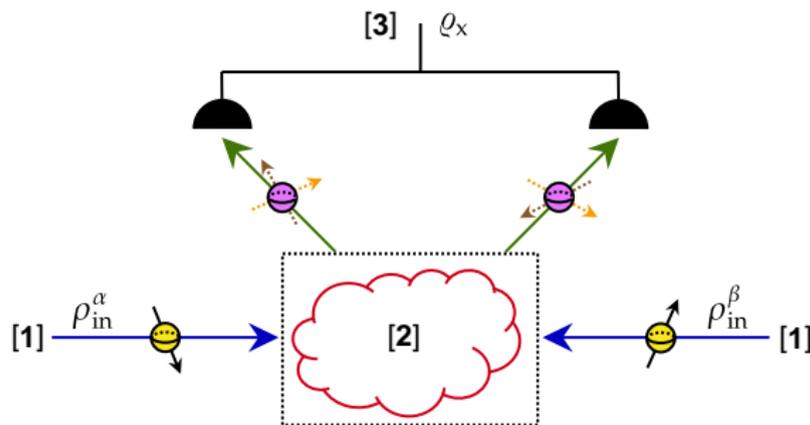
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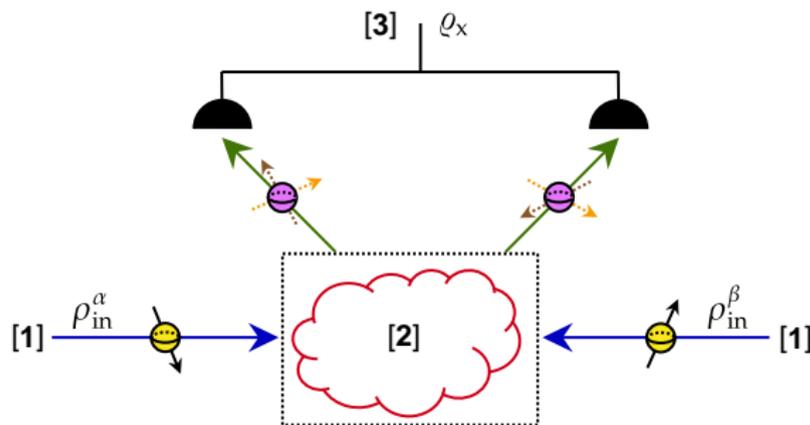
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Experimental Choi matrix reconstruction



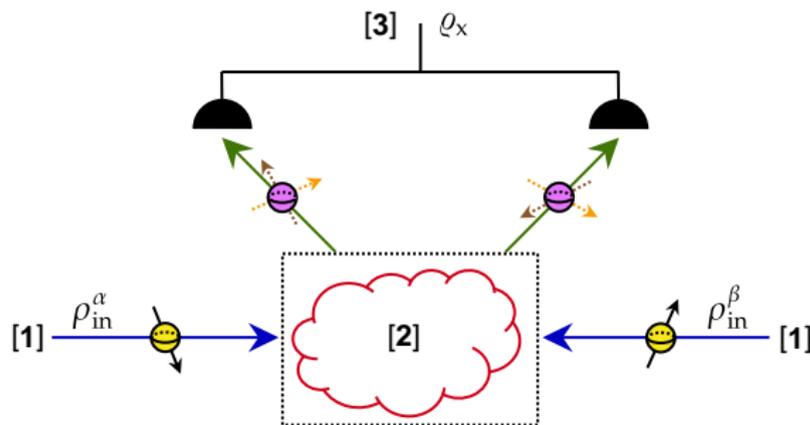
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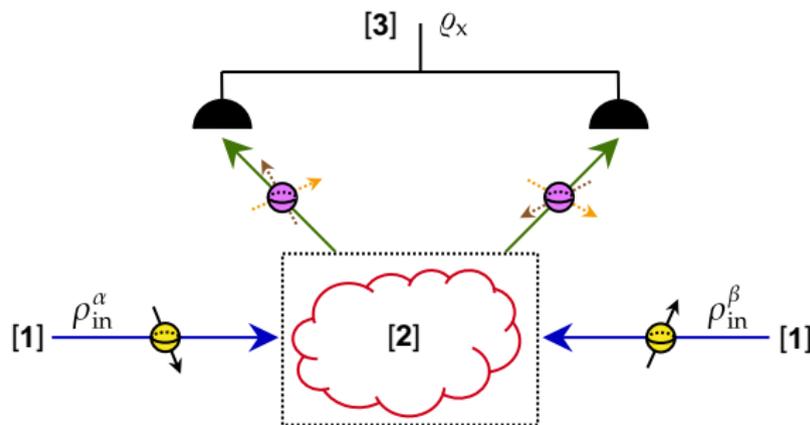
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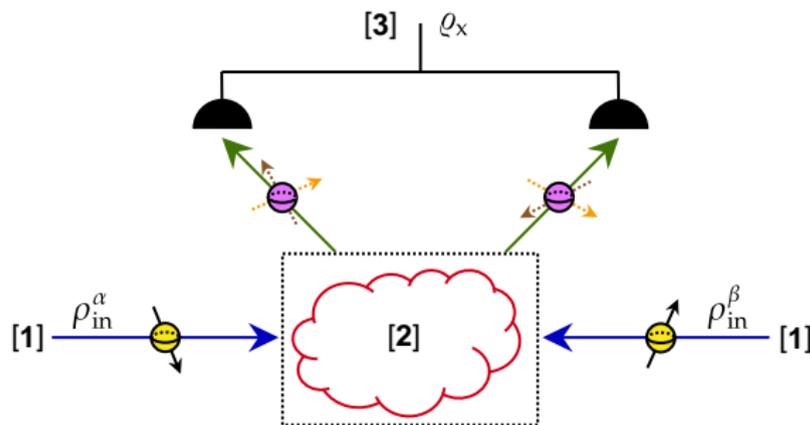
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Example: Polarised $e^-e^+ \rightarrow t\bar{t}$ scattering

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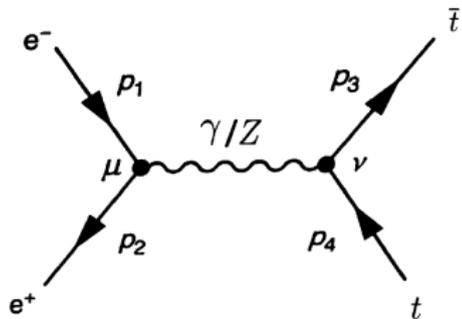
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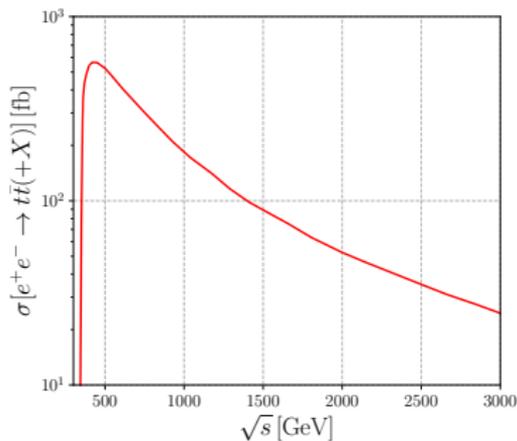
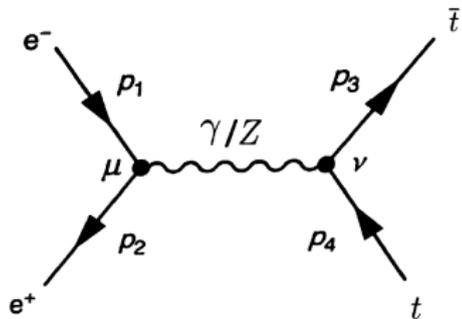
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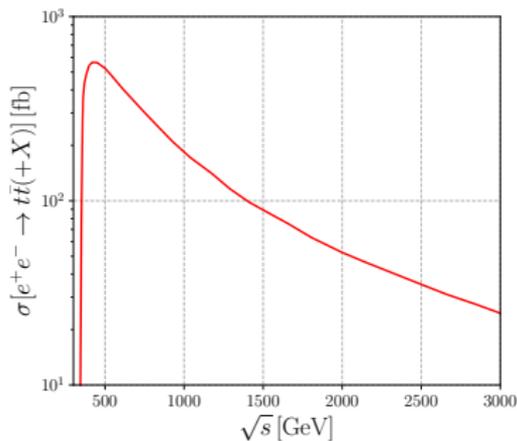
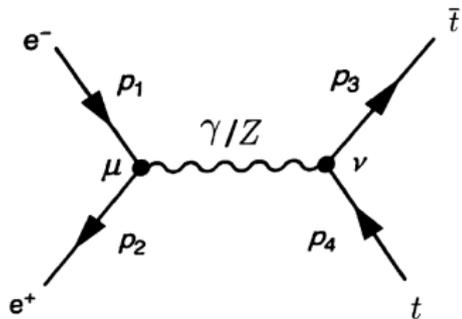
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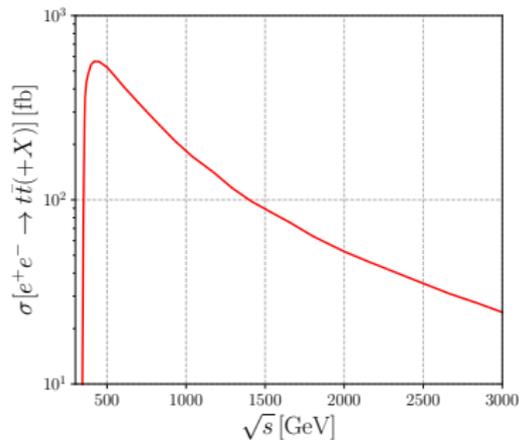
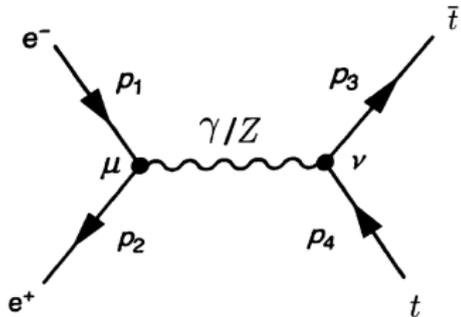
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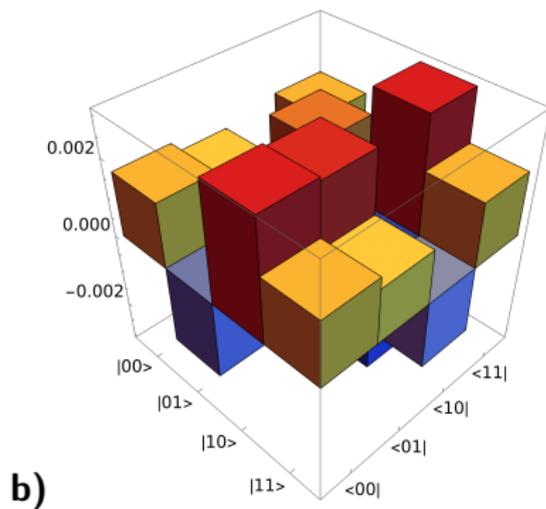
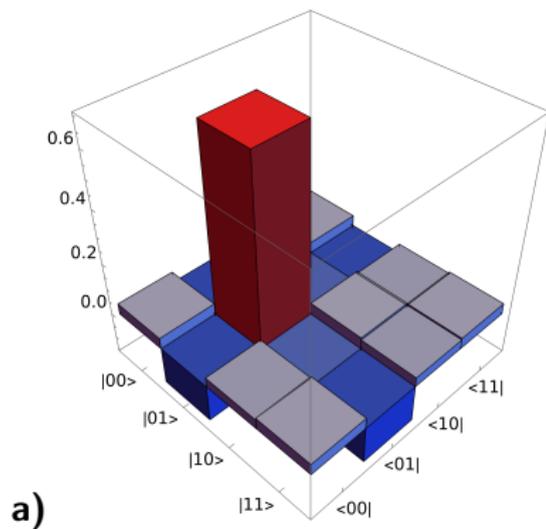
$$\mathcal{M}_{AB}^{+-} = \mathcal{M}_{AB}^{-+} = 0 \quad \Rightarrow \quad \tilde{\mathcal{I}}_x = \frac{1}{4} \begin{pmatrix} \mathcal{I}_x(|++\rangle\langle ++|) & 0 & 0 & \mathcal{I}_x(|++\rangle\langle --|) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \mathcal{I}_x(|--\rangle\langle ++|) & 0 & 0 & \mathcal{I}_x(|--\rangle\langle --|) \end{pmatrix}$$

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Example: Polarised $e^-e^+ \rightarrow t\bar{t}$ scattering



Block elements of the Choi matrix $\tilde{\mathcal{I}}_x$ for $e^-e^+ \rightarrow t\bar{t}$ evaluated at tree level in the Standard Model at center of mass energy $\sqrt{s} = 370$ GeV.

a) $\mathcal{I}_x(|++\rangle\langle++|)$ for $x = \{\theta \in [0, \pi], \phi \in [-\pi, \pi]\}$

b) $\mathcal{I}_x(|++\rangle\langle--|)$ for $x = \{\theta \in [2\pi/3, \pi], \phi \in [-\pi/4, \pi/4]\}$

Quantum process tomography at colliders

1 Motivation: *beyond* quantum theory

[M.E., P. Horodecki, *Proc. R. Soc. A.* **478**:20210806 (2022), arXiv:2103.12000]

- *Why* venturing beyond quantum theory?
- *What* might be out there?
- *How* to seek beyond-quantum effects?

2 Implementation: *quantum process tomography* at colliders

[C. Altomonte, A. Barr, M.E., P. Horodecki, K. Sakurai, arXiv:2412.01892]

- Quantum Field Theory prediction
- Procedure for experimental verification

3 Example: polarised $e^+e^- \rightarrow t\bar{t}$ process

4 Summary and prospects

Experimental prospects and opportunities

Search for **new physics** from quantum process tomography:

- Check the prediction of the Standard Model against BSM.
- Reconstruct quantum processes, which are *not* calculable perturbatively.
- Test the validity of quantum channel assumption (memory effects?)
- Foundational tests of QM: CP and linearity violations.

Experimental prospects:

- **Electron-Ion Collider:**
electron and proton beams with 70% polarisation and CME 20–140 GeV
- **International Linear Collider:**
 e^-e^+ collider with 80% e^- and 30% e^+ polarisation and CME 500 GeV
- **Future Circular Collider:**
 e^-e^+ collider with up to 10% polarisation at CME 45–80 GeV

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Take-home messages:

- Quantum mechanics is great, but we should never stop questioning it.
- We should remain open to 'beyond-quantum' physics, ...
- ... whatever it might be: *nonlinear?*, *supernonlocal?*, *beyond-spacetime?*
- It is possible to make foundational tests of QM in near-future colliders.
- A paradigm shift in collider physics — from observations to experiments.

Thank you for your attention!

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