The two photon transition form factor of η_c in the spacelike and timelike regions

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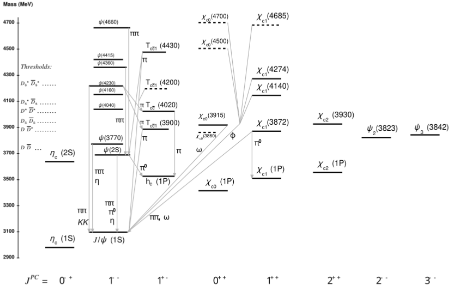
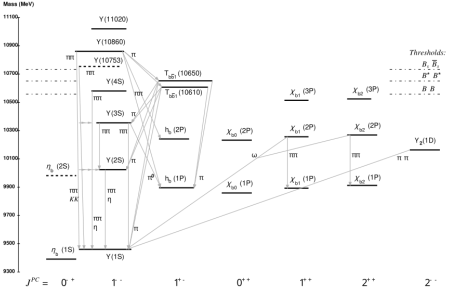
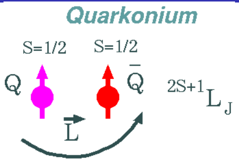
Introduction





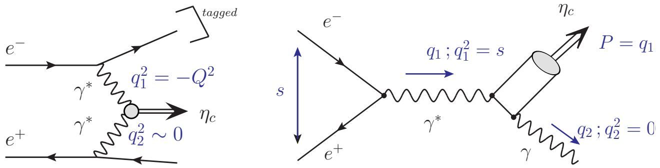


- Quarkonia $\equiv$ bound states of heavy quark and antiquark ($c\bar{c}$ or $b\bar{b}$).
- Spectra and many properties of states below open heavy flavor thresholds are well described by potential models of $Q\bar{Q}$ interactions.
- The ground state of the quarkonium system is a pseudoscalar $Q\bar{Q}$-state with vanishing orbital angular momentum (S–wave) in the spin–singlet.

$$\eta_Q : {}^1S_0, \quad J^{PC} = 0^{-+}$$

- The positive C-parity implies decay/coupling to two photons.

$\gamma^*\gamma^*$ transition form factor



- We are interested in the coupling of η_Q to two (off-shell) photons

$$\mathcal{M}(\gamma^*(q_1)\gamma^*(q_2) \to \eta_Q) = i\,4\pi\alpha_{\rm em}\,\varepsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu\epsilon_2^\nu q_1^\alpha q_2^\beta \underbrace{\mathcal{F}_{\eta_Q}(t_1,t_2)}_{\text{transition FF}} \qquad t_1 = \frac{q_1^2}{m_Q^2}, \quad t_2 = \frac{q_2^2}{m_Q^2}\,.$$

- the $\gamma^*\gamma^*$ transition form factor describes the following observables:

1. **on-shell**: $t_1 = t_2 = 0$: the decay width $\eta_Q \to \gamma\gamma$.
2. **space-like region**: $t_1 < 0, t_2 = 0$: exclusive production of η_Q in *single-tagged* e^+e^- collisions. $t_1 < 0, t_2 < 0 \to$ *double tagged* e^+e^- collisions.
3. **time–like region**: $t_1 > 0, t_2 = 0$: exclusive production of $\eta_Q\gamma$ in e^+e^- annihilation; Dalitz decay $\eta_Q \to \gamma\ell^+\ell^-$ or $\eta_Q \to 4\ell$.

Non-Relativistic Quantum Chromo-Dynamics (NRQCD)

- a good approximation to the potential between heavy quarks has a Coulomb and linear term (Cornell potential):

$$V(r) = -\frac{C_F \alpha_s}{r} + \sigma r$$

- From Ehrenfest's relation, we have for the relative velocity v of quarks in the bound state, $m_Q v^2 \sim V(r)$. For a Coulomb bound state using $r \sim 1/(m_Q v)$, we obtain $v \sim \alpha_s$. Empirically for $c\bar{c}$, $v^2 \sim \alpha_s \sim 0.3$.
- the **effective field theory** derived from QCD that allows for a systematic expansion in the small parameters v and α_s is NRQCD (Bodwin, Braaten & Lepage (1995)).
- The η_Q–state has an expansion, with a power counting in v motivated by a multipole expansion

$$\begin{aligned} |\eta_Q\rangle &= \mathcal{O}(v^0)\Big|[Q\bar{Q}](^1S_0^{[1]})\Big\rangle + \mathcal{O}(v)\Big|[Q\bar{Q}](^1P_1^{[8]})g\Big\rangle + \mathcal{O}(v^2)\Big|[Q\bar{Q}](^3S_1^{[8]})g\Big\rangle \\ &+ \mathcal{O}(v^2)\Big|[Q\bar{Q}](^1S_0^{[8]})gg\Big\rangle + \dots . \end{aligned}$$

  also operators made of quark field ψ and antiquark field χ have a definite power counting in v.
- short distance degrees of freedom $r < 1/m_Q$ are "integrated out" and described by perturbative QCD.

Relativistic corrections to the transition form factor

- The NRQCD factorization formula for the transition FF to relative order v^2 reads:

$$\mathcal{F}_{\eta_Q}(t_1,t_2) = \frac{C_0(t_1,t_2)}{m_Q^2}\Big\langle \eta_Q \Big| \psi^\dagger \chi \Big| 0 \Big\rangle + \frac{D_0(t_1,t_2)}{m_Q^4}\Big\langle \eta_Q \Big| \psi^\dagger \Big(-i\frac{\mathbf{D}}{2}\Big)^2 \chi \Big| 0 \Big\rangle + \dots .$$

- **long distance physics** is contained in the matrix elements (LDMEs):
  1. wave function at origin

$$\langle \eta_Q | \psi^\dagger \chi | 0 \rangle = \sqrt{2M_{\eta_Q}} \langle \eta_Q | \psi^\dagger \chi | 0 \rangle_{\rm BBL} = \sqrt{2M_{\eta_Q}} \sqrt{\frac{N_c}{2\pi}} R_{\eta_Q}(0) .$$

  2. effective expansion parameter:

$$\langle v^2 \rangle_{\eta_Q} = \frac{\Big\langle \eta_Q \Big| \psi^\dagger \Big(-i\frac{\mathbf{D}}{2}\Big)^2 \chi \Big| 0 \Big\rangle}{m_Q^2 \Big\langle \eta_Q \Big| \psi^\dagger \chi \Big| 0 \Big\rangle} .$$

- **short distance physics** is contained in the short-distance coefficients $C_0(t_1,t_2)$ and $D_0(t_1,t_2)$ which carry the dependence on photon virtualities t_1, t_2 and are calculable in perturbative QCD.
- as we assign the same smallness to v^2 and α_s, we should evaluate coefficient $C_0(t_1,t_2)$ to one–loop order in pQCD.

Short distance coefficients in the v^2 expansion

- To extract short distance coefficients C_0, D_0, one replaces the η_Q by a plane-wave $Q\bar{Q}$ state coupled to ${}^1S_0^{[1]}$ quantum numbers.
- calculate $\gamma^*\gamma^* \to [Q\bar{Q}]({}^1S_0^{[1]})$ in perturbative QCD
  1. We write the quark and antiquark momenta as

$$p_Q = \frac{1}{2}P + k\,, \quad p_{\bar{Q}} = \frac{1}{2}P - k\,, \quad Q\bar{Q} \text{ rest frame}: P = (2E, \vec{0})\,, \quad k = (0, \vec{k})\,, \quad E = \sqrt{m_Q^2 + \vec{k}^2}\,.$$

     and introduce

$$\tau = -\frac{t_1 + t_2}{4}\,, \quad \omega = \frac{t_1 - t_2}{t_1 + t_2}\,.$$

  2. the pQCD form factor has the same short distance coefficients as in the NRQCD expansion:

$$\mathcal{F}_{{}^1S_0}(t_1, t_2) = \frac{C_0(t_1, t_2)}{m_Q^2}\Big\langle [Q\bar{Q}]({}^1S_0^{[1]})\Big|\psi^\dagger\chi\Big|0\Big\rangle + \frac{D_0(t_1, t_2)}{m_Q^4}\Big\langle [Q\bar{Q}]({}^1S_0^{[1]})\Big|\psi^\dagger\Big(-i\frac{\mathbf{D}}{2}\Big)^2\chi\Big|0\Big\rangle + \dots$$

  3. expanding in $\vec{k}^2/m_Q^2$, we obtain:

$$C_0(t_1, t_2) = \frac{e_f^2}{2m_Q}\frac{1}{1+\tau}\,, \quad D_0(t_1, t_2) = \frac{\omega^2\tau^2 - 3\tau^2 - 7\tau - 5}{3(1+\tau)^2}\,C_0(t_1, t_2)\,.$$

Matching to NRQCD, transition FF of η_Q

$$\tau = -\frac{t_1 + t_2}{4}\,, \quad \omega = \frac{t_1 - t_2}{t_1 + t_2}\,.$$

LO TFF of η_Q

$$\mathcal{F}_0(t_1, t_2) = \frac{e_f^2}{2m_Q^3}\frac{1}{1+\tau}\left\langle \eta_Q \middle| \psi^\dagger \chi \middle| 0 \right\rangle .$$

η_Q TFF up to order v^2

$$\begin{aligned}
\mathcal{F}_{\eta_Q}(t_1, t_2) &= \frac{1}{m_Q^2}\left\langle \eta_Q \middle| \psi^\dagger \chi \middle| 0 \right\rangle \left(C_0(t_1, t_2) + D_0(t_1, t_2)\langle v^2 \rangle_{\eta_Q} + \dots \right) \\
&= \mathcal{F}_0(t_1, t_2)\left(1 + \frac{\omega^2\tau^2 - 3\tau^2 - 7\tau - 5}{3(1+\tau)^2}\langle v^2 \rangle_{\eta_Q} + \dots \right)
\end{aligned}$$

- for $t_1 = t_2 = 0$ we can calculate the $\eta_Q \to \gamma\gamma$ decay width. We checked that up to order v^4 we agree with the result of Bodwin & Petrelli (2009).
- the value of $\langle v^2 \rangle_{\eta_Q}$ must be determined by data or potential models.

All–order summation of v^2 corrections

- phenomenologically, in the charm sector v^2 corrections turn out to be large ($\sim 30 \div 40\%$ for the $\eta_c \to \gamma\gamma$ width). As suggested by Bodwin et al. (2008), one can sum up v^2–corrections that stem from a certain class of operators

$$\mathcal{F}_{\eta_Q}(t_1, t_2) = \sum_n c_n(t_1, t_2) \Big\langle \eta_Q \Big| \psi^\dagger \Big(-i \frac{\mathbf{D}}{2} \Big)^{2n} \chi \Big| 0 \Big\rangle = \Big\langle \eta_Q \Big| \psi^\dagger \chi \Big| 0 \Big\rangle \sum_n c_n(t_1, t_2) \, \langle \vec{k}^{2n} \rangle_{\eta_Q} \, .$$

- We exploit the fact, that we can obtain the pQCD transition–FF $\mathcal{F}_{^1S_0}(t_1, t_2)$ for $\gamma^*\gamma^* \to [Q\bar{Q}](^1S_0^{[1]})$ to all orders in $\vec{k}^2/m_Q^2$, so that we determine:

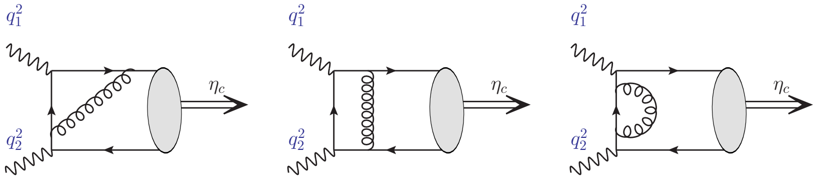
$$c_n(t_1, t_2) = \frac{1}{2\sqrt{2N_c}} \frac{1}{n!} \Big(\frac{\partial^n}{\partial \vec{k}^{2n}} \frac{\mathcal{F}_{^1S_0}(t_1, t_2)}{E(\vec{k}^2)} \Big) \Big|_{\vec{k}^2=0} \, .$$

- following Bodwin et al., we assume $\langle \vec{k}^{2n} \rangle_{\eta_Q} = \langle \vec{k}^2 \rangle^n_{\eta_Q} = (m_Q^2 \langle v^2 \rangle_{\eta_Q})^n$, so that we can sum up the series to

all-order v^2 TFF

$$\mathcal{F}_{\eta_Q}(t_1, t_2) \quad = \quad \frac{\langle \eta_Q | \psi^\dagger \chi | 0 \rangle}{2\sqrt{2N_c}} \, \frac{\mathcal{F}_{^1S_0}(t_1, t_2, \langle \vec{k}^2 \rangle_{\eta_Q})}{E(\langle \vec{k}^2 \rangle_{\eta_Q})} \, , \quad \langle \vec{k}^2 \rangle_{\eta_Q} = m_Q^2 \langle v^2 \rangle_{\eta_Q}$$

Perturbative α_s corrections



We can express the **perturbative correction** to the form-factor as follows

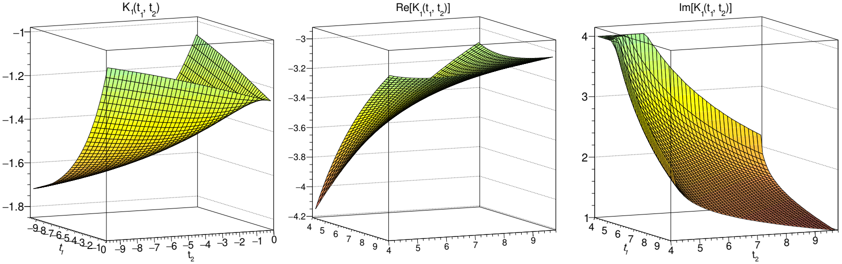
$$\mathcal{F}(t_1, t_2) = \mathcal{F}_0(t_1, t_2)\left(1 + \frac{\alpha_s}{\pi} C_F K_1(t_1, t_2)\right) + \mathcal{O}\left(\alpha_s^2\right), \quad C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}.$$

- Utilizing packages `FeynArts`, `FeynCalc`, `Apart`, `FIRE` we obtain a compact expression for $K_1(t_1, t_2)$ in terms of multiple polylogarithms

$$G(a_1, ..., a_n; x) = \int_0^x \frac{dt}{t - a_1} G(a_2, ..., a_n; t).$$

- We reproduce the well known on-shell-limit $K_1(0, 0) = \frac{\pi^2}{8} - \frac{5}{2}$, as well as $K_1(t, 0)$ in the space-like (Sang & Chen, 2010) and time–like (Feng et al. (2015)) regions.
- N.b.: on-shell (Abreu et al. (2023)) and for one virtual photon (Feng et al. (2010), numerically), the TFF is known to two–loops.

perturbative correction $K_1(t_1, t_2)$



- the perturbative correction as a function of *two virtualities* is obtained for the first time.
- For **spacelike** virtualities K_1 is a real function. It is negative in the whole domain.
- In the **timelike** region, for $t_i > 4$, we obtain also a nonvanishing imaginary part. It is a completely new result.
- As the Born result $\mathcal{F}_0(t_1, t_2)$ is *purely real*, both in spacelike and timelike domain, the imaginary part enters formally only at α_s^2.
- For charmonium, $\alpha_s \sim 0.3$, so that function K_1 enters with prefactor $\alpha_s C_F/\pi \sim 0.127$

Decay width for $\eta_c \to \gamma\gamma$

Table: Transition form factor at the on-shell point $F_{\eta_c}(0,0)$ and radiative decay width for two sets of parameters: (1) $|\mathrm{R}(0)|^2 = 0.9089$ GeV^3 and $< v^2 >= 0.226$, $m_c = 1.4$ GeV, (2) $|\mathrm{R}(0)|^2 = 0.881$ GeV^3 and $< v^2 >= 0.3$, $m_c = 1.5$ GeV . We used $\alpha_s = 0.3$ everywhere.

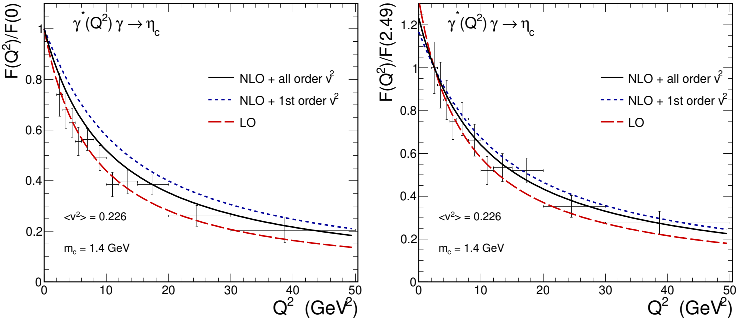
	F(0,0) [GeV^{-1}]	$\Gamma_{\eta_c \to \gamma\gamma}$ [keV]	F(0,0) [GeV^{-1}]	$\Gamma_{\eta_c \to \gamma\gamma}$ [keV]
LO	0.13	18.9	0.10	12.09
LO + 1st corr. v^2	0.081	7.33	0.052	3.02
LO + all order v^2	0.093	9.55	0.067	5.03
NLO	0.106	12.4	0.087	8.51
NLO + 1st corr. v^2	0.056	3.58	0.035	1.39
NLO + all order v^2	0.068	5.17	0.05	2.83
PDG		5.1 ± 0.4		5.1 ± 0.4

- two-photon decay width obtained from FF at on-shell point

$$\Gamma_{\eta_c \to \gamma\gamma} = \frac{\pi}{4} \alpha_{\mathrm{em}}^2 M_{\eta_c}^3 |\mathcal{F}_{\eta_c}(0,0)|^2$$

- large $\mathcal{O}(v^2)$ corrections are somewhat mitigated by all-order v^2 summation.

$\mathcal{F}_{\eta_c}(t,0) \equiv F(Q^2,0)$ from single-tagged e^+e^- collisions



- The BaBar collaboration published data for the ratio $F(Q^2,0)/F(0,0)$, thus normalizing to the rate for η_c production in *untagged* collisions.
- 1st order NLO and v^2 corrections worsen the description of data! Some mitigation through all-order v^2 resummation.
- We can get rid of the influence of untagged data, by normalizing data wrt. to the value in the first measured bin. Agreement with data is restored.

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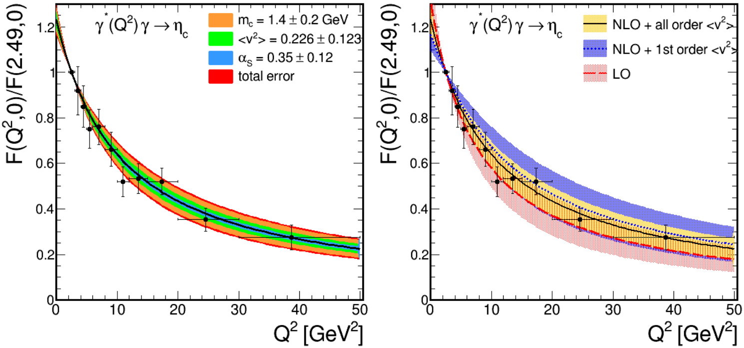

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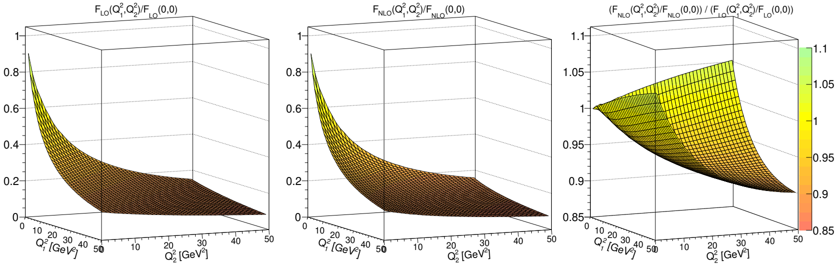
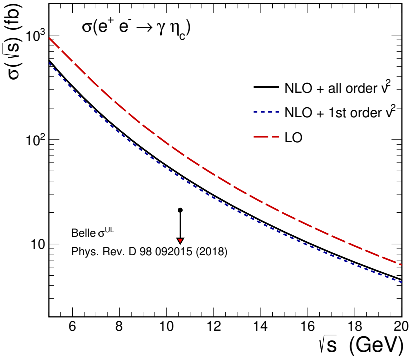
NLO vs LO in the spacelike region for η_c



- Combined v^2 and α_s corrections for the two-dimensional FF are presented for the first time.
- Predictions can be checked in future double–tagged e^+e^- experiments.

Cross section for exclusive $e^+e^- \to \eta_c\gamma$



$$\sigma = \frac{2\pi^2\alpha_{em}^3}{3}|\mathcal{F}(t,0)|^2\left(1-\frac{M^2}{s}\right)^3, \quad t = \frac{s}{m_Q^2}.$$

- at high enough $\sqrt{s}$, outside the resonance region, photons in the final state have large momentum and are emitted in the perturbative phase of the reaction $\longrightarrow$ calculable in terms of our transition FF.
- Measurements up to now exist only in the region of $\psi(4040), \psi(4415)$ resonances. At high energies $\sqrt{s} \sim 10$ GeV , the Belle collaboration cites an upper limit for the cross section of about 20 fb.

Summary

- We have studied the $\gamma^*\gamma^*$ transition form factor of η_Q in NRQCD factorization including perturbative α_s corrections as well as relativistic corrections in the v^2 expansion.
- We include for the first time NLO α_s and all-order v^2 corrections.
- Our results are in agreement with those in the literature in the appropriate limits.
- In contrast to the literature, our result also retains the imaginary part in the timelike region.
- Comparison with single-tag e^+e^- BaBar data for one virtual photon points to a problem with the normalization wrt. untagged data. After normalizing to the lowest bin of single-tagged data, we restore agreement of theory with data. The inclusion of all-order v^2 corrections has a major impact on the decay width.
- In the studied energy range, NLO and v^2–corrections to the exclusive cross section for $e^+e^- \to \gamma\eta_c$ are negative and move theory prediction closer to Belle upper limit.