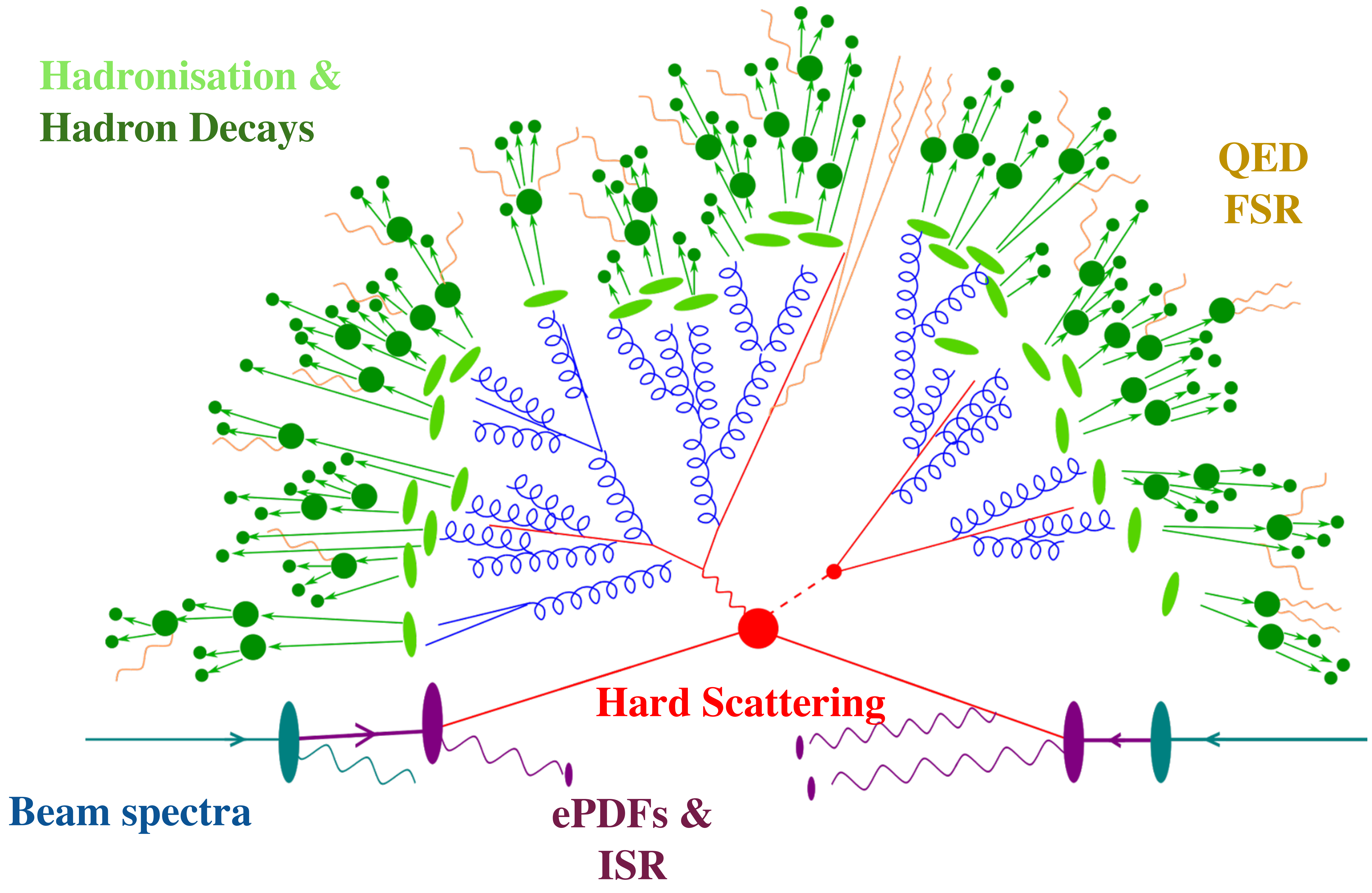
JAGIELLONIAN UNIVERSITY
IN KRAKÓW

YFS Generators for Low Energy e^+e^- Colliders

Alan Price

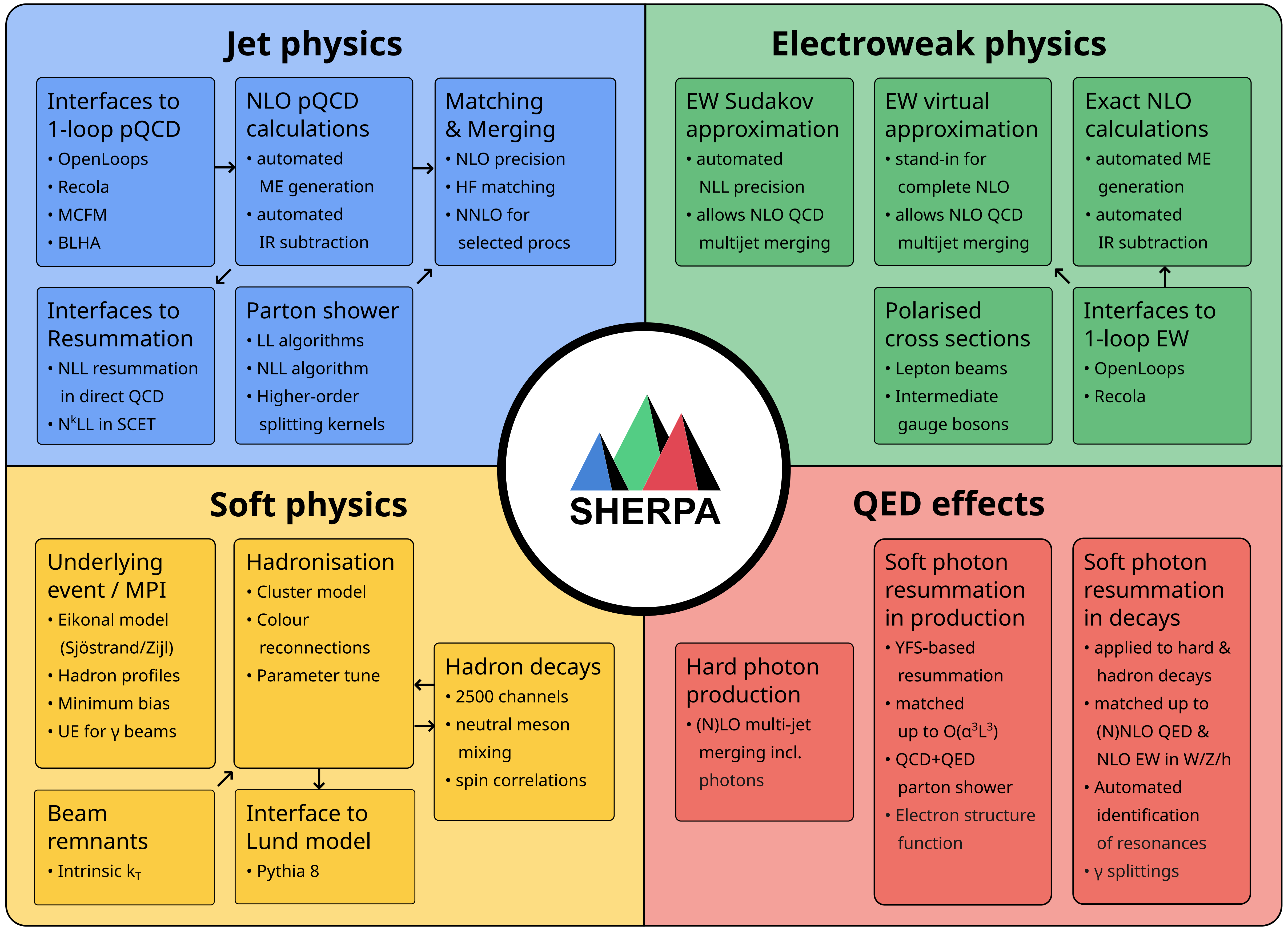
Disclaimer

Monte Carlo Events



SHERPA Framework

2410.22148   https://sherpa-team.gitlab.io/

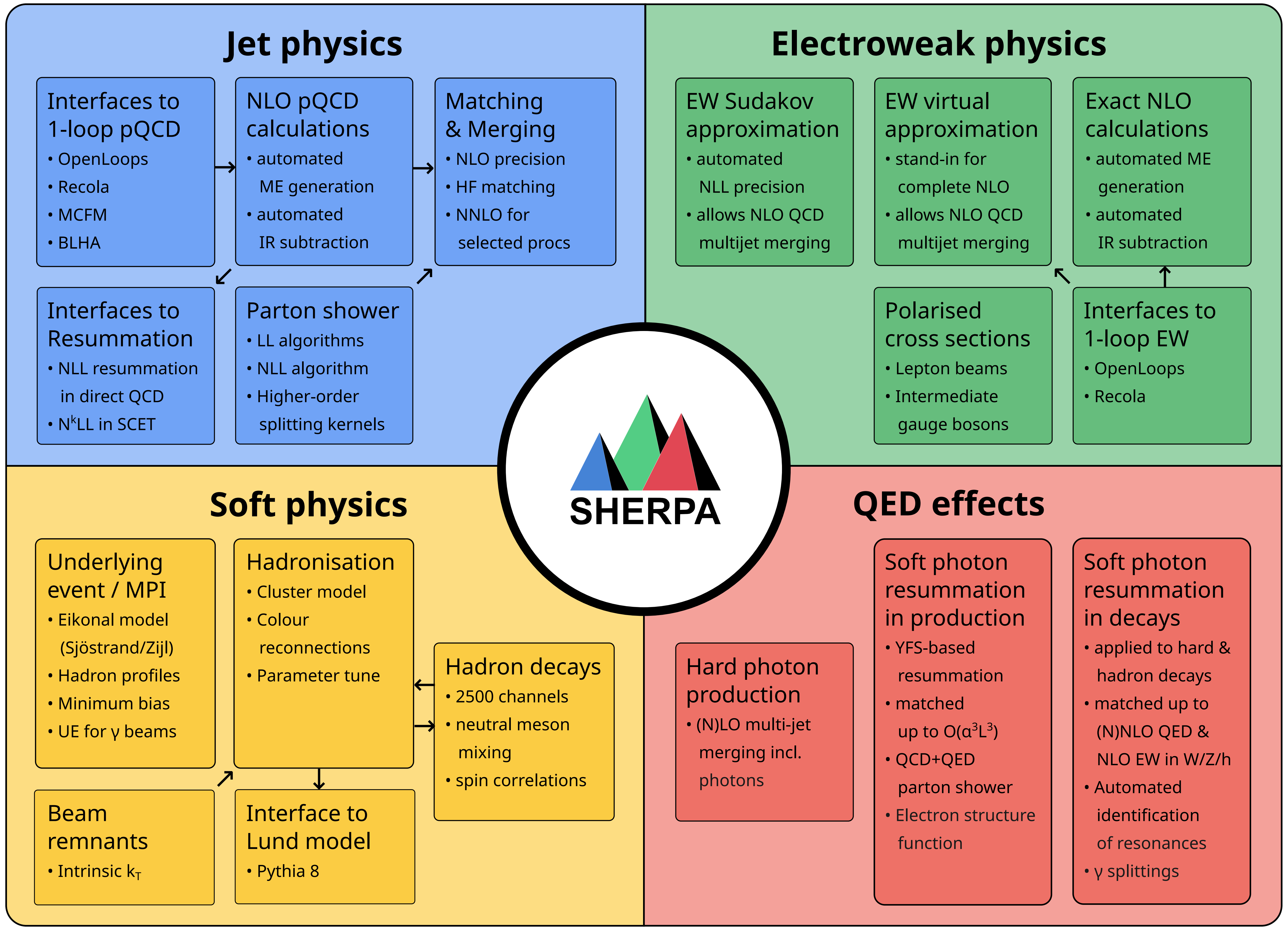


- Sherpa 3.0.0 released this summer
- Traditionally focused on LHC physics
- Sherpa3 contains many improvements for e^+e^- physics

SHERPA Framework

HADRONS++ module for hadron and τ decays

2410.22148 https://sherpa-team.gitlab.io/

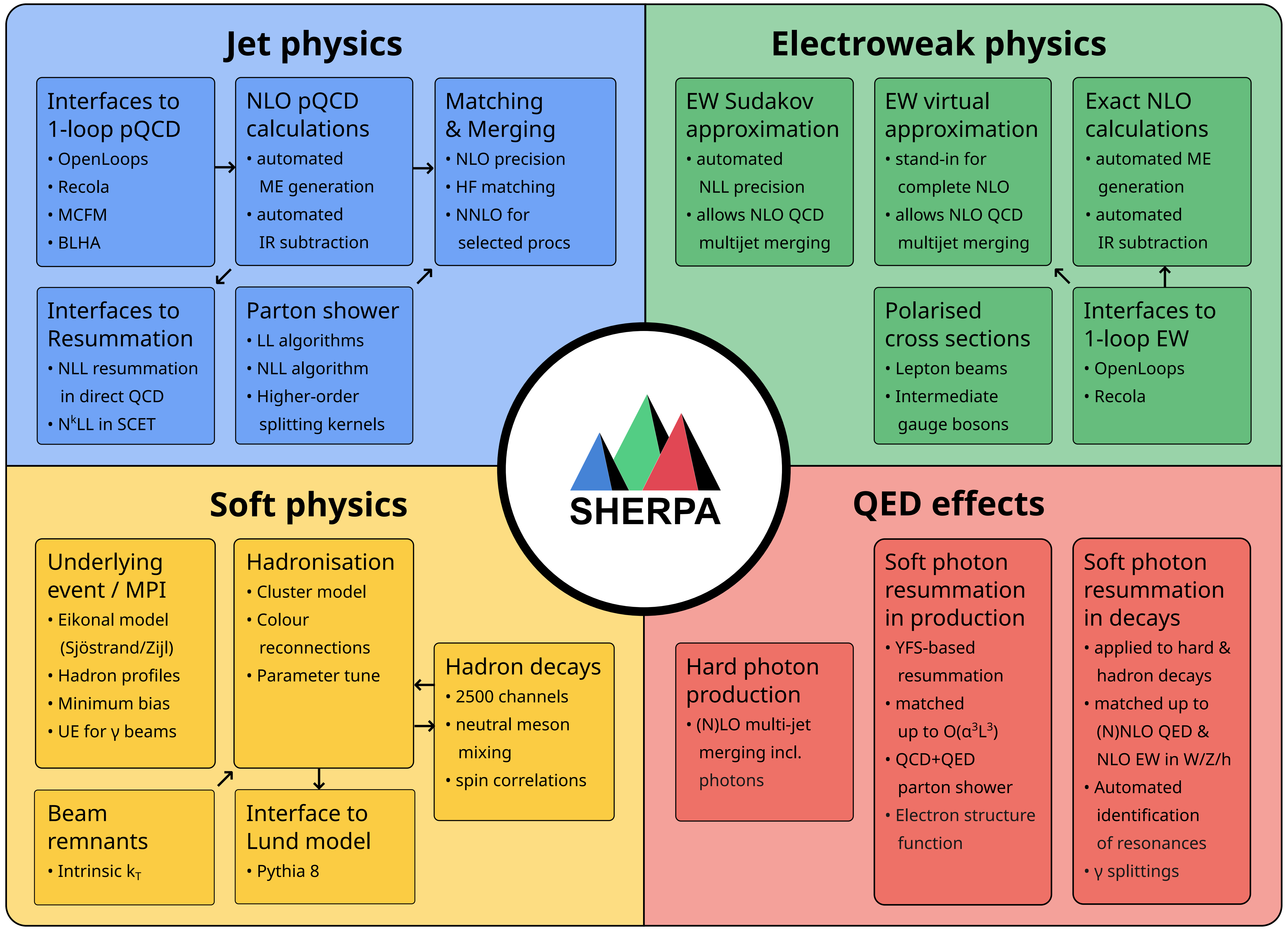


- Decay tables for τ and $\mathcal{O}(200)$ hadrons
- $\approx$ 2.6k decay channels
- accounts for $D-\bar{D}, B-\bar{B}, B_s-\bar{B}_s$, mixing, off-shell decays, form-factors
- "New" decay channels can be added dynamically

SHERPA Framework

QED Corrections

2410.22148 https://sherpa-team.gitlab.io/



- QED corrections are modelled using Yennie-Frauschi-Suura
- Soft Photons resummed to all orders
- Hard photon emissions corrected for order by order

KKMC Generator

KKMC is the MC event generator for the process

$$e^+e^- \rightarrow f\bar{f} + n\gamma$$

- New(ish) C++ version available at https://github.com/KrakowHEPSoft/KKMCee/tree/FCC_release_cpp
- The physics is identical to the F77 version
  - YFS Resummation + Amplitude based matching (CEEX)
- Interfaced with Tauola & Photos & Dizet
- Hadronization can be modelled with Pythia
- New Event Outputs available: HEPMC Format

Comput.Phys.Commun. 140 (2001) 475-512)

How to treat QED Corrections?

Collinear Resummation

- Collinear logs are resummed with universal PDF $(P_T = 0)$
- Recently matched to NLO
- Combined with Parton Shower to generate photon emissions
- Beyond NLO becomes tricky

Jadach et.al, Z.Phys.C 49 (1991) 577-584,Europhys. Lett.17(1992) 123–128

S.Frixone et.al JHEP 03 (2020)

Soft Resummation

- Soft logs resummed to infinite orde using the YFS theorem
- Correct soft limit achieved for n photons
- Provides a robust scheme for the inclusion of real and virtual corrections at any order.
- Provides an exact treatment of multi photon phasespace

$$d\sigma(L, \hat{L}) = \alpha^k \sum_n^{\infty} \alpha^n \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \hat{\sigma}_{n,i,j} L^i \hat{L}^j$$

$$\hat{L} = \log\left(\frac{Q^2}{E_\gamma^2}\right) \qquad L = \log\left(\frac{Q^2}{m_e^2}\right)$$

How to treat QED Corrections?

Collinear Resummation

- Collinear logs are resummed with universal PDF ($P_T = 0$)
- Recently matched to NLO
- Combined with Parton Shower to generate photon emissions
- Beyond NLO becomes tricky

Jadach et.al. Z.Phys.C 49 (1991) 577-584,Europhys. Lett.17(1992) 123–128

S.Frixone et.al JHEP 03 (2020)

Soft Resummation

- Soft logs resummed to infinite orde using the YFS theorem
- Correct soft limit achieved for n photons
- Provides a robust scheme for the inclusion of real and virtual corrections at any order.
- Provides an exact treatment of multi photon phasespace

$$d\sigma(L,\hat{L}) = \alpha^k \sum_n^{\infty} \alpha^n \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \hat{\sigma}_{n,i,j} L^i \hat{L}^j$$

$$\hat{L} = \log\left(\frac{Q^2}{E_\gamma^2}\right) \qquad L = \log\left(\frac{Q^2}{m_e^2}\right)$$

YFS Theorem

Yennie, Frautschi, and Suura showed how to reorder the entire perturbative series such that all IR divergences are resummed

It also provides an analytical treatment of the multi-photon phasespace

ANNALS OF PHYSICS: **13:** 379–452 (1961)

The Infrared Divergence Phenomena and High-Energy Processes*

D. R. YENNIE†

School of Physics, University of Minnesota, Minneapolis, Minnesota

S. C. FRAUTSCHI‡

Department of Physics, University of California, Berkeley, California

AND

H. SUURA

Department of Physics, Nihon University, Tokyo, Japan

A general treatment of the infrared divergence problem in quantum electrodynamics is given. The main feature of this treatment is the separation of the infrared divergences as multiplicative factors, which are treated to all orders of perturbation theory, and the conversion of the residual perturbation expansion into one which has no infrared divergence, and hence no need for an infrared cutoff. In the infrared factors, which are exponential in form, the infrared divergences arising from the real and virtual photons cancel out in the usual way. These factors can then be expressed solely in terms of the momenta of the initial and final charged particles and an integral over the region of phase space available to the undetected photons; they do not depend upon the specific details of the interaction. Electron scattering from a static potential is treated in considerable detail, and several other examples are discussed briefly. As an important byproduct of the general treatment, it is found that when the infrared contributions are separated in a particular way, they dominate the radiative corrections at high energies and together with certain "magnetic terms" and vacuum polarization corrections seem to give all the contributions proportional to ln (E/m). All of these corrections can be easily estimated (in most cases) simply from a knowledge of the external momenta of the charged particles; this then provides a very powerful and accurate way of estimating radiative corrections to high-energy processes.

* Supported in part by the U. S. Atomic Energy Commission, Contract AT(11-1)-50.

† Various refinements were added to the manuscript (particularly in Appendix C) during the academic year 1960–1961 while this author was a National Science Foundation Senior Fellow visiting the University of Paris. He is grateful to Professor M. M. Lévy for the hospitality afforded by the Laboratoire de Physique Théorique et Hautes Énergies at Orsay.

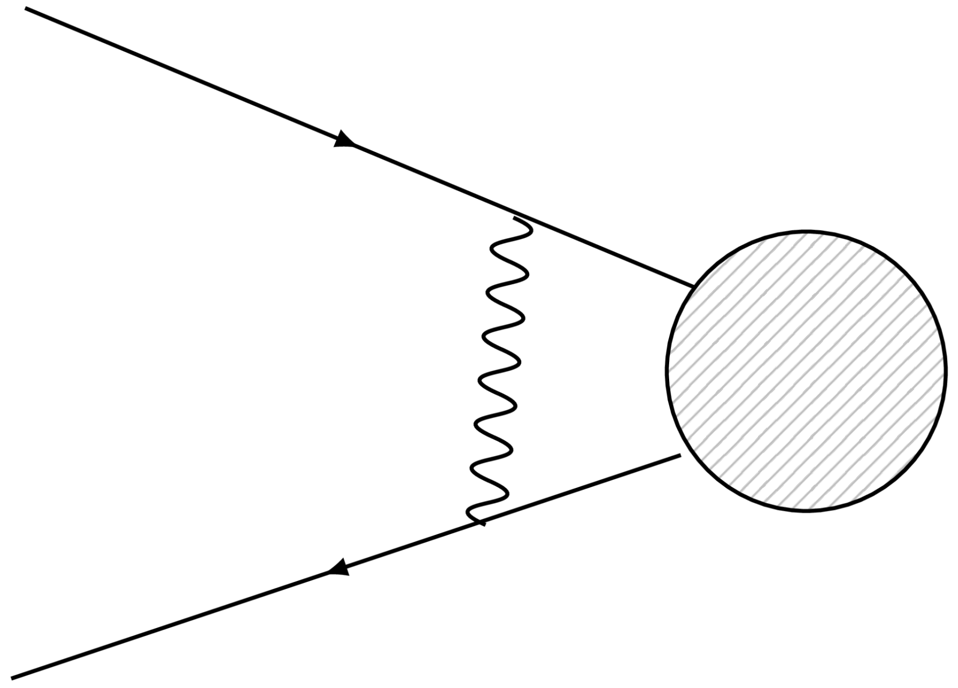
‡ Supported by National Science Foundation Grant.

379

Virtual Emissions

Taking the soft limit allows us to factorise out amplitude

$$\mathscr{M}_0^1 = \alpha B M_0^0 + M_0^1$$



Full One-loop amplitude, including IR divergence

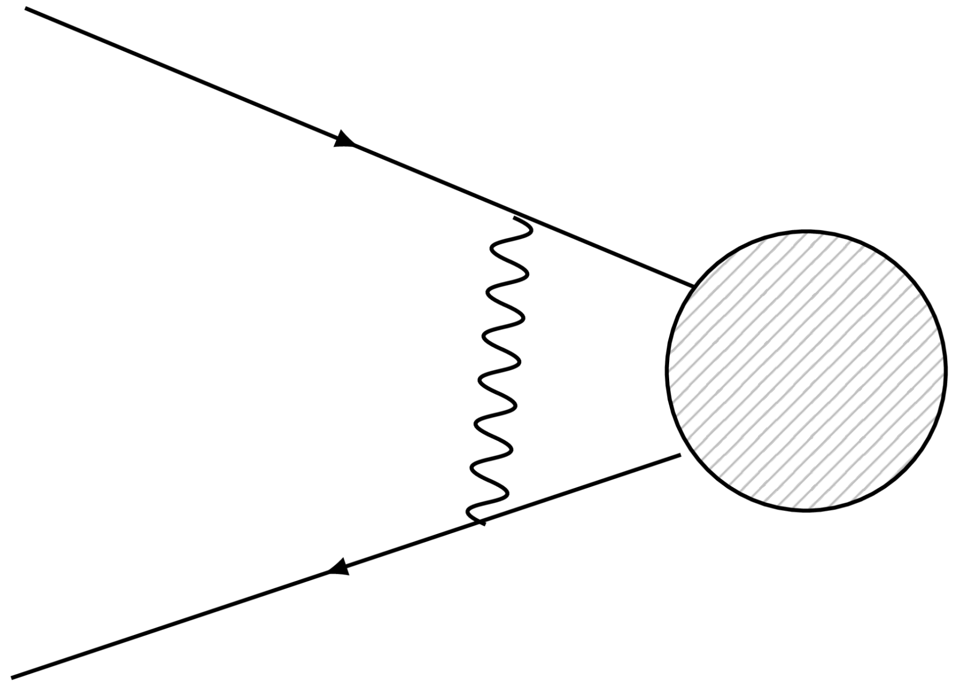
IR divergent contribution

IR finite contribution

Virtual Emissions

Taking the soft limit allows us to factorise out amplitude

$$\mathscr{M}_0^1 = \alpha B M_0^0 + M_0^1$$



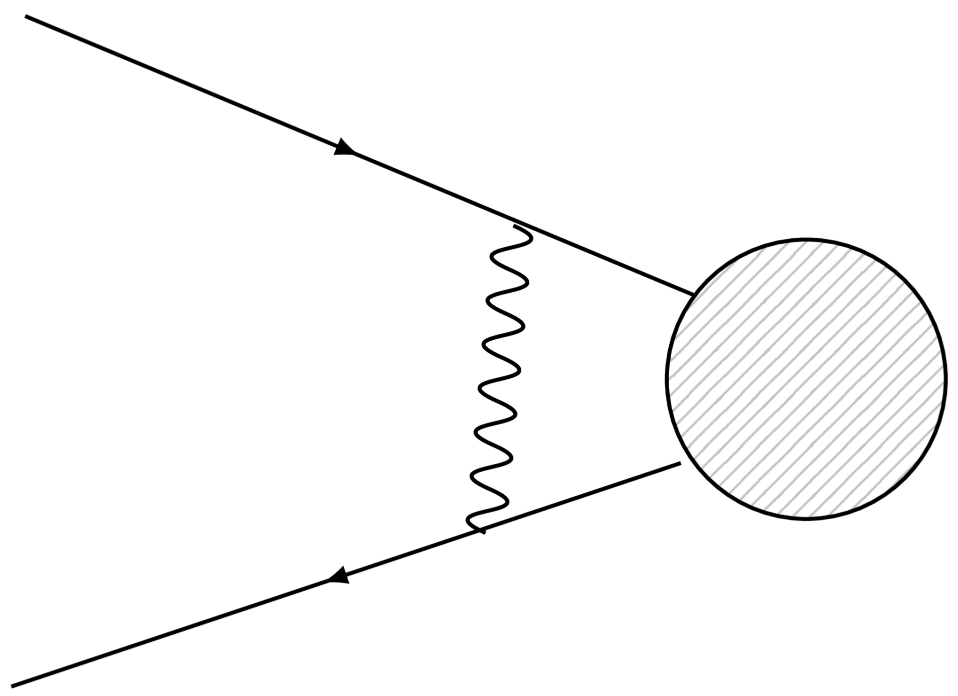
$$B_{ij} = -\frac{i}{8\pi^3} Z_i Z_j \theta_i \theta_j \int \frac{d^4k}{k^2} \left(\frac{2p_i\theta_i - k}{k^2 - 2(k \cdot p_i)\theta_i} + \frac{2p_j\theta_j + k}{k^2 + 2(k \cdot p_j)\theta_j} \right)^2$$

Universal Virtual

$Z_i =$ Particle Charge

$\theta_i = 1(-1)$ Incoming (Outgoing)

Virtual Emissions



This factorization continues at each order thanks to the abelian nature of QED

$$\mathscr{M}_0^0 = M_0^0$$

$$\mathscr{M}_0^1 = M_0^1 + \alpha B M_0^0$$

$$\mathscr{M}_0^2 = M_0^2 + \alpha B M_0^1 + \frac{(\alpha B)^2}{2!} M_0^0$$

$$\Downarrow$$

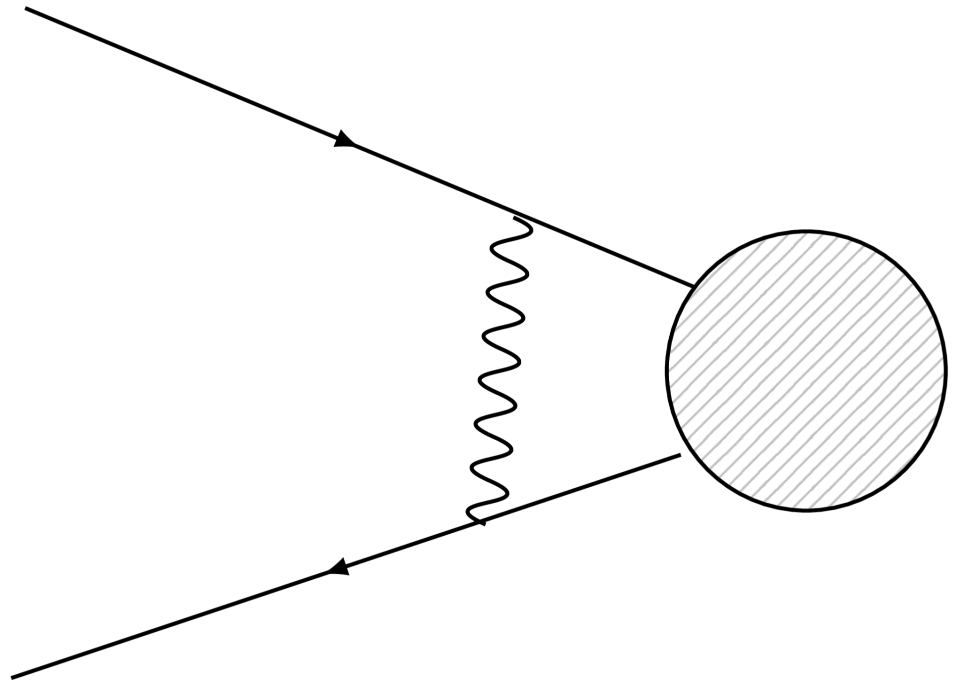
$$\mathscr{M}_0^{\bar{n}_\gamma} = \sum_{r=0}^{\bar{n}_\gamma} M_0^{\bar{n}_\gamma - r} \frac{(\alpha B)^r}{r!}$$

Virtual Emissions

This factorization continues at each order

We can then sum to infinity and due to the abelian nature of QED we can generalise to arbitrary real emissions

$$\left| \sum_{\bar{n}_\gamma=0}^{\infty} \mathcal{M}_{n_\gamma}^{\bar{n}_\gamma+\frac{1}{2}n_\gamma} \right|^2 = e^{2\alpha B} \left| \sum_{\bar{n}_\gamma}^{\infty} M_{n_\gamma}^{\bar{n}_\gamma+\frac{1}{2}n_\gamma} \right|^2$$



$$B_{ij} = -\frac{i}{8\pi^3} Z_i Z_j \theta_i \theta_j \int \frac{d^4k}{k^2} \left(\frac{2p_i\theta_i - k}{k^2 - 2(k\cdot p_i)\theta_i} + \frac{2p_j\theta_j + k}{k^2 + 2(k\cdot p_j)\theta_j} \right)^2$$

Real Emissions

$$\frac{1}{2(2\pi)^3}\left|\sum_{\bar{n}_\gamma=0}^{\infty}\mathscr{M}_1^{\bar{n}_\gamma+\frac{1}{2}}\right|^2=\tilde{S}(k)\left|\sum_{\bar{n}_\gamma=0}^{\infty}M_0^{\bar{n}_\gamma}\right|^2+\sum_{\bar{n}_\gamma=0}^{\infty}\tilde{\beta}_1^{\bar{n}_\gamma+1}(k)$$

For real emissions, we consider the factorization at the amplitude squared level

$$\tilde{S}(k)=\sum_{i,j}\frac{\alpha}{4\pi^2}Z_iZ_j\theta_i\theta_j\left(\frac{p_i}{p_i\cdot k}-\frac{p_j}{p_j\cdot k}\right)^2$$

$Z_i =$ Charge

$\theta_i = 1(-1)$ Incoming (Outgoing)

YFS Master Equation

Combining virtual and real emissions to all orders yields

$$\left(\frac{1}{2(2\pi)^3}\right)^{n_\gamma}\left|\sum_{\bar{n}_\gamma=0}^{\infty} M_{n_\gamma}^{\bar{n}_\gamma+\frac{1}{2}n_\gamma}\right|^2 = \tilde{\beta}_0\prod_{i=1}^{n_\gamma}\left[\tilde{S}(k_i)\right] + \sum_{i=1}^{n_\gamma}\left[\frac{\tilde{\beta}_1(k_i)}{\tilde{S}(k_i)}\right]\prod_{j=1}^{n_\gamma}\left[\tilde{S}(k_j)\right]$$

$$+\sum_{\substack{i,j=1\\ i<j}}^{n_\gamma}\left[\frac{\tilde{\beta}_2(k_i,k_j)}{\tilde{S}(k_i)\tilde{S}(k_j)}\right]\prod_{l=1}^{n_\gamma}\left[\tilde{S}(k_l)\right]$$

$$+\ldots+\tilde{\beta}_{n_\gamma-1}(k_1,\ldots,k_{i-1},k_{i+1},\ldots,k_{n_\gamma})\sum_{i=1}^{n_\gamma}\tilde{S}(k_i)+\tilde{\beta}_{n_\gamma}(k_1,\ldots,k_{n_\gamma})$$

$$\tilde{\beta}_{n_\gamma} = \sum_{\bar{n}_\gamma=0}^{\infty}\tilde{\beta}_{n_\gamma}^{\bar{n}_\gamma+n_\gamma}$$

This is essentially an all order subtraction scheme

Suppress the virtual multiplicity

YFS Master Equation

$$d\sigma = \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!} d\Phi_Q \left[\prod_{i=1}^{n_\gamma} d\Phi_i^\gamma \tilde{S}(k_i)\,\Theta(k_i,\Omega)\right]\left(\tilde{\beta}_0 + \sum_{j=1}^{n_\gamma}\frac{\tilde{\beta}_1(k_j)}{\tilde{s}(k_j)} + \sum_{\substack{j,k=1 \\ j<k}}^{n_\gamma}\frac{\tilde{\beta}_2(k_j,k_k)}{\tilde{s}(k_j)\tilde{s}(k_k)} + \cdots\right)$$

This expression contains **no approximations.**It does require any further matching. The accuracy is limited by how far you can calculate the betas

$$Y(\Omega) = \sum_{i<j} \mathscr{Re}\, B_{ij}(\Phi_n) + \tilde{B}_{ij}(\Phi_{n+1})$$

Hard emission corrections for $B \to X\ell\nu$

Full hard emission corrections are process dependent.

Computed NLO QED corrections to semileptonic B decays.
$B \to D\ell\nu$, $B \to D^*\ell\nu$, $B \to \pi\ell\nu$ (B^0, $B^\pm$) Bernlochner, MS '10

LO **hadronic current** is given by

$$H^\mu(p_B, p_X; q^2) = (p_B + p_X)^\mu f_+(q^2) + (p_B - p_X)^\mu f_-(q^2)$$

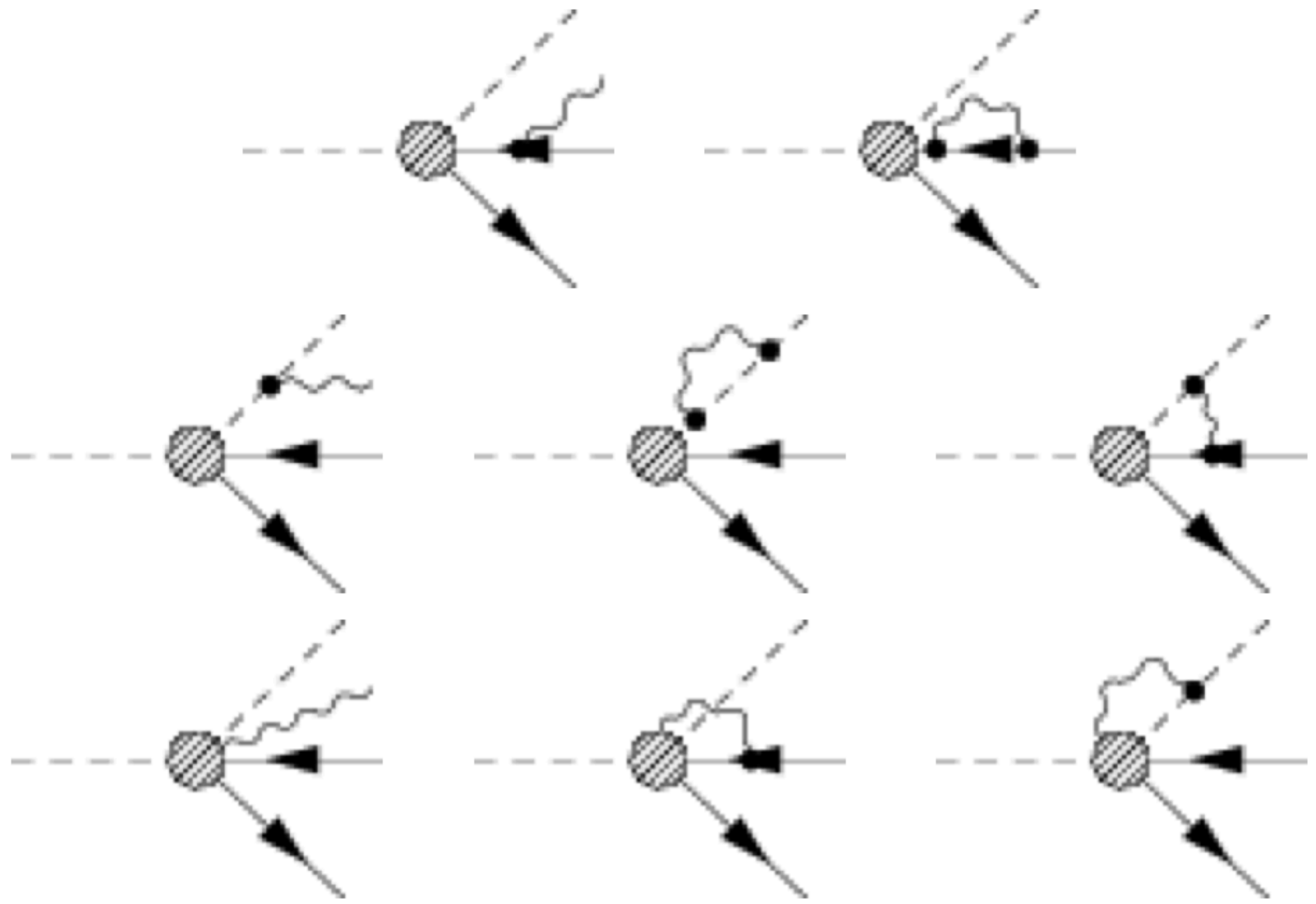
For constant form factors one can assign a **Lagrangian** for the weak decay

$$\mathcal{L}_W = \frac{G_F}{\sqrt{2}} V_{xb} \left[(f_+ + f_-)\phi_X \partial^\mu \phi_B + (f_+ - f_-)\phi_B \partial^\mu \phi_X \right] \overline{\psi}_\nu P_R \gamma_\mu \psi_\ell + \text{h.c.}$$

Requiring QED gauge invariance gives interaction terms

$$\mathcal{L}_{\text{int}}^{\text{QED}} = -e\overline{\psi}_\ell\gamma^\mu\psi_\ell A_\mu - ieQ_\phi\left(\phi^+\partial^\mu\phi^- - \phi^-\partial^\mu\phi^+\right)A_\mu + e^2Q_\phi^2\phi^+\phi^- A_\mu A^\mu$$
$$+ ie\sqrt{2}G_F V_{xb} f_\pm (Q_B \pm Q_X)\phi_B\phi_X\,\overline{\psi}_\nu P_R\gamma^\mu\psi_\ell A_\mu + \text{h.c.}$$

In addition to usual QED and scalar QED interactions an additional terms describing emissions off the vertex arises.

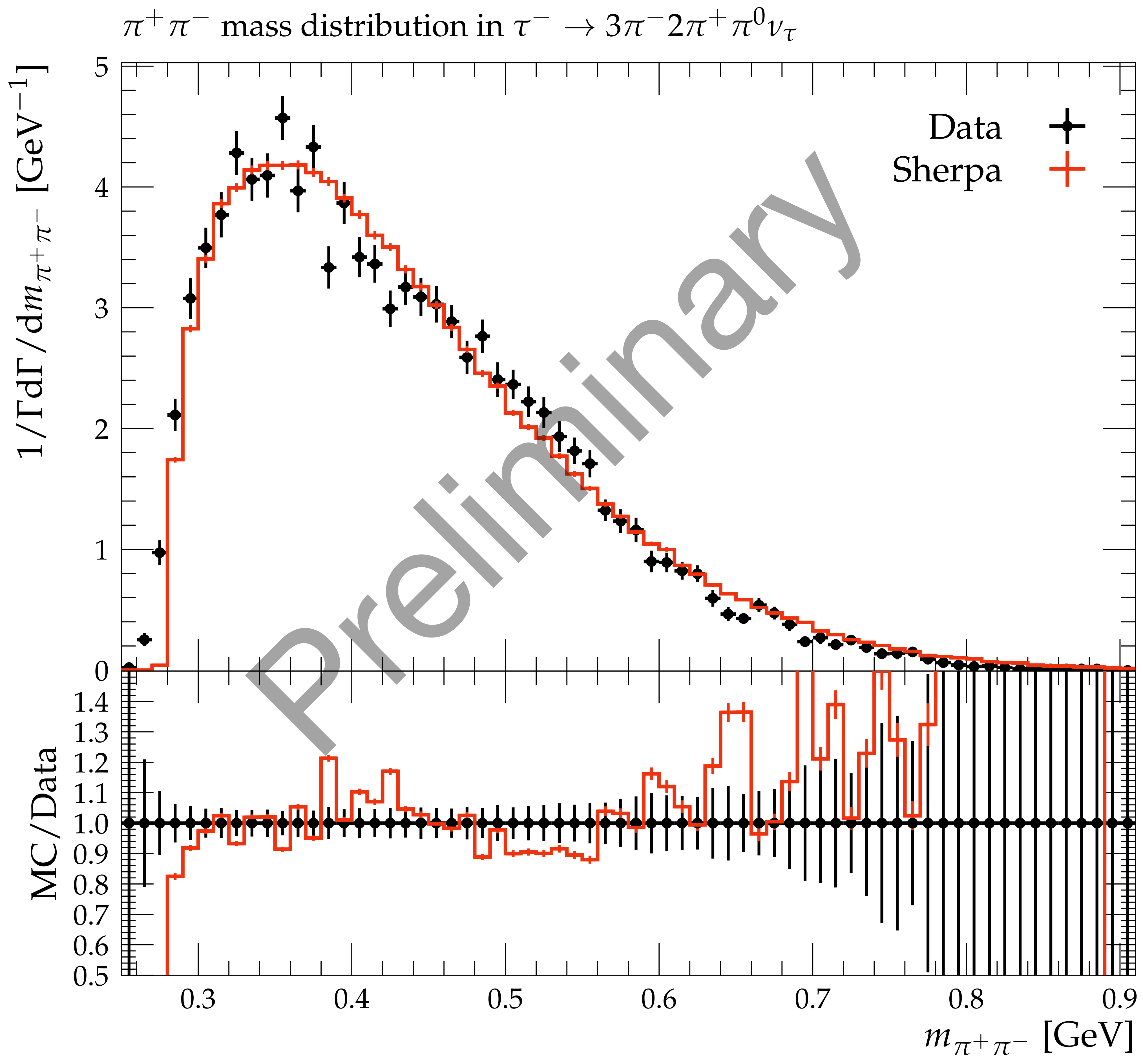


What is missing?

- off-shell form-factors in hadronic current
- proper treatment of hadronic resonances, eg. $B \to D^*\ell\nu \to D\gamma\,\ell\nu$
- hadron-photon interaction beyond point-like scalar QED

Example: $e^+e^- \to \tau^+\tau^-$

```
HADRON_DECAYS:
  Model: HADRONS++
  Max_Proper_Lifetime: 10.0
  QED_Corrections: 1
  Channels:
    15:
      211,211,-211,-211,-211,111,16:
        # BR: [0.00102, 0.00102]
        BR: [1, 1]
        IntResults: [2.293e-18, 4.514e-21, 3.598e-17]
```

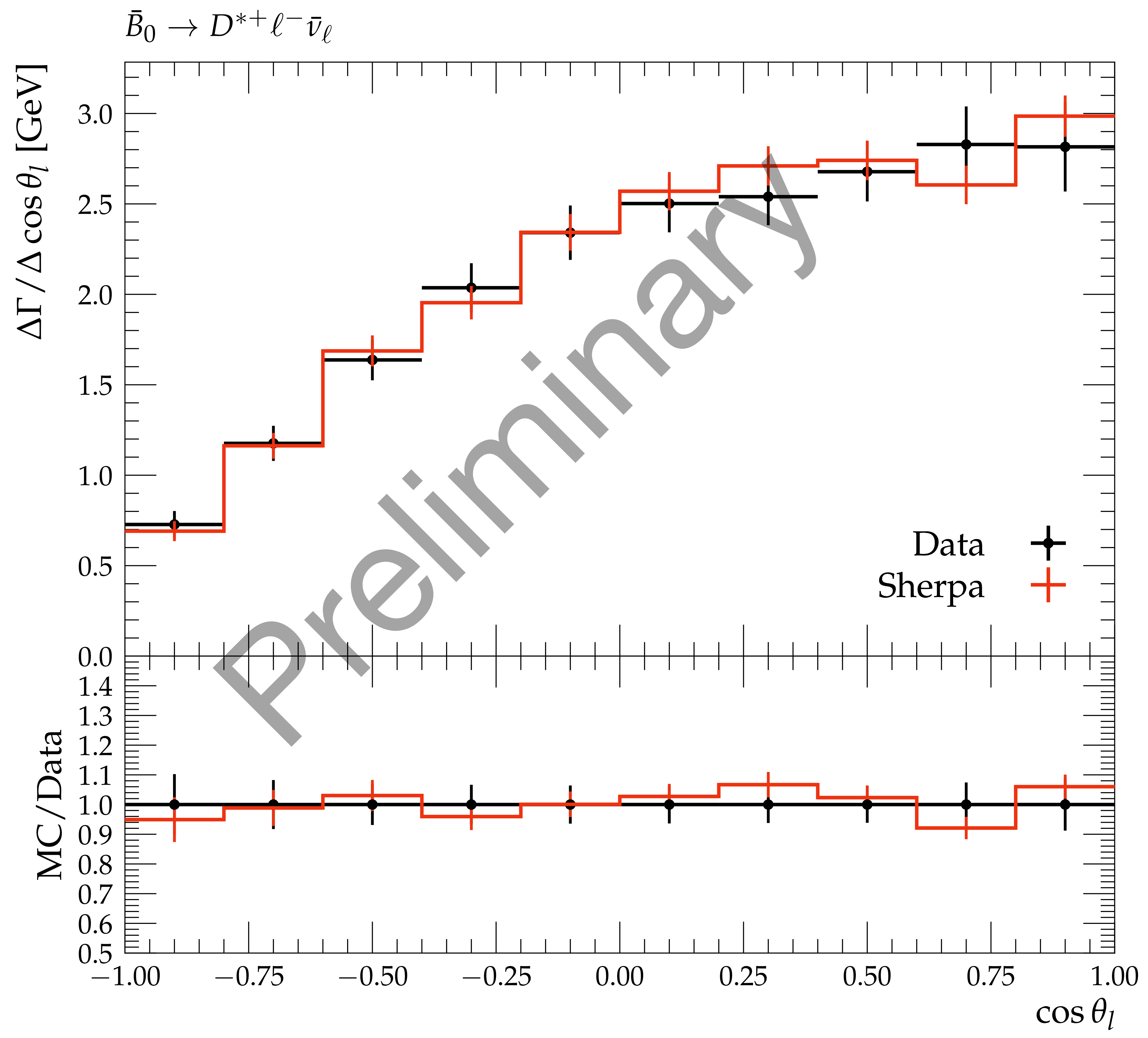


Data from BaBar Phys.Rev.D 86 (2012) 092010

Example: $e^+e^- \to \Upsilon(4S) \to B\bar{B}$

- Sherpa can directly simulate the “signal” process and the QCD background

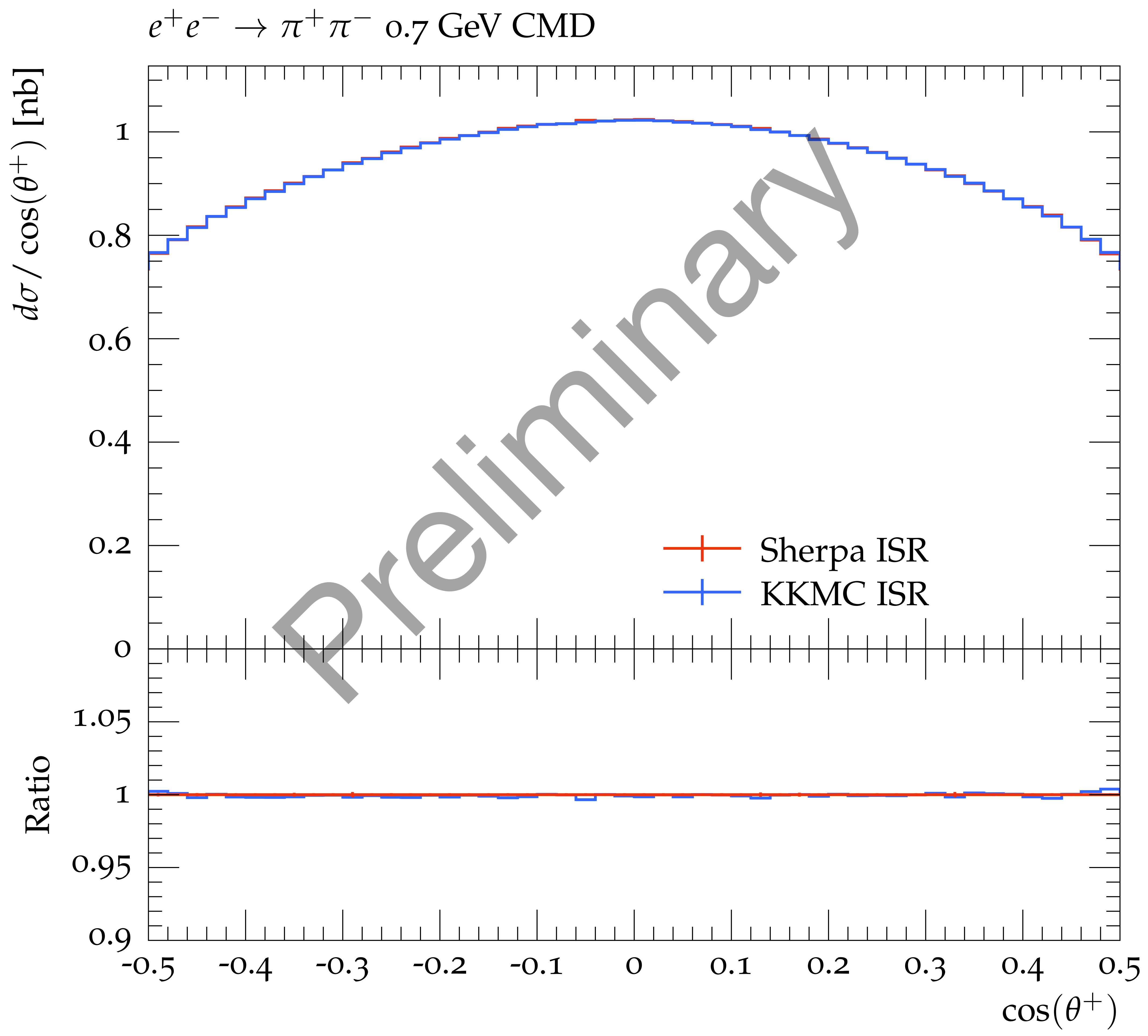
```
PROCESSES:
# electron positron -> Y(4S) -> B+ B-
- 11 -11 -> 300553[a]:
    Decay: "300553[a] -> 521 -521"
    Order: {QCD: 0, EW: 0}
# electron positron -> Y(4S) -> B0 B0bar
- 11 -11 -> 300553[a]:
    Decay: "300553[a] -> 511 -511"
    Order: {QCD: 0, EW: 0}
```



Data from BELLE [BELLE-CONF-1612]

Example: $e^+e^- \rightarrow \pi^+\pi^-$

- Direct production now in both Sherpa and KKMC with all order resummation of soft photons
- Sherpa supplemented with Pion Form factor and HPV
- KKMC HPV available via DIZET Package
- Next steps: Matching the resummation to higher order corrections



Conclusions

- Soft-Photon resummation captures the bulk of the QED corrections
- It can be further improved by matching to higher-order predictions
- Some work still needed for non-perturbative models e.g Form-Factors
- Parallel developments of Sherpa and KKMC will provide a powerful cross-check