The TAUOLA generator for τ lepton decays

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- **(1)** The τ lepton decays: fascinating laboratory for intermediate energy QCD; that may explain, why I am a bit biased against New Physics in τ decays, but ...
- **(2)** I will adress also TAUOLA in context of τ as precison tool to measure properties of SM physics (and beyond). This is mostly because of τ lepton production processes.
- **(3)** These two regimes separate perfectly, because τ leptons are extremely narrow resonances and production processes are relatively free of strong interactions.
- **(4)** How to optimize work of inhomogeneous community. From model builders to people managing large experimental data files. From F77 to C++ and Python.
- **(5)** My concern is also on how to handle different component of systematc errors: experiment, theor. Choice of quantities for measurements and tests.

Target points: what people may need

(1) Simulate detector response

(2) Provide distributions of τ decay products and of the τ itself: starting from lagrangian of Old and New physics

(3) Environment to study prototypes for matrix elements and prototypes for τ decay observables.

Technical detail: narrow width limit for intermediate resonances is often needed.

(4) For studies where τ leptons are used to constrain else, like Higgs CP or B physics.

(5) New challenges: multidimensional distributions? ML? Experimental systematic errors for that?

- **First some theory and software organization**

General formula for tau production and decay.

Formalism for $\tau^+\tau^-$

- Because narrow τ width approximation can be obviously used for phase space, cross-section for the process $f\bar{f} \to \tau^+\tau^- Y; \tau^+ \to X^+\bar{\nu}; \tau^- \to \nu\nu$ reads:

$$d\sigma = \sum_{spin} |\mathcal{M}|^2 d\Omega = \sum_{spin} |\mathcal{M}|^2 d\Omega_{prod}\ d\Omega_{\tau^+}\ d\Omega_{\tau^-}$$

- This formalism is fine, but, e.g. for 20 τ decay channels we would have 400 distinct processes. Also picture of production and decay are mixed.

- Below only τ spin indices are explicitly written:

$$\mathcal{M} = \sum_{\lambda_1\lambda_2=1}^{2} \mathcal{M}^{prod}_{\lambda_1\lambda_2}\ \mathcal{M}^{\tau^+}_{\lambda_1} \mathcal{M}^{\tau^-}_{\lambda_2}$$

- Cross section can be re-written into **core formula of spin algorithms**

$$d\sigma = \Big(\sum_{spin} |\mathcal{M}^{prod}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^+}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^-}|^2\Big) wt\ d\Omega_{prod}\ d\Omega_{\tau^+}\ d\Omega_{\tau^-}$$

General formula for tau production and decay.

- where

$$wt = \Big(\sum_{i,j=0,3} R_{ij} h^i h^j\Big)$$

$$R_{00} = 1, \quad < wt >= 1, \quad 0 \le wt \le 4.$$

R_{ij} can be calculated from $\mathcal{M}_{\lambda_1\lambda_2}$ by contraction with Pauli σ^i matrices and similarly h^i, h^j respectively from $\mathcal{M}^{\tau^+}$ and $\mathcal{M}^{\tau^-}$.

- Bell inequalities tell us that it is impossible to re-write wt in the following form

$$wt \neq \Big(\sum_{i,j=0,3} R^A_i h^i\Big)\Big(\sum_{i,j=0,3} R^B_j h^j\Big)$$

that means it is impossible to generate first τ^+ and τ^- first in some given ‘ quantum state’ and later perform separatelly decays of τ^+ and τ^-

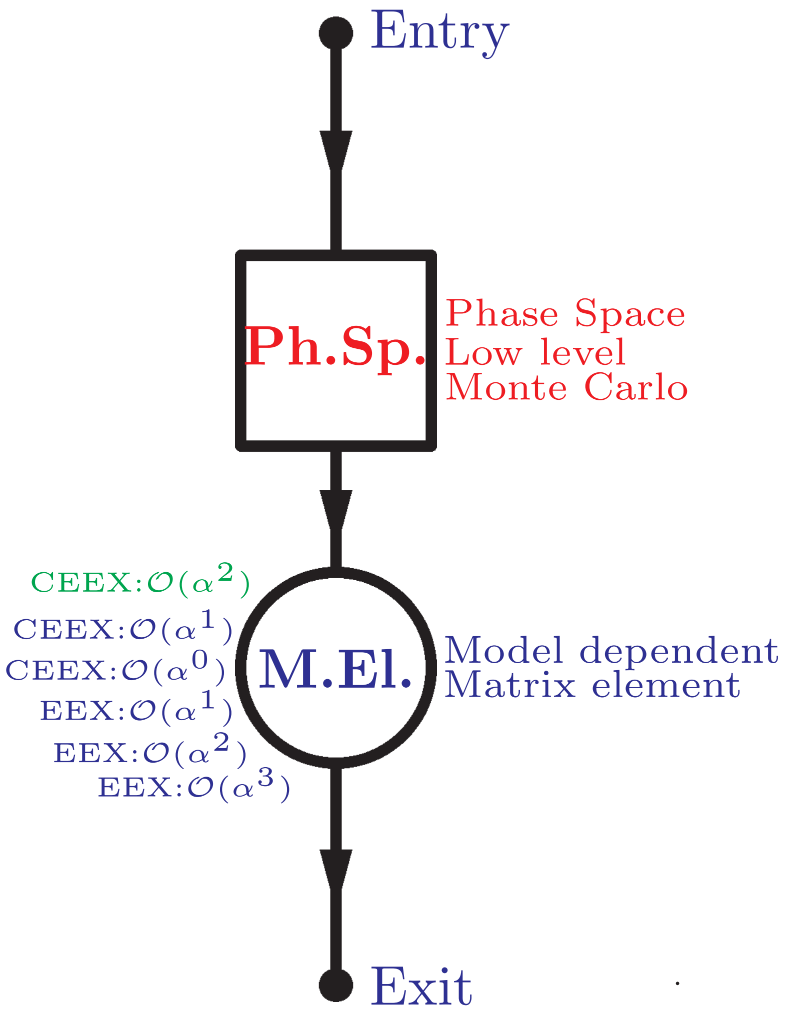
- It can be done only if approximations are used !!!
- May be reasonable in e.g. ultrarelativistic regime, but nonetheless approximation.

TAUOLA universal interface

- To run, generator for tau decays must be combined with production.
- In cases of packages for e^+e^- colliders, such as ours KORALB, KORALZ, KKMC, they provide environment for TAUOLA use.
- often information from event stored in production files can be used.
- I will adress only fraction of technicalities, there is a lot to that!
- In `KKMC` quantization frames, boosts to/from Lab to τ rest-frames are carefully prepared **HOWEVER:** τ lepton(s) spin states can be calculated from kinematical configurations of hard processes then it is up to user to control.
- Like in f77 version of `EvtGen`-interface to `TAUOLA` and `PHOTOS`. Also in our `TAUOLA universal interface`. Information from event record.
- It can work on stored production events. flexibility but require user attention.
- Algrithmic-wise, the same sophistication level, like solution in `KKMC` (e.g. quantum entanglement included).

General formula for tau production and decay.

Textbook principle “matrix element $\times$ full phase space” useful



- Phase-space Monte Carlo module producing “raw events”.
- Library of models for provides input for “model weight”
- **Useful for any application, not only τ production/decay.**
- Ratios of matrix elements squared define probability that event could be of model B if generated with mode A.
- Convenient for Machine Learning too.
- No compromises on precision are required.

Formalism for semileptonic decays at 0.2% precision level

- Matrix element used in `TAUOLA` for semileptonic decay of τ with P momentum and spin s

$$\tau(P,s) \to \nu_\tau(N) X$$
$$\mathcal{M} = \frac{G}{\sqrt{2}} \bar{u}(N)\gamma^\mu(v + a\gamma_5)u(P)J_\mu$$

- J_μ – the current, depends on the momenta of all hadrons ($h_\mu = H_\mu/H_t$)

$$|\mathcal{M}|^2 = G^2 \frac{v^2+a^2}{2}(\omega + H_\mu s^\mu)$$
$$\omega = P^\mu(\Pi_\mu - \gamma_{va}\Pi^5_\mu)$$
$$H_\mu = \frac{1}{M}(M^2\delta^\nu_\mu - P_\mu P^\nu)(\Pi^5_\nu - \gamma_{va}\Pi_\nu)$$
$$\Pi_\mu = 2[(J^* \cdot N)J_\mu + (J \cdot N)J^*_\mu - (J^* \cdot J)N_\mu]$$
$$\Pi^{5\mu} = 2\,\mathrm{Im}\,\epsilon^{\mu\nu\rho\sigma} J^*_\nu J_\rho N_\sigma$$

$$\gamma_{va} = -\frac{2va}{v^2+a^2}$$

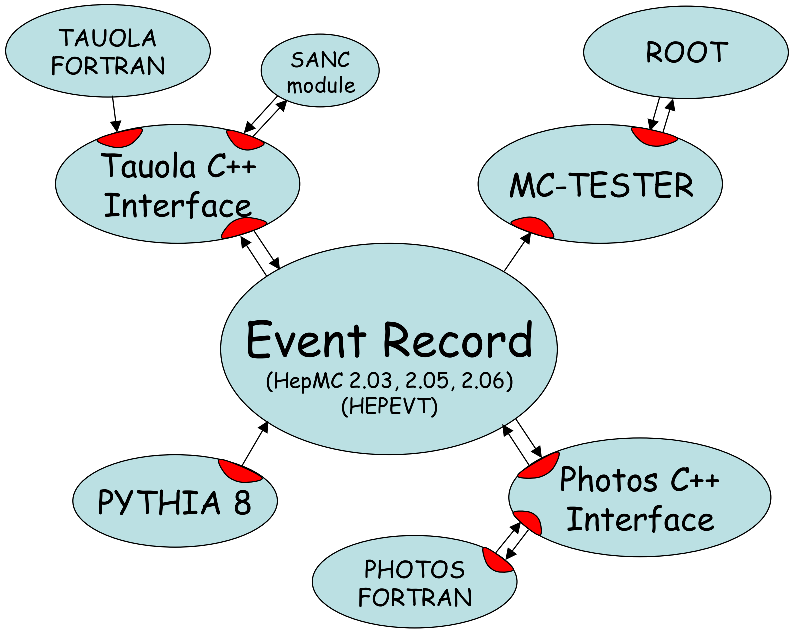
$$\hat{\omega} = 2\frac{v^2-a^2}{v^2+a^2} m_\nu M (J^* \cdot J)$$
$$\hat{H}^\mu = -2\frac{v^2-a^2}{v^2+a^2} m_\nu\, \mathrm{Im}\,\epsilon^{\mu\nu\rho\sigma} J^*_\nu J_\rho P_\sigma$$

TAUOLA perspective of τ decays themselves.

- Hadronic currents have to fulfil Lorentz invariance.
- In $\tau^\pm \to \rho^\pm \to \pi^\pm\pi^0\nu$ channel fits are technically straightforward: single distribution to be fitted with single real function to fit:
$J^\mu = (p_{\pi^\pm} - p_{\pi^0})^\mu F_V(Q^2) + (p_{\pi^\pm} + p_{\pi^0})^\mu F_S(Q^2)$ ($F_S \simeq 0$).
- For 3-scalar channels: 4 complex functions each of 3 variables to fit. Role of theoretical assumptions (oversimplifications?) is essential. Agreement on 1-dim distribution is just a consistency check. $J^\mu = J_1^\mu + J_2^\mu +_4^\mu + J_5^\mu$. CP breaking contribution installed so far in `TAUOLA` in some decays with K_L, K_S only. Directly or through event weights.
- **No go for model independent measurements? Not necessarily.** Use of all dimensions for data distributions: invariant masses Q^2, s_1, s_2 as arguments of form-factors. Angular asymmetries help to separate currents: scalar
$J_4^\mu \sim Q^\mu = (p_1 + p_2 + p_3)^\mu$, vector $J_1^\mu \sim (p_1 - p_3)^\mu|_{\perp Q}$ and
$J_2^\mu \sim (p_2 - p_3)^\mu|_{\perp Q}$ and finally pseudovector $J_5^\mu \sim \epsilon(\mu, p_1, p_2, p_3)$.
- Model independent methods, if: (i) enough data, (ii) absolute precision, (iii) no background, (iv) full detector coverage of decay phase-space helpful. We need that for orthogonality of fitted functions. ML techniques instead?
- It is a challenge but worth a try. I am ready to talk it over, any time any place.

An example when τ decays are modified.

Solutions: rigorous exact phase space $\times$ approximate $|M|^2$.



Parts:

- hard process: (Born, weak, new physics),
- parton shower,
- τ decays
- QED bremsstrahlung

- High precision achieved

- Detector studies: acceptance, resolution lepton with or without photon.

Such organization requires:

- Good control of factorization (theory)
- Good understanding of tools on user side.

Note:

(1) τ `leptons are long lived`

(2) QED low energy limit - solvable, analytycity helps semi-factorization too.

An example when τ decays are modified.

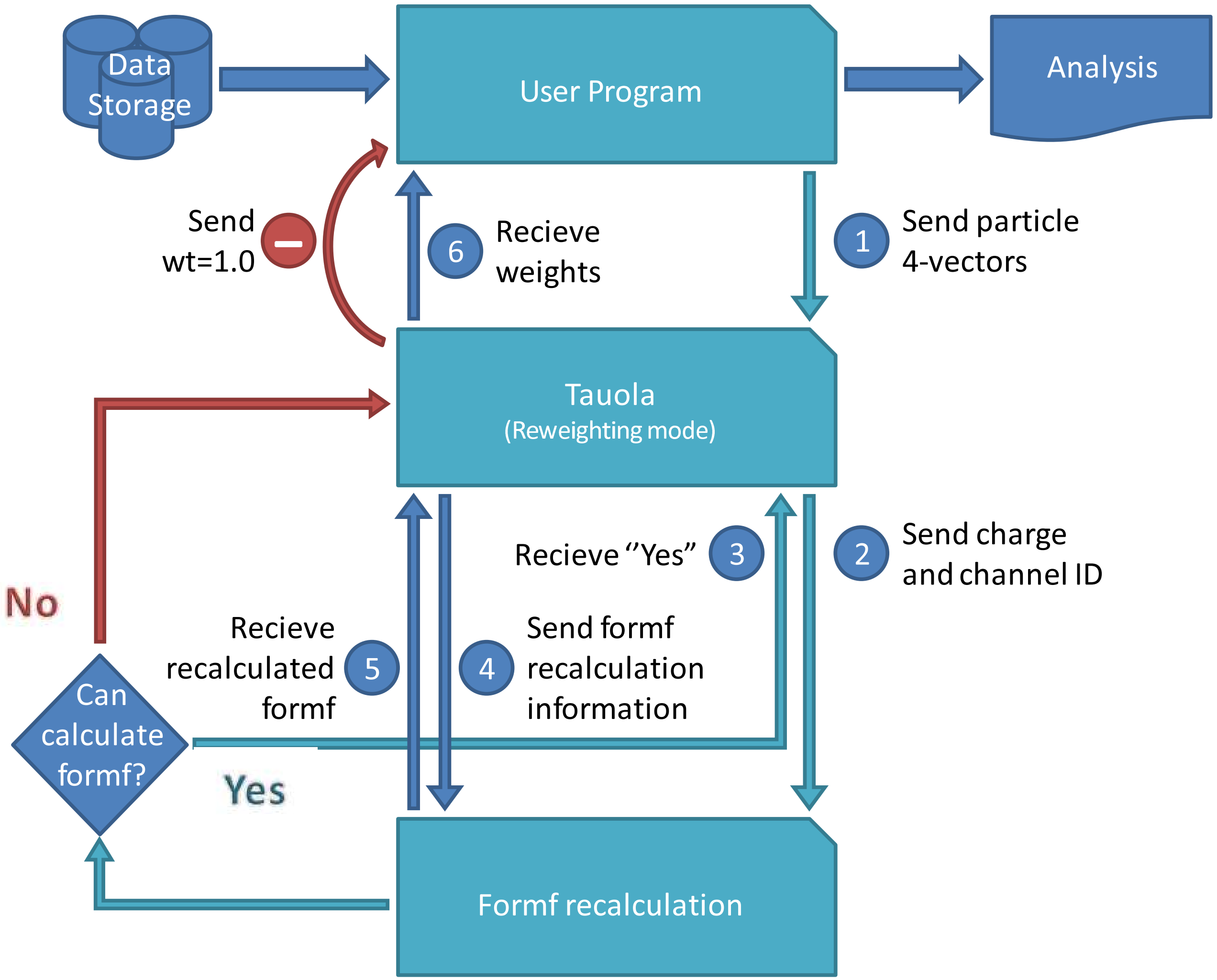


Figure 1: Flow chart for communication when already stored events are modified with the weights. Useful at LHC and at low energy applications as well.

New τ decays and precision simulation

1. **The following contexts: fit strategy, experimental, theoretical syst. errors., cooperation between sub-communities.**
2. Does it open a path to understand intermediate energy QCD?
   I am ready to talk it over, any time any place.
3. “Recently” I have introduced into TAUOLA some changes.
4. Quality stamps from the side of theory, experiment, technical precision.
5. TAUOLA of new hadronic currents, 200+ decay channels, which can be manipulated by user with c++ coded currents, ME and with any decay products: Comput.Phys.Commun. 232 (2018) 220
6. What should be included (acceptable in collaborations software) in standard initialization(s).
7. Constraints of software organization in Belle forced a step back: no user provided currents, migration to c++ is not completed.

New τ decays and precision simulation

- New hadronic currents (more than 88 % of hadronic τ decay width) version installed with the 0.05 % technical tag:
- **But** physics precision was definitely **NOT** as good as 0.05 %.
- Over two years we worked on preparing confrontation env. with the data keeping precision in mind, but ...
- Despite efforts , we are left as far from the complete solution as many years ago.
- We have investigated technical aspects for fitting using weights.
  Seemed interesting when experimental cuts are present, multidimensional distributions are used and no semi-analytical results can be easily prepared for fits.
- We have returned to the semi-analytical 1-dim distributions for fits. Because multi-dimensional data were not public, no systematic ambiguities evaluated.
- We had encountered difficulties with fits as well.
- **WARNING:** results of distinct fits gave at 10 % the same 1-dim distributions, but in more than 10 % of the contributions to total widths difGerencees were at a level of factor 2.
  I am ready to talk it over, any time any place.
- challenge for intermediate energy QCD?

New τ decays and precision simulation

To progress in case of $\tau \to 3\pi\nu_\tau$ we had to:

- Modify the model (contribution of σ or CP sensitive terms)
- Work simultaneously with fits using weights. Note difficulties with stability if strong fitted parameters correlations. Template method I have learned in Orsay (at ALEPH time) requires confirmation that model parameters dependencies are not correlated. Necessity to linearize dependencies because of CPU-time constraints in case when model was not giving perfect predictions complicated things further.
- Finally we relied on fitting semi-analytical formulas:
  - We had to assure that derivaties of results are continuous.
  - We had to speed up calculations using different methods of pretabulation/interpolation of results for Q-dependent a_1 width (unitarity constraint).
  - Only 1-dimensional invariant mass distributions were available.

New τ decays and precision simulation

- That was not enough constraining.
- What is the best, **<u>and still safe</u>**, input from experimental side

  Multidimensionality? Ambiguities controlled?
- I am ready to talk it over, any time any place.

Possible warning message

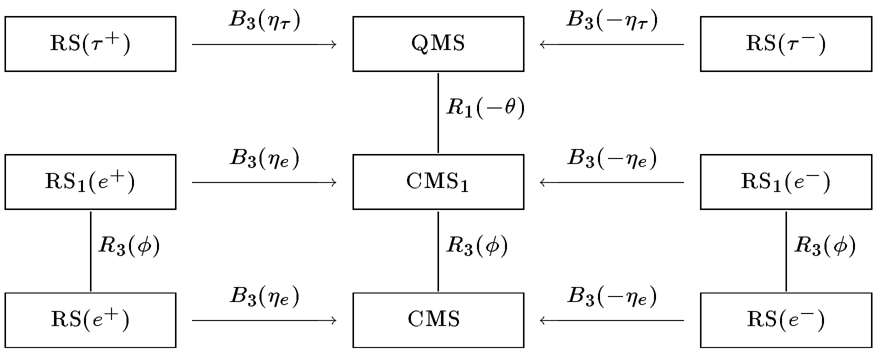
- Already for 3-scalar final states theoretical predictions and experimental data: distributions over 8-dimensional space. We fit 1- (2-) dim. histos. Result depend on model assumptions. Models inspired with results ... Fitting setup $\longrightarrow$ biases.
- Our pattern recognition algorithms are far less elaborate than human eye/brain.
- How to facilitate dialog, role of MC.

- Biases in art, Giuseppe Arcimboldo (1572 - 1593).

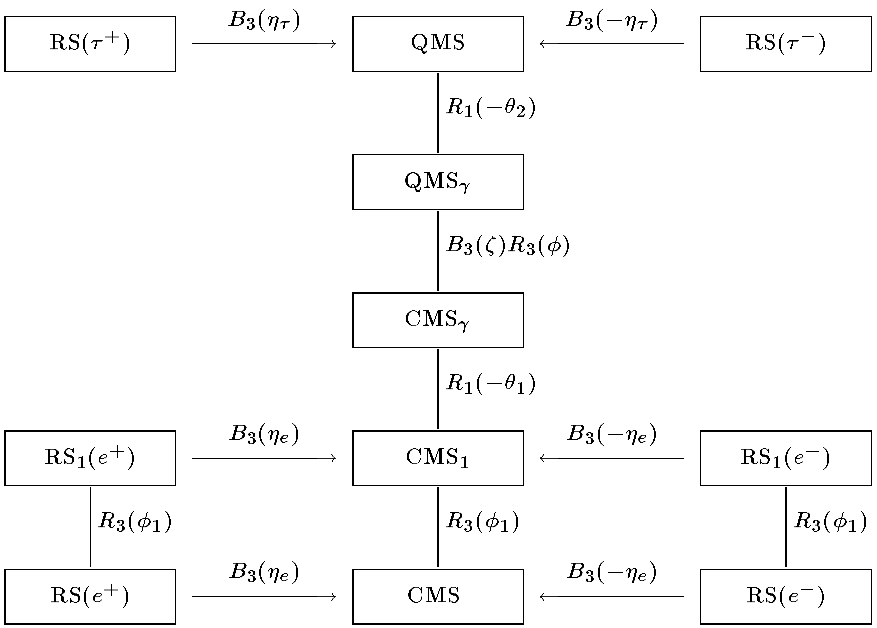
Matching production and decays

Ref. frames for spin; production, decay. Geometry of QED amplitudes optimized

Figure 2



(2a)



(2b)

Matching production and decays

- *Does it make sense to have so many frames.*
- *Automatizations make it obsolete.*
- *but it is helpful for intuition build and for construction of optimal variables*
- *Who needs them at a time of ML?*
- *But then, what about ambiguities?*

Phenomenology Of Mixed Parity: also from M.E.

- Higgs boson Yukawa coupling expressed with the help of the scalar–pseudo-scalar mixing angle ϕ

$$\bar{\tau} N (\cos\phi + i \sin\phi \gamma_5) \tau$$

- *Decay probability for the mixed scalar–pseudo-scalar case*

$$\Gamma(h_{mix} \to \tau^+ \tau^-) \sim 1 - s_{\parallel}^{\tau^+} s_{\parallel}^{\tau^-} + s_{\perp}^{\tau^+} R(2\phi)\, s_{\perp}^{\tau^-}$$

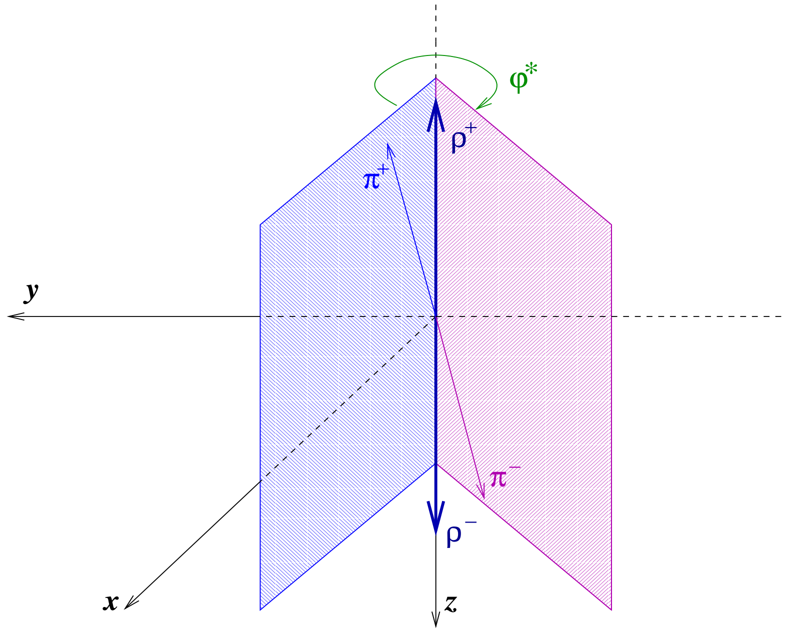
- $R(2\phi) -$ *operator for the rotation by angle* 2ϕ *around the* $\parallel$ *direction.*

$$R_{11} = R_{22} = \cos 2\phi \qquad R_{12} = -R_{21} = \sin 2\phi$$

- *Pure scalar case is reproduced for* $\phi = 0$.
- *For* $\phi = \pi/2$ *we reproduce the pure pseudo-scalar case.*

Optimal Observable Mixed Scalar—Pseudoscalar Case

- *For mixing angle ϕ, transverse component of τ^+ spin polarization vector is correlated with the one of τ^- rotated by angle 2ϕ.*
- *Acoplanarity $0 < \varphi^* < 2\pi$ is of physical interest, not just $\mathrm{arc\,cos}\,\mathbf{n}_- \cdot \mathbf{n}_+$.*
- *Distinguish between the two cases $0 < \varphi^* < \pi$ and $2\pi - \varphi^*$*
- *If no separation made the parity effect would wash itself out.*

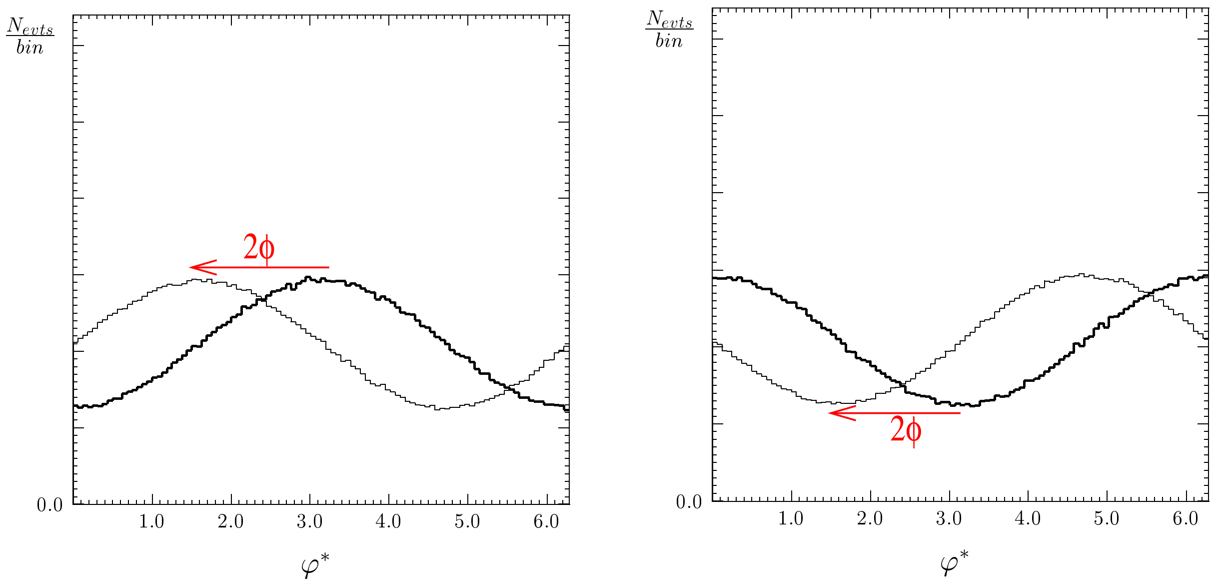


Normal to planes: $\mathbf{n}_\pm = \mathbf{p}_{\pi^\pm} \times \mathbf{p}_{\pi^0}$

Find the sign of $\mathbf{p}_{\pi^-} \cdot \mathbf{n}_+$

Negative $0 < \varphi^* < \pi$

Otherwise $2\pi - \varphi^*$

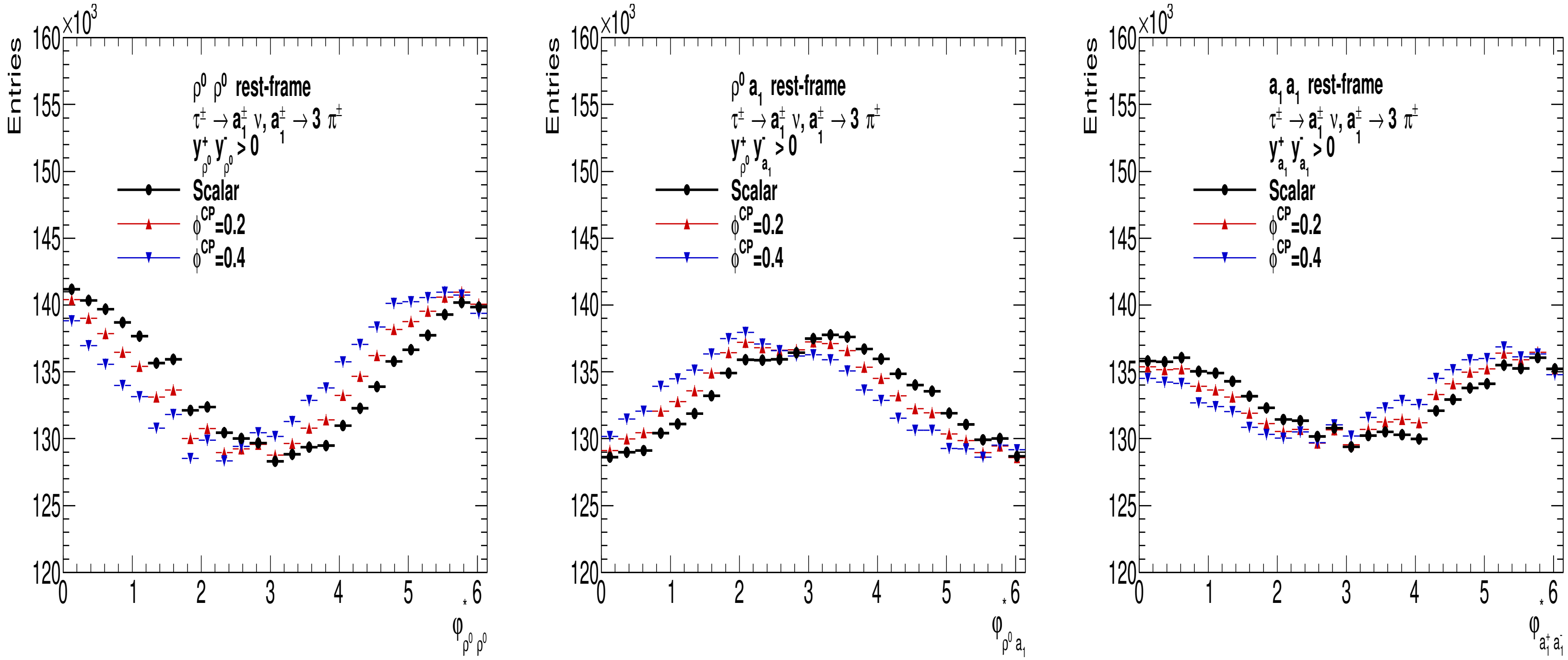


- *Only events where the signs of y1 and y2 are the same whether calculated using the method without or with the help of the τ impact parameter.*
- *The thick line corresponds to a scalar Higgs boson, the thin line to a mixed one.*
- *This Tesla-time observable , Phys.Lett. B 543 (2002) 227, B579 (2004) 157, survived into LHC times.*

Old time experience with frames was helpful

LHC, enrich small samples with $\tau \to 3\pi\nu$

Acoplanarity angles of oriented half decay planes: $\varphi^*_{\rho^0\rho^0}$ (left), $\varphi^*_{a_1\rho^0}$ (middle) and $\varphi^*_{a_1 a_1}$ (right), for events grouped by the sign of $y^+_{\rho^0} y^-_{\rho^0}$, $y^+_{a_1} y^-_{\rho^0}$ and $y^+_{a_1} y^-_{a_1}$ respectively. Three CP mixing angles ϕ^{CP} = 0.0 (scalar), 0.2 and 0.4. Note scale, effect on individual plot is so much smaller now. But up to **16 plots like that** have to be measured, correlations understood. Physics model depends on 1 parameter only ϕ^{CP} mixing scalar pseudo-scalar angle, which brings linear shift. I remained frustrated for 15 years, how to digest...



Summary

- The purpose of my talk was to push some ideas forward and what is needed for that.
- It was not disciplined talk. Sorry for that.
- (i) Experimental systematic errors (ii) Theoretical systematic errors
- What are the constraints on organization of Monte Carlo and fitting environments?
- We have collected some experience on requirements for building fitting environments.
- Systematic ambiguities, in case of fits to multi-dimensional representation of data, is a challenge.
- ML bring benefits but a price too ...
- Question of manpower and training as well as motivation of involved people.
- τ **leptons as tools for other high and medium energy physics points.**
- Easy to ignore: e.g. narrow width approximation availability in MC is useful for: (i) tests (ii) model development and tuning (iii) observable construction interpretation (iv) intuition buildup.

Outlook

- The purpose of my talk was to push some ideas forward and what is needed for that.
- It was not disciplined talk. Sorry for that.
- My aim was to underline directions, which in my opinion, are worth to follow.
- I have sketched also what are the main aspects of the activities till now.
- I hope it may be useful for somebody here.