e^+e^- low energy calculations with muons and pions

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Joint IJCLAB -IFJ PAN Heavy Flavour meeting

12–13 November 2024, Cracow

NATIONAL SCIENCE CENTRE POLAND

PHOKHARA MC generator

EVA: $e^+e^- \to \pi^+\pi^-\gamma$

- tagged photon ($\theta_\gamma > \theta_{cut}$)
- ISR at LO + Structure Function
- FSR: point-like pions

[Binner et al.]

$e^+e^- \to 4\pi + \gamma$

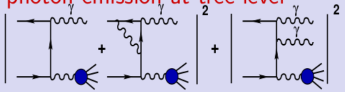
- ISR at LO + Structure Function

[Czyż, Kühn,2000]

F. Campanario, H.C., J. Gluza,
A. Grzelińska, M. Gunia, P. Kisza,
J. H. Kühn, E. Nowak-Kubat, T. Riemann,
G. Rodrigo, Sz. Tracz, A. Wapienik,
V. Yundin, D. Zhuridov

PHOKHARA 10.0: $\pi^+\pi^-, \mu^+\mu^-,$
4π, $\bar{N}N$, 3π, $KK, \Lambda\bar{\Lambda}$, $P\gamma$
$J/\psi, \psi(2S), \chi_{c_1}, \chi_{c_2}$

- **ISR at NLO:** virtual corrections to one photon events and two photon emission at tree level



- FSR at NLO: $\pi^+\pi^-$, $\mu^+\mu^-$, K^+K^-, $\bar{p}p$
- tagged or untagged photons
- $e^+e^- \to hadrons\ (muons)$ ISR at NNLO
- Modular structure

http://ific.uv.es/∼rodrigo/phokhara , http://czyz.phys.us.edu.pl/czyz/

"Standard model radiative corrections in the pion form factor measurements do not explain the a_μ anomaly",
F. Campanario et al, PRD100,076004(2019)

Phokhara, status

PHOKHARA

radiative return at flavour factories

Physics	Electron--positron annihilation into hadrons plus an energetic photon from initial state radiation (ISR) allows the hadronic cross-section to be measured over a wide range of energies at high luminosity flavour factories [DAPHNE, CESR, PEP-II, KEK-B, Super-KEKB, BESIII].
Content	**PHOKHARA** is a Monte Carlo event generator which simulates this process at the next-to-leading order (NLO) accuracy. This includes virtual and soft photon corrections to one photon emission events and the emission of two real hard photons.
Downloads	**VERSION 10.0 (October 2020):** Includes complete NLO radiative corrections for the extraction of the **pion form factor**. The new implementation is described in detail in Phys. Rev. D100 (2019) no.7, 076004 [arXiv:1903.10197 hep-ph]. • manual [PDF], source [.tar.gz]

Forthcoming features	• Further updates are not expected.

http://ific.uv.es/∼rodrigo/phokhara

The muon pair production with real photon emission $e^+e^- \to \mu^+\mu^-\gamma$ is an important background and normalization reaction in the measurement of the pion form-factor:

$$R_{exp} = \frac{\sigma(e^+e^- \to \pi\pi\gamma)}{\sigma(e^+e^- \to \mu^+\mu^-\gamma)}$$

which is necessary for an accurate determination of the anomalous magnetic moment of the muon $(g-2)_\mu$,

KLOE-2 uses both Bhabha and muon pair normalizations, Babar only radiative return

So, to discuss here e^+e^- with:
$\longrightarrow$ muons
$\longrightarrow$ pions
$\longrightarrow$ [Bhabha]

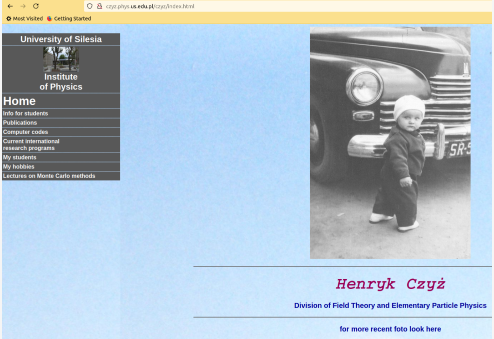
Based on

- Pion pair production with higher order radiative corrections in low energy e^+e^- collisions, Eur.Phys.J.C 24 (2002) 51 https://arxiv.org/abs/hep-ph/0107154
- Measuring the FSR inclusive $\pi^+\pi^-$ cross-section, Eur.Phys.J.C 28 (2003) 261-278 https://arxiv.org/abs/hep-ph/0212386
- NNLO leptonic and hadronic corrections to Bhabha scattering and luminosity monitoring at meson factories, JHEP 07 (2011) 126 https://arxiv.org/abs/1106.3178
- Complete QED NLO contributions to the reaction $e^+e^{\to}\mu^+\mu^-\gamma$ and their implementation in the event generator PHOKHARA, JHEP 02 (2014) 114 https://arxiv.org/abs/1312.3610
- Standard model radiative corrections in the pion form factor measurements do not explain the a_μ anomaly, Phys.Rev.D 100 (2019) 7, 076004 https://arxiv.org/abs/1903.10197

Additional material:

- Radiative Corrections to Hadron Production in e^+e Annihilations at DAΦNE Energies, **Axel Hoefer**, PhD thesis, link
- Working Group on Rad. Corrections and MC Generators for Low Energies, link: https://www.lnf.infn.it/wg/sighad/

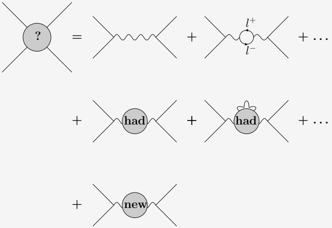
Some Phokhara MC related materials also at http://czyz.phys.us.edu.pl/czyz/



$\sigma(e^+e^- \to \text{hadrons})$ data to evaluate hadronic shift to $\alpha(s)$ and $g-2$

$\longrightarrow$ **Precision tests of the Standard Model**

Running $\alpha(s)$ determined by photon self energy contributions:

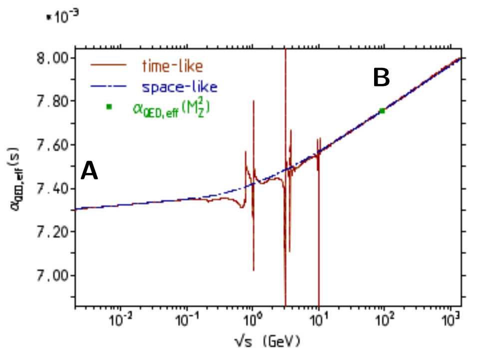


$$\alpha(s) = \frac{\alpha(0)}{1 + \Delta\alpha(s)}$$

$$\Delta\alpha(s) = \Delta\alpha_{\text{ew}}(s) + \Delta\alpha_{\text{had}}(s) + \Delta\alpha_{\text{new}}(s)$$

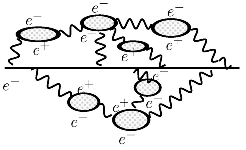
$\Delta\alpha_{\text{had}}(s) \longrightarrow$ non–perturbative QCD contributions $\Longrightarrow$ not calculable within perturbation theory $\Longrightarrow$ experimental input is needed

$\alpha_{QED}(s)$, vacuum polarisation



A: $\alpha_{QED}(0) \simeq 1/137$

B: $\alpha_{QED}(M_Z^2) \simeq 1/128$



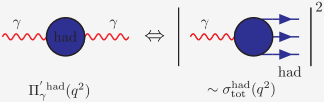
F. Jegerlehner, http://dx.doi.org/10.23731/CYRM-2020-003.9

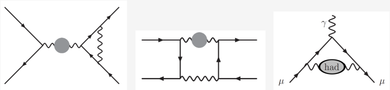
The effective $\alpha(s)$ in terms of the photon vacuum polarization (VP) self-energy correction $\Delta\alpha(s)$ by

$$\alpha(s) = \frac{\alpha}{1 - \Delta\alpha(s)} \; ; \quad \Delta\alpha(s) = \Delta\alpha_{\mathrm{lep}}(s) + \Delta\alpha^{(5)}_{\mathrm{had}}(s) + \Delta\alpha_{\mathrm{top}}(s) \, .$$

R-data evaluation of $\alpha(s)$

$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)} \; ; \quad \Delta\alpha(s) = \Delta\alpha_{\mathrm{lep}}(s) + \Delta\alpha^{(5)}_{\mathrm{had}}(s) + \Delta\alpha_{\mathrm{top}}(s) \, .$$





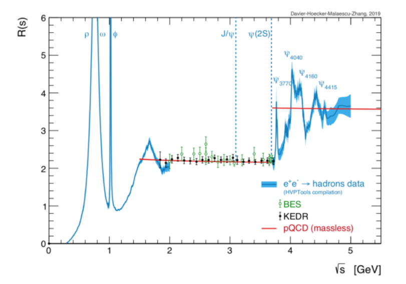
The non-perturbative hadronic piece from the five light quarks $\Delta\alpha^{(5)}_{\mathrm{had}}(s) = -\left(\Pi'_{\gamma}(s) - \Pi'_{\gamma}(0)\right)^{(5)}_{\mathrm{had}}$ can be evaluated in terms of $\sigma(e^+e^- \to \mathrm{hadrons})$ data via the dispersion integral **(s can be any, also negative!)**

$$\Delta\alpha^{(5)}_{\mathrm{had}}(s) = -\frac{\alpha\, s}{3\pi}\left(\int_{m^2_{\pi^0}}^{E^2_{\mathrm{cut}}} \mathrm{ds'}\, \frac{\mathrm{R}^{\mathrm{data}}_{\gamma}(\mathrm{s'})}{\mathrm{s'}(\mathrm{s'}-\mathrm{s})} + \int_{\mathrm{E}^2_{\mathrm{cut}}}^{\infty} \mathrm{ds'}\, \frac{\mathrm{R}^{\mathrm{pQCD}}_{\gamma}(\mathrm{s'})}{\mathrm{s'}(\mathrm{s'}-\mathrm{s})} \right),$$

$$a^{\mathrm{had}}_{\mu} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m^2_\pi}^{\infty} ds\, \frac{R(s)\,\hat{K}(s)}{s^2} \; , \hat{K}(s) \in 0.63 \div 1.$$

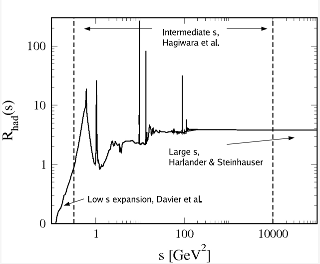
$$R_\gamma(s) \equiv \sigma^{(\mathbf{0})}(\mathbf{e}^+\mathbf{e}^- \to \gamma^* \to \mathrm{hadrons}) / \left(\frac{4\pi\alpha^2}{3s}\right)$$

Davier et al

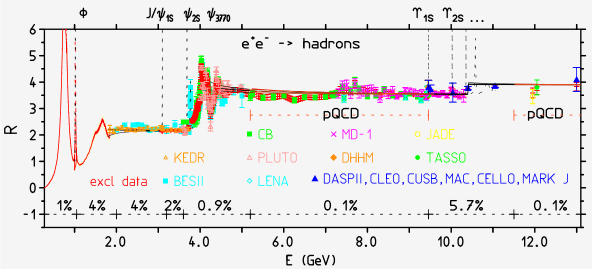


E.g. Parametrization used for Bhabha

Phys.Rev.D 78 (2008) 085019

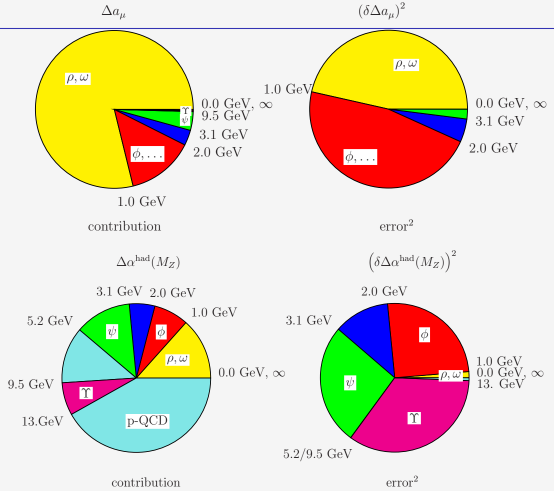


The compilation of $R(s)$-data utilized by F. Jegerlehner for $\Delta\alpha_{\rm had}$.



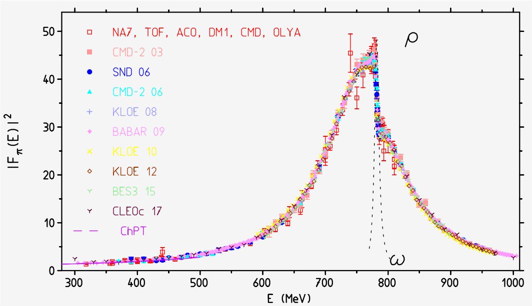
Nontrivial contributions from different energy regions for $a_\mu^{\rm had}$ and $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$

and square uncertainties



In contrast to the low energy dominated $a_\mu^{\rm had}$, $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ is sensitive to data from much higher energies. *Alternative methods to determine $\Delta\alpha^{(5)}_{\rm had}(s)$ at high energies have to be developed (Adler function etc.*

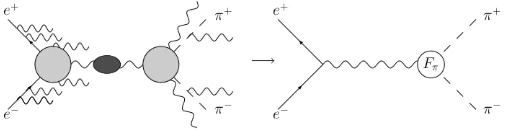
A piece of low energy mess: $\rho - \omega$



The low energy tail of R is provided by $\pi^+\pi^-$ production data.

$$R(s) = \frac{1}{4}\,\beta_\pi^3\,|F_\pi^{(0)}(s)|^2\,,\;\; \beta_\pi = (1 - 4m_\pi^2/s)^{1/2}$$

- $\sigma^{(0)}_{\rm had}(s)$ is a pseudo observable: can be extracted from e^+e^- data only via some theoretical input
- Observed data are dressed by radiative corrections
  $\longrightarrow$ Require undressing



– Photon self energy: to be subtracted to avoid double counting [multiplication with $(\alpha(0)/\alpha(s))^2$]

– ISR: Photonic corrections up to leading log $O(\alpha^3)$, leading + subleading IS e^+e^-–pair production

– **FSR**: **not** known → Model (e.g. sQED)

$\Longrightarrow$ Extraction of $\sigma^{(0)}_{\rm had}(s)$ is **model dependent**

- Consider channel $e^+e^- \to \pi^+\pi^-$:

$$\sigma_{\pi\pi}(s) = \sigma^{\rm point}_{\pi\pi}(s)\,|F_\pi(s)|^2 = \frac{\pi}{3}\frac{\alpha^2\beta_\pi^3}{s}\,|F_\pi(s)|^2$$

- Parameterization: Scalar QED modified by Pion Form Factor : Only correct to lowest order

1. $\pi^{\pm}$ are composite particles for which photonic radiation mechanism is not precisely known.
   $\longrightarrow$ Model Dependence

2. Intermediate hadronic composite state (mainly ρ, ω) can be probed by sufficiently hard photons.

- On the other hand

  — for sufficiently soft photons the intermediate state is neutral and pions are "elementary" scalar particles (factorization)

  — radiative corrections are dominated by ISR

- Question:

  How can we extract $\sigma_{\pi\pi}^{(0)}$ or even better $\sigma_{\pi\pi}^{(\gamma)}$ (= FSR–inclusive cross section) from experimental data with a minimum of model assumptions?

Scan Measurement:

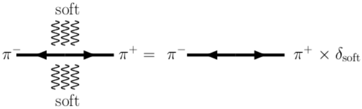
Measurement of the total cross section

$\sigma_{\text{obs}}(s) = \text{``}\sigma^{(0)}(s) + \text{rad. corr.''}$

- Exclusive Scenario:

  data selection (hard cuts) such that pions are back–to–back

  $\Longrightarrow$ phase space left only for soft real photons



– δ_{soft} is theoretically known (universal) and can be "subtracted"

– But: hard virtual FSR corrections are unknown!

$\longrightarrow$ 2 possibilities:

1. subtract virtual FSR corrections $\rightarrow \sigma^{(0)}(s)$
2. add real FSR corrections $\rightarrow \sigma^{(\gamma)}(s)$

$\longrightarrow$ For both possibilities ad hoc models (like sQED) are needed!

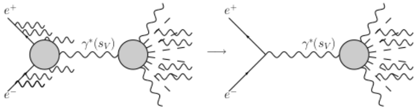
$\Longrightarrow$ Neither $\sigma^{(0)}$ nor $\sigma^{(\gamma)}$ can be obtained in a model independent way!

$\longrightarrow$ model error estimation within sQED: $O(1\%)$.

- Inclusive Scenario:

photon–inclusive measurement:

– "subtract" only ISR to obtain $\sigma_{\rm had}^{(\gamma)}$



- neutral current process $\longrightarrow$ decomposition into 2 separately gauge invariant, Lorentz–covariant Tensors

$\rightarrow$ formal phase space integration
$\Longrightarrow$ ISR $\otimes$ FSR–factorization[2]:

$$\sigma_{\rm obs}(s) \;=\; \quad ds_V \; \sigma^{(\gamma)}(s_V) \; \rho_{\rm ini}^{\rm incl}(s,s_V) + O(\alpha^2)_{\rm IFS}$$

– proof included summation over all orders (factorization hard to see to given perturbative order)

– s_V: not observable (formal integration variable) but boundaries (e.g. $4m_\pi^2 \le s_V \le s$) are!

$\Longrightarrow$ from the measured $\sigma_{\rm obs}$ and the theoretically known $\rho_{\rm ini}^{\rm incl}(s,s_V) \;\Longrightarrow\; \sigma^{(\gamma)}(s_V)$

$\longrightarrow$ best fit to data by choosing suitable parameterization for $\sigma^{(\gamma)}(s_V)$ (e.g. MINUIT package)

– no $O(\alpha)$ IFS (C–invariance)

– $O(\alpha^2)$ IFS: per mill level (no leading log's)

[2] hep-ph/0212386 (J. Gluza, A. Hoefer, S. Jadach, F. Jegerlehner)

- in real experiments: kinematical cuts
  $\Longrightarrow$ ISR $\otimes$ FSR breaking at $O(\alpha)$
- model error is small only if ISR $\otimes$ FSR is broken slightly by cuts

$\Longrightarrow$ Try to be as photon–inclusive as possible

- angular cuts: $\theta_\pi \geq \theta_\pi^{\min}$
  $\longrightarrow$ test model error due to ISR $\otimes$ FSR breaking within sQED:

$$\begin{aligned}
\sigma_{\rm obs}^{\rm cut}(s) &= \sigma_{\rm cut}^{(\gamma)}(s)\,[1+\delta_{\rm ini}(s,\Lambda)] \\
&+ \int_{4m_\pi^2}^{s-2\sqrt{s}\Lambda} ds_V\; \sigma_{\rm cut}^{(\gamma)}(s_V)\; \rho_{\rm ini}^{\rm cut}(s,s_V) - \delta_{\rm cut}^{\rm scan}(s) \\
\delta_{\rm cut}^{\rm scan}(s) &= \sigma_{\rm cut}^{(0)}(s)\,\frac{\alpha}{\pi}\,\{\eta(s)-\eta_{\rm cut}(s)\} + O(\alpha^2) \\
\sigma_{\rm cut}^{(\gamma)}(s) &= |F_\pi^{(\gamma)}(s)|^2\; \sigma_{\rm cut}^{0,\rm point}(s)
\end{aligned}$$

– $\eta_{(\rm cut)}(s)$: integrated $O(\alpha)$ FSR

- extract $|F_\pi^{(\gamma)}(s)|^2 \longrightarrow \sigma^{(\gamma)}(s)$ from data, once includin and once excluding $\delta_{\rm cut}^{\rm scan}(s)$ $[\sigma^{(\gamma)}(s)$ vs. $\hat{\sigma}^{(\gamma)}(s)]$

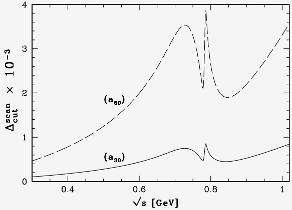
– data fitting with MINUIT

$\Longrightarrow$ model error estimate by comparison:

$$\Delta_{\rm cut}^{\rm scan}(s) = \frac{\sigma_{\rm cut}^{(\gamma)}(s) - \hat{\sigma}_{\rm cut}^{(\gamma)}(s)}{\sigma_{\rm cut}^{(\gamma)}(s)}$$

$\longrightarrow$

– model error is small for $\sigma^{(\gamma)}$ extraction.
– test of sQED by comparison with excl. scenario



$$\Delta^{scan}_{cut}(s) = \frac{\sigma^{(\gamma)}_{cut}(s) - \hat{\sigma}^{(\gamma)}_{cut}(s)}{\sigma^{(\gamma)}_{cut}(s)} = \frac{|F^{(\gamma)}_{\pi}(s)|^2 - |\hat{F}^{(\gamma)}_{\pi}(s)|^2}{|F^{(\gamma)}_{\pi}(s)|^2}$$

$$\sigma^{cut}_{obs}(s) = \hat{\sigma}^{(\gamma)}_{cut}(s)\,[1+\delta_{ini}(s,\Lambda)] + \int_{4m_\pi^2}^{s-2\sqrt{s}\Lambda} ds'\; \hat{\sigma}^{(\gamma)}_{cut}(s')\;\rho^{cut}_{ini}(s,s')$$

$$\sigma^{cut}_{obs}(s) = \sigma^{(\gamma)}_{cut}(s)\,[1+\delta_{ini}(s,\Lambda)] + \int_{4m_\pi^2}^{s-2\sqrt{s}\Lambda} ds'\; \sigma^{(\gamma)}_{cut}(s')\;\rho^{cut}_{ini}(s,s') - \delta^{scan}_{cut}$$

$$\delta^{scan}_{cut}(s) = \sigma^{(0)}_{cut}(s)\,\frac{\alpha}{\pi}\,\{\eta(s) - \eta_{cut}(s)\} + O(\alpha^2)$$

$\delta^{scan}_{cut}(s)$: ISR $\otimes$ FSR–breaking term

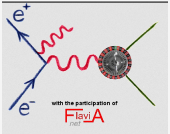
JG: ready for tests/comparisons ISR/FSR: files in fortran, + NAG, Vegas, ...

```
FF=gfortran
FFLAGS=-m32
#-m32

#LIBS = /products/naglib/libnag.a

%.o: %.f
	$(FF) $(FFLAGS) -c -o $@ $<

SOURCES=vegas7.f random.f cseepipi.f kincseepipi.f formkuehn.f elast.f \
int1.f int1neg.f

OBJECTS2=vegas7.o random.o main.o kinematic.o formkuehn.o elast.o hard.o

#LIBS=  /products/naglib/libnag.a

new: vegas7.o random.o main.o kinematic.o formkuehn.o elast.o hard.o
	$(FF) $(FFLAGS) -o csnew -O $(OBJECTS2)
```

Radiative Corrections to Hadron Production in e^+e Annihilations at DAΦNE Energies,

Axel Hoefer, PhD thesis, link

Radiative Return Measurement:



e.g. Φ factory like DAΦNE or B–factories like BARBAR

- e^+e^- collision energy $\sqrt{s}$ is fixed
- measurement of spectral function $d\sigma/ds'$ ($s' = M^2_{\rm had}$) (radiative return due to ISR)

⟶ obtain $\sigma^{(0)}$ or $\sigma^{(\gamma)}$ from data

- Question: why not take $d\sigma/ds_V$ to obtain $\sigma^{(\gamma)}(s_V)$ via

$$\frac{d\sigma_{\rm incl}}{ds_V} = \sigma^{(\gamma)}(s_V)\,\rho^{\rm incl}_{\rm ini}(s,s_V) + O(\alpha^2)_{\rm IFS}$$

- Problem: already to leading order $s_V = s'$ for ISR but $s_V = s$ for FSR (s_V is not observable!)

⟹ error one would make by identifying s_V with s': $O(1)$ FSR

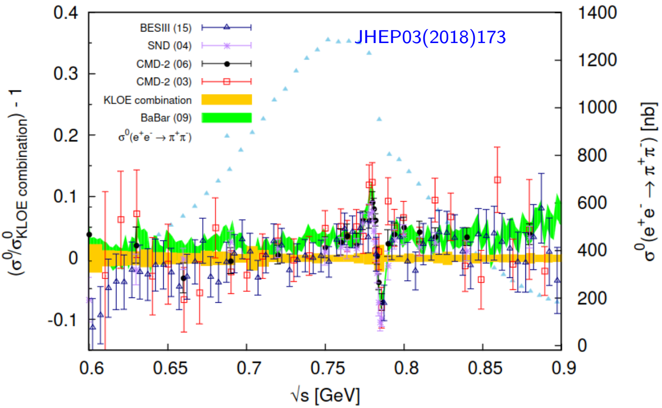
⟶ FSR $O(1)$ contribution is background ⟶ has to be subtracted!

- model error analysis within modified sQED (sQED $\times |F_\pi|^2$) → factorization of FSR

Basic conclusions

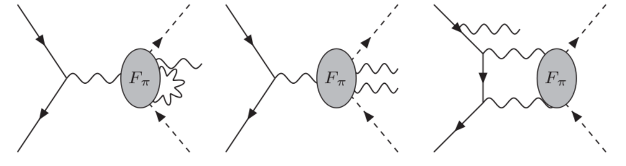
- accurate treatment of FSR in low energy σ_{had} measurements is crucial
  $\longrightarrow$ photon self energy $\longrightarrow \alpha(s)$, a_μ^{had}
- advocate FSR–inclusive measurement at scan experiments
  $\longrightarrow$ only way to obtain $\sigma^{(\gamma)}$ with acceptable model error
- Excl. Scan and Rad. Ret. : neither $\sigma^{(\gamma)}$ nor $\sigma^{(0)}$ without ad hoc model assumptions
- Radiative return measurements suffer from $O(1)$ FSR background
  $\longrightarrow$ unsurmountable problem for $\pi\pi$ measurements at Φ factories at $\sqrt{s'} \leq 0.5$ GeV
- new model independent approach for treating FSR is desirable, especially to controll $O(1)$ FSR background $\longrightarrow$ cross–check scan and τ–decay data

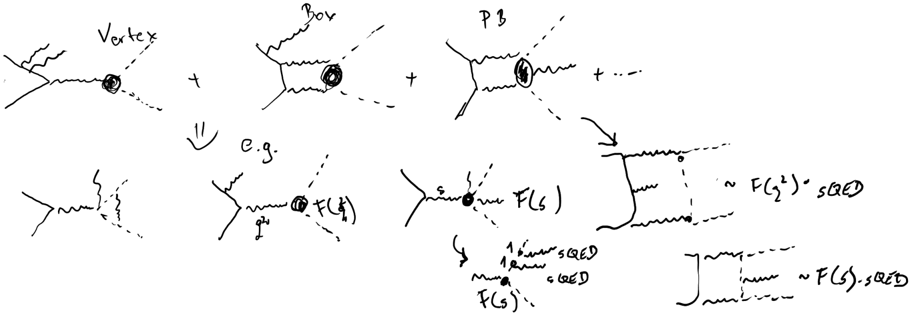
$e^+e^- \to \pi^+\pi^-$, motivation for further improvements



(a) KLOE combination vs. other experiments

The biggest difference between KLOE and BABAR measurements, amounts there to about 2%. It goes even up to 10% around the narrow ω resonance For higher $\pi^+\pi^-$ invariant masses (at 0.9 GeV) the difference raises to 5%.





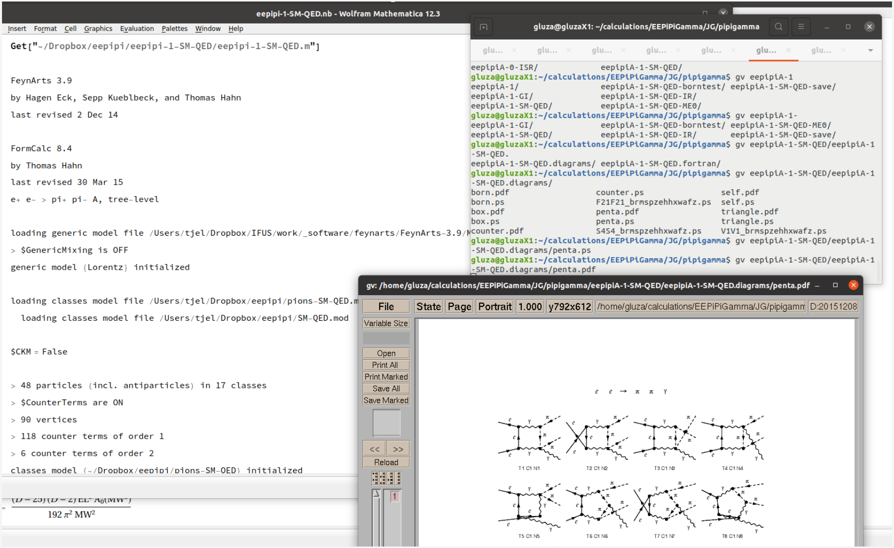
"Standard model radiative corrections in the pion form factor measurements do not explain the a_μ anomaly",

F. Campanario et al, PRD100,076004(2019)

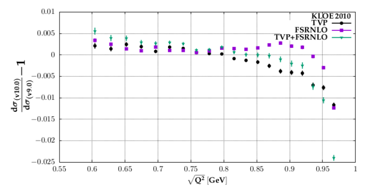
sQED + form factors: FSR at NLO and pentaboxes tested and implemented to Phokhara10.0

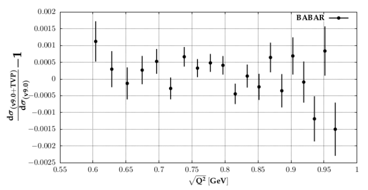
Available at: http://ific.uv.es/~rodrigo/phokhara

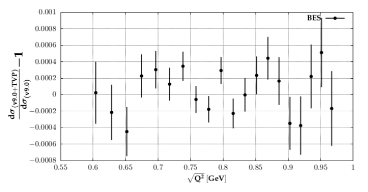
JG: ready for tests/comparisons



NLO pentabox corrections, results for KLOE, BABAR and BESS







- Missing NLO radiative corrections cannot be the source of the discrepancies between the different extractions of the pion form factor performed by BaBar, BES and KLOE.
- They cannot be the origin of the discrepancy between the experimental measurement and the SM prediction of a_μ (too small).

$e^+e^- \to \mu^+\mu^-\gamma$ process

F. Campanario, H. Czyż , JG, M. Gunia, T. Riemann, G. Rodrigo, V. Yundin
Complete QED NLO contributions to the reaction $e^+e^{\to}\mu^+\mu^-\gamma$ and their implementation in the event generator PHOKHARA, JHEP 02 (2014) 114 https://arxiv.org/abs/1312.3610

$e^+e^- \to \mu^+\mu^-\gamma$ — ideal benchmark process for massive tensor reduction

- Two different masses
- Large difference of scales (up to 7 orders in magnitude)
- Quasi-collinear region (due to small electron mass)
- Small number of diagrams

$e^+e^- \to \mu^+\mu^-\gamma$

- Diagram generation with DIANA [Tentyukov, Fleischer]
- Algebraic processing in FORM [Vermaseren]
- Tensor reduction PJFry [Yundin]
- Scalar integrals OneLOop [van Hameren]
- Monte-Carlo PHOKHARA [Rodrigo, Czyż, Kühn]

Compact result for squared one loop amplitude (~ 3 ms per point).

Monte-Carlo integration as a stability test

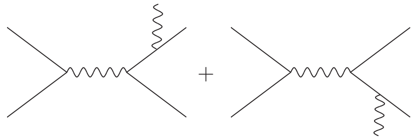
Two realistic sets of kinematical cuts

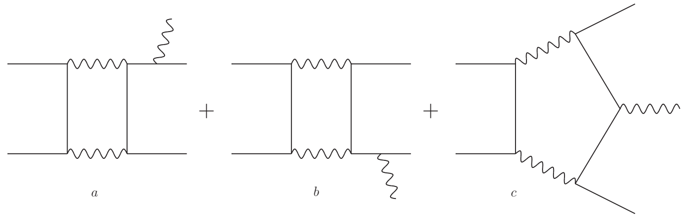
	BaBar	KLOE
E_{CMS}	10.56 GeV	1.02 GeV
$E_{\gamma,\text{min}}$	3 GeV	0.02 GeV
θ_γ	20°–138°	0°–15°, 165°–180°
Q^2	0.25–50 GeV^2	0.25–1.06 GeV^2
$\theta_{\mu^\pm}$	40°–140°	50°–130°

$$m_e = 0.5109989 \cdot 10^{-3}\ \text{GeV},\ m_\mu = 0.105658367\ \text{GeV},$$
$$\alpha(0) = 1/137.03599968.$$

Phase-space cuts for KLOE and BaBar settings.
Q^2 is the invariant mass squared of the muon pair.

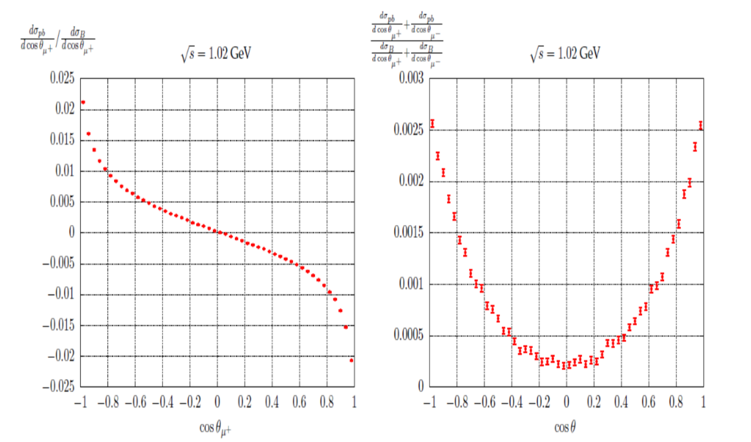
$e^+e^- \to \mu^+\mu^-\gamma$ KLOE Q^2





FSR gauge invariance between tree diagrams (upper picture), and gauge invariance among four and five point one-loop integrals (below). Here diagrams were limited to FSR cases, the same property is present for ISR amplitudes.
Trace and Quadrupole (helicity method) precision - pentagons, Gram determinants.
JG, Radcor 2011 talk, Theoretical improvements for luminosity monitoring at low energies
link

Complete NLO $e^+e^- \to \mu^+\mu^-\gamma$ KLOE Q^2



Phokhara - Relevance of NLO Penta-Box contributions

Staszek Jadach e-Print: hep-ph/0506180 [hep-ph]

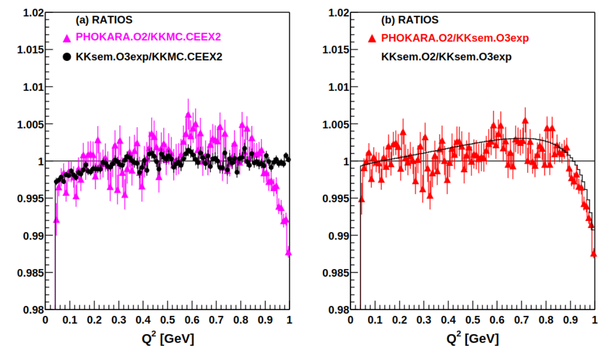


Fig. 1. Muon pair mass (square) spectrum in case of ISR only. $\sqrt{s} = 1.01942$ GeV.

- Agreement to within 0.3% with KKMC.
- Phokhara has no exponentiation, difference for high Q^2

Comment on BaBar arXiv:2308.05233

"Measurement of additional radiation in the initial-state-radiation processes $e^+e^- \to \mu^+\mu^-\gamma$ and $e^+e^- \to \pi^+\pi^-\gamma$ at BABAR"

Category	$\mu\mu$ $m_{\pi\pi} < 1.4\,\mathrm{GeV}/c^2$	$\pi\pi$ $0.6 < m_{\pi\pi} < 0.9\,\mathrm{GeV}/c^2$
LO	0.7716(4)(14)	0.7839(5)(12)
NLO SA-ISR	0.1469(3)(36)	0.1401(2)(16)
NLO LA-ISR	0.0340(2)(9)	0.0338(2)(9)
NLO ISR	0.1809(4)(35)	0.1739(3)(20)
NLO FSR	0.0137(2)(7)	0.0100(1)(16)
NNLO ISR [a]	0.0309(2)(38)	0.0310(2)(39)
NNLO FSR [b]	0.00275(6)(9)	0.00194(12)(50)
NNLO 2LA [c]	0.00103(3)(1)	0.00066(4)(4)

[a] NNLO ISR = 2SA-ISR or SA-ISR + LA-ISR
[b] NNLO FSR = SA-ISR + LA-FSR
[c] NNLO 2LA = 2LA-ISR, LA-ISR + LA-FSR or 2LA-FSR

NNLO effects visible.

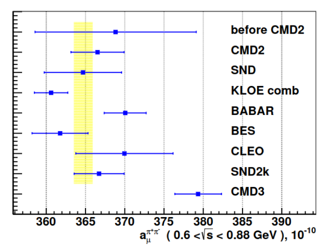
Comparisons with Phokhara, however

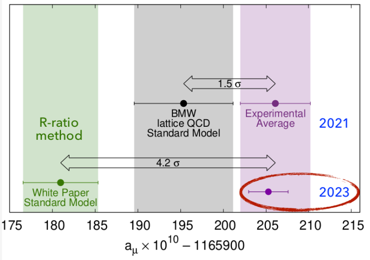
- The event selections used in arXiv:2308.05233 require to have at least 2 hard photons in the final state
- The matrix elements in Phokhara for $e^+e^- \to \pi^+\pi^-\gamma\gamma$ and $e^+e^- \to \mu^+\mu^-\gamma\gamma$ are calculated at LO , so no surprise the accuracy is not high

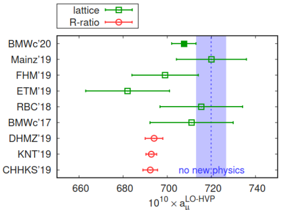
CMD3, new $\pi^+\pi^-$ results, latice QCD, smaller tensions

CMD3: https://arxiv.org/abs/2302.08834

"The CMD-3 result reduces the tension between the experimental value of the a_μ and its Standard Model prediction."







Summary

- Precision goals for SM theory high-energy studies at present and future colliders need progress in precision low energy input, e.g. $\alpha(s)$.
- Development of MC generators for low-energy studies is needed (e.g. ISR with 3 photons).
- Along with, inclusion of h.o. corrections, modeling hadron interactions.

Thank you for your attention.

The Bhabha scattering

- Precise calculations of higher order corrections for the process of Bhabha scattering ($e^+e^- \to e^+e^-$) are necessary for determine colliders luminosity with high accuracy.

$$L_{tot} = \frac{N}{\sigma_{theory}}$$

- High accuracy of luminosity in low energy region is necessary to research low energy hadron cross section from $e + e-$ annihilation process.

$$\sigma_{had} = \frac{N_{had}}{L_{tot}}$$

Carloni Calame, H. Czyz, JG, M. Gunia , G. Montagna, O. Nicrosini, F. Piccinini, T. Riemann, M. Worek,

NNLO leptonic and hadronic corrections to Bhabha scattering and luminosity monitoring at meson factories, JHEP 07 (2011) 126 https://arxiv.org/abs/1106.3178

Aim of the work: calculations at NNLO

- NNLO virtual corrections linked with real corrections and realistic experimental cuts for low energy machines:
  Φ factory Dafne at Frascati, B factories PEP-II (SLAC) and Belle (KEK) and at the charm/τ factory BEPC II, Beijing
- calculation of virtual corrections:
  package bha_nnlo_hf: Actis, Czakon, JG, Riemann
  calculation of real corrections:
  Monte Carlo generators EKHARA:, Czyż, Nowak BHAGHEN–1PH Czyż, Caffo
  Bhabha with additional pairs:
  HELAC–PHEGAS: Papadopoulos, Kanaki, Worek, Cafarella
- comparison complete calculations with approximate ones realized in the MC generator BabaYaga: C.C.Calame, C. Lunardini, G. Montagna, O. Nicrosini, F. Piccinini

- **KKMC** at low and high energies ... here low-energies

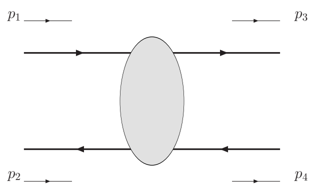
Available MC generators

- BHLUMI v.4.04: Jadach, Placzek, Richter-Was, Was: CPC 1997
- NLLBHA: Arbuzov, Fadin, Kuraev, Lipatov, Merenkov, Trentadue: NPB 1997, CERN 96-01
- SAMBHA: Arbuzov, Haidt, Matteuzzi, Paganoni, Trentadue: hep-ph/0402211
- BabaYaga: Calame, Montagna, Nicrosini, Piccinini, http://www2.pv.infn.it/ hepc omplex/babayaga.html

BabaYaga is presently the main tool for luminosity at flavor factories. BHLUMI was a main tool at LEP.

Kinematical Regions for Bhabha

Two regions where the Bhabha-scattering cross section is large and QED dominated



$$s = (p_1 + p_2)^2 = 4E^2 > 4m_e^2, \quad t = (p_1 - p_3)^2 = -4\,(E^2 - m_e^2)\sin^2\frac{\theta}{2} < 0$$

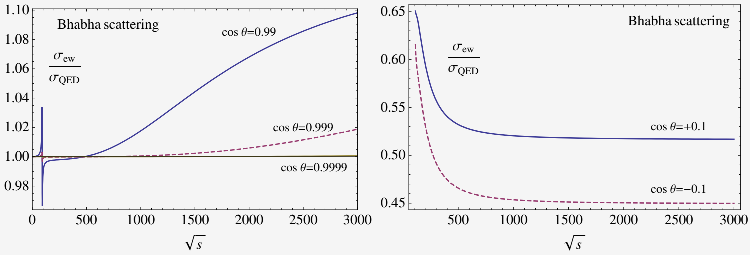
- $\sqrt{s} \sim 10^2$ GeV $\Rightarrow$ small θ
- SABS $\Rightarrow \mathcal{L}$ at LEP, ...

  $\sim$ a few degrees

- $\sqrt{s} \sim$ 1-10 GeV $\Rightarrow$ large θ
- LABS $\Rightarrow \mathcal{L}$ at KLOE, ...

  $\theta \sim 55^0 \ - \ 125^0$

Ratio of electroweak to QED Bhabha scattering cross-section at large and small angles as a function of CoM



Rough estimation, tree level calculation, no cuts etc,
PRD78 (2008) 085019

$$\cos(\theta) = 0.9999 \quad \rightarrow \quad \theta = 0.8^o$$
$$PRD78(2008)085019 \cos(\theta) = 0.999 \quad \rightarrow \quad \theta = 2.5^o$$
$$\cos(\theta) = 0.99 \quad \rightarrow \quad \theta = 8^o$$
$$\cos(\theta) = 0.1 \quad \rightarrow \quad \theta = 84^o$$

Cuts dependence study for different experiments

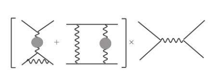
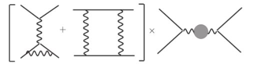
1.Φ factories KLOE/DAΦNE (Frascati)

(a) $\sqrt{s} = 1.02$ GeV
(b) $E_{min} = 0.4$ GeV
(c) For $\theta\pm$ two selections have to be checked
i. tighter selection $55^o < \theta\pm < 125^o$
ii. wider selection $20^o < \theta\pm < 160^o$
(d) ζ_{max}=4,5,6,7,8,...,14 deg., with reference value $\zeta_{max} = 9^o$

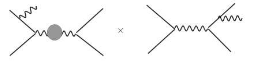
2. B-factories BABAR/PEP-II (SLAC) & BELLE/KEKB (KEK)

(a) $\sqrt{s} = 10.56$ GeV
(b) $|\vec{p}_+|/E_{beam} > 0.75$ and $|\vec{p}_-|/E_{beam} > 0.50$
or $|\vec{p}_-|/E_{beam} > 0.75$ and $|\vec{p}_+|/E_{beam} > 0.50$
(c) For $|cos(\theta\pm)|$ the following selections have to be checked
i. $|cos(\theta\pm)| < 0.65$ and $|cos(\theta+)| < 0.60$ or $|cos(\theta-)| < 0.60$
ii. $|cos(\theta\pm)| < 0.70$ and $|cos(\theta+)| < 0.65$ or $|cos(\theta-)| < 0.65$
iii. $|cos(\theta\pm)| < 0.60$ and $|cos(\theta+)| < 0.55$ or $|cos(\theta-)| < 0.55$
(d) ζ^{3d}_{max}= 20,22,24,...,40 deg., with reference value $\zeta^{3d}_{max} = 30^o$

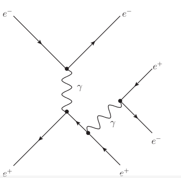
- the σ_{virt} consists of virtual two-loop corrections σ_{2L} and loop-by-loop corrections σ_{1L1L}

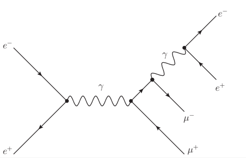



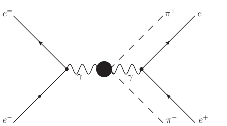
- contributions with real photon emission $\sigma_{\gamma} = \sigma_{\gamma,soft}(\omega) + \sigma_{\gamma,hard}(\omega)$



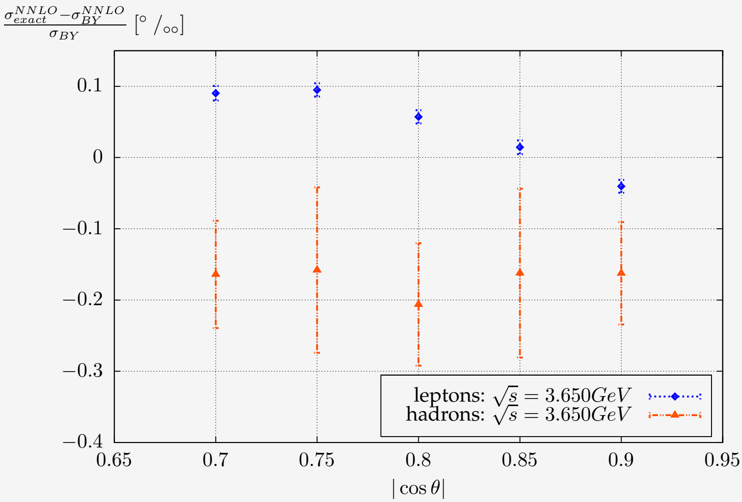
- contributions with real pair or hadron emission
  $\sigma_{real} = \sigma_{e^+e^-(e^+e^-)} + \sigma_{e^+e^-(f^+f^-)} + \sigma_{e^+e^-(hadrons)}$







BES III - relative difference in per-mile



Central reference cuts

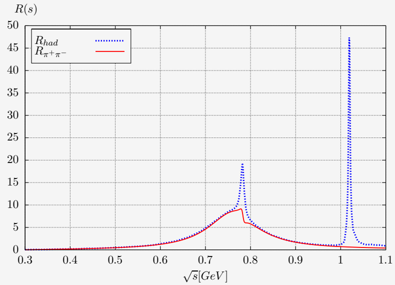
	$\sqrt{s}$		σ	$S_{e^+e^-}$	$S_{lep}[10^{-3}]$	S_{had}	S_{tot}
KLOE	1.020	NNLO		-3.935(4)	-4.472(4)	1.02(2)	-3.45(2)
		BY_{NLO}	455.71	-3.445(2)	-4.001(2)	0.876(5)	-3.126(5)
BES	3.097	NNLO		-2.246(8)	-2.771(8)	-	-
		BY_{NLO}	158.23	-2.019(3)	-2.548(3)	-	-
BES	3.650	NNLO		-1.469(9)	-1.913(9)	-1.3(1)	-3.2(1)
		BY_{NLO}	116.41	-1.521(4)	-1.971(4)	-1.071(4)	-3.042(5)
BES	3.686	NNLO		-1.435(8)	-1.873(8)	-	-
		BY_{NLO}	114.27	-1.502(4)	-1.947(4)	-	-
BaBar	10.56	NNLO		-1.48(2)	-2.17(2)	-1.69(8)	-3.86(8)
		BY_{NLO}	5.195	-1.40(1)	-2.09(1)	-1.49(1)	-3.58(2)
Belle	10.58	NNLO		-4.93(2)	-6.84(2)	-4.1(1)	-10.9(1)
		BY_{NLO}	5.501	-4.42(1)	-6.38(1)	-3.86(1)	-10.24(2)

The σ is the cross section in nb from BabaYaga(at)NLO, and $S_x = \frac{\sigma_x}{\sigma}$ in per-milles with $x = e^+e^-, lep, tot$, where tot stands for leptonic (lep) + hadronic corrections.

The vacuum polarisation function:

$$\Pi(q^2) = \frac{\alpha q^2}{3\pi} \int_{m_{\Pi 0}^2}^{\inf} \frac{dz}{z} \frac{R(z)}{q^2 - z + i\epsilon}$$

For leptons VP analytical expresions were used.
For pions VP numerical calculations of the integral were used.
For hadrons program VPHLMNT (T.Teubner et all.) was used.



resonance	$M_{\rm res}$ [GeV]	$\Gamma_{\rm res}^{e^+e^-}$ [keV]
J/ψ(1S)	3.096916	5.55
ψ(2S)	3.686093	2.33
Υ(1S)	9.46030	1.34
Υ(2S)	10.02326	0.612
Υ(3S)	10.3552	0.443
Υ(4S)	10.5794	0.272
Υ(5S)	10.865	0.31
Υ(6S)	11.019	0.13

	$\sqrt{s}$	$\sigma_{\rm rest,res}$	$\sigma_{\rm rest,res'}$	σ_B
KLOE	1.020	[all n.r.] -0.04538	[n.r. without J/ψ(1S)] -0.0096	529.5
BES	3.097	[all n.r.] 228.08	[n.r. without J/ψ(1S)] -0.0258	14.75
BES	3.650	[all n.r.] -0.1907	[n.r. without ψ(2S)] -0.023668	123.94
BES	3.686	[all n.r.] -62.537	[n.r. without ψ(2S)] -0.0254	121.53
BaBar	10.56	[all n.r.] -0.0163	[n.r. without Υ(4S)] -0.01438	6.744
Belle	10.58	[all n.r.] 0.04393	[n.r. without Υ(4S)] -0.0137	6.331

$$R_{res}(z) = \frac{9\pi}{\alpha^2} M_{res} \Gamma_{res}^{e^+e^-} \delta(z - M_{res}^2) \, .$$

$$\frac{d\sigma_{\rm rest}}{d\Omega} = \frac{9\alpha^2}{\pi \ s} \frac{\Gamma_{\rm res}^{e^+e^-}}{M_{\rm res}} \left\{ \frac{F_1(M_{\rm res}^2)}{t - M_{\rm res}^2} + \frac{1}{s - M_{\rm res}^2} \left[F_2(M_{\rm res}^2) + F_3(M_{\rm res}^2) \ln \left| 1 - \frac{M_{\rm res}^2}{s} \right| \right] \right\} .$$

Adaptive VEGAS is able to identify narrow resonances!

NNLO conclusions for meson factories Bhabha scattering

- Pions approximation is not enough.
- Exact calculations of NNLO massive corrections to Bhabha scattering were presented.
- The theoretical accuracy of the generator BabaYaga@NLO was tested. For reference realistic event selections the maximum observed difference is at the level of 0.07%. When cuts are varied the sum of the missing pieces can reach 0.1%, but for very tight acollinearity cuts only.
- Stability of the results with changing of the event selections was examined - there aren't dramatical changes of errors between points with real experimental cuts and their neighbours.
- NNLO massive corrections are relevant for precision luminosity measurements with 10^{-3} accuracy. The electron pair contribution is the largely dominant part of the correction. The muon pair and hadronic corrections are the next-to-important effects and quantitatively on the same grounds. The tau pair contribution is negligible for the energies of meson factories.