Heavy flavour physics

Lecture 1

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12 December 2024

Contents

Lecture 1

- What is flavour physics
- CKM mechanism
- SM flavour sector
- Flavour sector beyond the SM

What is flavour physics?

Fermions ("matter")		Bosons ("forces")
Quarks		$gggggggg$
uuu ccc ttt		γ
ddd sss bbb	× { MATTER, ANTIMATTER }	W^+
		W^-
Leptons		Z
e μ τ		
ν_e ν_μ ν_τ		H

Flavour physics:

- transitions between different kinds of quarks
- its all about weak interactions...
- strong interactions as a "background"

Parameters of the Standard Model

- 3 gauge couplings: α_{EM}, α_{weak}, α_{strong}
- 2 Higgs parameters: v, m_H
- 6 quark masses:
- 3 quark mixing angles + 1 phase *(CKM matrix)*
- 3 (+3) lepton masses
- (3 lepton mixing angles + 1 phase) *(PMNS matrix)*

( ) = with Dirac neutrino masses

FLAVOUR PARAMETERS

Open questions in flavour physics

- Why are there so many different fermions?
- What is responsible for their organisation into generations / families?
- Why are there 3 generations / families each of quarks and leptons?
- Why are there flavour symmetries?
- What breaks the flavour symmetries?
- What causes matter–antimatter asymmetry?

Flavour physics issues

Families / generations

3 pairs of quarks (are we sure?)

3 pairs of leptons (are we sure?)

Hierarchies

m(t) > m(c) > m(u) m(b) > m(s) > m(d)

m(τ) > m(μ) > m(e) $m(\nu_\tau) > m(\nu_\mu) > m(\nu_e)$?

Mixings & couplings

hierarchy in quark mixings

what about lepton mixings?

Flavour physics issues

Mixings & couplings

universality

(no) flavour changing neutral currents (at tree level in the Standard Model)

Symmetry principles & their violation

P violation / C violation

CP violation / T violation

baryon asymmetry of the universe

lepton flavour violation

Unification

Why is heavy flavour physics interesting?

- Hope to learn something about the mysteries of the flavour structure of the Standard Model
- CP violation and its connection to the matter–antimatter asymmetry of the Universe
- Discovery potential far beyond the energy frontier via searches for rare or SM forbidden processes

What breaks the flavour symmetries?

- In the Standard Model, the vacuum expectation value of the Higgs field breaks the electroweak symmetry

- Fermion masses arise from the Yukawa couplings of the quarks and charged leptons to the Higgs field *(taking* $m_\nu = 0$*)*

- The CKM matrix arises from the difference between weak eigenstates and mass eigenstates

- Consequently, the only flavour-changing interactions are the charged current weak interactions

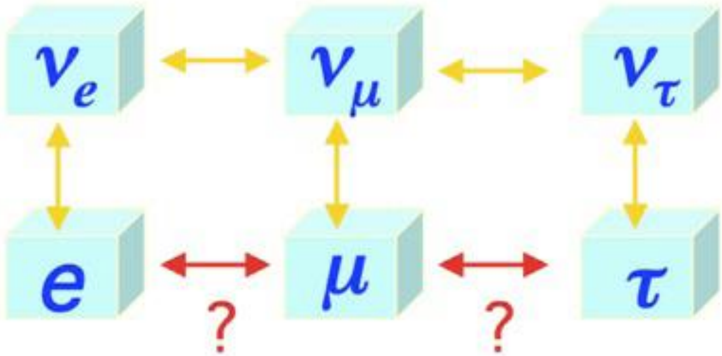
  $\rightarrow$ *no flavour-changing neutral currents (GIM mechanism)*

  $\rightarrow$ *not generically true in most extensions of the SM*

  $\rightarrow$ *flavour-changing processes provide sensitive tests*

Lepton flavour violation

- No right-handed neutrinos in the SM, implies they are massless
  - → neutrinos only left-handed (chirality)
  - → antineutrinos only right-handed (chirality)
- Neutrino oscillations show they have small but finite masses
  - → **where are the right-handed neutrinos?**
  - → charged lepton flavour violation
  - → physics beyond the Standard Model
- Why do we not observe the decay $\mu \rightarrow e\gamma$?
  - → exact (but accidental) lepton flavour conservation in the SM with $m_\nu=0$
  - → SM loop contributions suppressed by $(m_\nu / m_W)^4$
  - → LFV - a mechanism beyond the SM needed



What causes matter–antimatter asymmetry?

- The CKM matrix arises from the relative misalignment of the Yukawa matrices for the up- and down-type quarks

$$V_{CKM} = U_u U_d^+$$

(U - diagonalisation of mass matrices)

- It is a 3x3 complex unitary matrix

  $\rightarrow$ described by 9 (real) parameters

  $\rightarrow$ 5 can be absorbed as phase differences between the quark fields

  $\rightarrow$ 3 can be expressed as (Euler) mixing angles

  $\rightarrow$ the fourth makes the CKM matrix complex (i.e. gives it a phase)

  - weak interaction couplings differ for quarks and antiquarks
  - CP violation

Reminder - 1964: CP violation

Both $K^0 \to \pi\pi$ and anti-$K^0 \to \pi\pi$ occur

- K^0 may turn into its antiparticle, so are not mass eigenstates

The mass eigenstates are:

$$|K_S^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$$

$$|K_L^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$$

CP operator gives:

$$\mathbf{CP}|K^0\rangle = |\bar{K}^0\rangle, \mathbf{CP}|K_S\rangle = +|\bar{K}_S\rangle, \mathbf{CP}|K_L\rangle = -|\bar{K}_L\rangle$$

Thus:

$$\text{only } K_S \to \pi\pi, \text{ but } K_L \to 3\pi$$

Under CP symmetry:

K_S(CP=+1): can only decay (hadronically) to 2π's (CP=+1)

K_L(CP=-1): can only decay (hadronically) to 3π's (CP=-1)

If CP conserved, should *not* see the decay $K_L \to 2\pi$'s

Reminder - Cronin-Fitch experiment

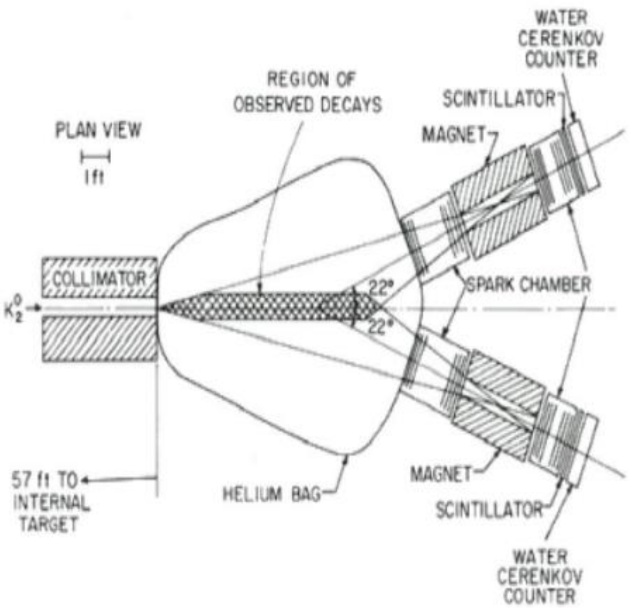
Observation of $K_2 \to \pi^+\pi^-$ $\to$ Christenson, Cronin, Fitch, Turlay (1964)

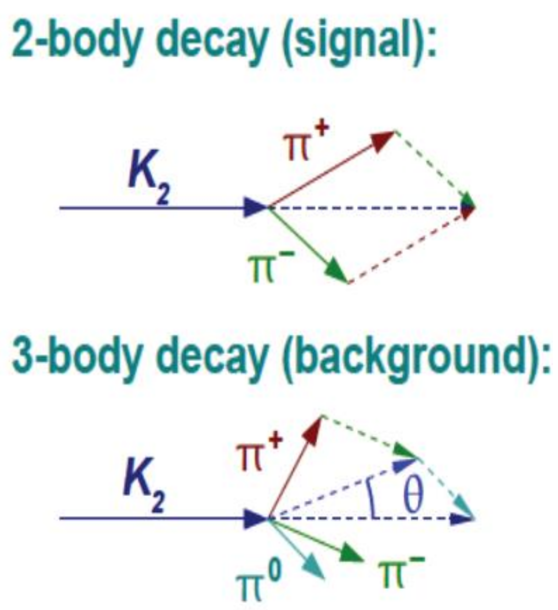
The experiment shoot protons on a target to produce K^0, after a long enough trip in a vacuum pipe, they achieved a pure K_2 beam.

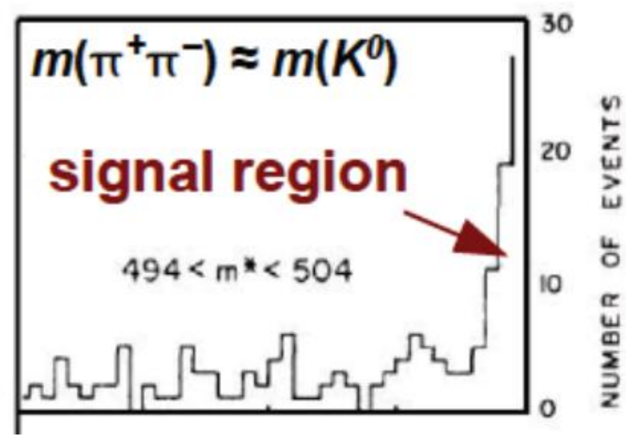
Experimentally use invariant mass (energy conservation) and angle between K_2 and $\pi^+\pi^-$ (momentum conservation).

Find excess of ~56 events in the signal region: **$BF(K_2 \to \pi^+\pi^-) \sim 2 \times 10^{-3}$**

$\to$ **CP violation!**





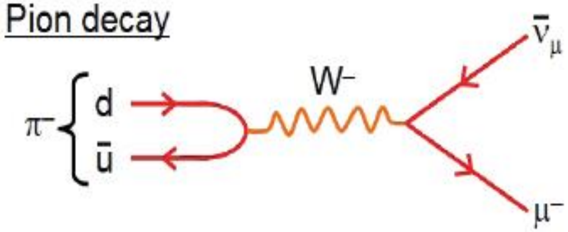


Reminder - 1963: Cabibbo mixing

The weak coupling did not look to be universal:

- $s \to u$ e.g. $K^+ \to \mu^+ \nu_\mu$
- $d \to u$ e.g. $\pi^+ \to \mu^+ \nu_\mu$
- $s \to u$ transitions suppressed by a factor ~20

Pion decay



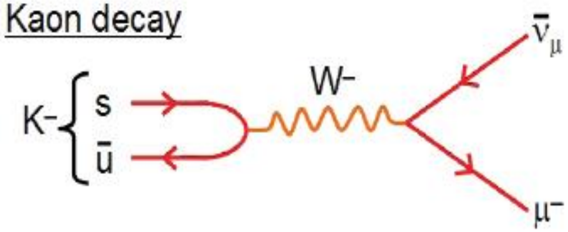
$\pi^-(d\bar{u}) \to \mu^- + \bar{\nu}_\mu$

Cabibbo (1963): **weak interactions couples to a linear combination:**

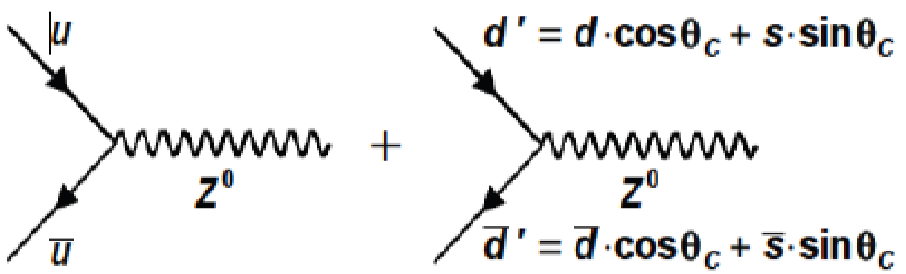
$$\mathbf{d' = d \cdot \cos\theta_c + s \cdot \sin\theta_c}$$

$\sin\theta_c = 0.22$ (empirically)

Kaon decay



$K^-(s\bar{u}) \to \mu^- + \bar{\nu}_\mu$



But, if the neutral weak currents also couple to *d'* expect large FCNC

Experimentally, however, $BR(K \to \mu\mu) \sim 7 \times 10^{-9}$

Reminder - 1970: GIM mechanism

$K^+ \to \mu^+\nu_\mu$ so why not $K^0 \to \mu^+\mu^-$?

$K^+ \to \pi^0\mu^+\nu_\mu$ so why not $K^0 \to \pi^0\mu^+\mu^-$?

$BR(K_L \to \mu^+\mu^-) \sim 7 \cdot 10^{-9}$ $BR(K_L \to e^+e^-) \sim 10^{-11}$

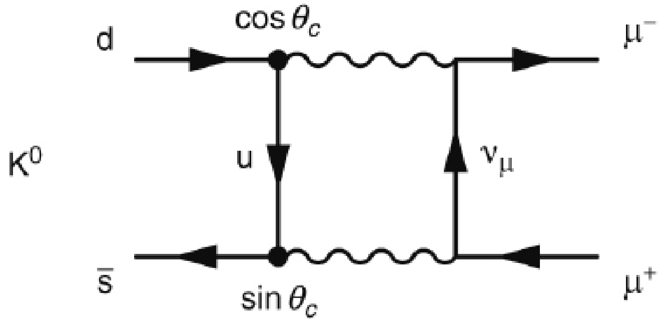
$BR(K^0 \to \pi^0\mu^+\mu^-) <\sim 10^{-10}$

GIM (Glashow, Iliopoulos, Maiani) mechanism (1970)

assume a **new (not yet observed) quark** in SU(2) quark doublets

no tree level flavour changing neutral currents

suppression of FCNC via loops



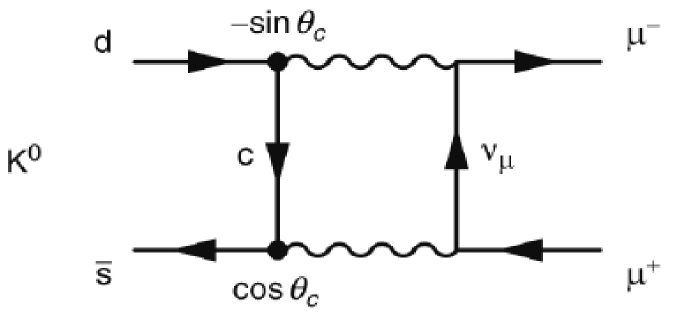
Requires that quarks come in pairs (doublets)

prediction of a **2nd** up-type quark

additional Feynman graph cancels the „*u*-box graph"

prediction of m(c) ≈ 1.5 GeV

→ *Gaillard and Lee (1974)*



Reminder - 1973: The CKM mechanism

1973: **Kobayashi & Maskawa** demonstrate that CP violation arises naturally from quark mixing if there are **3 generations of quarks**

CKM Matrix

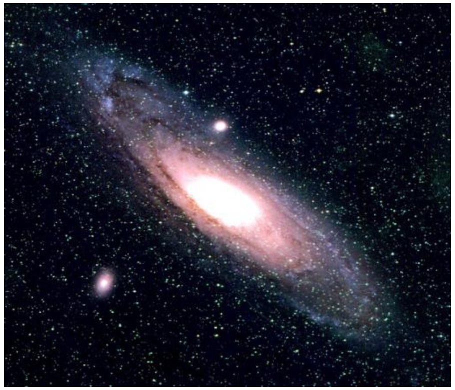
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

3x3 matrix of complex numbers

- ⇒ 18 parameters
- unitary ⇒ 9 parameters
- quark fields absorb unobservable phases ⇒ 4 parameters

3 mixing angles and 1 phase allowing for CP violation

Matter-antimatter asymmetry



- We know that the matter – anti-matter symmetry in the Universe is broken: the Universe consists of matter.
- But, shortly after the Big Bang, there should have been equal amounts of matter and anti-matter

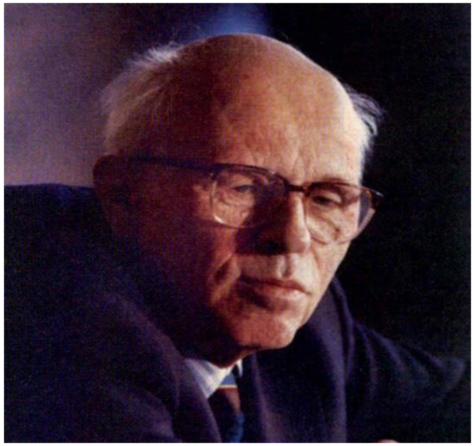
  → how did the Universe develop a preference of matter?

In 1966, Andrei Sakharov showed that necessary for evolution of matter dominated universe, from symmetric initial state, are:

(1) baryon number violation

(2) C & CP violation

(3) thermal inequilibrium



- No significant amounts of antimatter observed!
- (N(baryon) – N(antibaryon)) / $N_\gamma \sim 10^{-10}$

Standard Model *CPV* cannot explain matter asymmetry in the universe
→ the *only* CP violating phase in SM leads to $10^{-17}\ \Delta N_B/N_\gamma$

SM flavour sector

Flavour in Standard Model

- Higgs field was introduced to give masses to W^+, W^- and Z^0 bosons (after SBB)
- Since we have a Higgs field we can add (ad-hoc) interactions between the Higgs field ϕ and the fermions in a gauge invariant way (Yukawa couplings):

doublets, singlet

$$-L_{Yukawa} = Y_{ij}\left(\overline{\psi}_{Li}\,\phi\right)\psi_{Rj} + h.c.$$

- The quark flavour structure within the SM is described by 6 couplings and 4 CKM params
- It is convenient to move the CKM matrix from Yukawa sector to the weak current sector
- We can diagonalize the Y_{ij} matrices, such that we arrive in the „mass basis"

However, then the Lagrangian of the charged weak current should also be rewritten:

$$-L_{W^+} = \frac{g}{\sqrt{2}}\left(\overline{u},\overline{c},\overline{t}\right)_L \left(V_{CKM}\right)\begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \gamma^{\mu} W^+_{\mu}$$

CKM matrix (rotation matrix)

V_{CKM} originates from the diagonalization of the Yukawa couplings

Weak interactions in the SM

- After SSB, the charged current of a W^- exchange can be written as:

$$J^{\mu -} = (\overline{u}_L, \overline{c}_L, \overline{t}_L)\gamma^\mu V_{\text{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

- Weak interaction only couples to left-handed field
  $\rightarrow$ left-handed quarks or right-handed anti-quarks
  $\rightarrow$ manifestly violates parity

The weak eigenstates are related to the mass eigenstates by the CKM matrix:

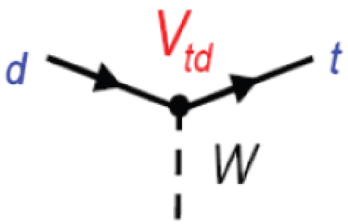
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak eigenstates — Mass eigenstates

CP transformation & the weak interaction

Quarks

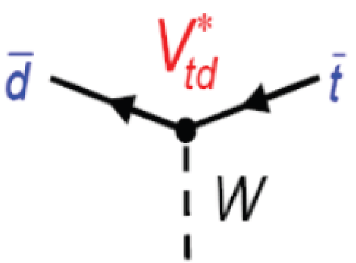
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



CP

Anti-quarks:

$$\begin{pmatrix} \bar{d}' \\ \bar{s}' \\ \bar{b}' \end{pmatrix} = \begin{pmatrix} V^*_{ud} & V^*_{us} & V^*_{ub} \\ V^*_{cd} & V^*_{cs} & V^*_{cb} \\ V^*_{td} & V^*_{ts} & V^*_{tb} \end{pmatrix} \begin{pmatrix} \bar{d} \\ \bar{s} \\ \bar{b} \end{pmatrix}$$



CP violation requires complex matrix elements

Relative phases

Q: ***How many parameters does the CKM matrix have?***

18 parameters (9 complex numbers):

- 9 unitary conditions: $V_{CKM} V_{CKM}^{\dagger} = 1$
- 5 relative phases of the quark fields
- 4 parameters (*)

(*) 3 (real) **Euler angles** and 1 **phase** (single source of CP violation in the SM)

- with 2 generations there is only one real (Euler) angle: the Cabbibo angle
- CP violation requires 3 generations!

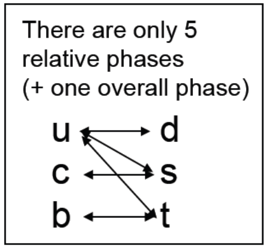
When I do a phase transformation of the (left-handed) quark fields:

$$u_{Lj} \to e^{i\phi_j} u_{Lj} \qquad d_{Lk} \to e^{i\phi_k} d_{Lk}$$

And a simultaneous transformation of the CKM matrix:

$$V \to \begin{pmatrix} e^{-i\phi_u} & & \\ & e^{-i\phi_c} & \\ & & e^{-i\phi_t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{-i\phi_d} & & \\ & e^{-i\phi_s} & \\ & & e^{-i\phi_b} \end{pmatrix}$$

or $V_{jk} \to \exp\left(-i\left(\phi_j + \phi_k\right)\right) V_{jk}$

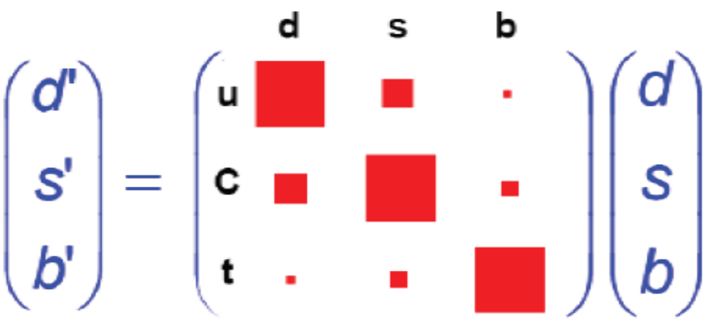


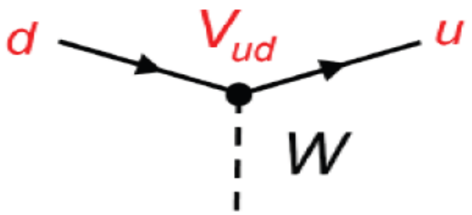
The charged current (i.e. the physics) remains invariant:

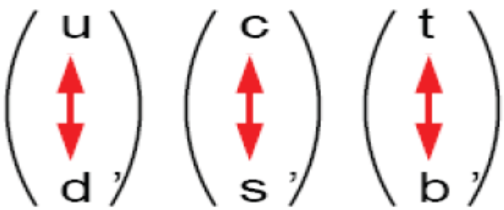
$$J^{\mu}_{CC} = \overline{u_{Li}} \gamma^{\mu} V_{ij} d_{Lj}$$

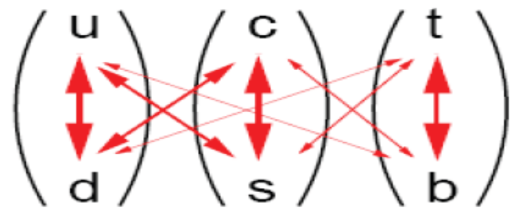
In other words, I can always absorb the 5 relative phases by redefining the quark fields → **these 5 phases are unobservable**

Hierarchy in quark mixing









- Diagonal elements of CKM matrix are close to one
- Only small off-diagonal contributions
- Mixing between quark families is „CKM suppressed”

Wolfenstein parametrization

Makes use of the fact that the off-diagonal elements are small compared to the diagonal elements

→ expansion in $\lambda \approx V_{us}$, $A \approx V_{cb} / \lambda^2$ and ρ, η

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} =$$

$\lambda \sim 0.22$ (=$\sin\theta_C$, sine of Cabibbo angle)
$A \sim 1$ (actually 0.80)
$\rho \sim 0.14$
$\eta \sim 0.34$

$$= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda^2 \equiv \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2} \qquad A^2\lambda^4 \equiv \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2} \qquad \bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

$$\rho + i\eta = \frac{\sqrt{1 - A^2\lambda^4}(\overline{\rho} + i\overline{\eta})}{\sqrt{1-\lambda^2}\,[1 - A^2\lambda^4(\overline{\rho} + i\overline{\eta})]}$$

CKM angles and unitarity triangle

Writing the complex elements explicitly:

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & \sim \lambda^3 e^{-i\gamma} \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ \sim \lambda^3 e^{-i\beta} & \sim -\lambda^2 e^{-i\beta_s} & 1 \end{pmatrix} + O(\lambda^4)$$

Definition of the angles:

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

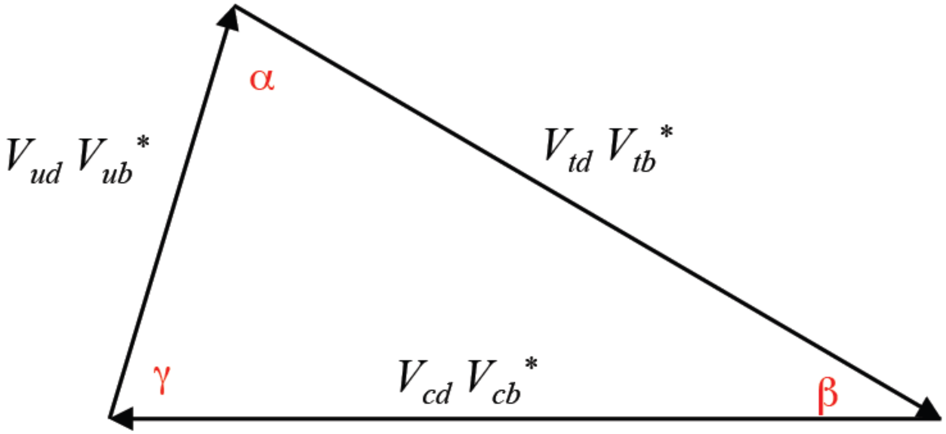
$$\gamma \equiv \arg\left(-\frac{V_{ud}V_{tb}^*}{V_{cd}V_{cb}^*}\right)$$

$$\beta_s \equiv \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right)$$

Using one of the 9 unitarity relations: $V_{\rm CKM} V_{\rm CKM}^\dagger = 1$

Multiply first „d” column with last „b” column:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



CKM angles and unitarity triangle

Writing the complex elements explicitly:

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & \sim \lambda^3 e^{-i\gamma} \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ \sim \lambda^3 e^{-i\beta} & \sim -\lambda^2 e^{-i\beta_s} & 1 \end{pmatrix} + O(\lambda^4)$$

Definition of the angles:

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

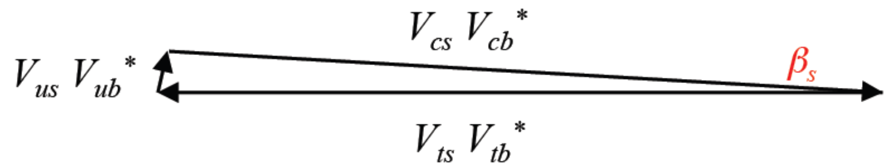
$$\gamma \equiv \arg\left(-\frac{V_{ud}V_{tb}^*}{V_{cd}V_{cb}^*}\right)$$

$$\beta_s \equiv \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right)$$

Using another unitarity relation: $V_{\mathrm{CKM}} V^{\dagger}_{\mathrm{CKM}} = 1$

Multiply second „s” column with last „b” column:

$$V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0$$



„Squashed unitarity triangle”

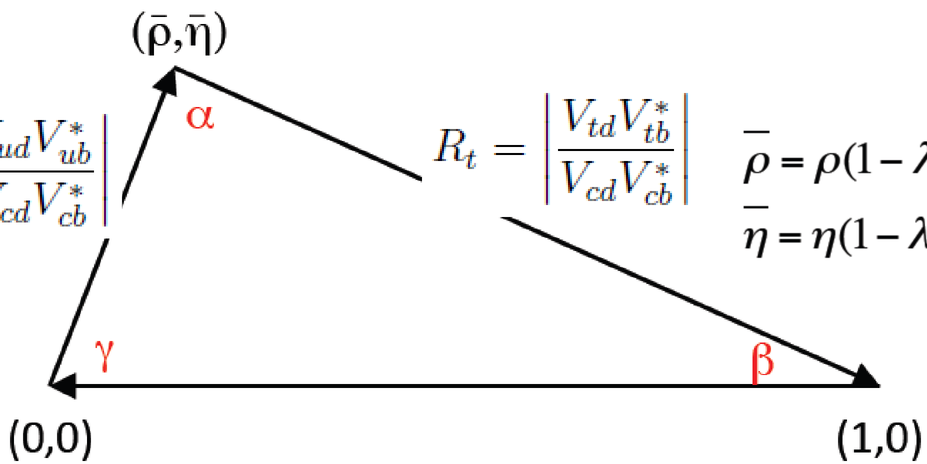
CKM angles and unitarity triangle

- Imposing unitarity to the CKM matrix results in six equations that can be seen as the sum of three complex numbers closing a triangle in the complex plane
- Two of these triangles are relevant for study of CP-violation in B-physics and define the angles

Normalized CKM triangle:

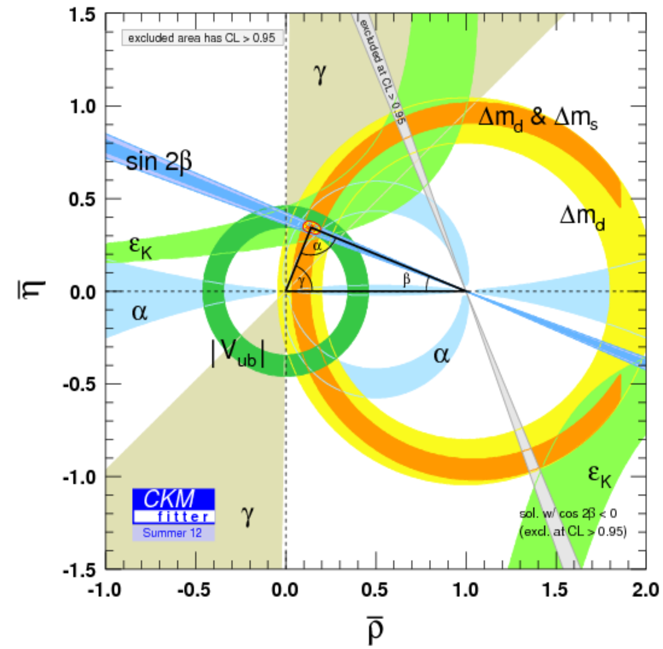
→ divide each side by $V_{cd}\, V^*_{cb}$

$$R_u = \left|\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}}\right| \qquad R_t = \left|\frac{V_{td}V^*_{tb}}{V_{cd}V^*_{cb}}\right|$$



$$\bar{\rho} = \rho(1-\lambda^2/2)$$
$$\bar{\eta} = \eta(1-\lambda^2/2)$$

Current knowledge of UT (from CKMFitter)

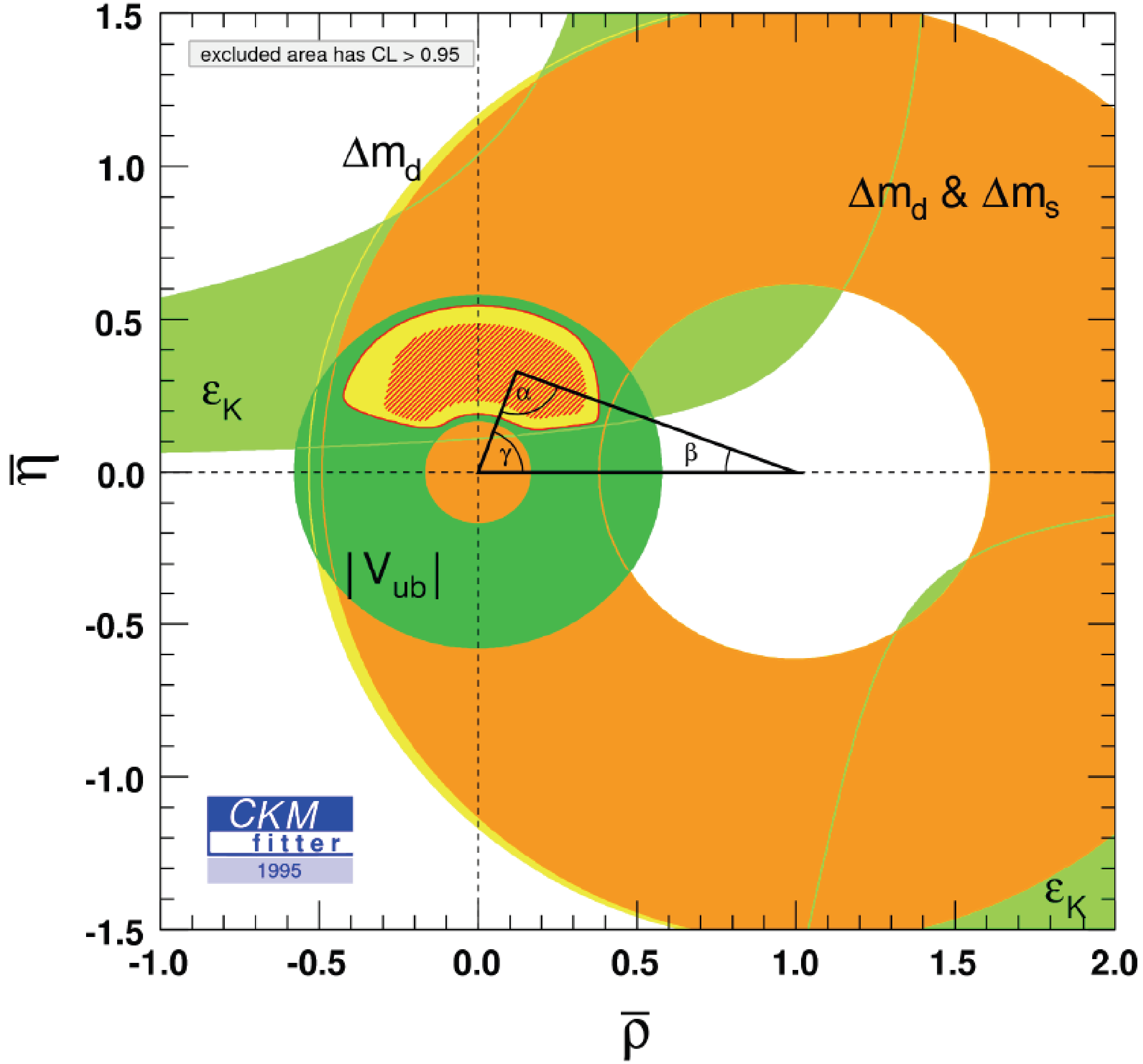


The unitarity triangle:

- Shows the size of the CP violation (no CPV means no triangle!)
- Presents our knowledge of CKM parameters
- Shows how consistent the measurements are!

Progress in UT

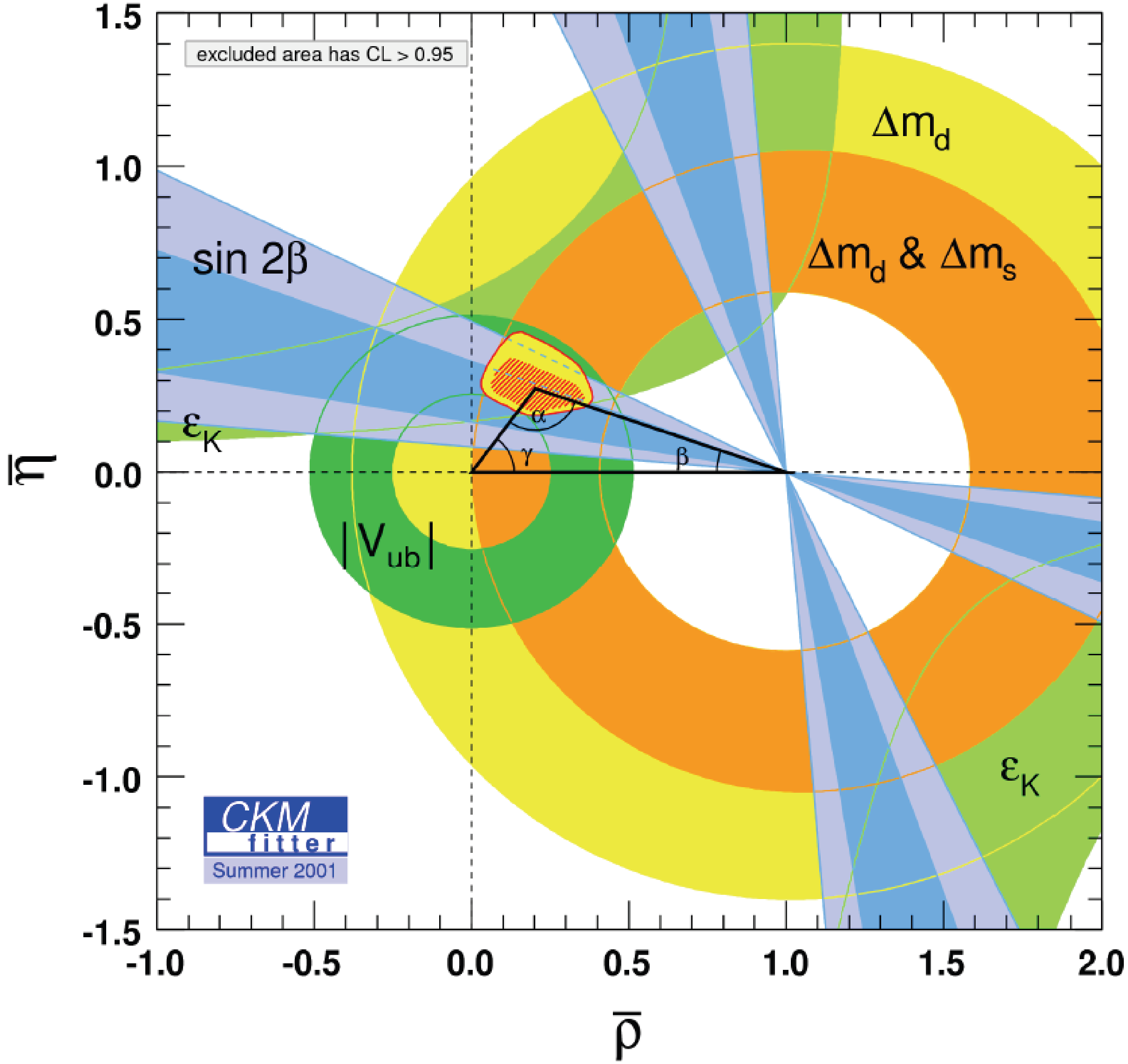
1995



Progress in UT

2001

first observation
of non-kaon CPV



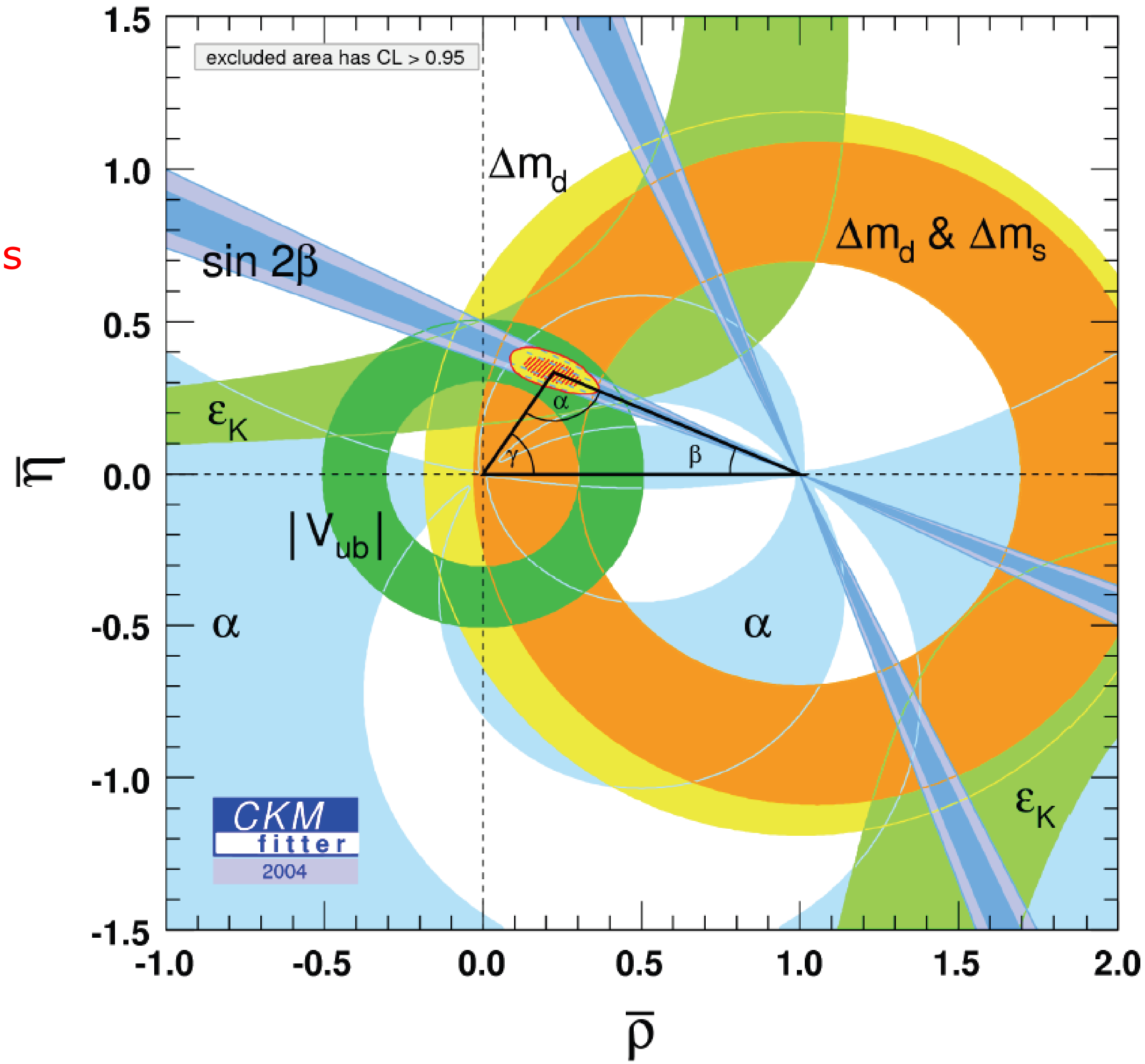
Progress in UT

2004

improvement in lattice calculations

more data on $A_{CP}(B \rightarrow J/\psi\ K_s)$

first constraints on the angle α from $B \rightarrow \rho\rho$



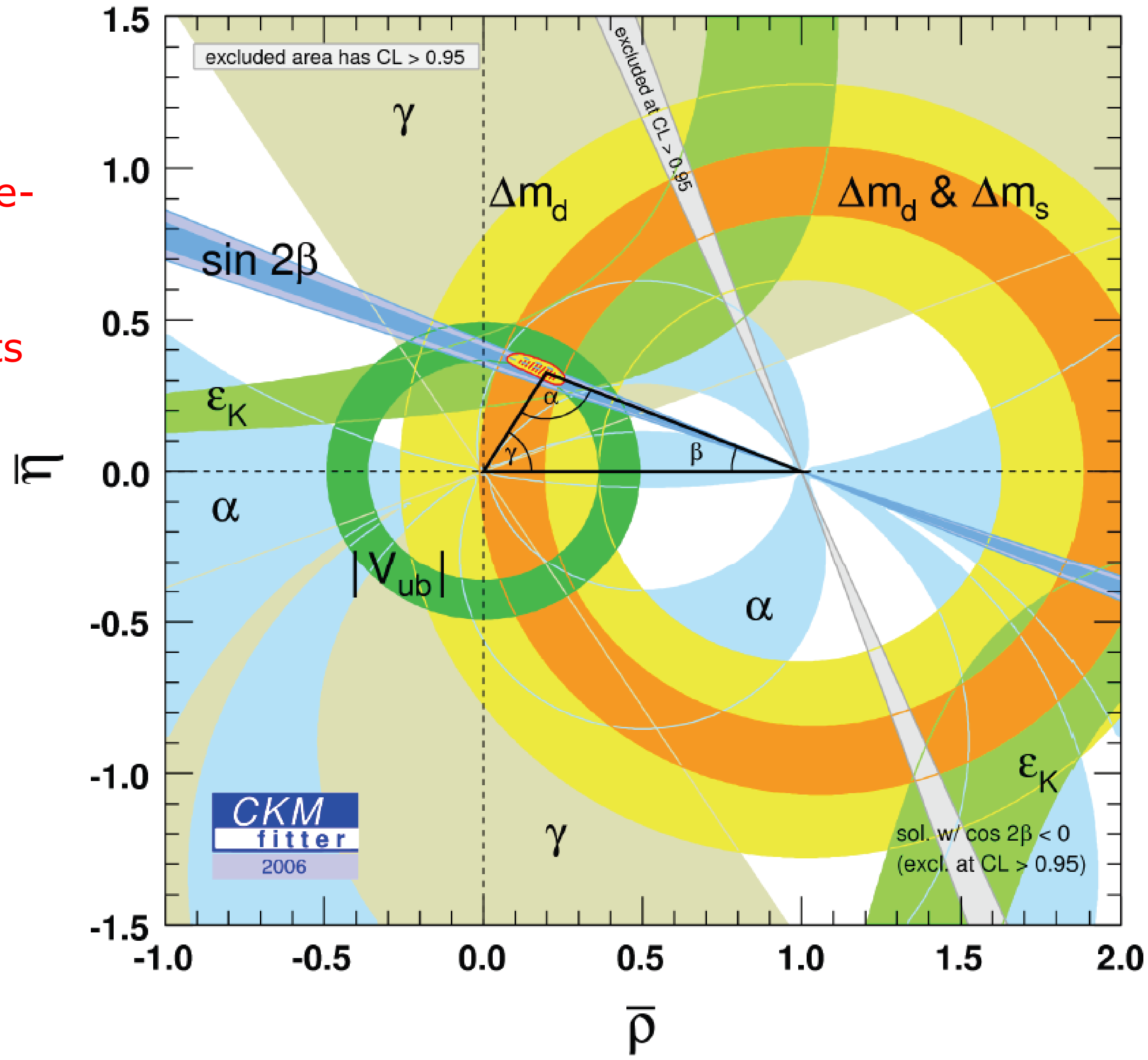
Progress in UT

2006

Tevatron measurement of Δm_s

tighter constraints on α

first constraints on γ from CPV in $B \to K \pi$

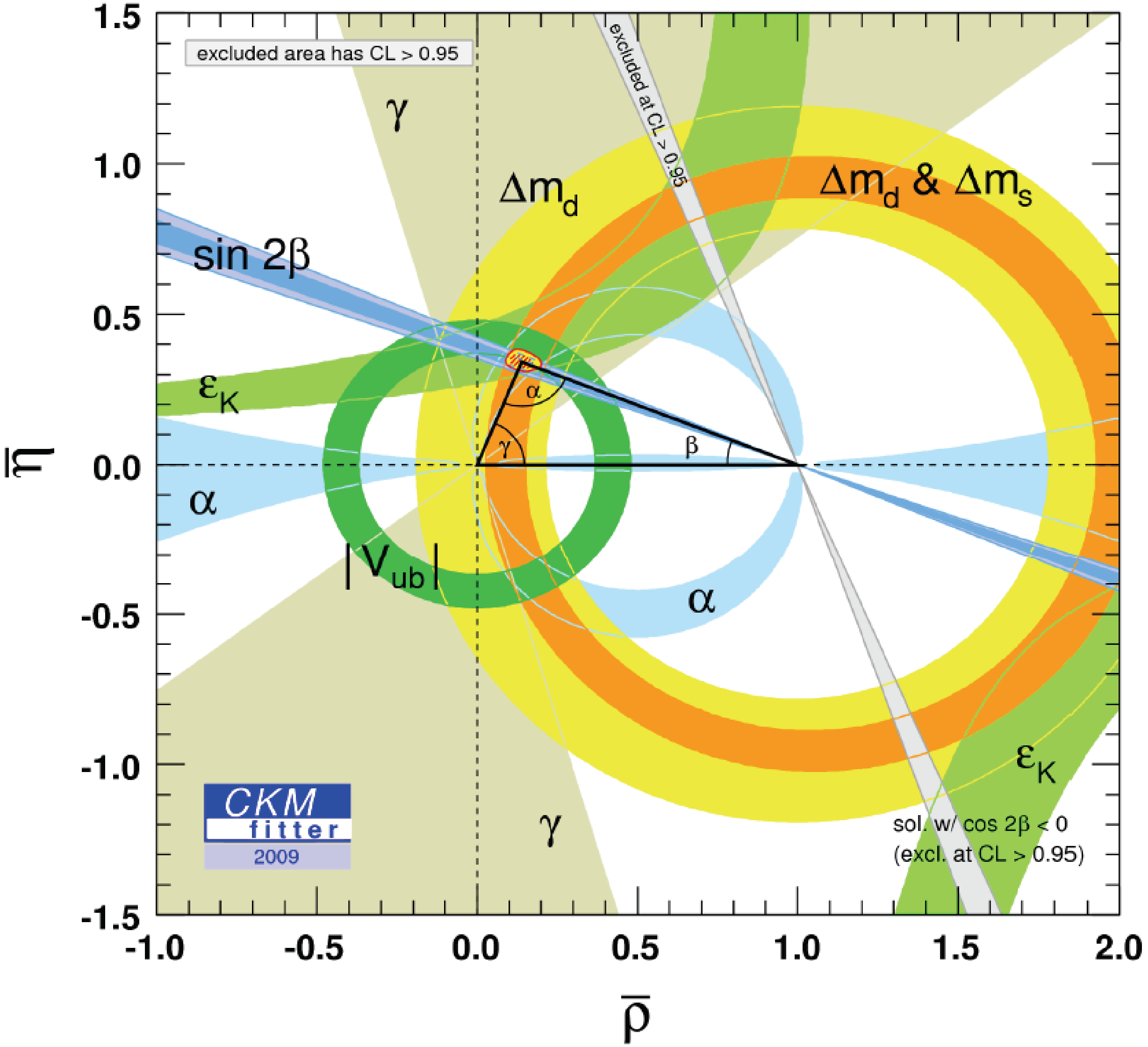


Progress in UT

2009

more contraints from B mixing

constraints from BaBar on β from CPV in $B \to \chi_{0c} K_s$

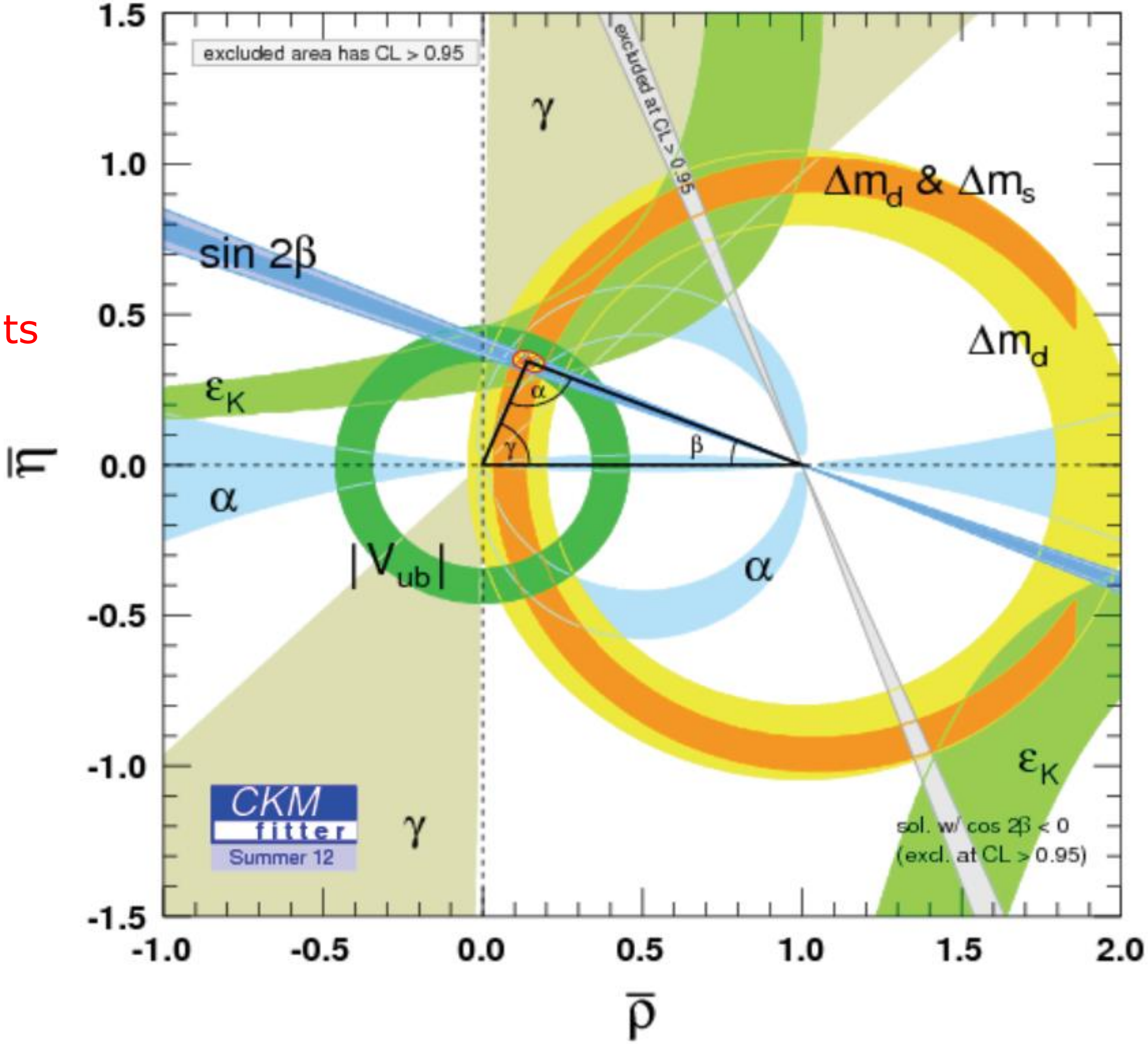


Progress in UT

2012

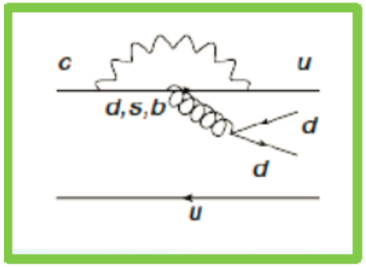
LHCb measurement of Δm_s

tighter constraints on γ

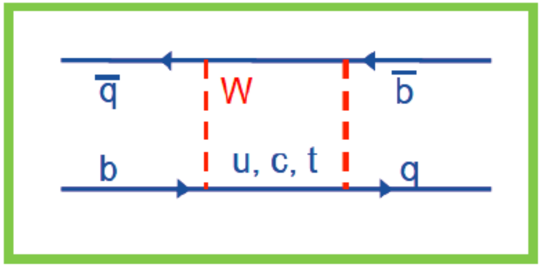


FCNC loops in the SM

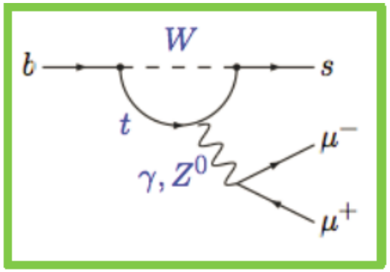
Map of flavour transitions and type of loop processes



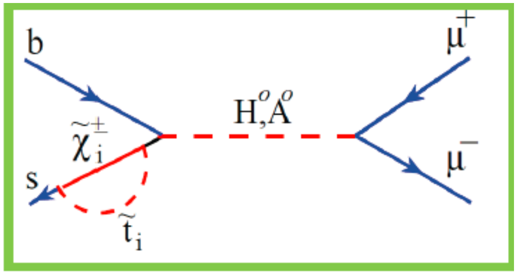
QCD penguin



ΔF=2 box



EW penguin



Higgs penguin

	b→s	b→d	c→u	s→d
QCD penguin	$A_{CP}(B_s \to hhh)$	$A_{CP}(B^0 \to hhh)$	$\Delta a_{CP}(D \to hh)$	$K \to \pi^0 ll$ ε'/ε
ΔF=2 box	ΔM_{Bs} $A_{CP}(B_s \to J/\psi\varphi)$	ΔM_{Bd} $A_{CP}(B^0 \to J/\psi K_s)$	x,y, q/p	ΔM_K ε_K
EW penguin	$B \to K^{(*)}\mu\mu$ $B \to X_s\gamma$	$B \to \pi\mu\mu$ $B \to X\gamma$	$D \to X_u l\, l$	$K \to \pi^0 ll$ $K \to \pi^{\pm}\nu\nu$
Higgs penguin	$B_s \to \mu\mu$	$B^0 \to \mu\mu$	$D \to \mu\mu$	$K^0 \to \mu\mu$

Flavour sector beyond the SM

Yukawa mechanism in the lepton sector

- in the SM the lepton Yukawa matrices can be diagonalized independently due to the global G_l symmetry of the Lagrangian, and therefore there are no FCNC

$$-\mathcal{L}^{\rm SM}_{\rm Yukawa} = Y_d^{ij}\bar{Q}_L^i D_R^j H + Y_u^{ij}\bar{Q}_L^i U_R^j H_c + Y_e^{ij}\bar{L}_L^i E_R^j H + \text{h.c.}$$

$$\mathcal{G}_q = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}, \qquad \mathcal{G}_\ell = SU(3)_{L_L} \otimes SU(3)_{E_R}$$

- however, the discovery that neutrinos oscillate (and are massive) implies that Lepton Flavour is not conserved
- the level of neutral Lepton Flavour Violation depends on the mechanism to generate neutrino masses (for instance **seesaw mechanism**)
- it could be just a copy of the quark sector, but it may be different due to the properties of the right-handed neutrino

Seesaw mechanism

Simplification: one family: ν_L and ν_R

- total mass term: Dirac and Majorana mass

$$\mathcal{L}_{mass} = -m(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L) - \frac{1}{2} M(\nu_R^T C \nu_R + \bar{\nu}_R C \bar{\nu}_R^T)$$

- diagonalization of the mass matrix:

  → Majorana mass eigenstates of the neutrinos

  for $M >> m$ we get $m_1 \approx \frac{m^2}{M} \quad m_2 \approx M$

- one very heavy, practically right handed neutrino
- one very light, practically left handed neutrino

At energies small compared to M, Majorana mass term for left handed neutrino:

$$\mathcal{L}_{mass} = -\frac{1}{2}\frac{m^2}{M}\left(\nu_L^T C \nu_L + \bar{\nu}_L C \bar{\nu}_L{}^T\right)$$

Majorana mass is small if $M >> m$

Seesaw mechanism

- In case of three families: **Neutrino Mixing**
- Compact notation for the Leptons:

$$\mathcal{N}_{L/R} = \begin{bmatrix} \nu_{e,L/R} \\ \nu_{\mu,L/R} \\ \nu_{\tau,L/R} \end{bmatrix} \qquad \mathcal{E}_{L/R} = \begin{bmatrix} e_{L/R} \\ \mu_{L/R} \\ \tau_{L/R} \end{bmatrix}$$

- Dirac masses are generated by the Higgs mechanism: (as for the quarks)

$$\mathcal{L}^N_{DM} = -\mathcal{N}_L m^N \mathcal{N}_R + h.c. \qquad \mathcal{L}^E_{DM} = -\mathcal{E}_L m^E \mathcal{E}_R + h.c.$$

- m^N: Dirac mass matrix for the neutrinos, m^E: (Dirac) mass matrix for e, μ, τ

- Right handed neutrinos → Majorana mass term:

$$\mathcal{L}_{MM} = -\frac{1}{2}\left(N_R^T M C N_R + \bar{N}_R M C \bar{N}_R^T\right)$$

- M: (symmetric) **Majorana Mass Matrix**
- this term is perfectly $SU(2)_L \otimes U(1)$ invariant

Implementation of the seesaw mechanism:

- assume that all eigenvalues of M are large

Effective theory at low energies → *only light, practically left handed neutrinos*

- effect of right handed neutrino: Majorana mass term for the light neutrinos

$$\mathcal{L}_{mass} = -\frac{1}{2}\left(N_L^T m^T M^{-1} m C N_L + \bar{N}_L m^T M^{-1} m C \bar{N}_L^T\right)$$

Lepton mixing: PMNS matrix

- we know there are FCNC in the lepton sector (analogous to the quark sector) because we have observed neutrino oscillations
- therefore the Yukawa couplings in lepton sector do contain also a mixing matrix

Pontecorvo Maki Nakagawa Sakata Matrix

- almost like CKM: Three Euler angles θ_{ij}

$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix}, \quad U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$$

- a Dirac phase δ and two Majorana phases α_1 and α_2

$$U_\delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix} \quad U_\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{-i\alpha_2} \end{bmatrix}$$

- PNMS parametrization: $V_{\text{PMNS}} = U_{23} U_\delta^\dagger U_{13} U_\delta U_{12} U_\alpha$
- V_{PMNS} is unitary like the CKM matrix
- left handed neutrinos are Majorana
  $\rightarrow$ *no freedom to rephase these fields!*

$$\theta_{12}[^\circ] = 33.36^{+0.81}_{-0.78}$$
$$\theta_{23}[^\circ] = 40.0^{+2.1}_{-1.5} \text{ or } 50.4^{+1.3}_{-1.3}$$
$$\theta_{13}[^\circ] = 8.66^{+0.44}_{-0.46}$$
$$\delta_{\text{CP}}[^\circ] = 300^{+66}_{-138}$$

No hierarchy observed!

Lepton mixing: PMNS matrix

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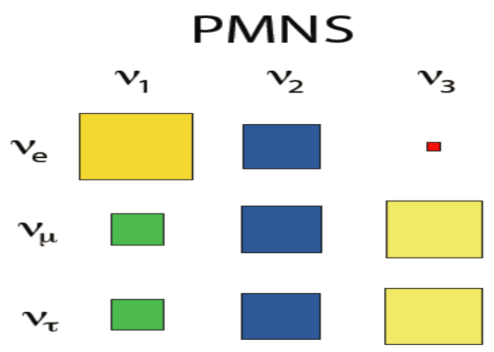

No hierarchy observed!

Lepton Flavour Violation

FCNC processes in the leptonic sector:

$$\tau \to \mu\gamma \quad \mu \to e\gamma \quad \tau \to eee \text{ etc.}$$

$$\nu_\tau \to \nu_e\gamma \quad \nu_\tau - \nu_e \text{ mixing}$$

Lepton Flavour Violation:

- right handed neutrinos are Majorana fermions:
  → no conserved quantum number corresponding to the rephasing of the right handed neutrino fields
- lepton flavour violation could feed via conserved B-L into baryon number violation
- if neutrinos are **Dirac particles**, expect **very small** (far from experimental sens.) **LFV**
- however, if neutrinos are **Majorana particles** and something like the **seesaw mechanism** is at work, **large values** (close to exp. sens.) are favoured
- in general, **any extension of the SM with new states at the TeV scale generates large charged LFV**

Many flavour related open questions

- Our understanding of Flavour is unsatisfactory:
  - $\rightarrow$ 22 (out of 27) free parameters of the SM originate from the Yukawa Sector (including Lepton Mixing)
  - $\rightarrow$ Why is the CKM Matrix hierarchical?
  - $\rightarrow$ Why is CKM so different from the PMNS?
  - $\rightarrow$ Why are quark masses (except top) so small compared with electroweak VEV?
  - $\rightarrow$ Why do we have three families?

- Why is CP Violation in flavour-diagonal processes not observed? (e.g. electric dipol moments of electron and neutron)

- Where is the CP violation needed to explain the matter-antimatter asymmetry of the Universe?

Strong CP remains mysterious

- flavour diagonal CP Violation is well hidden

$\rightarrow$ e.g electric dipole moment of the neutron:

$$
\begin{aligned}
d_e &\sim e\,\frac{\alpha_s}{\pi}\,\frac{G_F^2}{(16\pi^2)^2}\,\frac{m_t^2}{M_W^2}\,\mathrm{Im}\Delta\,\mu^3 \\
&\sim 10^{-32}\,e\,cm \quad \text{with } \mu \sim 0.3\,\mathrm{GeV} \\
d_{\mathrm{exp}} &\leq 3.0\times 10^{-26}\,e\,cm
\end{aligned}
$$

Many open questions

Standard Model

- does **not describe neutrino masses**
- does not have **a good DM candidate**
- **cannot explain the baryon asymmetry** in the Universe
- no explanation for the **flavour structure**
- does **not include gravity**
- suffers from **fine tuning issues in the Higgs sector**

Possible extensions

- SUSY, extra dimensions, hidden sectors, …..
- in general, the diagonalization of the mass matrix will not give diagonal Yukawa couplings $\rightarrow$ **large FCNC**

Needed

- precision measurements of flavour observables are generically sensitive to additions to the Standard Model
- precise measurements of the Higgs boson properties
- precise measurements of FCNC