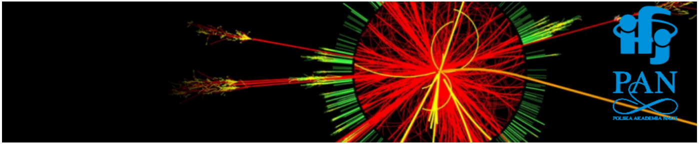


# Particle Physics- Standard Model(3)

A. Bożek, IFJ PAN Kraków

# Outline

This is an "introduction"

- Historical overview.
- the elementary particles
- the elementary forces
- Symmetries:
  - Gauge symmetry
  - Problem of mass
  - Spontaneous symmetry breaking

This lecture is not a complete course in particle physics and will only touch some most general problems.
Further reading:

- D. H. Perkins, "Introduction to High Energy Physics",
- F. Halzen, A. Martin: "Quarks and Leptons".

Further lecture to watch listen on SM and BSM physics:

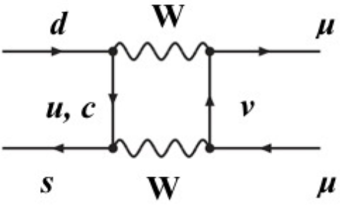
- Prof. Yuval Grossman (Cornell U.)
- Standard Model and Flavor - Lecture (`https://www.youtube.com/watch?v=GGzRdiBd8w8`)

# GIM mechanism

- Glashow, Iliopoulous, Maiani (GIM) proposed existence of this $4^{th}$ quark (charm)
- Charm couples to the $s'$ in same way $u$ couples to the $d'$
- Reason for introducing charm: to explain why flavor changing neutral currents (FCNC) are highly suppressed
- Two examples of FCNC suppression:
  1. $BR(K_L^0 \to \mu^+\mu^-) = 6.84 \times 10^{-9}$
  2. $BR(K^+ \to \pi^+\nu\nu)/BR(K^+ \to \pi^0\mu\nu) < 10^{-7}$
- Why are these decay rates so small?
- It turns out that there is also a $Z$ that couples to $f\bar{f}$ pairs, but it does not change flavor (same as $\gamma$)
- If only vector boson was the $W^\pm$, would require two bosons to be exchanged
  - Need second order charged weak interactions, but even this would give a bigger rate than seen unless there is a cancellation

# GIM mechanism box diagrams

- Consider the "box" diagram




- $\mathcal{M}$ term with $u$ quark $\propto \cos\theta_C \sin\theta_C$
- $\mathcal{M}$ term $c$ quark $\propto -\cos\theta_C \sin\theta_C$
- Same final state, so we add $\mathcal{M}$'s
- Terms cancel in limit where we ignore quark masses

# The cancelation is not accidental

- Matrix relating strong basis to weak basis is unitary

$$d'_i = \sum_j U_{ij} d_j$$

- Therefore is we sum over down-type quark pairs

$$\begin{aligned} \sum_i \overline{d}'_i d'_i &= \sim_{ijk} \overline{d}_j U^\dagger_{ji} U_{ik} d_k \\ &= \sum_j \overline{d}_j d_j \end{aligned}$$

- If an interaction is diagonal in the weak basis, it stays diagonal in the strong basis
- Independent of basis, there are no $d \longleftrightarrow s$ transitions

No flavor changing neutral current weak interactions
(up to terms that depend on the quark masses)

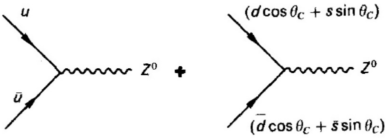
# GIM mechanism historical view

- it was noted that all neutral-current transitions had $\Delta S=0$
  - *strange* quark would transform into *up* but not into *down*
  - no "flavor-changing neutral currents"
    - … at tree level!
- Glashow, Iliopoulos and Maiani proposed a mechanism to explain this
  - now called the GIM mechanism

# GIM mechanism historical view

$$\underbrace{u\bar{u} + (d\bar{d}\cos^2\theta_C + s\bar{s}\sin^2\theta_C)}_{\Delta S=0} + \underbrace{(s\bar{d} + \bar{s}d)\sin\theta_C\cos\theta_C}_{\Delta S=1},$$

$$\begin{pmatrix} u \\ d_C \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix}$$




$$\underbrace{u\bar{u} + c\bar{c} + (d\bar{d} + s\bar{s})\cos^2\theta_C + (s\bar{s} + d\bar{d})\sin^2\theta_C}_{\Delta S=0}$$
$$+ \underbrace{(s\bar{d} + \bar{s}d - \bar{s}d - s\bar{d})\sin\theta_C\cos\theta_C}_{\Delta S=1}.$$

# More than two generation

- Generalize to $N$ families of quark ($N = 3$ as far as we know)
- $U$ is a unitary $N \times N$ matrix and $d'_i$ is an $N$-column vector

$$d'_i = \sum_{j=1}^{N} Y_{ij} d_j$$

- How many independent parameters do we need to describe $U$?
  - $N \times N$ matrix: $N^2$ elements
  - But each quark has an unphysical phase: can remove $2N - 1$ phases (leaving one for the overall phase of $U$)
  - So, $U$ has $N^2 - (2N - 1)$ independent elements
- However, an orthogonal $N \times N$ matrix has $\frac{1}{2}N(N-1)$ real parameters
  - So $U$ has $\frac{1}{2}N(N-1)$ real parameters
  - $N^2 - (2N-1) - \frac{1}{2}N(N-1)$ imaginary phases ($= \frac{1}{2}(N-1)(N-2)$)
- $N = 2$ 1 real parameter, 0 imaginary
- $N = 3$ 3 real parameters, 1 imaginary
- Three generations requires an imaginary phase: CP Violation inherent

# CKM matrix

- Write hadronic current

$$J^{\mu} = -\frac{g}{\sqrt{2}} \left( \overline{u} \; \overline{c} \; \overline{t} \right) \gamma_{\mu} \frac{(1-\gamma_5)}{2} V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- $V_{CKM}$ gives mixing between strong (mass) and (charged) weak basis
- Often write as

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Wolfenstein parameterization:

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Here $\lambda$ is the $\approx \sin\theta_C$.

# CKM matrix (2)

- From previous page

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- Impose Unitary and use all experimental measurements

$$\lambda = 0.22453 \pm 0.00044 \qquad A = 0.836 \pm 0.015$$
$$\rho = 0.122^{+0.018}_{-0.17} \qquad \eta = 0.355^{+0.12}_{-0.11}$$

- Result for the magnitudes of the elements is:

$$\begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359 \pm 0.00011 & 0.04214 \pm 0.00076 \\ 0.00896 \pm 00024 & 0.04133 \pm 0.00074 & 0.999105 \pm 000032 \end{pmatrix}$$

# b quark third family

Two families were known in 1977. In the weak interactions, the two families appear rotated (Cabibbo angle)

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix};$$

Third family

$$\begin{pmatrix} d_C \\ s_C \end{pmatrix} = \begin{bmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{bmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

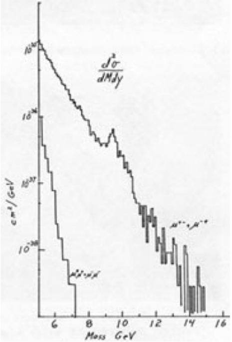
The mixing in fact, involves all three families (CKM matrix)

# Upsilon meson discovery : b quark

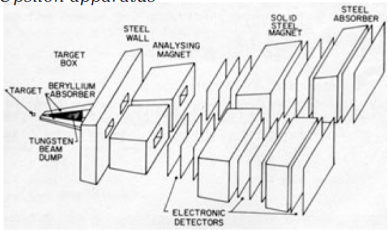
''*Observation of a Dimuon Resonance at 9.5 GeV in 400 GeV Proton-Nucleus Collisions*''

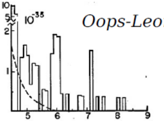
Summer of 1977, a team of physicists, led by Leon M. Lederman, working on experiment 288 in the proton center beam line of the Fermilab fixed target areas discovered the Upsilon $\Upsilon$

1970 proposal: study the rare events that occur when a pair of muons or electrons is produced in a collision of the proton beam from the acccelerator on a platinum target
Only one Upsilon is produced for every 100 billion protons which strike the target




*The Upsilon apparatus*

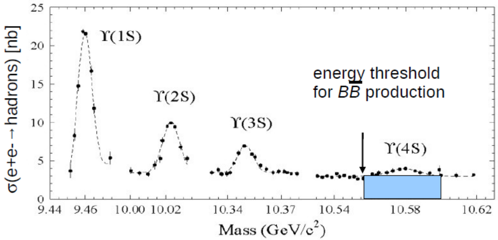


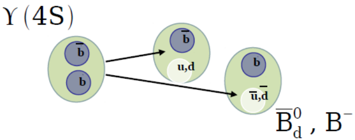




*Oops-Leon !!*

''*The Upsilon fits very nicely into the picture of a super-atom consisting of the bound state of a bottom quark and antiquark.*''

# Upsilon meson discovery : b quark

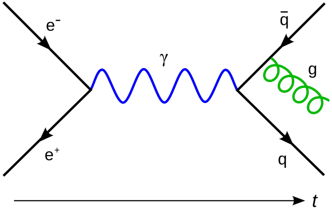



- 2 B's and nothing else !
- 2 B mesons are created simultaneously in a L=1 coherent state
  $\Rightarrow$ before first decay, the final states contains a B and a $\overline{B}$

# R ratio :Quarks electric charge, number of QCD charges (colors)

**R** is the ratio of the hadronic cross section to the muon cross section in electron–positron collisions:
$R = \frac{\sigma(e^+e^-)\rightarrow \text{hadron}}{\sigma(e^+e^-)\rightarrow \mu^+\mu^-}$




**R** also provides experimental confirmation of the electric charge of quarks, in particular the charm quark and bottom quark, and the existence of three quark colors. A simplified calculation of R yields
$R = 3\sum e_q^2$,
where the sum is over all quark flavors with mass less than the beam energy. $e_q$ is the electric charge of the quark, and the factor of 3 accounts for the three colors of the quarks. QCD corrections to this formula have been calculated.

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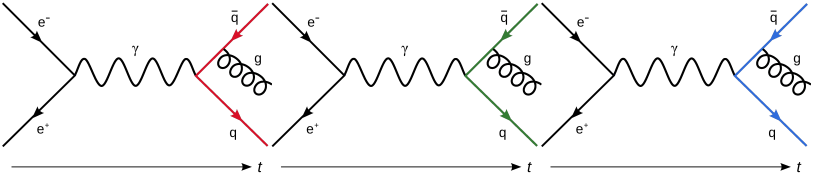


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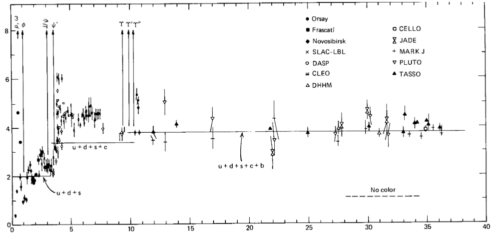


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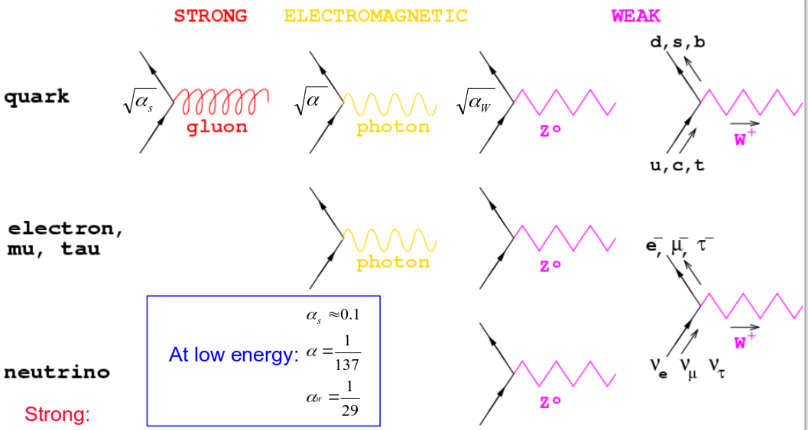
# The main standard model verticies




Strong:
All quarks (and anti-quarks)
No change of flavour

EM:
All charged particles
No change of flavour

Weak neutral current:
All particles
No change of flavour

Weak charged current:
All particles
Flavour changes

# Feynman diagrams revisited

$\sigma \propto |T_{fi}|^2$

The Feynman diagrams give us the amplitude, c.f. $\psi$ in QM whereas probability is $|\psi|^2$

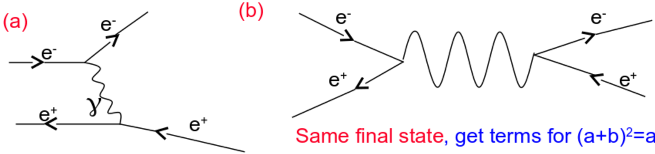
(1) So, two electro-magnetic vertices:
e.g. $e^-e^+ \to \mu^-\mu^+$ amplitude gets factor from each vertex $\sqrt{\alpha}\sqrt{\alpha} = \alpha$

Crosssection gets amplitude squared $\propto \alpha^2$

for $e^-e^+ \to q\bar{q}$ with quarks of charge q (1/3 or 2/3) $\propto (q\sqrt{\alpha}\sqrt{\alpha})^2 = q^2\alpha^2$

- Also remember : u,d,s,c,t,b quarks and they each come in 3 colours
- Scattering from a nucleus would have a Z term

(2) If we have several diagrams contributing to same process, we much consider *interference* between them e.g.




Same final state, get terms for $(a+b)^2 = a^2 + b^2 + ab + ba$

# Quantum Chromodynamics (QCD)

QED – mediated by spin 1 bosons (photons) coupling to conserved electric charge
QCD – mediated by spin 1 bosons (gluons) coupling to conserved colour charge

u,d,c,s,t,b have same 3 colours (red,green,blue), so identical strong interactions [c.f. isospin symmetry for u,d], leptons are colourless so don't feel strong force

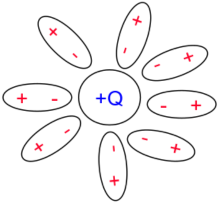
- Significant difference from QED:
  - photons have no electric charge
  - But gluons do have colour charge – eight different colour mixtures.

Hence, gluons interact with each other. Additional Feynman graph vertices:


Self-interaction

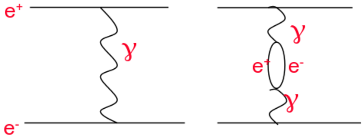
These diagrams and the difference in size of the coupling constants are responsible for the difference between EM and QCD

# Running constant in QED

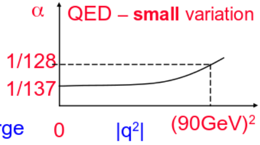



Charge +Q in dielectric medium
Molecules nearby screened,
At large distances don't see full charge
Only at small distances see +Q

Also happens in vacuum – due to spontaneous production of virtual $e^+e^-$ pairs




And diagrams with
two loops ,three loops….
each with smaller effect: $\alpha,\alpha^2$….

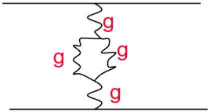



As a result coupling strength grows with $|q^2|$ of photon,

higher energy ⇒smaller wavelength gets closer to bare charge

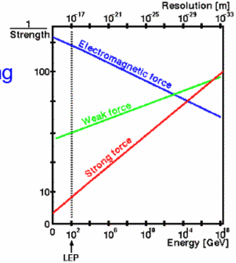
# Running constant in QCD

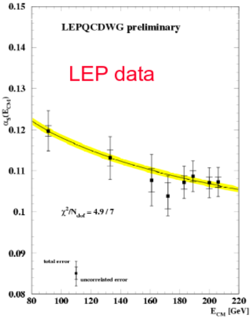
- Exactly same replacing photons with gluons and electrons with quarks
- But also have gluon splitting diagrams




This gives anti-screening effect.
Coupling strength **falls** as $|q^2|$ increases

Grand Unification ?




Strong variation in strong coupling
From $\alpha_s \approx 1$ at $|q^2|$ of 1 $GeV^2$
To $\alpha_s$ at $|q^2|$ of $10^4$ $GeV^2$

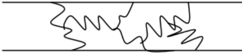
Hence:

- Quarks scatter freely at high energy

- Perturbation theory converges very Slowly as $\alpha_s \approx 0.1$ at current expts
And lots of gluon self interaction diagrams

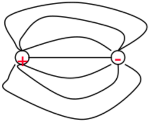
# Range of strong forces

Gluons are massless, hence expect a QED like long range force
But potential is changed by gluon self coupling



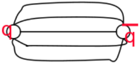
Qualitatively:

QED




Standard EM field

QCD




Field lines pulled into strings
By gluon self interaction

QCD – energy/unit length stored in field ~ constant.
Need infinite energy to separate qqbar pair.
Instead energy in colour field exceeds $2m_q$ and new q qbar pair created in vacuum

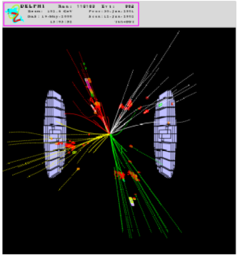
This explains absence of free quarks in nature.
Instead jets (fragmentation) of mesons/baryons

NB Hadrons are colourless, Force between hadrons due to pion exchange. 140MeV→1.4fm

Form of QCD potential:

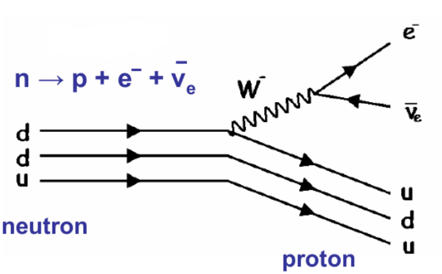
$$V_{QCD} = -\frac{4}{3}\frac{\alpha_s}{r} + kr$$

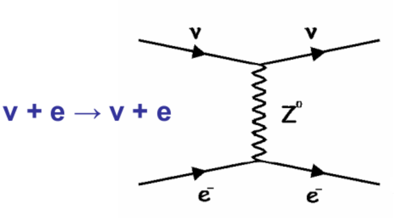
Coulomb like to start with,
but on ~1 fermi scale energy sufficient for fragmentation



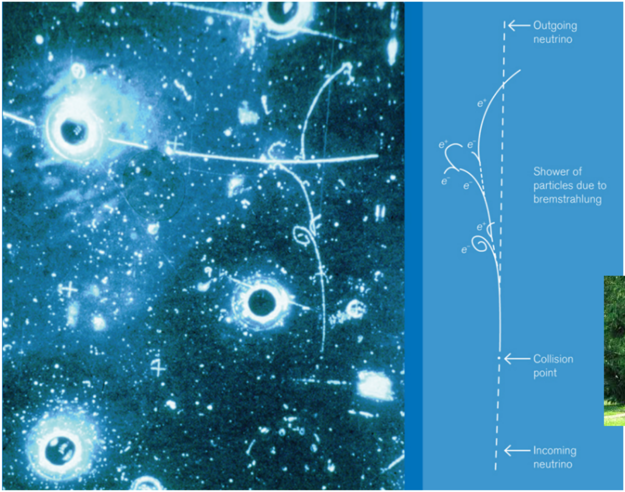
# Weak interaction

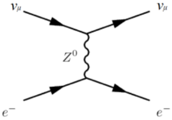
processes related to change flavor (quark decay)



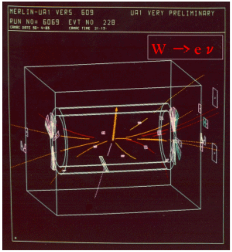
# Weak neutral current discovery - indirect evidence of $Z^0$

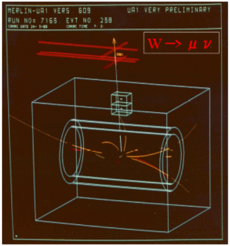


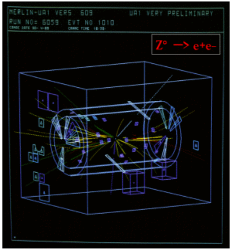



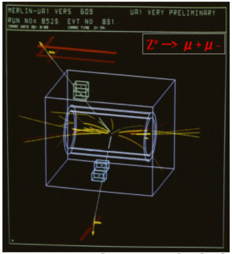

Gargamelle (Cern)

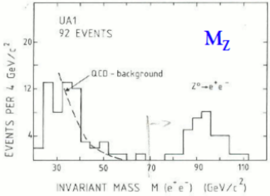
# $W$ and $Z^0$ discovery



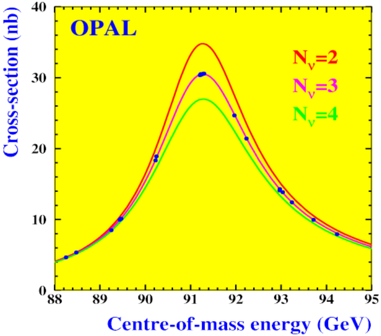




Rubbia and van der Meer were promptly awarded the 1984 Nobel Prize in Physics.

# Number of generations?

- Determination of the $Z^0$ line-shape:
  - Reveals the number of 'light neutrinos'
  - Fantastic precision on $Z^0$ parameters
    - Corrections for phase of moon, water level in Lac du Geneve, passing trains,…



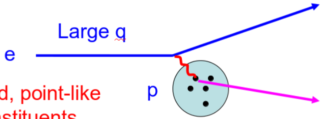

| | |
|---|---|
| $N_\nu$ | 2.984±0.0017 |
| $M_{Z0}$ | 91.1852±0.0030 GeV |
| $\Gamma_{Z0}$ | 2.4948 ±0.0041 GeV |

**Existence of only 3 neutrinos**

- Unless the undiscovered neutrinos have mass $m_\nu > M_Z/2$

# Partons are reality !!!

New “Rutherford” like experiments, but with much higher energy.
probing structure of proton itself




Hard, point-like
Constituents,
behaving as if they are free particles

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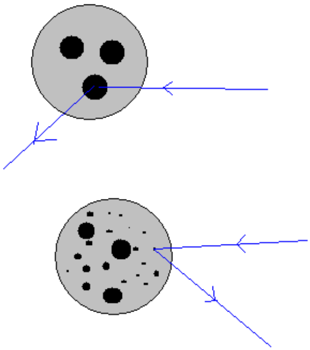
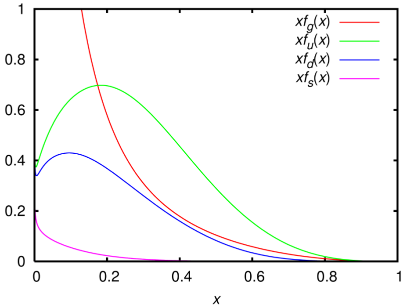


The scattering particle only sees the valence partons. At higher energies, the scattering particles also detects the sea partons.

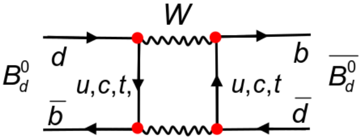


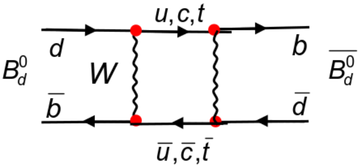

The probability density for finding a particle with a certain longitudinal momentum fraction x at resolution scale $q^2$. inside proton.

# Mixing of neutral mesons

As result of the quark mixing the Standard Model predicts oscillations of neutral mesons:

$$B_d^0 \Leftrightarrow \overline{B_d^0}$$




Similar graphs for other neutral mesons:

Neutral mesons:

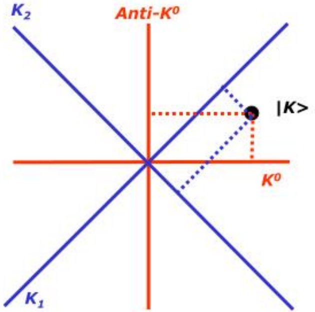
$$\left|P^0\right\rangle: \quad K^0 = \left|d\bar{s}\right\rangle \quad D^0 = \left|\bar{u}c\right\rangle \quad B_d^0 = \left|d\bar{b}\right\rangle \quad B_s^0 = \left|s\bar{b}\right\rangle$$

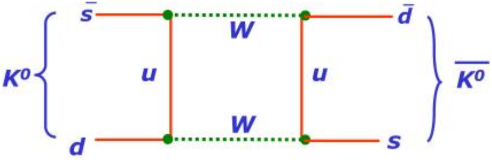
$$\left|\overline{P^0}\right\rangle: \quad \overline{K^0} = \left|\bar{d}s\right\rangle \quad \overline{D^0} = \left|\bar{u}c\right\rangle \quad \overline{B_d^0} = \left|d\bar{b}\right\rangle \quad \overline{B_s^0} = \left|s\bar{b}\right\rangle$$

discovery of mixing 1960 2019 1987 2006

# Mixing of neutral mesons

- $K^0 \to \overline{K^0}$ transition
  - Note 1: Two W bosons required ($\Delta S=2$ transition)
  - Note 2: many vertices, but still lowest order process...

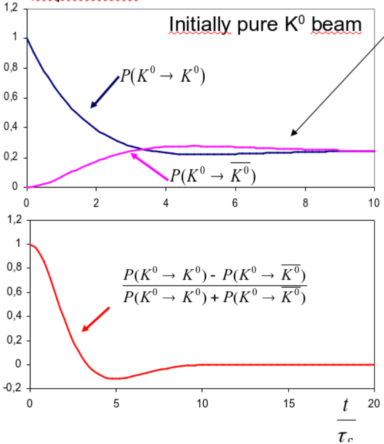


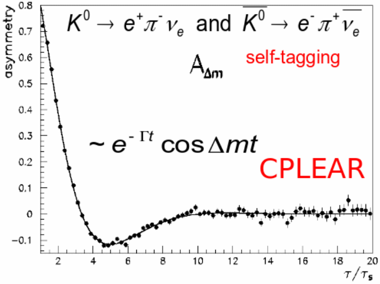
# Neutral Kaons system

After the lifetime of the $K_s$ the $K^0$ consists entirely out of K's, which are essentially an equal mixture of $K^0$ and $\overline{K^0}$.

Expectation




Measurement



# Discret Symmetries

- Three fundamental discrete Symmetries:
  - Parity (P) = Space inversion: $\vec{x} \to -\vec{x}$
  - Charge Conjugation (C) = particle $\to$ antiparticle
  - Time Reversal (T) = Time inversion: $x_0 \to -x_0$
- CPT Theorem:

Assuming only local interactions, Lorentz invariance and Causality the product of the three symmetries C $\times$ P $\times$ T is always a symmetry.

- ... this is always true for a Lagrangian field theory (with causal particle propagators)

# P transformation

- Parity P: $\vec{x} \to -\vec{x}$
- There has to be an operator P in Hilbert Space
- If P is a symmetry: $\mathrm{P}|0\rangle = |0\rangle \quad [H, \mathrm{P}] = 0$
- Scalar Field:

$$\mathrm{P}\phi(x_0, \vec{x})\mathrm{P}^\dagger = \phi(x_0, -\vec{x})$$

- Vector Field:

$$\left.\begin{array}{l} \mathrm{P}A^0(x_0, \vec{x})\mathrm{P}^\dagger = A^0(x_0, -\vec{x}) \\ \mathrm{P}A^i(x_0, \vec{x})\mathrm{P}^\dagger = -A^i(x_0, -\vec{x}) \end{array}\right\} \mathrm{P}A^\mu(x_0, \vec{x})\mathrm{P}^\dagger = A_\mu(x_0, -\vec{x})$$

# P transformation

- Spinor field:

$$\mathrm{P}\psi(x_0, \vec{x})\mathrm{P}^\dagger = \gamma_0\psi(x_0, -\vec{x})$$
$$\mathrm{P}\bar{\psi}(x_0, \vec{x})\mathrm{P}^\dagger = \bar{\psi}(x_0, -\vec{x})\gamma_0$$

- This is designed such that $\bar{\psi}(x)\gamma_\mu\psi(x)$ behaves like a vector field. Homework: check this!
- P invariance means that the action is invariant:

$$\mathrm{P}\,S\,\mathrm{P}^\dagger = S$$

# C transformation

- Charge Conjugation C: particle $\leftrightarrow$ antiparticle
- There has to be an operator C in Hilbert Space
- If C is a symmetry: $\mathrm{C}|0\rangle = |0\rangle \quad [H\,,\,\mathrm{P}] = 0$
- Scalar Field:

$$\mathrm{C}\phi(x)\mathrm{C}^{\dagger} = \phi(x)^{\dagger}$$

- Vector Field:

$$\mathrm{C}A^{\mu}(x)\mathrm{C}^{\dagger} = -A^{\mu}(x)$$

- Spinor Field:

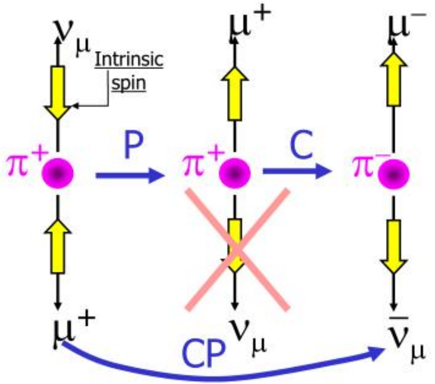
$$\mathrm{C}\psi(x)\mathrm{C}^{\dagger} = \mathcal{C}(\bar{\psi}(x))^{T}$$

# The History of CP

- Historically it was believed that all three discrete symmetries hold separately
- First revolution:
  1956 Lee and Yang suggest P violation
  quickly experimentally confimed by Wu et al.
- ... CP was still believed to be conserved,
- until 1964:
- Second Revolution:
  Cronin and Fitch discover CP violation
  CPT theorem: CP Violation = T Violation

# The Weak force and C,P parity violation

- What about C+P ≡ CP symmetry?
  - CP symmetry is parity conjugation (x,y,z → -x,-y,z) followed by charge conjugation ($X \rightarrow \overline{X}$)




100% P violation:
All ν's are lefthanded
All $\bar{\nu}$'s are righthanded

***CP appears to be preserved in weak interaction!***

# A first look at CP violation in Kaons

- There are two different neutral Kaons:

$$|K^0\rangle = |\bar{s}d\rangle \qquad \text{and} \qquad |\bar{K}^0\rangle = |\bar{d}s\rangle$$

- They are pseudoscalar particles:

$$\mathrm{P}|K^0\rangle = -|K^0\rangle \qquad \text{and} \qquad \mathrm{P}|\bar{K}^0\rangle = -|\bar{K}^0\rangle$$

- Charge conjugation is $q \leftrightarrow \bar{q}$, hence

$$\mathrm{C}|K^0\rangle = |\bar{K}^0\rangle \qquad \text{and} \qquad \mathrm{C}|\bar{K}^0\rangle = |K^0\rangle$$

$$|K_S\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \quad \text{and} \quad |K_L\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

# A first look at CP violation in Kaons

- The kaons are produced in mass eigenstates:
  - $|K^0>$: $s\bar{d}$
  - $|\bar{K}^0>$: $\bar{d}s$
- The CP eigenstates are:
  - CP=+1: $|K_1> = 1/\sqrt{2}\,(|K^0> - |\bar{K}^0>)$
  - CP= -1: $|K_2> = 1/\sqrt{2}\,(|K^0> + |\bar{K}^0>)$
- The kaons decay as short-lived or long-lived kaons:
  - $|K_S>$: predominantly CP=+1 $\quad |K_S\rangle = \dfrac{|K_1\rangle + \varepsilon|K_2\rangle}{\sqrt{1+|\varepsilon|^2}},$
  - $|K_L>$: predominantly CP= -1 $\quad |K_L\rangle = \dfrac{|K_2\rangle + \varepsilon|K_1\rangle}{\sqrt{1+|\varepsilon|^2}}.$

- $\eta_{+-} \equiv \dfrac{\langle \pi^+\pi^- | H | K_L \rangle}{\langle \pi^+\pi^- | H | K_S \rangle}$
- $\eta_{+-} = (2.236 \pm 0.007) \times 10^{-3}$
- $|\varepsilon| = (2.232 \pm 0.007) \times 10^{-3}$

# CP Violation in weak interactions

- Kaons decay either into two or three pions (in an *S* wave state)
- CP Quantum numbers of the (neutral) final states

$$\mathrm{CP}|\pi\pi\rangle = |\pi\pi\rangle \quad \text{and} \quad \mathrm{CP}|\pi\pi\pi\rangle = -|\pi\pi\pi\rangle$$

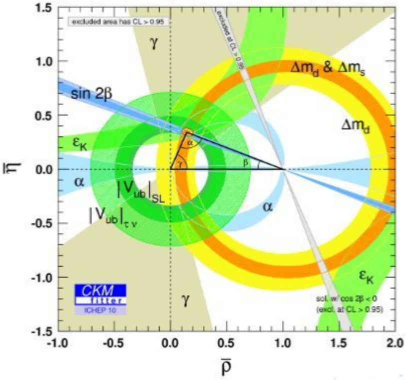
- Assuming CP Conservation:

$$|K_S\rangle \to |\pi\pi\rangle \quad \text{and} \quad |K_S\rangle \not\to |\pi\pi\pi\rangle$$
$$|K_L\rangle \not\to |\pi\pi\rangle \quad \text{and} \quad |K_L\rangle \to |\pi\pi\pi\rangle$$

- Cronin and Fitch: $|K_L\rangle \to |\pi\pi\rangle$
- We are back to CPT as the only real symmetry

# CP Violation in weak interactions B-factory

- In 2000: The *B* Factories go into operation:
- First observation of non-Kaon CP Violation
- CP Violation in the *B* system is in (almost too good) agreement with the predictions of KM:



# Why we need CP violation

- CP is needed to generate the Baryon-Antibaryon Asymmetry of the Universe: $\Delta = n_{\mathrm{Bar}} - n_{\overline{\mathrm{Bar}}} \neq 0$
- The Sakharov Conditions: (Sakharov 1967)

1. Baryon number violation: $H_{\mathrm{eff}}(\Delta \neq 0) \neq 0$
2. CP violation: $\Gamma(i \to f) \neq \Gamma(\bar{i} \to \bar{f})$
3. Absence of thermal equilibrium: Time is irrelevant in equilibrium, hence CPT implies CP

- The fundamental theory has to have CP violation
- NB: The SM has $H_{\mathrm{eff}}(\Delta \neq 0) \neq 0$

# CP violation from complex couplings

- Assume that the Lagrange operator is

$$\mathcal{L}(x) = \sum_i a_i \mathcal{O}_i(x) + \text{h.c.} = \sum_i \left( a_i \mathcal{O}_i(x) + a_i^* \mathcal{O}_i^\dagger(x) \right)$$

- Assume: The $\mathcal{O}_i$ behave like complex scalar fields

$$\mathrm{CP}\,\mathcal{L}(x)\,\mathrm{CP}^\dagger = \sum_i \left( a_i \mathcal{O}_i^\dagger(\bar{x}) + a_i^* \mathcal{O}_i(\bar{x}) \right) \qquad \bar{x} = (x_0, -\vec{x})$$

$$\mathrm{CP}\,S\,\mathrm{CP}^\dagger - S = -2i \int d^4x \sum_i \left( \mathrm{Im} a_i\, \mathcal{O}_i(x) - \mathrm{Im} a_i \mathcal{O}_i^\dagger(x) \right)$$

CP violation, if one of the couplings is complex !

# CP in the Standard Model

There are two sources of CP violation in the SM:

1. CKM CP violation: CP Violation encoded in the quark (and lepton) mass matrices
2. Strong CP violation: CP violation through the vacuum structure of QCD

- (1) is phenomenologically confirmed
- (2) remains an open question

# Structure of the Standard Model

- SM is a chiral gauge theory: Left and right handed components of fermions are in different multiplets
- $\rightarrow$ Implementation of Parity Violation
- $\rightarrow$ Fermion mass terms require symmetry breaking!

$$\mathcal{L}_{\text{mass}} = m\bar{\psi}_L \psi_R + \text{h.c.}$$

- There are three quarks with electric charge $+2/3e$: Up-type quarks
- There are three quarks with electric charge $-1/3e$: Down-type quarks

# Structure of the Standard Model

- All quarks are known to be massive
  $\rightarrow$ we need both left and right handed components

$$\mathcal{U}_L = \begin{bmatrix} u_L \\ c_L \\ t_L \end{bmatrix} \quad \mathcal{U}_R = \begin{bmatrix} u_R \\ c_R \\ t_R \end{bmatrix} \qquad \mathcal{D}_L = \begin{bmatrix} d_L \\ s_L \\ b_L \end{bmatrix} \quad \mathcal{D}_R = \begin{bmatrix} d_R \\ s_R \\ b_R \end{bmatrix}$$

- Mass terms: Two $3 \times 3$ mass matrices:

$$\mathcal{L}_{\text{mass}} = \bar{\mathcal{U}}_L \cdot M_u \cdot \mathcal{U}_R + \bar{\mathcal{D}}_L \cdot M_d \cdot \mathcal{D}_R$$

- $M_u$ and $M_d$ originate from spontaneous symmetry breaking:

$$M_u = Y_u \langle v \rangle \quad M_d = Y_d \langle v \rangle$$

# Structure of the Standard Model complex phases

- Origin of CKM-like CP violation:
  Quark Mass Matrices = Quark Yukawa Couplings
- The two mass matrices do not commute:

$$[M_u\,,\,M_d] \neq 0$$

- Relative rotation of the Eigenbases of $M_u$ vs. $M_d$:
  CKM matrix $V_{\rm CKM}$

$$M_u^{\rm diag} = V_{\rm CKM}^\dagger \cdot M_d^{\rm diag} \cdot V_{\rm CKM}$$

- The CKM matrix is unitary:

$$V_{\rm CKM}^\dagger \cdot V_{\rm CKM} = 1 = V_{\rm CKM} \cdot V_{\rm CKM}^\dagger$$

# Structure of the Standard Model CKM matrix

- Express everything in terms of mass eigenstates:
  Redefinition of the fields

$$\mathcal{D}' = V_{\mathrm{CKM}} \cdot \mathcal{D}$$

- The CKM matrix reappears ONLY in the charged current interaction

$$\mathcal{L}_{\mathrm{CC}} = \bar{U}_L(\gamma^\mu W_\mu^\pm) \cdot V_{\mathrm{CKM}} \cdot \mathcal{D}_L + \mathrm{h.c.}$$

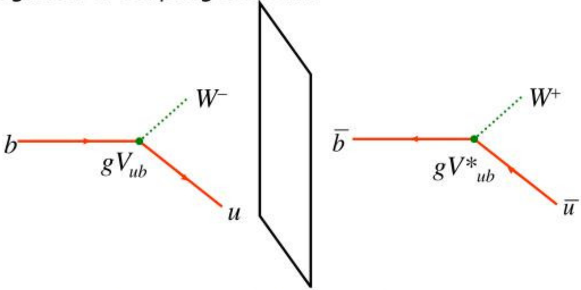
- Usual definition

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

# CKM complex phases and CP violation

## Why complex phases matter

- CP conjugation of a W boson vertex involves complex conjugation of coupling constant




***Above process violates CP if $V_{ub} \neq V_{ub}^*$***

- With 2 generations $V_{ij}$ is always real and $V_{ij} \equiv V_{ij}^*$
- *With 3 generations $V_{ij}$ can be complex* → ***CP violation built into weak decay mechanism!***

# CKM interpretation

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Off diagonal zeros of $V^\dagger_{CKM} V_{CKM} = 1 = V_{CKM} V^\dagger_{CKM}$
- $V^\dagger_{CKM} V_{CKM} = 1 : \begin{cases} V_{ub} V^*_{ud} + V_{cb} V^*_{cd} + V_{tb} V^*_{td} = 0 \\ V_{ub} V^*_{us} + V_{cb} V^*_{cs} + V_{tb} V^*_{ts} = 0 \\ V_{us} V^*_{ud} + V_{cs} V^*_{cd} + V_{ts} V^*_{td} = 0 \end{cases}$
- $V_{CKM} V^\dagger_{CKM} = 1 : \begin{cases} V_{ud} V^*_{td} + V_{us} V^*_{ts} + V_{ub} V^*_{tb} = 0 \\ V_{ud} V^*_{cd} + V_{us} V^*_{cs} + V_{ub} V^*_{cb} = 0 \\ V_{cd} V^*_{td} + V_{cs} V^*_{ts} + V_{cb} V^*_{tb} = 0 \end{cases}$

# Wolfenstein Parametrisation of CKM

- Diagonal CKM matrix elements are almost unity
- CKM matrix elements decrease as we move off the diagonal
- Wolfenstein Parametrization:

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & \lambda^2 A \\ \lambda^3 A(1-\rho-i\eta) & -\lambda^2 A & 1 \end{pmatrix}$$

- Expansion in $\lambda \approx 0.22$ up to $\lambda^3$
- $A$, $\rho$, $\eta$ of order unity

# CKM triangle interpretation

## Deriving the triangle interpretation

- Starting point: the 9 unitarity constraints on the CKM matrix

$$V^{+}V = \begin{pmatrix} V^{*}_{ud} & V^{*}_{cd} & V^{*}_{td} \\ V^{*}_{us} & V^{*}_{cs} & V^{*}_{ts} \\ V^{*}_{ub} & V^{*}_{cb} & V^{*}_{tb} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

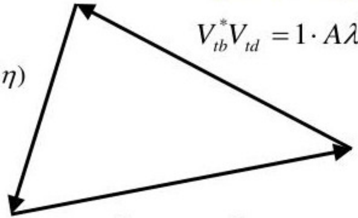
- Pick (arbitrarily) orthogonality condition with (i,j)=(3,1)

$$V^{*}_{ub}V_{ud} + V^{*}_{cb}V_{cd} + V^{*}_{tb}V_{td} = 0$$

# CKM triangle interpretation -visualisation

- Sum of three complex vectors is zero →
  Form triangle when put head to tail

**(Wolfenstein params to order $\lambda^4$)**



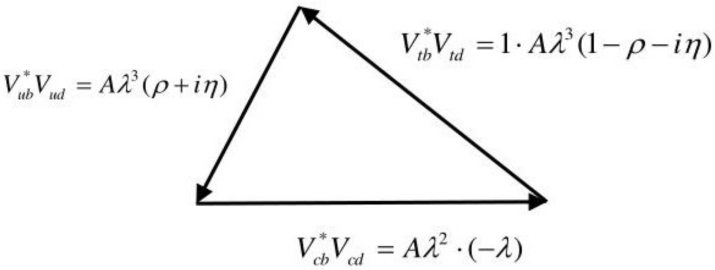
$$V_{tb}^* V_{td} = 1 \cdot A\lambda^3 (1 - \rho - i\eta)$$

$$V_{ub}^* V_{ud} = A\lambda^3 (\rho + i\eta)$$

$$V_{cb}^* V_{cd} = A\lambda^2 \cdot (-\lambda)$$

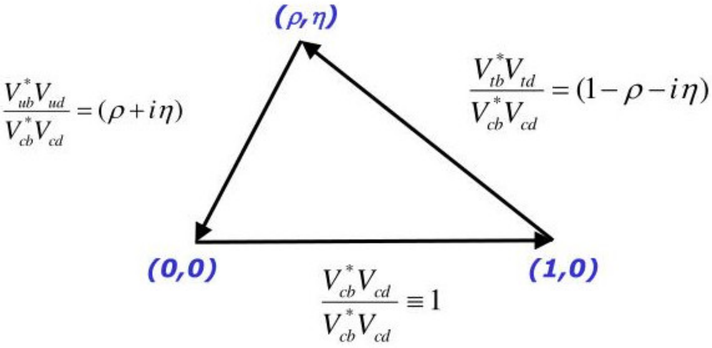
# CKM triangle interpretation -visualisation

- Phase of 'base' is zero → Aligns with 'real' axis,



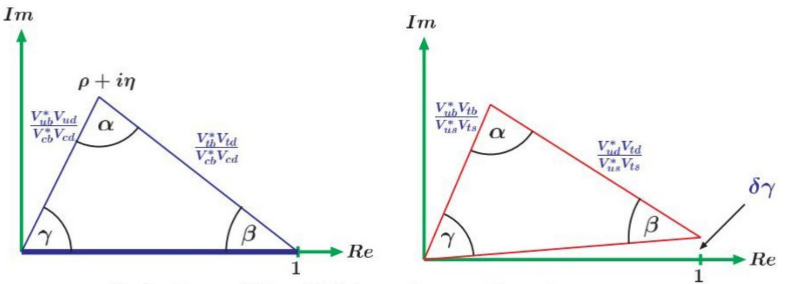
# CKM triangle interpretation -visualisation

- Divide all sides by length of base




- Constructed a triangle with apex $(\rho,\eta)$

# CKM triangle interpretation -visualisation



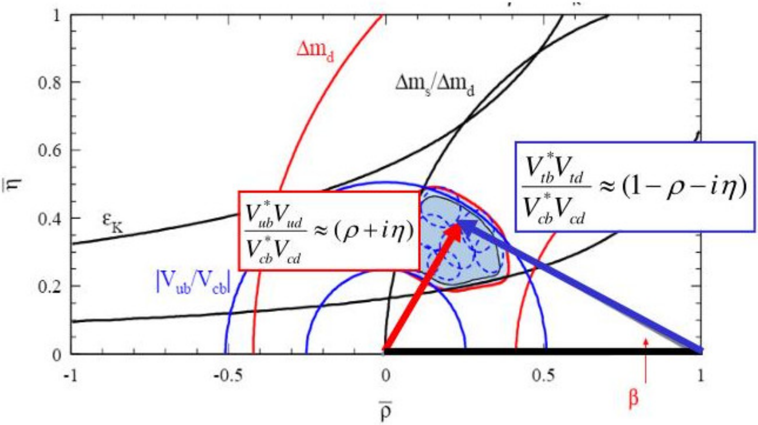

- Definition of the CKM angles $\alpha$, $\beta$ and $\gamma$
- To leading order Wolfenstein:

$$V_{ub} = |V_{ub}|e^{-i\gamma} \quad V_{tb} = |V_{tb}|e^{-i\beta}$$

  all other CKM matrix elements are real.
- $\delta\gamma$ is order $\lambda^5$

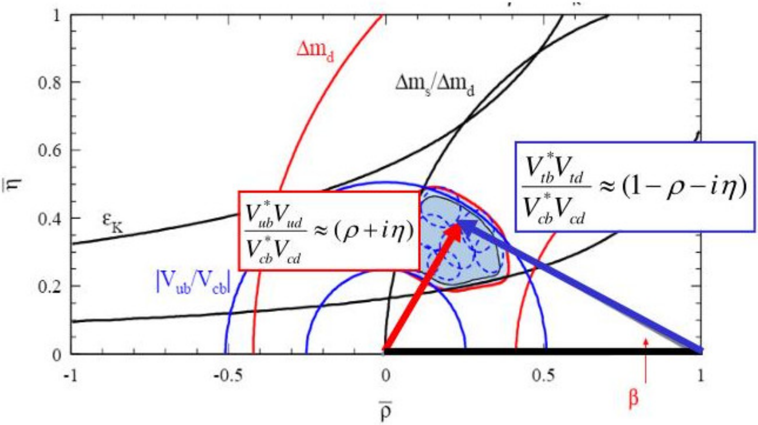
# CKM triangle interpretation -visualisation

- We can now put this triangle in the (ρ,η) plane



# CKM triangle interpretation -visualisation

- We can now put this triangle in the (ρ,η) plane



# The Standard Model Lagrangian

$$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$$

- $L_{Kinetic}$ : • Introduce the massless fermion fields
  • Require local gauge invariance ➔ gives rise to existence of gauge bosons
- $L_{Higgs}$ : • Introduce Higgs potential with $<\phi> \neq 0$
  • Spontaneous symmetry breaking

  } $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_Q$
  The $W^+$, $W^-$, $Z^0$ bosons acquire a mass
- $L_{Yukawa}$ : • Ad hoc interactions between Higgs field & fermions

# The Standard Model Lagrangian field notation

$Q = T_3 + Y$

**Fermions:** $\psi_L = \left(\frac{1-\gamma_5}{2}\right)\psi \;;\; \psi_R = \left(\frac{1+\gamma_5}{2}\right)\psi$ with $\psi = Q_L, u_R, d_R, L_L, l_R, \nu_R$

**Quarks:**

Under SU2:
Left handed doublets
Right hander singlets

- $\begin{pmatrix} u^I\,(3,2,1/6) \\ d^I\,(3,2,1/6) \end{pmatrix}_{Li} \equiv Q^I_{Li}\,(3,2,1/6)$

Interaction rep. ($I$); Left-handed ($L$); generation index ($i$); $SU(3)_C$; $SU(2)_L$; Hypercharge Y (=avg el.charge in multiplet)

- $u^I_{Ri}\,(3,1,2/3)$
- $d^I_{Ri}\,(3,1,-1/3)$

**Leptons:**

- $\begin{pmatrix} \nu^I\,(1,2,-1/2) \\ l^I\,(1,2,-1/2) \end{pmatrix}_{Li} \equiv L^I_{Li}\,(1,2,-1/2)$
- $l^I_{Ri}\,(1,1,-1)$
- $\left(\nu^I_{Ri}\right)$

**Scalar field:**

- $\phi\,(1,2,1/2) = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$

Note:
Interaction representation: standard model interaction is independent of generation number

# The Standard Model Lagrangian field notation

$Q = T_3 + Y$

Explicitly:

- The left handed quark doublet :

$$Q^I_{Li}(3,2,1/6) = \begin{pmatrix} u^I_r, u^I_g, u^I_b \\ d^I_r, d^I_g, d^I_b \end{pmatrix}_L, \begin{pmatrix} c^I_r, c^I_g, c^I_b \\ s^I_r, s^I_g, s^I_b \end{pmatrix}_L, \begin{pmatrix} t^I_r, t^I_g, t^I_b \\ b^I_r, b^I_g, b^I_b \end{pmatrix}_L \qquad \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix} \quad (Y = 1/6)$$

- Similarly for the quark singlets:

$$u^I_{Ri}(3,1,\ 2/3) = \left(u^I_r, u^I_r, u^I_r\right)_R, \left(c^I_r, c^I_r, c^I_r\right)_R, \left(t^I_r, t^I_r, t^I_r\right)_R \qquad (Y = 2/3)$$

$$d^I_{Ri}(3,1,-1/3) = \left(d^I_r, d^I_r, d^I_r\right)_R, \left(s^I_r, s^I_r, s^I_r\right)_R, \left(b^I_r, b^I_r, b^I_r\right)_R \qquad (Y = -1/3)$$

- The left handed leptons: $L^I_{Li}(1,2,-1/2) = \begin{pmatrix} \nu^I_e \\ e^I \end{pmatrix}_L, \begin{pmatrix} \nu^I_\mu \\ \mu^I \end{pmatrix}_L, \begin{pmatrix} \nu^I_\tau \\ \tau^I \end{pmatrix}_L \qquad \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix} \quad (Y = -1/2)$

- And similarly the (charged) singlets: $l^I_{Ri}(1,1,-1) = e^I_R, \mu^I_R, \tau^I_R \qquad (Y = -1)$

Niels Tuning (16)

# The Standard Model Lagrangian kinetic term

$$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

$\mathcal{L}_{Kinetic}$ : Fermions + gauge bosons + interactions

Procedure:
*Introduce the Fermion fields and* *<u>demand</u>* *that the theory is local gauge invariant under* $SU(3)_C x SU(2)_L x U(1)_Y$ *transformations.*

Start with the Dirac Lagrangian: $\mathcal{L} = i\bar{\psi}(\partial^\mu \gamma_\mu)\psi$

Replace: $\partial^\mu \rightarrow D^\mu \equiv \partial^\mu + ig_s G_a^\mu L_a + igW_b^\mu T_b + ig' B^\mu Y$

Fields:
- $G_a^\mu$ : 8 gluons
- $W_b^\mu$ : weak bosons: $W_1$, $W_2$, $W_3$
- $B^\mu$ : hypercharge boson

Generators:

| | | | | |
|---|---|---|---|---|
| $L_a$ | : Gell-Mann matrices: | $\frac{1}{2}\lambda_a$ | (3x3) | $SU(3)_C$ |
| $T_b$ | : Pauli Matrices: | $\frac{1}{2}\tau_b$ | (2x2) | $SU(2)_L$ |
| $Y$ | : Hypercharge: | | | $U(1)_Y$ |

*For the remainder we only consider Electroweak:* $SU(2)_L \times U(1)_Y$

Niels Tuning

# The Standard Model Lagrangian The Higgs potential

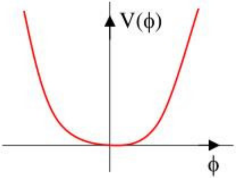
$$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$$

$$L_{Higgs} = D_\mu \phi^\dagger D^\mu \phi - V_{Higgs} \qquad V_{Higgs} = \frac{1}{2}\mu^2 \left(\phi^\dagger \phi\right) + |\lambda| \left(\phi^\dagger \phi\right)^2$$

**Symmetry**

$\mu^2 > 0:$

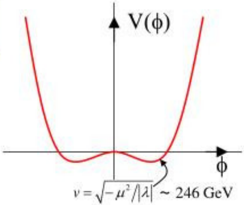
$<\varphi> = 0$




**Broken Symmetry**

$\mu^2 < 0:$

$<\varphi> = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$




Spontaneous Symmetry Breaking: The Higgs field adopts a non-zero vacuum expectation value

Procedure: $\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} \Re e\,\varphi^+ + i\Im m\,\phi^+ \\ \Re e\,\varphi^0 + i\Im m\,\phi^0 \end{pmatrix}$ Substitute: $\Re e\,\varphi^0 = \frac{v + H^0}{\sqrt{2}}$

(The other 3 Higgs fields are "eaten" by the W, Z bosons)

1. $G_{SM} : \left(SU(3)_C \times SU(2)_L \times U(1)_Y\right) \rightarrow \left(SU(3)_C \times U(1)_{EM}\right)$
2. The $W^+, W^-, Z^0$ bosons acquire mass
3. The Higgs boson $H$ appears

# The Standard Model Lagrangian

$$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

- $\mathcal{L}_{Kinetic}$ : •Introduce the massless fermion fields
  •Require local gauge invariance → gives rise to existence of gauge bosons
  ➔ CP Conserving
- $\mathcal{L}_{Higgs}$ : •Introduce Higgs potential with $<\phi> \neq 0$
  •Spontaneous symmetry breaking
  ➔ CP Conserving

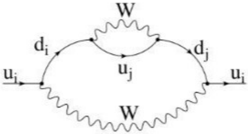
  } $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_Q$
  The $W^+$, $W^-$, $Z^0$ bosons acquire a mass
- $\mathcal{L}_{Yukawa}$ : •Ad hoc interactions between Higgs field & fermions
  ➔ CP violating with a single phase

---

- $\mathcal{L}_{Yukawa} \rightarrow \mathcal{L}_{mass}$ :
  - fermion weak eigenstates:
    - mass matrix is (3x3) non-diagonal } ➔ CP-violating
  - fermion mass eigenstates:
    - mass matrix is (3x3) diagonal } ➔ CP-conserving!
- $\mathcal{L}_{Kinetic}$ in mass eigenstates: CKM – matrix ➔ CP violating with a single phase

# Flavor diagonal CP violation

- Flavour diagonal CP Violation is well hidden: Check this!
  e.g electric dipole moment of the neutron:
  At least three loops (Shabalin)




$$d_e \sim e\frac{\alpha_s}{\pi}\frac{G_F^2}{(16\pi^2)^2}\frac{m_t^2}{M_W^2}\,\mathrm{Im}\Delta\,\mu^3$$
$$\sim 10^{-32}\,e\,cm \quad \text{with } \mu \sim 0.3\,\mathrm{GeV}$$
$$d_{\exp} \leq 3.0\times10^{-26}\,e\,cm$$

## Strong CP violation

$M_u^{\text{diag}}$ and $M_d^{\text{diag}}$ do not necessarily have real eigenvalues

- Using mass eigenstates

$$\mathcal{L}_{\text{mass}} = \bar{\mathcal{U}}_L \cdot M_u^{\text{diag}} \cdot \mathcal{U}_R + \bar{\mathcal{U}}_R \cdot M_u^{\text{diag}\,\dagger} \cdot \mathcal{U}_L + \mathcal{U} \leftrightarrow \mathcal{D}$$
$$= \bar{\mathcal{U}} \cdot \left( M_u^{\text{diag}} + M_u^{\text{diag}\,\dagger} \right) \cdot \mathcal{U} + \bar{\mathcal{U}} \gamma_5 \cdot \left( M_u^{\text{diag}} - M_u^{\text{diag}\,\dagger} \right) \cdot \mathcal{U}$$

- The term $\bar{\mathcal{U}} \gamma_5 \mathcal{U}$ should not be there!
- Can be removed by a chiral transformation:

$$\mathcal{U} \to \exp\left(-i\theta\gamma_5\right)\mathcal{U} \qquad \mathcal{U} \leftrightarrow \mathcal{D}$$

if

$$\theta = \mathrm{Arg\,Det} M \quad \text{with} \quad M = \begin{pmatrix} M_u^{\text{diag}} & 0 \\ 0 & M_d^{\text{diag}} \end{pmatrix}$$

# Strong CP violation

if the chiral transformations were a symmetry

- This is only classically true
- QFT: The Chiral symmetry is anomalous!

$$\partial_\mu \left(\mathcal{U}\gamma_\mu\gamma_5\mathcal{U}\right) = \mathcal{U}\gamma_5 \cdot M_u^{\mathrm{diag}} \cdot \mathcal{U} + \frac{\alpha_s}{4\pi} G^{\mu\nu,a}\tilde{G}^a_{\mu\nu}$$

and the same for $\mathcal{D}$.

- Hence: Removing the $\gamma_5$ term generates a new term in the SM action:

$$S \to S - i(\mathrm{Arg\,Det}M) \int d^4x \, \frac{\alpha_s}{8\pi} G^{\mu\nu,a}\tilde{G}^a_{\mu\nu}$$

# Strong CP violation

Quantum Theory modifies QCD

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD} + \theta \, \frac{\alpha_s}{8\pi} G^{\mu\nu,a} \tilde{G}^a_{\mu\nu}$$

- Hence the dynamics depend only on

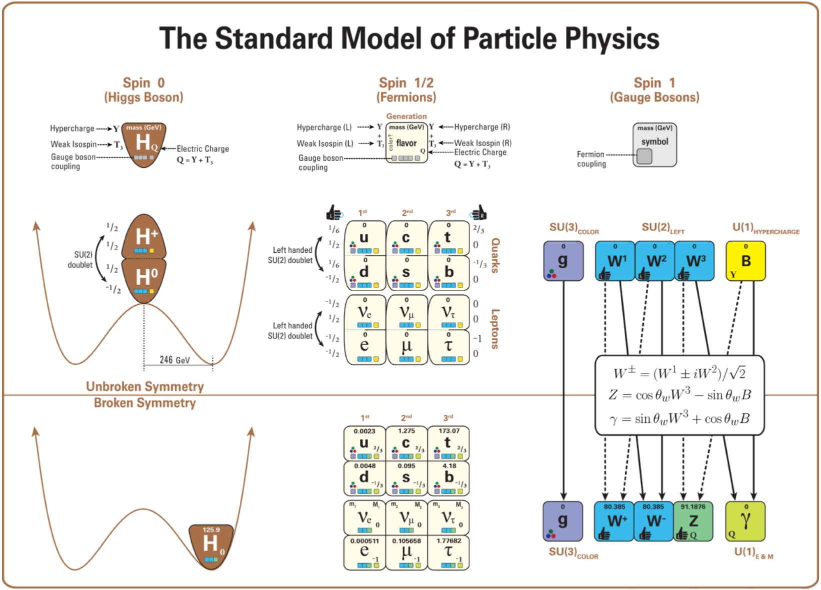
$$\bar{\theta} = \theta - \mathrm{Arg\,Det} M$$

- This solves certain problems, but it creates new ones
- $G^{\mu\nu,a} \tilde{G}^a_{\mu\nu}$ breaks P as well as CP!
- It generates a neutron electric dipole moment

$$d_N^{TH} \sim 10^{-16} \bar{\theta}\, \mathrm{e\,cm} \quad d_N^{exp} \leq 1.1 \times 10^{-25} \mathrm{e\,cm}$$

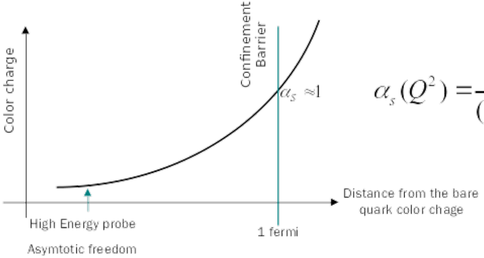
- Strong CP Problem: Why is $\bar{\theta} \leq 10^{-9\pm1}$ so small?

# The Standard Model



# Quark confinment

→ Quark confinement( Only colorless states are physically observable) is explained in QCD by infrared divergences due to the massless gluons




$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f)\log(Q^2/\Lambda^2_{QCD})}$$

# Heavy quark Efective Theory HQET

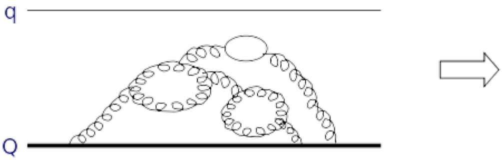
Heavy Quark : $m_Q > \Lambda_{QCD}$

Heavy Quark limit : $m_Q \to \infty$

Heavy Quark + light quark system


Comptom wavelength of Q : $\lambda_Q \sim \frac{1}{m_Q}$

To resolve the quantum number of Heavy quark, need a hard probe with $Q^2 \geq m_Q^2$




"Brown muck"

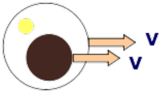
light quark q cannot see the quantum numbers of Heavy Quark

# Heavy quark Efective Theory HQET

The configuration light Degree of freedoms with different heavy quark flavor, spin system of hadron does not change if the velocity of heave quark is same.

We can regard heavy quark velocity as conserved quantity

Heavy Quark velocity ≒ Meson velocity
Momentum transfer ~ $\Lambda_{QCD}$ ⇒ velocity change ~ $\Lambda_{QCD}/m_Q$ ~ 0




Therefore this picture gives spin – flavor symmetry in QCD under $m_Q \to \infty$ limit.
$N_h$ heavy quark flavor → $SU(2N_h)$ spin-flavor symmetry group

It provide the relations between the properties of hadrons with different flavor and spin of heavy quark.
Such as B, D, $B^*$,$D^*$, $\Lambda_b$ $\Lambda_c$

# HQET spectroscopy

Strong Interaction dynamics is independent of the spin and mass of the heavy quark by heavy quark symmetry.

Therefore hadronic states can be classified by the quantum number of the light DOF s uch as flavor, spin, parity, etc.

Spin-flavor symmetry in HQET predict some relations of properties of hadron states, ty pically mass spectrum of different Hadrons states

| Meson | Constituent Quarks | J | P |
|---|---|---|---|
| D | c, (u or d) | 0 | - |
| D* | c, (u or d) | 1 | - |
| $D_1$ | c, (u or d) | 1 | + |
| $D_2^*$ | c, (u or d) | 2 | + |
| $D_s$ | c, s | 0 | - |
| $D_s^*$ | c,s | 1 | - |

| Meson | Constituent Quarks | J | P |
|---|---|---|---|
| B | b, (u or d) | 0 | - |
| B* | b, (u or d) | 1 | - |
| $B_1$ | b, (u or d) | ? | ? |
| $B_2^*$ | b, (u or d) | ? | ? |
| $B_s$ | b, s | 0 | - |
| $B_s^*$ | b,s | 1 | - |

# HQET spectroscopy

## 1. Ground state mesons

$$j_l = s_l = \frac{1}{2} \qquad J = j_l \pm \frac{1}{2} \qquad J = 0 \ or \ J = 1$$ degenerate states

Experimentally

$$m_{B^*} - m_B \approx 46\,\mathrm{MeV}$$
$$m_{D^*} - m_D \approx 142\,\mathrm{MeV}$$
$$m_{D_S^*} - m_{D_S} \approx 142\,\mathrm{MeV}$$

Quite small as expected

Need a hyperfine correction of order $1/m_Q$

$$m_{M^*} - m_M \sim \frac{1}{m_Q}$$

So we can expect $m^2_{B^*} - m^2_B \approx m^2_{D^*} - m^2_D \approx const.$

$$m^2_{B^*} - m^2_B \approx 0.49\,\mathrm{GeV}^2$$
$$m^2_{D^*} - m^2_D \approx 0.55\,\mathrm{GeV}^2$$

# HQET spectroscopy

## 2. Excited state mesons

$$s_l = \frac{1}{2}, \; j_l = \frac{3}{2} \qquad \left[ \begin{array}{l} J=1:\; D_1(2420) \\ J=2:\; D_2^*(2460) \end{array} \right.$$ degenerate states

$$m_{D_2^*} - m_{D_1} \approx 35\,\mathrm{MeV}$$

It is small mass splitting supporting our assertion

One can expect also

$$m^2_{B_2^*} - m^2_{B_1} \approx m^2_{D_2^*} - m^2_{D_1} \approx 0.17\ \mathrm{GeV}^2$$

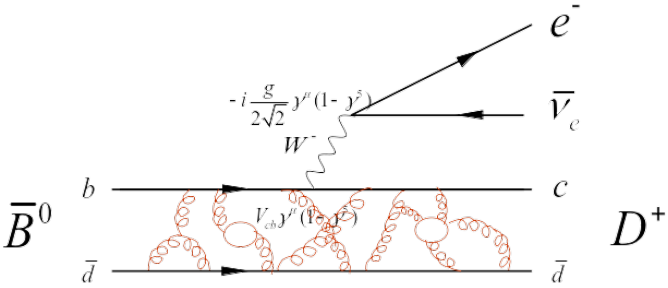
## 3. Excitation energy

$$m_{B_S} - m_B \approx m_{D_S} - m_D \approx 100\,\mathrm{MeV}$$

$$m_{B_1} - m_B \approx m_D - m_D \approx 557\,\mathrm{MeV}$$

# Weak decay form factors

Physical picture of weak decay



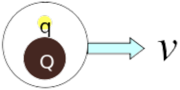

$$M = \frac{G_F}{\sqrt{2}} V_{cb} \left[ \overline{u}(p_e)\gamma^\mu(1-\gamma_5)v(p_{\nu_e}) \right] \left\langle D^{(*)}(p) \,|\, \overline{c}\gamma_\mu(1-\gamma_5)b \,|\, B(p) \right\rangle$$

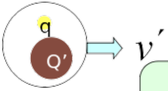
Hadronic matrix element $\left\langle D^{(*)}(p) \,|\, \overline{c}\gamma_\mu(1-\gamma_5)b \,|\, B(p) \right\rangle$

parameterized by several form factors.

# Weak decay form factors

Kinematical picture $M \to M' e \nu$



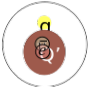

$t < t_0$ $\qquad$ $t > t_0$

$$q^2 = (P' - P)^2 \approx (m_{Q'} v' - m_Q v)^2$$
$$= m_{Q'}^2 + m_Q^2 - 2 m_{Q'} m_Q v' \cdot v$$

$$w \equiv v \cdot v' \qquad v \cdot v' = \frac{m_{M'}^2 + m_M^2 - q^2}{2 m_{M'} m_M} \qquad 0 \le q^2 \le (m_{M'} - m_M)^2$$

Maximum $q^2 = (m_{M'} - m_M)^2$ ; minimum $w = 1$ ⇨ Zero recoil


Minimum $q^2 = 0$ ; maximum w



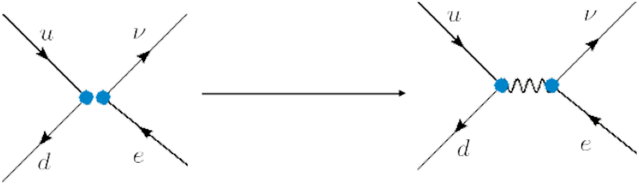
# Short and long interactions

- There is a hierarchy between the W/Z/t scales and the energies/masses of the external particles we are interested in:

$$m_W, m_Z, m_t \longleftrightarrow m_b, m_c, m_s$$

- This suggests that the physics at these two scales can be treated independently

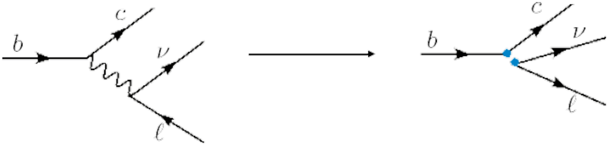
- Historically this is what happened for the $\beta$-decay



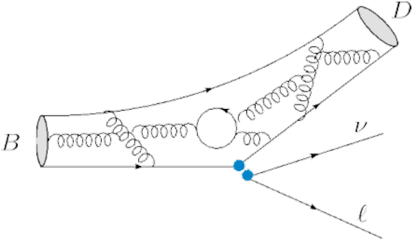

$$\frac{G_F}{\sqrt{2}} \qquad\qquad \frac{g_2^2}{8m_W^2}$$

# Example $B \to D\ell\nu$

- The first step is to identify the "core" short distance interaction:

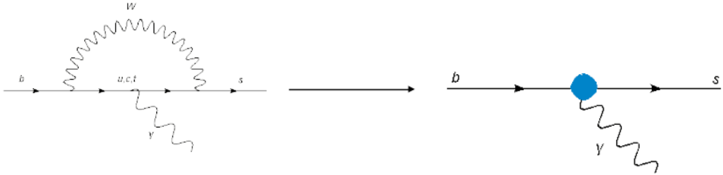



- The second step is to relate the $b$ and $c$ quarks to the particles we observe:



# Example - inclusive decays $B \to X_S\gamma$

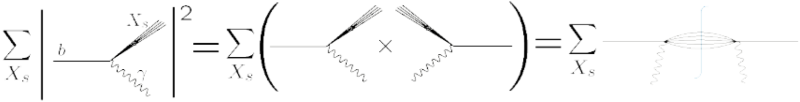
- The first step is to identify the "core" short distance interaction:




- Quark-hadron duality:

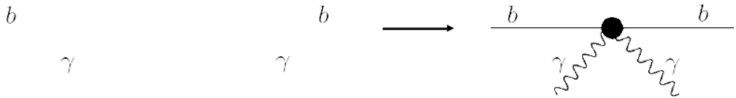
$$\sum_{X_s \in \text{hadrons}} \Gamma(b \to X_s\gamma) = \sum_{X_s \in \{\text{quarks,gluons}\}} \Gamma(b \to X_s\gamma)$$

- Optical theorem:



# Example - inclusive decays $B \to X_S\gamma$

- This part of the calculation is perturbative (hopefully):



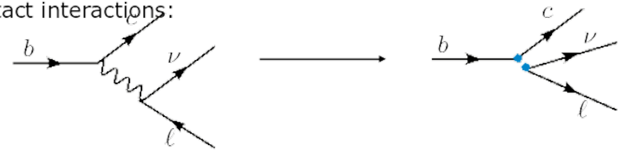

- The b quark has *non-perturbative* overlap with the B meson:

# Effective Hamiltonian

- The strategy described in the previous slides allows to decouple the problem of calculating short-distance effects from the complications of

  non-perturbative QCD

- Note that after "integrating out" the heavy field, we are left with new contact interactions:




$$\frac{g_2^2}{8}\frac{1}{m_W^2 - p_W^2}\left(\bar{c}_L\gamma^\mu b_L\right)\left(\bar{\nu}_L\gamma_\mu \ell_L\right) \qquad \frac{G_F}{\sqrt{2}}\left(\bar{c}_L\gamma^\mu b_L\right)\left(\bar{\nu}_L\gamma_\mu \ell_L\right) + O\left(\frac{p_W^2}{m_W^2}\right)$$

- We construct an effective Hamiltonian that does not contain the W, Z and t anymore, but has new operators.

# Effective Hamiltonian

- $\mathcal{L}_{SM} \longrightarrow \mathcal{L}_{eff} \equiv [\mathcal{L}_{SM}]_{\text{no W,Z,t}} - \sum C_i O_i$

  Wilson coefficients

- For example:

$$b \to c\ell\nu \Rightarrow \begin{cases} C = \frac{G_F}{\sqrt{2}} \\ O = (\bar{c}_L \gamma^\mu b_L)\,(\bar{\nu}_L \gamma_\mu \ell_L) \end{cases}$$

$$b \to s\gamma \Rightarrow \begin{cases} C = 4\frac{G_F}{\sqrt{2}}\, V_{tb}^* V_{ts}\, f(\frac{m_t^2}{m_W^2}) \\ O = \frac{e m_b}{16\pi^2}\, (\bar{s}_L \sigma^{\mu\nu} b_R)\, F_{\mu\nu} \end{cases}$$

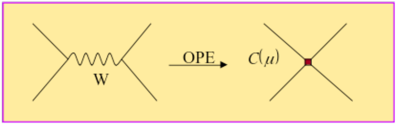
- One advantage is that a given operator contributes to many different processes

# Operators Products Expansion (OPE)

OPE allows to disentangle SD and LD effects by "integrating out" the W boson and other fields with mass larger than a certain factorization scale.

$$A = \langle H_{eff} \rangle = \sum_i C_i(\mu) \langle Q_i(\mu) \rangle$$

Wilson coefficients,
determined by matching

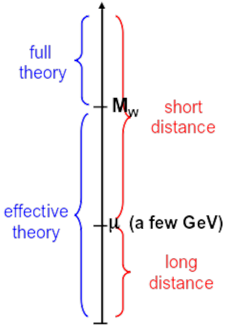



Due to asymptotic freedom of QCD, the strong interaction effects at short distances are calculable in perturbation theory.

However, as a result of matching procedure at the scale $M_W$ and RG equations:

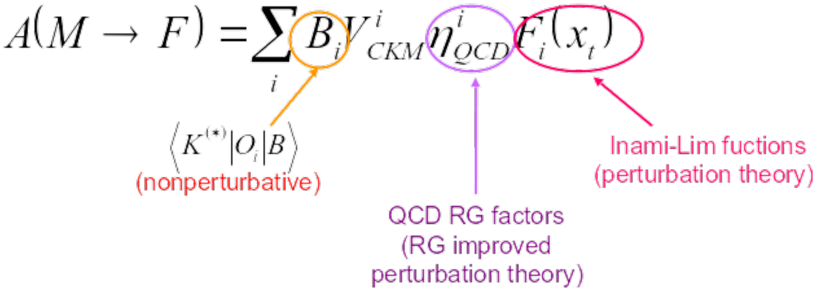
$$C_i(\mu) \quad \text{depend on} \quad \alpha_S(\mu) \log \frac{M_W}{\mu}$$

LARGE!
spoils the validity of
the usual perturbation theory



# Operators Products Expansion (OPE)

Basic structure of decay amplitudes:



# Recapitulation