Entropy production and dissipation in spin hydrodynamics

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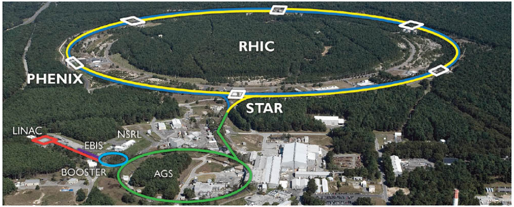
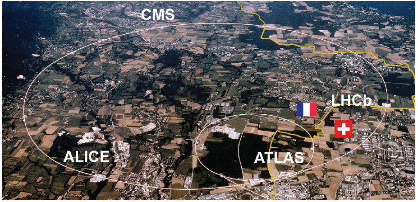
Outline

Probes of Quark-Gluon Plasma

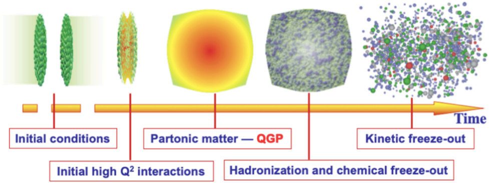
Relativistic heavy ion collisions





RHIC @ BNL

LHC @ CERN



Schematic picture of the evolution stages in relativistic heavy-ion collisions [Nucl.Phys.News30no.2,(2020)10–16].

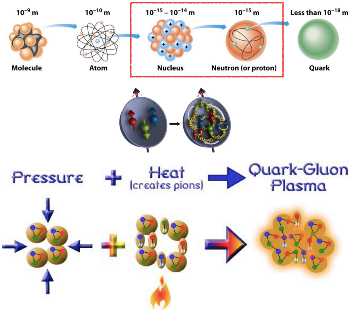
Quark-gluon plasma

- A thermalized collective deconfined phase of quarks and gluons.

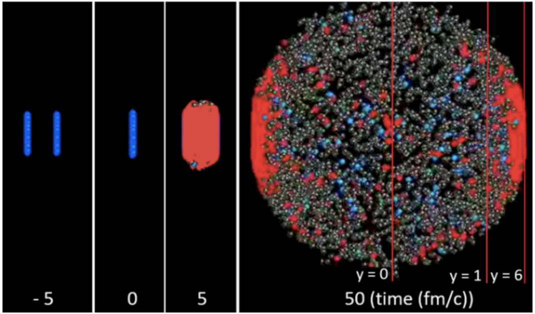
Deconfinement

Thermalization

Collective behavior



Deconfinement



Simulation of Pb+Pb collision at $\sqrt{s_{NN}} = 2.76$ TeV, showing hadrons (blue) and QGP (red) [HIC group @ MIT].

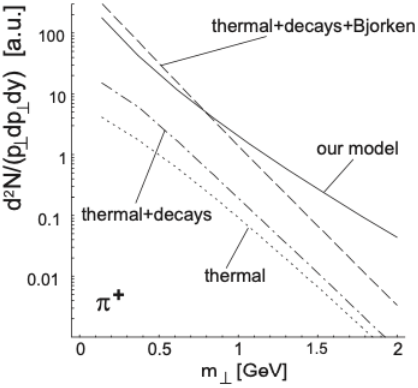
- At $t = 1\,\mathrm{fm/c}$ after the collision, the estimated average energy density is,

$$\varepsilon \approx 12\,\mathbf{GeV}/\mathbf{fm}^3$$

which is 20 $\times$ the typical hadron energy density of,

$$\varepsilon_{\mathrm{hadron}} \approx 500\,\mathbf{MeV}/\mathbf{fm}^3$$

Hence, the created matter is expected to behave differently from hadrons.



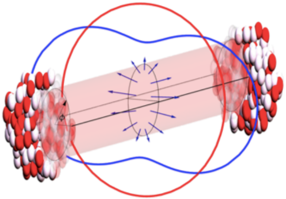
Contributions of various effects to the $p_\perp$-spectra of π^+ [Broniowski et al. Phys.Rev.Lett.87:272302,2001].

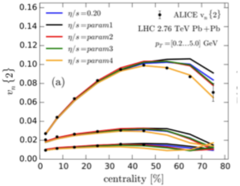
- **The QGP formed in the collision is locally equilibrated as suggested from the spectra.**

Collectivity

- **Anisotropic distribution signals collectivity ,**

$$dN/d\phi \simeq 1 + 2v_1 cos(\phi - \psi_{RP}) + 2v_2 cos[2(\phi - \psi_{RP})] + \ldots$$



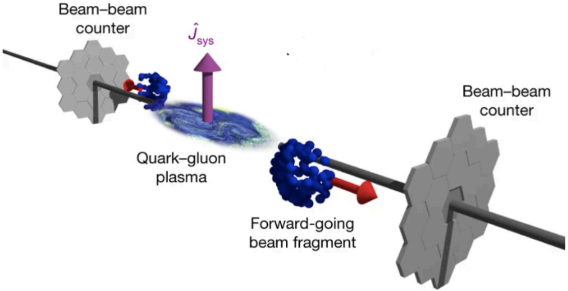


A hydrodynamic model of v_2 (top) is compared with ALICE measurements of the anisotropy [Ann.Rev.Nucl.Part.Sci.2018.68:1–49].

Initial coordinate anisotropy transforms into final momentum anisotropy in strongly interacting systems, as expected in hydrodynamic theory.

Spin polarization in heavy-ion collisions

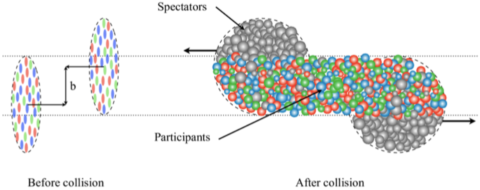
Total angular momentum of the QGP



A sketch of a Au+Au collision in the STAR detector system [Nature 548,62–65(2017)].

- **Non-central collisions involve substantial global angular momentum J_{sys} of the order of $10^3 - 10^4\hbar$.**

Vortical structures in the QGP?

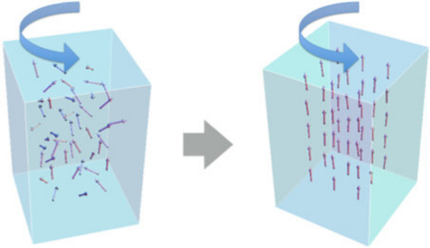


A schematic view of a heavy-ion collision. The impact parameter “b” is shown as well as the spectator nucleons and the participant nucleons [Aaij,Roel et al.arXiv:2111.01607CERN-LHCb].

- **Local vortical structures of the created fluid are suggested.**
- **The average (over the fluid elements) vorticity points along the direction of the angular momentum of the collision J_{sys} ,**

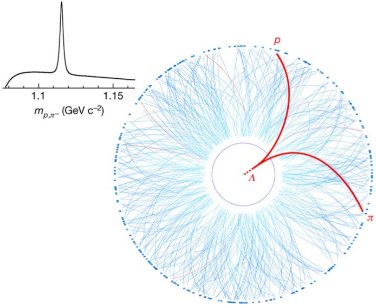
$$\omega_{kin} \simeq \nabla \times v \longrightarrow \omega_{\mu\nu} = \nabla_{(\nu} u_{\mu)} / T.$$

Hadron spin polarization

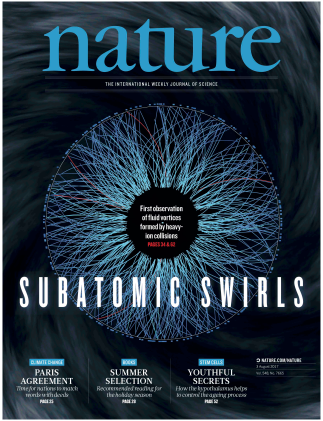


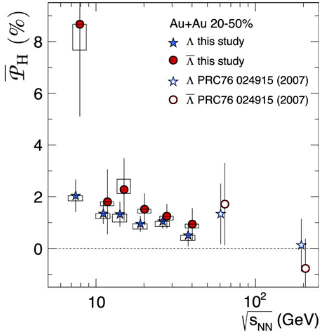
- If on average the spin of emitted hadrons tends to be polarized along to J_{sys}, signatures of local vortical structures are expected.

Global Λ polarization



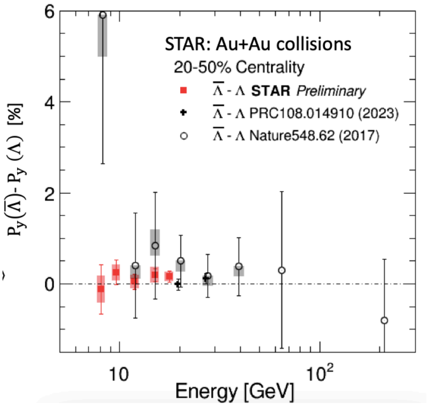
A single Au + Au collision in the STAR TPC [Nature548,62(2017)].





Average Λ global polarization [STAR, L. Adamczyk et al., Nature548,62(2017)].

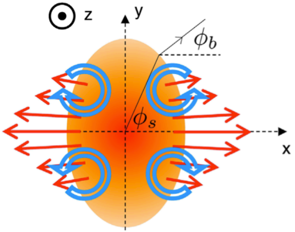
Λ-$\bar{\Lambda}$ splitting



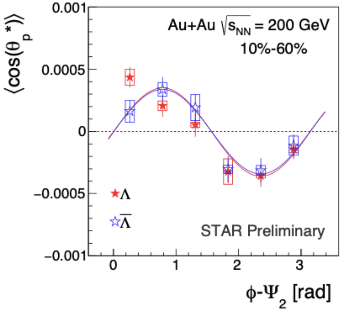
[SQM conference 2024]

- **Recent polarization results shows no splitting between Λ-$\bar{\Lambda}$.**
- **This result suggest that vortical structures does not differentiate between Λ and $\bar{\Lambda}$.**

Angle-dependent polarization along beam-direction

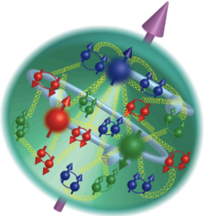


Vorticities along the beam direction (open arrows) induced by anisotropic flow (solid arrows) in the (x-y)-plane [Phys.Rev.Lett.123,132301].



Pz of Λ hyperons as a function of azimuthal angle ϕ in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV (Local Polarization) [Phys.Rev.Lett.123,132301].

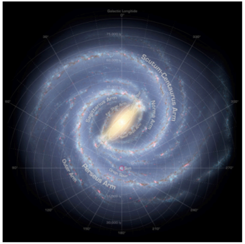
To understand the polarization measurements, and in turn the vortical structures, we need to extend relativistic hydrodynamics to spin hydrodynamics ,



neutron

$\sim 10^{-15}\mathrm{m}$

$\sim 1\mathrm{m}$



galaxy

$\sim 10^{21}\mathrm{m}$

[https://fizyka.ujk.edu.pl/files/seminars/UJK20Ryblewski20web.pdf]

What is spin hydrodynamics?

- **Spin hydrodynamics, emerging as an effective limit of quantum field theory, is a theoretical tool for describing the evolution of the collective partonic medium produced in non-central heavy-ion collisions throughout its lifetime, from its formation to the point at which it cools enough to hadronize into particles.**

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Covariant thermodynamics approach: results and outlook

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[2] A. Daher, A. Das, and R. Ryblewski, "Stability studies of first-order spin-hydrodynamic frameworks," *Phys. Rev. D* **107** no. 5, (2023) 054043, arXiv:2209.10460 [nucl-th].

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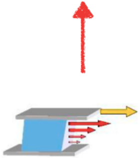
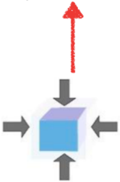
Non-relativistic hydrodynamics describes the evolution of energy density and fluid velocity using the Navier-Stokes equations,

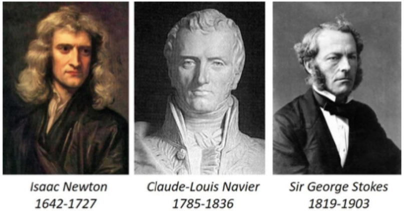
$$\partial_t \rho + \rho \vec{\partial} \cdot \vec{v} + \vec{v} \cdot \vec{\partial} \rho = 0$$

$$\frac{\partial v^i}{\partial t} + v^k \frac{\partial v^i}{\partial x^k} = -\frac{1}{\rho}\frac{\partial p}{\partial x^i} - \frac{1}{\rho}\frac{\partial \Pi^{ki}}{\partial x^k}$$

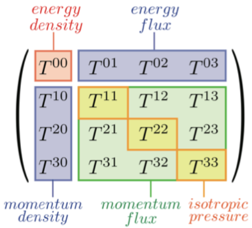
$$\Pi^{ki} = -\eta \left(\frac{\partial v^i}{\partial x^k} + \frac{\partial v^k}{\partial x^i} - \frac{2}{3}\delta^{ki}\frac{\partial v^l}{\partial x^l} \right) - \zeta\, \delta^{ik}\frac{\partial v^l}{\partial x^l}$$







For relativistic fluids one need to make some generalizations,



$$\begin{matrix} \mathbf{v}(t,\mathbf{x}) \\ \rho(t,\mathbf{x}) \end{matrix} \Rightarrow \begin{matrix} u^\mu = \gamma(\mathbf{v}) \begin{pmatrix} 1 \\ \mathbf{v} \end{pmatrix} \\ \epsilon(t,\mathbf{x}) \end{matrix}$$

$$\begin{aligned} u_\nu \partial_\mu T^{\mu\nu} &= D\epsilon + (\epsilon + p)\partial_\mu u^\mu + u_\nu \partial_\mu \Pi^{\mu\nu} = 0\,, \\ \Delta^\alpha_\nu \partial_\mu T^{\mu\nu} &= (\epsilon + p) D u^\alpha - \nabla^\alpha p + \Delta^\alpha_\nu \partial_\mu \Pi^{\mu\nu} = 0\,. \end{aligned}$$

$\xrightarrow{|\mathbf{v}| \ll 1}$

$$\begin{aligned} \partial_t \rho + \rho \vec{\partial} \cdot \vec{v} + \vec{v} \cdot \vec{\partial} \rho &= 0 \\ \frac{\partial v^i}{\partial t} + v^k \frac{\partial v^i}{\partial x^k} &= -\frac{1}{\rho}\frac{\partial p}{\partial x^i} - \frac{1}{\rho}\frac{\partial \Pi^{ki}}{\partial x^k} \end{aligned}$$

EOM are given by energy-momentum conservation, i.e.,

$$\partial_\mu \mathbf{T}^{\mu\nu} = 0\,, \quad \mathbf{T}^{\mu\nu} = \mathbf{T}_0^{\mu\nu} + \delta\mathbf{T}_S^{\mu\nu} + \delta\mathbf{T}_A^{\mu\nu}\,, \quad \mathbf{P}^\nu = \int \mathbf{d}\Sigma_\mu \mathbf{T}^{\mu\nu}\,, \quad \mathbf{T}^{\mu\nu} = \langle \hat{\rho}\, \hat{\mathbf{T}}^{\mu\nu} \rangle.$$

What is a fluid with spin ?

A fluid with spin is a fluid having a macroscopic spin density and which thus needs a spin tensor $\mathcal{S}^{\lambda,\mu\nu}$ to be described, besides the stress-energy tensor $T^{\mu\nu}$

New dynamical equation:

- $\mathbf{S}^{\lambda\mu\nu} \equiv$ **Spin current,**

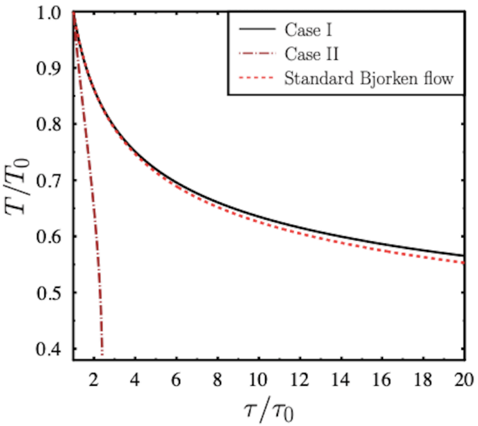
$$\partial_\lambda \mathbf{S}^{\lambda\mu\nu} = \mathbf{T}^{\nu\mu} - \mathbf{T}^{\mu\nu}\,, \quad \mathbf{S}^{\lambda\mu\nu} = \mathbf{S}_0^{\lambda\mu\nu} + \delta\mathbf{S}^{\lambda\mu\nu}\,, \quad \mathbf{S}^{\mu\nu} = \int \mathbf{d}\Sigma_\lambda \mathbf{S}^{\lambda\mu\nu}\,, \quad \mathbf{S}^{\lambda\mu\nu} = \langle \hat{\rho}\, \widehat{\mathbf{S}}^{\lambda\mu\nu} \rangle$$

$$\boldsymbol{u}^{\mu} \partial_\mu \mathbf{S}^{\alpha\beta} + \mathbf{S}^{\alpha\beta} \partial_\mu \mathbf{u}^{\mu} = -2\delta \mathbf{T}_A^{\mu\nu}$$

- **Euler equation with spin:**

$$\varepsilon + \mathbf{P} = \mathbf{Ts} + \mu \mathbf{n} + \boldsymbol{\omega}_{\mu\nu} \mathbf{S}^{\mu\nu}$$

Result: Temperature evolution



Red (dashed) line represents the variation of temperature for Bjorken flow in relativistic hydro without spin. Brown (dashed-dotted) line represents the variation evolution in Navier-stokes theory with spin. We consider $T_0 = 200$ MeV and $\tau_0 = 0.5$ fm.

Toolkit

One of the possible scenarios is to allow for every variable in $\mathbf{T}^{\mu\nu}$ and $\mathbf{S}^{\lambda\mu\nu}$ to be dynamical, i.e.,

$$\begin{cases} \mathbf{T}^{\mu\nu} & \longrightarrow \textbf{16 evolution equations}, \\ \mathbf{S}^{\lambda\mu\nu} & \longrightarrow \textbf{24 evolution equations.} \end{cases}$$

- **Muller-Israel-Stewart entropy production rate with spin,**

$$\partial_\mu \mathbf{s}^\mu_{\mathbf{IS}} = (\partial_\mu \boldsymbol{\beta}_\nu + 2\boldsymbol{\omega}_{\mu\nu}\boldsymbol{\beta})\delta\mathbf{T}_A^{\mu\nu} + \delta\mathbf{T}_S^{\mu\nu}\partial_\mu\boldsymbol{\beta}_\nu - \delta\mathbf{S}^{\mu\alpha\beta}\partial_\mu(\boldsymbol{\beta}\boldsymbol{\omega}_{\alpha\beta}) + \partial_\mu\mathbf{Q}^\mu \quad \geq \quad 0.$$

Toolkit

$$D\varepsilon + (\varepsilon + p)\theta = \pi^{\mu\nu}\partial_\mu u_\nu + \Pi\theta - \nabla \cdot q + \phi^{\mu\nu}\partial_\mu u_\nu \,,$$

$$\begin{aligned}(\varepsilon + p)Du^\alpha - \nabla^\alpha p = &- \Delta^\alpha_\nu \partial_\mu \pi^{\mu\nu} - \Delta^{\mu\alpha}\partial_\mu \pi + \pi D u^\alpha - q^\mu \partial_\mu u^\alpha \\ &+ \Delta^\alpha_\nu D q^\nu + q^\alpha \theta - \Delta^\alpha_\nu \partial_\mu \phi^{\mu\nu} \,,\end{aligned}$$

$$\tau_\Pi D\Pi + \Pi = \zeta \left[\theta + T a_1 \Pi\theta + T\Pi D a_1\right] ,$$

$$\tau_\pi \Delta^{\mu\nu}_{\alpha\beta} D\pi^{\alpha\beta} + \pi^{\mu\nu} = 2\eta \left[(\nabla^{(\mu}u^{\nu)} - \frac{1}{3}\theta\Delta^{\mu\nu}) + T a_2 \theta \pi^{\mu\nu} + T\pi^{\mu\nu} D a_2\right] ,$$

$$\tau_q \Delta^\mu_\nu D q^\nu + q^\mu = \lambda \left[(\beta\nabla^\mu T + Du^\mu - 4\omega^{\mu\nu}u_\nu) - T a_4 q^\mu \theta - T q^\mu D a_4\right] ,$$

$$\tau_\phi \Delta^{[\mu\nu]}_{[\alpha\beta]} D\phi^{\alpha\beta} + \phi^{\mu\nu} = \gamma \left[(\beta\nabla^{[\mu}u^{\nu]} + 2\beta\Delta^{\mu\alpha}\Delta^{\nu\beta}\omega_{\alpha\beta}) + a_5 \theta \phi^{\mu\nu} + \phi^{\mu\nu} D a_5\right] ,$$

$$DS^{\alpha\beta} + S^{\alpha\beta}\theta + \partial_\mu S_1^{\mu\alpha\beta} = -2(q^\alpha u^\beta - q^\beta u^\alpha + \phi^{\alpha\beta}),$$

$$\tau_\Phi D\Phi + \Phi = \chi_1 \left[-2u^\alpha \nabla^\beta(\beta\omega_{\alpha\beta}) + \tilde{a}_1 \theta\Phi + \Phi D\tilde{a}_1\right] ,$$

$$\tau_{\tau_s} \Delta^{\mu\nu}_{\alpha\beta} D\tau_s^{\alpha\beta} + \tau_s^{\mu\nu} = \chi_2 \left[- u^\alpha(\Delta^{\gamma\mu}\Delta^{\rho\nu} + \Delta^{\gamma\nu}\Delta^{\rho\mu} - \frac{2}{3}\Delta^{\gamma\rho}\Delta^{\mu\nu})\nabla_\gamma(\beta\omega_{\alpha\rho}) + \tilde{a}_2\theta\tau_s^{\mu\nu} + \tau_s^{\mu\nu} D\tilde{a}_2\right],$$

$$\tau_{\tau_a} \Delta^{[\mu\nu]}_{[\alpha\beta]} D\tau_a^{\alpha\beta} + \tau_a^{\mu\nu} = \chi_3 \left[-u^\alpha(\Delta^{\gamma\mu}\Delta^{\rho\nu} - \Delta^{\gamma\nu}\Delta^{\rho\mu})\nabla_\gamma(\beta\omega_{\alpha\rho}) + \tilde{a}_3\theta\tau_a^{\mu\nu} + \tau_a^{\mu\nu} D\tilde{a}_3\right] ,$$

$$\tau_\Theta \Delta^\alpha_\lambda \Delta^\mu_\sigma \Delta^\nu_\beta D\Theta^{\lambda\sigma\beta} + \Theta^{\alpha\mu\nu} = -\chi_4 \left[-\Delta^{\delta\mu}\Delta^{\rho\nu}\Delta^{\gamma\alpha}\nabla_\gamma(\beta\omega_{\delta\rho}) + \tilde{a}_4\theta\Theta^{\alpha\mu\nu} + \Theta^{\alpha\mu\nu} D\tilde{a}_4\right] ,$$

Outlook

Formulate analytically

Solve numerically

We are here ←

Calculate Observables at freeze-out

Compare with experiments

Relativistic quantum-statistical approach: Results and future directions

References

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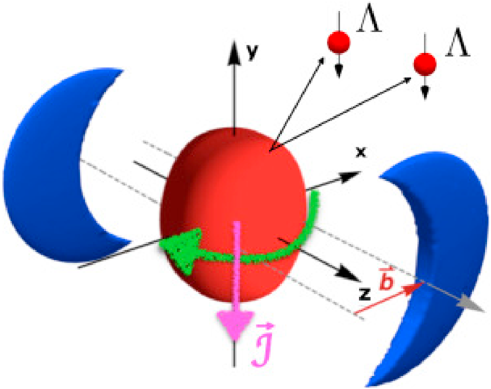
+ work in progress

Toolkit

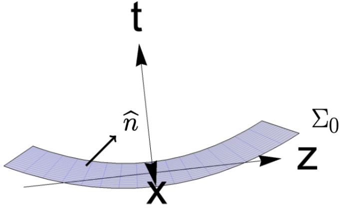
Two colliding nuclei form a strongly interacting mixed state $\widehat{\rho}_{(0)}$,

- **How to determine $\widehat{\rho}_{(0)}$?**
- **How to evolve $\widehat{\rho}_{(0)}$?**



Local equilibrium is achieved at initial hypersurface Σ_0, where entropy is maximum provided that the mean vales of energy, momentum, particle number, and spin currents are their actual values,

$$S = -\operatorname{Tr}(\widehat{\rho}\log\widehat{\rho})$$



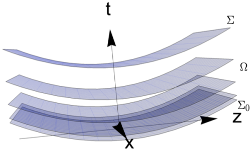
$$F[\widehat{\rho}] = -Tr[\widehat{\rho}\log\widehat{\rho}] - \int d\Sigma_0\, n_\mu\, (T^{\mu\nu}_{\rm LE} - T^{\mu\nu})\,\beta_\nu(x) - \int d\Sigma_0\, n_\mu\, (j^\mu_{\rm LE} - j^\mu)\,\zeta(x)$$
$$- \int d\Sigma_0\, n_\mu \left(S^{\mu\lambda\nu}_{\rm LE} - S^{\mu\lambda\nu}\right)\Omega_{\lambda\nu}(x)$$

$$T^{\mu\nu}_{\rm LE} \sim Tr\left[\widehat{\rho}\widehat{T}^{\mu\nu}\right]$$
$$T^{\mu\nu} \equiv \text{Actual Value}$$

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z}\exp\left[-\int_{\Sigma_0} {\rm d}\,\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\widehat{j}^\mu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu\lambda\nu}\right)\right]$$

Near local equilibrium at the hypersuface Σ, the entropy is defined as,

$$
\begin{aligned}
S &= -\operatorname{Tr}(\widehat{\rho}_{\mathrm{LE}} \log \widehat{\rho}_{\mathrm{LE}}) \\
&= \int_{\Sigma} \mathrm{d}\Sigma_{\mu} \left(\phi^{\mu} + T^{\mu\nu}_{\mathrm{LE}} \beta_{\nu} - \zeta j^{\mu}_{\mathrm{LE}} - \frac{1}{2} \Omega_{\lambda\nu} \mathcal{S}^{\mu\lambda\nu}_{\mathrm{LE}} \right)
\end{aligned}
$$



$$
\partial_{\mu} s^{\mu} = \delta T^{\mu\nu}_{S} \xi_{\mu\nu} - \delta j^{\mu} \partial_{\mu} \zeta + \delta T^{\mu\nu}_{A} (\Omega_{\mu\nu} - \varpi_{\mu\nu}) - \frac{1}{2} \delta \mathcal{S}^{\mu\lambda\nu} \partial_{\mu} \Omega_{\lambda\nu}
$$

The goal is to determine,

$$
\delta \mathbf{T}^{\mu\nu}_{S}\ , \quad \delta \mathbf{T}^{\mu\nu}_{A}\ , \quad \delta \mathbf{j}^{\mu}\ , \quad \delta \mathbf{S}^{\lambda\mu\nu}
$$

$$\delta T_S^{\mu\nu} = H^{\mu\nu\rho\sigma}\xi_{\rho\sigma} + K^{\mu\nu\rho}\partial_\rho\zeta + L^{\mu\nu\rho\sigma}\left(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}\right) + M^{\mu\nu\rho\sigma\tau}\partial_\rho\Omega_{\sigma\tau},$$

$$\delta T_A^{\mu\nu} = N^{\mu\nu\rho\sigma}\xi_{\rho\sigma} + P^{\mu\nu\rho}\partial_\rho\zeta + Q^{\mu\nu\rho\sigma}\left(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}\right) + R^{\mu\nu\rho\sigma\tau}\partial_\rho\Omega_{\sigma\tau},$$

$$\delta j^\mu = G^{\mu\rho\sigma}\xi_{\rho\sigma} + I^{\mu\rho}\partial_\rho\zeta + O^{\mu\rho\sigma}(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}) + F^{\mu\rho\sigma\tau}\partial_\rho\Omega_{\sigma\tau},$$

$$\delta S^{\mu\lambda\nu} = T^{\mu\lambda\nu\rho\sigma}\xi_{\rho\sigma} + U^{\mu\lambda\nu\rho}\partial_\rho\zeta + V^{\mu\lambda\nu\rho\sigma}\left(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}\right) + W^{\mu\lambda\nu\rho\sigma\tau}\partial_\rho\Omega_{\sigma\tau}.$$

Hence the goal reduces to determining the coefficient tensors,

$$H^{\mu\nu,\rho,\sigma}\ ,\quad K^{\mu\nu\rho}\ ,\quad L^{\mu\nu\rho\sigma}\ ,\quad M^{\mu\nu\rho\sigma\tau}$$

$$N^{\mu\nu\rho\sigma}\ ,\quad P^{\mu\nu\rho}\ ,\quad Q^{\mu\nu\rho\sigma}\ ,\quad R^{\mu\nu\rho\sigma\tau}$$

$$G^{\mu\rho\sigma}\ ,\quad I^{\mu\rho}\ ,\quad O^{\mu\rho\sigma}\ ,\quad F^{\mu\rho\sigma\tau}$$

$$T^{\mu\lambda\nu\rho\sigma}\ ,\quad U^{\mu\lambda\nu\rho}\ ,\quad V^{\mu\lambda\nu\rho\sigma}\ ,\quad W^{\mu\lambda\nu\rho\sigma\tau}$$

Using irreducible representation of $\mathbb{SO}(3)$ group,

$$\text{Vector:} \quad V^{\mu} = (0 \oplus 1)$$

$$\text{Symmetric 2-tensor:} \quad B^{\mu\nu} = (0 \oplus 0 \oplus 1 \oplus 2)$$

$$\text{Antisymmetric 2-tensor:} \quad A^{\mu\nu} = (1 \oplus 1)$$

Our hydrodynamics "tools" existing at global equilibrium,

$$u^{\mu}\ , \quad \Delta^{\mu\nu}\ , \quad \epsilon^{\mu\nu\alpha\beta}$$

Therefore, the irreducible representation, interms of our hydrodynamic variables,

$$\text{Vector:}\quad V^{\mu} = (u^{\mu} \oplus \Delta^{\mu}_{\alpha})$$

$$\text{Symmetric 2-tensor:}\quad B^{\mu\nu} = (u^{\mu}u^{\nu} \oplus \Delta^{\mu\nu} \oplus u^{\mu}\Delta^{\nu}_{\alpha} + u^{\nu}\Delta^{\mu}_{\alpha} \oplus \Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta} + \Delta^{\nu}_{\alpha}\Delta^{\mu}_{\beta}),$$

$$\text{Antisymmetric 2-tensor:}\quad A^{\mu\nu} = (u^{\mu}\Delta^{\nu}_{\alpha} - u^{\nu}\Delta^{\mu}_{\alpha} \oplus \epsilon^{\mu\nu\tau\alpha}u_{\tau}).$$

Outlook

Formulate analytically

We are here ⟵

Solve numerically

s

Calculate Observables at freeze-out

Compare with experiments

Conclusion and outlooks

Conclusion

1. **We develop Navier-Stokes-like and Muller-Israel-Stewart-like formulations to study a relativistic fluid of particles with spin such as the QGP produced in heavy-ion collision experiments.**

2. **We used a first-principle quantum-statistical methods to derive the entropy current, entropy production rate, and dissipative currents (Work in progress) for a relativistic fluid of particles with spin. Dissipative currents will allow us to develop the evolution equations in the system.**

Outlooks

1. **Solve numerically the Muller-Israel-Stewart-like formulations (16+24 dynamical equations). This will allow to calculate observables at the freeze-out hypersurface and compare with spin polarization measurements.**

2. **Finalize the analytical formulation of the dynamical equations in quantum-statistical approach, and check what results such method can offer.**

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THANK YOU