

Studying the QCD phase diagram with multi-messenger observations of compact stars



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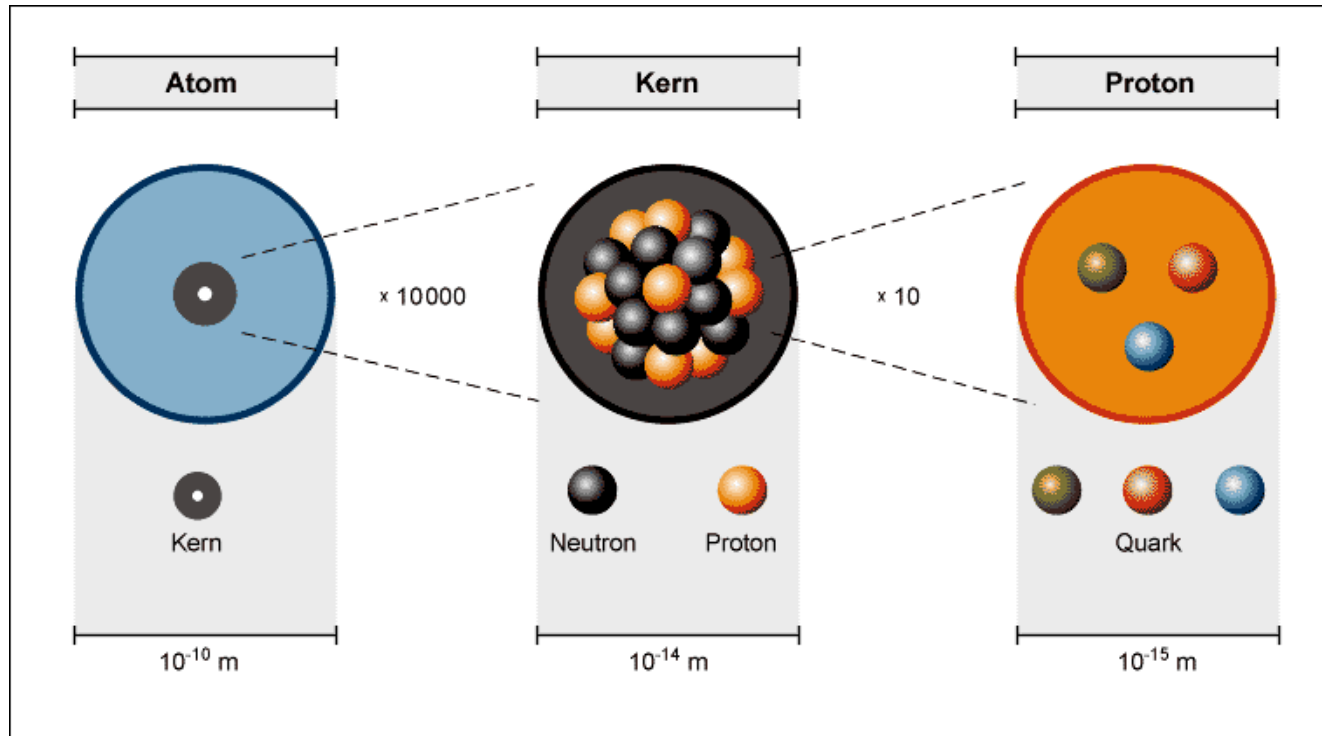
Outline

- A brief introduction to the physics of compact stars and their relation to the QCD phase diagram,.
- The compact star mass twins scenario.
- Multi-messenger observations of compact stars.
- Outlook and perspectives.

Motivation

- New channels of multi-messenger observations like gravitational radiation from merger events of binary systems of compact stars or radio and X-ray signals from isolated pulsars allow to study their most basic structural properties like mass, radius, compactness, cooling rates and compressibility of their matter.
- Nuclear measurement and experiments have narrowed the Equation of State (EoS) uncertainty in the lowest to intermediate density range.
- Violent, transient energetic emissions are associated not only with the strong magnetic fields and extreme gravity in the proximity of NS but with explosive, evolutionary stages often triggered by mass accretion from companion stars. It is expected that f -modes are excited in many of these astrophysical processes.

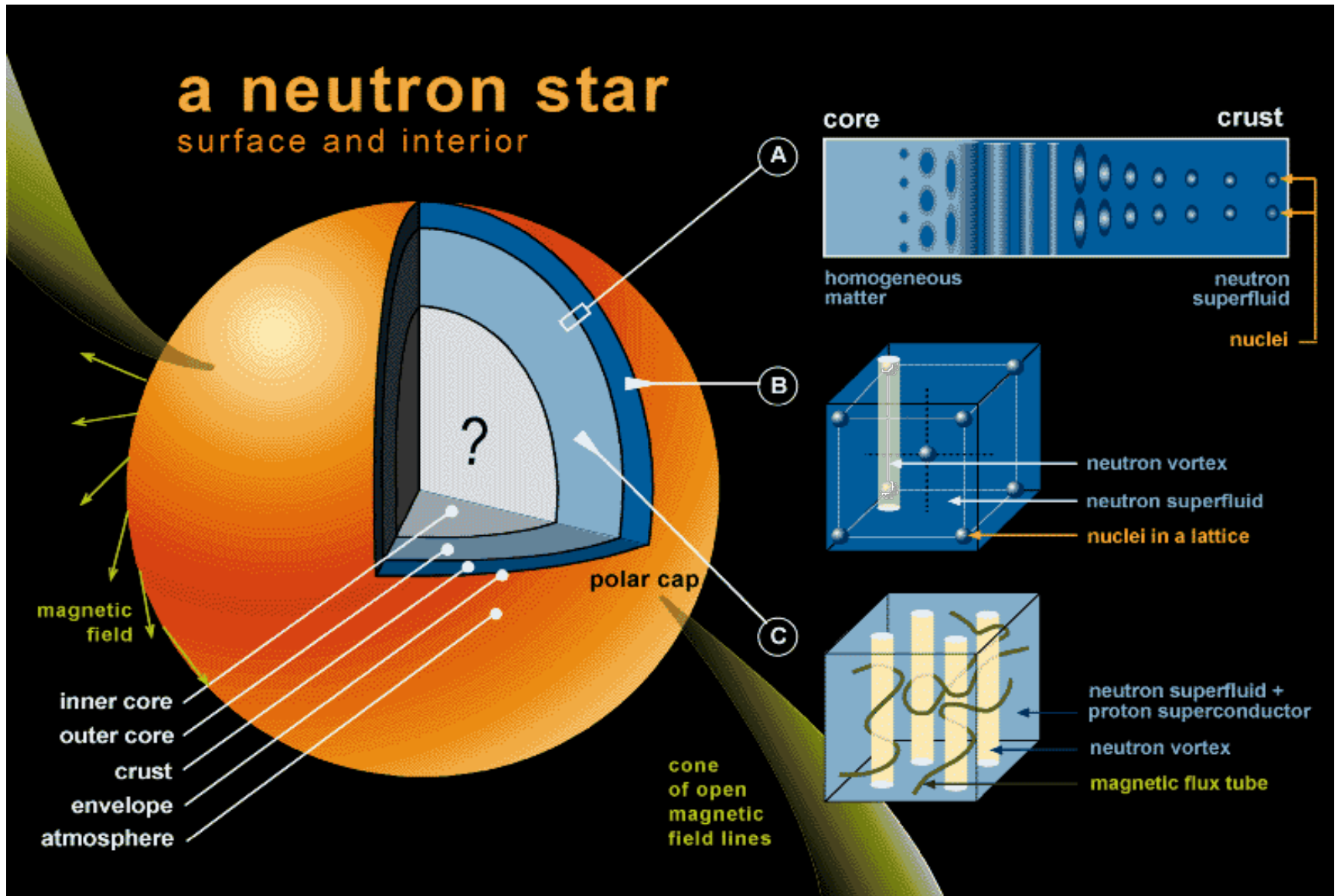
Superdense objects – what is inside?



Nucleus, A nucleons: $R_A = 1.2 \cdot 10^{-13} \text{ cm } A^{1/3}$; $\rho_0 = A \cdot 1.67 \cdot 10^{-24} \text{ g} / (4\pi/3 R_A^3) = 2.3 \cdot 10^{14} \text{ g/cm}^3$

Neutron star: $R = 10 \text{ km}$; $\rho = 2 \text{ Mo} / (4\pi/3 R^3) = 4 \cdot 10^{33} \text{ g} / (4 \cdot 10^{18} \text{ cm}^3) = 10^{15} \text{ g/cm}^3 = 4 \rho_0$

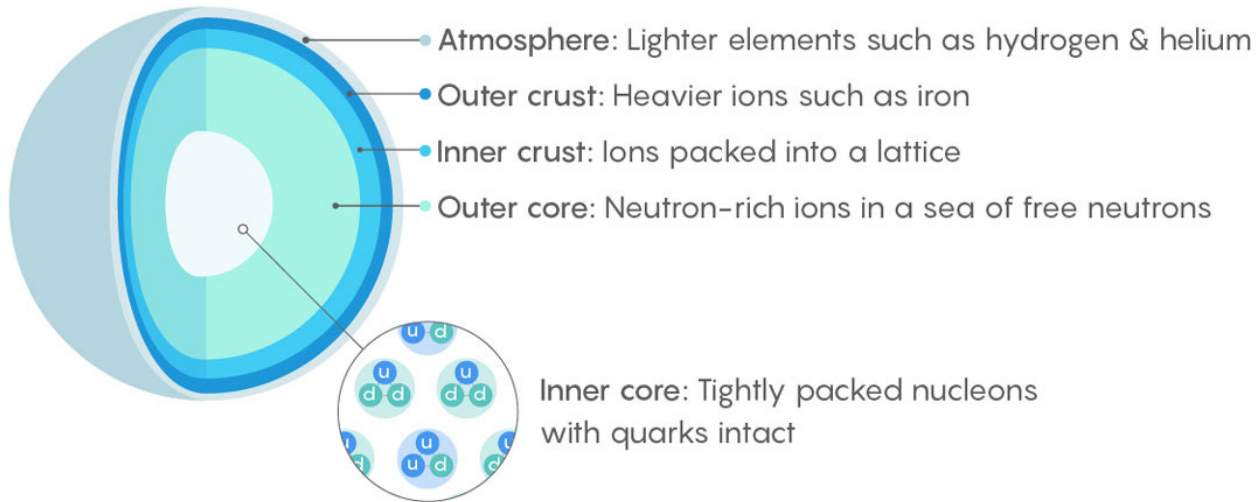
Superdense objects – what is inside?



The Extraordinary Core of a Neutron Star

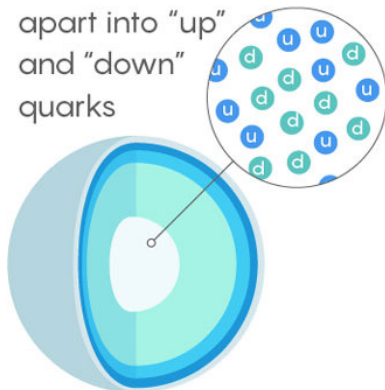
A neutron star's core is so dense that physicists aren't sure what happens inside. Researchers can't recreate the conditions in the lab, and even the theory of nuclear matter is of limited help. Here are some of the main ideas.

TRADITIONAL VIEW OF A NEUTRON STAR



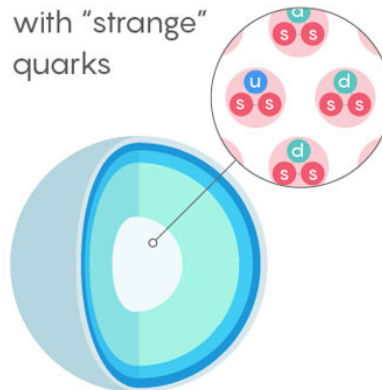
QUARK CORE

Nucleons break apart into "up" and "down" quarks



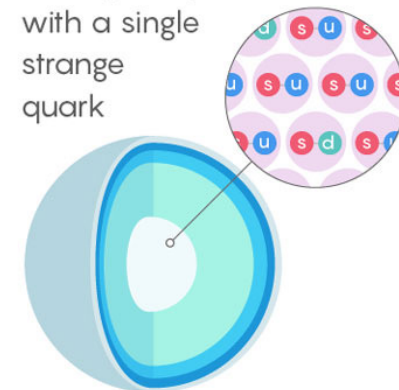
HYPERON CORE

Nucleons made with "strange" quarks

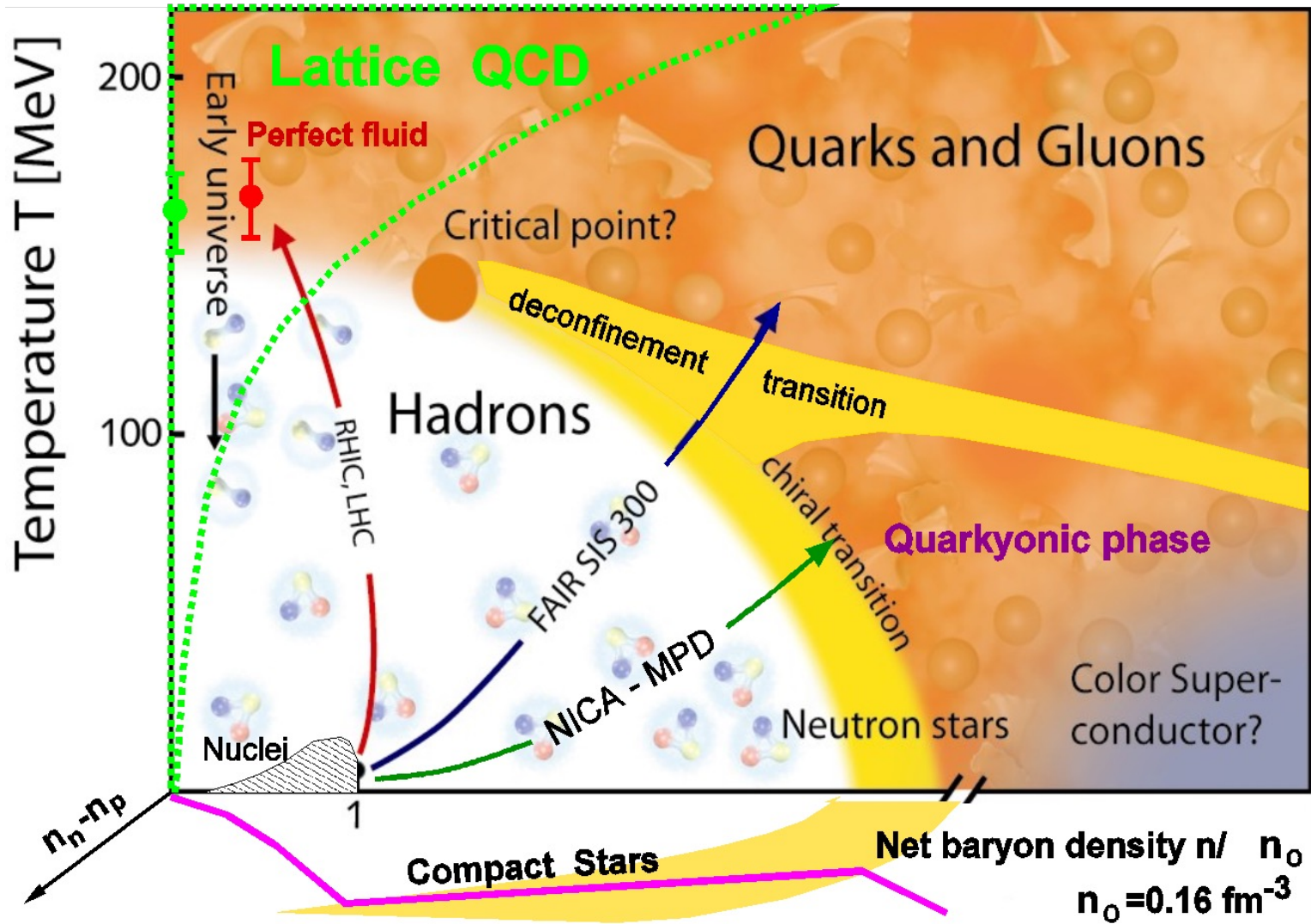


KAON CONDENSATE CORE

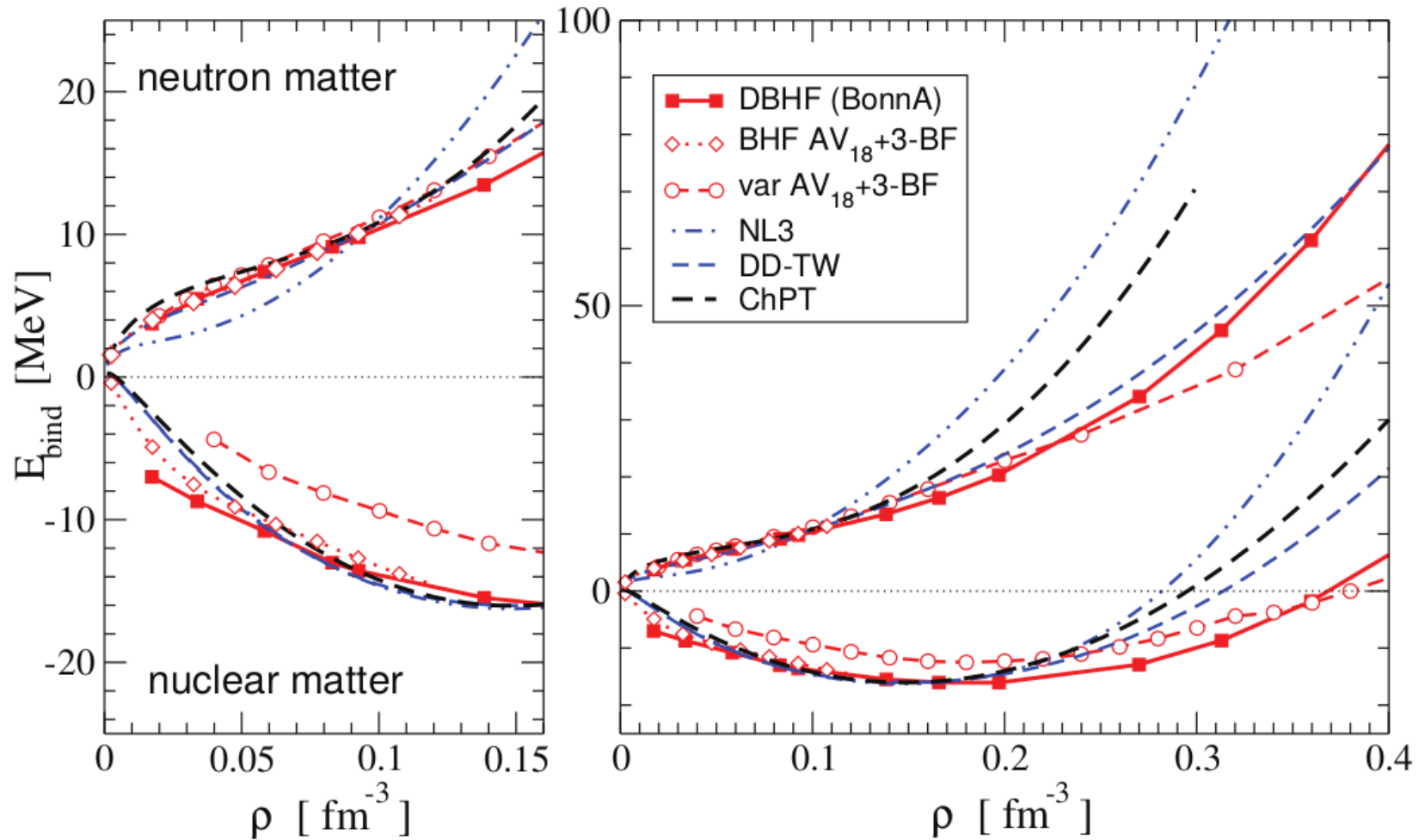
Two-quark particles with a single strange quark



Critical Endpoint in QCD



Nuclear Matter



Neutron Star Equation of State

The energy per nucleon in neutron star core matter is given by:

$$\begin{aligned} E_{\text{tot}}(n, \{x_i\}) &= E_{\text{b}}(n, x_p) + E_{\text{lep}}(n, x_e, x_\mu) , \\ E_{\text{b}}(n, x_p) &= E_0(n) + S(n, x_p) \\ E_{\text{lep}}(n, x_e, x_\mu) &= E_e(n, x_e) + E_\mu(n, x_\mu) , \end{aligned}$$

where $n = n_p + n_n$ is the total baryon density and $x_i = n_i/n$, $i = p, e, \mu$ are the fractions of protons, electrons and muons, respectively. The baryonic part is very well described by the parabolic approximation w.r.t. the asymmetry

$$\alpha = \frac{n_n - n_p}{n_n + n_p} = 1 - 2x_p,$$

resulting in $S(n, x_p) = (1 - 2x_p)^2 E_s(n)$. The leptonic contribution is a sum of the Fermi gas expressions for the contributing leptons $l = e, \mu$

$$E_l(n, x_l) = \frac{1}{n} \frac{p_{F,l}^4}{4\pi^2} \left[\sqrt{1 + z_l^2} \left(1 + \frac{z_l^2}{2} \right) - \frac{z_l^4}{2} \text{Arsinh} \left(\frac{1}{z_l} \right) \right] ,$$

where $z_l = m_l/p_{F,l}$. For massless leptons ($z_l \rightarrow 0$), this expression goes over to

$$E_l(n, x_l) \Big|_{m_l=0} = \frac{1}{n} \frac{p_{F,l}^4}{4\pi^2} = \frac{3}{4} (3\pi^2 n)^{1/3} x_l^{4/3} .$$

Charge neutrality and β -equilibrium

Under neutron star conditions charge neutrality holds,

$$x_p = x_e + x_\mu .$$

The β -equilibrium with respect to the weak interaction processes $n \rightarrow p + e^- + \bar{\nu}_e$ and $p + e^- \rightarrow n + \nu_e$ (and similar for muons), for cold neutron stars (temperature T below the neutrino opacity criterion $T < T_\nu \sim 1$ MeV) implies

$$\mu_n - \mu_p = \mu_e = \mu_\mu .$$

The chemical potentials are defined as

$$\mu_i = \frac{\partial \varepsilon_i}{\partial n_i} = \frac{\partial}{\partial x_i} E_i(n, \{x_j\}) , \quad i, j = n, p, e, \mu ,$$

where $\varepsilon_i = n E_i(n, \{x_j\})$ is the partial energy density of species i in the system. From the above equations:

$$\mu_e = 4(1 - 2x)E_s(n) .$$

Since electrons in neutron star interiors are ultrarelativistic,

$$\mu_e = \sqrt{p_{F,e}^2 + m_e^2} \approx p_{F,e}, \text{ and } p_{F,e} = (3\pi^2 n_e)^{1/3} = (3\pi^2 n)^{1/3} (x - x_\mu)^{1/3} ,$$

$$\frac{x - x_\mu}{(1 - 2x)^3} = \frac{64E_s^3(n)}{3\pi^2 n} , \quad (x - x_\mu)^{2/3} - x_\mu^{2/3} = \frac{m_\mu^2}{(3\pi^2 n)^{2/3}} .$$

The total pressure is then given as $P(n) = n^2 \left(\frac{\partial E_{\text{tot}}}{\partial n} \right) .$

NJL model with multiquark interactions

$$\mathcal{L} = \bar{q}(i\partial - m)q + \mu_q \bar{q}\gamma^0 q + \mathcal{L}_4 + \mathcal{L}_8, \quad \mathcal{L}_4 = \frac{g_{20}}{\Lambda^2} [(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2] - \frac{g_{02}}{\Lambda^2} (\bar{q}\gamma_\mu q)^2,$$

$$\mathcal{L}_8 = \frac{g_{40}}{\Lambda^8} [(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]^2 - \frac{g_{04}}{\Lambda^8} (\bar{q}\gamma_\mu q)^4 - \frac{g_{22}}{\Lambda^8} (\bar{q}\gamma_\mu q)^2 [(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]$$

Meanfield approximation: $\mathcal{L}_{\text{MF}} = \bar{q}(i\partial - M)q + \tilde{\mu}_q \bar{q}\gamma^0 q - U,$

$$M = m + 2\frac{g_{20}}{\Lambda^2} \langle \bar{q}q \rangle + 4\frac{g_{40}}{\Lambda^8} \langle \bar{q}q \rangle^3 - 2\frac{g_{22}}{\Lambda^8} \langle \bar{q}q \rangle \langle q^\dagger q \rangle^2,$$

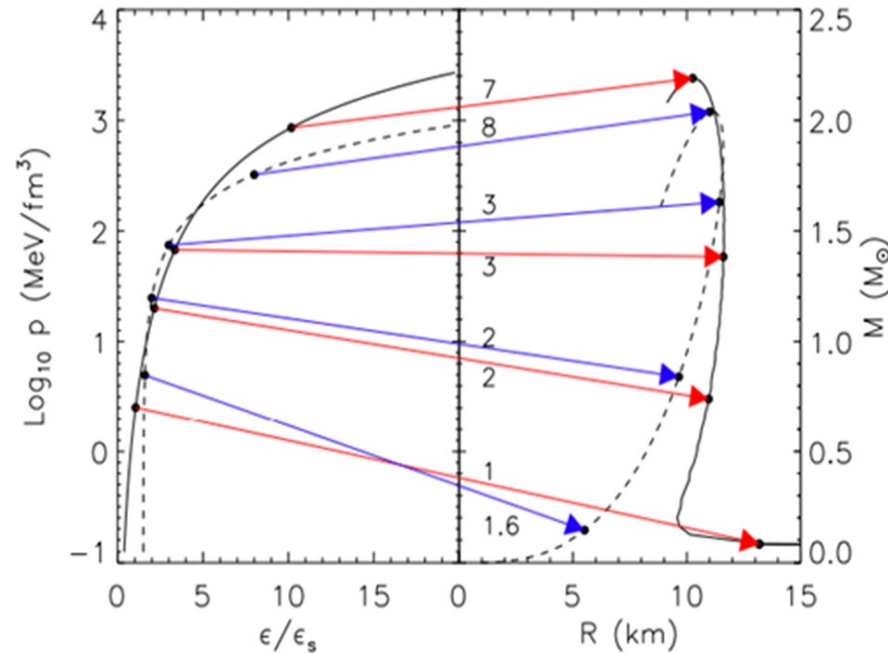
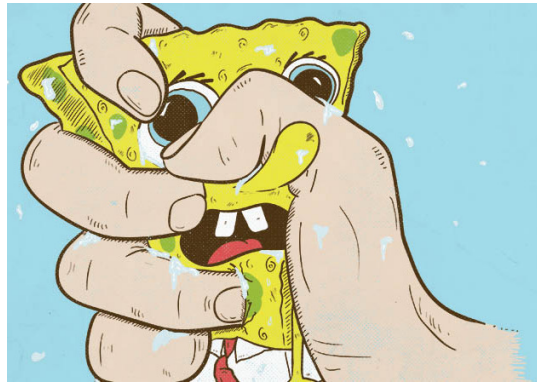
$$\tilde{\mu}_q = \mu_q - 2\frac{g_{02}}{\Lambda^2} \langle q^\dagger q \rangle - 4\frac{g_{04}}{\Lambda^8} \langle q^\dagger q \rangle^3 - 2\frac{g_{22}}{\Lambda^8} \langle \bar{q}q \rangle^2 \langle q^\dagger q \rangle,$$

$$U = \frac{g_{20}}{\Lambda^2} \langle \bar{q}q \rangle^2 + 3\frac{g_{40}}{\Lambda^8} \langle \bar{q}q \rangle^4 - 3\frac{g_{22}}{\Lambda^8} \langle \bar{q}q \rangle^2 \langle q^\dagger q \rangle^2 - \frac{g_{02}}{\Lambda^2} \langle q^\dagger q \rangle^2 - 3\frac{g_{04}}{\Lambda^8} \langle q^\dagger q \rangle^4.$$

Thermodynamic Potential:

$$\Omega = U - 2N_f N_c \int \frac{d^3 p}{(2\pi)^3} \left\{ E + T \log[1 + e^{-\beta(E - \tilde{\mu}_q)}] + T \log[1 + e^{-\beta(E + \tilde{\mu}_q)}] \right\} + \Omega_0$$

Compact Star Sequences (M-R \Leftrightarrow EoS)



James Lattimer,
Annu. Rev. Nucl. Part. Sci.
62, 485 (2012),
arXiv:1305.3510

$$\frac{dp}{dr} = - \frac{(\varepsilon + p/c^2)G(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)}$$

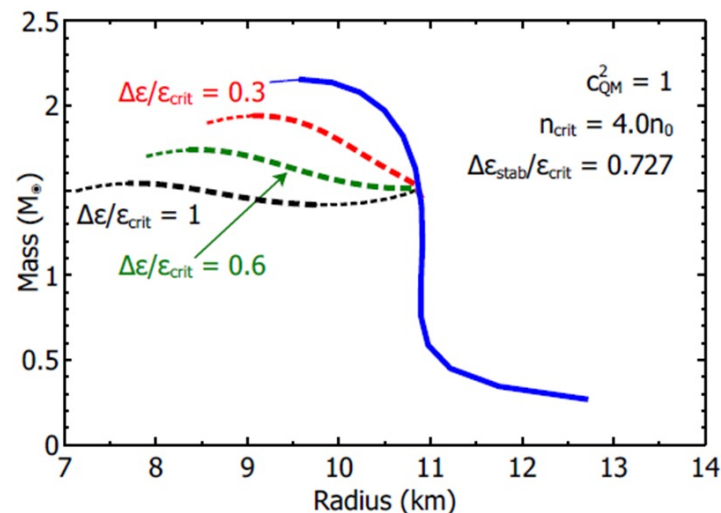
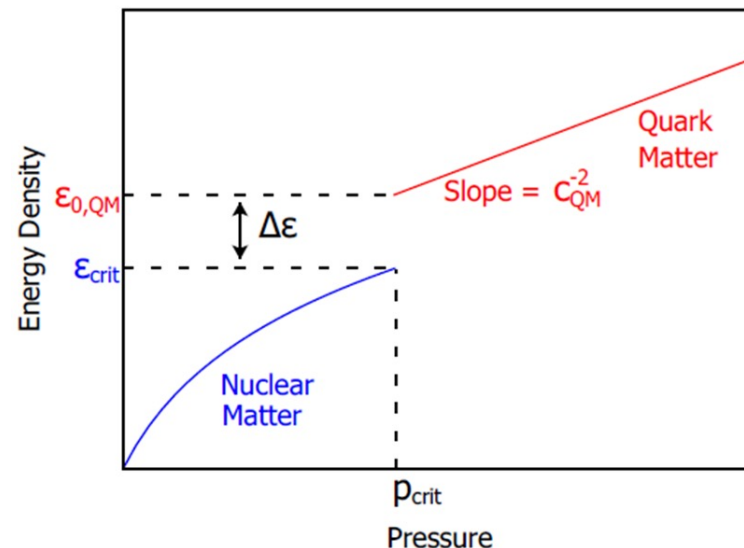
$$\frac{dm}{dr} = 4\pi r^2 \varepsilon \quad p(\varepsilon)$$

- TOV Equations
- Equation of State (EoS)

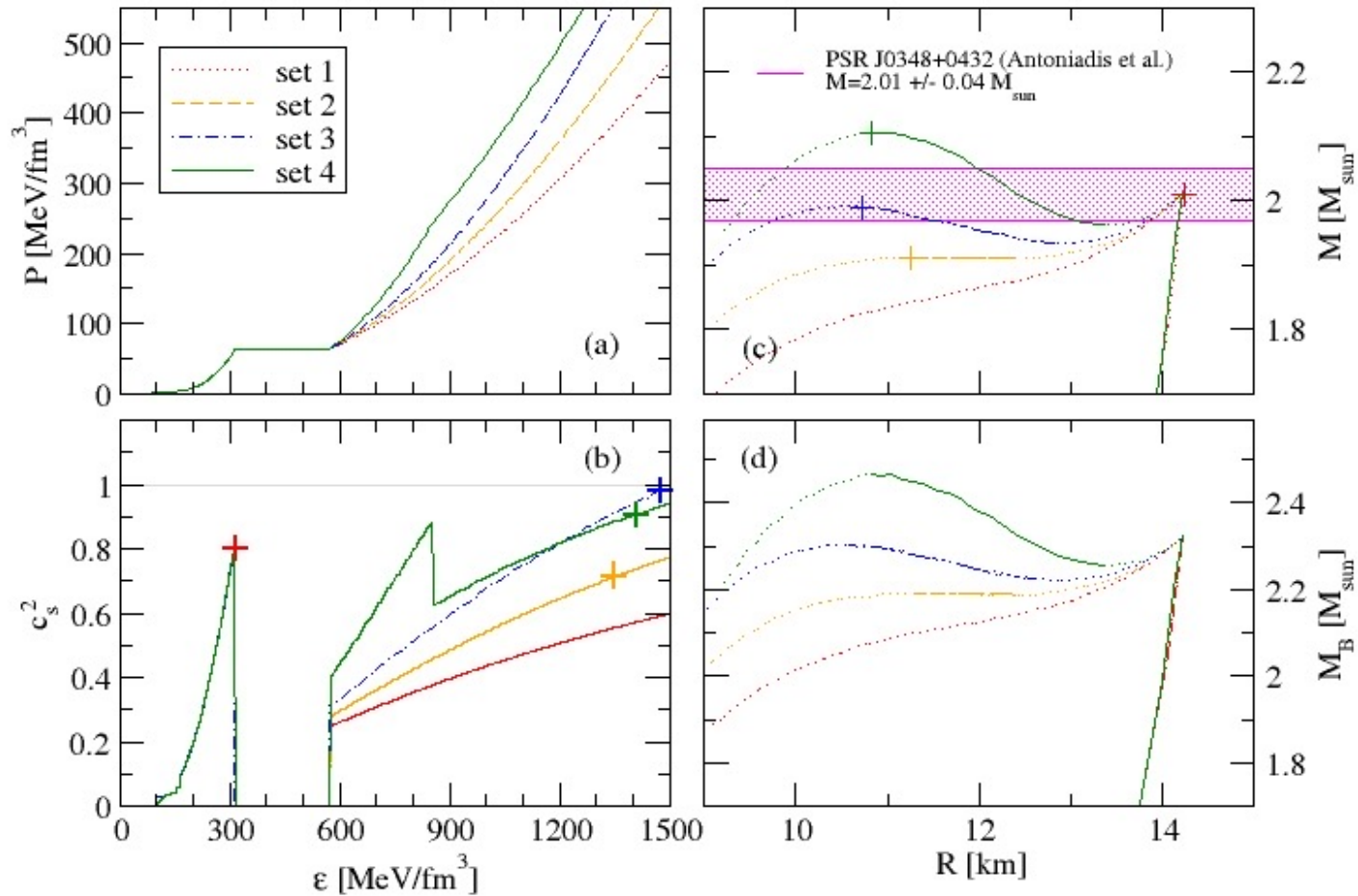
Compact Star Mass Twins and the AHP scheme

- First order PT can lead to a stable branch of hybrid stars with quark matter cores which, depending on the size of the “latent heat” (jump in energy density), can even be disconnected from the hadronic one by an unstable branch → “**third family of CS**”.
- Measuring two **disconnected populations** of compact stars in the M-R diagram would represent the **detection of a first order phase transition** in compact star matter and thus the indirect proof for the existence of a **critical endpoint (CEP)** in the QCD phase diagram!

Alford, Han, Prakash,
 Phys. Rev. D 88, 083013 (2013)
 arxiv:1302.4732



Compact Star Twins

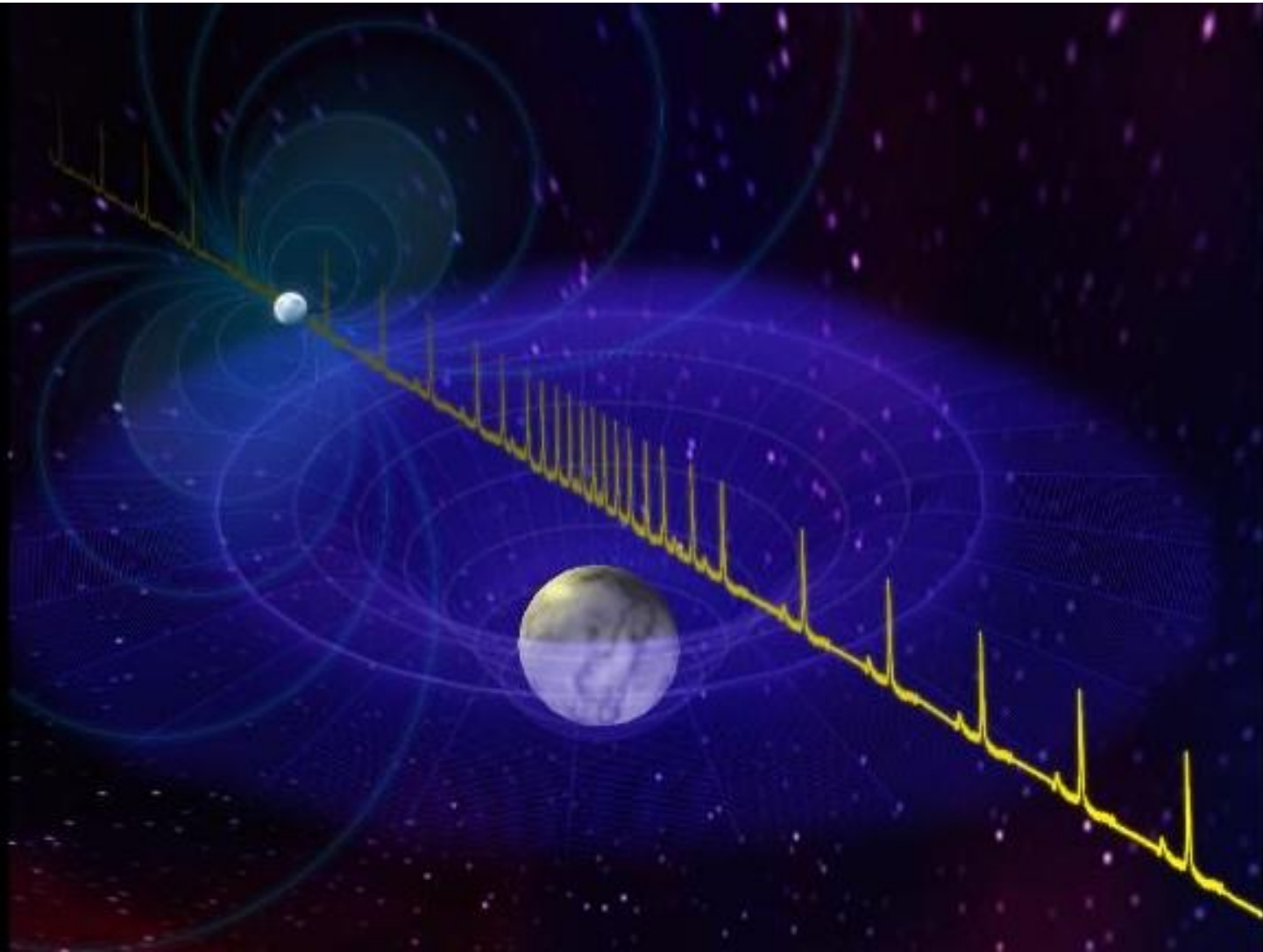


Alvarez-Castillo, Blaschke (2017)

High mass twins from multi-polytrope equations of state
arXiv: 1703.02681v2, Phys. Rev. C 96, 045809 (2017)

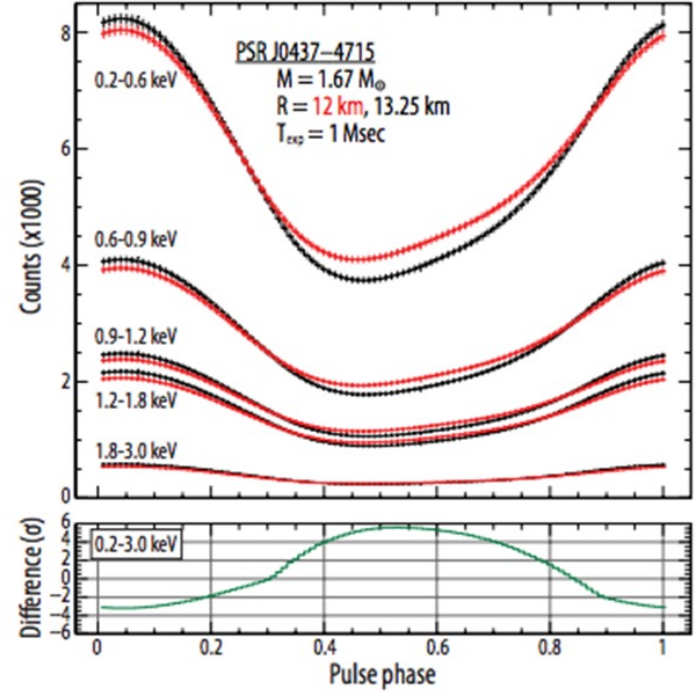
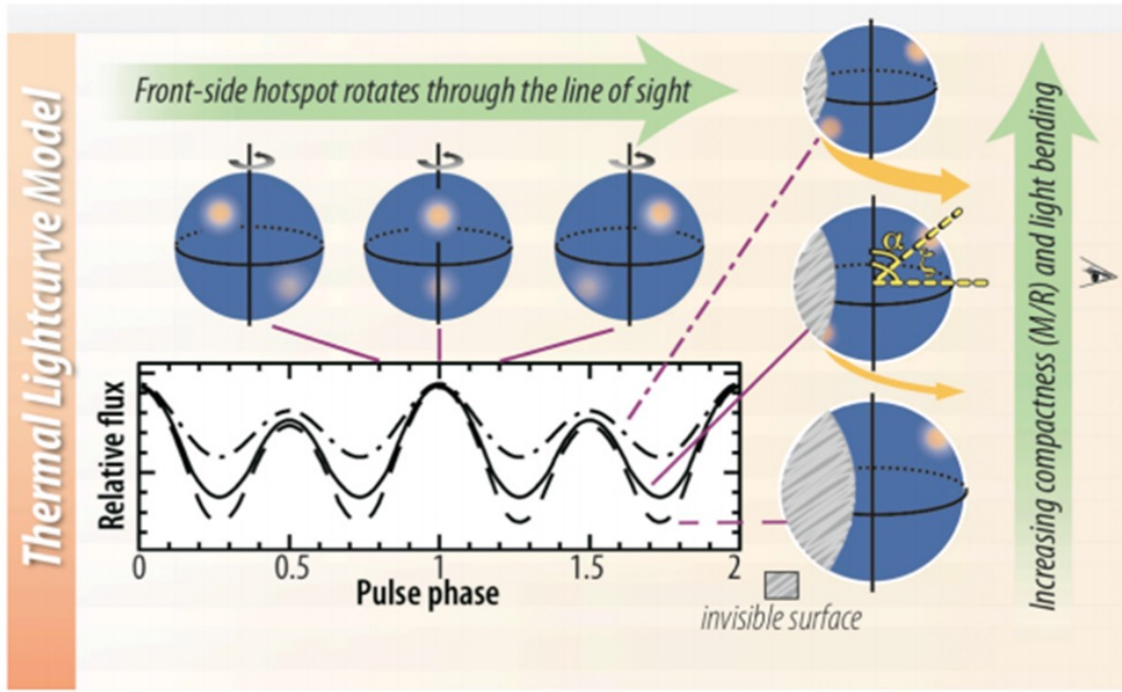
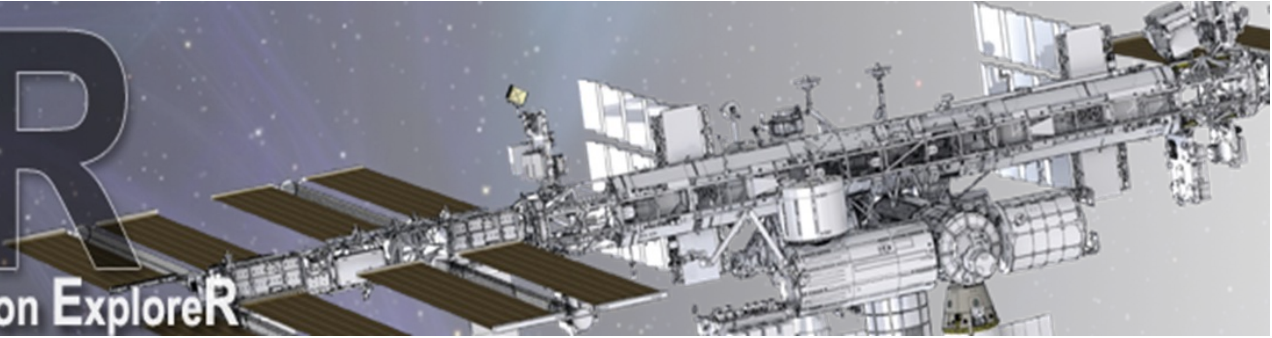
Multi-Messenger Observations

Massive Neutron Stars



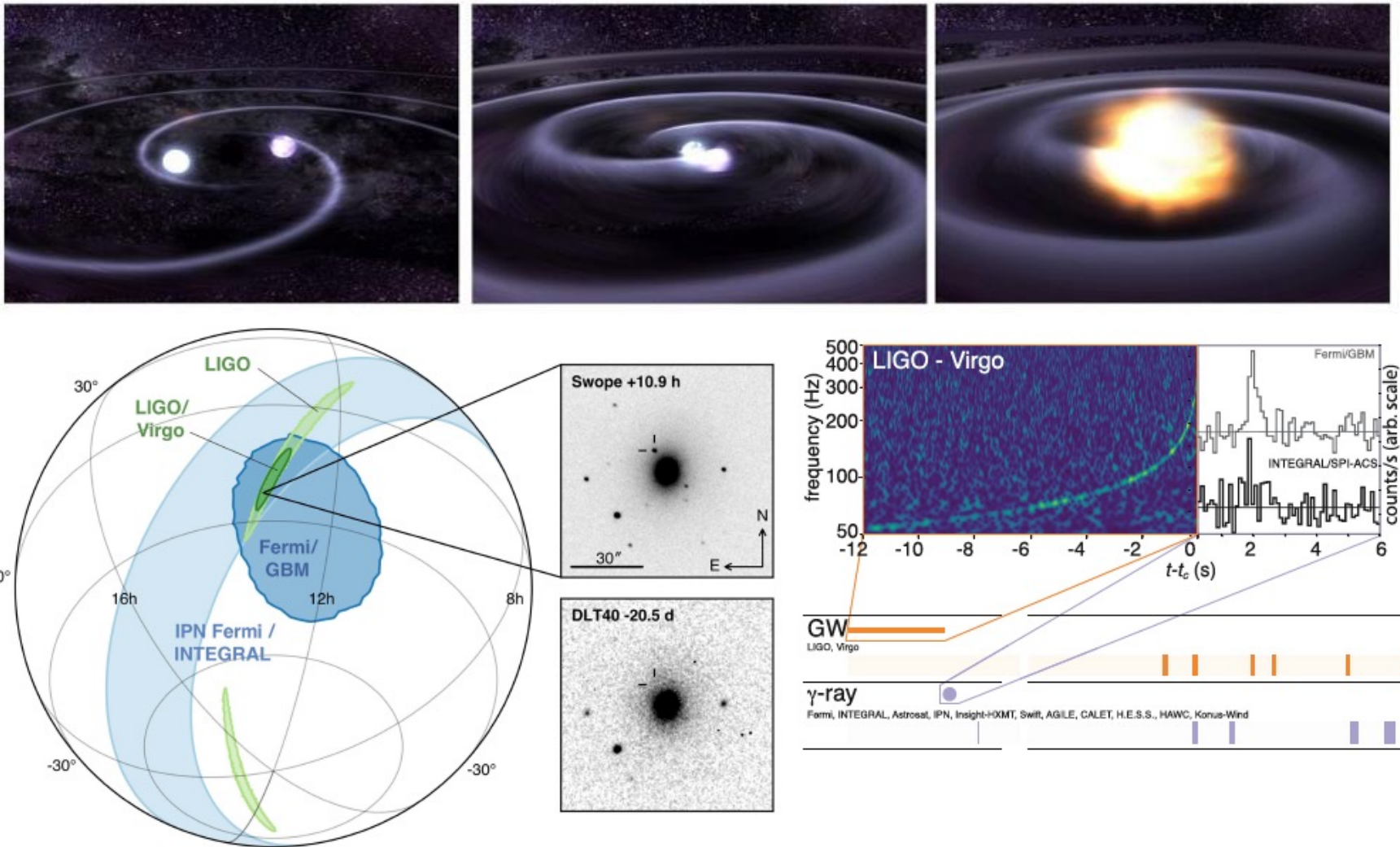
NICER

Neutron star Interior Composition Explorer



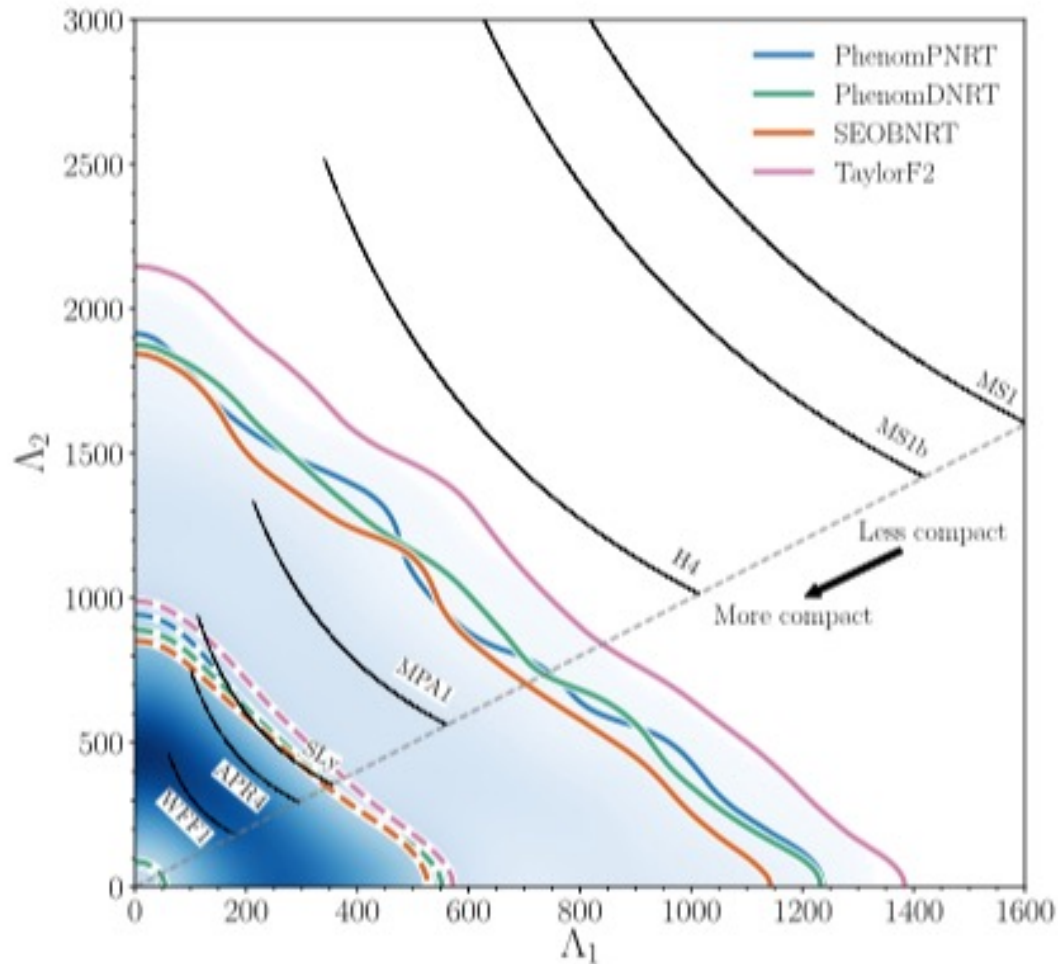
Hot Spots

GW170817: Neutron Star Merger

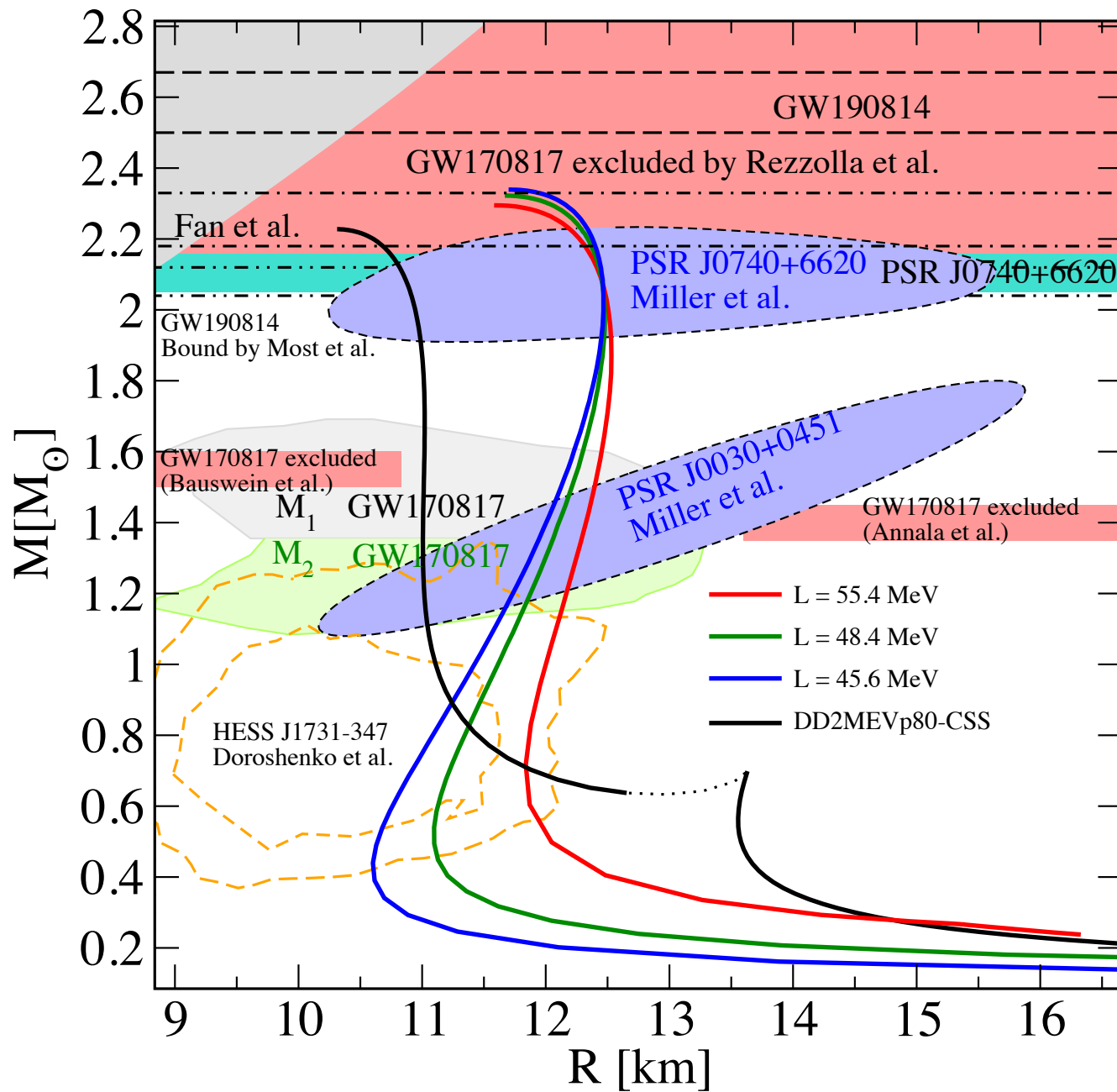


*) B.P. Abbott et al. [LIGO/Virgo Collab.], PRL 119, 161101 (2017); ApJLett 848, L12 (2017)

Implications from GW170817



Properties of the Binary Star Merger GW170817
B. P. Abbott et al., Phys. Rev. X 9, 011001 (2019)



Neutron Star Cooling Processes

Process Name	Process	Emissivity Q_ν (erg cm ⁻³ s ⁻¹)	Reference
Bremsstrahlung	$n + n \rightarrow n + n + \nu_e + \bar{\nu}_e$ $n + p \rightarrow n + p + \nu_e + \bar{\nu}_e$ $p + p \rightarrow p + p + \nu_e + \bar{\nu}_e$	$\simeq 10^{19} T_9^8$	Page, Geppert and Weber [92]
Modified Urca	$n + n \rightarrow n + p + e^- + \bar{\nu}_e$ $n + p + e^- \rightarrow n + n + \nu_e$	$\simeq 10^{20} T_9^8$	Friman and Maxwell [93]
Direct Urca	$n \rightarrow p + e^- + \bar{\nu}_e$ $p + e^- \rightarrow n + \nu_e$	$\simeq 10^{27} T_9^6$	Lattimer et al. [94]
Quark Urca	$d \rightarrow u + e^- + \bar{\nu}_e$ $u + e^- \rightarrow d + \nu_e$	$\simeq 10^{26} \alpha_c T_9^6$	Iwamoto [95]
Kaon Condensate	$n + K^- \rightarrow n + e^- + \bar{\nu}_e$ $n + e^- \rightarrow n + K^- + \nu_e$	$\simeq 10^{24} T_9^6$	Brown et al. [96]
Pion Condensate	$n + \pi^- \rightarrow n + e^- + \bar{\nu}_e$ $n + e^- \rightarrow n + \pi^- + \nu_e$	$\simeq 10^{26} T_9^6$	Maxwell et al. [97]

Direct Urca is the fastest cooling process.

Threshold for onset: $p_{F,n} < p_{F,p} + p_{F,e}$. For electrons only then $x_{DU} = 1/9$.

Neutron Star Cooling Processes

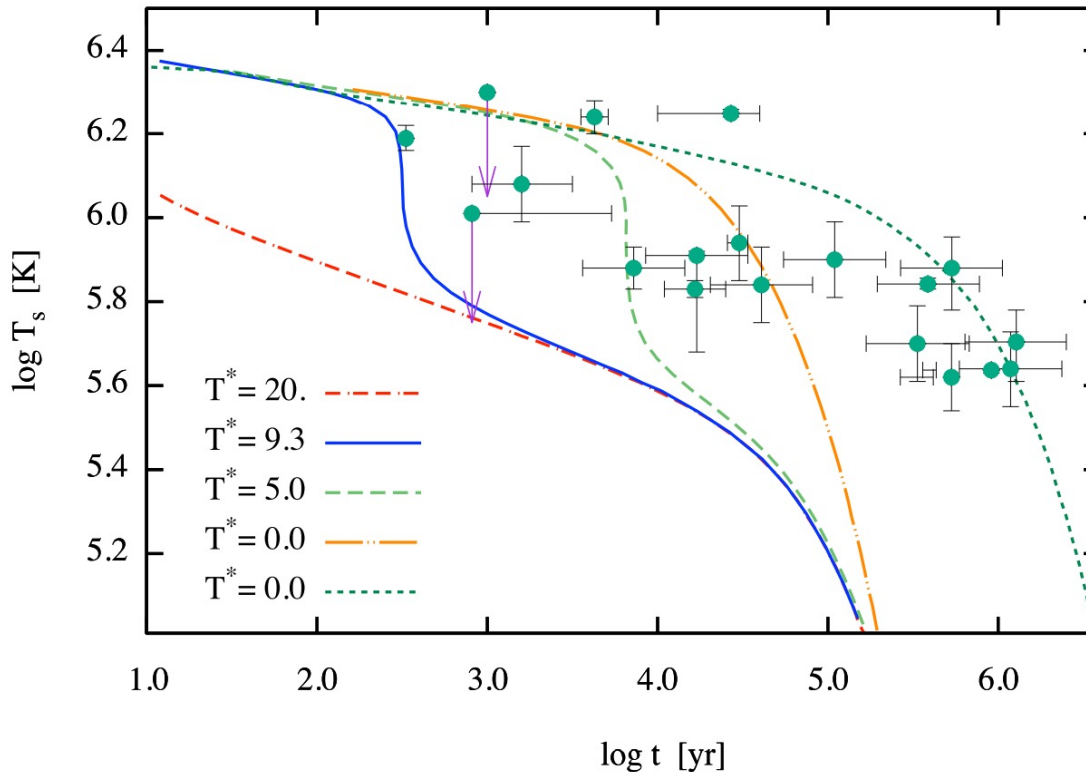
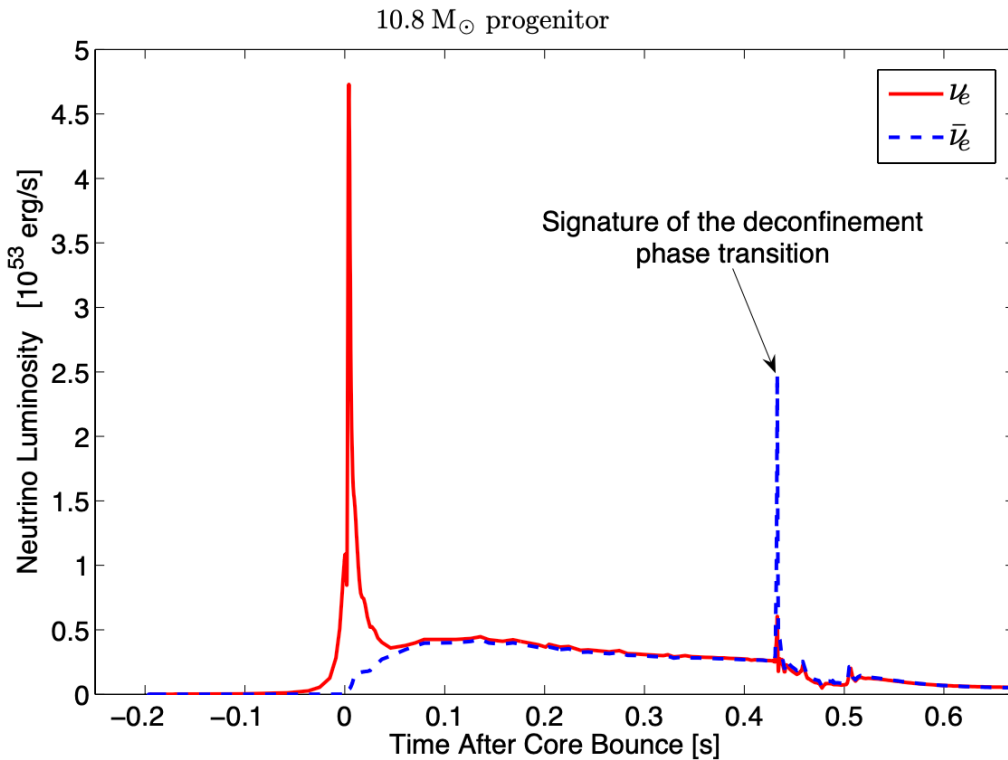


Figure 6. Cooling tracks of compact stars with quark cores in the surface-temperature–age diagram. The masses of the stars are the same, $M = 1.93 M_\odot$, and the different curves correspond to different values of the parameter T^* in units of keV, except the dotted line, which corresponds to $1 M_\odot$ mass nucleonic compact star without a quark core. The observational points with error bars are shown by green circles; the arrows show the upper limits on surface temperatures of known objects.

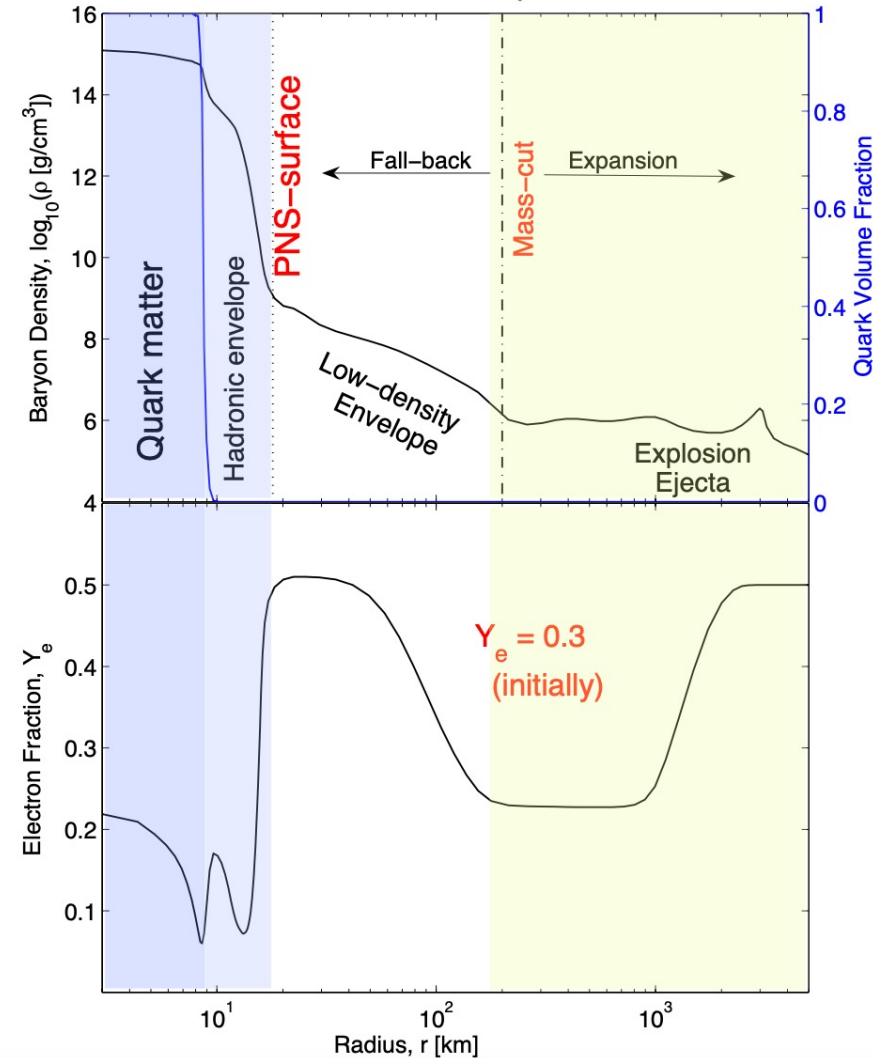
Observables in the quark-hadron phase transition

- Proton neutron star collapse and Shock formation
- Explosion mechanism of massive stars
- Observable in neutrino signal:
- Resolvable in (Super-K, Icecube) for a galactic supernova



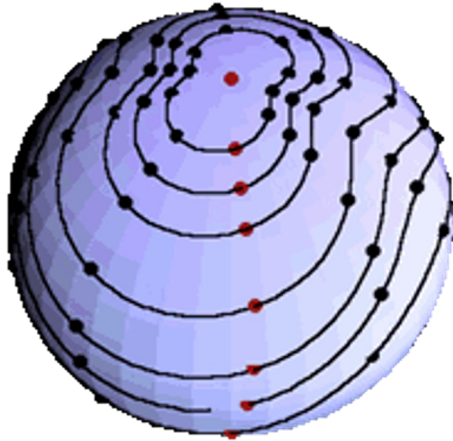
Sagert & T.F. et al. (2009), PRL 102, 081101
 Dasgupta & T.F. et al. (2010), PRD 81
 T.F. et al. (2011), ApJS 194, 39

The remnant hybrid star

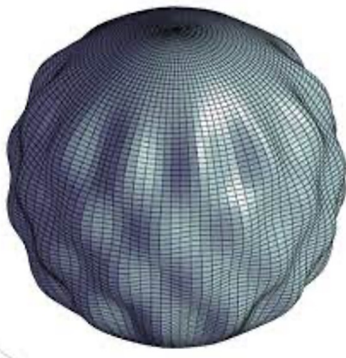


Credit: Tobias Fischer

Gravitational Waves from NS



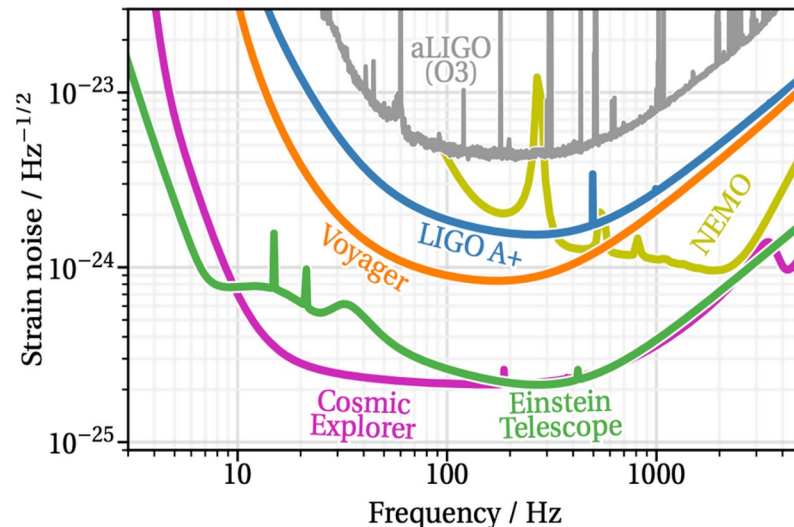
Credit: C. Hanna and B. Owen



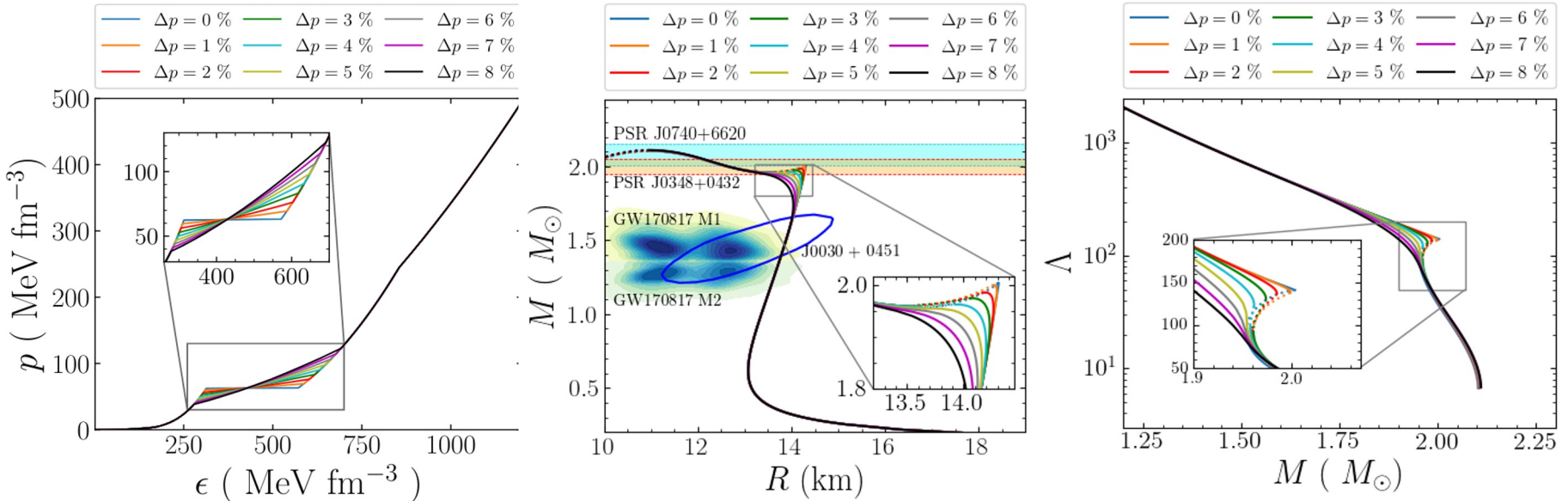
Credit: CERN/Indico

PC: cosmicexplorer.org/sensitivity

- Non-radial QNMs raised from time varying quadrupole deformations are source of GWs.
 - fundamental (f) mode,
 - no node, probe for mean density, ($1 \text{ kHz} < f < 3 \text{ kHz}$)
 - pressure (p) mode,
 - Sound speed, ($5 \text{ kHz} < f < 10 \text{ kHz}$)
 - gravity (g) mode,
 - ($50 \text{ Hz} < f < 500 \text{ Hz}$)
- R-mode, for rotating stars only.
 - Viscosity, ($0.5 \text{ kHz} < f < 2 \text{ kHz}$)
- Space-time (w) mode.
 - $5 \text{ kHz} < f$

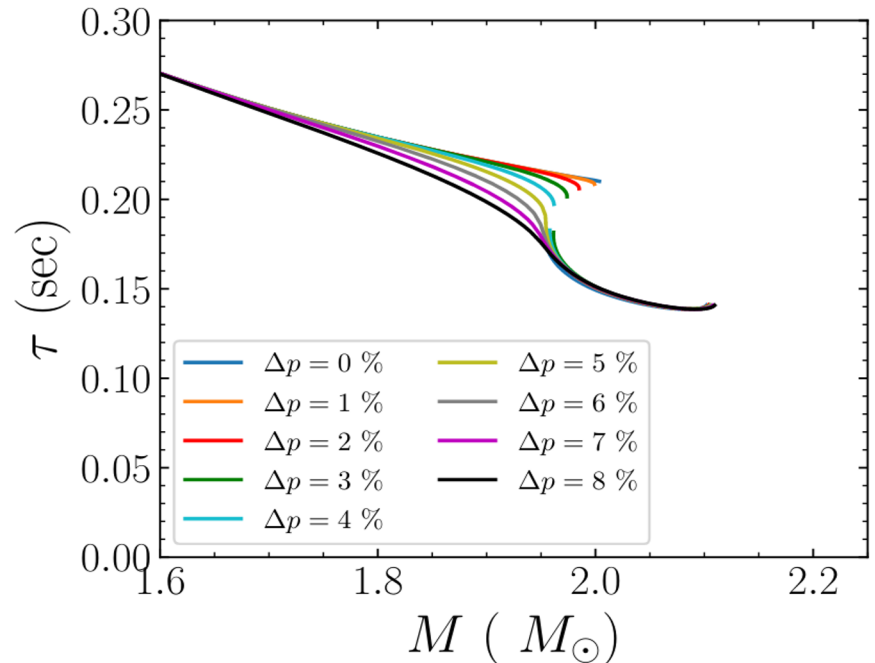
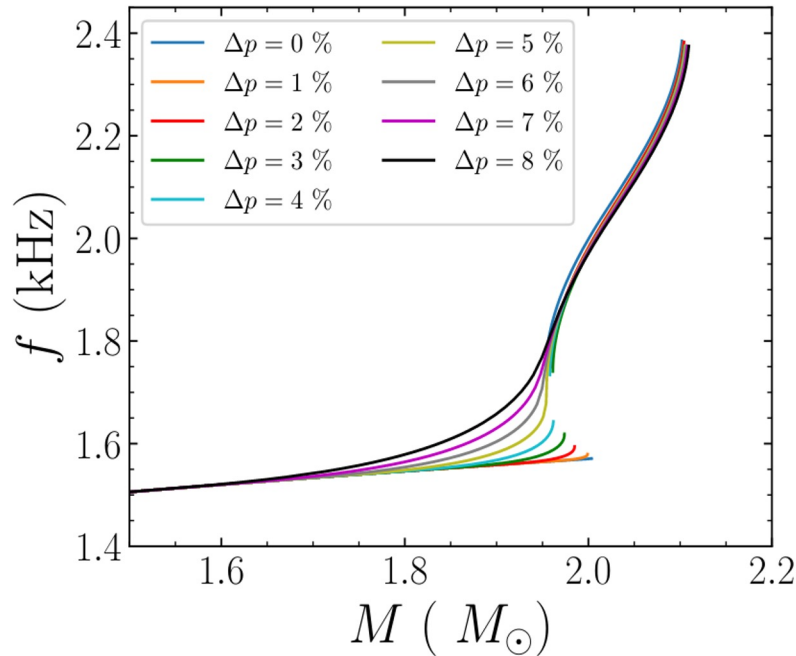


Stellar Properties



- ❖ The second and third family merge to form a single branch for $\Delta p > 4\%$.
- ❖ Precise measurement of M - R required for detection of twin star.
- ❖ The jump $\Delta\Lambda$ (if any) can be measured $\sim 15\%$ ($< 90\%$ CI) with next-generation GW detectors ([P. Landry & K. Chakravarti, arXiv:2212.09733, 2022](#)).

f-mode characteristics



- f-mode characteristics are obtained within **General relativistic** formalism.
- Sudden increase (decrease) in the frequency (damping time) observed with appearance of twin star.
- Detections of f-mode GWs from compact stars with known mass may reveal the presence of twin stars.
- Simultaneous measurement of M - f (from binary system) can be used to comment on twin stars.

David Alvarez-Castillo, Bikram Keshari Pradhan, Debarati Chatterjee – arXiv:2309.08775

Monthly Notices of the Royal Astronomical Society, Vol 531, Issue 4, Pag 4640–4655 (2024)

Asteroseismology and Universal Relations

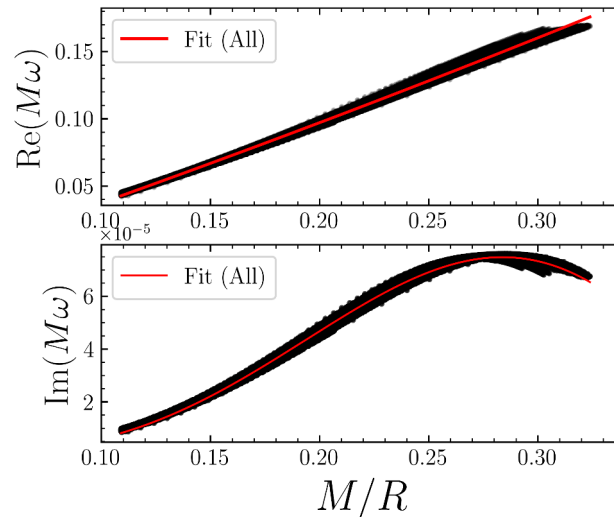
- URs among f-mode characteristics (f , τ_f or $\omega=2\pi f+1/\tau_f$) and NS observables.

$$f(\text{kHz}) = a_r + b_r \sqrt{\frac{M}{R^3}}$$

Empirical relations (EOS dependent)

$$\text{Re}(M\omega) = a_0 + a_1 \left(\frac{M}{R}\right) + a_2 \left(\frac{M}{R}\right)^2$$

$$\text{Im}(M\omega) = b_0 \left(\frac{M}{R}\right)^4 + b_1 \left(\frac{M}{R}\right)^5 + b_2 \left(\frac{M}{R}\right)^6$$



	Re($M\omega$)		Im($M\omega$)
a_0	$-0.027 \pm 9 \times 10^{-5}$	b_0	$(9.81 \pm 0.004) \times 10^{-2}$
a_1	0.610 ± 0.0015	b_1	$(-4.444 \pm 0.003) \times 10^{-1}$
a_2	0.049 ± 0.002	b_2	$(4.91 \pm 0.0045) \times 10^{-1}$

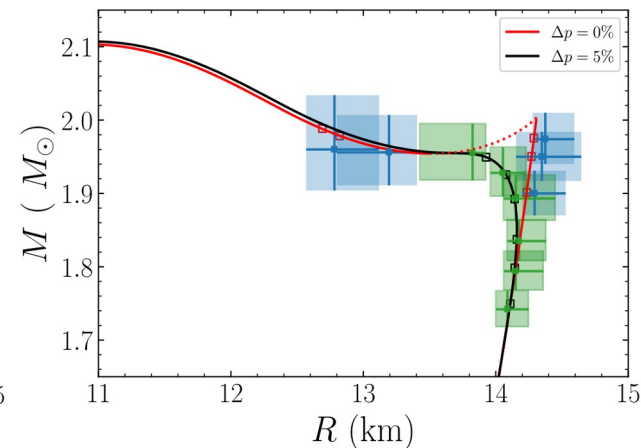
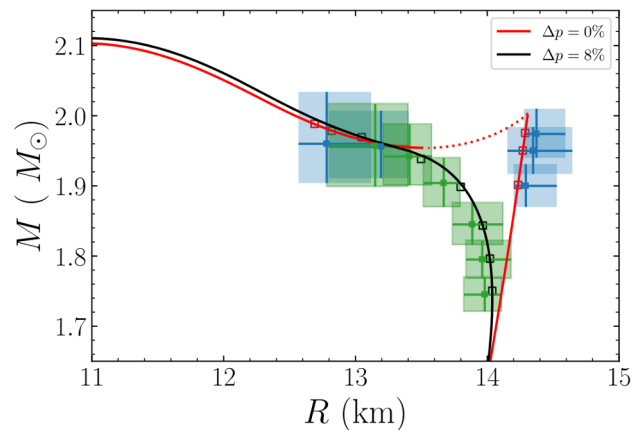
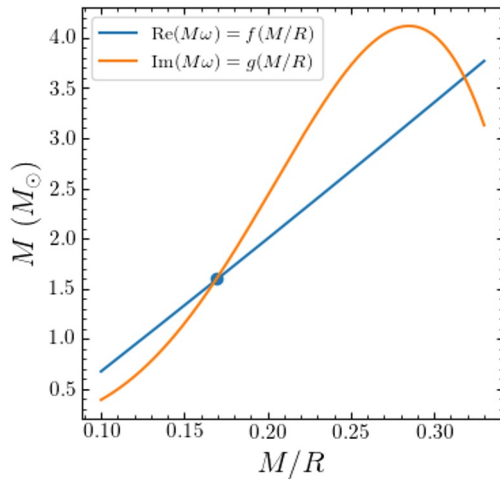
- Scaled Universal relations are more useful.
- The URs can be used for EoS inference.
- URs involving tidal deformability have also been examined.

David Alvarez-Castillo, Bikram Keshari Pradhan, Debarati Chatterjee – arXiv:2309.08775

Monthly Notices of the Royal Astronomical Society, Vol 531, Issue 4, Pag 4640–4655 (2024)

Compact star observables from f-mode observations: the role of UR Uncertainty

- ❖ Determination under the assumption that f , τ are measured precisely.
- ❖ Errors on UR results uncertainties on M - R .



- ★ The presence of the twins may be confirmed with exact measurement of f , and τ .
- ★ The unstable branch of $\Delta p = 0\%$ can be distinguished from the connecting stable branch of $\Delta p = 8\%$.
- ★ Differentiating among $\Delta p = 0\%$ and $\Delta p = 5\%$ is more challenging.

Inclusion of Observational Uncertainties

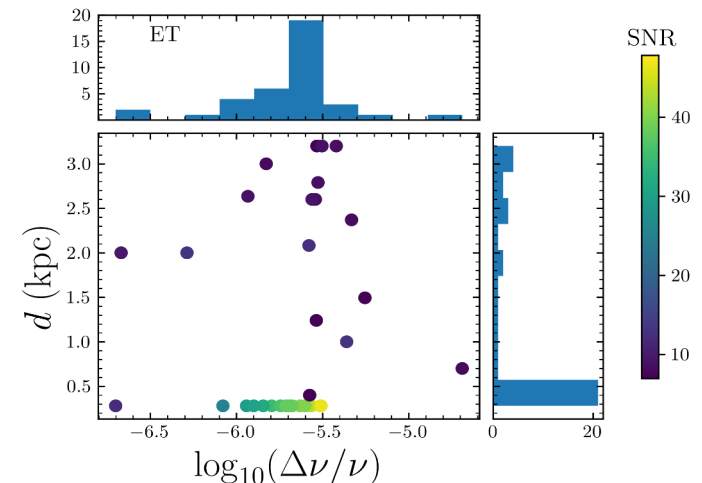
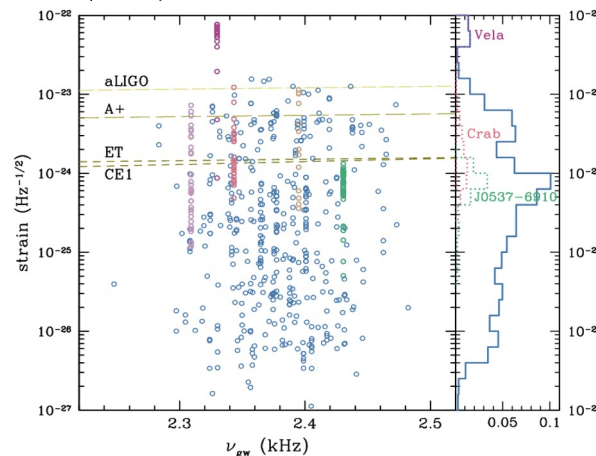
- F-mode being excited during pulsar glitches. All the energy radiated through GW.
- The burst waveform is modelled as an exponentially damped oscillation.

$$h(t) = h_0 \exp(-t/\tau_f) \sin(2\pi\nu_f t), \quad t > 0 \quad (\text{B.J. Owen,2010, Ho et al. 2020})$$

$$h_0 = 4.85 \times 10^{-17} \sqrt{\frac{E_{\text{gw}}}{M_{\odot} c^2}} \sqrt{\frac{0.1 \text{sec} \cdot 1 \text{kpc}}{\tau_f d}} \left(\frac{1 \text{kHz}}{\nu_f} \right)$$

f-modes GW

$$E_{\text{gw}} = E_{\text{glitch}} = 4\pi^2 I \nu^2 \left(\frac{\Delta\nu}{\nu} \right)^2$$



- B. Abbott et al., LVC, [ApJ 874 163, 2019.](#)
- R. Abbott et al., LVK, [PhRvD, 104, 122004, 2021.](#)
- R. Abbott, et al., LVK, [arXiv:2210.10931, 2022.](#)
- R. Abbott, et al., LVK, [arXiv:2203.12038, 2022.](#)
- D. Lopez et al., [PhRvD, 106, 103037, 2022](#)

[Ho et al ,PRD 101, 103009 \(2020\)](#)

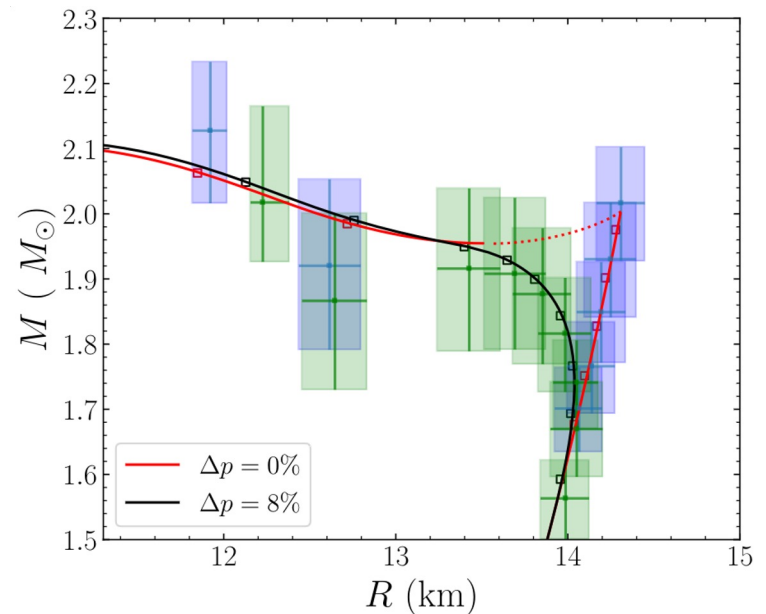
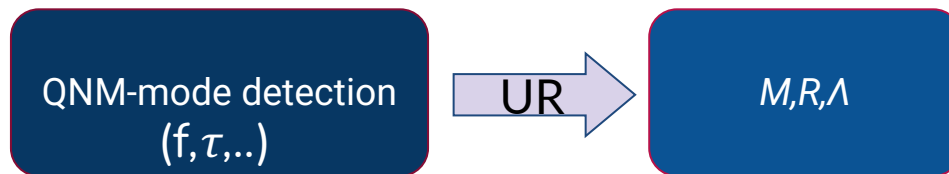
[B. K. Pradhan, D. Pathak, and D. Chatterjee, ApJ 956 38, \(2023\)](#)

David Alvarez-Castillo, Bikram Keshari Pradhan, Debarati Chatterjee – [arXiv:2309.08775](#)

Monthly Notices of the Royal Astronomical Society, Vol 531, Issue 4, Pag 4640–4655 (2024)

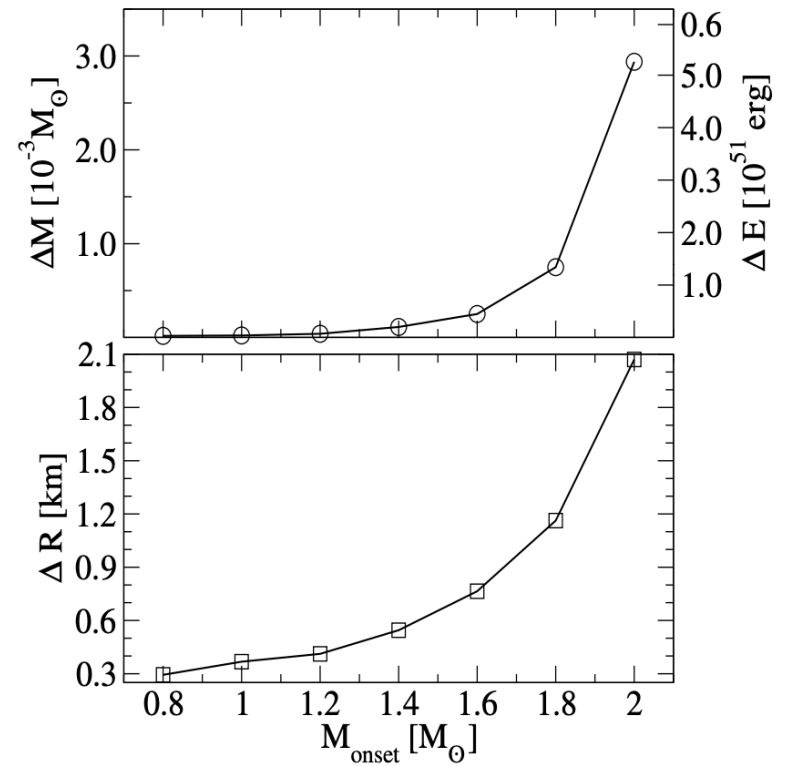
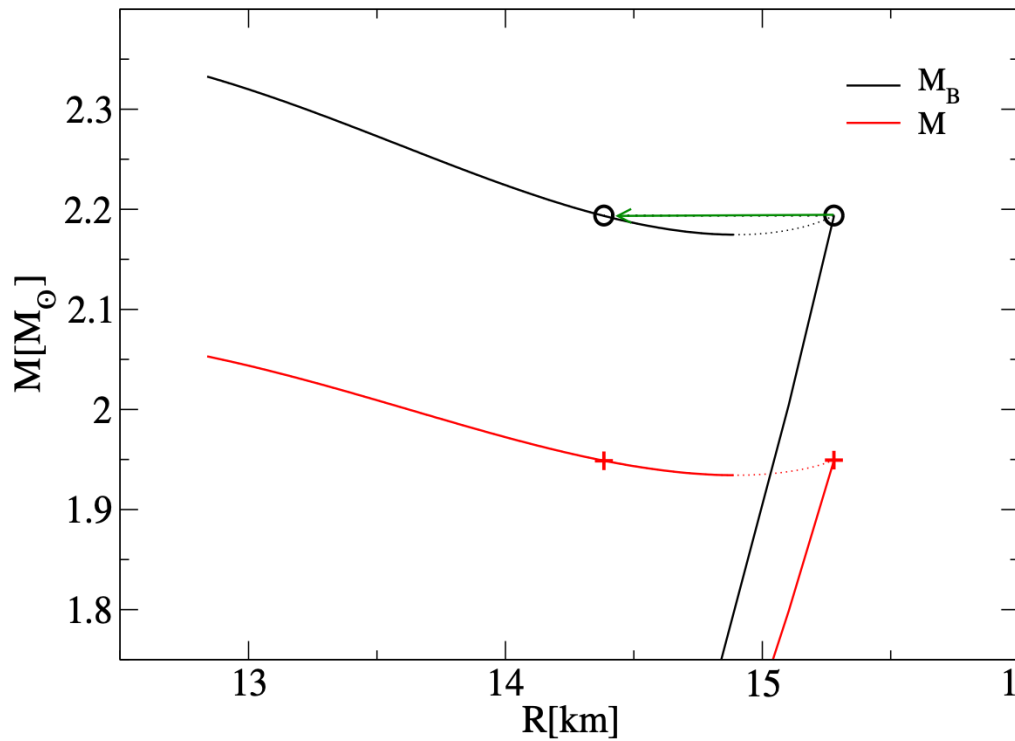
Inclusion of Observational Uncertainties

- Glitching pulsars data taken from the [Jodrell Bank Glitch catalogue](#).
- Spin frequency, distance (d) and sky position to each pulsar are assigned from [ATNF Pulsar Catalogue](#).
- Consider few random mass configurations with an assumed EOS model.
- Then f-mode frequency, damping time, moment of inertia to pulsars from the assumed EoS model.



- The measurement of R from f-mode observation may confirm the presence of twins.
- More challenging for low mass twins. However, we have more observations at low masses.
- Differentiating the nature of Δp is more challenging.

Mass Twins – Energy Released



Dynamical interaction of a binary with a circumbinary disc

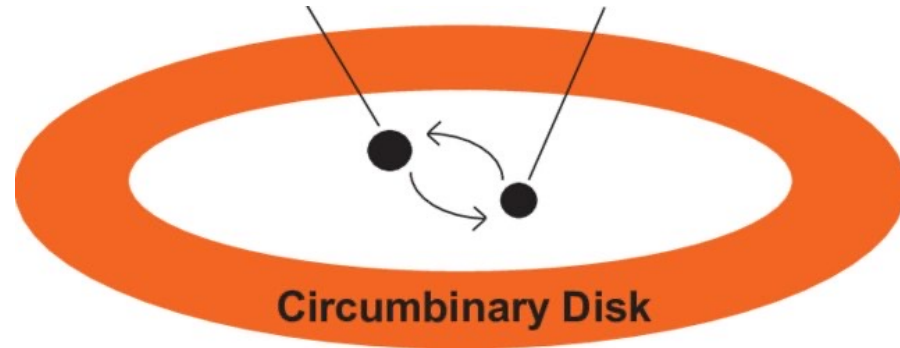
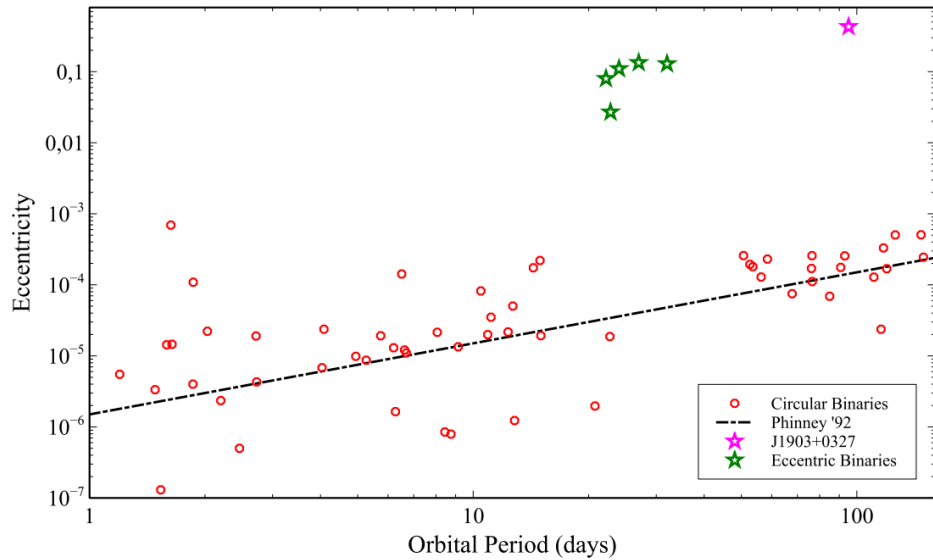


FIGURE 1 Eccentricity vs. orbital period for millisecond pulsars in binaries with white dwarf companions, see (J. Antoniadis, 2014; Stovall, 2019).

David Edwin Alvarez-Castillo, John Antoniadis, Alexander Ayriyan, David Blaschke, Victor Danchev, Hovik Grigorian, Noshad Khosravi Largani, Fridolin Weber.
Astron. Nachr. 2019;340:878-884. Eprint: arXiv: 1912.08782

Implications on eccentric and isolated millisecond pulsar populations

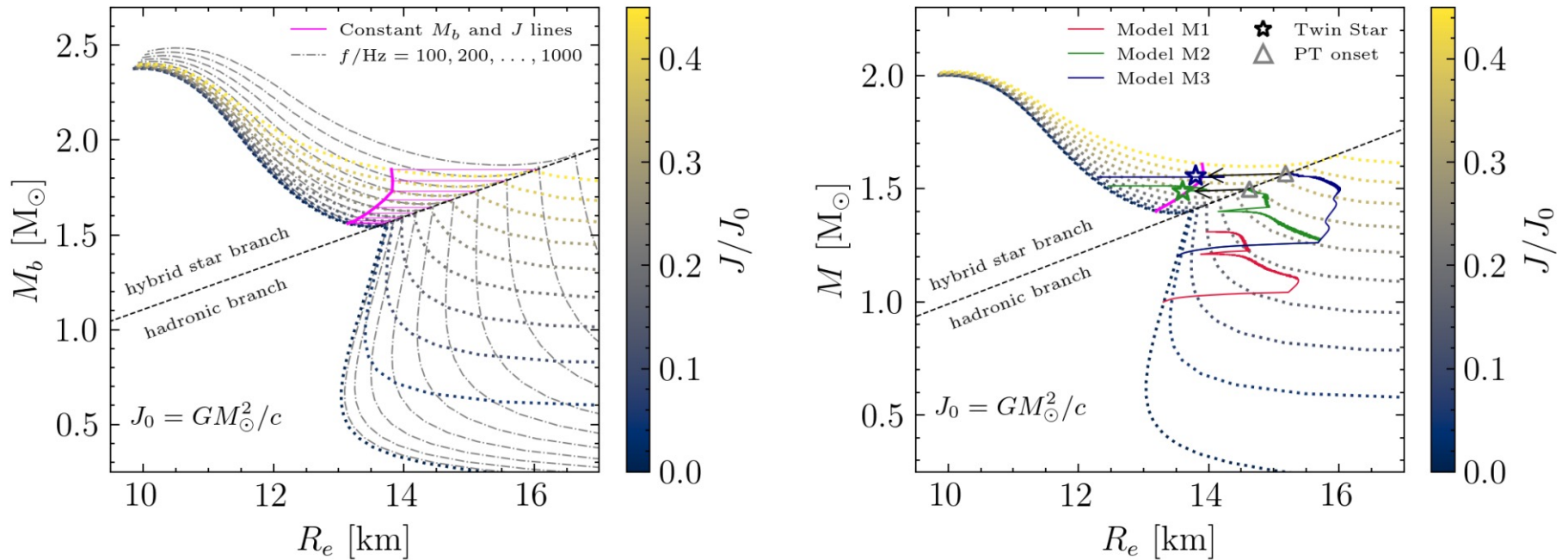


Fig. 1: *Left Panel:* Baryonic mass vs. equatorial radius. Dotted and dot-dashed curves indicate lines of constant angular momentum and constant frequency respectively. The black dashed line highlights the points of maximum stability in the hadronic branches, signaling the onset of a phase transition (PT). Thin, horizontal magenta lines show trajectories where angular momentum (J) and baryonic mass (M_b) are conserved. The thick magenta curve connects points on the hybrid star branches that can be reached through a collapse conserving both J and M_b . *Right Panel:* Similar to the left panel, but with the y-axis representing gravitational mass. Star markers denote the endpoint trajectories for direct (M2) and delayed (M3) collapse models. Here, the arrows are inclined because gravitational mass, unlike baryonic mass, is not conserved during the PT.

Implications on eccentric and isolated millisecond pulsar populations

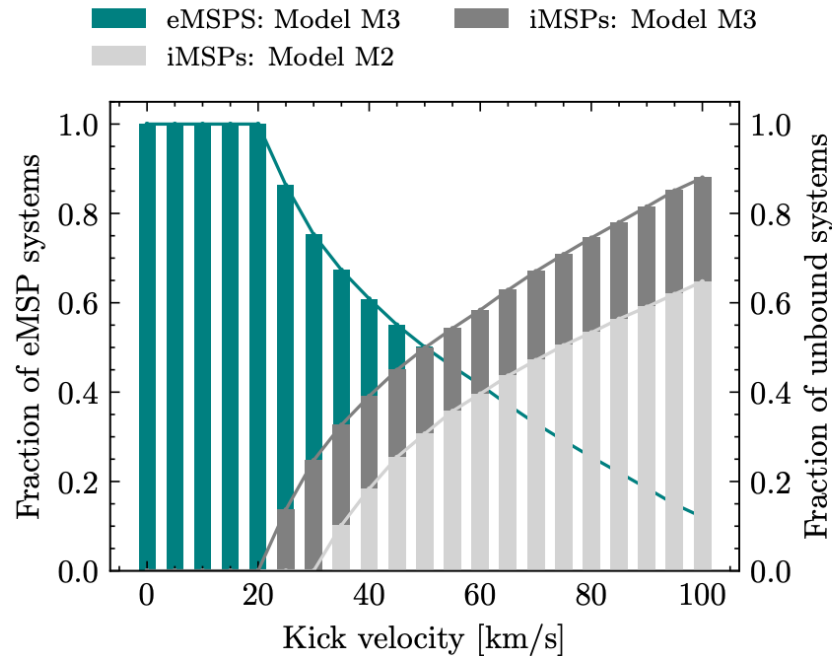
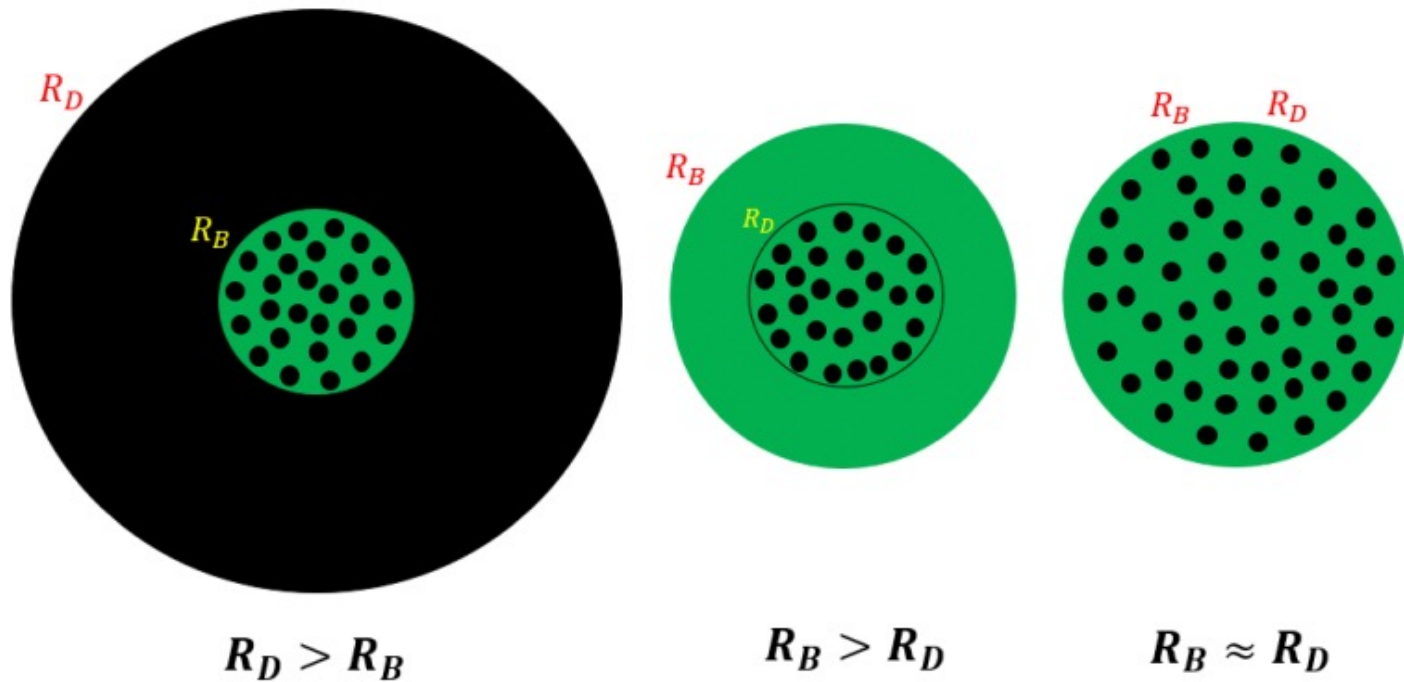
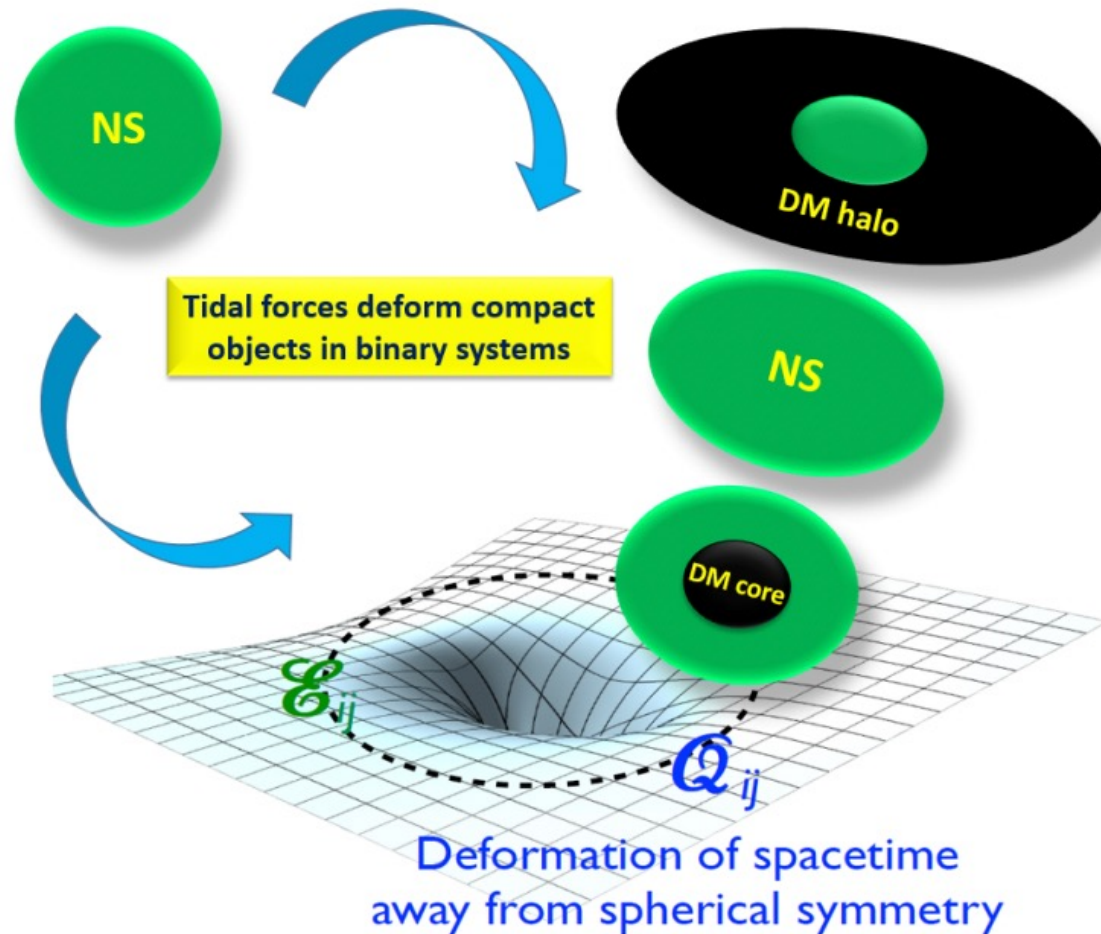


Fig. 6: Expected fraction of eMSPs (left y-axis) and iMSPs (hatched gray-colored bars, right y-axis) as a function of kick velocity, assuming a mass defect of $\Delta M = 0.016 M_{\odot}$. For iMSPs, we consider those systems with eccentricities greater than unity. Model m2 cannot create eccentric systems due to rapid re-circularization of the orbit.

Bosonic Dark Matter in NS



Bosonic Dark Matter in NS



The Axion

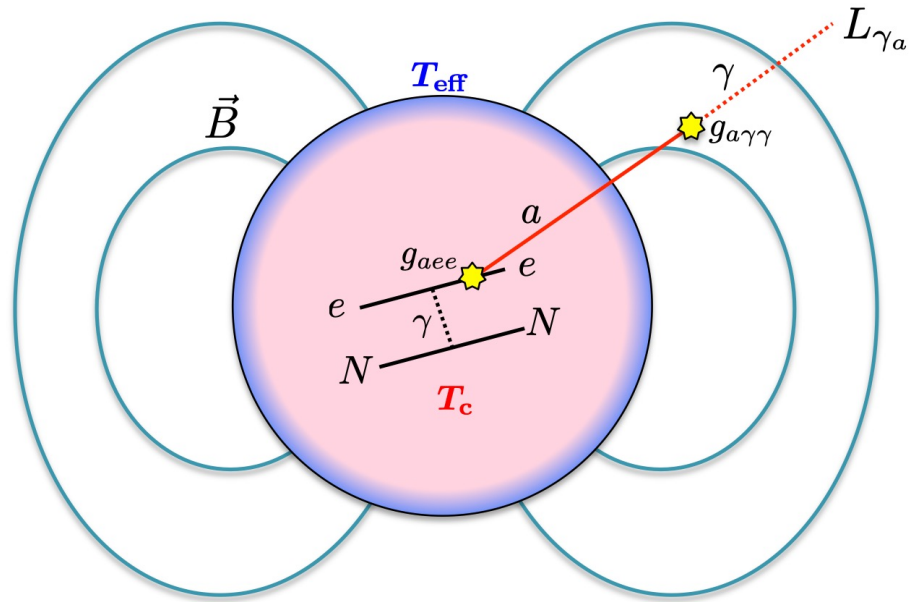


Figure 4: Eschematic dynamics of the conversion of an axion into a photon as it interacts with a white dwarf magnetic field. The core temperature T_c is higher than the surface temperature T_{eff} , see [1] for details.

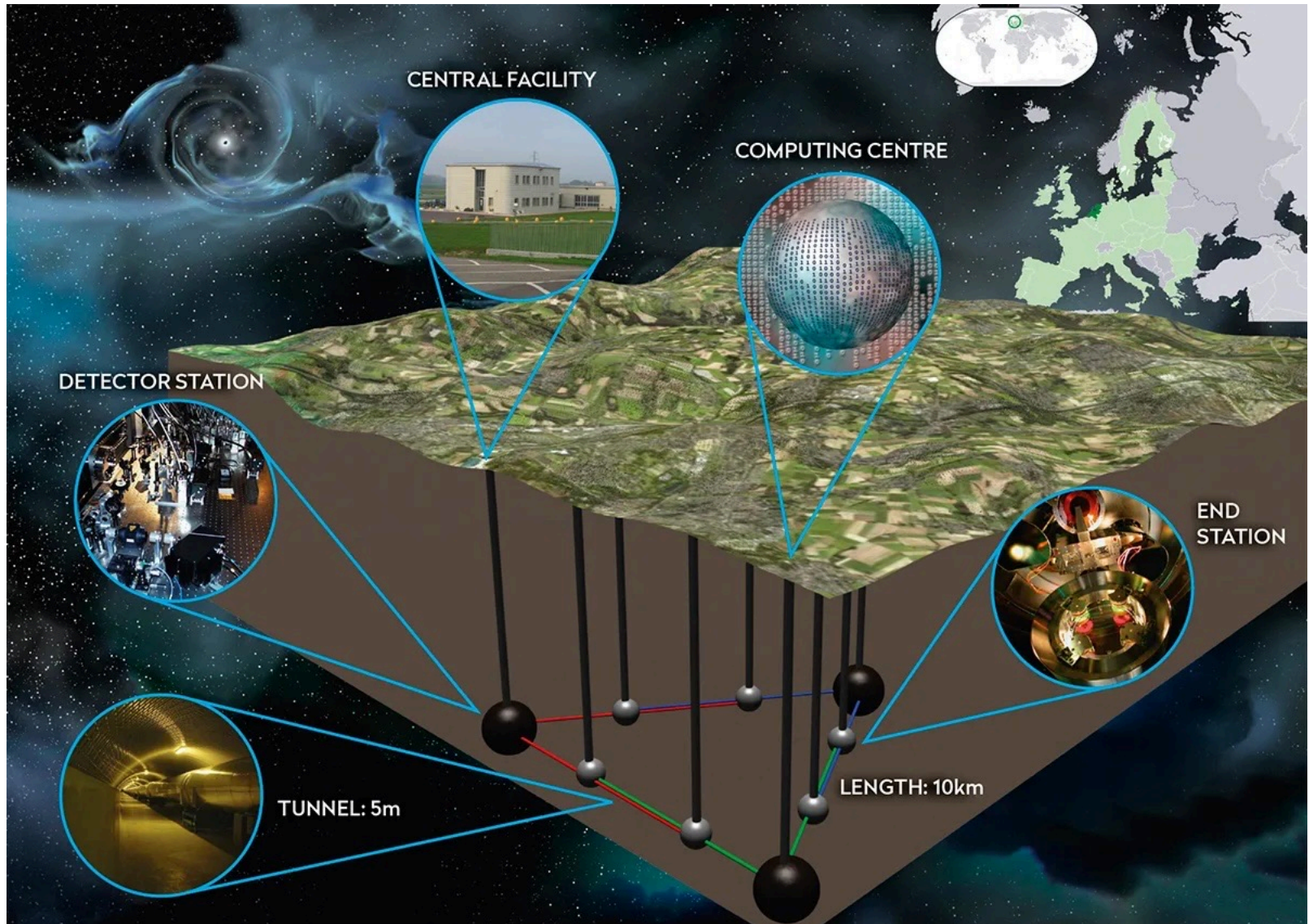
Outlook

- Multi-messenger astronomy and collider experiments will continue probing the properties of dense matter.
- Bayesian Analysis and Machine Learning methods are useful for estimation of unknown physical parameters.
- f-mode oscillation of hybrid stars and twin stars involving the “pasta phase” has been investigated.
- Re-examination of the asteroseismology problem considering the twin stars.
- Precise f-mode measurement provides suitable scenario for twin star detection.

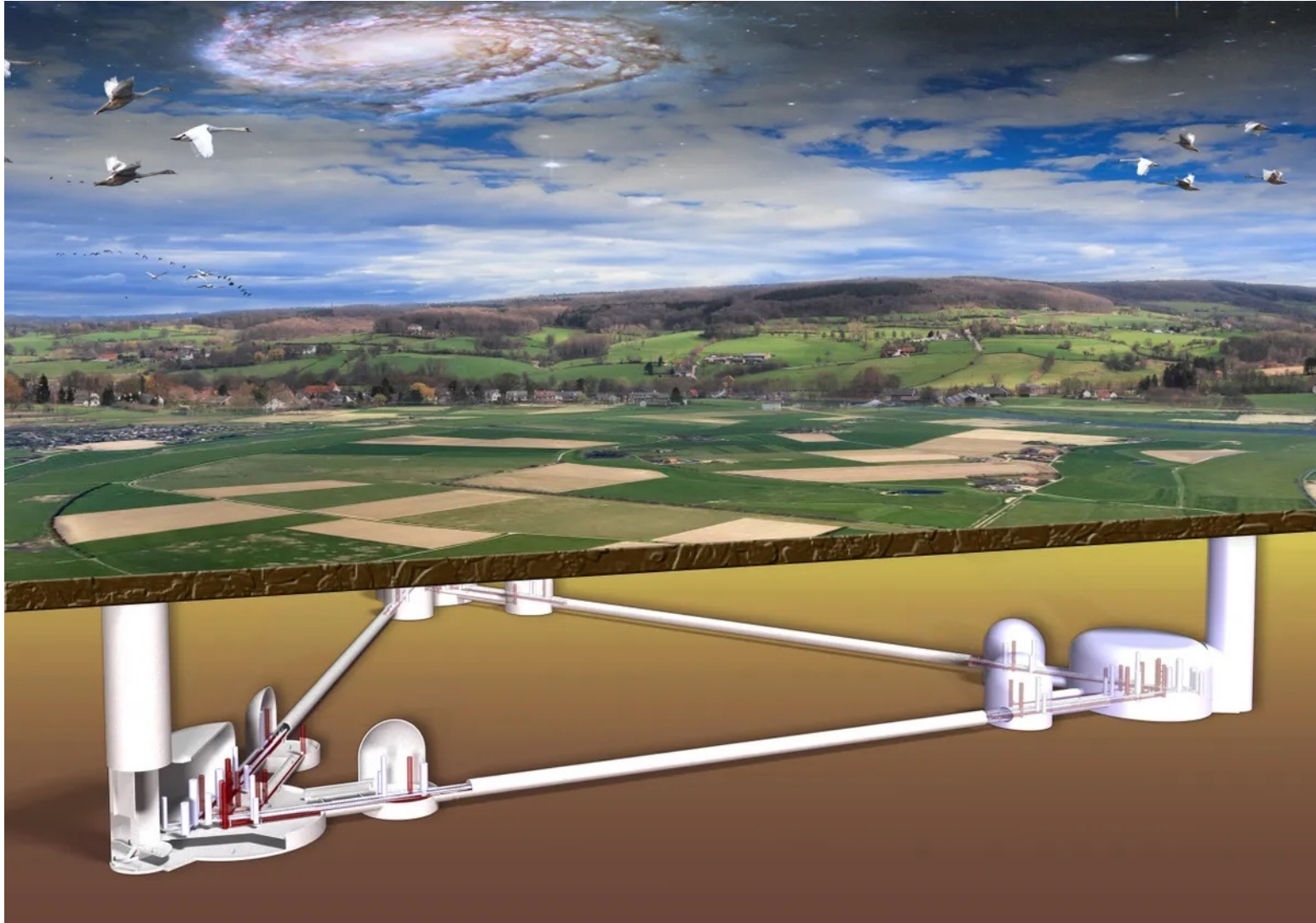
Outlook

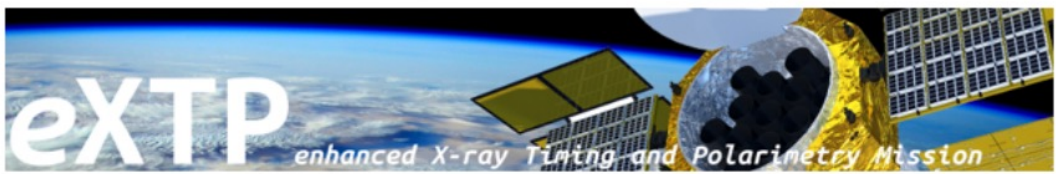
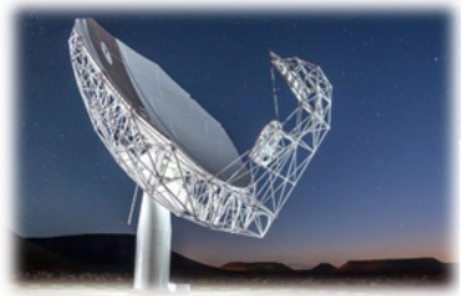
- f-mode GW detection with next-generation GW offers a promising scenario for confirming the existence of the twin stars.
- Distinguishing the nature of hadron-quark crossover phase transition requires further studies. Interestingly, accretion onto neutron stars can potentially excite fundamental modes in connection with neutrino emissions allowing for probing for phase transitions, see [arXiv:2311.15992](https://arxiv.org/abs/2311.15992).
- Consideration of the effect of rotation and magnetic fields can potentially improve this study.
- A detailed Bayesian study is in progress to constrain the pasta phase parameters from f-mode/binary observation.
- Cosmic Rays are potentially capable of bring insight into the solution of fundamental problems like the nature of dark matter or variation of fundamental constants in the entire universe.

Einstein Telescope



Einstein Telescope





Gracias