Oscillons in gapless scalar field theories

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11.09.2024

Plan

Plan:

Oscillons: overview and the gap

Oscillons: overview and the gap: Oscillons in general

Oscillons are spatially-localised, long-lived, oscillatory solutions of the field equation(s) of classical field theories

- nonlinearity of the field equation is essential
- no topological charge
- it is surprising that oscillons do not couple more strongly to radiation
- the fundamental reason for their existence remains somewhat mysterious
- sometimes the oscillon can be thought of as a decaying **sphaleron** of the field theory [1].

[1] N. Manton and TR, *The Simplest Oscillon and its Sphaleron*, Phys. Rev. D 107, 085012 (2023)

Oscillons: overview and the gap: Oscillons in general

- They are found in many models and in various dimensions,
  – phase transitions in condensed matter
  – astrophysics (dark matter candidates)
  – cosmology (inflation)[2] [3]
  – fundamental theory of electroweak interactions
  – graphene ribbons
  – can be created directly from radiation, as an intermediate stage for kink-antykink creation [4]

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[2] J. Sakstein and M. Trodden, *Oscillons in higher-derivative effective field theories*, Phys. Rev. D 98, 123512 (2018)
[3] Kaloian D. Lozanov and Mustafa A. Amin, *End of inflation, oscillons, and matter-antimatter asymmetry*, Phys. Rev. D 90, 083528 (2014)
[4] TR and Ya. Shnir, *Phys. Rev. Lett. 105, 081601*, (2010)

Oscillons: overview and the gap: Oscillons in general

- A basic oscillon is periodic, with a fundamental frequency $\omega < m$, where m is the threshold for radiation modes.
- A model has to have a frequency gap (counterexamples in the future).
- The oscillon has an arbitrary amplitude lying in some finite range.
- As the amplitude increases, the frequency ω decreases away from the threshold m.
- In 1+1d $\omega \to m$ but in higher dimensions $\omega \to \omega_{\text{max}} < m$.
- The main channel of losing energy to radiation is usually through 2ω (but there are exceptions):
  – large oscillons with $\omega < m/2$
  – models without quadratic terms in the e.o.m.
  – breathers in the sine-Gordon model.

Oscillons: overview and the gap: Example: ϕ^4

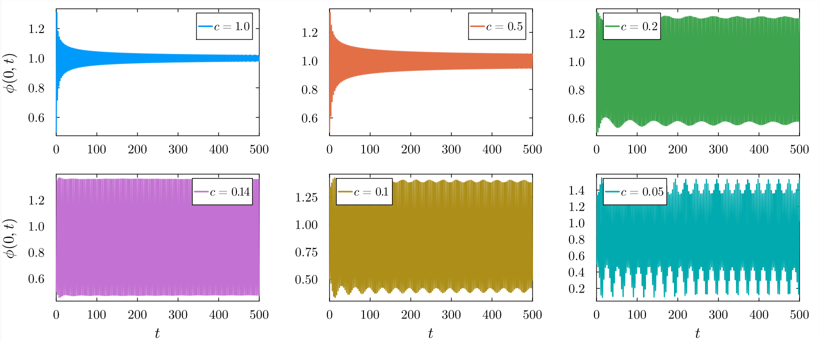
Let us consider the standard ϕ^4 model:

$$L = \int \frac{1}{2}\phi_t^2 - \frac{1}{2}\phi_x - \frac{1}{2}(\phi^2 - 1)^2 \; dx$$

Starting from gaussian initial conditions

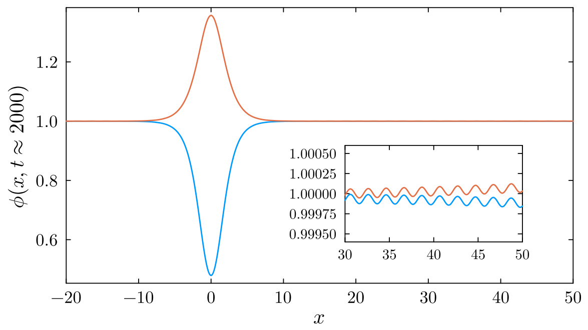
$$\phi(x,0) = 1 - Ae^{-cx^2}, \quad \phi_t(x,0) = 0$$

for $A = 0.5$ we see different behaviour for different values of c.



Oscillons: overview and the gap: Example: ϕ^4

Oscillon profile and small radiation tails for $A = 0.5, c = 0.14$ and $t \approx 2000$



Oscillons: overview and the gap: Example: ϕ^4

Similar behaviour is observed for **arbitrary small** value of A, with a certain scaling $c(A)$.
Small oscillations around vaccua obey (?) Klein-Gordon equation

$$\psi_{tt} - \psi_{xx} - 4\psi + \mathcal{O}(\psi^2) = 0$$

for $\phi = \pm 1 + \psi$

- For large times solutions of KG equations behave as

$$\psi(0,t) \sim \frac{\cos(2t+\delta)}{\sqrt{t}}$$

- This was true for $c = 1$ and $c = 0.5$, but for $c \approx 0.14$ oscillations did not change visibly.
- Sometimes almost quasi-periodic oscillations were visible.
- Scalability of oscillons means that linearization can be incorrect even as $A \to 0$.

Oscillons: overview and the gap: Example: ϕ^4

Naive explanation:
Assuming that the field profile does not change with time we can plug:

$$\phi(x,t) = 1 - A(t)e^{-cx^2}$$

into the field theory lagrangian and integrate over x. This gives us mechanical lagrangian for anharmonic oscillator:

$$L = \sqrt{\frac{\pi}{8c}}\left[\dot{A}^2 - A^2(c+4) + 4\sqrt{\frac{2}{3}}A^3 - \frac{A^4}{\sqrt{2}}\right]$$

Small oscillations oscillate with frequency $\omega_0 = \sqrt{c+4} > 2$. In field theory such oscillations frequencies are above the mass threshold and disperse.

However, nonlinear corrections lower the frequency below the mass threshold

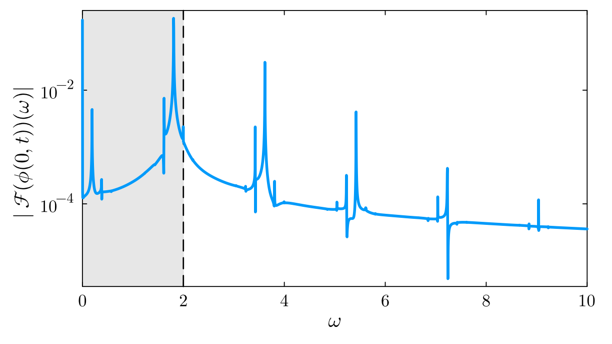
$$\omega = \omega_0 - \frac{80 + 3\sqrt{2}\omega_0^2}{8\omega_0^3}A_0^2 + \mathcal{O}(A_0^4)$$

Critical amplitude, for $\omega < m$:

$$A_{crit} = \sqrt{\frac{\omega_0 - 2}{\omega}}$$

Oscillons: overview and the gap: Example: ϕ^4

Power spectrum of the field at centre for $A = 0.5, c = 0.1$



- Basic frequency is below the mass threshold $m_{\phi^4} = 2$
- Higher harmonics are in the continuous spectrum (radiation).
- Smaller peaks (for example at the threshold) are responsible for the modulations.

Oscillons: overview and the gap: Relation to breathers

In the integrable sine-Gordon model

$$L = \int \frac{1}{2}\phi_t^2 - \frac{1}{2}\phi_x^2 - V_{sG}(\phi)\,dx\,, \qquad V_{sG}(\phi) = 1 - \cos(\phi)$$

there exists a periodic finite-energy breather solution

$$\phi_B(x,t) = 4\,\mathrm{atan}\left(\frac{\sqrt{1-\omega^2}}{\omega}\,\frac{\cos(\omega t)}{\cosh\left(\sqrt{1-\omega^2}\,x\right)}\right)$$

with $\omega = \cos(A/4) \in (0,1)$ with mass threshold $m_{sG} = 1$.
Potentials in many field theory models around vacua (after shift and rescaling) can be expressed as

$$V(\phi) = 1 - \cos(\phi) + \mathcal{O}(\phi^n) \quad \text{where } n > 2$$

therefore, in some sense, small amplitude oscillons owe their existence to sine-Gordon integrability.

Massless model

Massless model: Construction

Let us consider a real scalar field theory in $(d+1)$ dimensions

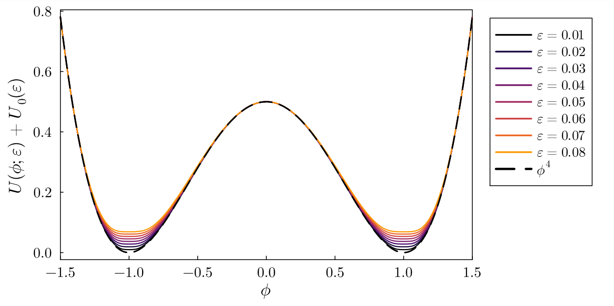
$$L = \int \left(\frac{1}{2}\phi_t^2 - \frac{1}{2}(\nabla\phi)^2 - U(\phi) \right) d^d x.$$

Here we will assume $d = 1$ or 3. Let $U(\phi)$ have a form very similar to the simplest, most prototypical, double vacuum potential, i.e., ϕ^4 but *flattened* near the vacua, i.e. so that the potential is *quartic* near the vacua. This can be achieved with a rational modification

$$U(\phi) = \frac{W^2}{W+\epsilon}, \qquad W = \frac{1}{2}(1-\phi^2)^2,$$

where ϵ is a small positive parameter, which controls the size of the deformed region.

Massless model: Construction



where the shift $U_0(\epsilon) = -1/(4\epsilon + 2) + 1/2$ is just for better visualization. Near vacua:

$$U \approx \frac{1}{4\epsilon}(1-\phi)^4(1+\phi)^4$$

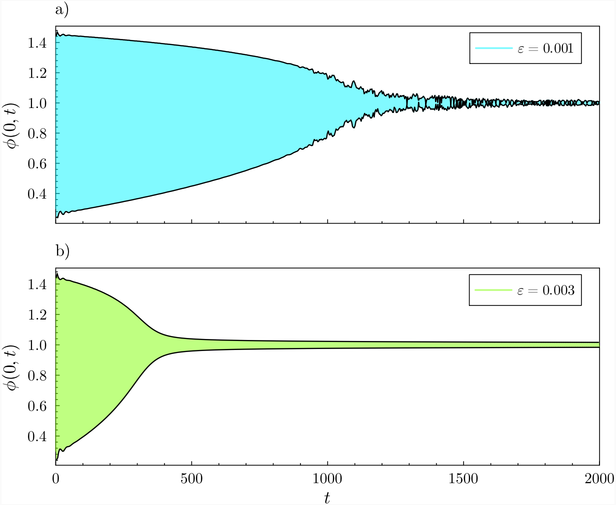
so the mass of (infinitesimally) small perturbations vanishes,

$$m^2 = \left.\frac{d^2U}{d\phi^2}\right|_{\phi=\pm 1} = 0.$$

This model has no mass gap, so oscillons should not exist.

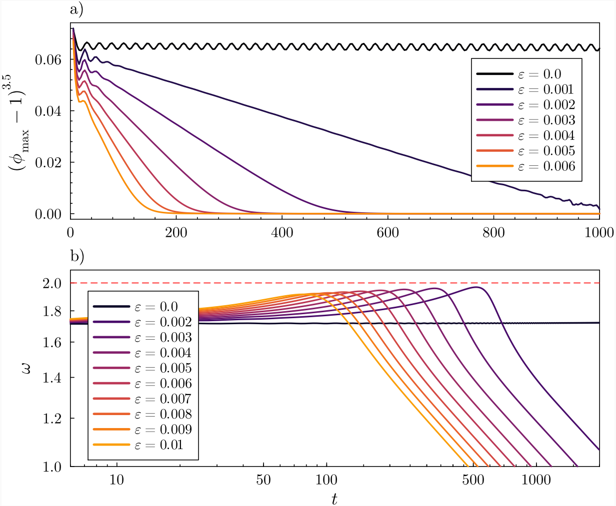
Massless model: Numerical results

However, large amplitude oscillons "see" the potential as if it was the ϕ^4 model, and only the low amplitude tails "feel" that the model is massless.



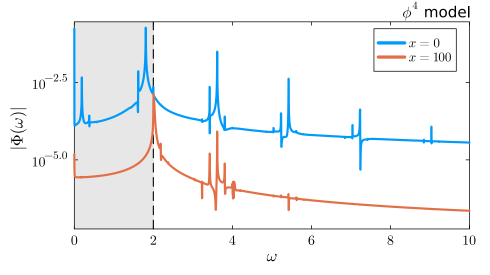
Massless model: Numerical results

Oscillons in the modified model definitely exist, although they live shorten than usual oscillons.

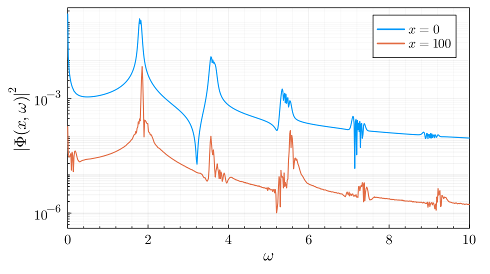


Massless model: Numerical results

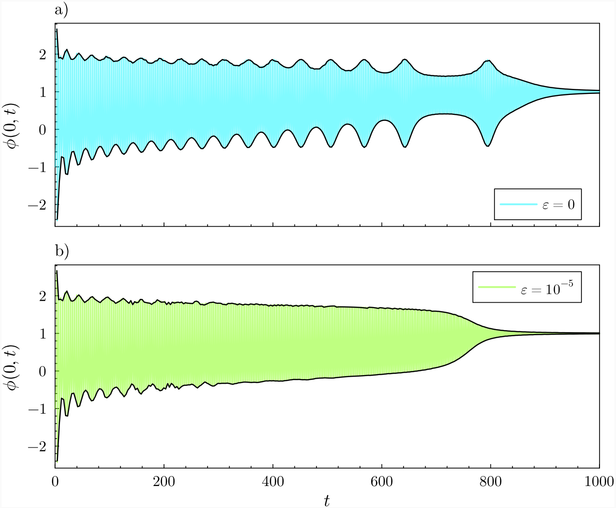
- similar spectra, but
- ϕ^4: radiation through 2$^{\text{nd}}$ harmonic
- ϕ^4: slow threshold radiation (residue from bad IC)



- mod: radiation through 1$^{\text{st}}$ harmonic (speed of light)
- mod: no threshold radiation



Massless model: Oscillons in 3+1d



Massless model: Oscillons in 3+1d

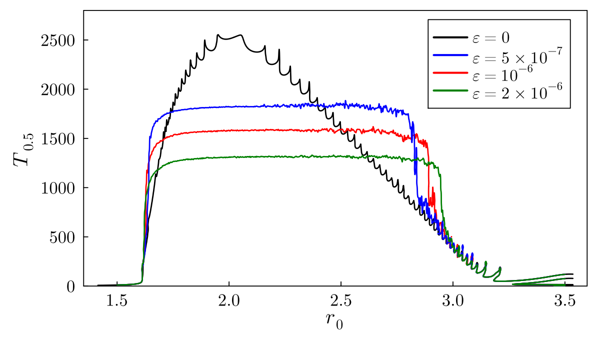
- In 3+1D oscillons in ϕ^4 are less stable
- No integrable sine-Gordon model (no 3d breathers)
- Additional upper frequency $0 < \omega_{cr,1} < \omega < \omega_{cr,2} < m$
- Modulations grow with time
- Lifetimes form resonant patters (with modulation[5])
- After reaching $\omega_{cr,2}$ fast decay is observed
- Inthe deformed model modulations vanish

[5] E. P. Honda, M. W. Choptuik, *Fine structure of oscillons in the spherically symmetric ϕ^4 Klein-Gordon model*, Phys. Rev. D 65 084037 (2002).

Massless model: Oscillons in 3+1d

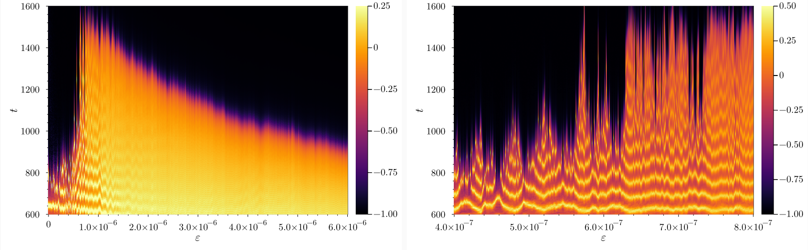
Lifetimes of oscillons from gaussian initial conditions:

$$\phi(x,0) = 1 - 2e^{-r^2/r_0^2}$$



- Sometimes oscillons in the modified model live longer than in the non-modified model.
- More radiation in the deformed model
- But no instability due to the modulations.

Massless model: Oscillons in 3+1d



For fixed r_0 but changing ϵ we can see another resonant (fractal-like?) structures

Summary

Summary: Oscillons in 3+1d

- Oscillons are interesting and important objects
- Nonlinearities are essential for their existence
- In standard theories their frequency is below the mass threshold
- But they also can exits in gap-less models
  – only finite amplitude oscillons exist (similar to usual 3+1d)
  – can have more applications then previously expected

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