Gravitational form factors of the pion: lattice QCD meets meson dominance

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Polish Particle and Nuclear Theory Summit, Cracow, 11-13 September 2024

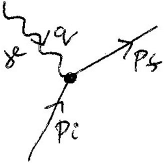
Based on 2405.07815 [hep-ph]

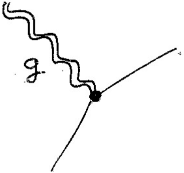
Outline

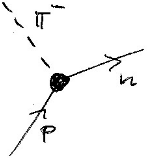
- Form factors
- Gravitational form factor (GFF) of the pion
- Explanation of recent lattice QCD data with meson dominance
- σ meson in the scalar channel
- Comparison to pQCD asymptotics
- Sizes of distributions

Different probes of the structure

electromagnetic . . .   mass (gravitational) . . .   composite operators (hadronic)







scattering amplitude $= \sum$ tensorial structure $\times$ form factor (scalar function)
Extracted from scattering data and lattice QCD

"Gravitational": no need for gravitons, e.g., $\gamma\gamma^* \to \pi^0\pi^0$ or lattice, the probe is the energy-momentum tensor $T_{\mu\nu}$

Recent activity: GFF of the electron [Berends, Gasmans, 1976, Freese, Metz, Pasquini, Rodini, 2022] ... deuteron [He, Zahed, 2024] ... light nuclei [García Martín-Caro, Huidobro, Hatta, 2023] ... charmonium [Xu et al., 2024] ... (pion, nucleon - lots)

Definition of GFF (here for the pion)

... from $\langle h(p')|T^{\mu\nu}(0)|h(p)\rangle$. The stress-energy-momentum tensor is conserved, $\partial_\mu T^{\mu\nu}(x) = 0$

For the pion (spin-0) two tensor structures allowed by Lorentz covariance and conservation:

$$\langle \pi^a(p')|T^{\mu\nu}(0)|\pi^b(p)\rangle = \delta_{ab}\left[2P^\mu P^\nu A(t) + \tfrac{1}{2}\left(q^\mu q^\nu - g^{\mu\nu} q^2\right) D(t)\right]$$

a, b - isospin, $P = \frac{1}{2}(p' + p)$, $q = p' + p$, $t = q^2 = -Q^2$

Long history: Pagels 1965, K. Raman 1971, Łopuszański 1974, ..., χPT: Donoghue, Leytwyler 1991

Properties:

$$A(0) = 1, \quad D(0) = -1 + \mathcal{O}(m_\pi^2)$$

Relation to GPD

$\int_{-1}^{1} dx\, x H_\pi^{I=0}(x,\xi,t)) = A(t) + \xi^2 D(t)$, $\quad \xi$ - skewness

$$\langle \pi^a(p')|\Theta^{\mu\nu}(0)|\pi^b(p)\rangle = \delta_{ab}\left[2P^\mu P^\nu A(t) + \tfrac{1}{2}\left(q^\mu q^\nu - g^{\mu\nu}q^2\right) D(t)\right]$$

$$\Theta^\mu_\mu \equiv \Theta(t) = 2\left(m_\pi^2 - \frac{t}{4}\right) A(t) - \frac{3t}{2} D(t)$$

$\Theta^{\mu\nu}$ can be decomposed into a sum of two separately conserved irreducible tensors corresponding to well-defined total angular momentum, $J^{PC} = 0^{++}$ (scalar) and 2^{++} (tensor):

$$\Theta^{\mu\nu} = \Theta_S^{\mu\nu} + \Theta_T^{\mu\nu}\,, \qquad \begin{cases} \Theta_S^{\mu\nu} = \frac{1}{3}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\Theta \\ \Theta_T^{\mu\nu} = 2\left[P^\mu P^\nu - \frac{P^2}{3}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\right] A \end{cases}$$

Since Θ and A carry the information on good J^{PC} channels, they should be regarded as the primary objects, whereas D mixes the quantum numbers:

$$D = -\frac{2}{3t}\left[\Theta - \left(2m_\pi^2 - \tfrac{1}{2}\,t\right) A\right]$$

Mechanistic interpretation

Physical meaning: “energy, pressure, stress distribution”
T^{00} – energy, , T^{ii} – pressure, , $T^{ij}, i \neq j$ – stress
[M. Polyakov 2003, Polyakov, Schweitzer 2018, Ji 2021, Lorcé, Metz, Pasquini, Rodini 2021, ...]

$$\frac{\langle \pi_{\rm rest} | \int d^3 r\, T^{\mu\nu}(\vec{r}) | \pi_{\rm rest} \rangle}{\langle \pi_{\rm rest} | \pi_{\rm rest} \rangle} = {\rm diag}(m_\pi, 0, 0, 0)$$

Balance of pressure, $\int d^3 r\, p(r) = 0$ ($p(r)$ must change sign)

Also, $m_\pi \int d^3 r\, r^2 p(r) = D(t=0)$ and for the shear forces $-\frac{4}{15} m_\pi \int d^3 r\, r^2 s(r) = D(0)$

D - Druck term
Since $D(0) < 0$, $p(r)$ must change from $+$ at low r to $-$ and high r

QCD

Trace anomaly
$$\partial^\mu D_\mu = \Theta^\mu_\mu \equiv \Theta = \frac{\beta(\alpha)}{2\alpha} G^{\mu\nu a} G^a_{\mu\nu} + \sum_q m_q \left[1+\gamma_m(\alpha)\right] \bar{q}q$$
$$\beta(\alpha) = \mu^2 d\alpha/d\mu^2,\ \alpha = g^2/(4\pi),\ \gamma_m(\alpha) = d\log m_q / d\log\mu^2,\ \alpha \sim 1/\log Q^2$$

Scale dependent decomposition $\Theta^{\mu\nu} = \Theta^{\mu\nu}_q + \Theta^{\mu\nu}_g$ (not analyzed here)

pQCD ($t \to \infty$) [Tong, Ma, Yuan, 2021, 2022]:
$$A(t) = -48\pi f_\pi^2 \alpha(t)/t + \mathcal{O}(\alpha(t)^2/t)$$
$$D(t) = +16\pi f_\pi^2 \alpha(t)/t + \mathcal{O}(\alpha(t)^2/t)$$
$$\Theta(t) = -t/2(A+3D) = 16\pi f_\pi^2 \beta[\alpha(t)] \simeq -4 f_\pi^2 \beta_0 \alpha(t)^2 + \mathcal{O}(\alpha(t)^3)$$

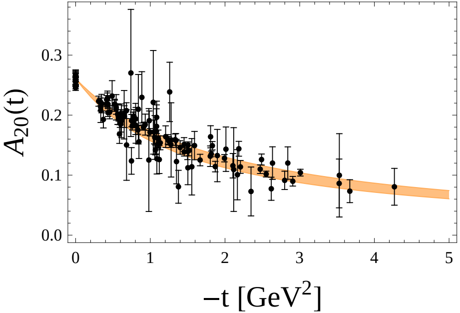
Recall that the EM ff is $F_\pi(t) = -16\pi\alpha(t) f_\pi^2/t + \mathcal{O}(\alpha(t)^2/t)$

$$m_q \langle \pi(p') | \bar{q}q(0) | \pi(p) \rangle \sim m_\pi^2 f_\pi^2 \alpha(t)/t$$

Low-energy constraints $A(0) = 1,\ D(0) = -1 + \mathcal{O}(m_\pi^2) \to \Theta(0) = 2m_\pi^2, \quad d\Theta(t)/dt|_{t=0} = 1 + \mathcal{O}(m_\pi^2)$

Early model estimate

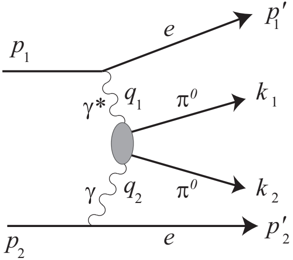
Chiral quark models: $\langle r_2 \rangle_A = \frac{1}{2}\langle r_2 \rangle_{EM}$ - A distribution more compact than charge [WB, ERA, 2008]



Lattice [Brommel 2007] vs meson dominance [Masjuan, ERA, WB, Phys.Rev.D 87 (2013) 1, 014005]
($A_{20}(t) \equiv \frac{1}{2} A_q(t)$ - quark part)

At that time $D_q(t)$ very noisy, no gluons

Determination from the Belle data



[Belle, 2015]

Prospects at super-KEKB, ILC!

[Kumano, Song, Teryaev, 2015] (GDAs) $\rightarrow$

$$\langle r^2 \rangle_A = (0.32 - 0.39 \text{ fm})^2$$
$$\langle r^2 \rangle_D = (0.82 - 0.88 \text{ fm})^2$$

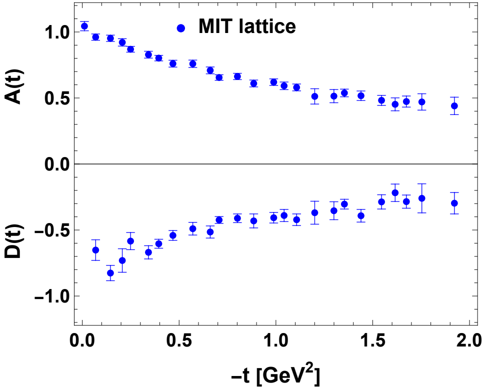
recall $\langle r^2 \rangle_{EM} = (0.656 \pm 0.005 \text{ fm})^2$ (PDG 2021)

Recent MIT data

[Phys.Rev.D 108 (2023) 11, 114504 & D. Pefkou, PhD Thesis]

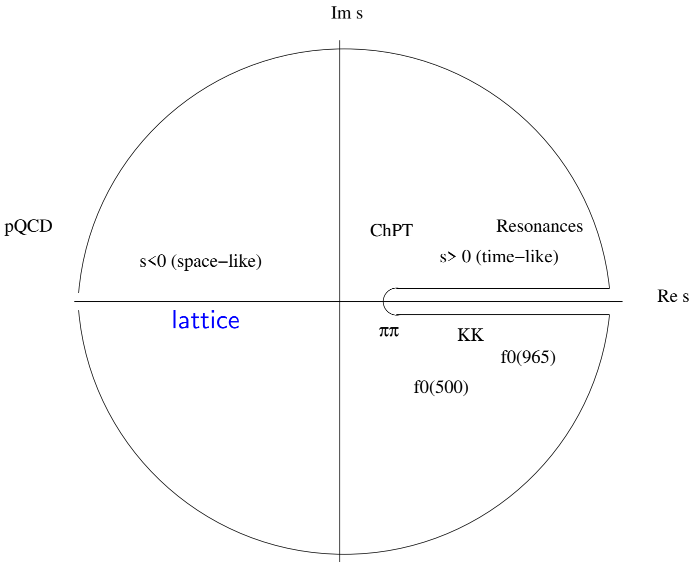
Unprecedented accuracy, both quarks and gluons, $m_\pi = 170$ MeV

(below the total $q+g$ used, as it corresponds to the conserved current $\rightarrow$ renorm invariant)



Allows for more stringent tests and general understanding

Analyticity



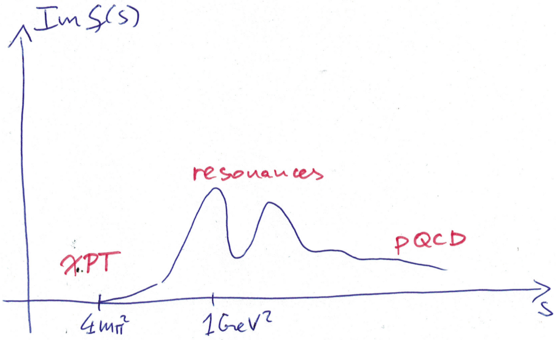
Question of obtaining the spectral density

$\rho(s) = \mathrm{Im} f(s) = [f(s+i\epsilon) - f(s-i\epsilon)]/(2i)$

$$\rho(s) = \begin{cases} \rho_{\mathrm{ChPT}}(s) & 4m_\pi^2 \le s \lesssim m_\rho^2 \\ \rho_{\mathrm{R}}(s) & m_\rho^2 \lesssim s \le \Lambda_{pQCD}^2 \\ \rho_{\mathrm{pQCD}}(s) & \Lambda_{pQCD}^2 \le s \end{cases}$$

Contributions to spectral density

Typically, one expects

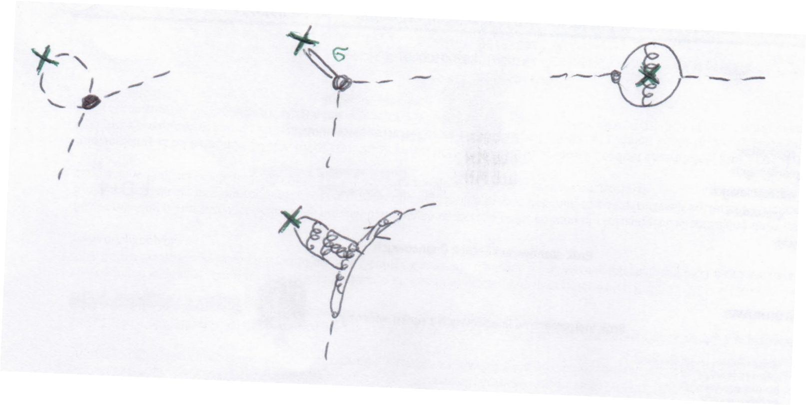


(not necessarily positive-definite)

Meson dominance = resonances dominate (at some intermediate Q^2) the dispersion relations

Cartoon view

Chiral Perturbation Theory   meson dominance   pQCD



Dispersion relations

According to analyticity, GFFs satisfy dispersion relations. Once-subtracted form:

$$A(-Q^2) = 1 + \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{s}{s'} \frac{\mathrm{Im} A(s')}{s' + Q^2 - i\epsilon}, \qquad D(-Q^2) = D(0) + \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{s}{s'} \frac{\mathrm{Im} D(s')}{s' + Q^2 - i\epsilon}$$

pQCD: $\lim_{Q^2\to\infty} A(-Q^2) = \lim_{Q^2\to\infty} D(-Q^2) = 0$ (vanish as $1/Q^2$ mod logs) $\to$ sum rules

$$0 = 1 - \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im} A(s')}{s'}, \quad 0 = D(0) - \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im} D(s')}{s'}$$

$$\text{Similarly} \quad \Theta(-Q^2) = 2m_\pi^2 + \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{s}{s'} \frac{\mathrm{Im}\Theta(s')}{s' + Q^2 - i\epsilon} \to 0 = 2m_\pi^2 - \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\Theta(s')}{s'}$$

Also $\quad \lim_{Q^2\to\infty} Q^2 f(-Q^2) = 0 \to 0 = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \, \mathrm{Im} f(s'), \quad f = A, D$

hence spectral densities of A, D (and also Θ) must change sign! $\leftarrow$ from QCD constraints

Large N_c and meson dominance

t Hooft, Witten: At large N_c, amplitudes are saturated by tree-level diagrams with towers of mesons in intermediate states $\to$

$$\mathrm{Im}A(s) = \sum_T c_T M_T^2 \pi \delta(M_T^2 - s), \quad \mathrm{Im}\Theta(s) = \sum_S c_S M_S^4 \pi \delta(M_S^2 - s)$$

Dispersion relations $\to$

$$A(-Q^2) = 1 - \sum_T \frac{c_T Q^2}{M_T^2 + Q^2}, \quad \Theta(-Q^2) = 2m_\pi^2 - \sum_S \frac{c_S Q^2 M_S^2}{M_S^2 + Q^2}$$

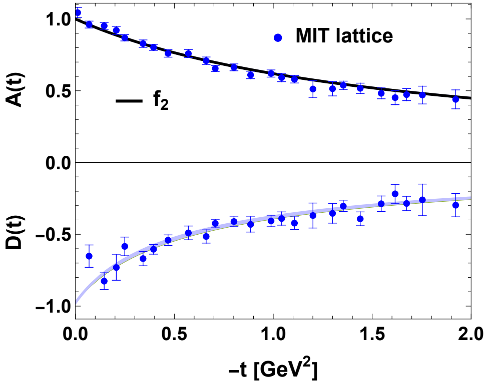
$\sum_T c_T = 1$ (since $A(-\infty) = 0$ and (in the chiral limit) $\sum_S c_S = 1$ ($\Theta'(0) = 1$)
We take one meson per channel, $f_2(1275)$ and $f_0(975)$

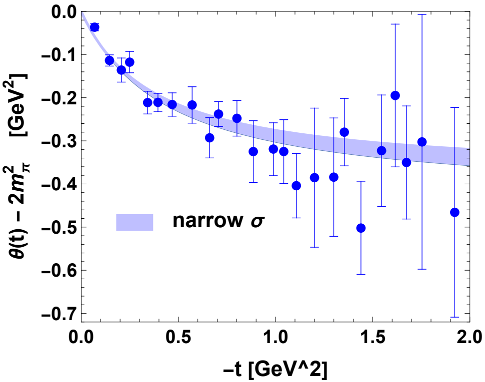
$$A(-Q^2) = \frac{m_{f_2}}{m_{f_2}^2 + Q^2}, \quad \Theta(-Q^2) = 2m_\pi^2 - \frac{Q^2 m_\sigma^2}{m_\sigma^2 + Q^2}$$

Formula for $D(-Q^2)$ follows. Only m_σ is fitted

Result of the fit of m_σ

(our way of taking advantage of the MIT data)



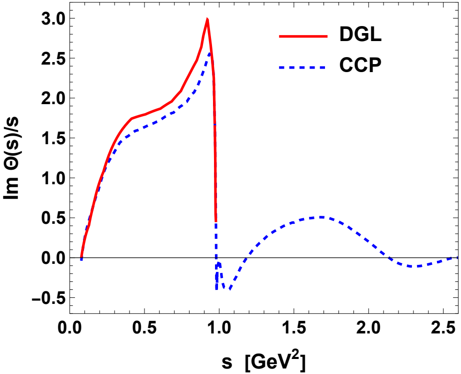


$m_{f_2} = 1.275$ MeV, $m_\sigma = 0.64(2)$ GeV, band width in D and Θ - 68% CL

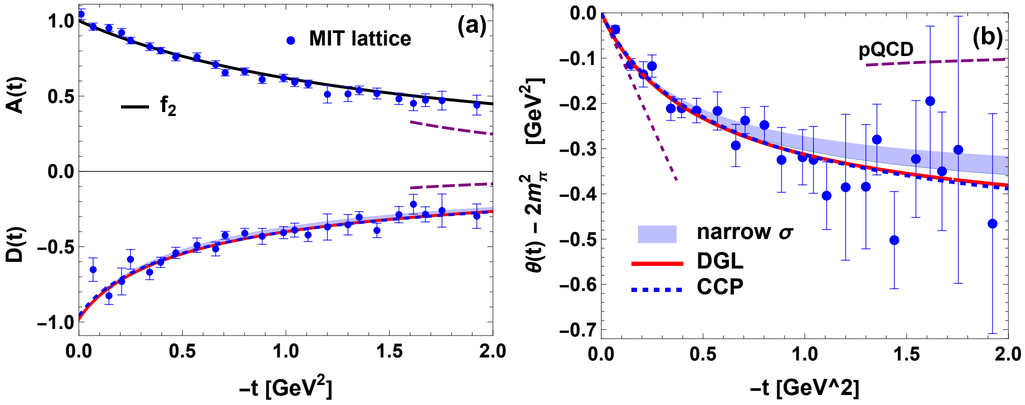
Scalar spectral function

$$\Theta(-Q^2) = 2m_\pi^2 - \frac{Q^2}{\pi}\int_{4m_\pi^2}^{\infty} ds \frac{\mathrm{Im}\Theta(s)}{s(s+Q^2)}$$

Spectral density from physical phase shifts [Donoghue, Gasser, Leutwyler, 1990] or [Celis, Cirigliano, Passemar, 2013] (Watson's theorem, Omnès-Muskhelishvili–type coupled-channel equations)



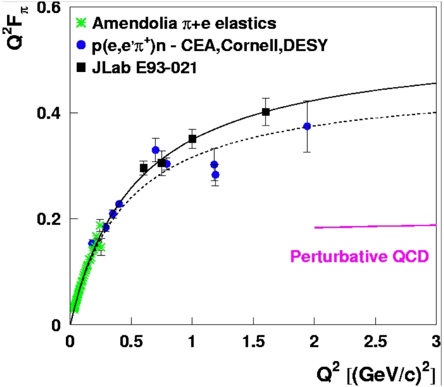
Spectral modeling vs lattice (main slide)



- Lattice consistent with "meson physics", also and in particular in the scalar channel
- pQCD far away!

Retrospect: EM ff of the pion

- low Q^2 - χPT $\qquad Q^2 = q^2 = -t$
- high Q^2 - pQCD: $F_\pi(Q^2)Q^2 \to 16\pi\alpha(Q^2)f_\pi^2\left[1+6.58\alpha(Q^2)/\pi+\ldots\right]$ (far away!)
- intermediate Q^2 - meson dominance (Sakurai ...): $F(Q^2)=\frac{1}{1+Q^2/m_\rho^2}$ (or sum over resonances)



Gravitational radii from our approach

$$\langle r^2 \rangle_A = (0.38 \text{ fm})^2, \ \ \langle r^2 \rangle_D = (0.71 \text{ fm})^2, \ \ \text{also } D(0) = -0.98$$

where

$$\langle r^2 \rangle_f = 6 \left. \frac{df(t)/dt}{f(t)} \right|_{t=0} \quad \left(= \frac{\int d^3r \, r^2 \hat{f}(r)}{\int d^3r \, \hat{f}(r)} \right)$$

Since $\langle r^2 \rangle_{EM} = (0.659(4) \text{ fm})^2$, the spatial distribution of A is much more compact than charge [WB, ERA, 2008], while D it is a bit less compact than charge

Conclusions

1. Recent lattice results for gravitational ff of the pion fully compatible with meson dominance at "intermediate" values of Q^2
2. Important to look at the data in good spin channels
3. Tensor channel on the lattice: just $f_2(1275)$
4. Scalar channel (Θ) modeled with dispersion relations and physical spectral density (a way to include a broad σ meson
5. Spectral densities must change sign
6. Much higher Q^2 needed to approach pQCD (similarly to EM ff)
7. Spatial distribution of A more compact than charge, charge more compact than D
8. Super-KEKB and ILC can measure GDAs

9. For the nucleon similar story expected (3 structures, more mesons per channel needed to satisfy the QCD constraints), cf. [Masjuan, ERA, WB, 2013]

One sees mesons all over the lattice!