Politechnika Krakowska
im. Tadeusza Kościuszki

Comparison of in-medium interaction and gluon saturation effects on jet-production

Martin Rohrmoser

Politechnika Krakowska, Kraków

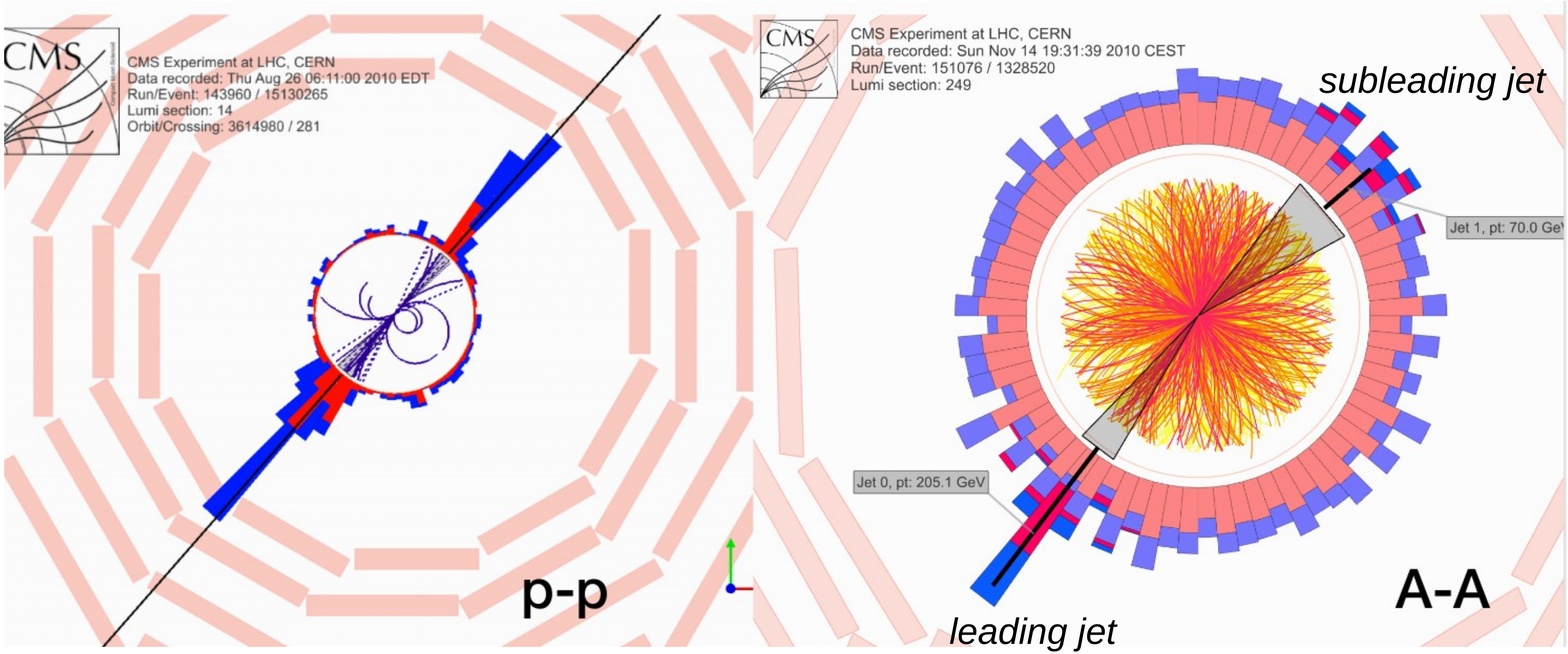
Collaborators:

Souvik Adhya, Krzysztof Kutak, Wiesław Płaczek, Konrad Tywoniuk,
Andreas van Hameren, Etienne Blanco, Robert Straka

based on:

[S. Adhya, K. Kutak, W. Płaczek, MR, K. Tywoniuk, arxiv: 2409.06675]

Jet Quenching



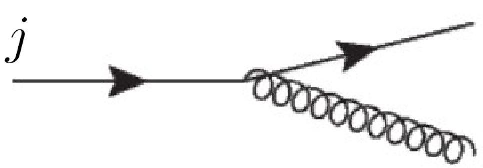
Jets interact with medium → Jet Quenching!

↓

probe of the medium

Processes in jets in the medium

Splitting:



Bremsstrahlung as
in vacuum.

Momentum distribution:

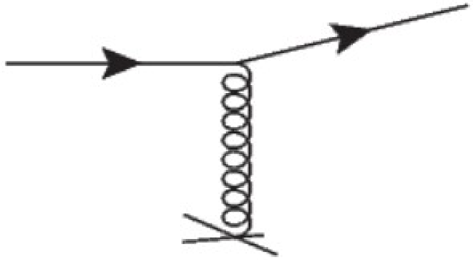
$$p \to zp$$

DGLAP-Kernel:

$$\frac{\partial^2 \mathcal{P}_{\text{split}}}{\partial \hat{t} \partial z} \propto \frac{1}{\hat{t}} P_{ij}(z)$$

Emissions in Vacuum,
Vacuum Like
Emissions in medium

Scattering:



Momentum transfer!

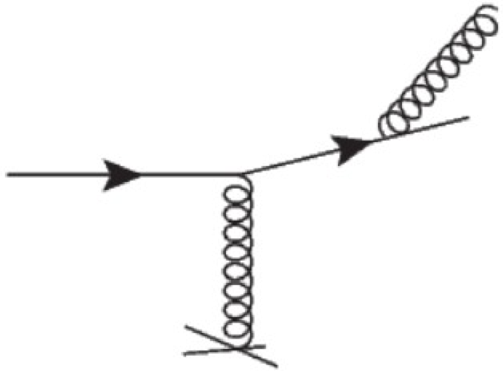
$$p \to p + Q$$

Scattering Kernel:

$$\frac{\partial^3 \mathcal{P}_{\text{scat}}}{\partial t \partial^2 \mathbf{Q}} = \frac{1}{(2\pi)^2} w(\mathbf{Q})$$

Average transfer: $\hat{q}$

Induced radiation:



Momentum distribution:

$$p \to zp$$

+Momentum transfer:

$$p \to zp + Q$$

Kernel: $\frac{\partial^4 \mathcal{P}_{\text{split}}}{\partial t \partial z \partial^2 \mathbf{Q}} = \frac{\alpha_s}{(2\pi)^2} \mathcal{K}(\mathbf{Q}, z, p_+)$

Jet-Medium interactions

Coherent emission

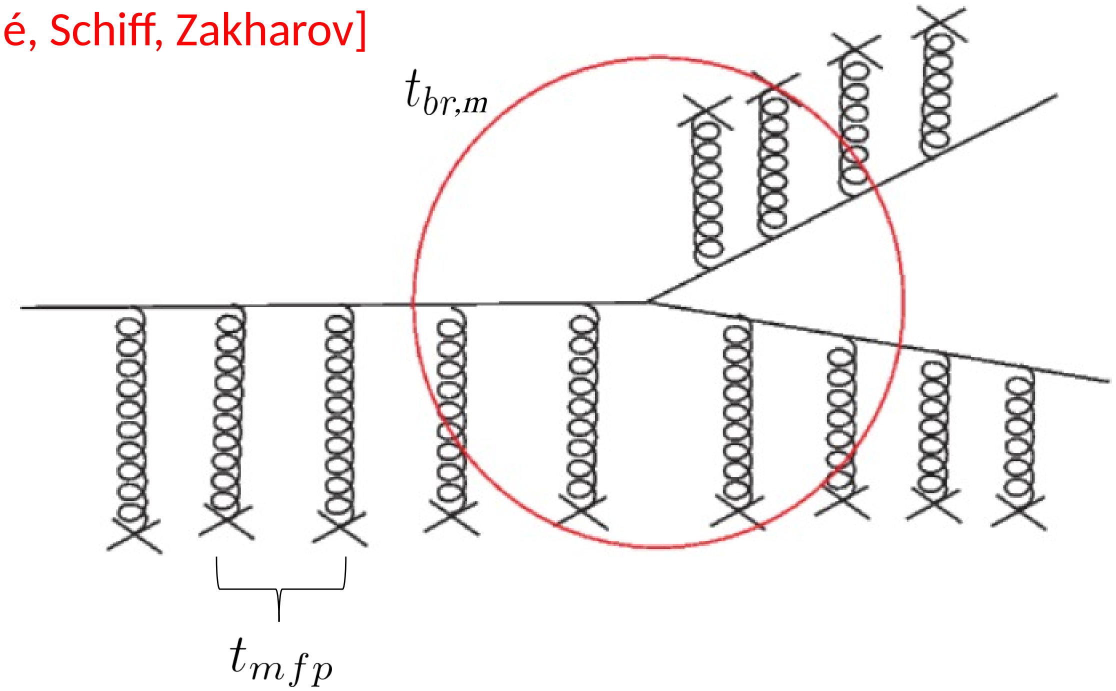
…à la BDMPS-Z [Baier, Dokshitzer, Mueller, Peigné, Schiff, Zakharov]

$$t_{br,m} \sim \sqrt{\frac{2\omega}{\hat{q}}}$$

$t_{br,m} \sim t_{mfp}$: one scattering + radiation
…Bethe-Heitler spectrum

$t_{br,m} \gg t_{mfp}$: coherent radiation

$$\omega \frac{dI}{d\omega} \sim \alpha_s \frac{L}{t_{br,m}} = \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$



Look at range: $\omega_{BH} < \omega < \omega_c$

need effective kernel: $\mathcal{K}(z, k_T)$

cf. [Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1301 (2013) 143]
[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

Splitting Kernels for Quarks and Gluons

[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918], [Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1301 (2013) 143]

$$\mathcal{K}_{ij}(\boldsymbol{Q}, z, p_0^+) = \frac{2P_{ij}(z)}{z(1-z)p_0^+} \sin\left(\frac{\boldsymbol{Q}^2}{2k_{\rm br}^2}\right) \exp\left(-\frac{\boldsymbol{Q}^2}{2k_{\rm br}^2}\right)$$

$$k_{\rm br}^2 = \sqrt{z(1-z)p_0^+ f_{ij}(z)\frac{\hat{q}}{N_c}}$$

$$f_{gg}(z) = (1-z)C_A + z^2 C_A$$

$$f_{qg}(z) = C_F - z(1-z)C_A\,,$$

$$f_{gq}(z) = (1-z)C_A + z^2 C_F$$

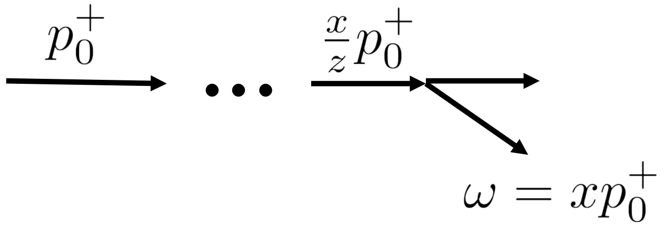
$$f_{qq}(z) = zC_A + (1-z)^2 C_F$$

$$\frac{\partial^4 \mathcal{P}_{\rm split}}{\partial t \partial z \partial^2 \mathbf{Q}} = \frac{\alpha_s}{(2\pi)^2}\mathcal{K}(\mathbf{Q}, z, p_+)$$

$$\frac{\partial^5 \mathcal{P}_{\rm split}}{\partial t \partial x \partial z \partial^2 \mathbf{Q}} = \frac{\alpha_s}{(2\pi)^2}\mathcal{K}(\mathbf{Q}, z, \tfrac{x}{z}p_0^+)$$

$$\Big\downarrow \int_0^\infty d^2\mathbf{Q}\times$$

$$\frac{\partial^3 \mathcal{P}_{\rm split}}{\partial t \partial x \partial z} = \frac{2\pi}{\sqrt{x}t^*}\sqrt{z}\mathcal{K}(z)$$



$$\frac{\partial^2 \mathcal{P}_{\rm split}}{\partial t \partial x} = 2\pi\frac{1}{\sqrt{x}t^*}\int dz\sqrt{z}\mathcal{K}(z)$$

$$\frac{1}{t^*} = \frac{\alpha_s}{\pi}\sqrt{\frac{\hat{q}}{p_0^+}}$$

$$\sqrt{x}t^* \propto t_{{\rm br},m}$$

Generalization of BDMPS-Z approach

Scattering Kernels

Used right now:

$$w_g(\mathbf{q}) = \frac{16\pi^2\alpha_s^2 N_c n_{\text{med}}}{\mathbf{q}^4}$$

$$w_g(\mathbf{q}) = \frac{g^2 m_D^2 T}{\mathbf{q}^2(\mathbf{q}^2 + m_D^2)}\,, \qquad g^2 = 4\pi\alpha_s$$

$n_{\text{med}} \ldots$ density of scatterers
$m_D \ldots$ Debye mass
$T \ldots$ medium temperature

$$w_q(\mathbf{q}) = \tfrac{C_F}{C_A} w_g(\mathbf{q})$$

Sudakov factors

Probabilities of interaction:

$$\Phi_g(x) = \alpha_s \int_\epsilon^{1-\epsilon} dz \int_{q>0} \frac{d^2\mathbf{q}}{(2\pi)^2} \left[\mathcal{K}_{gg}(\mathbf{q}, z, xp_+) + \mathcal{K}_{qg}(\mathbf{q}, z, xp_+) \right] + \int_{q>q_{\min}} \frac{d^2\mathbf{q}}{(2\pi)^2} w_g(\mathbf{q}) \,,$$
$$\Phi_q(x) = \alpha_s \int_\epsilon^{1-\epsilon} dz \int_{q>0} \frac{d^2\mathbf{q}}{(2\pi)^2} \mathcal{K}_{qq}(\mathbf{q}, z, xp_+) + \int_{q>q_{\min}} \frac{d^2\mathbf{q}}{(2\pi)^2} w_q(\mathbf{q}) \,,$$

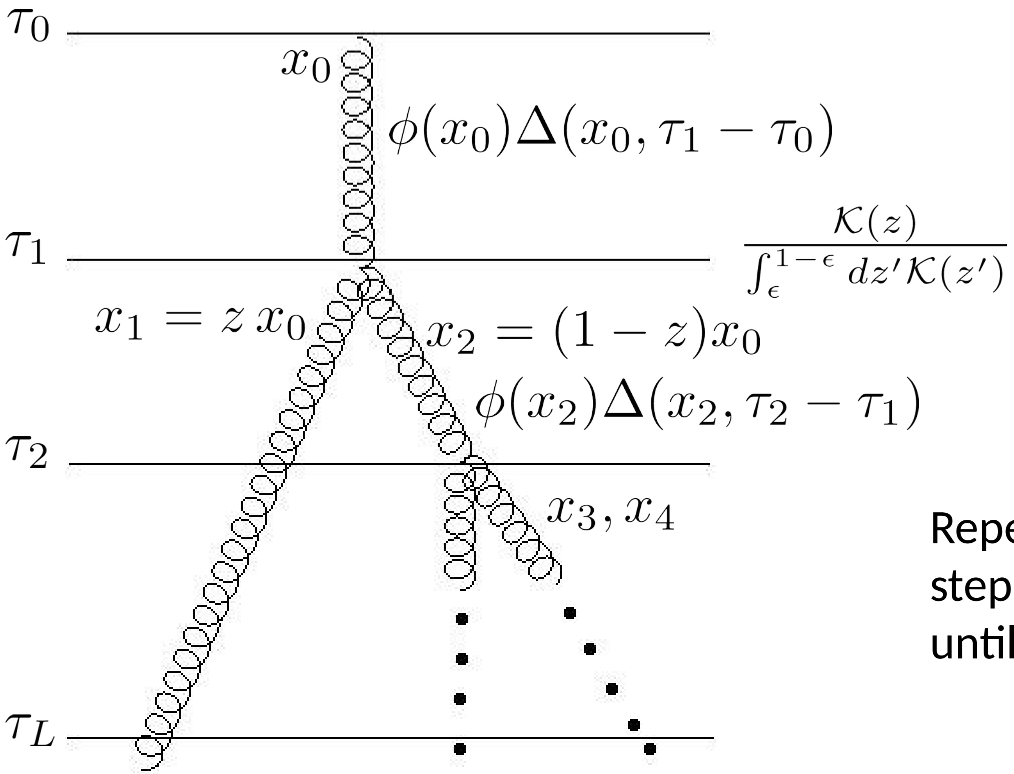
Probability of no interaction for particle A over time (t_2-t_1):

$$\Delta_A(x, t_2 - t_1) = \exp\left(-\Phi_A(x)(t_2 - t_1)\right)$$ … Sudakov factor

Monte-Carlo algorithm TMDICE

Other codes implementing BDMPS-Z spectra:

MARTINI, JEWEL, QPYTHIA, …



Analogous for the k_T dependent equation in x, k_T, and, τ and system of equations!

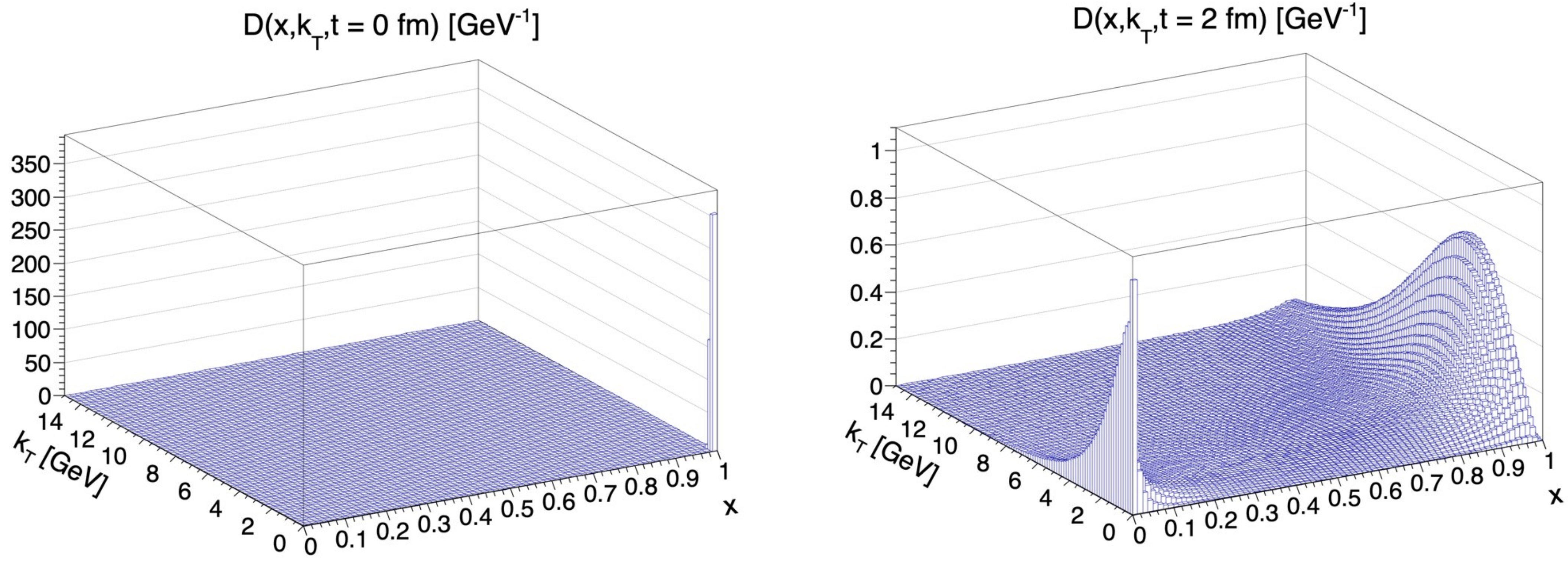
Repeat for all steps in τ and x until $\tau > \tau_L$

TMDICE code: [MR, arxiv: 2111.00323]
- Written in C++
- Source code available at

https://github.com/Rohrmoser/TMDICE

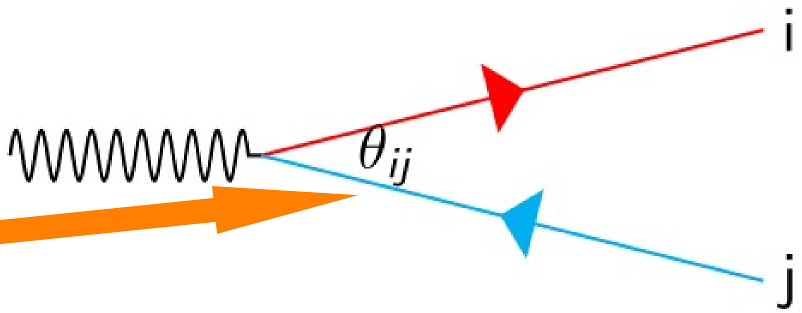
Evolution of $D(x,k_T,t)$

$$D(x,k_T,t) = x\frac{\partial^2 N(t)}{\partial x \partial k_T}$$



[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Color Coherence



Individual colors of partons may not be resolvable by medium particles

Phenomenological estimation of color resolution via Decoherence time:

$$t_{\mathrm{decoh}} \approx \left(\frac{12}{\hat{q}\theta_{ij}^2} \right)^{1/3} , \qquad \hat{q} = \frac{\mathrm{d}\langle k_\perp^2 \rangle}{\mathrm{d}t} .$$

and Branching time: $t_{\mathrm{br}} \approx 2E_i/Q_i^2 .$

$$t_{\mathrm{decoh}} < t_{\mathrm{L}},$$

No color resolution if $t_{\mathrm{br}} < t_{\mathrm{decoh}}$
=> Branching as in Vacuum

Vacuum Like Emissions

DGLAP-Evolution as in Vacuum
Branching probability:

$$P_{qq}(z) = C_F \frac{1+z^2}{1-z},$$
$$P_{gq}(z) = P_{qq}(1-z),$$
$$P_{qg}(z) = T_R\left[z^2 + (1-z)^2\right],$$
$$P_{gg}(z) = C_A\left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right].$$

$$\frac{d^2\mathcal{P}_{ji}}{dQ^2 dz} = \frac{\alpha_s}{2\pi}\frac{1}{Q^2}P_{ji}(z),$$

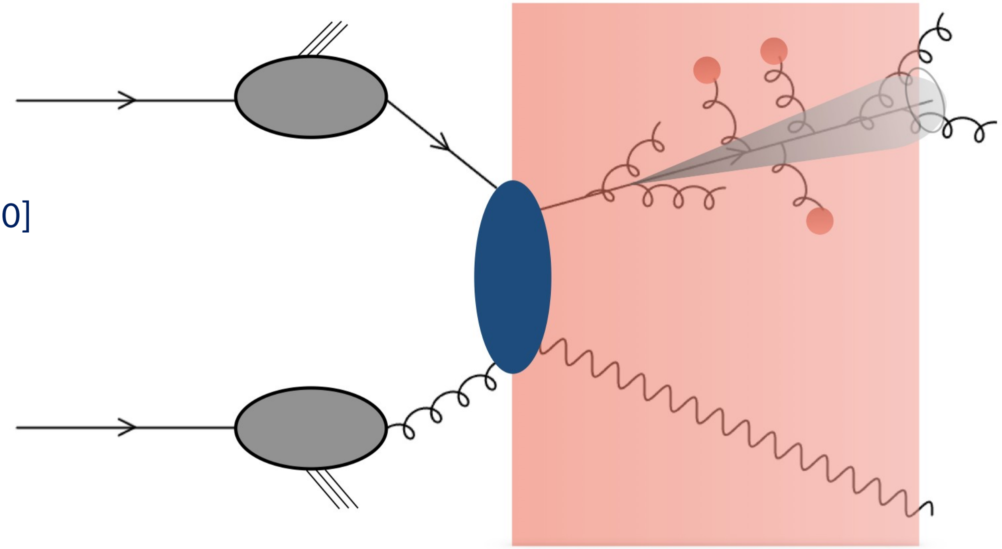
Stop Evolution once $t_{\mathrm{br}} < t_{\mathrm{decoh}}$ or $t_{\mathrm{decoh}} < t_{\mathrm{L}},$

Then: In-Medium Evolution

Jet-photon Production(1/2)

Cross section =
PDF*TMD
*hard cross section } Here via KATIE
*fragmentation of jet

[van Hameren: Comput.Phys.Commun. 224 (2018) 371-380]



Jet-photon Production (2/2)

k_T factorization:

$$\frac{d\sigma^{AA\to\gamma+\text{jet}+X}}{dy_1 dy_2 dp_{1T} dp_{2T} d\Delta\phi} = \frac{p_{1T}p_{2T}}{8\pi^2(x_1x_2s)^2}\sum_a x_1 f_{a/A}(x_1,\mu_F^2)$$
$$\times |\mathcal{M}^{\text{off}-\text{shell}}_{ag^*\to\gamma a}|^2 \mathcal{F}(x_2,k_{2T}^2,\mu_F^2)\,,$$

$\mathcal{F}_g(x,k_T^2,\mu_F^2)$ ...transverse momentum distribution (TMD)

full phase space access at LO
particularly relevant at low x

TMDs used:

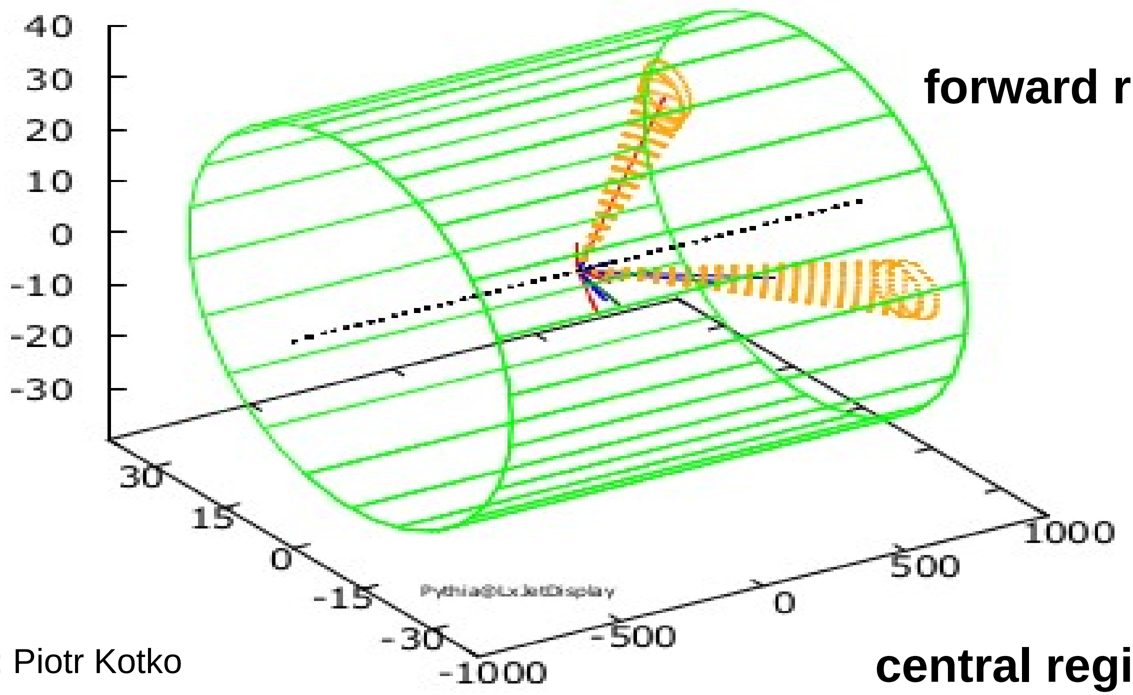
- NCTEQ [K. Kovarik, et al.,Phys. Rev. D 93 (8)(2016) 085037]
- Pb KS [K. Kutak, S.Sapeta: Phys. Rev. D 86 (2012) 094043, M. A. Al-Mashad, A. van Hameren, H. Kakkad, P. Kotko, K. Kutak,P. van Mechelen, S. Sapeta, JHEP 12 (2022) 131]
- p KS [K. Kutak, S.Sapeta: Phys. Rev. D 86 (2012) 094043, M. A. Al-Mashad, A. van Hameren, H. Kakkad, P. Kotko, K. Kutak,P. van Mechelen, S. Sapeta, JHEP 12 (2022) 131]

cf. [I. Ganguli, A. van Hameren, P. Kotko, K. Kutak, Eur.Phys.J.C 83 (2023) 9, 868]

and for dijets:

[A. van Hameren, H. Kakkad, P. Kotko, K. Kutak, S. Sapeta,Eur.Phys.J.C 83 (2023) 10, 947]

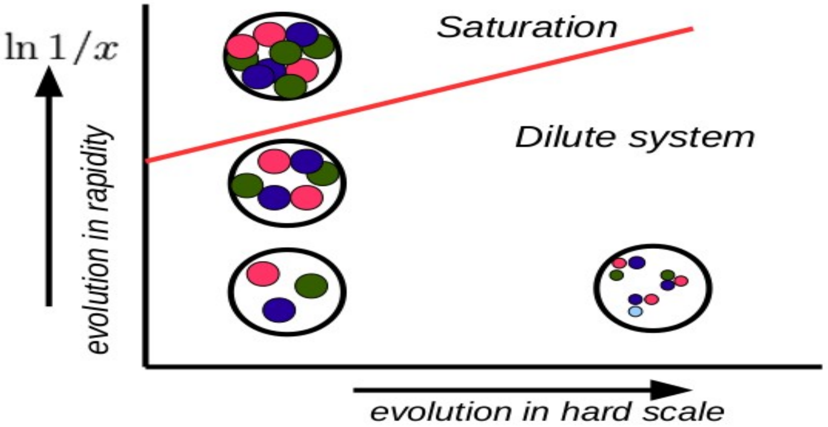
p – A (dilute-dense) forward-forward di-jets



forward region

central region

From: Piotr Kotko
LxJet



It originated from the aim to provide predictions for forward-forward jet production at the LHC

The saturation problem: supressing gluons below Q_s

Originally formulated in coordinate space
Balitsky '96, Kovchegov '99

Fit AAMQS '10

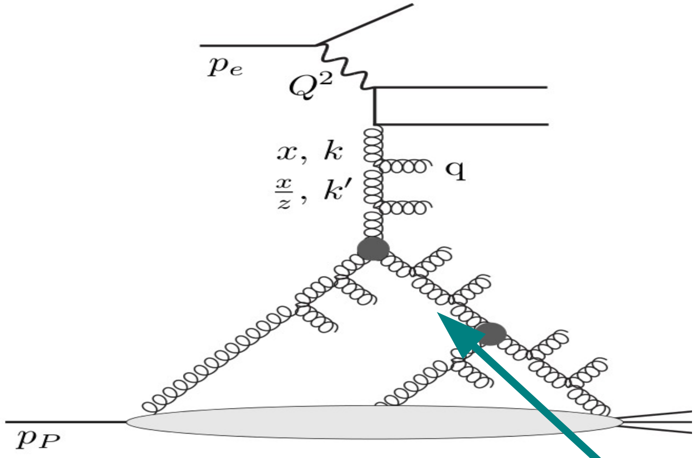
NLO accuracy
Balitsky, Chirilli '07

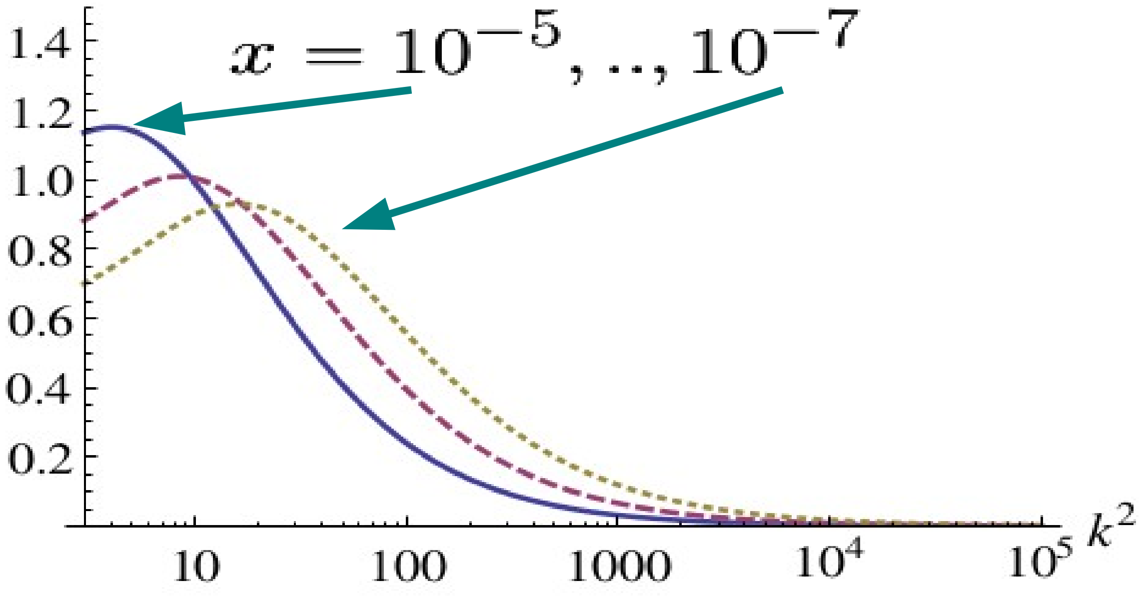
and solved
Lappi, Mantysaari '15

Kinematic corrections
Iancu at el

Solved b dependent
Stasto, Golec-Biernat '02

with kinematic
corrections and b
Cepila, Contrares, Matas '18





solution of Balitsky-Kovchegov directly for dipole gluon density

Kwiecinski, Kutak '02
Nikolaev, Schafer '06

Fit to F_2 data
KK. Sapeta '12

The momentum space BK equation for dipole gluon density

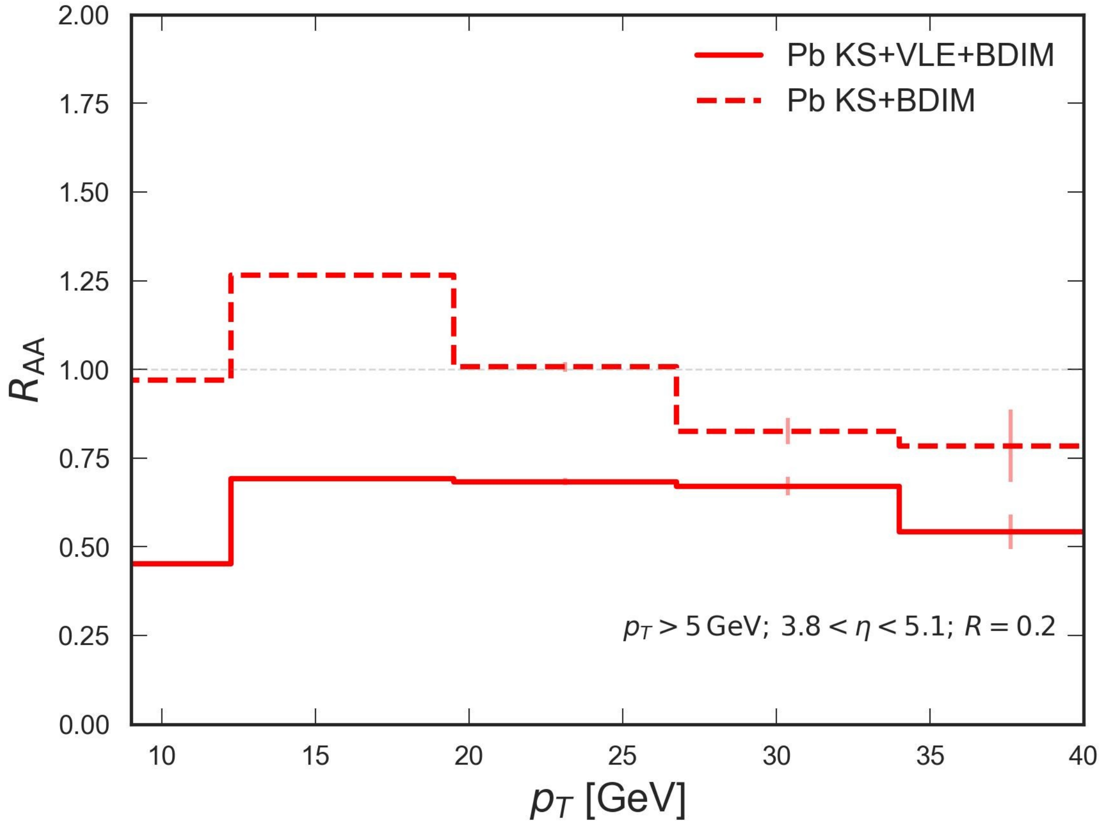
$$\mathcal{F} = \mathcal{F}_0 + K \otimes \mathcal{F} - \frac{1}{R^2} V \otimes \mathcal{F}^2$$

hadron's radius

Accounts for kinematical constraint,

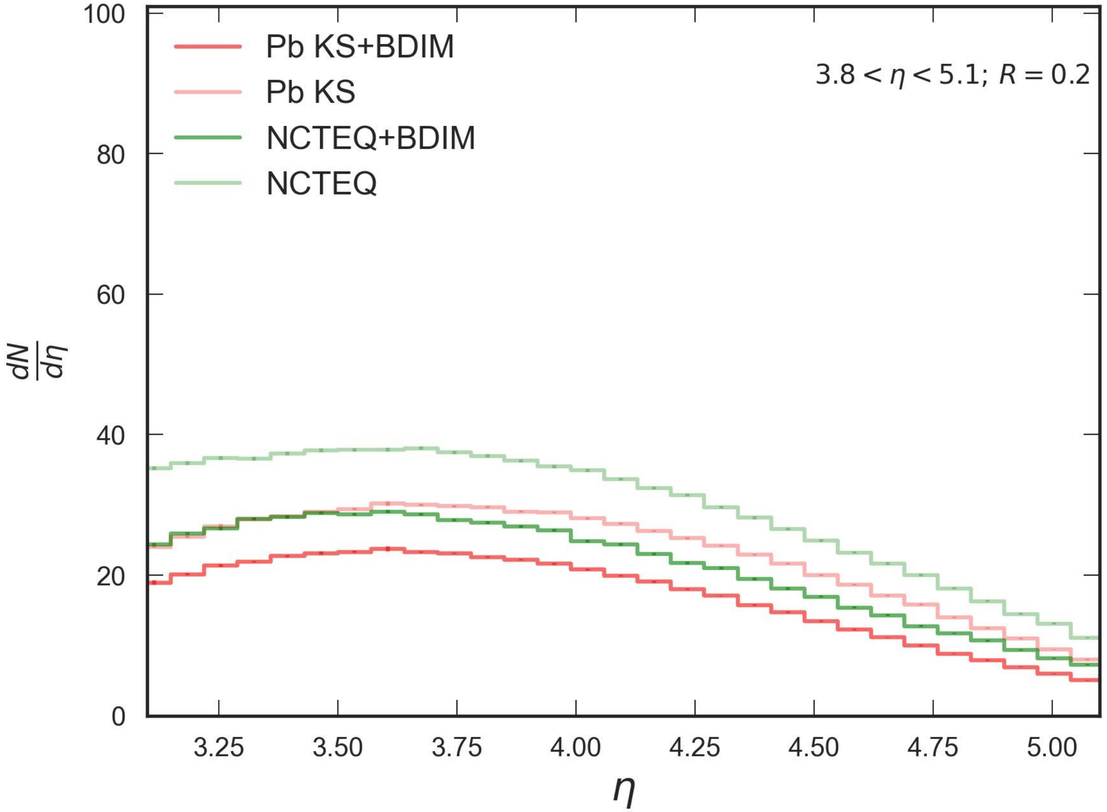
Nonsingular parts of splitting function,
running coupling

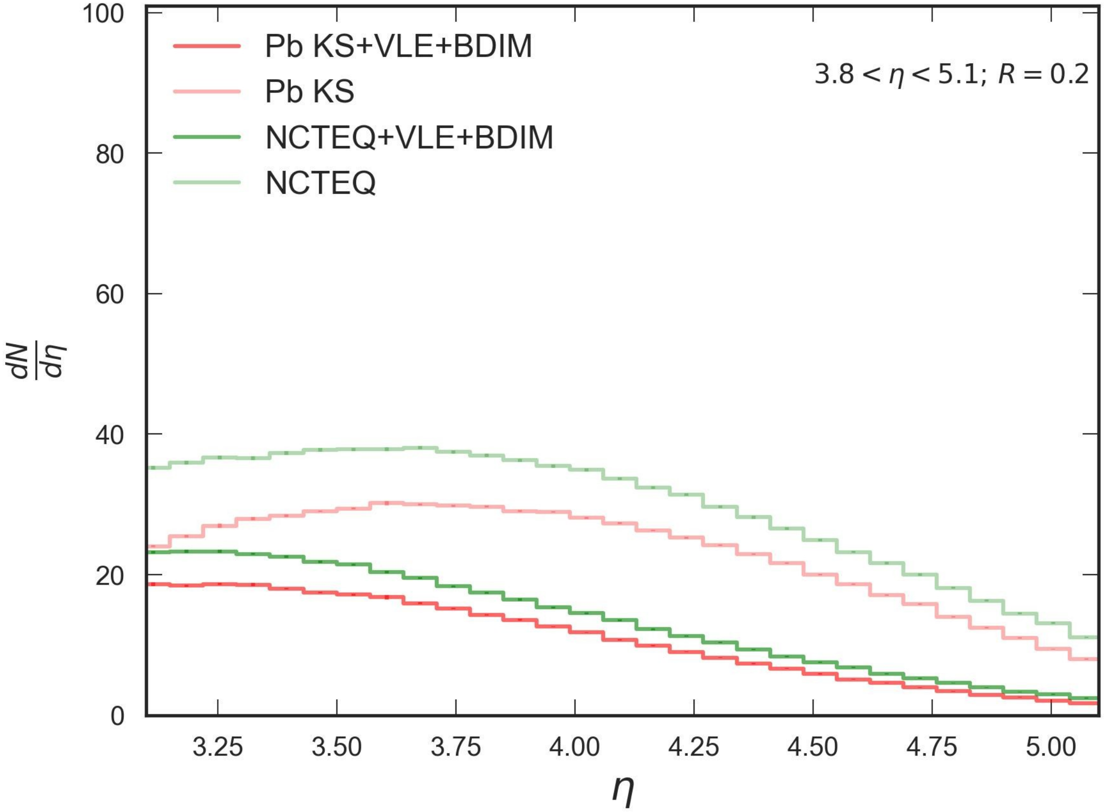
Photon-jet production



[Adhya, Kutak,Płaczek, MR, Tywoniuk: 2409.06675]

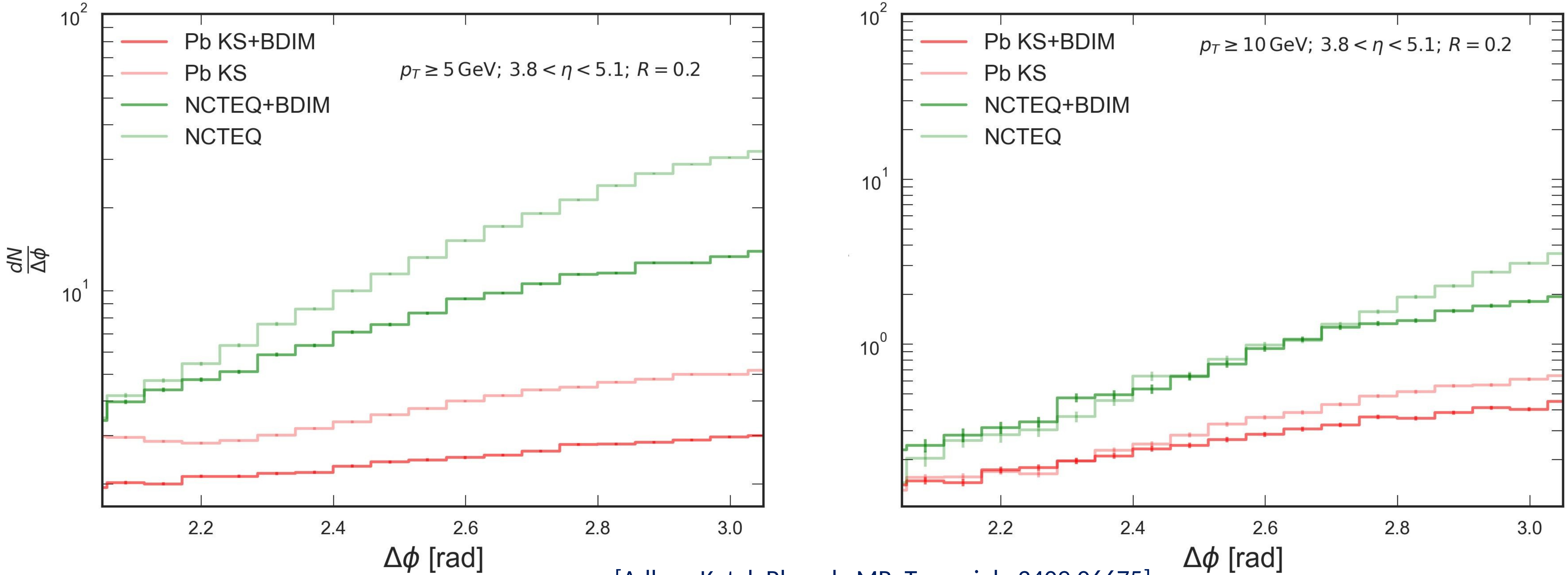
Rapidity spectra





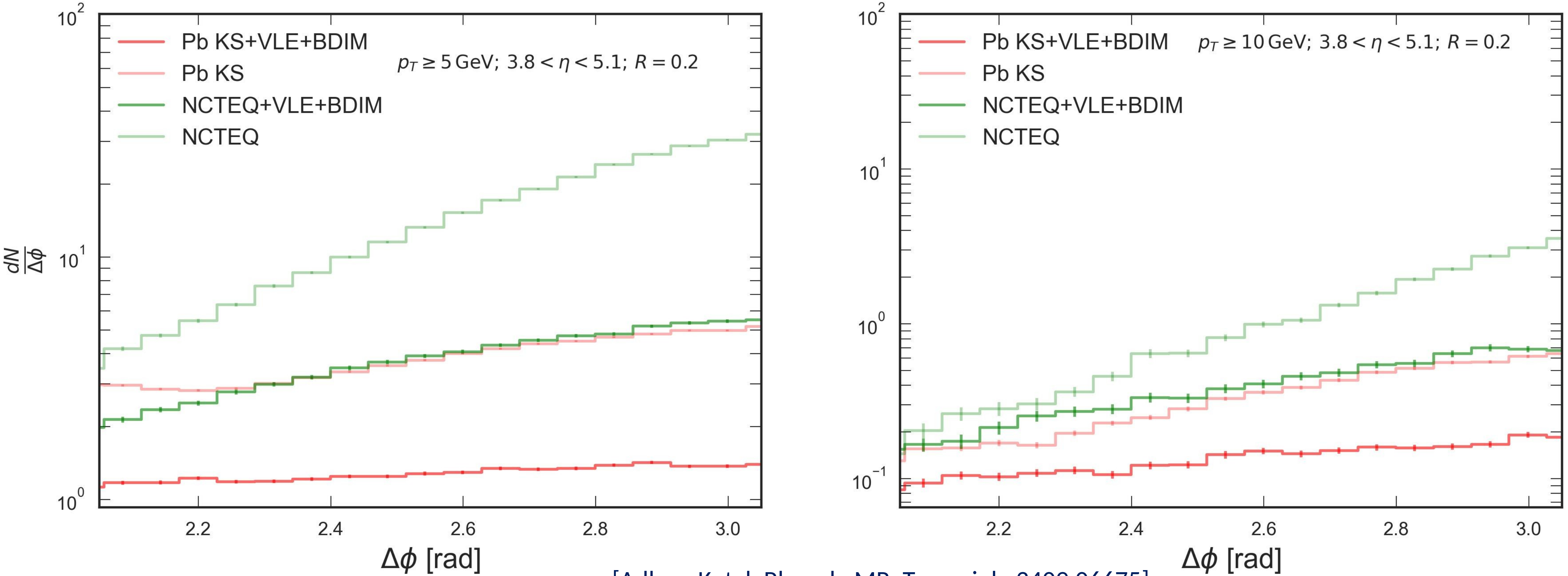
[Adhya, Kutak,Płaczek, MR, Tywoniuk: 2409.06675]

Azimuthal decorrelations (1/2)



[Adhya, Kutak,Płaczek, MR, Tywoniuk: 2409.06675]

Azimuthal decorrelations (2/2)



[Adhya, Kutak,Płaczek, MR, Tywoniuk: 2409.06675]

Summary & Outlook:

- Description of photon-jet events in forward direction (FOCAL-range) via Monte-Carlo algorithms (saturation + quenching)
- Inclusion of VLE in quenching
- Comparison with and without saturation and quenching: Saturation effects survive.
- Quenching: k_T Broadening and jet suppression.
- Strong suppression due to VLE and strong broadening effects

Outlook:

- More realistic Media (e.g.: expanding media; Temperature profiles)
- Study dijets.

Thank you for your attention!

Back-up slides

Jet Production (3/3)

Factorization for AA collisions:

$$\frac{d\sigma_{AA}}{d\Omega_p} = \int d\Omega_q \int d^2\boldsymbol{l} \int_0^1 \frac{d\tilde{x}}{\tilde{x}} \delta(p^+ - \tilde{x}q^+)\delta^{(2)}(\boldsymbol{p} - \boldsymbol{l} - \boldsymbol{q}) D(\tilde{x}, \boldsymbol{l}, \tau(q^+)) \frac{d\sigma_{pp}}{d\Omega_q}$$

$$\Big\downarrow$$

$$d\Omega_q = dq^+ d^2\boldsymbol{q} \qquad \tau(q^+) = \frac{\alpha_s N_c}{\pi}\sqrt{\frac{\hat{q}}{q^+}}L$$

$$\frac{d^2\sigma_{AA}}{d\Omega_{p_1} d\Omega_{p_2}} = \int d\Omega_{q_1} \int d\Omega_{q_2} \int d^2\boldsymbol{l}_1 \int d^2\boldsymbol{l}_2 \int_0^1 \frac{d\tilde{x}_1}{\tilde{x}_1} \delta(p_1^+ - \tilde{x}_1 q_1^+) \int_0^1 \frac{d\tilde{x}_2}{\tilde{x}_2} \delta(p_2^+ - \tilde{x}_2 q_2^+)$$

$$\delta^{(2)}(\boldsymbol{p}_1 - \boldsymbol{l}_1 - \boldsymbol{q}_1)\delta^{(2)}(\boldsymbol{p}_2 - \boldsymbol{l}_2 - \boldsymbol{q}_2) D(\tilde{x}_1, \boldsymbol{l}_1, \tau(q_1^+)) D(\tilde{x}_2, \boldsymbol{l}_2, \tau(q_2^+)) \frac{d^2\sigma_{pp}}{d\Omega_{q_1} d\Omega_{q_2}}$$

Rohrmoser kT broadening: scattering in jets 22

Algorithm: KATIE+MINCAS

[v. Hameren, Kutak, Płaczek, MR, Tywoniuk, Phys. Rev. C 102, 044910]

- Use KATIE for hard initial collisions:
    - PDFs/TMDs for colliding nucleons
    - Hard collision cross-section (Monte-Carlo simulation)
    - Resulting particles→initial particles of jets

[van Hameren: Comput.Phys.Commun. 224 (2018) 371-380]

- Jets: by MINCAS
    - Monte-Carlo simulation of BDIM equation
    - Time-evolution of jets in medium

[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Vacuum like emissions

$$\frac{\mathrm{d}^2\mathcal{P}_{ji}}{\mathrm{d}Q^2\mathrm{d}z} = \frac{\alpha_s}{2\pi}\frac{1}{Q^2}P_{ji}(z)\,,$$

$$\begin{aligned}
P_{qq}(z) &= C_F\,\frac{1+z^2}{1-z}\,,\\
P_{gq}(z) &= P_{qq}(1-z)\,,\\
P_{qg}(z) &= T_R\left[z^2+(1-z)^2\right]\,,\\
P_{gg}(z) &= C_A\left[\frac{1-z}{z}+\frac{z}{1-z}+z(1-z)\right]\,.
\end{aligned}$$

Departure from Gaussian broadening

always same distribution for changes $p \rightarrow p + \boldsymbol{q}$
→central limit theorem

$$\frac{\partial}{\partial t}D(x,\boldsymbol{k},t) = \int \frac{d^2\boldsymbol{q}}{(2\pi)^2}\boxed{C(\boldsymbol{q})D(x,\boldsymbol{k}-\boldsymbol{q},t)}$$

$$+\frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)$$

$$\left[\boxed{\frac{1}{z^2}\sqrt{\frac{z}{x}}D\left(\frac{x}{z},\frac{\boldsymbol{k}}{z},t\right)\theta(z-x)} - \boxed{\frac{z}{\sqrt{x}}D(x,\boldsymbol{k},t)}\right]$$

Splitting à la $p \rightarrow zp$
→ perturbations of different sizes
→non Gaussian behavior

Virtual emissions

For example:

$$p \rightarrow z_1 p \rightarrow z_1 p + \boldsymbol{q}_1$$

$$\rightarrow z_1 p + \boldsymbol{q}_1 + \boldsymbol{q}_2$$

$$\rightarrow z_2(z_1 + \boldsymbol{q}_1 + \boldsymbol{q}_2) \rightarrow \dots$$

$\mathcal{K}(z)$ $\quad w(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$

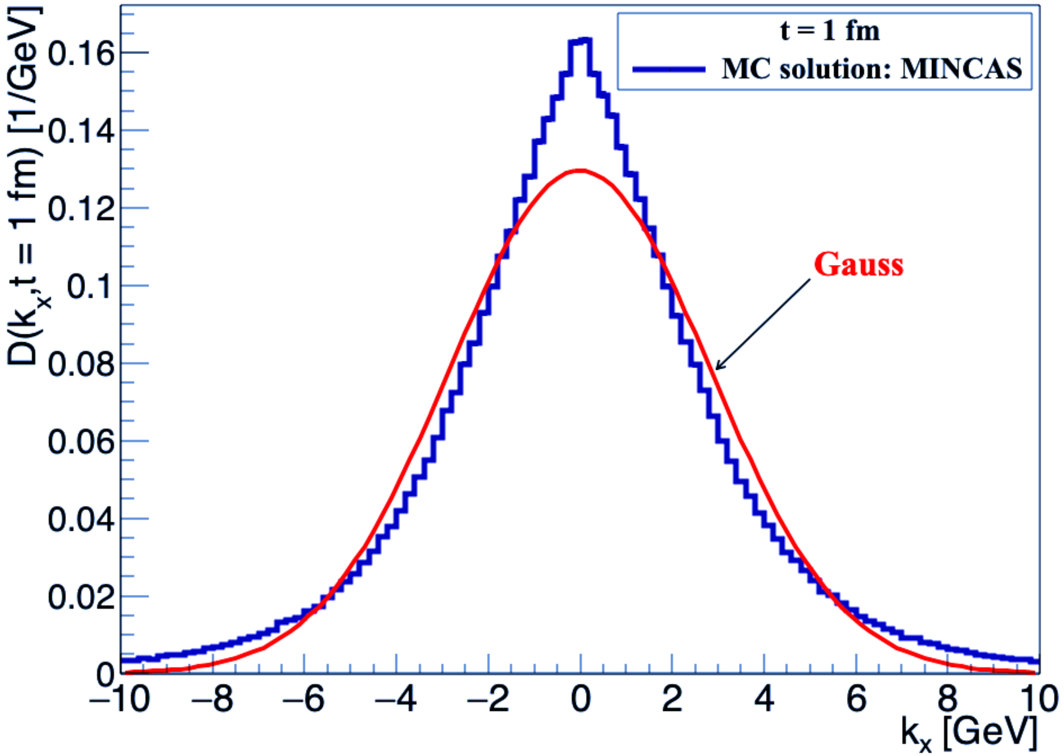


Figure: [Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Link to Evolution equations

[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Collinear evolution: $\mathcal{K}(z)$, $w(\mathbf{q}) = 0$

$$D(x,\tau) = x\frac{dN}{dx}$$

$$D(x,\tau) = e^{-\phi(x)(\tau-\tau_0)}D(x,\tau_0) + \int_{\tau_0}^{\tau} d\tau' \int_{\epsilon}^{1-\epsilon} dz \int_0^1 dy\delta(x-zy)\sqrt{\frac{z}{x}}z\mathcal{K}(z)e^{-\phi(x)(\tau-\tau')}D(y,\tau')$$

$$\frac{\partial}{\partial t}D(x,t) = \frac{1}{t*}\int_0^1 dz\mathcal{K}(z)\left[\sqrt{\frac{z}{x}}D\left(\frac{x}{z},t\right) - \frac{z}{\sqrt{x}}D(x,t)\right]$$

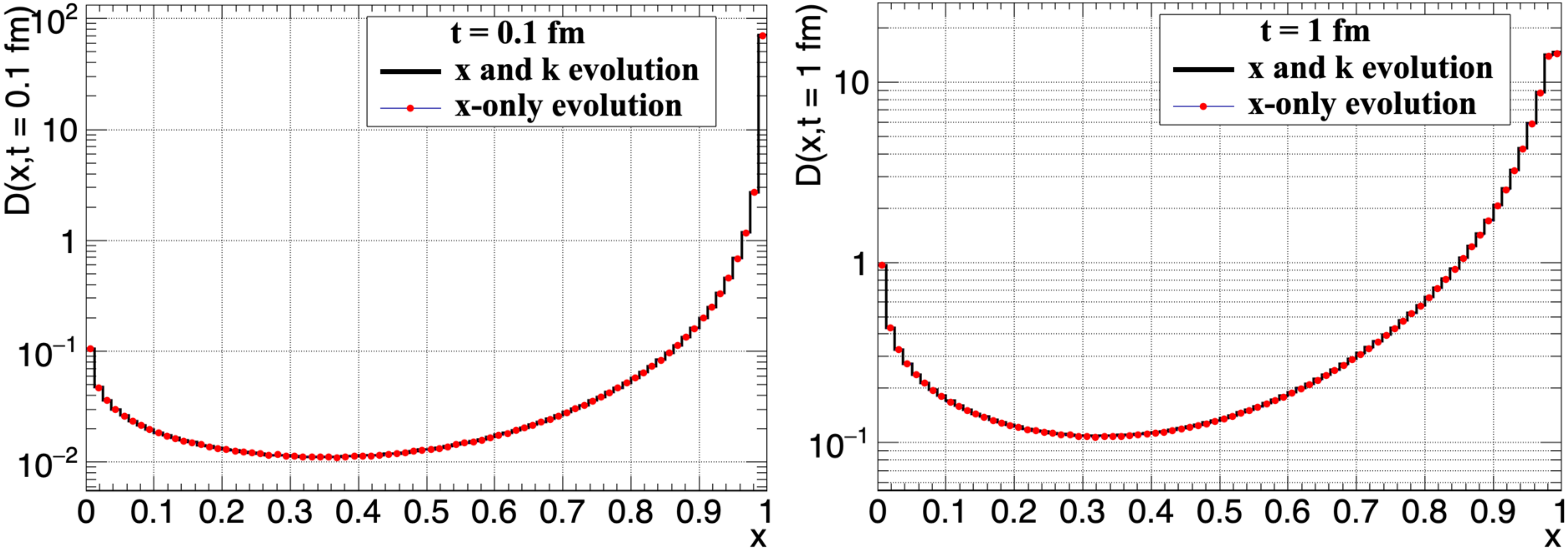
$$\tau = \frac{t}{t*}$$

Monte-Carlo algorithm that solves these evolution equations:

MINCAS
[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

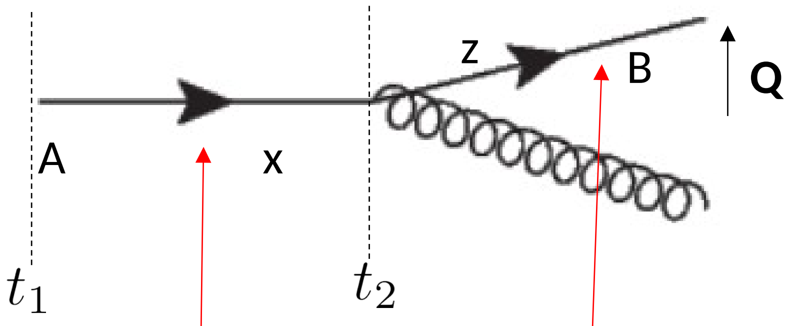
Exist direct methods: Chebyshev method, Runge Kutta... [Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

Turbulent behavior (1/2)



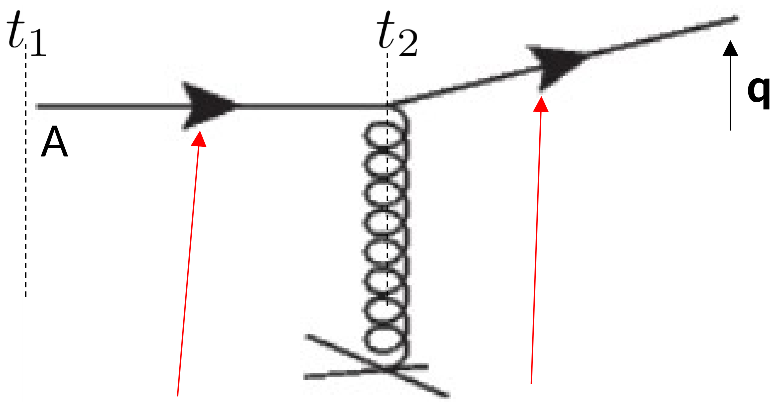
[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Probabilities for interactions



$$\Delta_A(x,t_2-t_1)\left(\frac{\alpha_s}{(2\pi)^2}\mathcal{K}_{BA}(\mathbf{Q},z,xp_+)\right)$$

$$(\Delta_A(x,t_2-t_1)\phi_A(x))\underbrace{\left(\frac{\sum_B\rho_{BA}(x)}{\phi_A(x)}\right)}_{\text{Splitting probability}}\underbrace{\left(\frac{\rho_{BA}(x)}{\sum_B\rho_{BA}(x)}\right)}_{\text{Probability for splitting into B}}\left(\frac{1}{\rho_{BA}(x)}\frac{\alpha_s}{(2\pi)^2}\mathcal{K}_{BA}(\mathbf{Q},z,xp_+)\right)$$



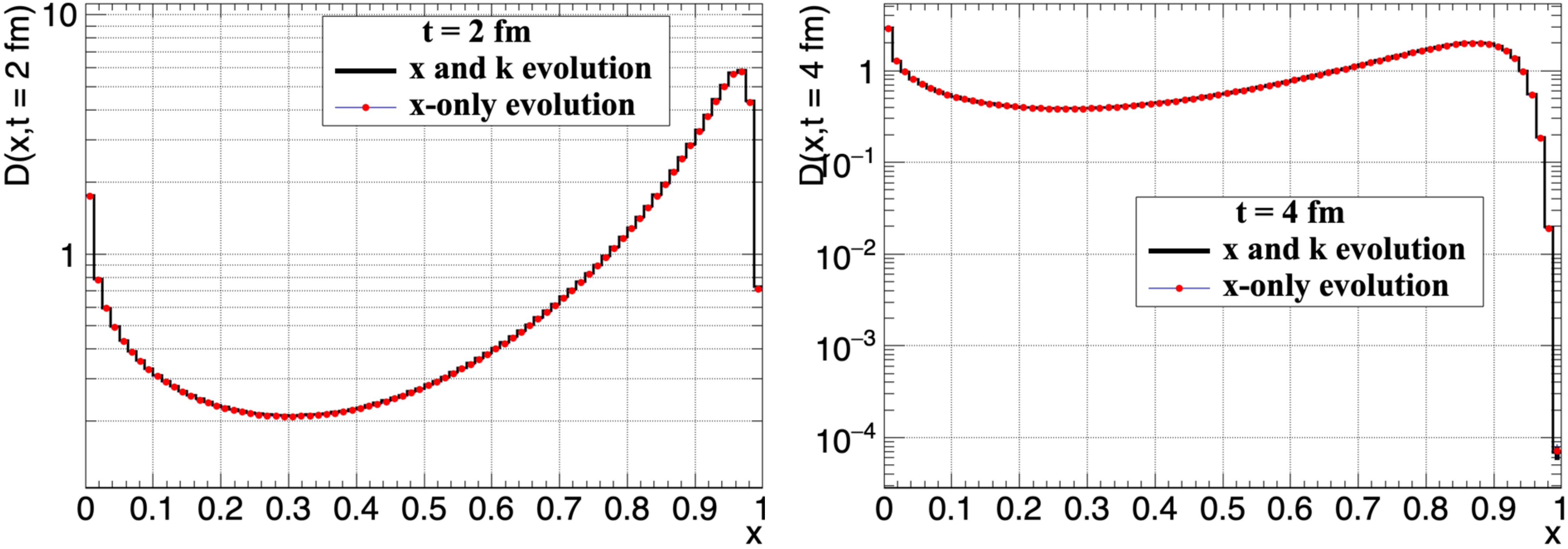
$$\Delta_A(x,t_2-t_1)\left(\frac{1}{(2\pi)^2}w_A(\mathbf{q})\right)$$

$$(\Delta_A(x,t_2-t_1)\phi_A(x))\underbrace{\left(\frac{W_A}{\phi_A(x)}\right)}_{\text{Scattering probability}}\left(\frac{1}{W_A}\frac{1}{(2\pi)^2}w_A(\mathbf{q})\right)$$

$$\rho_{BA}(x)=\alpha_s\int_\epsilon^{1-\epsilon}dz\int_{q>0}\frac{d^2q}{(2\pi)^2}\mathcal{K}_{BA}(\mathbf{q},z,xp_+)$$

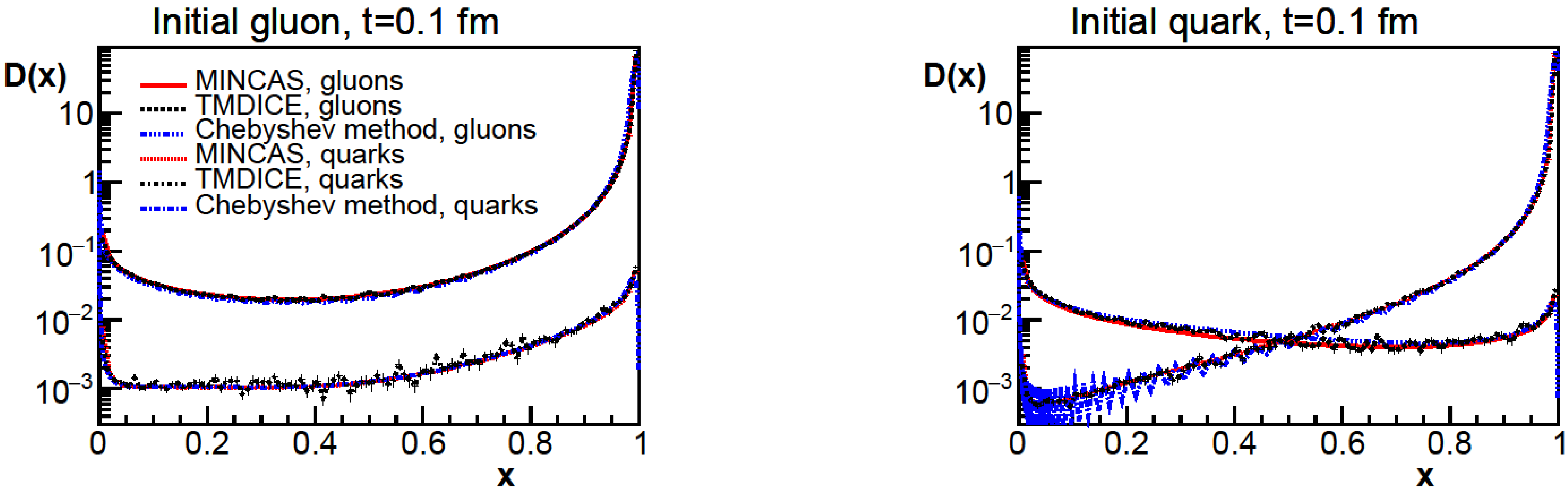
$$W_A=\int\frac{d^2\mathbf{q}}{(2\pi)^2}w_A(\mathbf{q})$$

Turbulent behavior (2/2)



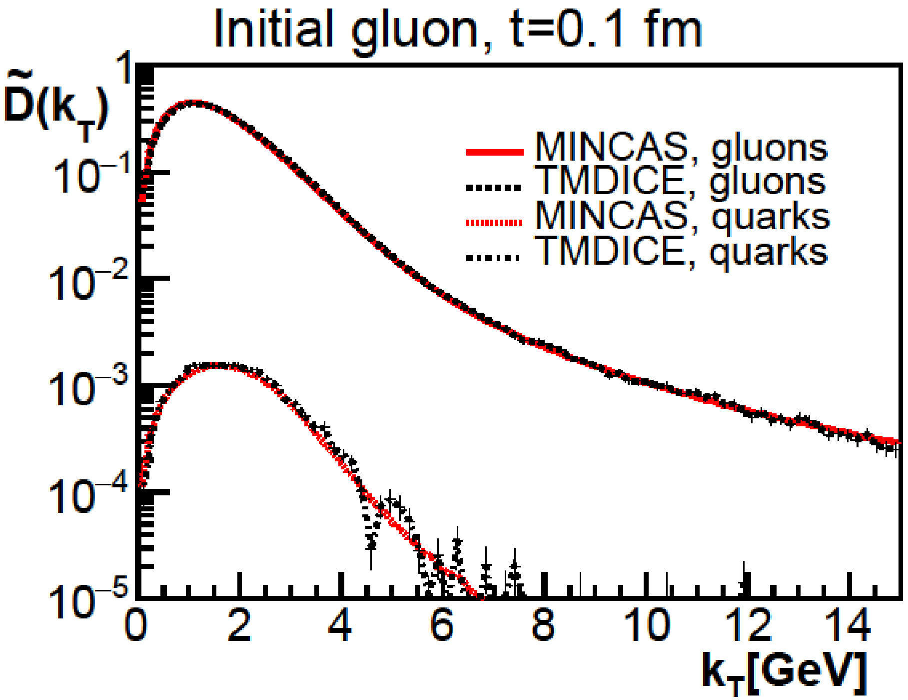
[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

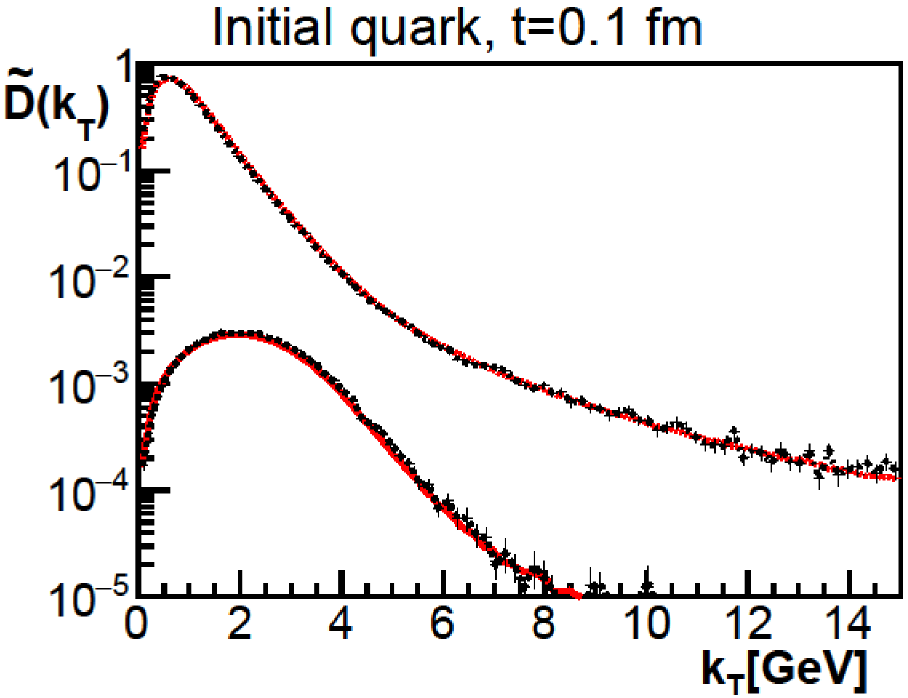
Fragmentation functions (1/2)



[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

Fragmentation functions (2/2)





[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

Different models

- Broadening in branching: $\mathcal{K}_{ij}(\mathbf{Q}, z, xp_0^+)$
  - No scattering
  - Scattering: $w_g(\mathbf{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\mathbf{q}^4}$
  - Scattering: $w_g(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$
- No broadening in branching: $\mathcal{K}_{ij}(z)$
  - Scattering: $w_g(\mathbf{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\mathbf{q}^4}$
  - Scattering: $w_g(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$
- Gaussian broadening:

  x given by collinear evolution without scattering via $\mathcal{K}_{ij}(z)$

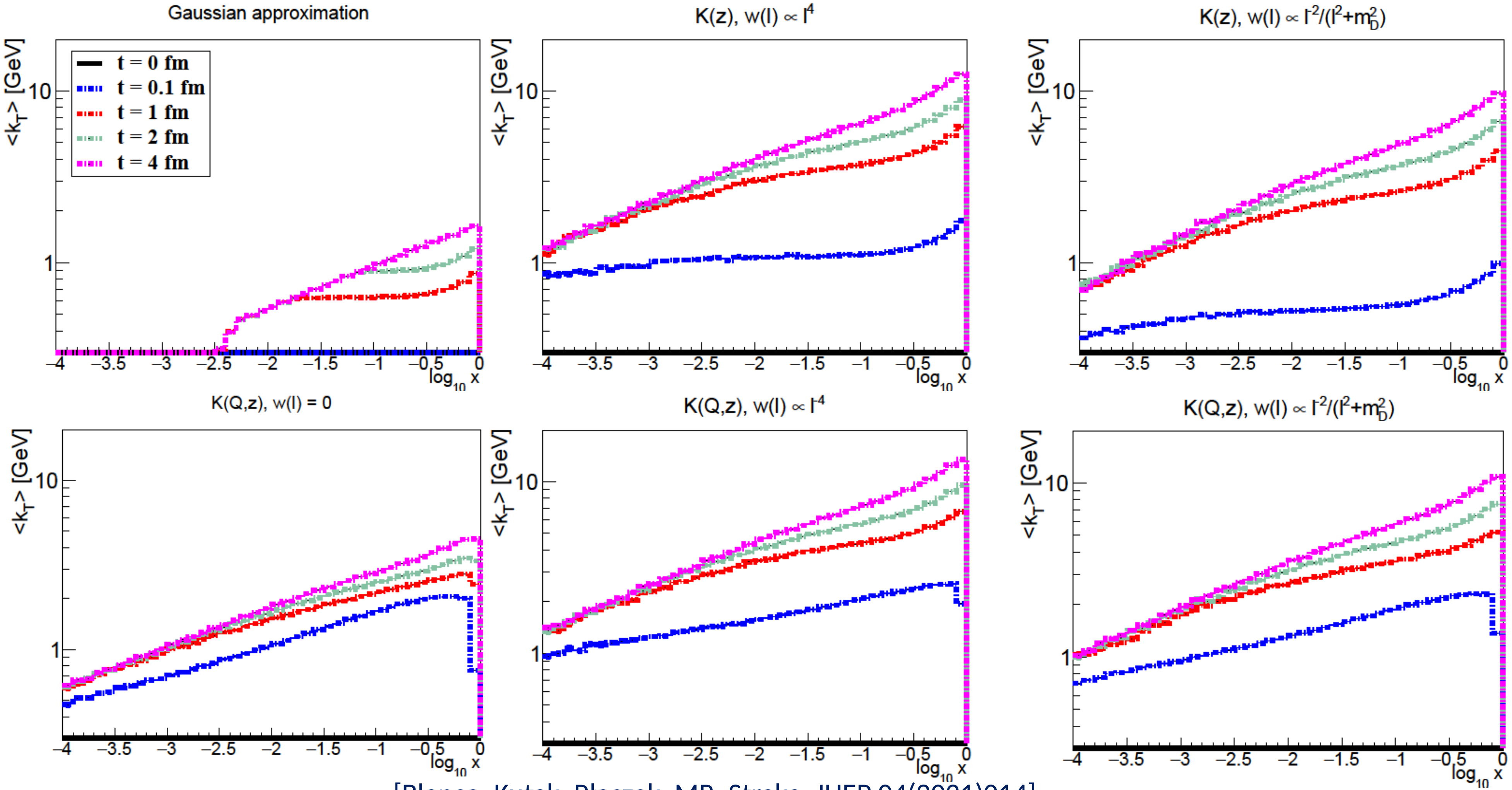
  $\boldsymbol{k}$ given by Gaussian distribution with variance $\sigma^2 \sim \hat{q}L$

All models yield the same k_T averaged splitting kernel $\mathcal{K}_{ij}(z)$!

Constant medium parameters:
L, $\hat{q}$, n, m_D

k_T Broadening

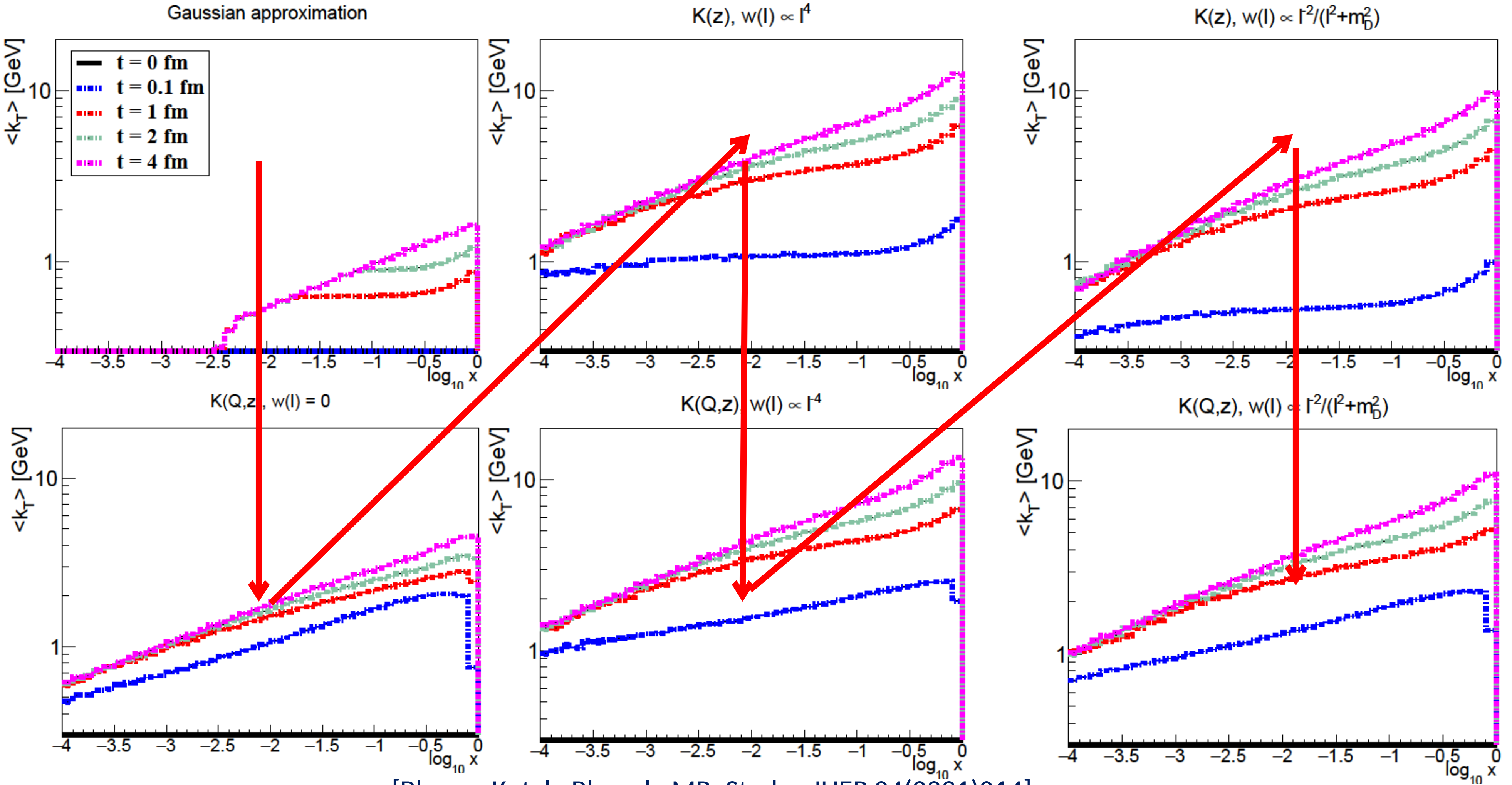
$$\tilde{D}(x, k_T, t) = 2\pi k_T D(x, k_T, t)$$



[Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

k_T Broadening

$$\tilde{D}(x, k_T, t) = 2\pi k_T D(x, k_T, t)$$



[Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

System of Equations for quarks and gluons

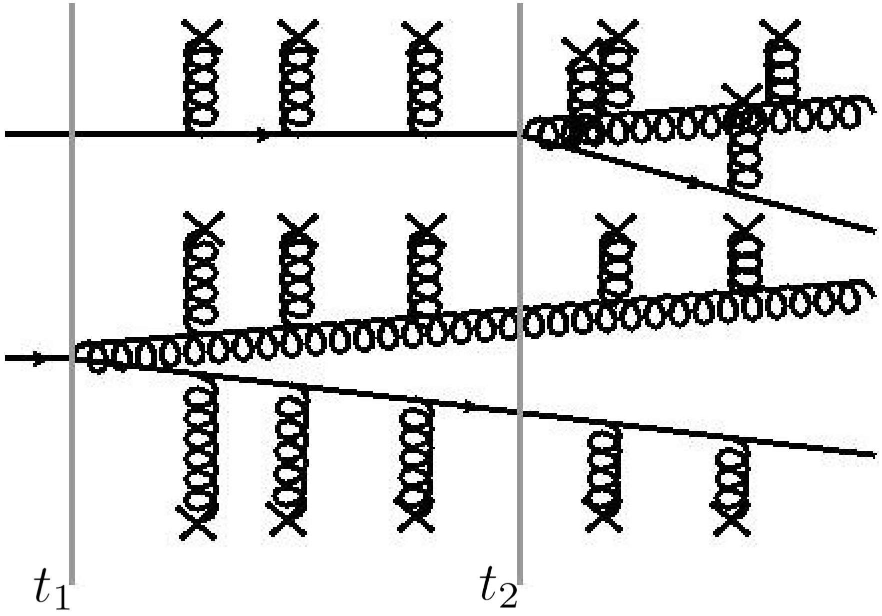
[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

$$
\begin{aligned}
\frac{\partial}{\partial t} D_g(x,\boldsymbol{k},t) = {} & \int_0^1 \mathrm{d}z \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}\alpha_s \Big\{ 2\mathcal{K}_{gg}\left(\boldsymbol{Q}, z, \frac{x}{z}p_0^+\right) D_g\left(\frac{x}{z},\boldsymbol{q},t\right) + \mathcal{K}_{gq}\left(\boldsymbol{Q},z,\frac{x}{z}p_0^+\right)\sum_i D_{q_i}\left(\frac{x}{z},\boldsymbol{q},t\right) \\
& - \left[\mathcal{K}_{gg}(\boldsymbol{q},z,xp_0^+) + \mathcal{K}_{qg}(\boldsymbol{q},z,xp_0^+)\right] D_g(x,\boldsymbol{k},t)\Big\} + \int \frac{\mathrm{d}^2\boldsymbol{l}}{(2\pi)^2}\, C_g(\boldsymbol{l})\, D_g(x,\boldsymbol{k}-\boldsymbol{l},t), \\
\frac{\partial}{\partial t} D_{q_i}(x,\boldsymbol{k},t) = {} & \int_0^1 \mathrm{d}z \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}\alpha_s \Big\{ \mathcal{K}_{qq}\left(\boldsymbol{Q}, z, \frac{x}{z}p_0^+\right) D_{q_i}\left(\frac{x}{z},\boldsymbol{q},t\right) + \frac{1}{N_F}\mathcal{K}_{qg}\left(\boldsymbol{Q},z,\frac{x}{z}p_0^+\right) D_g\left(\frac{x}{z},\boldsymbol{q},t\right) \\
& - \mathcal{K}_{qq}(\boldsymbol{q},z,xp_0^+)\, D_{q_i}(x,\boldsymbol{k},t)\Big\} + \int \frac{\mathrm{d}^2\boldsymbol{l}}{(2\pi)^2}\, C_q(\boldsymbol{l})\, D_{q_i}(x,\boldsymbol{k}-\boldsymbol{l},t)\,,
\end{aligned}
$$

$$
C_{q(g)}(\boldsymbol{l}) = w_{q(g)}(\boldsymbol{l}) - \delta(\boldsymbol{l}) \int \mathrm{d}^2\boldsymbol{l}'\, w_{q(g)}(\boldsymbol{l}')
$$

Effective Splitting Kernels

$$\mathcal{K}_{ij}(\mathbf{Q}, z, p_0^+) \sim P_{ij}(z) \times \mathcal{I}_{ij}$$



- Assumptions:
  Transverse momentum transfer only,
  harmonic oscillator approximation,
  static medium,
  static scattering centers.

$$\mathcal{I}_{ij}(\boldsymbol{u}_2, t_2; \boldsymbol{u}_1, t_1) = \int_{\boldsymbol{u}(t_1)=\boldsymbol{u}_1}^{\boldsymbol{u}(t_2)=\boldsymbol{u}_2} \mathcal{D}\boldsymbol{u}\, \mathrm{e}^{i\frac{\omega_0}{2}\int_{t_1}^{t_2} \mathrm{d}s\, \dot{\boldsymbol{u}}^2(s) - \int_{t_1}^{t_2} \mathrm{d}s\, n(s)\sigma_{\mathrm{eff}}(\boldsymbol{u}(s),\boldsymbol{v})}$$

$$\boldsymbol{u} = \boldsymbol{r}_i - \boldsymbol{r}_k$$

$$\boldsymbol{v} = z\boldsymbol{r}_i + (1-z)\boldsymbol{r}_k - \boldsymbol{r}_j$$

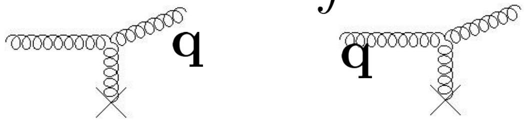
$$\sigma_{\mathrm{eff}}(\boldsymbol{u}, \boldsymbol{v}) = \frac{C_i + C_k - C_j}{2}\,\sigma(\boldsymbol{u}) + \frac{C_i + C_j - C_k}{2}\,\sigma(\boldsymbol{v} + (1-z)\boldsymbol{u}) + \frac{C_k + C_j - C_i}{2}\,\sigma(\boldsymbol{v} - z\boldsymbol{u})$$

BDIM Equation for Gluons

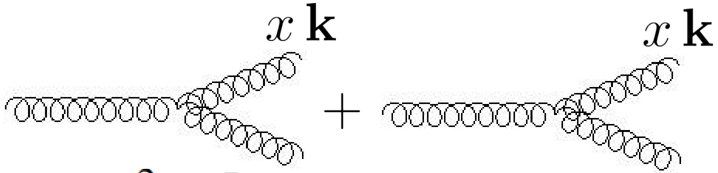
[Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1406 (2014) 075]

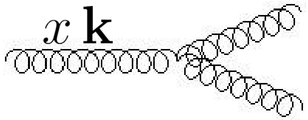
Scattering:

$$C(\boldsymbol{q}) = w(\boldsymbol{q}) - \delta(\boldsymbol{q}) \int d^2\boldsymbol{q}' w(\boldsymbol{q}')$$



$$D(x, \mathbf{k}, t) = x \frac{\partial^3 N(x, \mathbf{k}, t)}{\partial x \partial^2 \mathbf{k}}$$





For gluon-jets:

$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \alpha_s \int_0^1 dz \int \frac{d^2 q}{(2\pi)^2} \left[2\mathcal{K}(\mathbf{Q}, z, \frac{x}{z} p_0^+) D\left(\frac{x}{z}, \mathbf{q}, t\right) - \mathcal{K}(\mathbf{q}, z, x p_0^+) D(x, \mathbf{k}, t) \right] + \int \frac{d^2 \mathbf{l}}{(2\pi)^2} C(\mathbf{l}) D(x, \mathbf{k} - \mathbf{l}, t).$$

Average Kernels over $\mathbf{Q}$

$$\frac{\partial}{\partial t} D(x, \boldsymbol{k}, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\boldsymbol{k}}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, \boldsymbol{k}, t) \right] + \int \frac{d^2 \boldsymbol{q}}{(2\pi)^2} C(\boldsymbol{q}) D(x, \boldsymbol{k} - \boldsymbol{q}, t)$$

Integrate over $\mathbf{k}$

$$\frac{\partial}{\partial t} D(x, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t\right) - \frac{z}{\sqrt{x}} D(x, t) \right]$$

$$D(x, t) = \int d^2\mathbf{k} D(x, \mathbf{k}, t)$$

Departure from Gaussian broadening

always same distribution for changes $p \rightarrow p + \boldsymbol{q}$
→central limit theorem

$$\frac{\partial}{\partial t}D(x,\boldsymbol{k},t)=\int\frac{d^2\boldsymbol{q}}{(2\pi)^2}C(\boldsymbol{q})D(x,\boldsymbol{k}-\boldsymbol{q},t)$$
$$+\frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)$$
$$\left[\frac{1}{z^2}\sqrt{\frac{z}{x}}D\left(\frac{x}{z},\frac{\boldsymbol{k}}{z},t\right)\theta(z-x)-\frac{z}{\sqrt{x}}D(x,\boldsymbol{k},t)\right]$$

Splitting à la $p \rightarrow zp$
→ perturbations of different sizes
→non Gaussian behavior

Virtual emissions

For example:

$$p \rightarrow z_1 p \rightarrow z_1 p + \boldsymbol{q}_1$$
$$\rightarrow z_1 p + \boldsymbol{q}_1 + \boldsymbol{q}_2$$
$$\rightarrow z_2(z_1 + \boldsymbol{q}_1 + \boldsymbol{q}_2) \rightarrow \ldots$$

$\mathcal{K}(z)$ $\quad w(\boldsymbol{q}) = \dfrac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2+m_D^2)}$

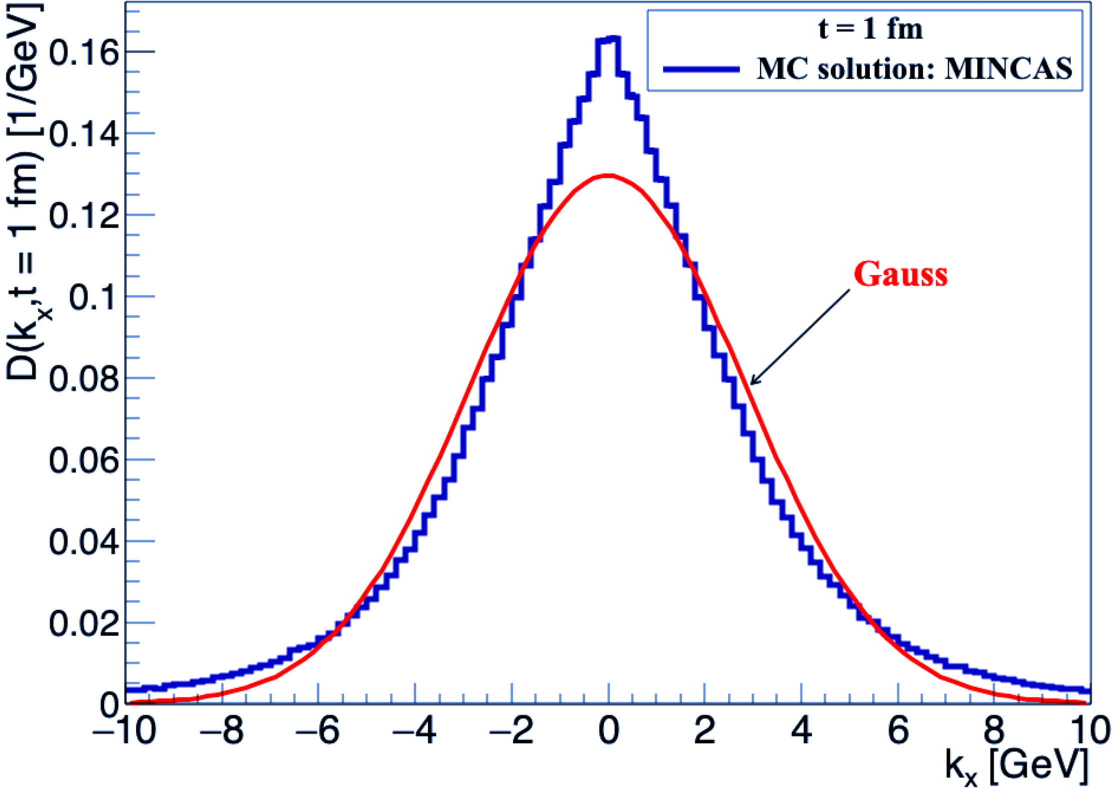
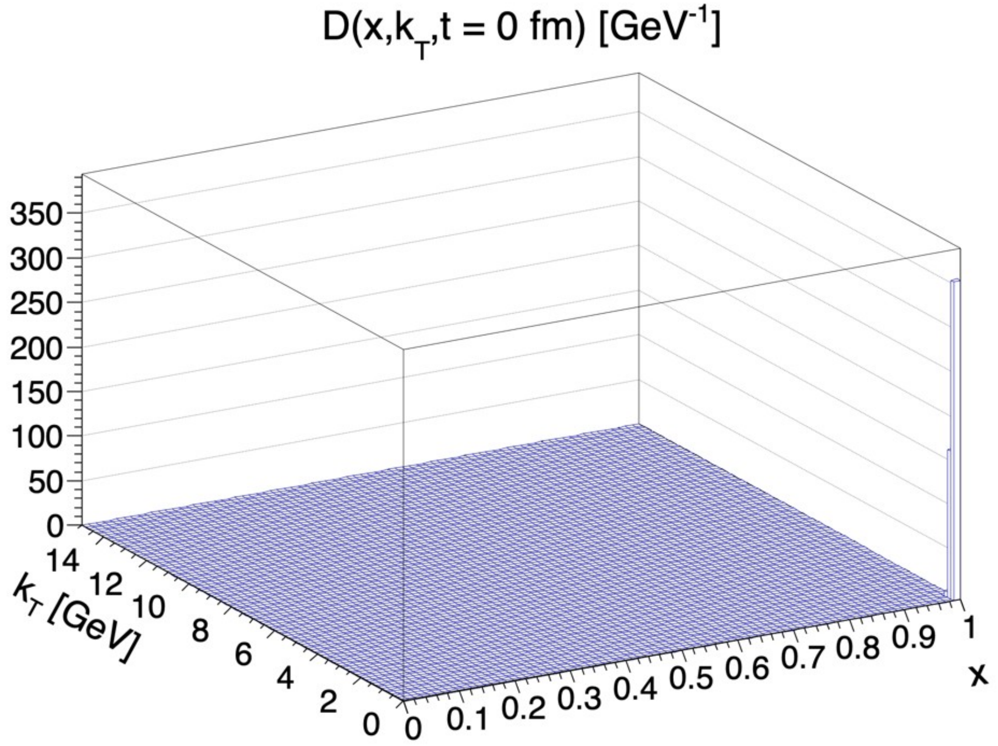
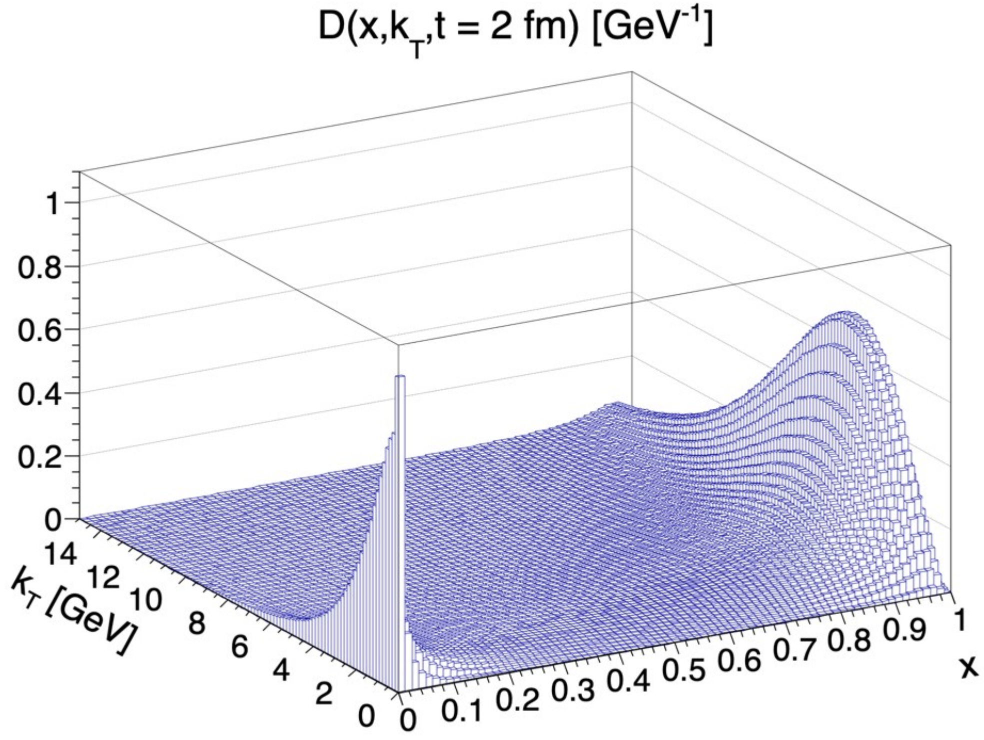


Figure: [Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Evolution of D(x,k_T,t) (1/2)

$\mathcal{K}(z)$ $\quad$ $w(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$

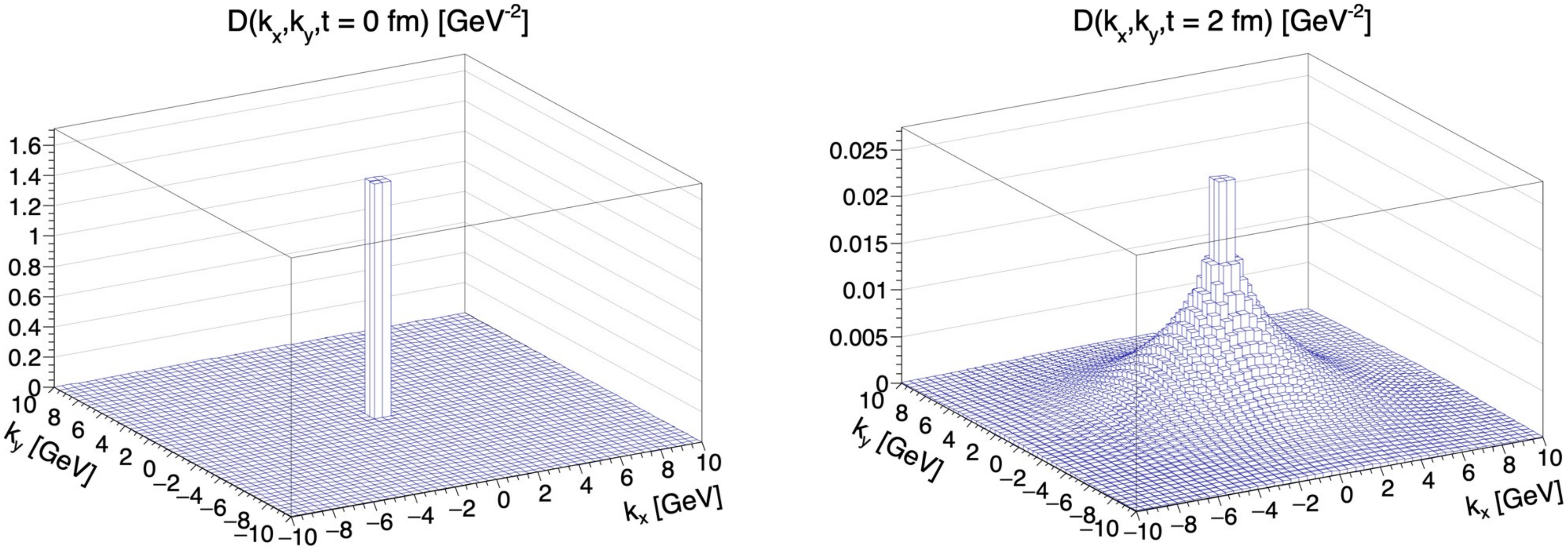

D(x,k_T,t = 0 fm) [GeV^{-1}]


D(x,k_T,t = 2 fm) [GeV^{-1}]

[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

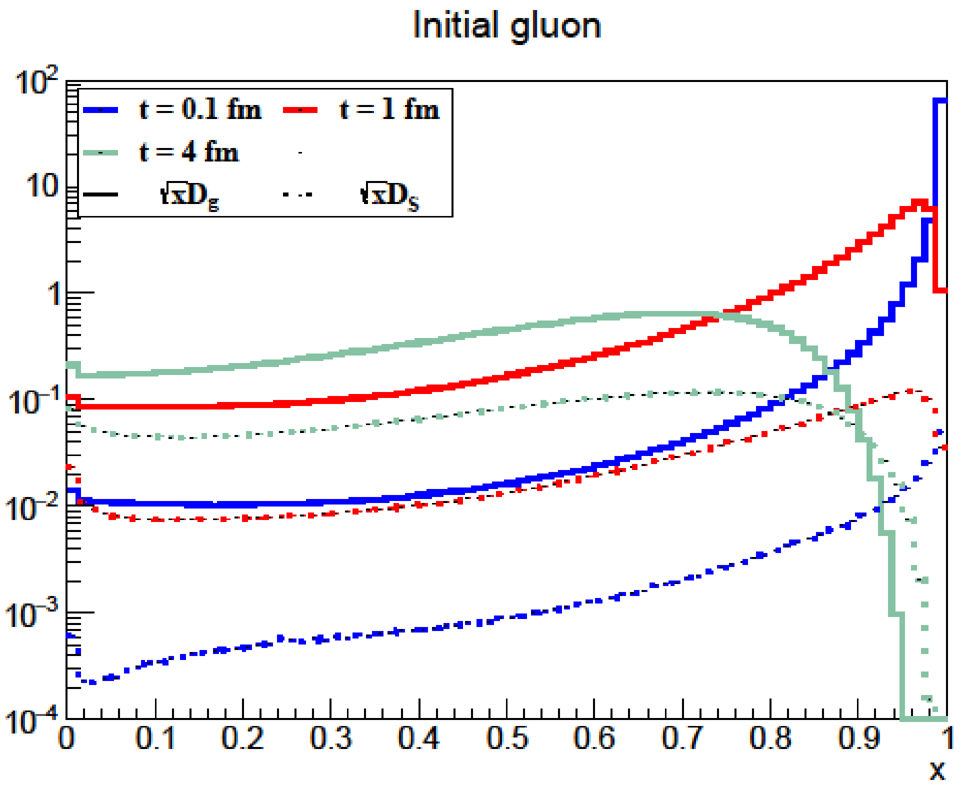
Evolution of D(x,k$_T$,t) (2/2)

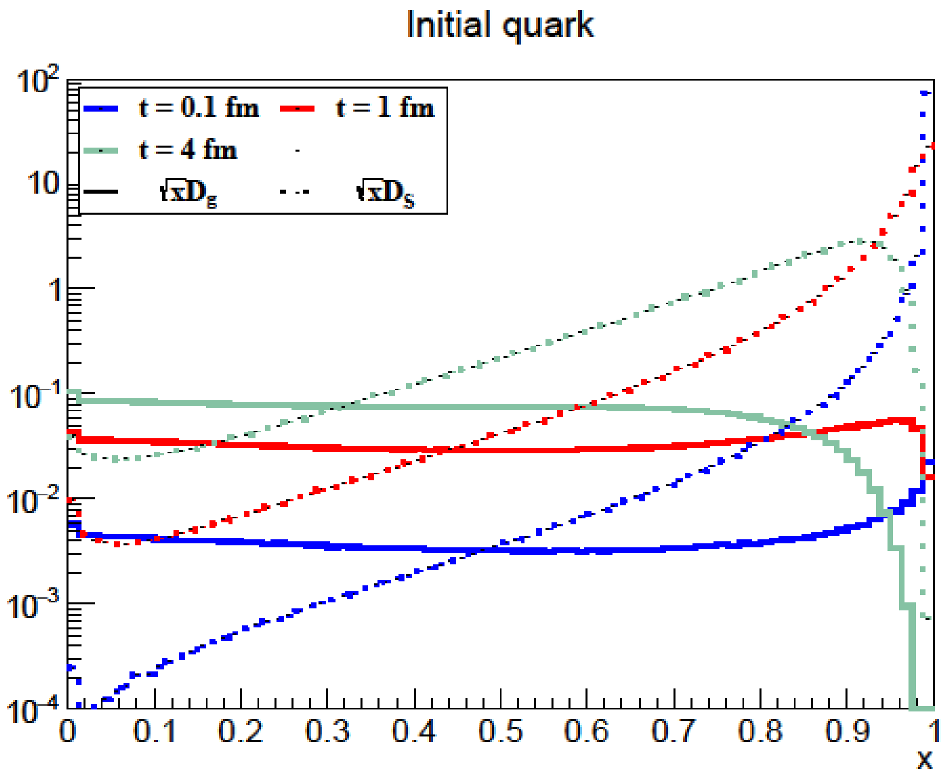
$$\mathcal{K}(z) \quad w(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$$



[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Evolution in x

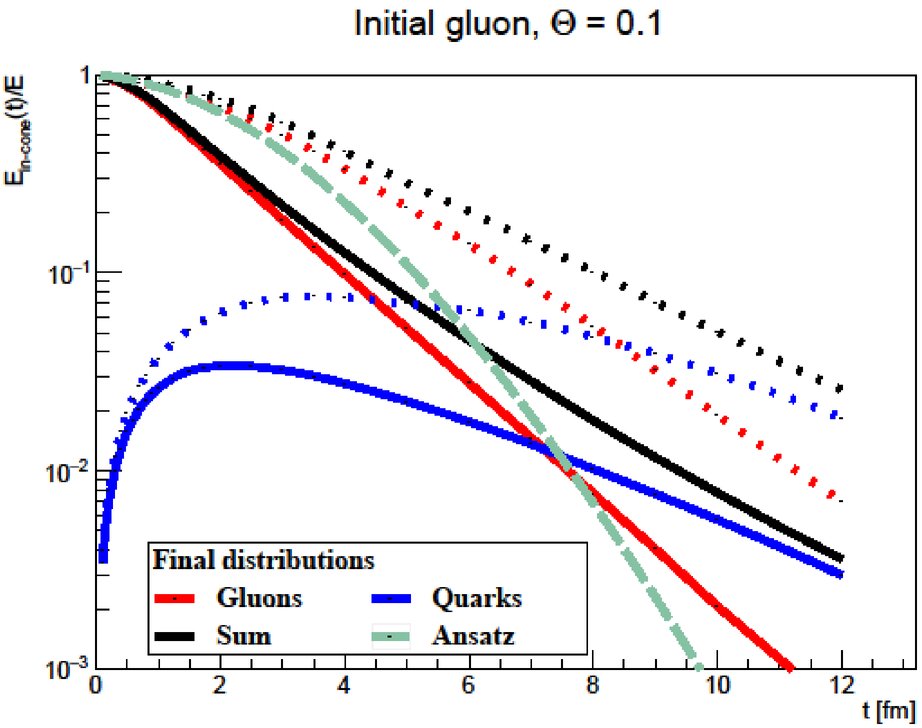


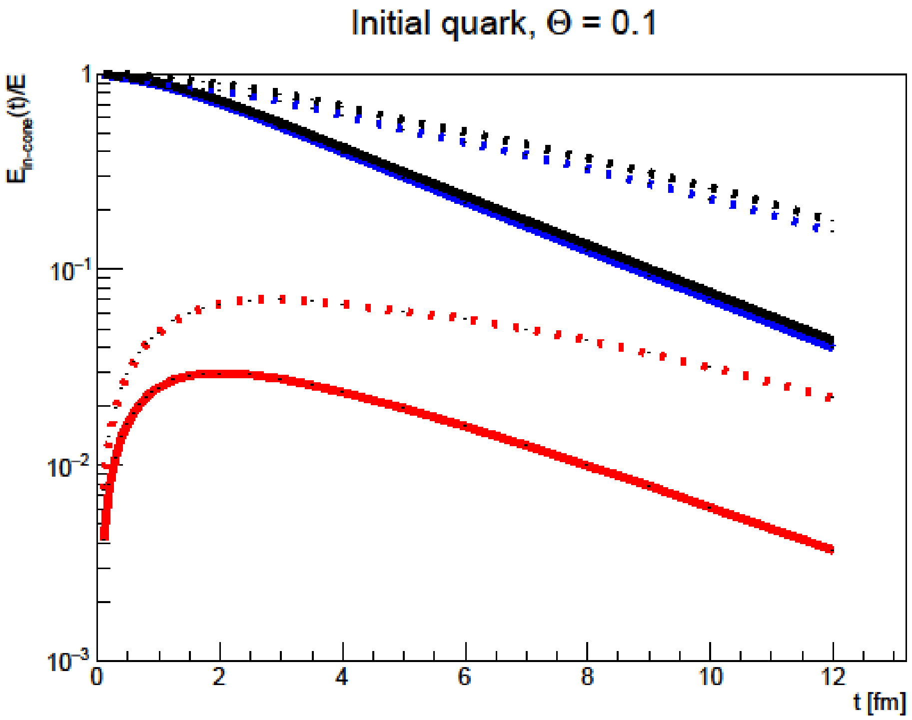


[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

In cone energy

$$E_{\mathrm{in-cone}}(\Theta) = \int_0^1 \mathrm{d}x \int_0^{2\pi} d\varphi \int_0^{xE\sin\Theta} \mathrm{d}k_T k_T \, D(x,\mathbf{k},t)$$

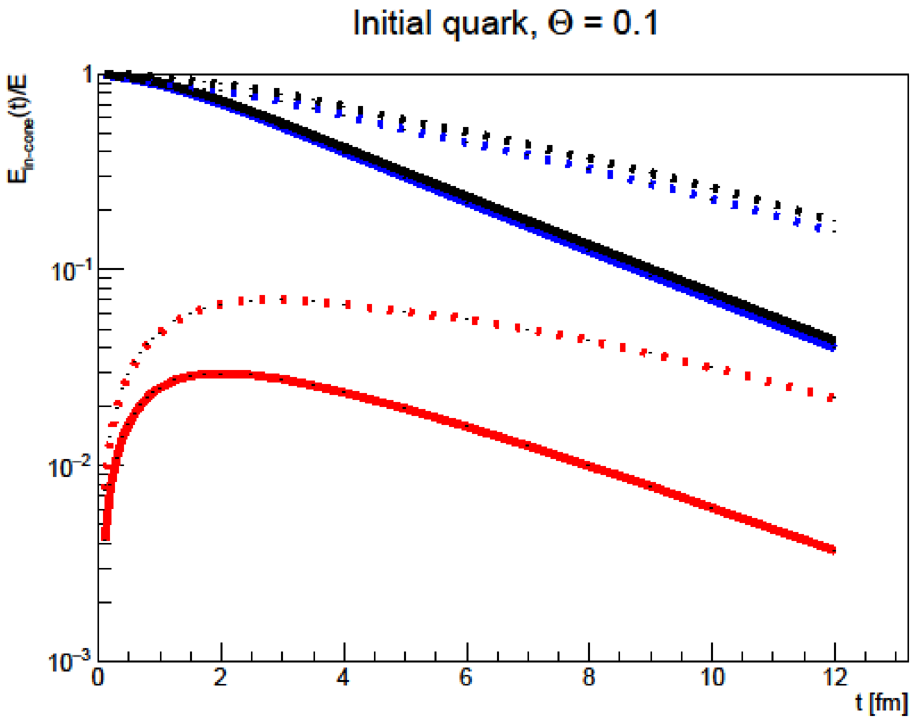

Initial gluon, Θ = 0.1

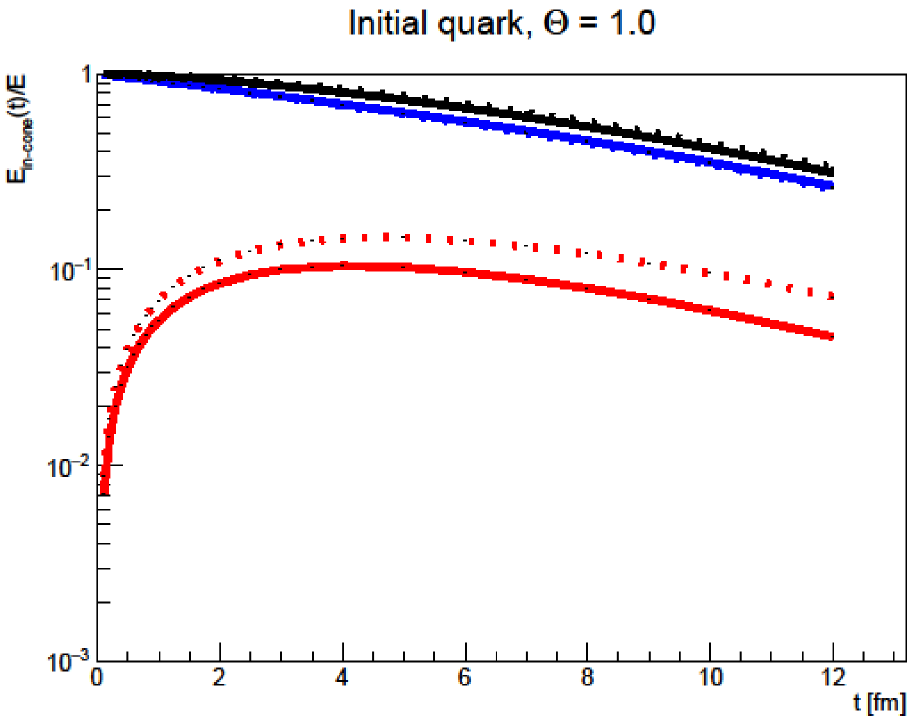

Initial quark, Θ = 0.1

[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

In cone energy

$$E_{\mathrm{in-cone}}(\Theta) = \int_0^1 \mathrm{d}x \int_0^{2\pi} d\varphi \int_0^{xE\sin\Theta} \mathrm{d}k_T k_T \, D(x, \mathbf{k}, t)$$


Initial quark, Θ = 0.1


Initial quark, Θ = 1.0

[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]