Isolated gauge boson production in *pp* collisions at forward rapidities

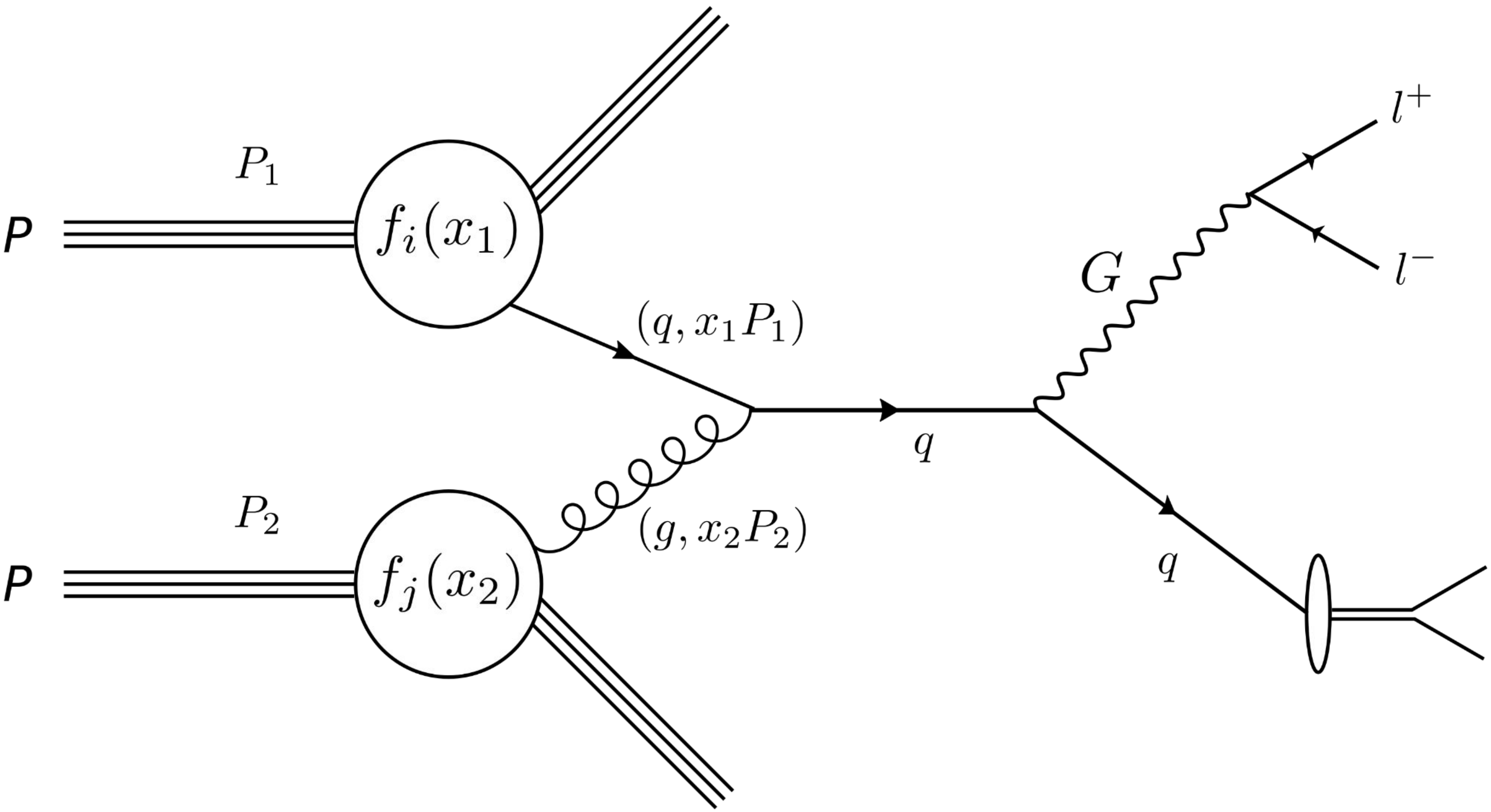
Yan B. Bandeira

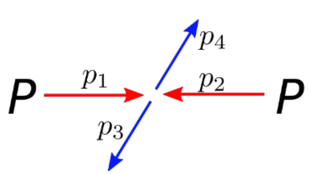
Based on: *Y. B. Bandeira, V. P. Gonçalves and W. Schäfer* [JHEP 07 (2024) 171]

$$\sigma_{AB\to QX} = \int_{z_{\min}}^{1} \frac{\mathrm{d}z}{z^2} \mathcal{D}^{k/Q}(z,\mu^2) \sum_{i,j} \int_0^1 \mathrm{d}x_a \mathrm{d}x_b f_{i/A}(x_i,\mu_F) f_{j/B}(x_j \mu_F) \hat{\sigma}_{ij\to QX}(\mu_F,\mu)$$

Collinear factorization

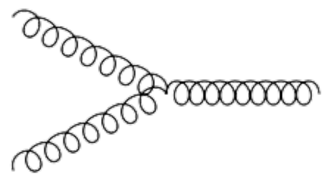
$$G = \gamma,\ Z,\ W^{\pm}$$

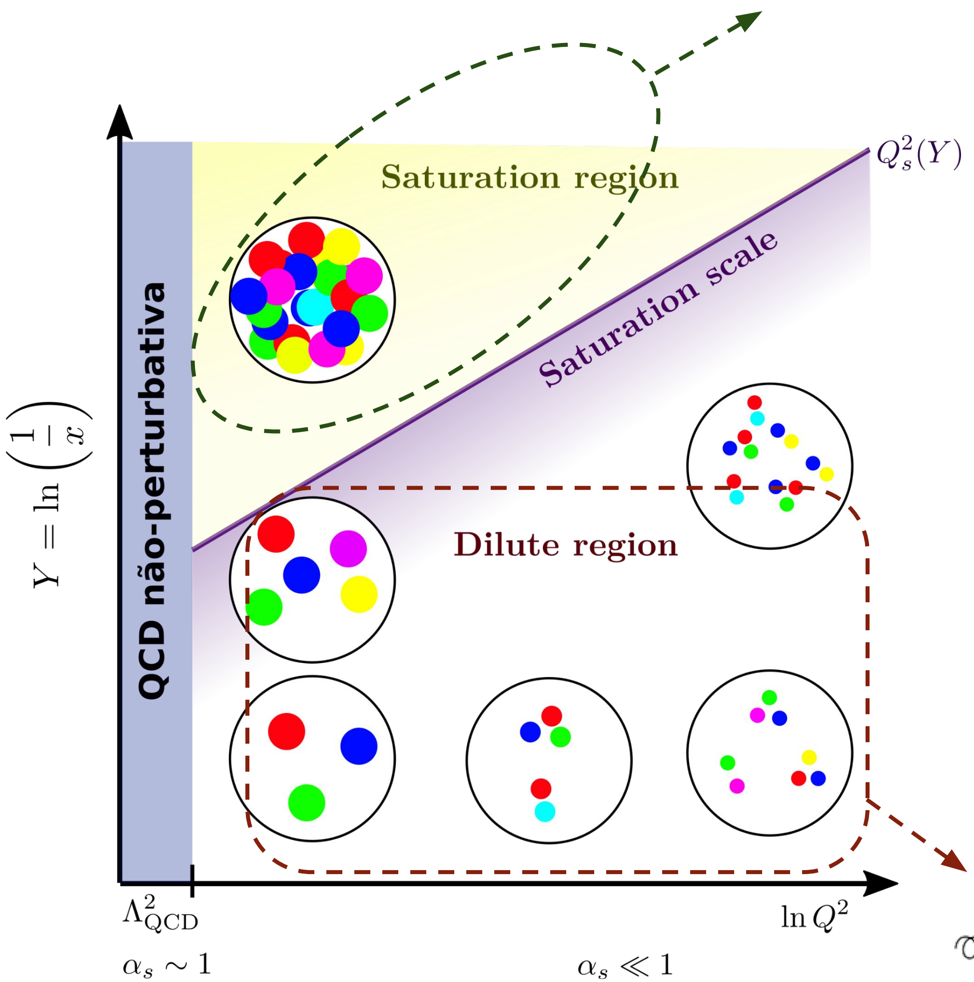


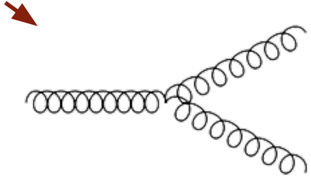


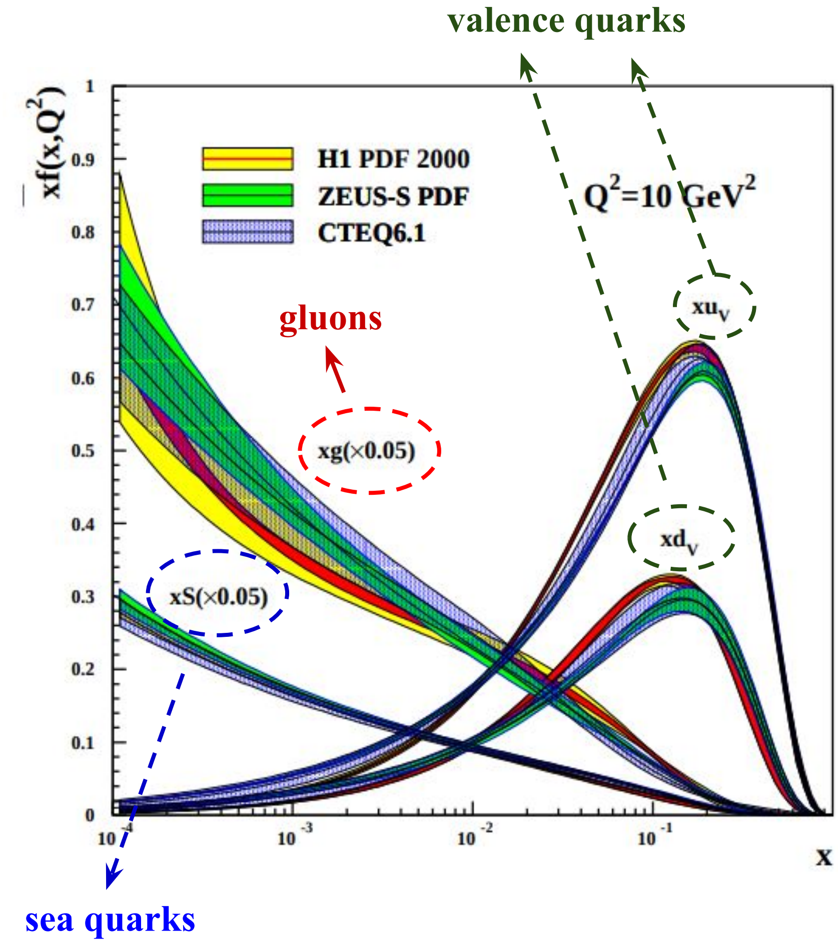
dilute region

Proton dynamics





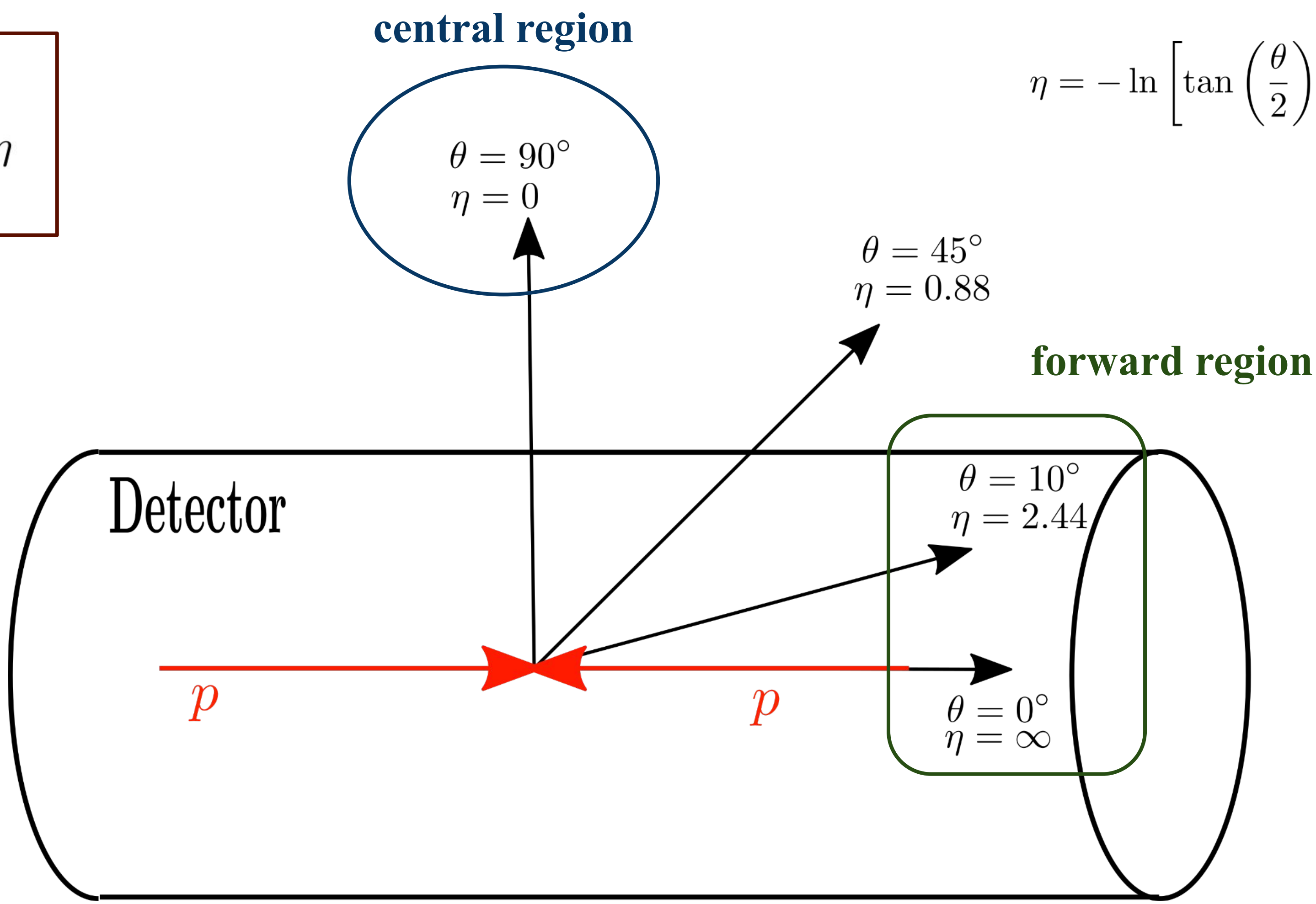




Forward rapidity

$$x_1 \propto e^{\eta}$$
$$x_2 \propto e^{-\eta}$$

$$\eta = -\ln\left[\tan\left(\frac{\theta}{2}\right)\right]$$



In the forward rapidity region, one expect the violation of collinear factorization

Color-dipole *S*-matrix framework

$$ag \to bc$$

The master dijet production in the color - dipole S - matrix framework is given by

$$\frac{d\sigma(a\to b(p_b)c(p_c))}{dz d^2\boldsymbol{p}_b d^2\boldsymbol{p}_c} = \frac{1}{(2\pi)^4} \int d^2\boldsymbol{b}_b d^2\boldsymbol{b}_c d^2\boldsymbol{b}'_b d^2\boldsymbol{b}'_c \exp[i\boldsymbol{p}_b \cdot (\boldsymbol{b}_b - \boldsymbol{b}'_b) + i\boldsymbol{p}_c \cdot (\boldsymbol{b}_c - \boldsymbol{b}'_c)]$$

$$\times \Psi(z, \boldsymbol{b}_b - \boldsymbol{b}_c)\Psi^*(z, \boldsymbol{b}'_b - \boldsymbol{b}'_c) \left\{ S^{(4)}_{\bar{b}\bar{c}cb}(\boldsymbol{b}'_b, \boldsymbol{b}'_c, \boldsymbol{b}_b, \boldsymbol{b}_c) + S^{(2)}_{\bar{a}a}(\boldsymbol{b}', \boldsymbol{b}) - S^{(3)}_{\bar{b}\bar{c}a}(\boldsymbol{b}, \boldsymbol{b}'_b, \boldsymbol{b}'_c) - S^{(3)}_{\bar{a}bc}(\boldsymbol{b}', \boldsymbol{b}_b, \boldsymbol{b}_c) \right\}$$

N. N. Nikolaev, W. Schäfer, B. G. Zakharov and V. R. Zoller [*J. Exp. Theor. Phys. 97, 441-465 (2003)*]

N. N. Nikolaev and W. Schäfer [*Phys. Rev. D 71, 014023 (2005)*]

N. N. Nikolaev, W. Schäfer, B. G. Zakharov and V. R. Zoller [*Phys. Rev. D 72, 034033 (2005)*]

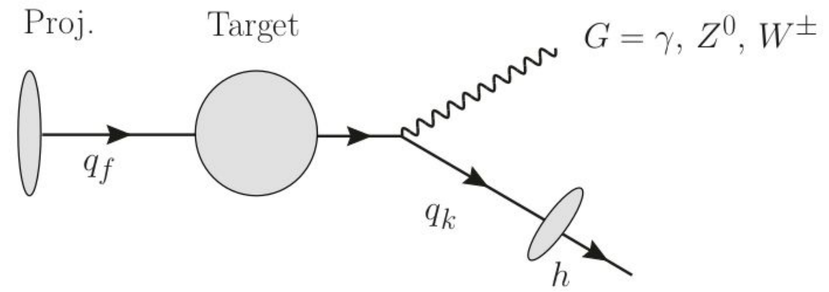
N. N. Nikolaev, W. Schäfer and B. G. Zakharov, [*Phys. Rev. D 72, 114018 (2005)*]

N. N. Nikolaev, W. Schäfer, B. G. Zakharov and V. R. Zoller [*JETP Lett. 82, 325-334 (2005)*]

N. N. Nikolaev, W. Schäfer and B. G. Zakharov [*Phys. Rev. Lett. 95, 221803 (2005)*]

Our work was evaluated the case: $q \to Gq$

Gauge boson production



Our work was use this formalism to a electroweak gauge boson production, for this case the cross-section expression simplifies as:

$$\frac{d\sigma^f_{T,L}(q_f N \to GX)}{dz d^2\boldsymbol{p}} = \frac{1}{(2\pi)^2} \overline{\sum}_{\text{quark pol.}} \int d^2\boldsymbol{r} d^2\boldsymbol{r}' \exp[i\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')]\, \Psi_{T,L}(z,\boldsymbol{r})\Psi^*_{T,L}(z,\boldsymbol{r}')$$
$$\times \frac{1}{2}\left[\sigma_{q\bar{q}}(z\boldsymbol{r},x) + \sigma_{q\bar{q}}(z\boldsymbol{r}',x) - \sigma_{q\bar{q}}(z|\boldsymbol{r}-\boldsymbol{r}'|,x)\right]$$

where, we connect the S-matrix with dipole cross-section by

$$\sigma(\boldsymbol{r}) = 2\int d^2\boldsymbol{B}\left[1 - S^{(2)}_{q\bar{q}}\left(\boldsymbol{B}+\frac{\boldsymbol{r}}{2}, \boldsymbol{B}-\frac{\boldsymbol{r}}{2}\right)\right]$$

the dipole cross-section is model dependent. Therefore,

The unknown ingredient is the Wave Function!

$$n^-_\mu A^\mu = n^-_\mu W^{\mu\pm} = n^-_\mu Z^\mu$$

LF - gauge

Light Front Wave Function

We calculate a general WF in the Light - Front approach for all gauge fields!

$$\Psi_V(z,\boldsymbol{k}) = C_f^G g_{V,f}^G \sqrt{z(1-z)}\frac{\Gamma_V}{\boldsymbol{k}^2+\epsilon^2}$$

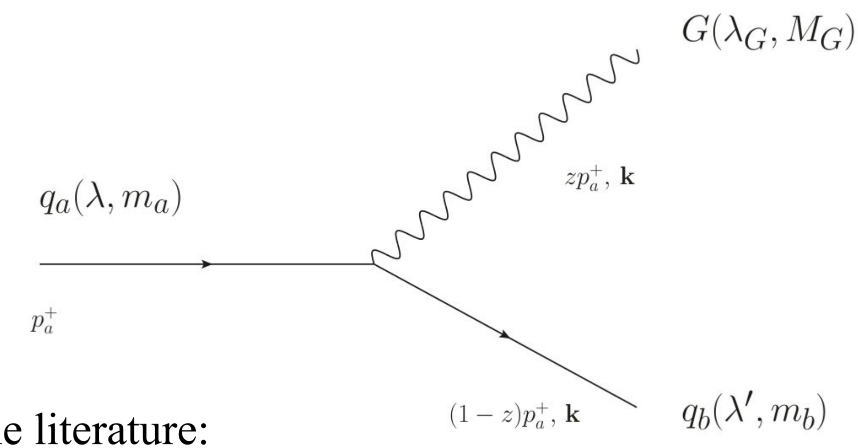
$$\Psi_A(z,\boldsymbol{k}) = C_f^G g_{A,f}^G \sqrt{z(1-z)}\frac{\Gamma_A}{\boldsymbol{k}^2+\epsilon^2}$$

$$\Gamma_V = E^*_\mu(k,\lambda)\overline{u}(p_b,\lambda',m_b)\left\{\gamma^\mu + (m_b - m_a)\frac{k^\mu}{M^2}\right\}u(p_a,\lambda,m_a)$$

$$\Gamma_A = E^*_\mu(k,\lambda)\overline{u}(p_b,\lambda',m_b)\left\{\left[\gamma^\mu + (m_b + m_a)\frac{k^\mu}{M^2}\right]\gamma_5\right\}u(p_a,\lambda,m_a)$$

Gauge Boson	C_f^G	$g_{v,f}^G$	$g_{a,f}^G$
Z^0	$C_f^Z = \frac{\sqrt{\alpha_{em}}}{\sin 2\theta_W}$	$g_{v,f_u}^Z = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W$ $g_{v,f_d}^Z = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$	$g_{a,f_u}^Z = \frac{1}{2}$ $g_{a,f_d}^Z = -\frac{1}{2}$
$W^\pm$	$C_f^{W^+} = \frac{\sqrt{\alpha_{em}}}{2\sqrt{2}\sin\theta_W}V_{f_u f_d}$ $C_f^{W^-} = \frac{\sqrt{\alpha_{em}}}{2\sqrt{2}\sin\theta_W}V_{f_d f_u}$	$g_{v,f}^W = 1$	$g_{a,f}^W = 1$
Photon	$C_f^\gamma = \sqrt{\alpha_{em}}e_f$	$g_{v,f}^\gamma = 1$	$g_{a,f}^\gamma = 0$

LFWF



As a result, we presented for the first in the literature:

$$\Psi_V^T(z,\boldsymbol{k}) = C_f^G g_{V,f}^G \frac{\sqrt{z}}{\boldsymbol{k}^2+\epsilon^2}\chi_{\lambda'}^\dagger\left\{\left(\left(\frac{2-z}{z}\right)\boldsymbol{k}\cdot\boldsymbol{E}^*(\lambda_G)+i\lambda\left[\boldsymbol{k},\boldsymbol{E}^*(\lambda_G)\right]\right)I-\lambda\left[m_b-(1-z)m_a\right]\boldsymbol{\sigma}\cdot\boldsymbol{E}^*(\lambda_G)\right\}\chi_\lambda,$$

$$\Psi_A^T(z,\boldsymbol{k}) = C_f^G g_{V,f}^G \frac{\sqrt{z}}{\boldsymbol{k}^2+\epsilon^2}\chi_{\lambda'}^\dagger\left\{\left[\frac{2-z}{z}\boldsymbol{k}\cdot\boldsymbol{E}^*(\lambda_G)+i\lambda\left[\boldsymbol{k},\boldsymbol{E}^*(\lambda_G)\right]\right]\lambda I-(m_b+(1-z)m_a)\boldsymbol{\sigma}\cdot\boldsymbol{E}^*(\lambda_G)\right\}\chi_\lambda,$$

$$\Psi_V^L(z,\boldsymbol{k}) = C_f^G g_{V,f}^G \frac{1}{\sqrt{z}M}\chi_{\lambda'}^\dagger\left\{I\left[\frac{z^2 m_a\left(m_b-m_a\right)-z\left(m_b^2-m_a^2\right)-2(1-z)M^2}{\boldsymbol{k}^2+\epsilon^2}\right]+\frac{\left[z(m_b-m_a)\right]}{\boldsymbol{k}^2+\epsilon^2}\lambda\left(\boldsymbol{\sigma}\cdot\boldsymbol{k}\right)\right\}\chi_\lambda,$$

$$\Psi_L^A(z,\boldsymbol{k}) = C_f^G g_{A,f}^G \frac{1}{\sqrt{z}M}\chi_{\lambda'}^\dagger\left\{\lambda I\left[-\frac{z^2 m_a\left(m_b+m_a\right)+z\left(m_b^2-m_a^2\right)+2(1-z)M^2}{\boldsymbol{k}^2+\epsilon^2}\right]-z\left(m_b+m_a\right)\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{k})}{\boldsymbol{k}^2+\epsilon^2}\right\}\chi_\lambda.$$

$$\rho^{T,L}_{V,A} = \frac{1}{2}\sum_{\lambda\lambda'\lambda_G} \psi^{T,L}_{V,A}(z,\boldsymbol{r})\psi^{T,L,*}_{V,A}(z,\boldsymbol{r}')$$

Parton -level cross-section

$$\left.\frac{d\sigma^f_{T,L}}{dz\, d^2\boldsymbol{p}}\right|_{V,A} = \frac{1}{2(2\pi)^2}\int d^2\boldsymbol{r}\int d^2\boldsymbol{r}' e^{i\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')}\rho^{T,L}_{V,A}\left[\sigma_{q\bar{q}}(z\boldsymbol{r},x)+\sigma_{q\bar{q}}(z\boldsymbol{r}',x)-\sigma_{q\bar{q}}(z|\boldsymbol{r}-\boldsymbol{r}'|,x)\right]$$

↓

$$\left.\frac{d\sigma^f_T}{dzd^2\boldsymbol{p}}\right|_V = \frac{(C^G_f)^2(g^G_{f,V})^2}{(2\pi)^2}\left\{z[(m_b-m_a)+zm_a]^2\mathcal{D}_1(z,p,\epsilon)+\frac{[1+(1-z)^2]}{z}\epsilon^2\mathcal{D}_2(z,p,\epsilon)\right\},$$

$$\left.\frac{d\sigma^f_T}{dzd^2\boldsymbol{p}}\right|_A = \frac{(C^G_f)^2(g^G_{f,A})^2}{2\pi^2}\left\{z[(m_b+m_a)-zm_a]^2\mathcal{D}_1(z,p,\epsilon)+\frac{1+(1-z)^2}{z}\epsilon^2\mathcal{D}_2(z,p,\epsilon)\right\}.$$

$$\left.\frac{d\sigma^f_L}{dzd^2\boldsymbol{p}}\right|_V = \frac{(C^G_f)^2(g^G_{f,V})^2}{(2\pi)^2}\left\{\frac{\left[z^2m_a(m_b-m_a)-z(m_b^2-m_a^2)-2(1-z)M_G^2\right]^2}{zM_G^2}\mathcal{D}_1(z,p,\epsilon)+\frac{z(m_b-m_a)^2}{M_G^2}\epsilon^2\mathcal{D}_2(z,p,\epsilon)\right\}.$$

$$\left.\frac{d\sigma^f_L}{dzd^2\boldsymbol{p}}\right|_A = \frac{(C^G_f)^2(g^G_{f,A})^2}{(2\pi)^2}\left\{\frac{\left[z^2m_a(m_b+m_a)+z(m_b^2-m_a^2)+2(1-z)M_G^2\right]^2}{zM_G^2}\mathcal{D}_1(z,p,\epsilon)+\frac{z(m_b+m_a)^2}{M_G^2}\epsilon^2\mathcal{D}_2(z,p,\epsilon)\right\}.$$

for

$$\mathcal{D}_1(z,p,\epsilon) = \frac{1}{p^2+\epsilon^2}I_1(z,p)-\frac{1}{4\epsilon}I_2(z,p)$$

$$\mathcal{D}_2(z,p,\epsilon) = \frac{1}{\epsilon}\frac{p}{(p^2+\epsilon^2)}I_3(z,p)-\frac{1}{2\epsilon^2}I_1(z,p)+\frac{1}{4\epsilon}I_2(z,p).$$

$$I_1(z,p) = \int dr r \mathrm{J}_0(pr)\mathrm{K}_0(\epsilon r)\sigma_{q\bar{q}}(z\boldsymbol{r})$$

$$I_2(z,p) = \int dr r^2 \mathrm{J}_0(pr)\mathrm{K}_1(\epsilon r)\sigma_{q\bar{q}}(z\boldsymbol{r})$$

$$I_3(z,p) = \int dr r \mathrm{J}_1(pr)\mathrm{K}_1(\epsilon r)\sigma_{q\bar{q}}(z\boldsymbol{r})$$

A generalized description for all electroweak gauge boson

Particular cases

Real photon production

$$\left.\frac{d\sigma_T^f}{d\ln z\, d^2\boldsymbol{p}}\right|_{qp\to\gamma X} = \frac{\alpha_{em} e_f^2}{2\pi^2}\{m_f^2 z^4\,\mathcal{D}_1(z,p,\epsilon) + [1+(1-z)^2]\epsilon^2\,\mathcal{D}_2(z,p,\epsilon)\}$$

J.Jalilian-Marian and A.H.Rezaeian [*Phys.Rev.D* 86 (2012) 034016]

V. P. Goncalves, Y.Lima, R.Pasechnik and M.Šumbera [*Phys.Rev.D* 101 (2020) 9, 094019]

B.Ducloué, T.Lappi and H.Mäntysaari [*Phys.Rev.D* 97 (2018) 5, 054023]

Drell-Yan process

$$\frac{d\sigma(qp\to[G\to l\bar{l}]X)}{dzd^2\boldsymbol{p}dM^2} = \mathcal{F}_G(M)\,\frac{d\sigma(qp\to GX)}{dzd^2\boldsymbol{p}}$$

F. Gelis and J. Jalilian-Marian, Phys. Rev. D 66, 094014 (2002)

R. Baier, A. H. Mueller and D. Schiff, Nucl. Phys. A 741, 358-380 (2004)

A. Stasto, B. W. Xiao and D. Zaslavsky, Phys. Rev. D 86, 014009 (2012)

Z. B. Kang and B. W. Xiao, Phys. Rev. D 87, no.3, 034038 (2013)

E. Basso, V. P. Goncalves, J. Nemchik, R. Pasechnik and M. Sumbera, Phys. Rev. D 93, no.3, 034023 (2016)

Our general formalism cover particular case present in the literature.

$$\mathcal{E}_1(\boldsymbol{p},\boldsymbol{k},z,\epsilon)=\left[\frac{(\boldsymbol{p}-z\boldsymbol{k})}{[(\boldsymbol{p}-z\boldsymbol{k})^2+\epsilon^2]}-\frac{\boldsymbol{p}}{(p^2+\epsilon^2)}\right]^2$$

$$\mathcal{E}_2(\boldsymbol{p},\boldsymbol{k},z,\epsilon)=\left[\frac{1}{[(\boldsymbol{p}-z\boldsymbol{k})+\epsilon^2]^2}-\frac{1}{(p^2+\epsilon^2)}\right]^2$$

Next steps

Work in progress

- The associated gauge boson + jet differential spectrum:

$$\left.\frac{\mathrm{d}\sigma_T}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}\mathrm{d}^2\boldsymbol{\Delta}}\right|_V=\frac{1}{2}\frac{(C_f^G)^2(g_{V,f})^2}{2\pi^2}f(x,\boldsymbol{\Delta})\left\{\frac{1+(1-z)^2}{z}\mathcal{E}_1(\boldsymbol{p},\boldsymbol{\Delta},z,\epsilon)+z[(m_b-m_a)+zm_a]^2\mathcal{E}_2(\boldsymbol{p},\boldsymbol{\Delta},z,\epsilon)\right\},$$

$$\left.\frac{\mathrm{d}\sigma_T}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}\mathrm{d}^2\boldsymbol{\Delta}}\right|_A=\frac{1}{2}\frac{(C_f^G)^2(g_{A,f})^2}{2\pi^2}f(x,\boldsymbol{\Delta})\left\{\frac{1+(1-z)^2}{z}\mathcal{E}_1(\boldsymbol{p},\boldsymbol{\Delta},z,\epsilon)+z[(m_b-m_a)+zm_a]^2\mathcal{E}_2(\boldsymbol{p},\boldsymbol{\Delta},z,\epsilon)\right\},$$

$$\left.\frac{\mathrm{d}\sigma_L}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}\mathrm{d}^2\boldsymbol{\Delta}}\right|_V=\frac{1}{2}\frac{(C_f^G)^2(g_{f,V}^G)^2}{4\pi^2}f(x,\boldsymbol{\Delta})\left\{\frac{z(m_b-m_a)^2}{M_G^2}\mathcal{E}_1(\boldsymbol{p},\boldsymbol{\Delta},z,\epsilon)+\frac{\left[z^2m_a(m_b-m_a)-z(m_b^2-m_a^2)-2(1-z)M_G^2\right]^2}{zM_G^2}\mathcal{E}_2(\boldsymbol{p},\boldsymbol{\Delta},z,\epsilon)\right\},$$

$$\left.\frac{\mathrm{d}\sigma_L}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}\mathrm{d}^2\boldsymbol{\Delta}}\right|_A=\frac{1}{2}\frac{(C_f^G)^2(g_{f,A}^G)^2}{4\pi^2}f(x,\boldsymbol{\Delta})\left\{\frac{z(m_b-m_a)^2}{M_G^2}\mathcal{E}_1(\boldsymbol{p},\boldsymbol{\Delta},z,\epsilon)+\frac{\left[z^2m_a(m_b-m_a)-z(m_b^2-m_a^2)-2(1-z)M_G^2\right]^2}{zM_G^2}\mathcal{E}_2(\boldsymbol{p},\boldsymbol{\Delta},z,\epsilon)\right\}.$$

Which, once more, cover the expressions in the literature for particular cases!

F. Dominguez, C. Marquet, B. W. Xiao and F. Yuan, Phys. Rev. D 83, 105005 (2011)
E. Basso, V. P. Goncalves, J. Nemchik, R. Pasechnik and M. Sumbera, Phys. Rev. D 93, no.3, 034023 (2016)

- Leptons angular distribution;
- Lam-Tung relation;

Summary

- We derived, for the *first time*, the generic expressions for the **LFWF's**.
  - We have estimated the *vector* and *axial* contributions for the description of the longitudinal and transverse spectra associated with the isolated gauge boson production in the impact parameter and transverse momentum spaces.
- We demonstrated that our results reduce to expressions previously used in the literature for the description of the *real photon production* and *Drell - Yan* process at forward rapidities in some particular limits.
- As seen, the expressions obtained are the main ingredients for the calculation of the *pp* cross - sections, which can be compared with the current and forthcoming LHC data.

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Thank you for your attention!