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Photon-photon transition form factor for tensor meson quarkonium

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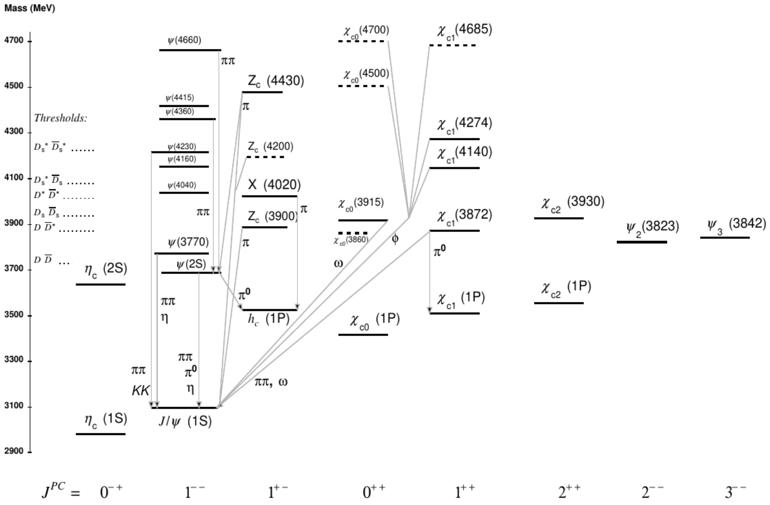
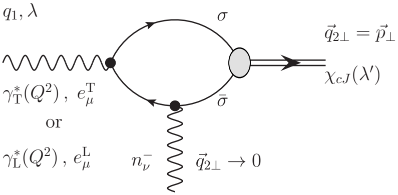
Charmonium family



J^{PC} : 0^{-+} - pseudoscalar, 0^{++} - scalar ,
1^{--} - vector , 1^{++} - axial vector,
2^{++} - tensor meson

spectroscopic notation:
$\ell = 0, 1, 2$
S, P, D

Transition matrix element



In Drell-Yan frame one of the photons carries vanishing light-front plus momentum:

$$q_1 = \{q_1^+, q_1^- = -\frac{Q^2}{2q_1^+}, \vec{q}_{1\perp} = 0\},$$

$$q_2 = \{q_2^+ = 0, q_2^- = P^- - q_1^-, \vec{q}_{2\perp}\},$$

Q^2 - photon virtuality, $q_1^2 = -Q^2$, $Q^2 \geq 0$, and $q_2^2 = 0$

Possibilities of virtual photon polarizations:
transverse : $\gamma^*_{\rm T}(Q^2)$,
longitudinal: $\gamma^*_{\rm L}(Q^2)$.

Helicity amplitudes related to matrix elements of the LF-plus component of the current:

$$\mathcal{M}(\lambda \to \lambda') \equiv \langle \chi_{cJ}(\lambda')|J_+(0)|\gamma^*_{\rm T,L}(Q^2)\rangle$$

$$= 2q_1^+ \sqrt{N_c}\, e^2 e_f^2 \int \frac{dz d^2\vec{k}_\perp}{z(1-z)16\pi^3} \sum_{\sigma,\bar{\sigma}} \underbrace{\Psi^{\lambda' *}_{\sigma\bar{\sigma}}(z,\vec{k}_\perp)}_{\rm Q\bar{Q}\ wave\ function} (\vec{q}_{2\perp}\cdot\nabla_{\vec{k}_\perp}) \underbrace{\Psi^{\gamma_{\rm T,L}}_{\sigma\bar{\sigma}}(z,\vec{k}_\perp,Q^2)}_{\rm photon\ wave\ function}$$

$$\Psi^{\gamma_T}_{\sigma\bar{\sigma}}(z,\vec{k}_\perp,Q^2) = \sqrt{z(1-z)}\,\frac{\delta_{\sigma,-\bar{\sigma}}\,(\vec{e}_\perp\cdot\vec{k}_\perp)\left(2(1-z)\delta_{\bar{\sigma},\lambda} - 2z\delta_{\sigma,\lambda}\right) + \delta_{\sigma\bar{\sigma}}\delta_{\sigma\lambda}\sqrt{2}m_f}{\vec{k}_\perp^2 + m_f^2 + z(1-z)Q^2},$$

$$\Psi^{\gamma_L}_{\sigma\bar{\sigma}}(z,\vec{k}_\perp,Q^2) = \left(\sqrt{z(1-z)}\right)^3 \frac{2Q\,\delta_{\sigma,-\bar{\sigma}}}{\vec{k}_\perp^2 + m_f^2 + z(1-z)Q^2},$$

$\sigma(\bar{\sigma})$quark(antiquark) polarization

Light Front Wave Function - tensor meson

$$\Psi^{(\lambda)}_{\sigma\bar\sigma}(z,\vec{k}_\perp) = \sqrt{\frac{3}{2}}\,\Phi^{(\lambda)}_{\sigma\bar\sigma}\,\phi(z,k_\perp)\,\frac{2}{\sqrt{M_0^2-4m_f^2}}, \qquad \phi(z,k_\perp) = \pi\sqrt{M_0}\,\frac{u_1(k)}{k}$$

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$$\Phi^{(0)}_{\sigma\bar\sigma} = \begin{pmatrix} \frac{k_\perp e^{-i\varphi}}{M_0}\left[m_f + \frac{2k_\perp^2 - (2z-1)^2 M_0^2}{M_0+2m_f})\right] & (2z-1)\left[\frac{k_\perp^2}{M_0+2m_f} + \frac{1}{2}M_0 - \frac{(2z-1)^2 M_0^2}{2(M_0+2m_f)}\right] \\ (2z-1)\left[\frac{k_\perp^2}{M_0+2m_f} + \frac{1}{2}M_0 - \frac{(2z-1)^2 M_0^2}{2(M+2m_f)}\right] & -\frac{k_\perp e^{i\varphi}}{M_0}\left[m_f + \frac{2k_\perp^2 - (2z-1)^2 M_0^2}{M_0+2m_f}\right] \end{pmatrix},$$

$$\Phi^{(+1)}_{\sigma\bar\sigma} = \frac{1}{2\sqrt{z(1-z)}}\begin{pmatrix} m_f(2z-1) + 2k_\perp^2\frac{(2z-1)}{M_0+2m_f} & k_\perp e^{i\varphi}\left[\frac{(2z-1)^2 M_0}{M_0+2m_f} + (z-1)\right] \\ k_\perp e^{i\varphi}\left[\frac{(2z-1)^2 M_0}{M_0+2m_f} - z\right] & -2k_\perp^2 e^{i2\varphi}\frac{(2z-1)}{M_0+2m_f} \end{pmatrix},$$

$$\Phi^{(-1)}_{\sigma\bar\sigma} = \frac{1}{2\sqrt{z(1-z)}}\begin{pmatrix} -2k_\perp^2 e^{-i2\varphi}\frac{(2z-1)}{M_0+2m_f} & -k_\perp e^{-i\varphi}\left[\frac{(2z-1)^2 M_0}{M_0+2m_f} - z\right] \\ -k_\perp e^{-i\varphi}\left[\frac{(2z-1)^2 M_0}{M_0+2m_f} + (z-1)\right] & m(2z-1) + 2k_\perp^2\frac{(2z-1)}{M_0+2m_f} \end{pmatrix},$$

$$\Phi^{(+2)}_{\sigma\bar\sigma} = \frac{-k_\perp e^{i\varphi}}{M_0\sqrt{z(1-z)}}\begin{pmatrix} m_f + \frac{k_\perp^2}{M_0+2m_f} & \frac{1}{2}k_\perp e^{i\varphi}\left(1 + \frac{(2z-1)M_0}{M_0+2m_f}\right) \\ -\frac{1}{2}k_\perp e^{i\varphi}\left(1 - \frac{(2z-1)M_0}{M_0+2m_f}\right) & -k_\perp^2 e^{i2\varphi}\frac{1}{M_0+2m_f} \end{pmatrix},$$

$$\Phi^{(-2)}_{\sigma\bar\sigma} = \frac{k_\perp e^{-i\varphi}}{M_0\sqrt{z(1-z)}}\begin{pmatrix} -k_\perp^2 e^{-i2\varphi}\frac{1}{M_0+2m_f} & \frac{1}{2}k_\perp e^{-i\varphi}\left(1 - \frac{(2z-1)M_0}{M_0+2m_f}\right) \\ -\frac{1}{2}k_\perp e^{-i\varphi}\left(1 + \frac{(2z-1)M_0}{M_0+2m_f}\right) & m_f + \frac{k_\perp^2}{M_0+2m_f} \end{pmatrix}.$$

Transition amplitude in the Drell-Yan frame

The covariant amplitude for the process $\gamma^*(q_1)\gamma(q_2) \to 2^{++}$:

$$\mathcal{M}_{\mu\nu\alpha\beta} = 4\pi\alpha_{em}\Big[\delta^{\perp}_{\mu\nu}(q_2-q_1)_\alpha(q_2-q_1)_\beta\, F_{\mathrm{TT},0}(Q^2)$$

$$+\frac{1}{2}\left(\delta^{\perp}_{\mu\alpha}\delta^{\perp}_{\nu\beta}+\delta^{\perp}_{\nu\alpha}\delta^{\perp}_{\mu\beta}-\delta^{\perp}_{\mu\nu}\delta^{\perp}_{\alpha\beta}\right) F_{\mathrm{TT},2}(Q^2)+\left(q_{1\mu}-\frac{q_1^2}{q_1\cdot q_2}q_{2\mu}\right)\delta^{\perp}_{\nu\alpha}(q_2-q_1)_\beta\, F_{\mathrm{LT}}(Q^2)\Big],$$

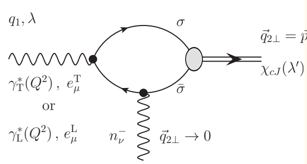
and $\delta^{\perp}_{\mu\nu} = g_{\mu\nu} - \frac{1}{(q_1\cdot q_2)^2}\left((q_1\cdot q_2)(q_{2\mu}q_{1\nu}+q_{1\mu}q_{2\nu}) - q_1^2 q_{2\mu}q_{2\nu}\right)$.

- three independent form factors - each of them contain wave function of the proper meson polarization

The covariant amplitude definition $\leftrightarrow$ transition amplitude in the Drell-Yan frame

$$\mathcal{M}(\boldsymbol{\lambda}\to\boldsymbol{\lambda'}) = e_\mu(\lambda)n^-_\nu\,\mathcal{M}^{\mu\nu\alpha\beta}\,E^*_{\alpha\beta}(\lambda')$$

$$\begin{aligned}
\mathcal{M}(+1\to 0) &= 2q_1^+\,e^2\,(\vec{e}_\perp(+1)\cdot\vec{q}_{2\perp})\frac{2}{\sqrt{6}}\frac{M^2+Q^2}{M^2}\,F_{\mathrm{TT},0}(Q^2)\,,\\
\mathcal{M}(+1\to +2) &= -2q_1^+e^2\,(\vec{e}^{\,*}_\perp(+1)\cdot\vec{q}_{2\perp})\frac{1}{M^2+Q^2}\,F_{\mathrm{TT},2}(Q^2)\,,\\
\mathcal{M}(0\to +1) &= 2q_1^+e^2(\vec{e}^{\,*}_\perp(+1)\cdot\vec{q}_{2\perp})\,\frac{Q}{\sqrt{2}M}\,F_{\mathrm{LT}}(Q^2)\,.
\end{aligned}$$



Transition Form Factors: $F_{TT,0}$, $F_{TT,2}$, F_{LT}

Form factors with light-front wave functions

$$F_{\rm TT,0}(Q^2) = \sqrt{6N_c}e_f^2 \frac{M^2}{M^2+Q^2} \int \frac{dz\, k_\perp\, dk_\perp}{\sqrt{z(1-z)}8\pi^2} \frac{1}{[k_\perp^2+\varepsilon^2]^2} \Bigg[m_f k_\perp \tilde{\psi}^{0}_{\uparrow\uparrow}(z,k_\perp) - \frac{\varepsilon^2}{2}\Big((2z-1)\Big(\tilde{\psi}^{0}_{\uparrow\downarrow}(z,k_\perp)+\tilde{\psi}^{0}_{\downarrow\uparrow}(z,k_\perp)\Big) + \Big(\tilde{\psi}^{0}_{\uparrow\downarrow}(z,k_\perp)-\tilde{\psi}^{0}_{\downarrow\uparrow}(z,k_\perp)\Big)\Big)\Bigg],$$

$$F_{\rm TT,2}(Q^2) = -2\sqrt{N_c}e_f^2 (M^2+Q^2) \int \frac{dz\, k_\perp\, dk_\perp}{\sqrt{z(1-z)}8\pi^2} \frac{1}{[k_\perp^2+\varepsilon^2]^2} \Bigg[m_f k_\perp \tilde{\psi}^{+2}_{\uparrow\uparrow}(z,k_\perp) + \frac{k_\perp^2}{2}\Big((2z-1)\Big(\tilde{\psi}^{+2}_{\uparrow\downarrow}(z,k_\perp)+\tilde{\psi}^{+2}_{\downarrow\uparrow}(z,k_\perp)\Big) + \Big(\tilde{\psi}^{+2}_{\uparrow\downarrow}(z,k_\perp)-\tilde{\psi}^{+2}_{\downarrow\uparrow}(z,k_\perp)\Big)\Big)\Bigg],$$

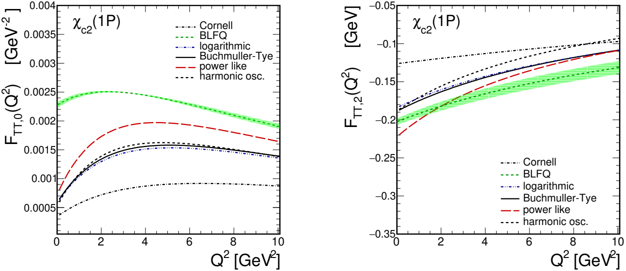
$$F_{\rm LT}(Q^2) = 4\sqrt{N_c}e_f^2\, M \int \frac{dz k_\perp\, dk_\perp}{\sqrt{z(1-z)}8\pi^2} \frac{z(1-z)k_\perp}{[k_\perp^2+\varepsilon^2]^2} \Big(\tilde{\psi}^{+1}_{\uparrow\downarrow}(z,k_\perp)+\tilde{\psi}^{+1}_{\downarrow\uparrow}(z,k_\perp)\Big).$$

In the limit of nonrelativistic motion of quarks in the bound state

- expand $z=\frac{1}{2}, \implies z=\frac{1}{2}-\xi$, $1-z=\frac{1}{2}+\xi$, $\xi\to 0$,
- expand $k_\perp = 0$

$$\text{NRQCD} \implies \begin{cases} F_{\rm TT,0}(Q^2) = e_f^2\,(-4)\sqrt{\frac{3N_c M}{\pi}}\frac{Q^2}{(M^2+Q^2)^3}R'(0) \\ F_{\rm TT,2}(Q^2) = e_f^2\, 8\sqrt{\frac{3N_c M}{\pi}}\frac{1}{M^2+Q^2}R'(0), \\ F_{\rm LT}(Q^2) = e_f^2\,(-8)\sqrt{\frac{3N_c M}{\pi}}\frac{1}{(M^2+Q^2)^2}R'(0). \end{cases}$$

Contribution to radiative decay width: $\Gamma(\chi_{c2} \to \gamma\gamma)$



- the χ_{c2} helicity $\lambda = 0$ is strongly suppresed.
- $\frac{\Gamma_{\gamma\gamma}(\lambda=0)}{\Gamma_{\gamma\gamma}(\lambda=\pm 2)} =$ $(0.0 \pm 0.6 \pm 1.2) \times 10^{-2}$, BESIIICollaboration, M.Ablikim et al. Phys.Rev.D96(2017)9, 092007

$$\Gamma_{\gamma\gamma}(\chi_{c2}) = (4\pi\alpha_{\rm em})^2 \left[\frac{|F_{\rm TT,0}(0)|^2 \cdot M^3_{\chi_{c2}}}{120\pi} + \frac{|F_{\rm TT,2}(0)|^2}{80\pi M_{\chi_{c2}}} \right].$$

Helicity decomposition of the two-photon decay width of $\chi_{c2}(1P)$.

	$\Gamma_{\gamma\gamma}(\lambda = 0)$ [keV]	$\Gamma_{\gamma\gamma}(\lambda = \pm 2)$ [keV]	$\frac{\Gamma(\lambda=0)}{\Gamma(\lambda=\pm 2)}$	$\Gamma_{\gamma\gamma}$ [keV]
Cornell	1.18×10^{-4}	0.15	0.7×10^{-3}	0.15
logarithmic	3.37×10^{-4}	0.32	0.3×10^{-3}	0.32
Buchmüller-Tye	3.36×10^{-4}	0.34	1.0×10^{-3}	0.34
power like	5.18×10^{-4}	0.47	1.1×10^{-3}	0.47
harmonic osc.	2.80×10^{-4}	0.33	0.8×10^{-3}	0.33
BLFQ	$(5.2 \pm 0.2) \times 10^{-3}$	0.39 ± 0.01	$(1.3 \pm 0.1) \times 10^{-2}$	0.39 ± 0.01

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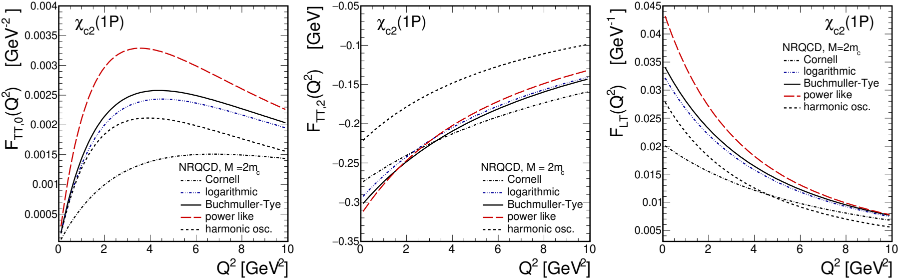
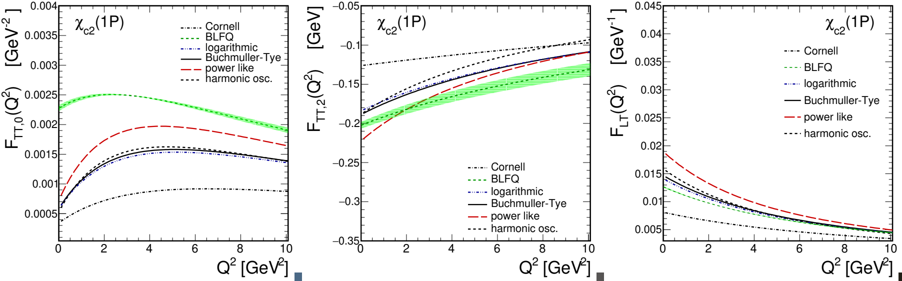
Transition form factors: NRQCD vs LFWF approach

NRQCD



LFWF



The off-shell decay width

The definition of off-shell widths that we were using comes from writing the $\gamma^*\gamma$ cross-section for photons as $(i, j \in T, L)$ $\underline{\mathrm{J.Olsson\,Nucl.Phys.BProc.Suppl.3(1988)613-637}}$

$$\sigma_{ij} = \frac{32\pi}{N_i N_j}(2J+1)\frac{W^2}{2\sqrt{X}}\frac{\Gamma\Gamma^*_{ij}(Q^2)}{(W^2-M^2)^2+M^2\Gamma^2} = \frac{32\pi}{N_i N_j}(2J+1)\frac{W^2}{2M\sqrt{X}}\,\mathrm{BW}(W^2, M^2)\,\Gamma^*_{ij}(Q^2)\,.$$

The kinematical factor $\sqrt{X} = \frac{1}{2}(M^2+Q^2)$, $N_T = 2$, $N_L = 1$, and J is the spin of the resonance, the Breit-Wigner distribution, in the narrow approx. : $\mathrm{BW}(W^2, M^2) \to \frac{\pi}{2M}\delta(W-M)$.
The TT and LT cross sections are obtained from the c.m.-frame helicity amplitudes as

$$\begin{aligned}
\sigma_{\mathrm{TT}} &= \frac{1}{4\sqrt{X}}\Big(\mathcal{M}^*(++)\mathcal{M}(++)+\mathcal{M}^*(+-)\mathcal{M}(+-)\Big)\,\mathrm{BW}(W^2, M^2)\\
&= \frac{(4\pi\alpha_{\mathrm{em}})^2}{4\sqrt{X}}\left\{|F_{\mathrm{TT},2}(Q^2)|^2+\frac{2}{3}\left(1+\frac{Q^2}{M^2}\right)^4 M^4\,|F_{\mathrm{TT},0}(Q^2)|^2\right\}\mathrm{BW}(W^2, M^2)\,,\\
\sigma_{\mathrm{LT}} &= \frac{1}{2\sqrt{X}}\,\mathcal{M}^*(0+)\mathcal{M}(0+)\,\mathrm{BW}(W^2, M^2) = \frac{Q^2\sqrt{X}}{W^2}\,(4\pi\alpha_{\mathrm{em}})^2\,|F_{\mathrm{LT}}(Q^2)|^2\,BW(W^2, M^2)\,,
\end{aligned}$$

The single-tag cross-section, which we write as:

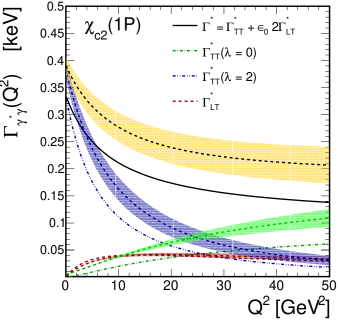
$$\frac{d\sigma}{dQ^2} = 2\int dW\,\frac{dL}{dWdQ^2}\,\Big(\sigma_{\mathrm{TT}}(W^2, Q^2)+\epsilon_0\sigma_{\mathrm{LT}}(W^2, Q^2)\Big)\,.$$

$$\frac{d\sigma}{dQ^2} = 4\pi^2\frac{(2J+1)}{M^2}\left(1+\frac{Q^2}{M^2}\right)^{-1}\frac{2\,dL}{dWdQ^2}\Big|_{W=M}\Gamma_{\gamma^*\gamma}(Q^2)\,,$$

with the effective off-shell width defined as

$$\Gamma_{\gamma^*\gamma}(Q^2) = \Gamma^*_{\mathrm{TT}}(Q^2)+\epsilon_0 2\Gamma^*_{\mathrm{LT}}(Q^2)\,.$$

The off-shell decay width



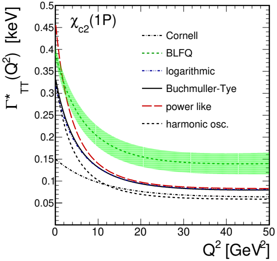
The effective off-shell width

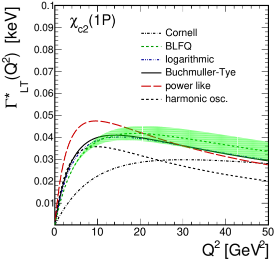
$$\Gamma_{\gamma^*\gamma}(Q^2) = \Gamma^*_{\mathrm{TT}}(Q^2) + \epsilon_0 2\Gamma^*_{\mathrm{LT}}(Q^2)$$

$$\Gamma^*_{\mathrm{TT}}(Q^2) = (4\pi\alpha_{\mathrm{em}})^2 \left\{ \frac{|F_{\mathrm{TT}2}(Q^2)|^2}{80\pi M} + \frac{M^3 |F_{\mathrm{TT}0}(Q^2)|^2}{120\pi}\left(1+\frac{Q^2}{M^2}\right)^4 \right\}.$$

$$\Gamma^*_{\mathrm{LT}}(Q^2) = (4\pi\alpha_{\mathrm{em}})^2 \frac{1}{160\pi}\left(1+\frac{Q^2}{M^2}\right)^2 M Q^2 |F_{\mathrm{LT}}(Q^2)|^2 .$$

For the χ_{c2} we took $\epsilon_0 = 1$, and $M = M_{\chi_{c2}}$.





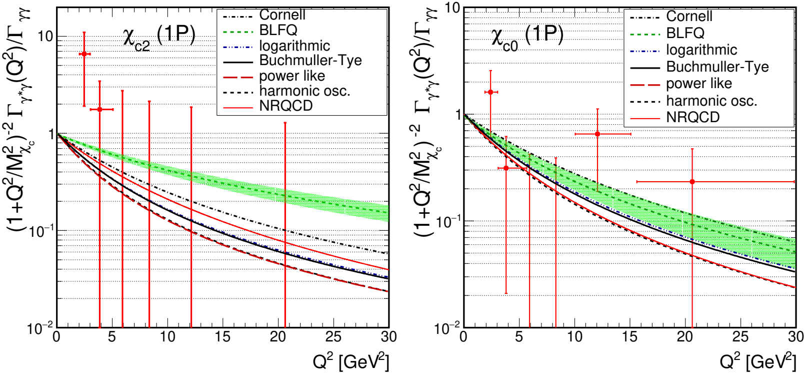
- for $Q^2 < 20$ GeV the dominant is $\lambda = 2$
- for $Q^2 > 50$ GeV the dominant is $\lambda = 0$

The off-shell decay width

Off-shell widths are convention-dependent, and to compare to the experimental data from Ref. M. Masuda et. al Phys. Rev. D 97, 052003(2018) , we note that the Belle collaboration writes

$$\frac{d\sigma}{dQ^2} = 4\pi^2 \frac{(2J+1)}{M^2} \left(1 + \frac{Q^2}{M^2}\right) \frac{2\,dL}{dWdQ^2}\bigg|_{W=M} \Gamma^{\rm Belle}_{\gamma^*\gamma}(Q^2)\,,$$

$$\frac{d\sigma}{dQ^2} = 4\pi^2 \frac{(2J+1)}{M^2} \left(1 + \frac{Q^2}{M^2}\right)^{-1} \frac{2\,dL}{dWdQ^2}\bigg|_{W=M} \Gamma_{\gamma^*\gamma}(Q^2)\,,$$



Summary

- The transition form factors expressed by the Light Front Wave Functions were presented. The wave functions from two different approaches were used: • the Light Front Wave Functions (LFWF) obtained through the Melosh Spin-Rotation transform (for the spin-orbit part), and the solution from the Schrödinger equation for several models of the central potential models of $Q\bar{Q}$ interaction; • Basis Light Front Quantization (BLFQ) approach
- We have found rather wide spread results of radiative decay width $\Gamma_{\gamma\gamma}$: $(0.15 - 0.47)\mathrm{keV}$
- We find the ratio $\Gamma(\lambda = 0)/\Gamma(\lambda = \pm 2)$ of the order of 10^{-3}, which is in agreement with the current experimental agrrement.
- We have defined and calulated the so-called Q^2-dependent off-shell diphoton width and compared to the Belle data. It is rather difficult to conclude on the consistency of the model with rather low statistic available data. Future Belle II or Super τ-Charm facility (STCF) high-statistics data on $\gamma^*\gamma \to \chi_{c2}$ would be very useful to test the wave function and the formalism discussed in our studies.