Entropy production and dissipation in spin hydrodynamics

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Based on: Phys.Lett.B850(2024)138533 + work in progress

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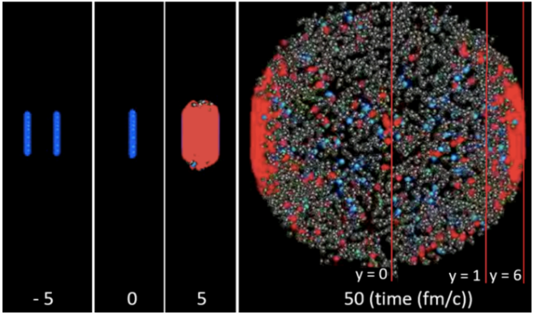
Outline

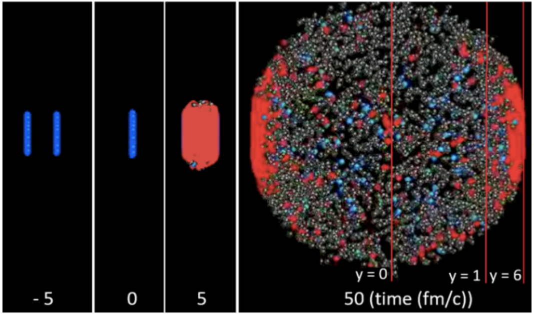
1. **QGP probes and relativistic hydrodynamics**
2. **Spin polarization measurements**
3. **Quantum-based formulation of spin hydrodynamics**
4. **Dissipative currents: Method and results (Ongoing work)**
5. **Conclusions and what's next?**

QGP probes and relativistic hydrodynamics



Snapshot of heavy ion event: Pb+Pb 2.76 TeV with hadrons (blue and grey spheres) and QGP (red) [Ann.Rev.Nucl.Part.Sci.68(2018)339-376]

- **LHC Pb-Pb Collision: For $\sqrt{s_{NN}} = 2.76\,TeV$, the average density at 1 fm/c after impact is approximately 12 GeV/fm^3, which is 20x hadron energy density** [Ann.Rev.Nucl.Part.Sci.68(2018)339-376]



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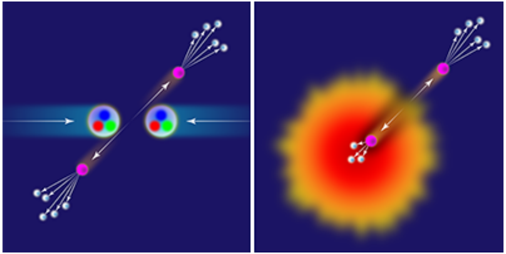
Quarks and gluons produced can't be described as a collection of individual hadrons, but rather a collective medium.

One of the various experimental evidences of the existence of QGP is:

- Jet Quenching

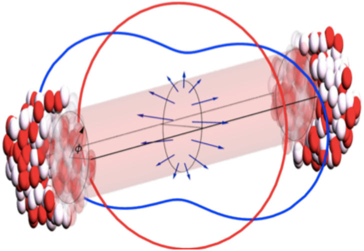


Evidence of b-Jet Quenching in Pb-Pb at $\sqrt{2.76}$ TeV [Phys.Rev.Lett.113,132301]

The results shows a strong suppression in the b-jet yield in Pb-Pb collisions relative to the yield observed in p-p collisions at the same energy.

The collective medium expands and flows as a relativistic hydrodynamic fluid. One of the evidences for the hydrodynamic behavior is:

- Elliptic flow



[Ann.Rev.Nucl.Part.Sci.68(2018)339-376]

A peripheral heavy ion collision creates an elliptical region. For interacting particles model, the final distribution is expected roughly isotropic, while in a fluid, anisotropic particle distribution occurs due to varying pressure gradients.

Machinery [Rept.Prog.Phys.81(2018)4,046001 Florkowski, Spalinski et al.]

1. **Hydrodynamic evolution**

$$\partial_\mu T^{\mu\nu} = 0 \; , \quad \partial_\mu j^{\,\mu} = 0$$

2. **Particle spectra using the Cooper-Frye formula at freeze-out hypersurface**

$$E_p dN/d^3 p$$

3. **Experimental measurements were used to verify these models**

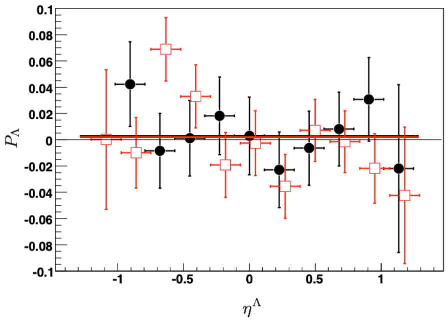
Spin polarization measurements

Globally Polarized Quark-Gluon Plasma in Noncentral $A + A$ Collisions

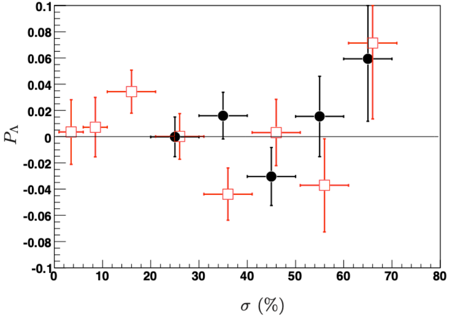
Zuo-Tang Liang and Xin-Nian Wang
Phys. Rev. Lett. **94**, 102301 – Published 14 March 2005; Erratum Phys. Rev. Lett. **96**, 039901 (2006)

- **Produced partons have a large local OAM opposite to the reaction plane**
- **Parton scattering is shown to polarize quarks due to spin-orbital coupling**
- **Values of polarization: 30%**

STAR-RHIC meaurements

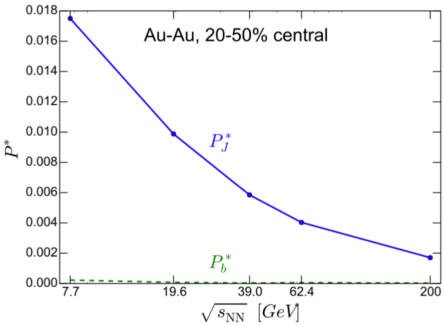


Global polarization of Λ-hyperons as a function of pseudo-rapidity η^Λ. Filled circles for $\sqrt{s_{NN}}$=200 GeV and open squares for $\sqrt{s_{NN}}$=62.4 GeV. [Phys.Rev.C76,024915].

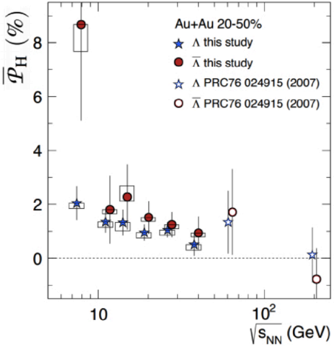


Global polarization of Λ-hyperons as a function of centrality. Filled circles for $\sqrt{s_{NN}}$=200 GeV and open squares for $\sqrt{s_{NN}}$=62.4 Gev [Phys.Rev.C76,024915]

“Spin polarization is driven by hydrodynamic vorticity”

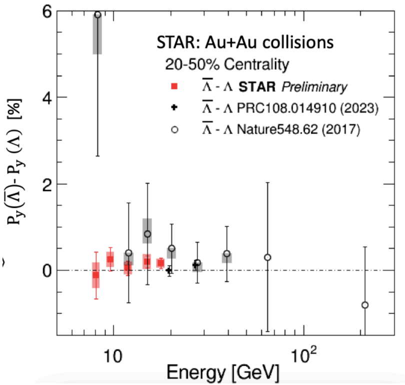


Global polarization of Λ [Eur.Phys.J.C(2017)77:213]



Average Λ global polarization [STAR, L. Adamczyk et al., Nature 548, 62 (2017)]

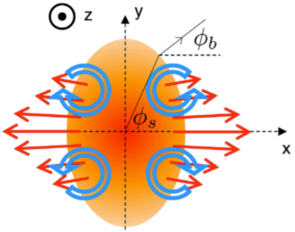
Recent average Λ global polarization-STAR RHIC



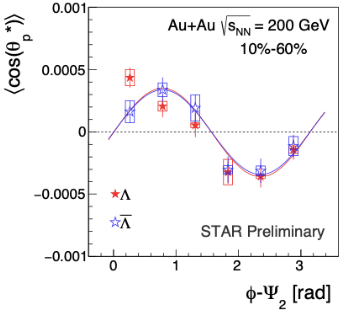
[SQM conference 2024]

No splitting between Λ and $\bar{\Lambda}$ global polarization

Angle-dependent polarization along beam-direction



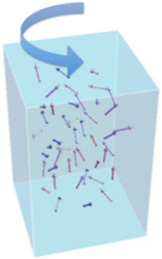
Vorticities along the beam direction (open arrows) induced by anisotropic flow (solid arrows) in the (x-y)-plane [Phys. Rev. Lett. 123, 132301]



Pz of Λ hyperons as a function of azimuthal angle ϕ in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV (Local Polarization) [Phys.Rev.Lett.123,132301]

Quantum-based formulation of spin hydrodynamics

Given the polarization measurements, spin hydrodynamics emerges as a potential theory that takes into account the particles' spin degrees of freedom (D.O.F.), in addition to their energy and momentum, before the fluid evolves hydro-dynamically [Florkowski, Friman et. al Phys.Rev.C97(2018)4,041901]



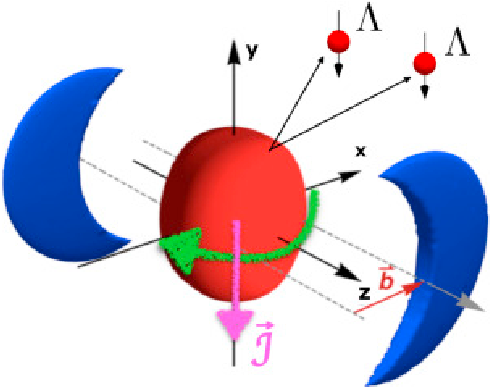
What is a fluid with spin ?

A fluid with spin is a fluid having a macroscopic spin density and which thus needs a spin tensor $\mathcal{S}^{\lambda,\mu\nu}$ to be described, besides the stress-energy tensor $T^{\mu\nu}$

$$T^{\mu\nu}(x) = \mathrm{tr}[\widehat{\rho}\widehat{T}^{\mu\nu}(x)]$$

$$\mathcal{S}^{\lambda,\mu\nu}(x) = \mathrm{tr}[\widehat{\rho}\widehat{\mathcal{S}}^{\lambda,\mu\nu}(x)]$$

- **Initial state : 2 nuclei colliding forming a strongly interacting system described by mixed state $\widehat{\rho}_{(0)}$**
- **How to evolve $\widehat{\rho}_{(0)}$? (Typical QFT problem)**



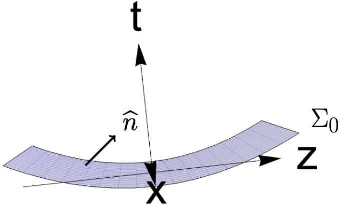
R.Ryblewski et al. [Prog.Part.Nucl.Phys.108(2019)103709]

At initial hyperspace Σ_0, local equilibrium is achieved, where entropy is maximum provided that the mean vales of energy, momentum, particle number, and spin densities are their actual values:

$$S = -\operatorname{Tr}(\widehat{\rho}\log\widehat{\rho})$$

$$F[\widehat{\rho}] = -Tr[\widehat{\rho}\log\widehat{\rho}] - \int d\Sigma_0\, n_\mu\, (T^{\mu\nu}_{\rm LE} - T^{\mu\nu})\,\beta_\nu(x) - \int d\Sigma_0\, n_\mu\, (j^\mu_{\rm LE} - j^\mu)\,\zeta(x)$$
$$- \int d\Sigma_0\, n_\mu \left(S^{\mu\lambda\nu}_{\rm LE} - S^{\mu\lambda\nu}\right)\Omega_{\lambda\nu}(x)$$

$$T^{\mu\nu}_{\rm LE} \sim Tr\left[\widehat{\rho}\,\widehat{T}^{\mu\nu}\right]$$
$$T^{\mu\nu} \equiv \text{Actual Value}$$



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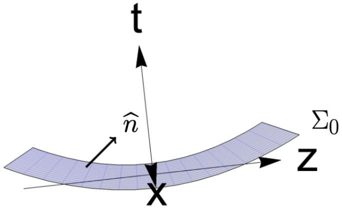


$$F\,[\widehat{\rho}] = -\,Tr\,[\widehat{\rho}\log\widehat{\rho}] - \int d\Sigma_0\; n_\mu\,(T^{\mu\nu}_{\mathrm{LE}} - T^{\mu\nu})\,\beta_\nu(x) - \int d\Sigma_0\; n_\mu\,(j^\mu_{\mathrm{LE}} - j^\mu)\,\zeta(x)$$
$$- \int d\Sigma_0\; n_\mu\left(S^{\mu\lambda\nu}_{\mathrm{LE}} - S^{\mu\lambda\nu}\right)\Omega_{\lambda\nu}(x)$$

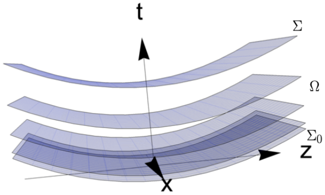
$$T^{\mu\nu}_{\mathrm{LE}} \sim Tr\left[\widehat{\rho}\,\widehat{T}^{\mu\nu}\right]$$
$$T^{\mu\nu} \equiv \text{Actual Value}$$

$$\widehat{\rho}_{\mathrm{LE}} = \frac{1}{Z}\exp\left[-\int_{\Sigma_0} \mathrm{d}\,\Sigma_\mu\,\left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\widehat{j}^\mu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu\lambda\nu}\right)\right]$$

The Lagrange multipliers are obtained by solving the constraint equations at Σ_0. Their evolution is determined by solving the conservation equations:

- $\beta^\mu \to u^\mu = \beta^\mu/\sqrt{\beta^2} \quad T = 1/\sqrt{\beta^2}$
- $\zeta = \mu/T$
- $\Omega_{\mu\nu} = \omega_{\mu\nu}/T$
- Thermal Shear: $\xi_{\mu\nu} = \frac{1}{2}\left(\nabla_\mu\beta_\nu + \nabla_\nu\beta_\mu\right)$ Thermal Vorticity: $\varpi_{\mu\nu} = \frac{1}{2}\left(\nabla_\nu\beta_\mu - \nabla_\mu\beta_\nu\right)$

Using Gauss and linear response theorems:



$$\widehat{\rho} = \frac{1}{Z}\exp\left[\underbrace{-\int_\Sigma \mathrm{d}\,\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\widehat{j}^\mu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu\lambda\nu}\right)}_{\widehat{\rho}_{\mathrm{LE}}(t)\ at\ \Sigma} + \underbrace{\int_\Omega d\Omega\ \widehat{T}_S^{\mu\nu}\xi_{\mu\nu} + \widehat{T}_A^{\mu\nu}(\Omega_{\mu\nu} - \varpi_{\mu\nu}) - \frac{1}{2}\widehat{\mathcal{S}}^{\mu\lambda\nu}\nabla_\mu\Omega_{\lambda\nu}}_{\text{Dissipative Corrections}}\right]$$

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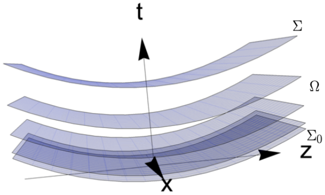


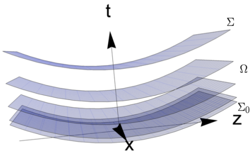
$$\widehat{\rho} = \frac{1}{Z} \exp \left[\underbrace{- \int_{\Sigma} \mathrm{d}\,\Sigma_{\mu} \left(\widehat{T}^{\mu\nu} \beta_{\nu} - \zeta \widehat{j}^{\mu} - \frac{1}{2} \Omega_{\lambda\nu} \widehat{\mathcal{S}}^{\mu\lambda\nu} \right)}_{\widehat{\rho}_{\mathrm{LE}}(t)\ at\ \Sigma} + \underbrace{\int_{\Omega} d\Omega\ \widehat{T}_{S}^{\mu\nu} \xi_{\mu\nu} + \widehat{T}_{A}^{\mu\nu} (\Omega_{\mu\nu} - \varpi_{\mu\nu}) - \frac{1}{2} \widehat{\mathcal{S}}^{\mu\lambda\nu} \nabla_{\mu} \Omega_{\lambda\nu}}_{\text{Dissipative Corrections}} \right]$$

This implies that dissipation in spin hydrodynamics occurs when:

$$\xi \neq 0 \quad \Omega \neq \varpi \quad \nabla\Omega \neq 0$$

Near local equilibrium at the hypersuface Σ, the entropy is defined as:

$$
\begin{aligned}
S &= -\operatorname{Tr}\left[\widehat{\rho}_{\mathrm{LE}}(t) \log \widehat{\rho}_{\mathrm{LE}}(t)\right] \\
&= \log Z_{\mathrm{LE}} + \int_{\Sigma} \mathrm{d}\Sigma_{\mu} \left[\operatorname{Tr}\left(\widehat{\rho}_{\mathrm{LE}} \widehat{T}^{\mu\nu}\right)\beta_{\nu} - \zeta \operatorname{Tr}\left(\widehat{\rho}_{\mathrm{LE}} \widehat{j}^{\mu}\right) - \frac{1}{2}\Omega_{\lambda\nu} \operatorname{Tr}\left(\widehat{\rho}_{\mathrm{LE}} \widehat{\mathcal{S}}^{\mu\lambda\nu}\right)\right]
\end{aligned}
$$



Can we define an entropy current out of S ? In other words, is it possible to show that $\log Z_{\mathrm{LE}}$ is an extensive quantity?

Therefore, entropy current exists:

$$S = \int_\Sigma \mathrm{d}\Sigma_\mu \, \phi^\mu + T^{\mu\nu}_{\mathrm{LE}} \beta_\nu - \zeta j^\mu_{\mathrm{LE}} - \frac{1}{2} \Omega_{\lambda\nu} \mathcal{S}^{\mu\lambda\nu}_{\mathrm{LE}}$$

$$s^\mu_{LE} = \phi^\mu + T^{\mu\nu}_{\mathrm{LE}} \beta_\nu - \zeta j^\mu_{\mathrm{LE}} - \frac{1}{2} \Omega_{\lambda\nu} \mathcal{S}^{\mu\lambda\nu}_{\mathrm{LE}} \, .$$

In quantum theory, we only have the total entropy, not the entropy current. We need to construct an entropy current through an integral. However, this introduces ambiguities, as several fields can lead to the same integral. However if $s^\mu - s^\mu_{LE} \perp n^\mu$,

$$s^\mu = \phi^\mu + T^{\mu\nu} \beta_\nu - \zeta j^\mu - \frac{1}{2} \Omega_{\lambda\nu} \mathcal{S}^{\mu\lambda\nu} \qquad \phi^\mu = \int_0^T \frac{\mathrm{d}T'}{T'^2} \left(T^{\mu\nu}[T'] u_\nu - \mu j^\mu[T'] - \frac{1}{2} \omega_{\lambda\nu} \mathcal{S}^{\mu\lambda\nu}[T'] \right)$$

$$
\begin{aligned}
\partial_\mu s^\mu = &\left(T_S^{\mu\nu} - T_{S(\mathrm{LE})}^{\mu\nu}\right)\xi_{\mu\nu} - (j^\mu - j^\mu_{\mathrm{LE}})\,\partial_\mu\zeta + \left(T_A^{\mu\nu} - T_{A(\mathrm{LE})}^{\mu\nu}\right)(\Omega_{\mu\nu} - \varpi_{\mu\nu}) \\
&- \frac{1}{2}\left(\mathcal{S}^{\mu\lambda\nu} - \mathcal{S}^{\mu\lambda\nu}_{\mathrm{LE}}\right)\partial_\mu\Omega_{\lambda\nu}
\end{aligned}
$$

$$
\varpi_{\mu\nu} = -\frac{\partial_{[\mu}u_{\nu]}}{T}, \quad \eta_{\mu\nu} = \frac{\partial_{(\mu}u_{\nu)}}{T}, \quad \Omega_{\mu\nu} = \omega_{\mu\nu}/T
$$

The goal now is to determine the dissipative currents:

$$\delta T_S^{\mu\nu} = \left(T_S^{\mu\nu} - T_{S(\mathrm{LE})}^{\mu\nu} \right) \ , \quad \delta T_A^{\mu\nu} \ , \quad \delta j^\mu \ , \quad \delta \mathcal{S}^{\lambda\mu\nu}$$

In general the average values can be decomposed into equilibrium part and dissipative part. For example,

$$T^{\mu\nu} = T_{eq}^{\mu\nu} + \delta T_S^{\mu\nu} + \delta T_A^{\mu\nu}$$
$$S^{\lambda\mu\nu} = S_{eq}^{\lambda\mu\nu} + \delta S^{\lambda\mu\nu}$$
$$j^\mu = j_{eq}^\mu + \delta j^\mu$$

To then solve,

$$\partial_\mu T^{\mu\nu} = 0 \quad , \quad \partial_\lambda S^{\lambda\mu\nu} = T^{\nu\mu} - T^{\mu\nu} \quad , \quad \partial_\mu j^\mu = 0$$

Dissipative currents: Method and results (Ongoing work)

- **The main goal (as mentioned in the beginning) is to find:**

$$\delta T_s^{\mu\nu} = (T_s^{\mu\nu} - T_{s(LE)}^{\mu\nu}) \,, \quad \delta T_A^{\mu\nu} \,, \quad \delta S^{\lambda\mu\nu}, \quad \delta j^{\mu}$$

- **Hence we expand the above interms of all gradients in the system. For example:**

$$\delta T_S^{\mu\nu} = H^{\mu\nu\rho\sigma}\xi_{\rho\sigma} + K^{\mu\nu\rho}\partial_\rho\zeta + L^{\mu\nu\rho\sigma}(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}) + M^{\mu\nu\rho\sigma\tau}\partial_\rho\Omega_{\sigma\tau}$$

- **The goal thus is to determine:**

$$H^{\mu\nu\rho\sigma},\ K^{\mu\nu\rho},\ L^{\mu\nu\rho\sigma},\ M^{\mu\nu\rho\sigma\tau}$$

- **We expand from $SO(3)$ invariant global equilibrium**

$$u^{\mu}\ , \quad \Delta^{\mu\nu}\ , \quad \epsilon^{\lambda\mu\nu\gamma}$$

- **Using the irreducible representation of $SO(3)$, a vector, symmetric rank-2 tensor, and antisymmetric tensor can be written as:**

$$V^{\mu} = (0 \oplus 1) = (u^{\mu} \oplus \Delta^{\mu}_{\alpha}),$$
$$S^{\mu\nu} = (0 \oplus 0 \oplus 1 \oplus 2) = (u^{\mu}u^{\nu} \oplus \Delta^{\mu\nu} \oplus u^{\mu}\Delta^{\nu}_{\alpha} + u^{\nu}\Delta^{\mu}_{\alpha} \oplus \Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta} + \Delta^{\nu}_{\alpha}\Delta^{\mu}_{\beta}),$$
$$A^{\mu\nu} = (1 \oplus 1) = (u^{\mu}\Delta^{\nu}_{\alpha} - u^{\nu}\Delta^{\mu}_{\alpha} \oplus \epsilon^{\mu\nu\tau\alpha}u_{\tau}).$$

- **Matching Condition**

$$n_\mu(\delta T_S^{\mu\nu} + \delta T_A^{\mu\nu}) = 0, \quad n_\mu \delta j^\mu = 0, \quad n_\mu \delta S^{\mu\lambda\nu} = 0$$

- **Entropy positivity**

$$\partial_\mu s^\mu \geq 0$$

- **We were able to reproduce what is the in the literature so far:**

$$\delta T_S^{\mu\nu} = \bar{h}_2 \frac{\Delta^{\mu\nu}}{T}\theta + \frac{2h_3}{T}\sigma^{\mu\nu}$$

$$\delta T_A^{\mu\nu} = q_4 \Delta^{[\mu[\sigma}\Delta^{\nu]\rho]}(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}) + ...$$

$$\delta j^\mu = i_2 \nabla^\mu \zeta$$

$$\delta S^{\lambda\mu\nu} = ...$$

Conclusions and what's next?

- We used first-principle density operator method to derive the entropy current and the entropy production rate in relativistic fluids for particles with spin.
- Established a method based on $SO(3)$ irreducible representations to drive the dissipative currents.

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