Asymptotic Safety and the Litim Sannino Model

Daniele Rizzo

Last year PhD student @ NCBJ
Warsaw - Poland

Work in collaboration with

Daniel Litim

During my visit at

Sussex University
Brighton, UK

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Asymptotic Behaviors

$$\beta(g) \equiv \frac{dg}{d\log\mu} = A\,g^2$$

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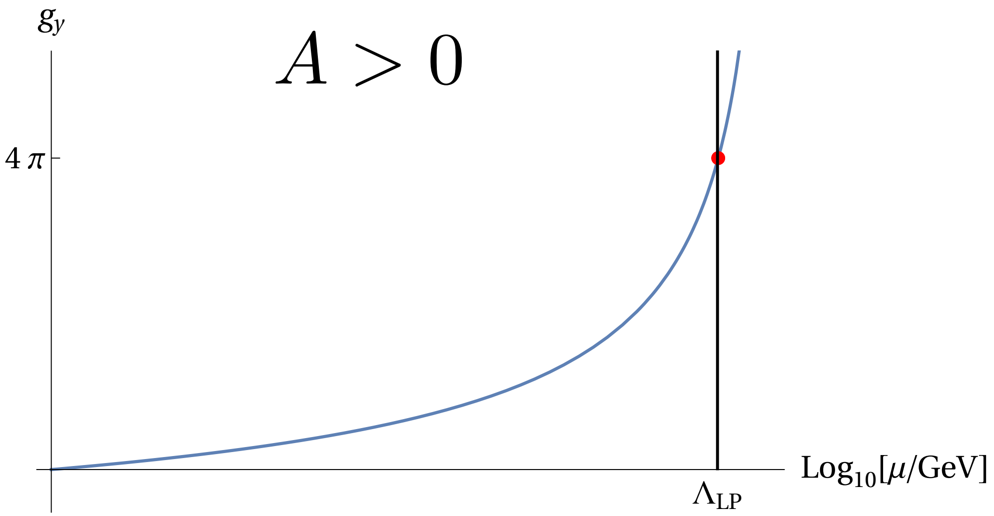
$$A > 0$$



Landau pole

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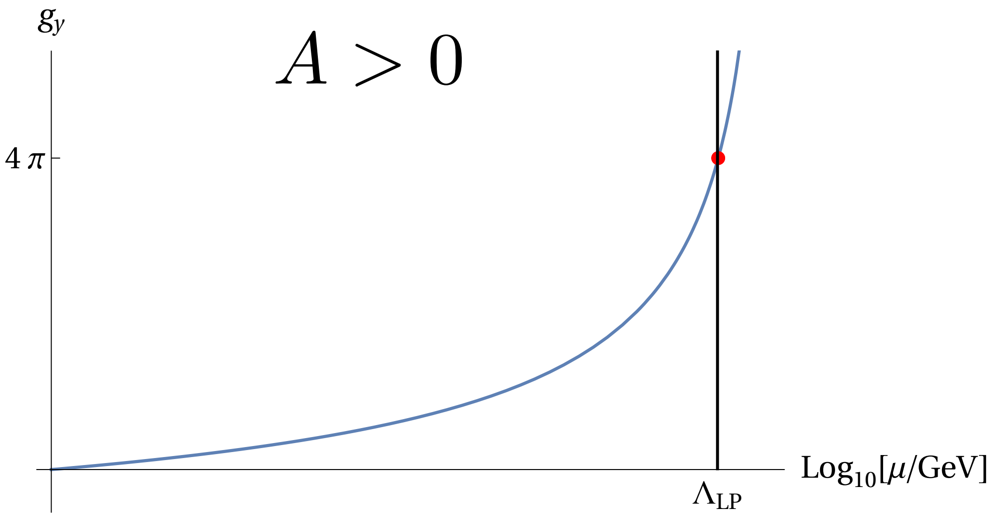

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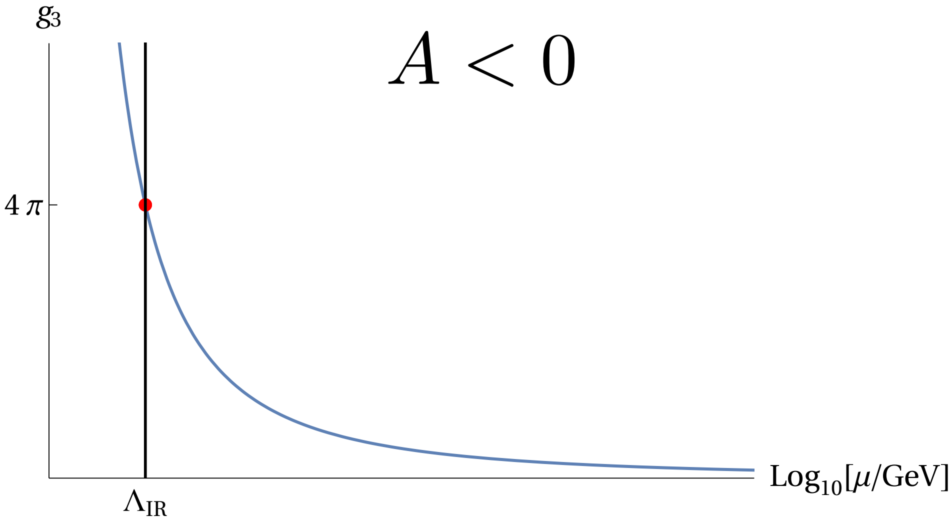

Landau pole

Asymptotic freedom

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Asymptotic Safety

$$\beta(g) \equiv \frac{dg}{d\log\mu} = A\,g^2 + B\,g^3$$

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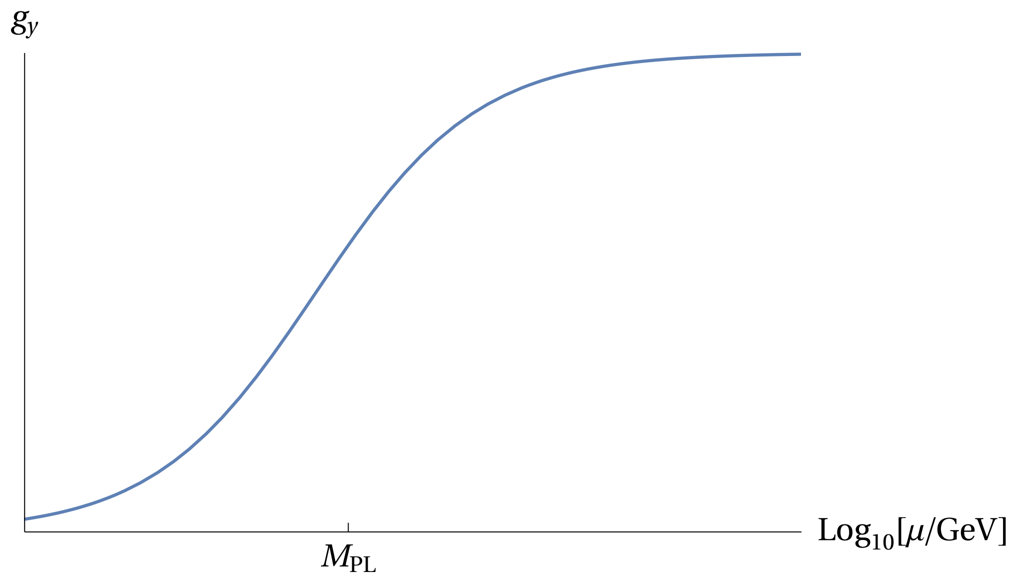
Asymptotic Safety

$$\beta(g) \equiv \frac{dg}{d\log\mu} = A\,g^2 + B\,g^3$$

There is a specific value $g^* = -B/A$

$$\beta(g^*) = 0$$

Fixed Point!



Asymptotic safety

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Gauge-Yukawa Theory

Litim, Sannino (2014)

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Gauge-Yukawa Theory

Gauge $F^a_{\mu\nu}\,(a = 1, ..., N_C^2 - 1)$

Fermions $Q_i\,(i = 1, ..., N_F)$

Scalars $H \in N_F \times N_F$

$$\epsilon \equiv \frac{N_F}{N_C} - \frac{11}{2}$$

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} + \mathrm{Tr}\left(\partial_\mu H^\dagger \partial^\mu H\right) + \mathrm{Tr}\left(\bar{Q}\,i\not{D}\,Q\right)$$

$$-y\,\mathrm{Tr}\left(\bar{Q}_L H Q_R + \bar{Q}_R H^\dagger Q_L\right) - u\,\mathrm{Tr}\left(H^\dagger H\right)^2 - v\left(\mathrm{Tr}\,H^\dagger H\right)^2$$

Litim, Sannino (2014)

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Conformal Window

Under perturbative expansion, the theory has an Ultra Violet Fixed Point:

$$g^* = +0.456\epsilon + 0.781\epsilon^2 + 6.610\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$y^* = +0.211\epsilon + 0.508\epsilon^2 + 3.322\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$u^* = +0.200\epsilon + 0.440\epsilon^2 + 2.693\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$v^* = -0.137\epsilon - 0.632\epsilon^2 - 4.313\epsilon^3 + \mathcal{O}(\epsilon^4)$$

Bond et al. 1710.07615
Litim et al. 2307.08747

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Let us be a little bit more quantitative and ask the questions:

- For what values of the Veneziano parameter do we actually have a fixed point?
- What can cause a fixed point to disappear?

The values of the Veneziano parameter for which the fixed point exist is called

CONFORMAL WINDOW

Vacuum Stability at the Fixed Point

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Vacuum Stability at the Fixed Point

$$u^* = +0.200\epsilon + \mathcal{O}(\epsilon^2)$$

$$v^* = -0.137\epsilon + \mathcal{O}(\epsilon^2)$$

At the fixed point, the potential is given by:

$$V^*(H) = u^* \, \mathrm{Tr} \left(H^\dagger H \right)^2 + v^* \left(\mathrm{Tr} \, H^\dagger H \right)^2$$

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It was shown that the vacuum stability is obtained provided

$$\begin{cases} u^* > 0 \\ u^* + v^* \geq 0 \end{cases}$$

OR

$$\begin{cases} u^* < 0 \\ u^* + v^*/N_F \geq 0 \end{cases}$$

Litim, Mojaza, Sannino (2015)

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2-1-1 $$u^* + v^* = 0.063\epsilon \geq 0$$

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4-3-3 $\quad u^* + v^* = 0.063\epsilon - 0.192\epsilon^2 - 1.62\epsilon^3 \geq 0$

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4-3-3 $\quad u^* + v^* = 0.063\epsilon - 0.192\epsilon^2 - 1.62\epsilon^3 \geq 0 \qquad \epsilon \leq 0.147$

Using Padè resummation this number is tighten further more to be $\epsilon \leq 0.087$

Fixed Point Merger

$$A + B\,g + g^2 = 0$$

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Fixed Point Merger

$$A + B\,g + g^2 = 0 \qquad\qquad g^*_\pm = \frac{-B \pm \sqrt{B^2 - 4A}}{2}$$

- If the expression inside the squared root is negative, we have a pair of complex conjugate poles.
- On the other hand, if the expression inside squared root is positive, we have two real solutions, with a split given by the squared root term.

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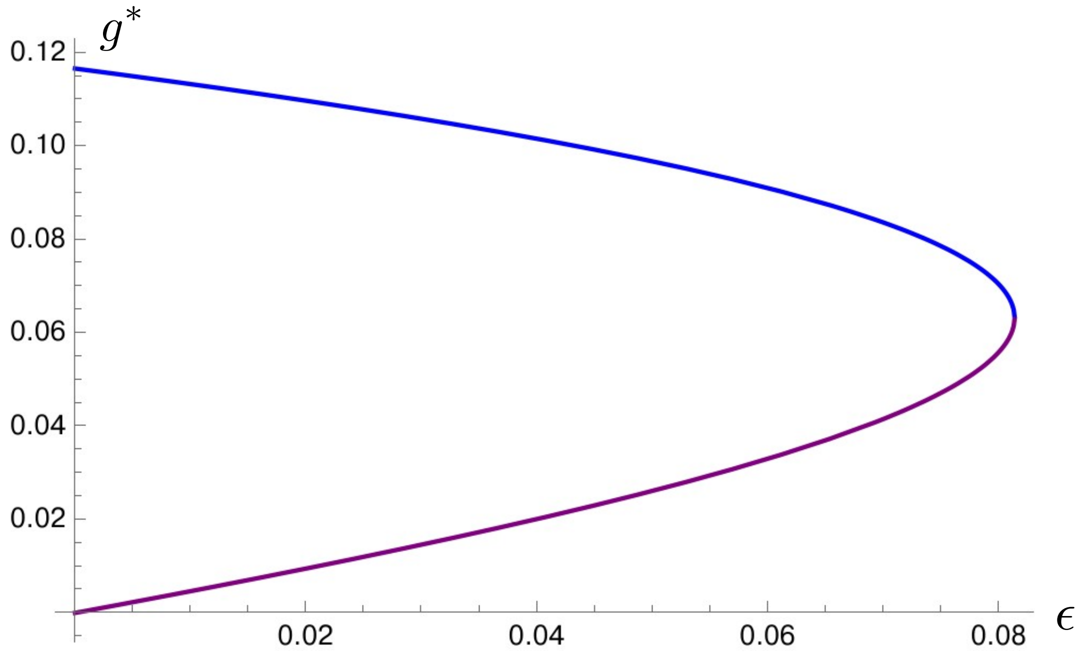

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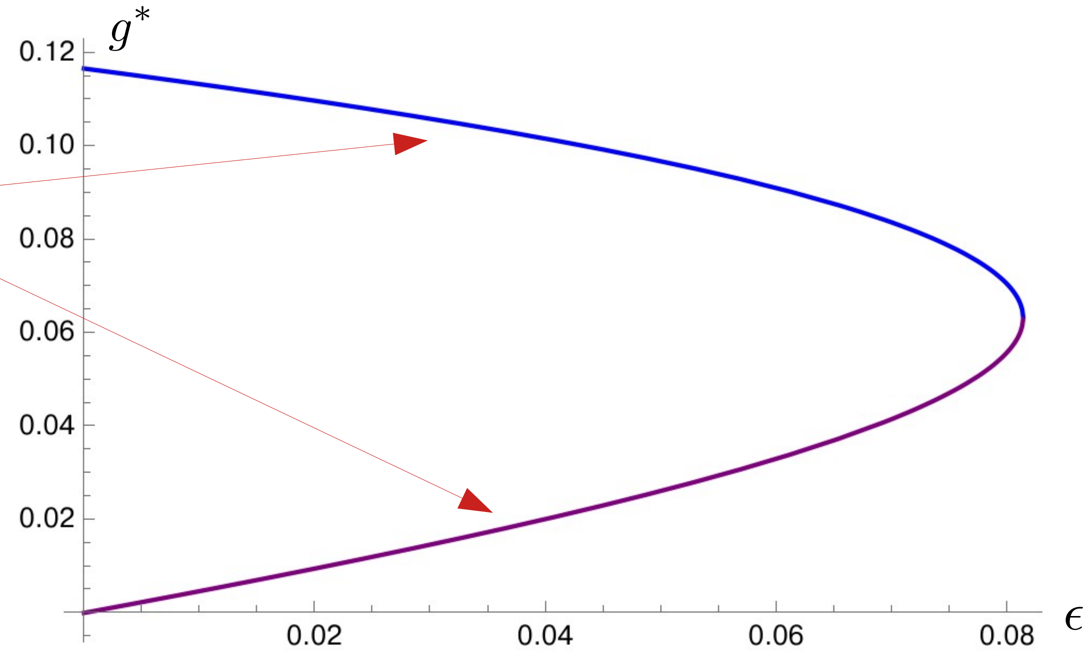
For small values of the Veneziano parameter the FPs are real



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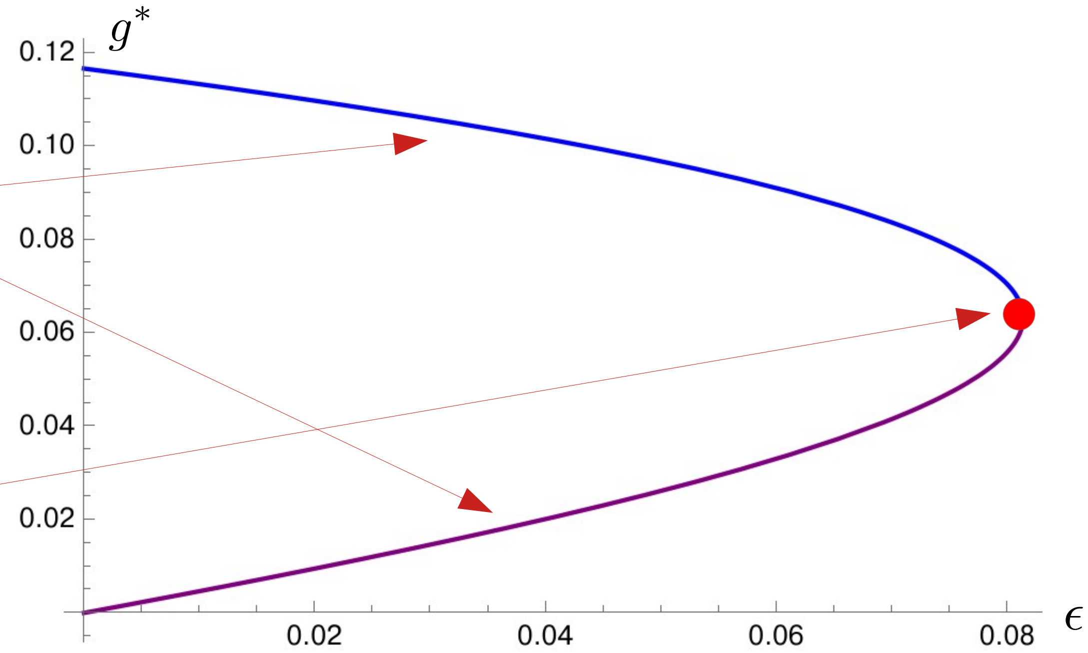
For small values of the Veneziano parameter the FPs are real

At a certain value, the solutions become complex and the FPs disappear



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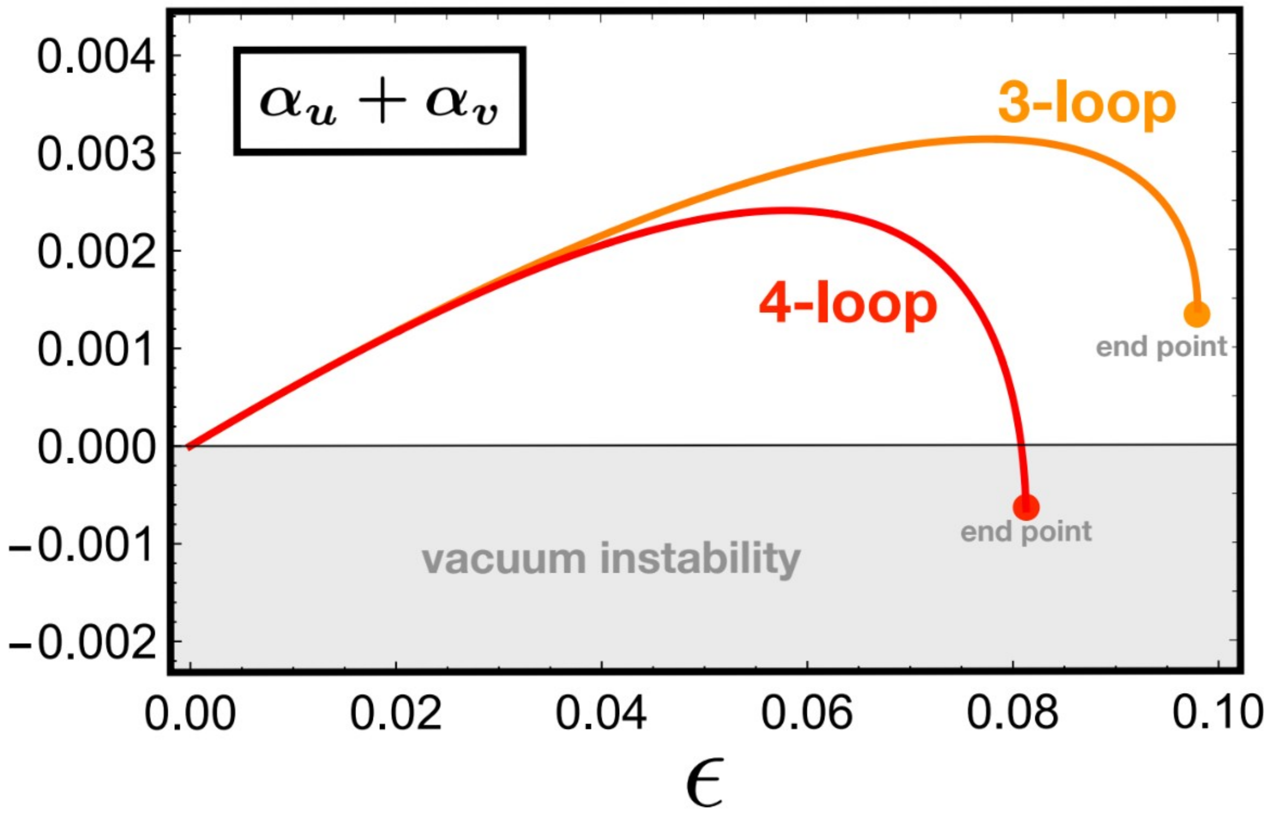
Conformal Window

Is the conformal window closing because of Vacuum Stability or a Fixed Point merger?



Plot kindly shared by Nahzaan Riyaz.

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Beyond marginal operators

$$v\left(\mathrm{Tr}\, H^\dagger H\right)^2$$

$$u\,\mathrm{Tr}\left(H^\dagger H\right)^2$$

$$y\,\mathrm{Tr}(\bar{Q}HQ)$$

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Beyond marginal operators

$$v\left(\mathrm{Tr}\,H^\dagger H\right)^2 \longrightarrow \sum_n \gamma_n (\mathrm{Tr} H^\dagger H)^{n-2} (\mathrm{Tr} H^\dagger H)^2$$

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Functional Renormalization Group

$\partial_t U(\mathrm{Tr} H^\dagger H)$ $\partial_t C(\mathrm{Tr} H^\dagger H)$ $\partial_t Y(\mathrm{Tr} H^\dagger H)$

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Functional Renormalization Group

$$\partial_t U(\mathrm{Tr} H^\dagger H) \qquad \partial_t C(\mathrm{Tr} H^\dagger H) \qquad \partial_t Y(\mathrm{Tr} H^\dagger H)$$

Wetterich equation

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{STr}\left[\partial_t R_k \cdot \left(\Gamma_k^{(2)} + R_k\right)^{-1}\right]$$

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Regulator

$$R_k = Z_k\,(k^2 - q^2)\,\Theta(k^2 - q^2)$$

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Flow

We define dimension-less couplings:

$$U = k^4\, u \qquad \mathrm{Tr}\, H^\dagger H = \rho\, k^2 \qquad \mathrm{Tr}(H^\dagger H)^2 = \tau\, k^4$$

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Flow

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And compute the flow:

$$\partial_t u = -4u + (2+\eta_H)\rho u' + \frac{1}{2}\left(\frac{1}{1+u'+4\rho\, c} + \frac{1}{1+u'}\right) - \frac{2N_C}{N_F}\frac{1}{1+\rho\, y^2}$$

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Canonical dimension

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Anomalous dimension

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Quantum corrections from the scalar potential

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Canonical dimension

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Quantum corrections from the scalar potential

Quantum corrections from the yukawa

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Flow

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Tuğba Büyükbeşe, PhD Thesis

$$\partial_t c = 2\eta_H c + (2+\eta_H)\rho c' - \frac{2N_C}{N_F}\frac{y^4}{(1+\rho y^2)^3}$$
$$+\frac{1}{2}\left(-\frac{128\rho^3 c^5}{(1+u')^3(1+4\,\rho\, c+u')^3} + \frac{64\rho^2 c^3(c-\rho c')}{(1+u')^2(1+4\,\rho\, c+u')^3} - \frac{8\rho c c'}{(1+4\,\rho\, c+u')^3} - \frac{48\rho^2 c^2 c'}{(1+u')(1+4\,\rho\, c+u')^3} + \frac{16c^2}{(1+4\,\rho\, c+u')^3} - \frac{2c'}{(1+4\,\rho\, c+u')^2}\right)$$

Tuğba Büyükbeşe, PhD Thesis

$$\partial_t y = -3\alpha_g y(0) + \frac{1}{2}(2\eta_\psi + \eta_H)y + (2+\eta_\phi)\rho y' - \frac{1}{2}\left(\frac{y'}{(1+4\,\rho\, c+u')^2} + \frac{y'}{(1+u')^2}\right)$$
$$+\frac{y^3}{2\,(1+\rho y^2)\,(1+4\rho\, c+u')}\left(\frac{1}{1+4\rho\, c+u'} + \frac{1}{1+\rho y^2}\right) - \frac{y^3}{2\,(1+u')\,(1+\rho y^2)}\left(\frac{1}{1+\rho y^2} + \frac{1}{1+u'}\right)$$

Fixed Point

$$\partial_t u = 0 \qquad \partial_t c = 0 \qquad \partial_t y = 0$$

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$$\partial_t u = 0 \qquad \partial_t c = 0 \qquad \partial_t y = 0$$

$$u(\rho) = \sum_{n=0}^{N\to\infty} \alpha_n \rho^{n+1} \qquad c(\rho) = \sum_{n=1}^{N\to\infty} \gamma_n \rho^{n-1} \qquad y(\rho) = \sum_{n=0}^{N\to\infty} y_n \rho^{n}$$

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$$\partial_t \alpha_n = 0 \qquad \partial_t \gamma_n = 0 \qquad \partial_t y_n = 0$$

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Fixed Point & Power Counting in ε

Coupling	FP	Coupling	FP	Coupling	FP
γ_1	$+0.199781\epsilon$	α_1	$+0.0625304\epsilon$	y_0	$+0.458831\sqrt{\epsilon}$
γ_2	$-0.404135\epsilon^3$	α_2	$-0.0844283\epsilon^3$	y_1	$+0.318417\sqrt{\epsilon^5}$
γ_3	$+0.558651\epsilon^4$	α_3	$+0.0721923\epsilon^4$	y_2	$-0.468528\sqrt{\epsilon^7}$
γ_4	$-0.812282\epsilon^5$	α_4	$-0.0699564\epsilon^5$	y_3	$+0.626392\sqrt{\epsilon^9}$
γ_5	$+1.16104\epsilon^6$	α_5	$+0.0706016\epsilon^6$	y_4	$-0.798058\sqrt{\epsilon^{11}}$
	$\vdots$		$\vdots$		$\vdots$

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Vacuum stability at the UV fixed point

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At leading order in ε a re-summation of the couplings can be performed:

$$u^*(\rho) = \alpha_1^* \rho^2 + \frac{A^2\,\rho^2}{4}\log(1 + A\,\rho) + \frac{B^2\,\rho^2}{4}\log(1 + B\,\rho) - \frac{N_c}{N_F} D^2\,\rho^2 \log(1 + D\,\rho)$$

$A \equiv 2\alpha_1^*$ $\quad$ $B \equiv 2\alpha_1^* + 4\gamma_1^*$ $\quad$ $D \equiv \alpha_y^*$

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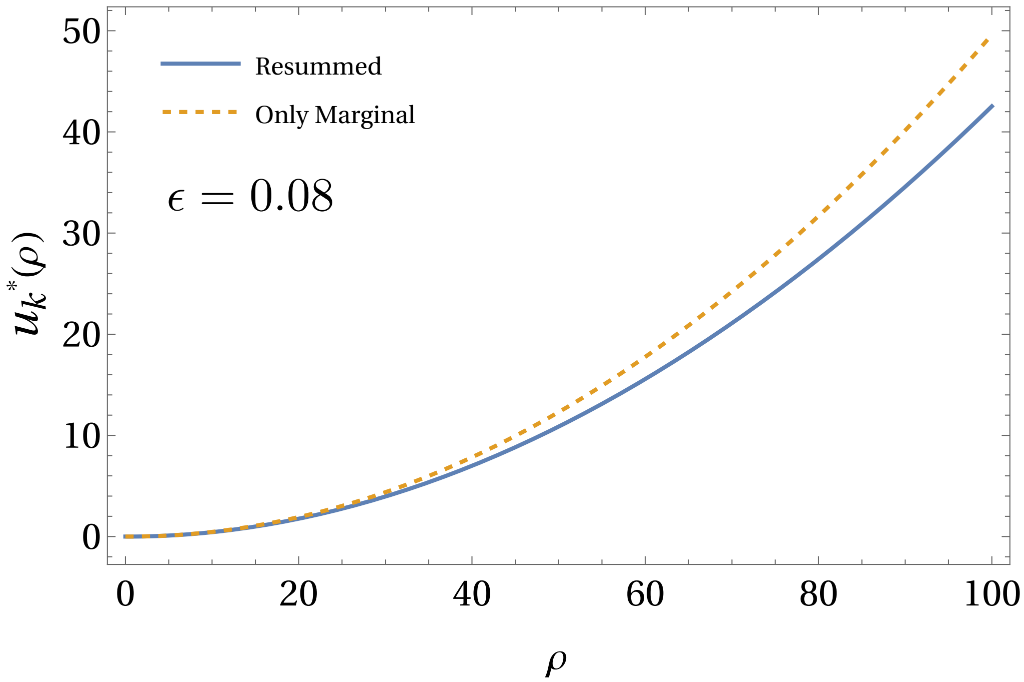

$$A \equiv 2\alpha_1^* \qquad B \equiv 2\alpha_1^* + 4\gamma_1^*$$

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Conclusion

- It is not completely understood whether the conformal window of gauge-Yukawa theories closes because of vacuum instability or because of two FPs merging.

- The inclusion of beyond marginal operators can spoil the stability of the scalar potential.

- We have computed the FP of infinitely many higher dimensional operators at leading order in ϵ and found a power counting argument.

- Because of the power counting, it was possible to perform a re-summation of the scalar potential and study the stability for high values of the field: the potential remains stable!

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