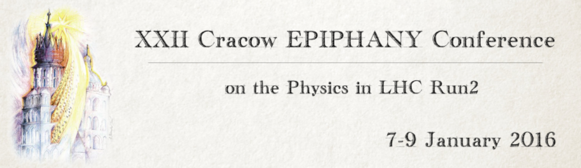
Splitting Functions for High-Energy Factorization

O.Gituliar, M.Hentschinski, K.Kutak

based on arXiv:1511.08439



In this talk

- How are hadron collisions described nowadays?

  Here I consider only *long-distance aspects*. See the next talk by M.Serino on *short-distance aspects*.
- Is there some room for improvements?
- What is our contribution?

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of Krzysztof Kutak

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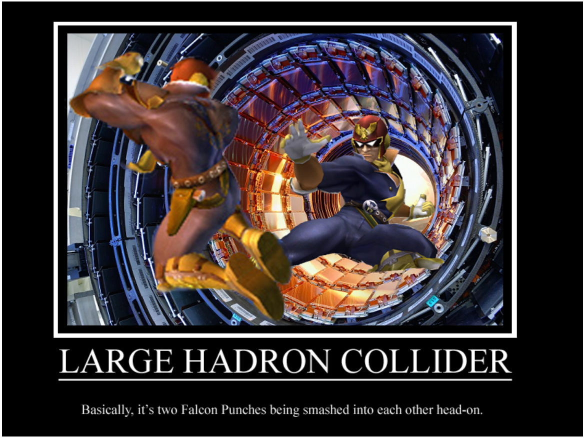

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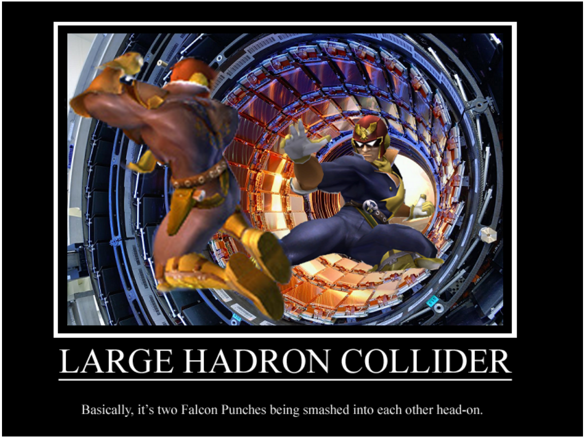
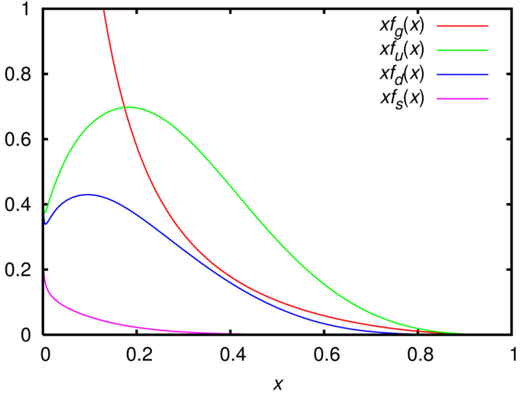


Falcon Punches? REALLY?!

How are hadron collisions described nowadays?

But we know that it is like...



The CTEQ6 parton distribution functions in the $\overline{\text{MS}}$ renormalization scheme and Q = 2 GeV for gluons, up, down, and strange quarks.

Parton Distribution Functions.

A brief reminder: Collinear Factorization and PDFs

Let us consider PDFs $f_i(x, \mu)$ which depend only on

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  - **Collinear Factorization** framework with 2 master formulae
    Hadronic Cross-Section

$$\sigma = \sum_i \int \frac{dx}{x} \boxed{f_i(x, \mu)} \, \hat{\sigma}_i(x, \alpha_s(\mu), Q^2/\mu^2) + O(\Lambda_{\text{QCD}}/Q)$$

DGLAP Evolution Equations

$$\frac{\partial}{\partial \ln \mu} \boxed{f_i\,(x, \mu)} = \sum_{j=g,q,\bar{q}} P_{ij}(x) \otimes \boxed{f_j(x, \mu)}$$

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It makes possible to

- calculate collinear-unsafe observables
- build collinear parton shower Monte-Carlo
- simulate jets
- ...

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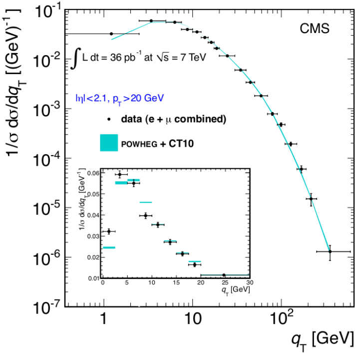
Is there some room for improvements?

Do we know enough about PDFs to describe hadron collisions?

Classical example

- The Z-boson q_T spectrum in pp collisions at LHC in the lepton pair's invariant mass range $60\text{GeV} < M < 120\text{GeV}$.[1]



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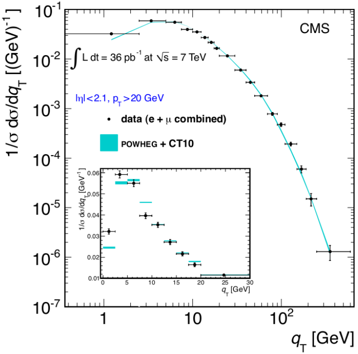

- high-q_T region, for $q_T > 10GeV$
  Collinear PDFs work well for any fixed-order in pQCD (LO, NLO, etc.)

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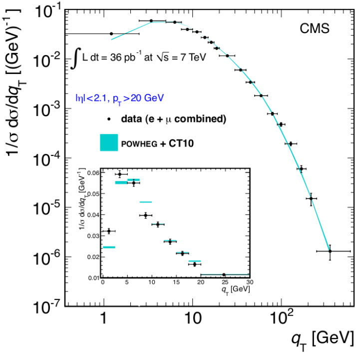

- high-q_T region, for $q_T > 10GeV$
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- low-q_T region, for $q_T \approx 10GeV$ and below
  Fixed-order pQCD with collinear PDFs fail, hence resummation to infinite order in pQCD is required.

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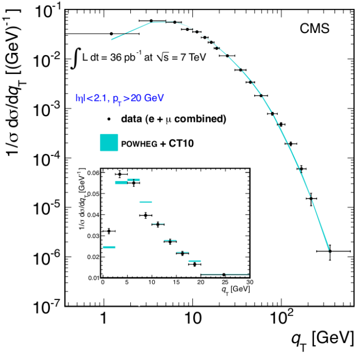

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  Collinear PDFs work well for any fixed-order in pQCD (LO, NLO, etc.)
- <u>low-q_T region</u>, for $q_T \approx 10 GeV$ and below
  Fixed-order pQCD with collinear PDFs fail, hence resummation to infinite order in pQCD is required.

  It can be fixed by introducing a generalized form of QCD factorization[a], i.e. **low-q_T factorization**, with PDFs dependent on **transverse momentum**[b] and polarization.

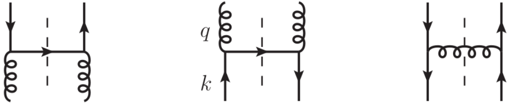
[a] Collins, Soper, Sterman (1983, 1985)
[b] Hautmann, Jung, et al. (2014)

[1] Chatrchyan, et al. (2012)

Project description: The Task

Describe evolutions of TMD PDFs.

- Calculate missing TMD Splitting Functions, i.e. P_{gq} and P_{qq}.



(a) P_{qg} (b) P_{gq} (c) P_{qq}

Figure 2: *Matrix elements for the determination of splitting functions. Lower (incoming) lines carry always momentum k, upper (outgoing) lines carry momentum q.*

Sudakov parametrization for incoming and outgoing momenta, k and q (see fig. 2), reads

$$k^\mu = yp^\mu + k_\perp^\mu, \qquad q^\mu = xp^\mu + q_\perp^\mu + \frac{q^2 + \mathbf{q}^2}{2x\, p\cdot n} n^\mu, \qquad \tilde{\mathbf{q}} = \mathbf{q} - z\mathbf{k}, \tag{3.2}$$

Use the approach of Curci, Furmanski, Petronzio (1980)
modified to be compatible within High-Energy Factorization framework [2]

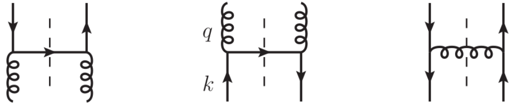
- light-cone **axial gauge**
- **projectors** for incoming and outgoing legs

[2] Catani, Hautmann (1994)

Project description: Some Formulas

Master Formula

$$\hat{K}_{ij}\left(z, \frac{\mathbf{k}^2}{\mu^2}, \epsilon, \alpha_s\right) = \int \frac{dq^2 d^{2+2\epsilon}\boldsymbol{q}}{2(2\pi)^{4+2\epsilon}} \Theta(\mu_F^2 - q^2)\mathbb{P}_{j,\,\mathrm{in}} \otimes \hat{K}_{ij}^{(0)}(q,k) \otimes \mathbb{P}_{i,\,\mathrm{out}}\,.$$



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Projectors

$$\mathbb{P}_{g,\,\mathrm{in}}^{\mu\nu} = \frac{k_\perp^\mu k_\perp^\nu}{\mathbf{k}^2}\,, \qquad \mathbb{P}_{q,\,\mathrm{in}} = \frac{y\,\not{p}}{2}\,.$$

$$\mathbb{P}_{g,\,\mathrm{out}}^{\mu\nu} = -g^{\mu\nu} \qquad \mathbb{P}_{q,\,\mathrm{out}} = \frac{\not{n}}{2\,q\cdot n}$$

Note that momenta k and q are off-shell

- usual Feynman rules brake gauge invariance
- modification for vertices with incoming and outgoing lines is required

Project description: Modified Vertices

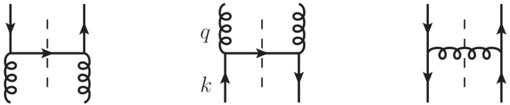

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Figure 2: *Matrix elements for the determination of splitting functions. Lower (incoming) lines carry always momentum k, upper (outgoing) lines carry momentum q.*

$$\Gamma^{\mu}_{q^*g^*q}(q,k,p') = igt^a \left(\gamma^\mu - \frac{n^\mu}{k\cdot n}\not{q}\right),$$

$$\Gamma^{\mu}_{g^*q^*q}(q,k,p') = igt^a \left(\gamma^\mu - \frac{p^\mu}{p\cdot q}\not{k}\right),$$

$$\Gamma^{\mu}_{q^*q^*g}(q,k,p') = igt^a \left(\gamma^\mu - \frac{p^\mu}{p\cdot p'}\not{k} + \frac{n^\mu}{n\cdot p'}\not{q}\right).$$

Pay attention to the linear propagators in the **above vertices**

- together with linear propagators from **axial denominators** they make life really complicated (fortunately at higher orders).

Results: P_{gq}

Complete Result

$$\hat{K}_{ij}\left(z, \frac{\mathbf{k}^2}{\mu_F^2}, \alpha_s, \epsilon\right) = \frac{\alpha_s}{2\pi} z \int_0^{(1-z)(\mu_F^2 - z\mathbf{k}^2)} \frac{d\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2} \left(\frac{\tilde{\mathbf{q}}^2}{\mu^2}\right)^{\epsilon} \frac{e^{-\epsilon\gamma_E}}{\Gamma(1+\epsilon)} P_{ij}^{(0)}\left(z, \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}, \epsilon\right)$$

$$P_{gq}^{(0)}\left(z, \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}, \epsilon\right) = C_F \left[\frac{2\tilde{\mathbf{q}}^2}{z|\tilde{\mathbf{q}}^2 - (1-z)^2\mathbf{k}^2|} - \frac{\tilde{\mathbf{q}}^2(\tilde{\mathbf{q}}^2(2-z) + \mathbf{k}^2 z(1-z^2)) - \epsilon z(\tilde{\mathbf{q}}^2 + (1-z)^2\mathbf{k}^2)}{(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)^2}\right]$$

Collinear limit $\mathbf{k}^2 \to 0$ leads to classical Altarelli-Parisi kernels

$$P_{gq}^{(0)}(z, 0, \epsilon) = C_F \frac{1 + (1-z)^2 + \epsilon z^2}{z}$$

Results: P_{qq}

Complete Result

$$\hat{K}_{ij}\left(z, \frac{\mathbf{k}^2}{\mu_F^2}, \alpha_s, \epsilon\right) = \frac{\alpha_s}{2\pi} z \int_0^{(1-z)(\mu_F^2 - z\mathbf{k}^2)} \frac{d\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2} \left(\frac{\tilde{\mathbf{q}}^2}{\mu^2}\right)^{\epsilon} \frac{e^{-\epsilon\gamma_E}}{\Gamma(1+\epsilon)} P_{ij}^{(0)}\left(z, \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}, \epsilon\right)$$

$$P_{qq}^{(0)}\left(z, \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}, \epsilon\right) = C_F \left(\frac{\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2}\right) \left[\frac{\tilde{\mathbf{q}}^2 + (1-z^2)\mathbf{k}^2}{(1-z)|\tilde{\mathbf{q}}^2 - (1-z)^2\mathbf{k}^2|} + \frac{z^2\tilde{\mathbf{q}}^2 - z(1-z)(1-3z+z^2)\mathbf{k}^2 + (1-z)^2\epsilon(\tilde{\mathbf{q}}^2 + z^2\mathbf{k}^2)}{(1-z)(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)}\right]$$

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$$P_{qq}^{(0)}(z, 0, \epsilon) = C_F \frac{1 + z^2 + \epsilon(1-z)^2}{1-z}$$

Summary

Conclusions

- calculated a complete set of TMD Splitting Functions
  - in agreement with collinear limit
  - in agreement with known results
  - ensure gauge invariance
- automated Mathematica package for calculations

Future Prospects

- define evolution equations for TMD Splitting Functions
- calculate TMD Matrix Elements (next talk by M.Serino)
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Thank You!
Any questions?