Searching for new physics - muon anomaly and MUonE experiment.

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Presentation schedule

The Standard Model of particle physics is a collection of theories that embodies all our current understanding of fundamental particles and forces.
It is supported by a great deal of experimental evidence. One can predict various observables to a very high precision.

However...

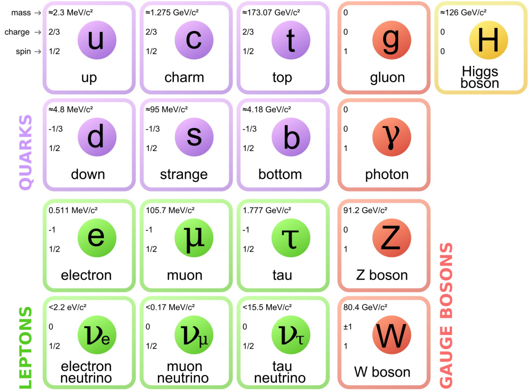


Figure: Fundamental particles of Standard Model.

...Despite a great success of Standard Model, there are strong suggestions that it is incomplete! Dark matter, gravity and its weakness, the disproportion between matter and antimatter and more.

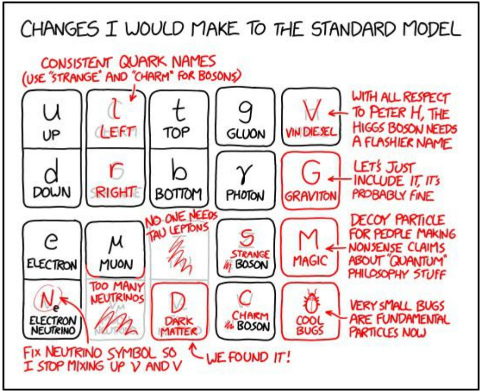
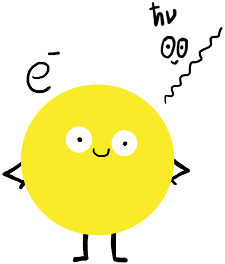


Figure: Randall Munroe's graphic inspired by problems of Standard Model.

Whats more, some experiments are showing discrepancies from its prediction. One of these issues may be anomalous magnetic moment of a muon (a_μ). But lets start from the beginning.



Electron interacts only electromagnetically. Anomalous magnetic moment of an electron was measured in 1948 and agreed perfectly with theoretical prediction.

Physicists expected the same in the case of muon. Standard model is always correct, right? Well, everything is a bit more complicated here.

- muon is unstable particle with a short lifetime
- magnetic moment of a muon is hard to measure
- muon interacts not only electromagnetically, but also strongly and weakly
- theoretical predictions of a_μ are very complicated

First measurements of a_μ took place in 1958 at CERN and are continued until today.

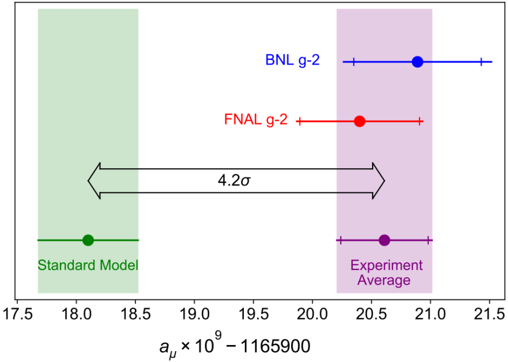


Figure: Experimental results on muon anomaly compared to the Standard Model predictions.

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (251 \pm 59) \times 10^{-11}$$

The average of experimental results differs from the Standard Model predictions by 4.2 standard deviations (we need 5σ to claim a discovery).

Theoretical predictions of a_μ

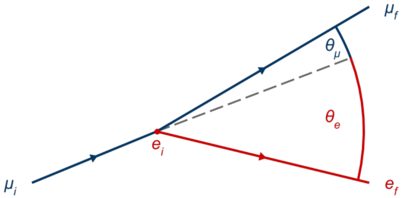
The non-zero value of a_μ comes from the following factors:

$$a_\mu = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{QCD} + a_\mu^{NP}.$$

The last term may be caused by phenomena beyond the Standard Model.

Problem: The biggest uncertainty comes from theoretical predictions on hadronic contribution. QCD is a non-perturbative theory - a_μ^{QCD} can not be calculated analytically.

Recently, MUonE experiment proposed a novel method to get a_μ^{HLO} term by measuring the shape of differential cross section of high-energy muons elastic-scattering on atomic electrons.



Combining expected results from MUonE with the results from muon g-2 experiments may increase the significance of a possible discovery to around 7σ.

The aim of MUonE experiment is to directly measure the hadronic contribution to running fine structure constant, which enables precise determination of a_μ^{HLO} contribution to anomalous magnetic moment of a muon.

$$a_\mu^{HVP,\,LO} = \left(\frac{\alpha_0}{\pi}\right)^2 \int_0^{0.932} dx(1-x)\Delta\alpha_{had}[t(x)],$$

The crucial goal is to ensure effective control of systematic effects, the reduction of which is critical to the success of accurate measurement.

As the MUonE experiment measures the scattering angles of outgoing particles, the well performed track reconstruction is essential.

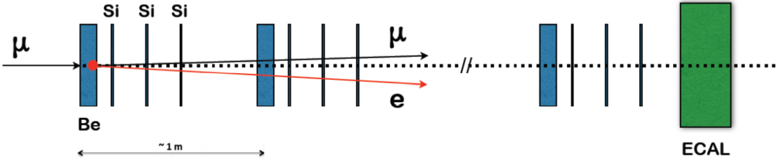


Figure: Detector scheme.

No external magnetic field - particles trajectories are expected to be the straight lines.
Fit parameters: x_0, t_x, y_0, t_y.

Alignment procedure allows determining the detector geometry with maximum possible precision.

Differences between assumed and actual detector geometry can significantly worsen the overall precision of track reconstruction. It is important to control this effect by using alignmet algorithms.

Each module consists of two sensors, which measure in the same direction. For reconstruction and alignment purposes, positions from two sensors are averaged.

Such system, with z position being the position in the center of the module and averaged hit position will be called a Layer.

Global alignment is a track-based method that minimises the χ^2 constructed from a large number of tracks.

- χ^2_{Track} describes the fit quality and is determined from residuals - distances between hits and fitted tracks.

$$\chi^2_{Track} = \sum_{Hits} \left(\frac{r_i}{\sigma_i}\right)^2, \chi^2_{Track} = \mathrm{r}^T V^{-1} \mathrm{r},$$

where r is a vector of residuals, V is a measurement covariance matrix.

- The global χ^2 method uses a χ^2_{Global} built from a large number of tracks:

$$\chi^2_{Global} = \sum_{Tracks} \chi^2_{Track}.$$

- The χ^2_{Global} minimisation allows achieving set of alignment parameters (the corrections to the detector geometry) for each detector module.

In general, those corrections reffer to 6 degrees of freedom - 3 coordinates (dx, dy, dz) and 3 orientations in space (dtx, dty, dtz).

- Searching for alignment parameters that minimise χ^2_{Global} involves the first and second derivatives of χ^2_{Global} with respect to alignment parameters α:

$$\mathrm{Y} \equiv \left(\frac{d\chi^2_{Global}}{d\alpha}\right)^T = 2\sum_{Tracks} A^T V^{-1}(V - HCH^T)V^{-1}\mathrm{r}$$

$$\mathrm{M} \equiv \left(\frac{d^2\chi^2_{Global}}{d\alpha^2}\right) = 2\sum_{Tracks} A^T V^{-1}(V - HCH^T)V^{-1}A,$$

First set of corrections is given by:

$$\mathrm{X} \equiv \Delta\alpha = -\left(\left.\frac{d^2\chi^2_{Global}}{d\alpha^2}\right|_{\alpha_0}\right)^{-1} \left.\left(\frac{d\chi^2_{Global}}{d\alpha}\right)^T\right|_{\alpha_0} = -\mathrm{M}^{-1}\mathrm{Y}$$

$$Y \equiv \left(\frac{d\chi^2_{Global}}{d\alpha}\right)^T = 2\sum_{Tracks} A^T V^{-1}(V - HCH^T)V^{-1}\mathrm{r}$$

$$M \equiv \left(\frac{d^2\chi^2_{Global}}{d\alpha^2}\right) = 2\sum_{Tracks} A^T V^{-1}(V - HCH^T)V^{-1}A,$$

Set of corrections is given by:

$$X \equiv \Delta\alpha = -\left(\left.\frac{d^2\chi^2_{Global}}{d\alpha^2}\right|_{\alpha_0}\right)^{-1}\left.\left(\frac{d\chi^2_{Global}}{d\alpha}\right)^T\right|_{\alpha_0} = -M^{-1}Y$$

- V - measurement covariance matrix
- C - fit covariance matrix
- H - matrix of residuals' derivatives with respect to track parameters (x, y, tx, ty)
- A - matrix of residuals' derivatives with respect to alignment parameters (dx, dy, dz, dtx, dtz, dty)
- r - vector of residuals

In general, M matrix is singular.

Weak (Singular) modes - associated with zeros on M diagonal - caused by the detector movements that does not change the χ^2.

There are two considerable approaches to handle this problem:

- Matrix diagonalisation

The best option for simple geometries. Provides the full information about singular modes. Some conclusions can be done by looking on diagonalized matrix - no need to perform many steps of alignment and reconstruction every time.

- Elimination of singularities

Adding the value relaitive much smaller than the corrections on the M diagonal makes matrix not singular and does not affect them significantly.

Matrix diagonalization cd.

Inverting diagonalized matrix is trivial:

$$M_D = \begin{pmatrix} \lambda_0 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & \dots \end{pmatrix}, M_D^{-1} = \begin{pmatrix} 1/\lambda_0 & 0 & 0 & 0 \\ 0 & 1/\lambda_1 & 0 & 0 \\ 0 & 0 & 1/\lambda_2 & 0 \\ 0 & 0 & 0 & \dots \end{pmatrix}$$

Alignment formula for the final corrections has to be transformed to the diagonal base:

$$-MX = Y \rightarrow -EME^T EX = EY,$$

where E - matrix of M eigenvectors. This lead to the same equation, expressed in diagonal base:

$$-M_D X_D = Y_D$$

To obtain alignment parameters:

$$-M_D^{-1} Y_D = X_D$$

Going back to original base:

$$X = E^T X_D$$

Eigen software

Root does not handle singular matrices well.

- Inverting irreversible matrix without any warning, returning arbitrary values(!)
- Troubles with satisfying diagonalization condition ($EE^T = E^T E = I$)

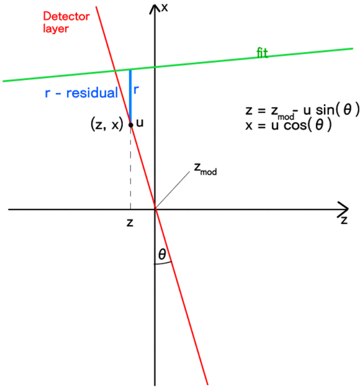
Eigen is a c++ template library for linear algebra.

- Very easy to use within fairmuone,
- satisfies the diagonalization condition,
- allows to have numerical aspects under control.

The global alignment algorithm workflow in fairmuone:

- track reconstruction
- perform the global alignment equations and calculate the firs set of corrections to detector geometry.
- perform track reconstruction with updated geometry (new track parameters)
- perform alignment procedure with geometry updated in previous step and with new track parameters; achieve more accurate set of corrections
- continue until meet the convergence.

- Defining the residuals...

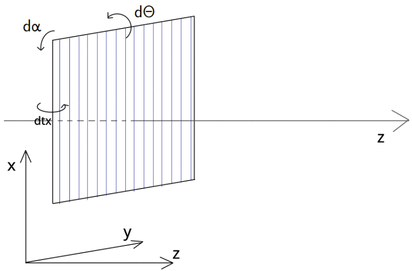


$$\mathrm{r} = u \cdot \cos\left(\widehat{\theta}\right) - [(x_0 + t_x \cdot (\widehat{z_{mod}} - u \cdot \sin\left(\widehat{\theta}\right) - z_0) - \widehat{x_{mod}}) \cdot \cos(\widehat{\alpha})$$

$$+ (y_0 + t_y \cdot (\widehat{z_{mod}} - u \cdot \sin\left(\widehat{\theta}\right) - z_0) - \widehat{y_{mod}}) \cdot \sin(\widehat{\alpha})]$$

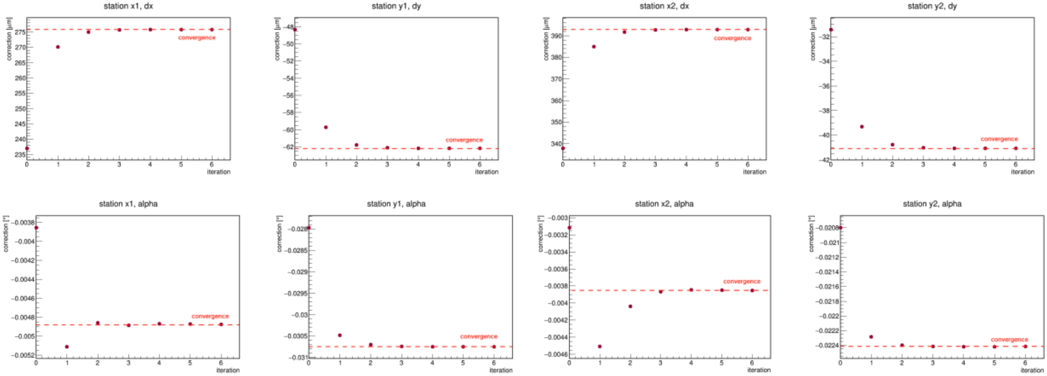
- Current version uses 3 degrees of freedom (dx, dy, dα).
- The z problem

The positions of the stations in z direction are not available at this moment. Thus, this degree of freedom is not active in the global alignment implementation.



Diagonalization method - Low intensity sample, 100 000 events, rootfile - 3086_034a000a0_034a0409f_4201232.root

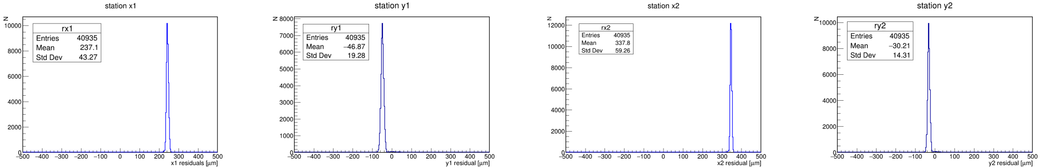
At this moment Global alignment works for x1, y1, x2, y2 layers and three degrees of freedom (dx, dy, dα).



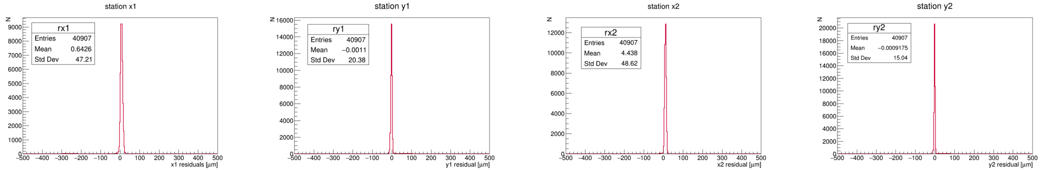
Final corrections:

station	x correction [μm]	y correction [μm]	αcorrection [$^\circ$]
x1	275.79	-	-0.0049
y1	-	-62.17	-0.0307
x2	392.80	-	-0.0039
y2	-	-41.07	-0.0224

Residuals before alignment:



Residuals after global alignment:



MUonE schedule

The results from MUonE testbeam 2018 appeared to be very successful and promising.

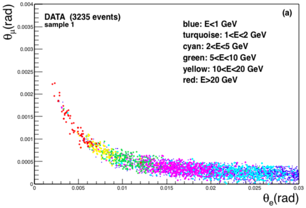


Figure: Results from testbeam 2018 - dependence between muons and electrons scattering angles.

Pilot runs in 2022/23 towards final data taking in 2026.

perspectives for muon g-2

The mystery of anomalous magnetic moment of a muon may be solved soon.

Precise measurements together with better than before determination of hadronic leading order contribution from MUonE experiment are expected to reach the 7σ level.

The global alignment method is precise method of detector positioning and provides full information about the system, e.g. weak modes.

Results for x1, y1, x2, y2 layers using Eigen software is estabilished.

Numerical aspects under control - M eigenvalues positively defined, $EE^T = E^T E = I$ condition satisfied*.

All calculations are performed using Eigen software.

Corrections for alpha angle successfully implemented into Global Alignment.

? Current work on GA algorithm focuses on solving the problem with stereo layers.

*E - matrix of M eigenvectors