Exploitation of Cyclostationarity and its Generalizations for Science Data Analysis

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Motivations

- The combination of a random phenomenon and a periodic one gives rise to a process which is not periodic but whose statistical characteristics vary periodically with time. This process is called ***cyclostationary (CS)***.
- Almost all modulated signals adopted in **telecommunications, telemetry, radar**, and **sonar** can be modeled as cyclostationary processes.
- In **radio astronomy**, periodicity results from revolution and rotation of planets and on pulsation of stars.

Motivations (cont'd)

- If more periodicities with incommensurate periods are present, then the statistical characteristics are almost-periodic functions of time and the process is called ***almost-cyclostationary (ACS)***.
- Many physical phenomena give rise to signals with statistical functions exhibiting *irregular cyclicity*:
  - Intentional or unintentional **time- or frequency-warping** due to variations ot timing parameters;
  - **Doppler effect** due to relative motion between transmitter and receiver with generic motion law.

Almost-Cyclostationary (ACS) Signals

$$\mathrm{E}\{x(n+m)\,x(n)\} = \sum_{\alpha} R_x^{\alpha}(m)\, e^{j2\pi\alpha n}$$

The autocorrelation function of the signal $x(n)$ ***is an almost-periodic function of*** n having Fourier series expansion with frequencies α *(cycle frequencies)* and coefficients $R_x^{\alpha}(m)$ *(cyclic autocorrelation functions)*.

$$R_x^{\alpha}(m) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \mathrm{E}\{x(n+m)\,x(n)\}\; e^{-j2\pi\alpha n}$$

Jointly Almost-Cyclostationary Signals

$$\mathrm{E}\{y_1(n+m)\, y_2(n)\} = \sum_{\beta} R^{\alpha}_{y_1y_2}(\tau)\, e^{j2\pi\alpha t}$$

The cross-correlation function of the processes $y_1(n)$ ***and*** $y_2(n)$ ***is an almost-periodic function of*** n having Fourier series expansion with frequencies β *(cycle frequencies)* and coefficients $R^{\alpha}_{y_1y_2}(m)$ *(cyclic cross-correlation functions).*

$$R^{\alpha}_{y_1y_2}(m) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \mathrm{E}\{y_1(n+m)\, y_2(n)\}\, e^{-j2\pi\alpha n}$$

Fraction-of-Time (FOT) Probability

- In several applications, the signal is a single time series. That is, an ensemble of realizations, namely a stochastic process, does not exist.
- The statistical characterization is more suitably made in the functional of fraction-of-time (FOT) approach.
- In the FOT approach, starting from a single time series, all familiar probabilistic parameters such as mean, autocorrelation, distribution, moments, and cumulants, are constructed starting from the unique available time series.
- A new operator, the almost-periodic component extraction operator $\mathrm{E}^{\{\alpha\}}\{\cdot\}$, replaces the ensemble average $\mathrm{E}\{\cdot\}$.

Fraction-of-Time (FOT) Probability (cont'd)

The function

$$F_x(n;\xi) = \mathrm{E}^{\{\alpha\}}\{\mathrm{u}(\xi - x(n))\}$$
$$= \sum_{\alpha} F_x^{\alpha}(\xi)\, e^{j2\pi\alpha n}$$

as a function of ξ, is a valid cumulative distribution function, almost-periodic with respect to n.

- $\mathrm{u}(\cdot)$ = *unit step function*: $\mathrm{u}(\xi) = 1$ for $\xi \geqslant 0$ and $\mathrm{u}(\xi) = 0$ for $\xi < 0$
- $\mathrm{E}^{\{\alpha\}}\{\cdot\}$ = *almost-periodic component extraction operator*: it extracts all the finite-strength additive sine-wave components of its argument. It is the expectation operator in the FOT approach.

Fraction-of-Time (FOT) Probability (cont'd)

Joint cumulative distribution function (CDF) of $y_1(n+m)$ and $y_2(n)$

$$F_{y_1y_2}(n,m;\xi_1\xi_2) = \mathrm{E}^{\{\alpha\}}\left\{\mathrm{u}(\xi_1 - y_1(n+m))\,\mathrm{u}(\xi_2 - y_2(n))\right\}$$
$$= \sum_{\alpha} F^{\alpha}_{y_1y_2}(m;\xi_1\xi_2)\, e^{j2\pi\alpha n}$$

Fourier coefficients

$$F^{\alpha}_{y_1y_2}(m;\xi_1\xi_2) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \mathrm{u}(\xi_1 - y_1(n+m))\,\mathrm{u}(\xi_2 - y_2(n))\, e^{-j2\pi\alpha n}$$

CREDO Collaboration

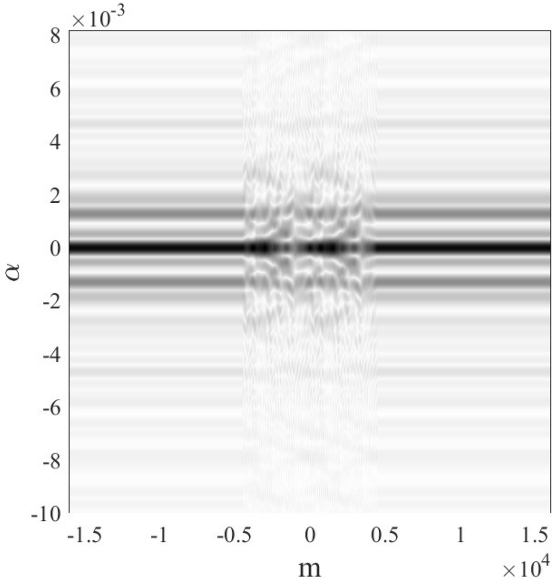
- The cyclostationary model has been exploited to confirm the main results presented in

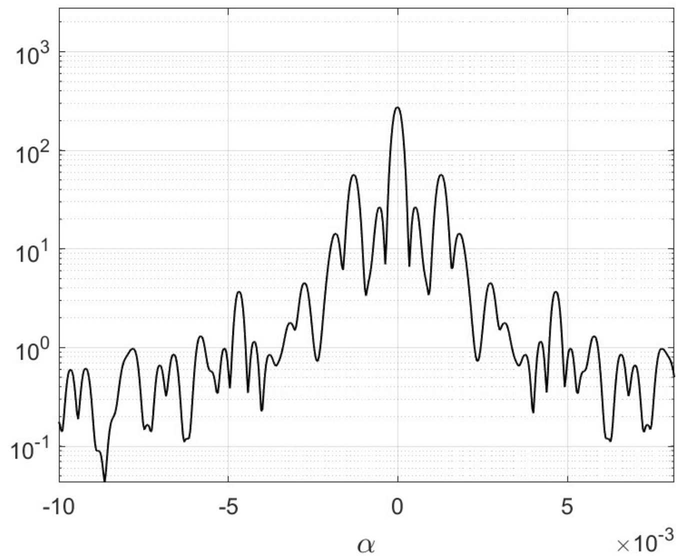
  P. Homola, V. Marchenko, A. Napolitano, et al., “Observation of large scale precursor correlations between cosmic rays and earthquakes with a periodicity similar to the solar cycle”, *Journal of Atmospheric and Solar-Terrestrial Physics (JASTP)*, Vol. 247, art. 106068, 2023.

- The following time series
  - average variation of the cosmic ray detection rate
  - earthquake sum magnitude
  - Sunspot monthly mean

  are shown to be pairwise jointly cyclostationary time series and the Fourier coefficients of their cross statistical functions are estimated. The results show the existence of periodic correlation or statistical dependence between pairs of these time series.

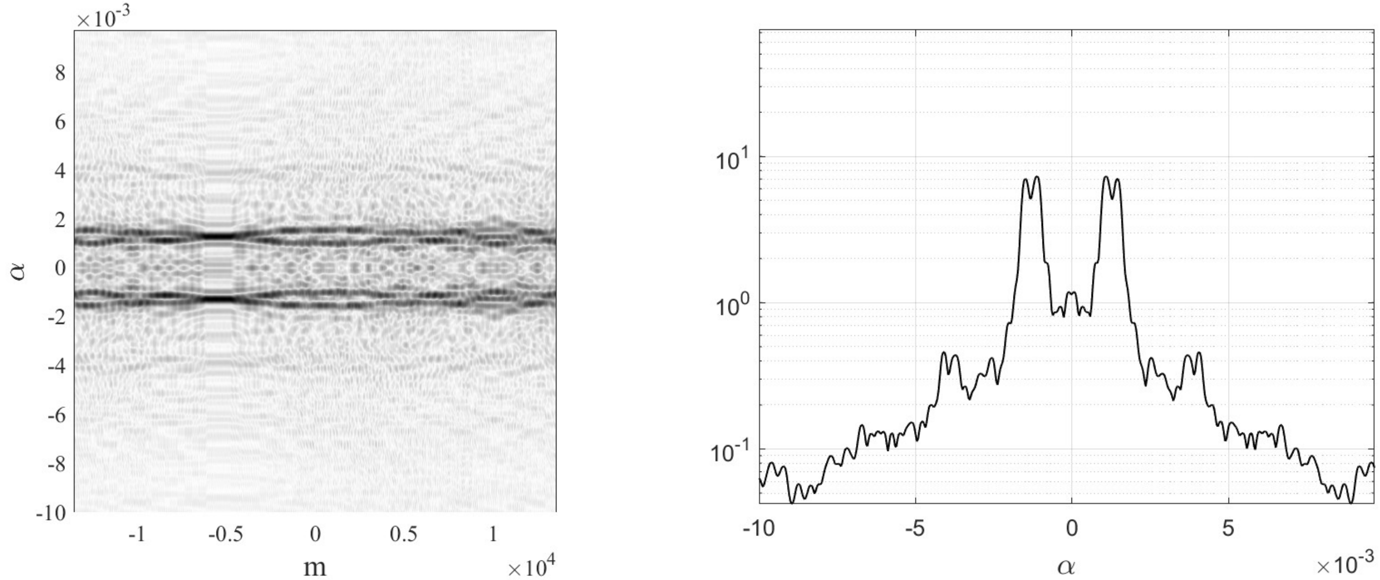
JASTP 2023





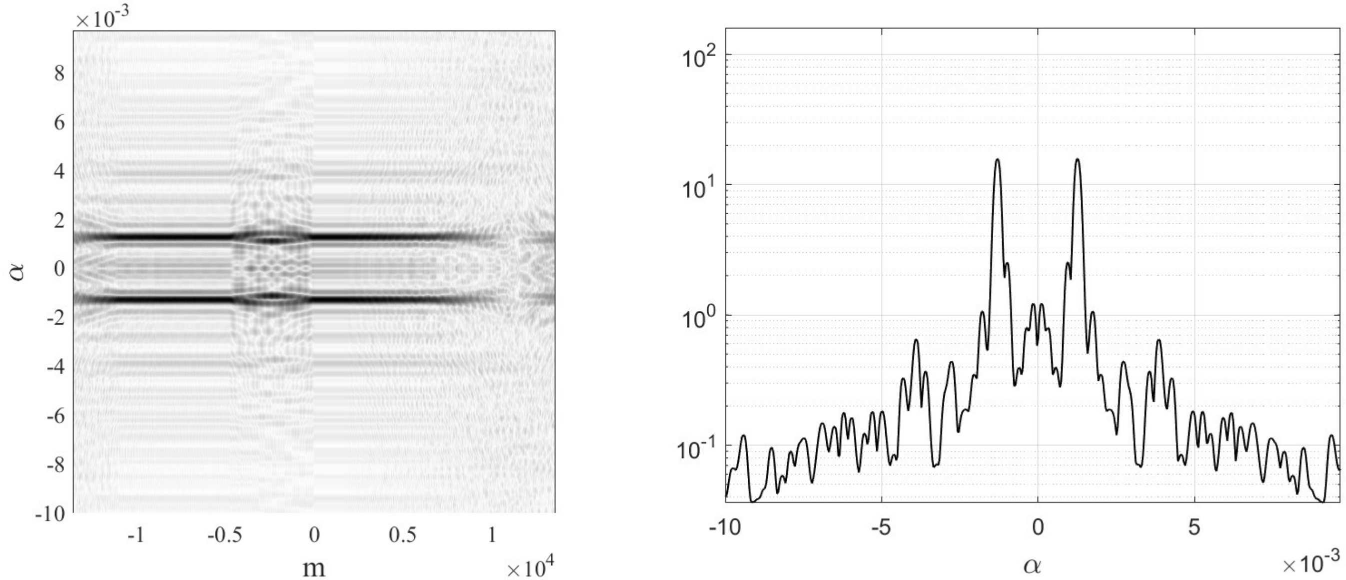
Statistical Dependence Between Cosmic Rays (y_1) and Earthquakes (y_2). (Left) magnitude of the estimated cyclic joint CDF $F^{\alpha}_{y_1 y_2}(m;\xi_1,\xi_2)$ as a function of α and m (2-dimensional grayscale elevation map); (Right) energy of the estimated joint CDF as a function of α.

JASTP 2023 (cont'd)



Cyclic Cross-Correlation Between Cosmic Rays (y_1) and Sunspots (y_2). (Left) magnitude of the estimated cyclic cross-correlation function $R^{\alpha}_{y_1 y_2}(m)$ as a function of α and m (2-dimensional grayscale elevation map); (Right) energy of the estimated cyclic cross-correlation as a function of α.

JASTP 2023 (cont'd)



Cyclic Cross-Correlation Between Earthquakes (y_1) and Sunspots (y_2). (Left) magnitude of the estimated cyclic cross-correlation function $R^{\alpha}_{y_1 y_2}(m)$ as a function of α and m (2-dimensional grayscale elevation map); (Right) energy of the estimated cyclic cross-correlation as a function of α.

Generalized Almost-Cyclostationarity (GACS) Signals

- Time-stretching and chirp modulation of $x(t)$:

$$y(t) = a\, x(s(t-\tau_0))\, e^{j2\pi\nu t}\, e^{j\pi\gamma t^2}$$

  - s = time-stretch factor
  - τ_0 = time delay
  - ν = frequency shift
  - γ = chirp rate
- $x(t)$ ACS $\Longrightarrow$ $y(t)$ GACS
- Multivariate statistical functions of $y(t)$ are almost-periodic functions of time whose generalized Fourier series have both coefficients and frequencies depending on lag parameters.

Spectrally Correlated (SC) Signals

- Frequency warping and filtering of $x(t)$:

$$Y(f) = H(f)\,X(\Psi(f))$$

  - $X(f)$ = Fourier transform of $x(t)$
  - $\Psi(f)$ = frequency-warping function
  - $H(f)$ = harmonic response of a linear time-invariant filter
  - $Y(f)$ = Fourier transform of $y(t)$
- $x(t)$ ACS $\implies$ $y(t)$ SC
- Statistical correlation exists between distinct spectral components of $y(t)$ at frequencies f_1 and f_2 with $f_2 = \Psi(f_1)$.

Oscillatory Almost-Cyclostationary (OACS) Signals

- Time-warping and amplitude modulation of $x(t)$:

$$y(t) = a(t)\, x(\psi(t))$$

  - $a(t)$ = amplitude-modulation function
  - $\psi(t)$ = time-warping function

- $x(t)$ ACS $\implies$ $y(t)$ OACS
- multivariate statistical functions of $y(t)$ are amplitude and angle modulated sinusoidal functions.
- **Doppler effect** due to relative motion between transmitter and receiver with generic motion law.

Bibliography

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[2] A. Napolitano, *Cyclostationary Processes and Time Series: Theory, Applications, and Generalizations*. Elsevier, 2019.