A survey of double logarithmical asymptotics of space-like parton correlators

Yizhuang Liu, Jagiellonian University

The talk is based on speaker's theoretical works (together with Ji & Su & Liu & others) in:

1. *Threshold resummation for computing large-x parton distribution through large-momentum effective theory (arxiv: 2305.04416)*
   (Threshold limit of perturbative quark quasi-PDF)
2. *Computing Light-Front Wave Functions Without Light-Front Quantization: A Large-Momentum Effective Theory Approach (Phys. Rev. D 105 (2022))*
   (Large $2xP_z$ limit of quasi-LFWF amplitudes)
3. *Transverse-momentum-dependent parton distribution functions from large-momentum effective theory (Phys.Lett.B 811 (2020), 135946) && TMD soft function from large-momentum effective theory (Nucl.Phys.B 955 (2020) 115054)*
   (Large $2xP_z$ limit of quasi-TMDPDFs)
4. *Large-momentum effective theory (Rev. Mod. Phys. 93 (2021))*

Outline

- Introduction to marginal-PT near UV fixed point.
- The HL Sudakov universality class .

1. Overview.
2. Quasi-TMDPDF && LFWF amplitudes.
3. Reduced soft factor. Lattice application.
4. Threshold limit of quark quasi-PDF.

UV fixed-point of local-QFT

- Successful of QFT in late 1940s: $g - 2$; lamb shift.
- Development of renormalization perturbation theory: Power-counting theorems/Symanzik polynomials/BPHZ....
- However, $-Q^2 \rightarrow \infty$ limit not under control. Large logarithms/Landau poles. Proper interpretation not clear.
- Early 1970s, multiple clues: Perturbative asymptotic freedom && RGE-analysis /Scaling theory..... strongly suggest local-QFTs, defined through scaling limits, have universal short distance limit in terms of relevant/marginally-relevant perturbation to UV CFT.
- QCD is the most well-known example which approaches the UV limit marginally.

General features of Marginal Perturbation Theory

- Euclidean correlator: $G(zm) = z^{2d_O}\langle 0|O(z)O(0)|0\rangle$. Universal $z^2 \to 0$ asymptotic expansion:

$$G(zm) \to \alpha(z)^{\frac{-2\gamma_O^1}{\beta_0}} \exp\int_0^{\alpha(z)} \left(\frac{2\gamma_O^1}{\beta_0\alpha} - \frac{2\gamma_O(\alpha)}{\beta(\alpha)}\right) d\alpha \, G_{PT}[\alpha(z)] + O(mz).$$

1. $\alpha(z) \to \frac{1}{\beta^0 \ln(\frac{1}{mz})}$: the running coupling constant.
2. Minimal scheme: $\frac{1}{\alpha} + \frac{\beta_1}{\beta_0} \ln\alpha = \beta_0 \ln\frac{1}{m^\star z}$. Each $m^\star$ specifies a scheme (such as $\overline{MS}$).
3. γ_O^1: the LO anomalous dimension.
4. REG resumed form of the perturbative series calculated through Feynman diagrams.

General features of PT

- Anomalous dimension $\gamma_0(\alpha)$ has no explicit $\ln \mu$ dependence. Single log evolution.
- Requires specific singularity structure of Feynman integrals. Key structure: Natural factorization in each *Hepp sector* through maximal-forest of singularity.
- UV-IR conspiracy between leading-power and high-power. Example: pole mass vs linear divergence in HQET.

The Bjorken limit.

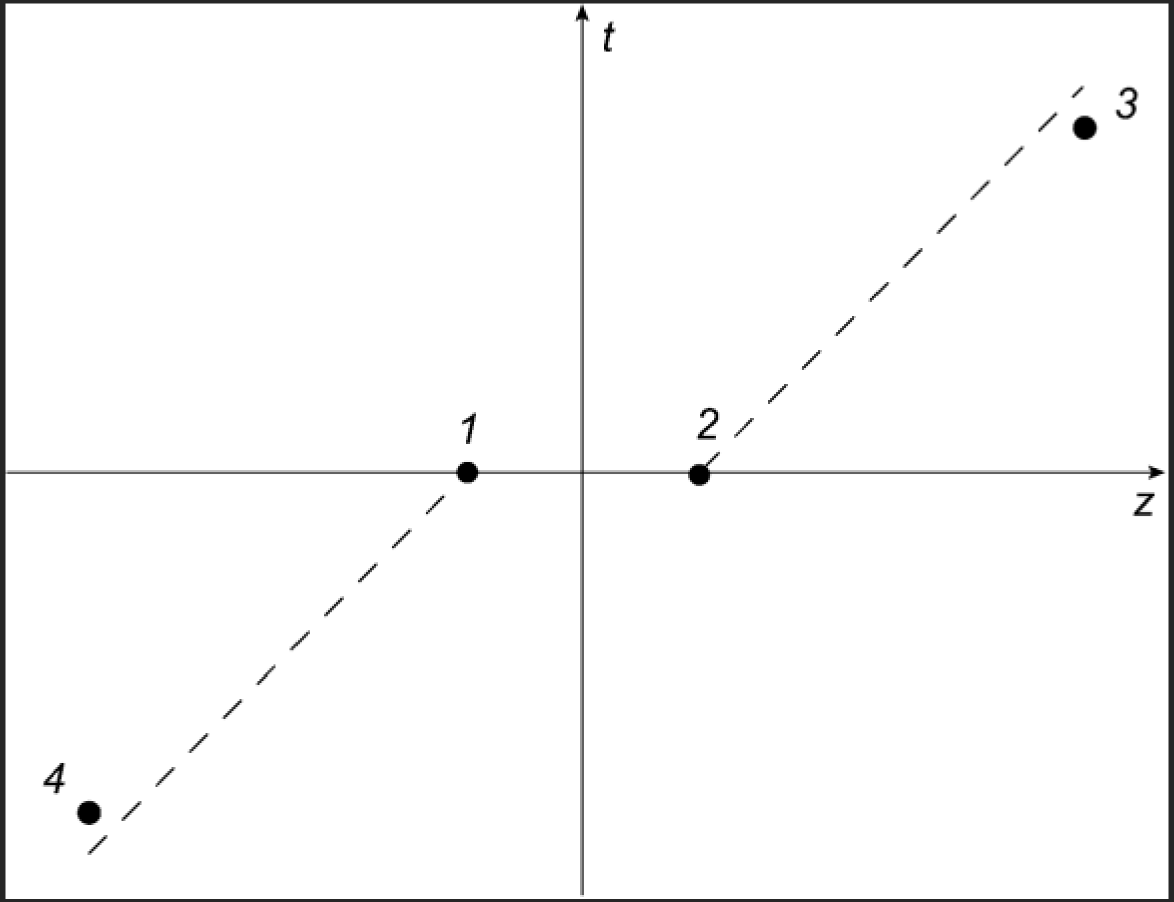
- Next simplest object. Structure functions . Not completely Euclidean.

$$F(Q^2, P\cdot Q) = \int \frac{d^4z}{(2\pi)^4}\, e^{iz\cdot Q}\langle P|J(z)J(0)|P\rangle_c.$$

- The Bjorken limit : $-Q^2 \to \infty$ at fixed $x_B = -\frac{Q^2}{2P\cdot Q}$ && $-z^2 \to 0$ at fixed $\lambda = z\cdot P$.
- This limit can still be controlled by marginal perturbation to UV CFT.

1. $F_n(Q^2) \sim C_n^q(\alpha(Q))\, \exp\left(\int_{\alpha(\mu)}^{\alpha(Q)} \frac{\gamma_n(\alpha)-2\gamma_J(\alpha)}{\beta(\alpha)} d\alpha\right)\; O_n(\mu)$ + high power
2. $F_n(z^2) \sim \widetilde{C_n}^q(\alpha(z))\; \exp\left(\int_{\alpha(\mu)}^{\alpha(z)} \frac{\gamma_n(\alpha)-2\gamma_J(\alpha)}{\beta(\alpha)} d\alpha\right)\; O_n(\mu)$ + high power

The Bjorken limit in CFT.



The Bjorken limit in coordinate space for $\langle O(x_3)O(x_2)O(x_1)O(x_4)\rangle$

- $x_1 = -x_2 = (0,1)$
- $x_3 = -x_4 = (t, t + \lambda)$
- $t \to +\infty, \lambda > 1$ fixed.
- Controlled by OPE with $min\{\Delta - J\}$.

Bjorken limit in marginal theory: taming logarithms

- Additional structure to control the logarithms.

1. PT version $F_n^q(Q^2, \alpha(\mu))$ does satisfies single logarithm RGE in IR (DGLAP).
2. IR anomalous dimension of $F_n^q(Q^2, \alpha(\mu))$ matches with UV of $O_n(\mu)$.
3. $f(\lambda, \mu) = \sum_{n=0} \frac{1}{n!} O_n(\mu) \lambda^n = \frac{1}{2P^+} \langle P|\bar{\psi}(x^-) W(x^-, 0) \gamma^+ \psi(0)|P\rangle$ naturally interpretated as parton distribution function. Consistent with parton picture.
4. Supported by CSS leading region analysis && SCET && LR OPE.

Beyond single-log asymptotics

- Are there other limits which can be controlled by marginal perturbation to UV CFT? (universal asymptotics)
- Certainly, not all "high-energy limits" are universal. Example:

  Regge limit in large N_c 2D QCD. $Im\, A(s,0) \sim s^{-2\beta-1}$.

  $$\pi\beta \cot \pi\beta = -\frac{m_q^2}{2\sigma_T} + 1.$$

- Nevertheless, there does exists universal "high-energy" limits which are not single-log type.

Double log asymptotics && Light-Light Sudakov

The famous double-log object: light-quark Sudakov form factor.

- $\langle p' | \bar{\psi} \Gamma \psi | p \rangle$ with $Q^2 = (p - p')^2 \to \infty$, $p^2 = p'^2 = 0$.

1. Double logarithm at one-loop.
2. No power-counting theorem proven yet. RGE conjectured and verified to four loops.
3. Key feature: anomalous dimension contains logarithm, but only one!

$$\mu \frac{d \ln H_{LL}(Q^2, \alpha(\mu))}{d\mu} = 2\Gamma_{cusp}(\alpha) \ln \frac{Q^2}{\mu^2} + 2\gamma_V(\alpha).$$

Exact integral equation in planar N=4.

Light-Light Sudakov universality class: TMD factorization

- Appearing in TMD factorizations && threshold limits.
- TMD factorization/resummation

$$F(Q^2, b_\perp, x_A, x_B) = H_{LL}(\alpha(Q)) \exp[S_{suda}(\alpha(Q))] \exp[2K(b_\perp) \ln Q^2 b_\perp^2]$$
$$\times f_{TMD}(x_A, b_\perp) f_{TMD}(x_B, b_\perp) + \text{high power}$$

1. The $H_{LL} \exp S_{suda}$: RGE resumed Sudakov form factor.
2. The $f_{TMD}(x, b_\perp)$: universal transverse-momentum dependent PDFs.
3. New feature: the Rapidity anomalous dimension (Collins-Soper kernel) $2K(b_\perp)$ for light-like gauge-link staples at separation $b_\perp$.
4. Heuristic "proofs": CSS argument based on reduced diagram of leading region && SCET.

Light-Light Sudakov universality class: threshold limit

1. The same LL Sudakov form factor also appears in threshold limits of various objects. For example: quark structure function in PT && hard kernel for DIS.

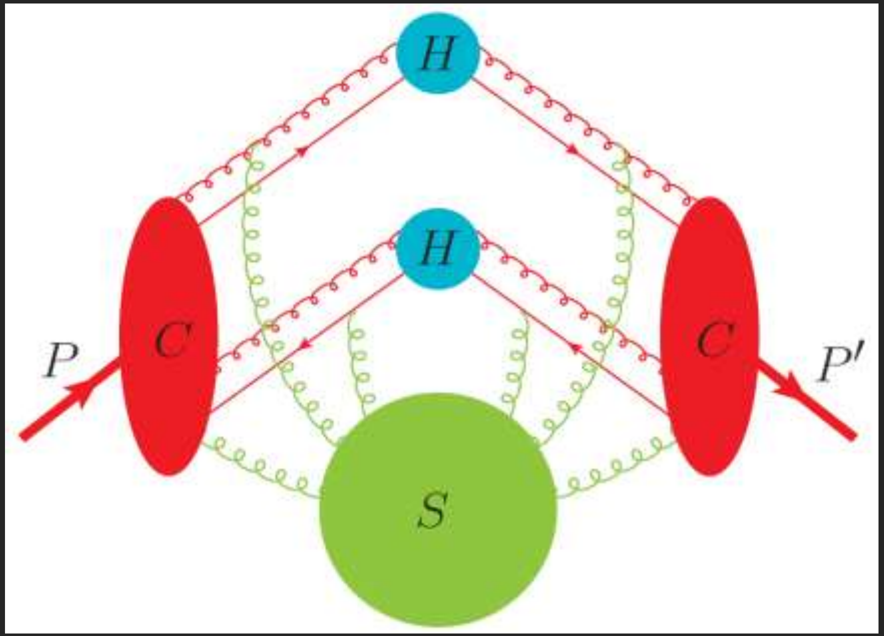
2. Threshold limit: $x \to 1$ in $F_q(\frac{Q^2}{\mu^2}, x, \alpha(\mu))$.

3. To leading power $(1-x)^{-1}$, one has threshold factorization

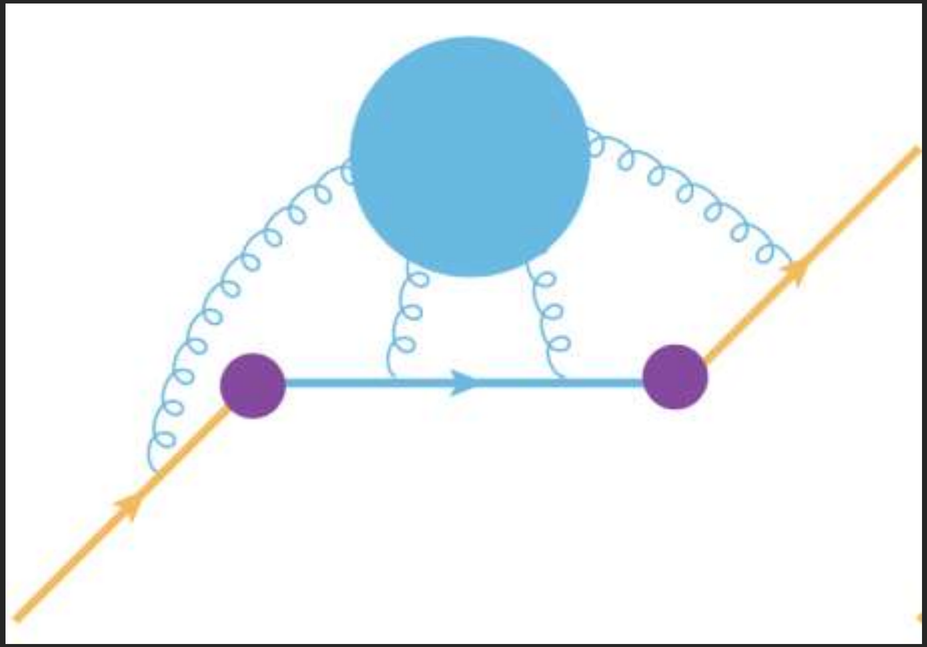
$$F_q\left(\frac{Q^2}{\mu^2}, x, \alpha(\mu)\right) = H_{LL}\left(\frac{Q^2}{\mu^2}, \alpha(\mu)\right) J_q(Q^2(1-x), \alpha(\mu)) + O(1-x)$$

4. LL Sudakov form factor && quark jet function J_q.

LL Sudakov. TMD && Threshold



TMD factorization for a space-like form-factor



Threshold limit of quark structure function

Outline

- High energy limit of QFT : factorization && scaling && universality.
- The HL Sudakov universality class .

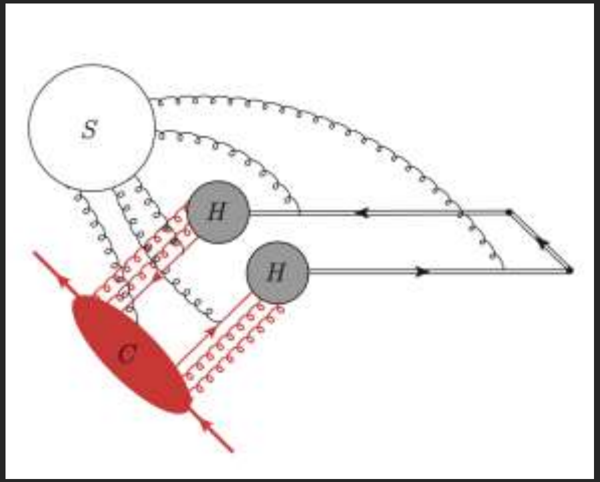
1. Overview
2. Quasi-TMDPDF && LFWF amplitudes.
3. Reduced soft factor. Lattice calculation.
4. Threshold limit of quark quasi-PDF.

Overview of the class

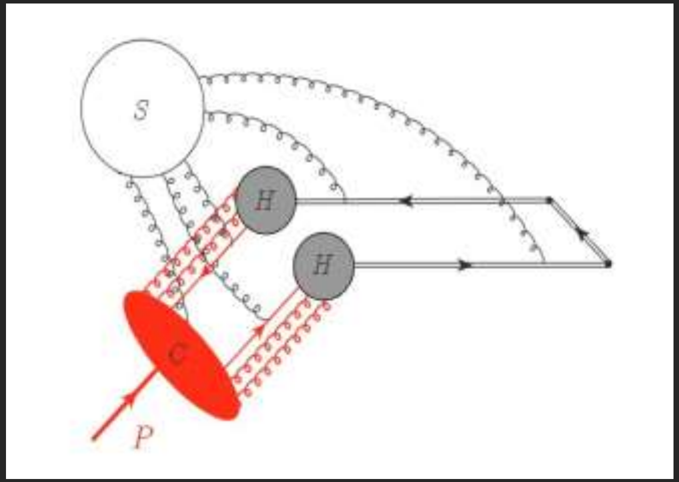
- Correlators with space-like gauge-links naturally arise in lattice parton distributions.
- LL current: $J^\mu(x) = \bar{q}\gamma^\mu q \rightarrow$ HL current: $\tilde{J}(x) = \bar{q}Q_{n_z}$.
- $Q_{n_z}(z)\overline{Q}_{n_z}(0) = Pexp\ ig \int_0^z dz' A_z(z')$: space-like gauge-link.

1. Quark quasi-PDF: $\tilde{f}_q(x, \zeta_z = 4x^2P_z^2)$ in threshold limit $x \rightarrow 1$.
2. Quasi-TMDPDF: $\tilde{f}(x, \zeta_z = 4x^2P_z^2, b_\perp)$ at large ζ_z.
3. Quasi-LFWF of light meson: $\tilde{\psi}_{\bar{q}q}\ (x, \bar{x}, \zeta_z, \bar{\zeta}_z, b_\perp)$ at large ζ_z.

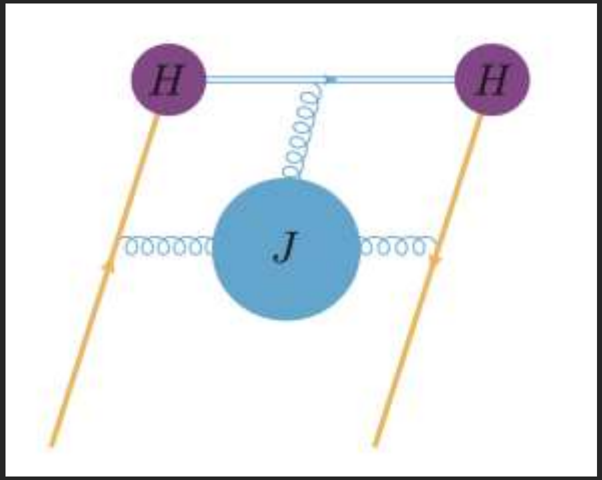
Overview of the class



quasi-TMDPDF



quasi-LFWF



threshold limit of quark quasi-PDF

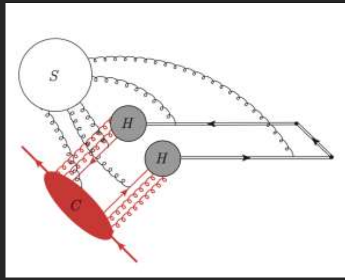
Space-time pictures of the HL Sudakov Universality Class

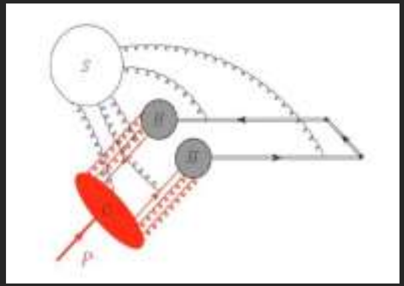
The HL Sudakov form factor

- HL Sudakov form factor: $H_{HL}^{\sigma}(\ln\frac{\zeta_z}{\mu^2}, \alpha(\mu))$.

1. External $p^2 = 0$ && $\mathrm{n}^2 = -1, \zeta_z = 4(p\cdot n)^2 \neq 0$.
2. NL: $H_{HL}^{\pm} = 1 + \frac{\alpha C_F}{4\pi}\left(-4 - \frac{5\pi^2}{6} + 2L_\pm - L_\pm^2\right)$ with $L_\pm = \ln\frac{\zeta_z}{\mu^2} \pm i\pi$.
3. RGE: $\mu\frac{d}{d\mu}\ln H_{HL}^{\sigma}\left(\ln\frac{\zeta_z}{\mu^2}, \alpha(\mu)\right) = \Gamma_{cusp}(\alpha)\ln\frac{\zeta_z}{\mu^2} + \tilde{\gamma}_H(\alpha) - i\pi\sigma\,\Gamma_{cusp}(\alpha)$.
4. Imaginary part depends on $\sigma = sign(-n\cdot p)$.
5. Single log anomalous dimension $\tilde{\gamma}_H = 2\gamma_F + \gamma_V + 2\gamma_{HL} - \gamma_S$.

Quasi-TMDPDF/LFWF at large ζ_z





- Quasi TMDPDF factorizes in the large $\zeta_z = 4x^2 P_z^2$ limit as

$$\tilde{f}(x, b_\perp, \zeta_z) = |H_{HL}(\zeta_z)|^2 \exp(\tilde{S}(\zeta_z)) \exp(\tfrac{1}{2} K(b_\perp) \ln \zeta_z b_\perp^2) \times S_r^{-\frac{1}{2}}(b_\perp) f_{TMD}(x, b_\perp) + \text{high power}$$

- Quasi LFWF amplitudes factorizes similarly:

$$\tilde{\psi}_{q\bar{q}}(x, b_\perp, \zeta_z) = H_{HL}(\zeta_z) H_{HL}(\bar{\zeta}_z) \exp(\tilde{S}(\zeta_z, \bar{\zeta}_z)) \exp(\tfrac{1}{2} K(b_\perp) \ln \sqrt{\bar{\zeta}_z \zeta_z b_\perp^2}) \times S_r^{-\frac{1}{2}}(b_\perp) \psi_{q\bar{q}}(x, b_\perp) + \text{high power}$$

Quasi-TMDPDF/LFWF at large ζ_z

$$\tilde{f}(x, b_\perp, \zeta_z) = |H_{HL}(\zeta_z)|^2 \exp(\tilde{S}(\zeta_z)) \exp(\tfrac{1}{2}K(b_\perp)\ln \zeta_z b_\perp^2) \times S_r^{-\frac{1}{2}}(b_\perp) f_{TMD}(x, b_\perp) + \text{high power}$$

1. The HL form factor at scale $\mu = \zeta_z$: $H_{HL}(\zeta_z)$.
2. The RGE re-summation factor: $\exp \tilde{S}(\zeta_z)$.
3. Asymptotically, $\tilde{S}(\zeta_z) \to -\frac{\Gamma_0}{\beta_0^2 \alpha(\zeta_z)} \ln \frac{e}{\alpha(\zeta_z)}$. Allows systematic expansion in increasing orders of $\alpha(\zeta_z)$.

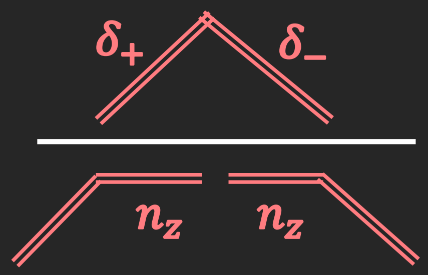
(Items 1–3: HL Sudakov)

- The rapidity evolution factor: $\exp \frac{1}{2} K(b_\perp) \ln \zeta_z b_\perp^2$.
- The standard TMDPDF/LFWF amplitudes : $f_{TMD}(x, b_\perp)$ && $\psi_{q\bar{q}}(x, \bar{x}, b_\perp)$.

The reduced soft factor S_r

- The $S_r(b_\perp)$ is a new NP function: the reduced soft factor.
- Defined through ratio of three TMD soft factors.

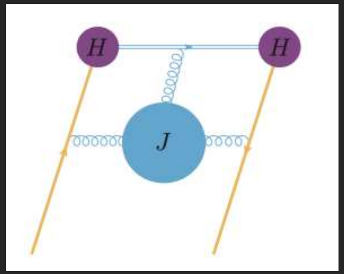
$$S_r(b_\perp, \mu) = \lim_{\delta^+,\delta^- \to 0} \frac{S(b_\perp, \mu, \delta^+, \delta^-)}{S(b_\perp, \mu, \delta^+, n_z) S(b_\perp, \mu, \delta^-, -n_z)}$$



1. Standard TMD soft factor: $S(b_\perp, \mu, \delta^+, \delta^-)$.
2. The HL soft factor: $S(b_\perp, \mu, \delta^+, n_z)$. Both light ($\delta^+$) and heavy ($n_z$) Wilson-lines.
3. Only a single log anomalous dimension: $\gamma_{HH} \equiv 2\gamma_{HL} - \gamma_s \equiv \Gamma_S$.

- Insensitive to time-ordering. Interpreted as LFWF of a $\bar{Q}Q$ pair.

Threshold limit of quark quasi-PDF



- The perturbative quark quasi-PDF has threshold logarithms as $y \to 1$.
- Factorizable in terms of HL Sudakov hard kernel and space-like jet function at power $(y-1)^{-1}$. Applies to quasi-GPD/DA as well.
- One has

$$\tilde{f}(y, L_z, \alpha(\mu))|_{y\to 1} = \left|H_{HL}(L_z, \alpha(\mu))\right|^2 J_f((1-y)p_z, L_z, \alpha(\mu)).$$

1. Reproduces the correct $y \to 1$ limit from exact NNLO quark quasi-PDF.
2. Allowing extraction of H_{HL} at NNLO.

$$L_z = \ln\frac{4p_z^2}{\mu^2}$$

Space-like jet function

- The J_f combines a space-like jet function $J(|z|)$ with phase factor of HL Sudakov form factor.
- In coordinate space,

$$J_f(|z|, sign(z), L_z) = J(|z|) \exp[iArg(H_{HL}^{sign(z)}(L_z))]$$

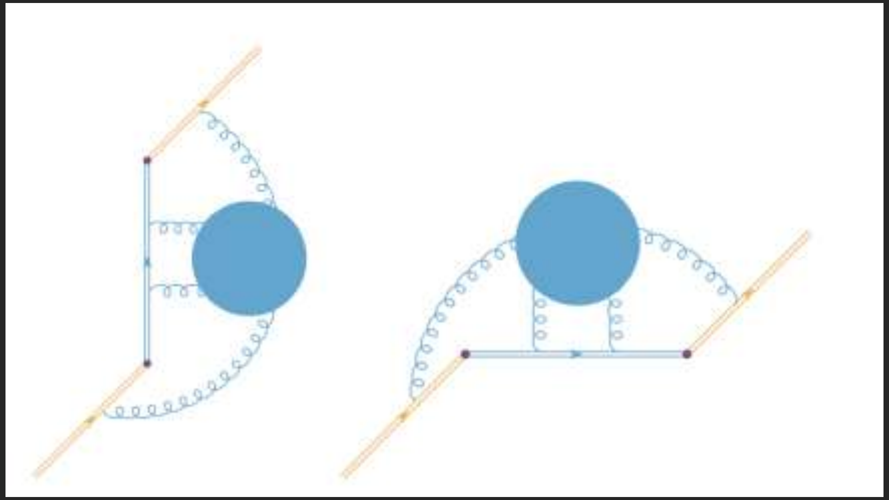
1. Imaginary part crucial to match to the threshold limit DGLAP $P(z)|_{z\to 1} = \frac{2\Gamma_{cusp}}{(1-z)^+} - (\gamma_V + \gamma_s)\delta(1-z)$.
2. $J(|z|)$ relates to a time-like version $J(t)$ through analytic continuation $t \to -i|z|$ to all orders.
3. $J(t)$ equals to a known heavy-quark jet function at NNLO**.

**Jain && Scimemi && Stewart arxiv:0801.0743

Space-like jet function

- RGE of space-like jet function

1. $\mu \frac{d}{d\mu} \ln J(|z|\mu, \alpha(\mu)) = \Gamma_{cusp}(\alpha) \ln \frac{e^{2\gamma_E}\mu^2 z^2}{4} - \tilde{\gamma}_J(\alpha).$
2. Single log anomalous dimension $\tilde{\gamma}_J = 2\gamma_{HL} - 2\gamma_S$.
3. Allows threshold RGE re-summation.



The time-like jet function $J(t)$ (left) and space like jet function $J(|z|)$ (right).

Application: HL Sudakov form factor at NNLO

- As an application, we extracted the NNLO HL Sudakov.
- All logarithms are determined by the β function and anomalous dimensions.
- The constant term of $\ln|H_{HL}|^2$ and the phase angle at NNLO reads

1. $$c_H = \left(\frac{241\zeta_3}{144\pi^2} + \frac{11\pi^2}{320} - \frac{559}{1728} - \frac{971}{324\pi^2}\right) C_F C_A + \left(\frac{-45\zeta_3 - 2\pi^4 + 30\pi^2 - 30}{24\pi^2}\right) C_F^2 + \left(\frac{36\zeta_3 + 51\pi^2 + 1312}{1296\pi^2}\right) C_F n_f T_F$$

2. $$c_a = \left(\frac{11}{4\pi^2} - \frac{11}{24} - \frac{475}{108\pi^2}\right) C_F C_A + \left(-\frac{3\zeta_3}{\pi^2} + \frac{7}{12} - \frac{1}{2\pi^2}\right) C_F^2 + \left(\frac{1}{6} + \frac{38}{27\pi^2}\right) C_F n_f T_F$$

- Agrees with direct calculation.

X.J&&Y.L&&Y.S, arxiv: 2305.04416

Relationship of Anomalous Dimensions

- Double log part: light-like cusp anomalous dimension $\Gamma_{cusp}(\alpha)$.
- Single log part: linear combination of four independent ones.

Quantity	Single-Log anomalous dimensions
Light-light Sudakov form factor H_{LL}	$2\gamma_V$
TMD soft factor $S(\delta, b)$	$-2\gamma_s$
Quasi TMDPDF $\tilde{f}(x, b)$	$2\gamma_F$
Space like jet function $J(z)$	$-2\gamma_{HL} + 2\gamma_s$
Quark splitting function $P_{qq}(t)\vert_{t\to 1}$	$-(\gamma_V + \gamma_s)\delta(t-1)$
Light-cone TMDPDF $f_{TMD}(x, \xi, b)$	$-\gamma_V$
Reduced soft factor $S_r(b)$	$2\gamma_{HL} - \gamma_s$
Heavy-light Sudakov form factor H_{HL}	$\gamma_V + 2\gamma_F + 2\gamma_{HL} - \gamma_s$

X.J&&Y.L&&Y.S, arxiv: 2305.04416

Linear renormalon of HL Sudakov && Twist-three soft factor

- The HL Sudakov has a new feature: Linear IR-renormalon.
- $B(u) = R(u) + \frac{C_F}{2}\frac{G_0(u) - uG_0'(0) - G_0(0)}{u^2}$, $G_0(u) = \frac{1}{2}\frac{e^{\frac{5u}{3}}}{u-1}\left(\frac{1}{\sin \pi u} - \frac{i}{\cos \pi u}\right)\left(\frac{\mu^2}{4p_z^2}\right)^u$.
- The $u = \frac{1}{2}$ singularity: $B(u) \to -i\frac{\mu}{p_z}\frac{C_F e^{\frac{5}{6}}}{2\pi\left(u - \frac{1}{2}\right)}$. Linear power correction.
- Must cancel with UV renormalon of NLP soft contributions.
- For LFWF factorization, $\psi^1(u, b_\perp) = \frac{C_F}{2p_z b_\perp} e^{\frac{5u}{3}} (\mu^2 b_\perp^2)^u \frac{(2u+1)\sin \pi u \Gamma(-2u)}{\pi}$.
- HL soft factor with single $D_\perp^2$ insertion on the light Wilson-lines.
- Linear rapidity divergence → UV renormalon.
- Similar cancellation in the threshold limit. NLP space-like jet function.

2311.06907

Summary and Outlook

- Double log evolution limit.

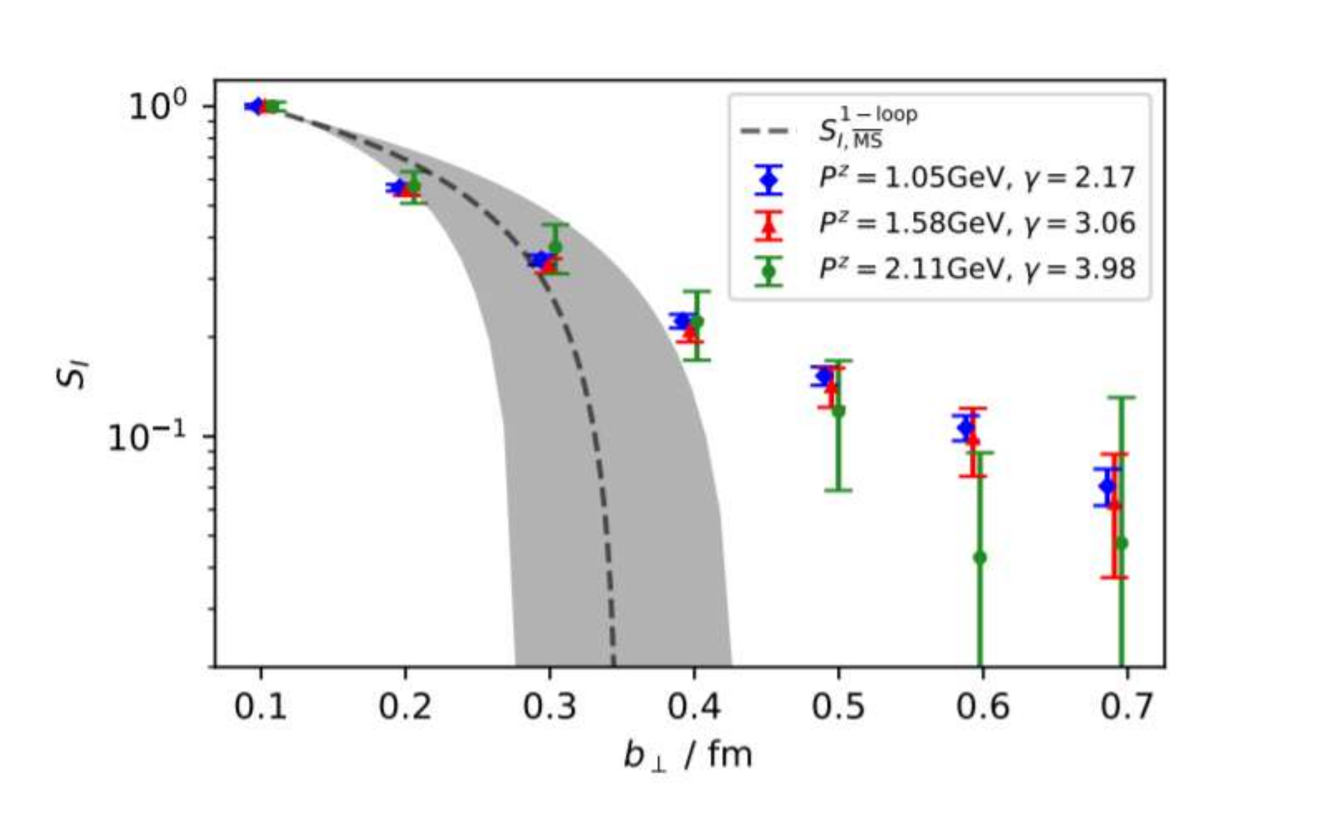
1. Universal light-like cusp anomalous dimension Γ_{cusp}.
2. TMD factorization && threshold limit of quark structure function.

- Heavy-light Sudakov universality class appears naturally in lattice parton distribution.

1. Quasi-TMDPDF && LFWF amplitude && threshold limit of quark quasi-PDF.
2. Allows lattice extraction of TMD parton densities.
3. NNLO Heavy-light Sudakov form factor extracted from threshold limit.

S_r from lattice

Lattice Parton Collaboration, arXiv:2005.14572



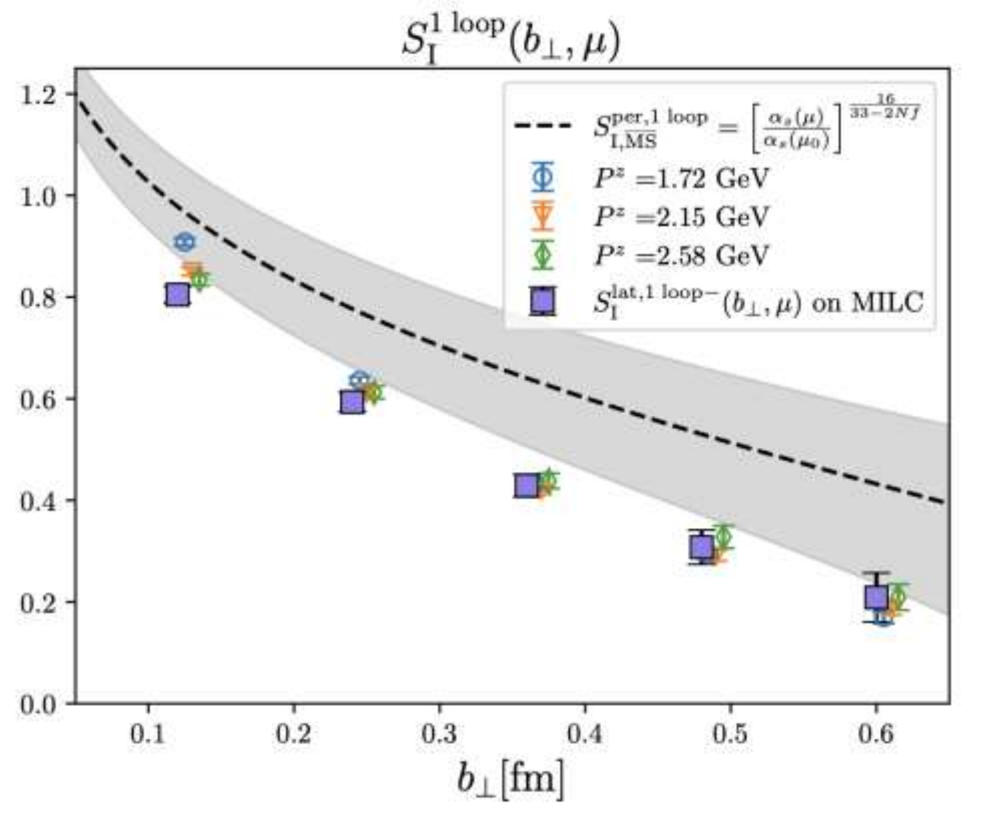
β	$L^3 \times T$	a (fm)	c_{sw}	$\kappa_l^{\rm sea}$	$m_\pi^{\rm sea}$(MeV)
3.34	$24^3 \times 48$	0.098	2.06686	0.13675	333
			N_{cfg}	κ_l^v	m_π^v (MeV)
			864	0.13622	547

CLS A654

Preliminary results of S_r by LPC . Only tree-level matching.

S_r from lattice

Lattice Parton Collaboration, arXiv:2306.06488



Ensemble	a(fm)	$N_\sigma^3 \times N_\tau$	m_π^{sea}	m_π^{val}	Measure
X650	0.098	$48^3 \times 48$	333 MeV	662 MeV	911×4
A654	0.098	$24^3 \times 48$	333 MeV	662 MeV	4923×20
a12m130	0.121	$48^3 \times 64$	132 MeV	310 MeV	1000×4
				220 MeV	1000×16
a12m310	0.121	$24^3 \times 64$	305 MeV	670 MeV	1053×8

MILC a12m310

S_r from lattice

Yuan Li and others, arxiv:2106.13027

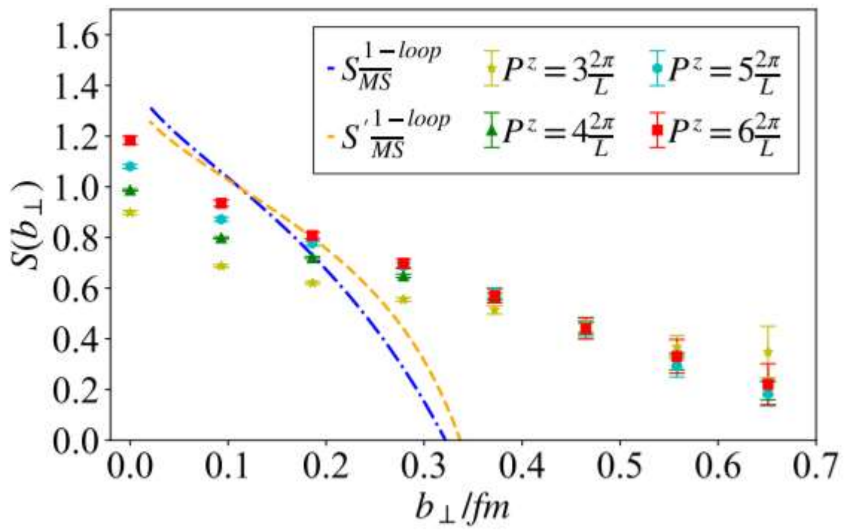


Figure 2. The lattice results of $S(b_\perp)$ for various momenta, together with the one-loop perturbative result $S^{\text{1-loop}}_{\overline{\text{MS}}}$ and its variant $S'^{\text{1-loop}}_{\overline{\text{MS}}}$ with α_s including up to 4 loops. The scale μ in Eq. (17) is set as $\mu = 2$ GeV.

$(L/a)^3 \times T/a$	a (fm)	$a\mu_{sea}$	m^π_{sea}	N_{conf}
$24^3 \times 48$	0.093	0.0053	350	126

$a\mu_{v0}$	m^π_{v0}	$a\mu_{v1}$	m^π_{v1}	$a\mu_{v2}$	m^π_{v2}	$a\mu_{v3}$	m^π_{v3}
0.0053	350	0.013	545	0.018	640	0.03	827

Table I. Parameters of the ensemble used in this work. We list the spatial and temporal extents, L/a and T/a, the lattice spacing a, the sea quark mass μ_{sea}, the pion mass m^π_{sea}, the number of configurations used, N_{conf}, and four valence quark masses μ_{vi} for $i = 0, 1, 2, 3$ together with the associated pion masses m^π_{vi}. All the pion masses are given in units of MeV.

ETMC configuration

Preliminary results of S_r . Only tree-level matching.