Spin hydrodynamics

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Outline

1 Hydrodynamics with spin

1.1 Is QGP the most vortical fluid?

First positive measurements of Λ spin polarization

Non-central heavy-ion collisions create fireballs with large global angular momenta which may generate a spin polarization of the hot and dense matter in a way similar to the Einstein-de Haas and Barnett effects

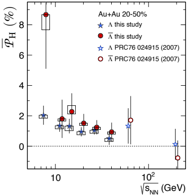
Much effort has recently been invested in studies of polarization and spin dynamics of particles produced in high-energy nuclear collisions, both from the experimental and theoretical point of view

L. Adamczyk et al. (**STAR**), (2017), **Nature 548 (2017) 62-65**, arXiv:1701.06657 (nucl-ex)

Global Λ hyperon polarization in nuclear collisions: evidence for the most vortical fluid

www.sciencenews.org/article/smashing-gold-ions-creates-most-swirly-fluid-ever

The $\sqrt{s_{NN}}$-averaged polarizations indicate a vorticity of $\omega \approx (9 \pm 1) \times 10^{21}\,\mathrm{s}^{-1}$, with a systematic uncertainty of a factor of two, mostly owing to uncertainties in the temperatur This far surpasses the vorticity of all other known fluids, including solar subsurface flow[23] ($10^{-7}\,\mathrm{s}^{-1}$); large-scale terrestrial atmospheric patterns[24] (10^{-7}–$10^{-5}\,\mathrm{s}^{-1}$); supercell tornado cores[25] ($10^{-1}\,\mathrm{s}^{-1}$); the great red spot of Jupiter[26] (up to $10^{-4}\,\mathrm{s}^{-1}$); and the rotating, heated soap bubbles ($100\,\mathrm{s}^{-1}$) used to model climate change[27]. Vorticities of up to $150\,\mathrm{s}^{-1}$ have been measured in turbulent flow[28] in bulk superfluid He II, and Gomez *et al.*[29] have recentl produced superfluid nanodroplets with $\omega \approx 10^{7}\,\mathrm{s}^{-1}$.

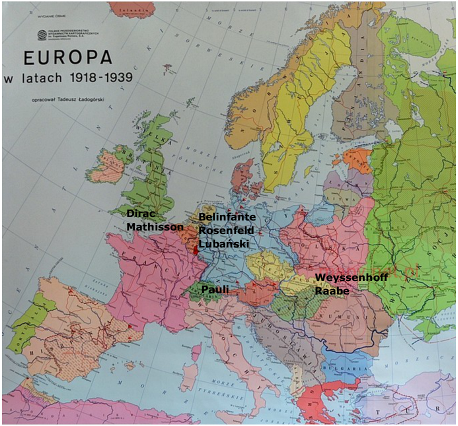


$\Delta t = 1$ fm/c $= 3 \times 10^{-24}$ s, $\quad \Delta t\, \omega_{\rm max} = 27 \times 10^{-24} \times 10^{21} = 2.7 \times 10^{-2}$

Large angular momentum does not mean large rotation!

1.2 Weyssenhoff's spinning fluid

Weyssenhoff's scientific contacts, years 1930s - 1940s



Jan Weyssenhoff
1889-1972

J. Weyssenhoff and A. Raabe, Acta Phys. Pol. 9 (1947) 7

Vol. IX (1947) Acta Physica Polonica Fasc. 1

RELATIVISTIC DYNAMICS OF SPIN-FLUIDS AND SPIN-PARTICLES*

By Jan WEYSSENHOFF and A. RAABE †, Institute of Theoretical Physics, Jagellonian University, Kraków **

1. Following up a train of thought inaugurated by Einstein and Gromer, Mathisson *(1, 1937)* and Lubański *(2, 1937)* deduced the equation of motion of a material particle endowed with spin from the general principles of the theory of relativity. Even for a free particle in Galileian domains these equations do not coincide with the Newtonian laws of motion; there remains an additional term depending on the internal angular momentum or spin of the particle, which raises the order of these differential equations to three. Mathisson did not notice the fact that for the above conditions his equations are equivalent to the equations which were previously found by Frenkel *(3, 1926)* for a spinning electron (if only their terms depending on the electromagnetic field are dropped). This is all the more excusable as Frenkel considered throughout the additional term in his equations as an infinitely small perturbation, and did not even mention the fact that they disagree with Newton's First Law for a free particle.

In the present paper we give a third method of obtaining the same equations by establishing first the laws of the dynamics of an incoherent spin-fluid and passing then to the limit. Strictly speaking, we obtain different and much simpler equations, which prove however to be equivalent to Frenkel's and Mathisson's. The simplification is due to the explicit introduction of the 4-vector of linear monumentum and energy G^{α}.

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* Presented at a meeting of the Cracow Section of the Polish Physical Society on February 28, 1945. The main contents of this paper and the next one, as well as most of the results of the three following, were a subject of a lecture at a secret meeting of physicists at Prof. Pieńkowski's home in Warsaw, October 1942.

** Mr. Raabe was a highly gifted young physicist with whom I outlined in all its main features the contents of this paper and the next one in 1940/41 in Lwów. We tried to pursue our work in 1942 in Cracow, but unfortunately in June 1942 Mr. Raabe fell victim of a man-hunt in the streets of Cracow; he died four months later in the German concentration camp of Oświęcim. (J. W.)

$N^\mu = n u^\mu$ **current = density × flow vector**
analogies for energy, momentum and spin

1) conservation of energy and momentum with an asymmetric energy-momentum tensor

$$T^{\mu\nu}(x) = g^\mu(x) u^\nu(x), \quad \partial_\nu T^{\mu\nu}(x) = 0 \qquad (1)$$

u^μ is the four-velocity of the fluid element, while g^μ is the density of four-momentum
with the notation $\partial_\nu(f u^\nu) \equiv Df$ we may write $Dg^\mu = 0$

2) conservation of total angular momentum $J^{\lambda,\mu\nu} = L^{\lambda,\mu\nu} + S^{\lambda,\mu\nu}$ (orbital and spin parts)

$$L^{\lambda,\mu\nu}(x) = x^\mu T^{\nu\lambda}(x) - x^\nu T^{\mu\lambda}(x), \quad S^{\lambda,\mu\nu}(x) = s^{\mu\nu}(x) u^\lambda(x) \qquad (2)$$

$s^{\mu\nu} = -s^{\nu\mu}$ describes the spin density

$$\partial_\lambda J^{\lambda,\mu\nu} = 0 \to Ds^{\mu\nu} = g^\mu u^\nu - g^\nu u^\mu \qquad (3)$$

3) 10 equations for 13 unknown functions: g^μ, $s^{\mu\nu}$ and u^i ($i = 1, 2, 3$)
additional constraint has been adopted, the Frenkel (or Weyssenhoff) condition
$s^{\mu\nu} u_\mu = 0$

ideas still frequently cited in the context of the Einstein-Cartan theory

1.3 Equilibrated spin

revival of interest in hydrodynamics of spin polarized systems
works of Francesco Becattini and collaborators

Connection betweeen theory and experiment by the "spin Cooper-Frye formula" Pauli-Lubański vector defined by the spin chemical potential $\omega^{\mu\nu}$

$$\pi^\mu(p) = -\frac{1}{8m}\varepsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int d\Sigma_\lambda p^\lambda n(1-n)\omega_{\rho\sigma}}{\int d\Sigma_\lambda p^\lambda n} \qquad (4)$$

integral over freeze-out hypersurface Σ

- spin degrees of freedom are equilibrated, spin-orbit coupling interaction included, asymmetric energy-momentum tensor, the spin chemical potential is equal to thermal vorticity

  $$\omega_{\mu\nu} = \varpi_{\mu\nu} = -1/2\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right) \qquad \beta^\mu = u^\mu/T, \quad \beta = 1/T$$

  the spin chemical potential is not an independent hydrodynamic variable
- standard (dissipative) hydro is used, $\omega_{\mu\nu}$ determined by the standard hydrodynamic variables such as T and u^μ
- recent works on extension to include effects of the shear stress tensor
- forms of global and local distribution functions obtained from QFT (Dirac field under rotation and acceleration)
- **great success in decribing global polarization, ongoing works aiming at understanding the longitudinal polarization**

1.4 Spin conserved

idea of a perfect-fluid hydrodynamics with spin
WF, B. Friman, A. Jaiswal, E. Speranza, Phys. Rev. C97 (2018) 041901

Review: Relativistic hydrodynamics for spin-polarized fluids

WF, A. Kumar, R. Ryblewski, Prog.Part.Nucl.Phys. 108 (2019) 103709 general concept of hydrodynamics with spin: conservation of energy, linear momentum, total angular momentum, and charge:

$$\partial_\mu T^{\mu\nu}[\beta^\alpha, \omega^{\alpha\beta}, \xi] = 0, \tag{5}$$

$$\partial_\lambda J^{\lambda,\mu\nu}[\beta^\alpha, \omega^{\alpha\beta}, \xi] = 0, \tag{6}$$

$$\partial_\mu j^\mu[\beta^\alpha, \omega^{\alpha\beta}, \xi] = 0. \tag{7}$$

Here $\beta^\alpha = u^\alpha/T$, $\xi = \mu/T$, where μ is the chemical potential

$\omega^{\alpha\beta}$ - new chemical potential connected with the angular momentum conservation
$J^{\lambda,\mu\nu} = L^{\lambda,\mu\nu} + S^{\lambda,\mu\nu}$ - sum of the orbital and spin parts

$$\partial_\lambda J^{\lambda,\mu\nu} = 0 \iff \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu} \tag{8}$$

spin-orbit interaction, quantum energy-momentum tensors have asymetric parts
conservation of angular momentum for particle with spin is non-trivial

S. Bhadury, WF, B. Friman, A. Jaiswal, A. Kumar, R. Ryblewski, R. Singh (GSI-Kraków-NISER framework)

- spin tensor is separately conserved, makes sense for s-wave dominated scattering processes, one can use the equation
  $\partial_\lambda S^{\lambda,\mu\nu} = 0$
  $\omega_{\mu\nu}$ plays a role of the traditional Lagrange multiplier
- the forms of the energy-momentum and spin tensor are those derived by de Groot, van Leuveen, van Weert (canonical book on the relativistic kinetic theory), **semiclassical expansion of the Wigner function**
- for small spin polarization, energy-momentum evolution has no correction from spin, the spin dynamics can be considered in a given hydrodynamic background
- **inclusion of dissipation by using an RTA (relaxation time approximation) collisional integral;** Phys. Lett. B814 (2021) 136096
- **inclusion of magnetic fields (spin MHD):** Phys. Rev. Lett. 129 (2022) 192301
- **numerical solutions found for simple expansion geometries** (mostly one-dimensional expansion, problems with stability...)

1.5 Pseudo-gauge freedom

Pseudo-gauge transformation (QCD language in the context of the proton spin puzzle: adding boundary terms)

$$T'^{\mu\nu} = T^{\mu\nu} + \frac{1}{2}\partial_\lambda\left(\Phi^{\lambda,\mu\nu} + \Phi^{\nu,\mu\lambda} + \Phi^{\mu,\nu\lambda}\right) \quad (9)$$

$$S'^{\lambda,\mu\nu} = S^{\lambda,\mu\nu} - \Phi^{\lambda,\mu\nu} + \partial_\rho Z^{\mu\nu,\lambda\rho} \quad (10)$$

One most often considers free Dirac field, should describe a gas of fermions, good starting point for thermodynamics and/or hydrodynamics

Canonical forms (directly obtained from Noether's Theorem): asymmetric energy-momentum tensor, spin tensor directly expressed by axial current (couples to weak interactions)

Belinfante-Rosenberg version, $\Phi^{\lambda,\mu\nu} = S^{\lambda,\mu\nu}, Z^{\mu\nu,\lambda\rho} = 0$, (couples to classical gravity); spin tensor appears in modified theories of gravity, couples to torsion

de Groot, van Leuveen, van Weert (GLW) forms: symmetric energy-momentum tensor and conserved spin tensor

Hilgevoord and Wouthuysen (HW) choice: symmetric energy-momentum tensor and conserved spin tensor

1.6 Local vs. non-local effects

dissipative hydrodynamics and kinetic theory with spin
N. Weickgenannt, E. Speranza, X.-l. Sheng, Q. Wang, and D. H. Rischke
Phys. Rev. Lett. 127 (2021) 052301
Phys. Rev. D104 (2021) 016022.

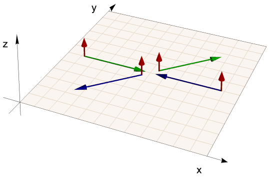
Classical spins

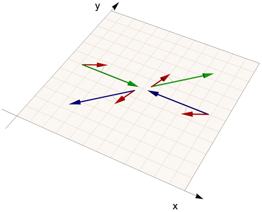
Internal angular momentum (Mathisson), classical spin vector, extended phase-space

Review: WF, A. Kumar, R. Ryblewski, Prog.Part.Nucl.Phys. 108 (2019) 103709

$$s^{\alpha\beta} = \frac{1}{m}\varepsilon^{\alpha\beta\gamma\delta} p_\gamma s_\delta, \quad p_\alpha s^\alpha = 0, \quad s^\alpha = \frac{1}{2m}\varepsilon^{\alpha\beta\gamma\delta} p_\beta s_{\gamma\delta} \tag{11}$$

$$f^{\pm}_{\rm eq}(x,p,s) = \exp\left(-p\cdot\beta(x) \pm \xi(x) + \frac{1}{2}\omega_{\alpha\beta}(x) s^{\alpha\beta}\right) \tag{12}$$





Local scattering = s-wave scattering
classical treatment of spin, local collisions $\Longleftrightarrow$ semiclassical approach based on the Wigner function, GLW tensors

N. Weickgenannt, E. Speranza, X.-l. Sheng, Q. Wang, and D. H. Rischke

- classical treatment of spin with non-local effects
- use of the HW pseudogauge
- spin chemical potential agrees with thermal vorticity in global equilibrium
- discussion about the concept of local equilibrium (of particles with spin)
- reproduces non-relativistic mechanics of polar fluids (G. Łukasiewicz, *Micropolar fluids, Theory and Applications*)
- the method of moments used to derive hydrodynamic equations, spin version of DNMR (Denicol-Niemi-Molnar-Rischke)
- extensions to spin-1 particles

at the moment a very promising approach to develop spin hydrodynamics although non-local effects may potentially lead to problems with causality (?)

1.7 Entropy production arguments

methods of Israel and Stewart applied to hydrodynamics with spin
K. Hattori, M. Hongo, X.-G. Huang, M. Matsuo, H. Taya
Phys. Lett. B 795 (2019) 100

- combined ideas of **Weyssenhoff + Navier/Stokes + Israel/Stewart** (phenomenological formulation)

$$T^{\mu\nu}_{\rm ph} = T^{\mu\nu}_{\rm perfect\,fluid} + T^{\mu\nu}_{\rm ph,1}\,, \quad S^{\lambda,\mu\nu}_{\rm ph} = u^\lambda S^{\mu\nu} + S^{\lambda,\mu\nu}_{\rm ph,1} \tag{13}$$

- **thermodynamic relations**

$$\varepsilon + P = Ts + \omega_{\alpha\beta} S^{\alpha\beta}\,, \quad dP = sdT + S^{\alpha\beta} d\omega_{\alpha\beta} \tag{14}$$

- entropy current analysis

$$S^{\mu}_{\rm ph} = T^{\mu\nu}_{\rm ph} + P\beta^\mu - \omega_{\alpha\beta} S^{\alpha\beta}\beta^\mu + ... \tag{15}$$

- symmetric and asymmetric contributions to the energy-momentum tensor

$$T^{\mu\nu}_{\rm ph,1s} = h^\mu u^\nu + h^\nu u^\mu + \tau^{\mu\nu}\,, \quad T^{\mu\nu}_{\rm ph,1a} = q^\mu u^\nu + q^\nu u^\mu + \phi^{\mu\nu} \tag{16}$$

- new kinetic coefficients should be introduced (as shear viscosity in $\tau^{\mu\nu} = 2\eta\sigma^{\mu\nu}$)

- discussion of the pseudo-gauge invariance - our freedom to choose different forms of the energy-momentum and spin tensors

  K. Fukushima. S. Pu, Phys. Lett. B 817 (2021) 136346

  A. Daher, A. Das, WF, R. Ryblewski, Phys. Rev. C 108 (2023) 024902

- stability and causality checks

  G. Sarwar, M. Hasanujjaman, J. R. Bhatt, H. Mishra, J.-e. Alam, Phys. Rev. D107 (2023) 54031

  A. Daher, A. Das, R. Ryblewski, Phys. Rev. D107 (2023) 54043

- numerical solutions for simple expansion geometries

  D.-L. Wang, S. Fang, S. Pu, Phys. Rev. D104 (2021) 114043

  R. Biswas, A. Daher, A. Das, WF, R. Ryblewski, Phys. Rev. D107 (2023) 094022

- second-order theory has been constructed

  R. Biswas, A. Daher, A. Das, WF, R. Ryblewski, Phys. Rev. D108 (2023) 014024

2. Summary and Outlook

- Spin hydrodynamics is a new territory (New World) with many groups that have made progress going in different directions (conquering different continents). It is time now to make some efforts to synthetize those various efforts.
- Basic issues with pesudo-gauge (in)dependence seem to be (at least) partially solved.
- An old concept of a fluid is rapidly changing...