Twist decomposition of non-linear effects in Balitsky-Kovchegov evolution of proton structure functions

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Introduction: the subject — the dense partonic system

- The key problem: understanding of nucleus or nucleon structure in terms of partons probed in high energy scattering of small (hard) probes
- Two main regimes:

  Linear: at high probe scale Q^2 and moderate x the parton evolution with scale and x is described by linear DGLAP evolution equations.

  Well measured and understood, the domain of precision physics

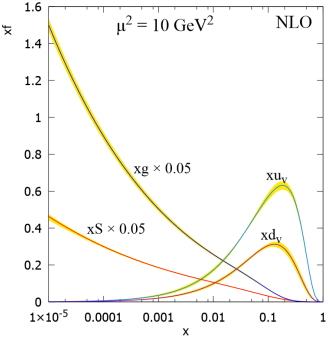
  Non-linear: at moderate Q^2 and very small x, say $x < 10^{-3}$, the number of partons in the nucleon grows rapidly and the partons start recombining. Alternatively: at scattering multiple partons couple to the probe. Eventually parton saturation / unitarity limit approached

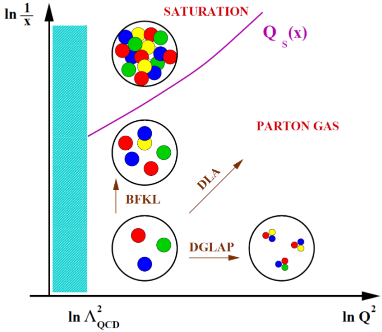
  Color Glass Condensate: complex, many open questions

Introduction: understanding the dense partonic systems

- When a dense system is probed in high energy scattering it is essential to take into account multiple scattering effects entangled with evolution effects. The emerging structures are complex, for instance parton number in the exchange is not conserved
- In the collinear picture, multiple scattering is (in part) described by operators with higher twist, and those are not known experimentally. Even to construct the complete basis for twist 4-operators for the nucleon took more than two decades (Jaffe, Soldate; Ellis, Furmanski, Petronzio 1982; Braun, Manashov, Rohrwild 2009)
- Problem: experimentally, the scales where these effects are important are at the onset of perturbative regime. These scales become harder at smaller x and for denser targets, like large nuclei

Gluon growth and dominance at small x





The relevance

- The collinear factorization framework at the leading twist is an excellent theoretical tool, but it is limited to a subclass of observables and to domain of hard physics
- Complete description of hadron scattering events requires treatment of multiple scattering. This is part of existing event generators like e.g. PYTHIA or HERWIG, but currently a lot of phenomenological modeling is involved. Need better theory
- The collisions with nuclei are even more difficult: nuclear PDFs are different from those for a nucleon, multiple scattering is a dominant regime
- Understanding the proton structure: collinear PDFs are just parton densities, multiple scattering (or higher twist operators) probes correlations of partons in the nucleon wave function

Experimental evidence

Can we probe the high parton density regime and how?

- The most convincing evidence for the transition from the linear to non-linear evolution still comes from the family of models originating from the famous Golec-Biernat–Wüsthoff model (1998), describing in a unified way DIS at HERA down to small scales and the hard inclusive diffractive scattering.
- The key features: the power like growth of the onset of the dense regime $Q_s(x)$ with decreasing x and the emergent geometric scaling phenomenon, $\sigma(Q_s^2(x)/Q^2)$ [Golec-Biernat, Kwieciński, Staśto, 2000]
- HERA data require $Q_s^2(x) \sim 1$ GeV^2 at $x \sim 10^{-4}$, but at EIC higher saturation scales are expected to appear due to the nuclear enhancement factor $\sim A^{1/3}$.
- Other observables under study: e.g. dijet p_T imbalance in ep, pp and pA collisions

The fundamental underlying concept is the unitarity of the S–matrix

- Theory description of saturation: non-linear evolution equations for gluon densities at small x. First approach: collinear — extensions of DGLAP to Gribov–Levin–Ryskin (GLR) equation (1983) and Bukhvostov, Frolov, Kuraev, Lipatov equation (1985)
- More insight: non-linear evolution equations for k_T dependent PDFs, resuming powers of $\log(1/x)$. Based on the Balitsky–Fadin–Kuraev–Lipatov approach
- Milestones: Bartels triple pomeron vertex (1993), Balitsky hierarchy (1995), JIMWLK evolution (1998) Kovchegov evolution (1999)
- Balitsky–Kovchegov evolution explains the key features of the GBW model, and can fit the data
- Currently NLL$(1/x)$ non-linear evolution is available, and a handful of NLO calculations in this framework are available

Our approach: the goal and outline

- The goal: use information provided by the low-x BFKL/BK framework to input the collinear framework:

  $K_T \to$ collinear

  BFKL $\to$ DGLAP + Higher Twist corrections

  Balitsky–Kovchegov $\to$ "GLR"

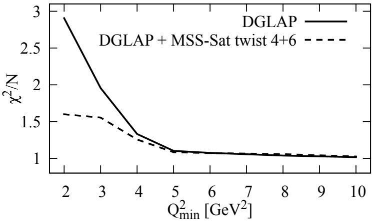
- Why? — because in this way the power and precision of DGLAP can be employed at the onset of non-linear effects

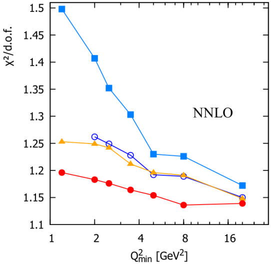
- Outline: use the Mellin moment space conjugate to scale to isolate contributions of subsequent twist operators in BFKL / BK. Identify effects of nonlinearity at all twists

Based on LM, M. Sadzikowski, 2306.02118, EPJ C

Puzzle: problems of DGLAP in precision HERA data at low x and moderate Q^2

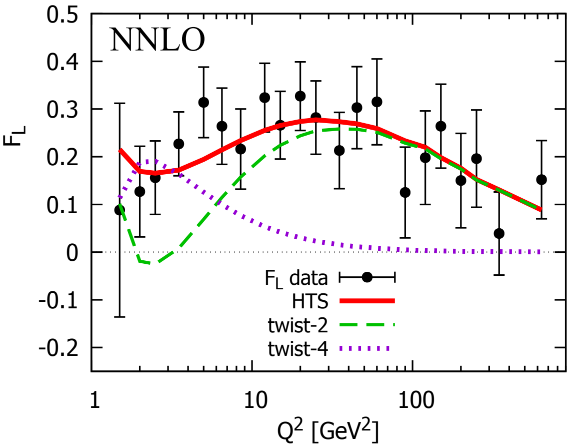
- The final HERA data show problems of DGLAP fits of DIS and DDIS when $Q^2 < 5$ GeV^2 data are included fits, problems at small x
- Explanations proposed:
  1. Higher Twist corrections [LM, M. Sadzikowski, W. Słomiński, 2012, L. Harland–Lang et al., 2016; I. Abt et al. 2016; LM, M. Sadzikowski, W. Słomiński, K. Wichman, 2017]
     Expectations: twist 4 / twist 2 $\sim (Q_0^2/Q^2)(x_0/x)^\lambda$



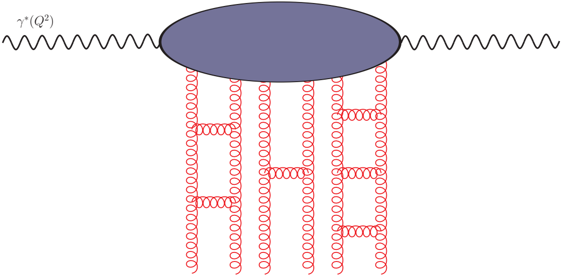


  2. Small x resummation beyond DGLAP [R. Ball et. al, 2017]

Somewhat surprising finding from DGLAP + Higher Twist fits: small corrections in F_2, significant and **positive** corrections in F_L [arXiv:1707.05992]

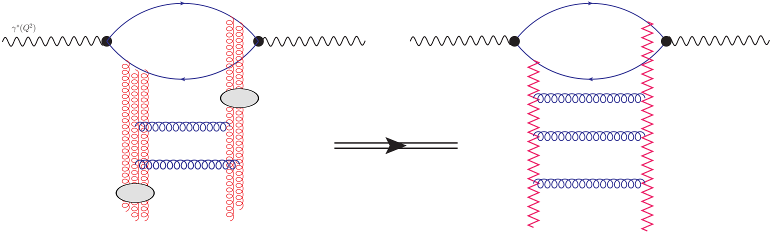


- Golec-Biernat–Wüsthoff or Bartels–Golec-Biernat–Kowalski saturation models assume multiple independent (eikonal) scattering: $\sigma(x, Q^2) = \sigma_0 \left[1 - \exp(-\sigma_1(x, Q^2)/\sigma_0)\right]$
- HT contributions found to increase quickly with decreasing x, as $\sigma^{(\text{twist}=2n)} \sim \sigma_1^n$
- This is, however a model that does not fully agree with QCD-based analyzes concerning the HT components.



Independent multiple scattering
— eikonal approximation —
GBW saturation model

- QCD: HT from BFKL equation [LM, M. Sadzikowski, arXiv:1411.7774] — at LL$(1/x)$ the HT contributions found to decrease with decreasing x in the asymptotic regime.
- The reason: gluon Reggeization binds t-channel gluons that couple to color dipole produced by γ^* into two Reggeized gluons, that span a single gluon ladder dominated by twist 2 contribution.



BFKL bootstrap — gluon Reggeization

BFKL equation — two interacting Reggeized gluons

The Higher Twist puzzle

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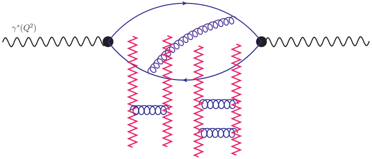
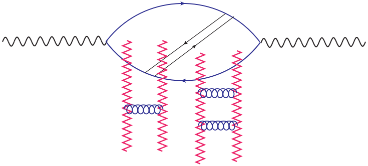
A possible solution: coupling of two BFKL ladders to a splitted γ^* dipole:



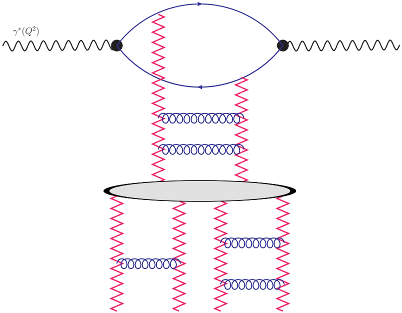


To be consistent, one should include the BFKL evolution both in the dipole wave function and in the ladders. At LL($1/x$): one iteration of nonlinearity in the BK equation

Higher Twists from LL Baltisky–Kovchegov (BK) equation

Triple Pomeron Vertex allows for a transition from single BFKL ladder to two and more ladders, that carry the HT contributions with the strongest enhancement due to evolution.

This mechanism should provide the most reliable estimate of HT effects in the proton structure in the LL$(1/x)$ approximation in QCD.



Triple Pomeron (BK) vertex — transition to 4 Reggeized gluons

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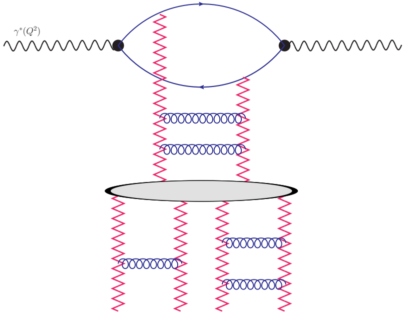


Triple Pomeron (BK) vertex — transition to 4 Reggeized gluons

Note however: the BK Triple Pomeron Vertex vanishes at LL(Q^2)
[J. Bartels, K. Kutak, 2007]

Calculational method

- The HT determinations are performed in the space of Mellin moments conjugate to the hard scale.
- The total cross sections related to F_2 and F_L structure functions may be decomposed into twist components by isolating the singularities in the Mellin plane, that at small α_s lead to terms with the canonical Q^2 scalling, $(Q^2)^{-n}$.
- For GBW saturation model one finds simple poles in the Mellin plane for integer n.
- For LL BFKL equation essential singularities appear at integer n.
- For LL BK equation there are multiple ladders, that lead to multiple Mellin variables, and convolutions of the Mellin integrals. The convoluted expressions have essential singularities in all Mellin variables.
- Present results on HT from BK: one Triple Pomeron Vertex included, leading to two independent Mellin variables. Enough to estimate the leading twist 4 contribution.

Expansion of Balitsky–Kovchegov equation in nonlinearity

The basic object: impact parameter density of dipole scattering amplitude $N(y, \mathrm{r})$, which is related to unintegrated and collinear gluon densities by linear integral transformations. It is convenient to use $\phi(y, k^2)$ instead

$$\phi(y, k_\perp^2) = \int \frac{d^2 r}{2\pi} e^{-i\mathrm{k}_\perp \cdot \mathrm{r}} \frac{N(y, \mathrm{r})}{r^2}.$$

The Balitsky–Kovchegov equation for $\phi(y, k_\perp^2)$

$$\frac{\partial \phi(y, k_\perp^2)}{\partial y} = \bar{\alpha}_s \int_0^\infty \frac{dq_\perp^2}{q_\perp^2} \left\{ \frac{q_\perp^2 \phi(y, q_\perp^2) - k_\perp^2 \phi(y, k_\perp^2)}{|q_\perp^2 - k_\perp^2|} + \frac{k_\perp^2 \phi(y, k_\perp^2)}{\sqrt{4q_\perp^4 + k_\perp^2}} \right\} - \bar{\alpha}_s \phi^2(y, k_\perp^2)$$

The solution in terms of power series of in nonlinearity — expansion in the solutiuon ϕ_0 to the linear equation

$$\phi \;=\; \phi_0 + \phi_1 + \phi_2 + \ldots \;=\; \sum_{n=0}^{\infty} \phi_n, \qquad \phi_n \sim \phi_0^{n+1}$$

The order n is equal to number of Triple Pomeron Vertices. For $n = 0$ the linear — i.e. the BFKL equation — is reproduced, for $n = 1$ the $1 \to 2$ BFKL ladders transition is described.

The iterative solution

We solve the equation directly in the Mellin space:

$$\begin{aligned}
\frac{\partial \tilde{\phi}_0(y,\gamma)}{\partial y} &= \bar{\alpha}_s \chi(\gamma) \tilde{\phi}_0(y,\gamma) \\
\frac{\partial \tilde{\phi}_1(y,\gamma)}{\partial y} &= \bar{\alpha}_s \chi(\gamma) \tilde{\phi}_1(y,\gamma) \\
& -2\pi i\, \bar{\alpha}_s \int_{c_1 - i\infty}^{c_1 + i\infty} \frac{d\gamma_1}{2\pi i} \int_{c_2 - i\infty}^{c_2 + i\infty} \frac{d\gamma_2}{2\pi i} \delta(\gamma - \gamma_1 - \gamma_2) \tilde{\phi}_0(y,\gamma_1) \tilde{\phi}_0(y,
\end{aligned}$$

The solution is obtained iteratively,

$$\phi_0(y, k_\perp^2) = \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{4k_\perp^2}{Q_0^2} \right)^{-\gamma} C_0(\gamma) \exp[\bar{\alpha}_s \chi(\gamma) y]$$

$$\begin{aligned}
\tilde{\phi}_1(y,\gamma) &= \int \frac{d\gamma_1}{2\pi i} \frac{d\gamma_2}{2\pi i} 2\pi i \delta(\gamma - \gamma_1 - \gamma_2) C_0(\gamma_1) C_0(\gamma_2) \\
& \frac{\exp(\bar{\alpha}_s y \chi(\gamma)) - \exp(\bar{\alpha}_s y \chi(\gamma_1) + \bar{\alpha}_s y \chi(\gamma_2))}{\chi(\gamma_1) + \chi(\gamma_2) - \chi(\gamma)}
\end{aligned}$$

Mellin representation of γ^* cross sections

The γ^*_T and γ^*_L cross sections may be also expanded $\sigma^{\gamma^* A}_{T,L} = \sum_{i=0}^{\infty} \sigma^{(i)\,\gamma^* A}_{T,L}$

$$\sigma^{(i)\,\gamma^* A}_{T,L}(x, Q^2) = \sigma_0 \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{Q_0^2}{Q^2}\right)^{\gamma} \tilde{H}_{T,L}(\gamma) \frac{\Gamma(1-\gamma)}{2^{2\gamma-1}\Gamma(\gamma)} \tilde{\phi}_i(y, \gamma)$$

The Mellin fundamental strip is located in $-3/4 < \Re c < 0$. Functions $\tilde{H}_{T,L}$ are Mellin transforms of the photon wave functions.
The linear (BFKL) component

$$\sigma^{(0)\,\gamma^* A}_{T,L} = \sigma_0 \int_{-1/2-i\infty}^{-1/2+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{Q_0^2}{Q^2}\right)^{\gamma} \tilde{H}_{T,L}(\gamma)\Gamma(-\gamma) e^{\bar{\alpha}_s y \chi(\gamma)},$$

The first nonlinear correction:

$$\begin{aligned} \sigma^{(1)\,\gamma^* A}_{T,L} &= \sigma_0 \int_c \frac{d\gamma}{2\pi i} \left(\frac{Q_0^2}{Q^2}\right)^{\gamma} \int_{c_1} \frac{d\gamma_1}{2\pi i} \tilde{H}_{T,L}(\gamma) \frac{\Gamma(1-\gamma) B(\gamma_1, \gamma-\gamma_1)}{2\gamma_1(\gamma-\gamma_1)} \\ &\times \frac{\exp(\bar{\alpha}_s y \chi(\gamma)) - \exp(\bar{\alpha}_s y \chi(\gamma_1) + \bar{\alpha}_s y \chi(\gamma-\gamma_1))}{-\chi(\gamma) + \chi(\gamma_1) + \chi(\gamma-\gamma_1)} \end{aligned}$$

The double Mellin form of γ^* cross sections

- The cross sections

$$\sigma^{(1)\,\gamma^* A}_{T,L} = \sigma_0 \int_c \frac{d\gamma}{2\pi i} \left(\frac{Q_0^2}{Q^2}\right)^{\gamma} \int_{c_1} \frac{d\gamma_1}{2\pi i} \tilde{H}_{T,L}(\gamma) \frac{\Gamma(1-\gamma)B(\gamma_1, \gamma-\gamma_1)}{2\gamma_1(\gamma-\gamma_1)}$$
$$\times \frac{\exp(\bar{\alpha}_s y \chi(\gamma)) - \exp(\bar{\alpha}_s y \chi(\gamma_1) + \bar{\alpha}_s y \chi(\gamma-\gamma_1))}{-\chi(\gamma) + \chi(\gamma_1) + \chi(\gamma-\gamma_1)}$$

  have isolated singularities in the double Mellin plane (γ, γ_1) in the right half-planes

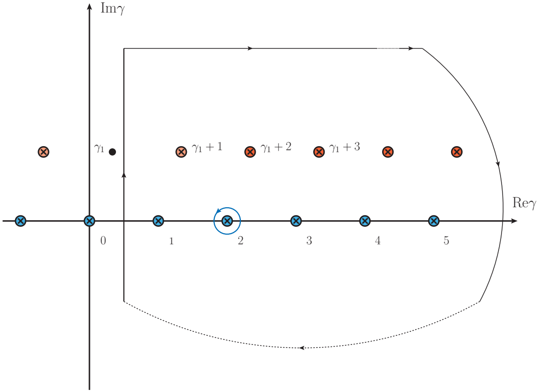
- The first line contributes with poles while essential singularities appear in the second one

- Singularities from $\Gamma(\gamma_i)$ and $\chi(\gamma_i)$ are found for integer γ and γ_1, but also for integer $\gamma - \gamma_1$

- Recall that the eigenvalue of the BFKL kernel $\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$ has simple poles for all integer values of γ. In our convention the twist poles correspond to singularities in positive integer values of γ.

Twist decomposition of γ^* cross sections

- Decomposition strategy: represent the cross sections as sums of contributions coming from singularities in γ at fixed γ_1

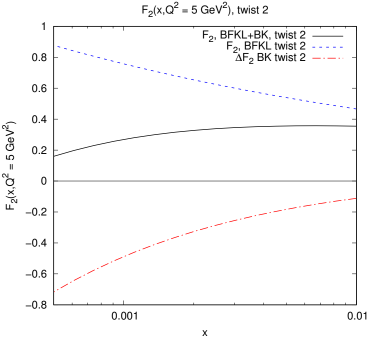


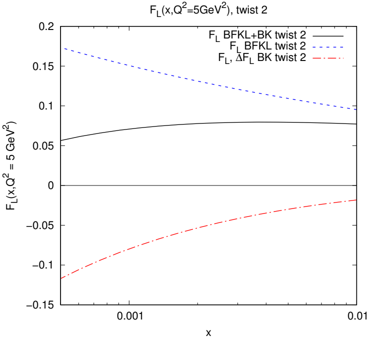
- The integrations around γ-singularities performed numerically. The resulting expression has terms with γ_1 dependence factored out from Q^2 dependence and terms of the form of $\int \frac{d\gamma_1}{2\pi i}(Q_0^2/Q^2)^{\gamma_1} G(\gamma_1)$ where $G(\gamma_1)$ has singularities at integer γ_1

The leading twist shadowing in BK in proton F_2 (left) and F_L (right)

$Q^2 = 5\ \mathrm{GeV}^2$, twist 2 only

The curves: **BFKL + BK**, **BFKL**, **BK correction**





Note: the calculations are performed for the BFKL term adjusted to data. The plots show the strength of the BK effects at twist 2 assuming a single iteration of the Triple Pomeron Vertex.

$Q^2 = 5$ GeV^2, the ratio of higher twist contributions to the BFKL+BK twist-2 contribution

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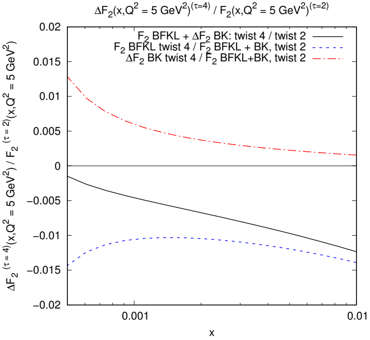
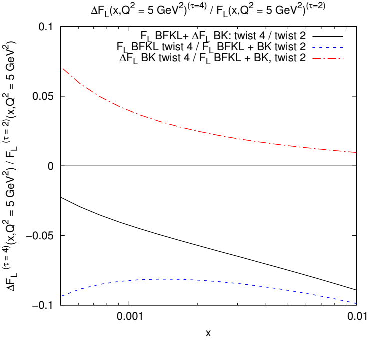




The BFKL/BK higher twist corrections are found to be a small (F_2) or moderate (F_L) fraction of the leading twist contributions.

Surprises

1. Surprise: approach to the non-linear regime from above (in Q^2) in proton structure functions occurs mostly by non-linear corrections at the leading twist. This is in a seeming conflict with the non-linearity beeing related to multiple scattering.

   Solution: multiple scattering is absorbed into the initial condition for twist-2 operators, provided the input scale is larger than the saturation scale

2. Surprise: the Balitsky–Kovchegov equation does not lead to the exact GLR equation in the double logarithmic limit: BK gives two powers of $\log(Q^2)$ less than GLR — it enters at NNLL(Q^2). This is a direct consequence of vanishing triple pomeron vertex for k_T-ordering [J. Bartels, K. Kutak] and it weakens non-linear corrections to the DGLAP evolution

Disclaimer

The presented results focus on the BK equation, describing total cross section for color dipole scattering. This is a particular inclusive observable related to the dipole TMD that probe the non–linear effects in a particular way. Impact of non-linearity on other observables / TMDs may be stronger.

Non-linear effects in collinear evolution

BK equation for unintegrated gluon density $f(x,k^2)$ (kernels K_1 and K_2)

$$\frac{xdf(x,k^2)}{dx} = \alpha_s K_1[f](x,k^2) - \alpha_s^2 K_2[f \otimes f](x,k^2)$$

may be transformed at DLA to GLR-like equation for
$G(x,Q^2) \equiv xg(x,Q^2)$

$$\frac{\mu^2\, dG(x,\mu^2)}{d\mu^2} = \alpha_s P_1[G](x,\mu^2) - \frac{\alpha_s^2}{\mu^2 R^2} P_2[G \otimes G](x,\mu^2)$$

The nonlinear term $\sim Q_s^2/\mu^2$ affects evolution between Q_0^2 and Q^2 mostly for $\mu^2 \sim Q_0^2$

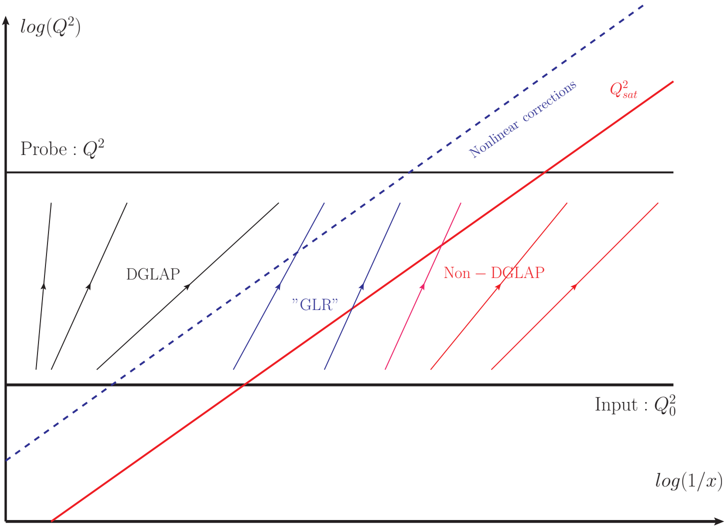
Represent $G(x,\mu^2)$ as $G(x,\mu^2) = h(x,\mu^2) G_{\rm linear}(x,\mu^2)$, solve non-linear equation. Neglecting x-dependence:

$$\frac{h(Q^2)-1}{h(Q^2)} = -c\left[\frac{G_{\rm linear}(Q_0^2)}{R^2 Q_0^2} - \frac{G_{\rm linear}(Q^2)}{R^2 Q^2}\right] \equiv \frac{\delta G}{G_{\rm linear}}$$

At $Q^2 \to \infty$: $\delta G/G_{\rm linear} \sim -c[G_{\rm linear}(Q_0^2)/Q_0^2]$ — not vanish.

The effects on the shape of $G(Q^2)$ are, however, strongest for $Q^2 \to Q_0^2$.

The three evolution regimes



Conclusions and outlook

- We have performed the twist decomposition of the proton structure functions at small x from the LL$(1/x)$ Balitsky–Kovchegov equation, assuming a single iteration of the Triple Pomeron Vertex
- Non-linear (saturation) corrections enter strongly at twist 2, the unitarization is driven by the twist 2 contributions
- The BK introduces small higher twist corrections in F_2 and moderate corrections in F_L
- We have obtained a GLR-like equation from BK for the dipole gluon density. The non-linear corrections in the evolution may affect precision DIS data

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THANKS!