Naturally small neutrino mass from asymptotic safety

Enrico Maria Sessolo

National Centre for Nuclear Research (NCBJ)
Warsaw, Poland

Based on

JHEP 08 (2022) 262 (2204.00866)
and 2308.06114 (accepted in JHEP)

in collaboration with
Abhishek Chikkaballi, Kamila Kowalska, Soumita Pramanick

National Science Centre Poland

2PiNTS Kraków

23.11.2023

NATIONAL CENTRE FOR NUCLEAR RESEARCH ŚWIERK

Neutrino mass

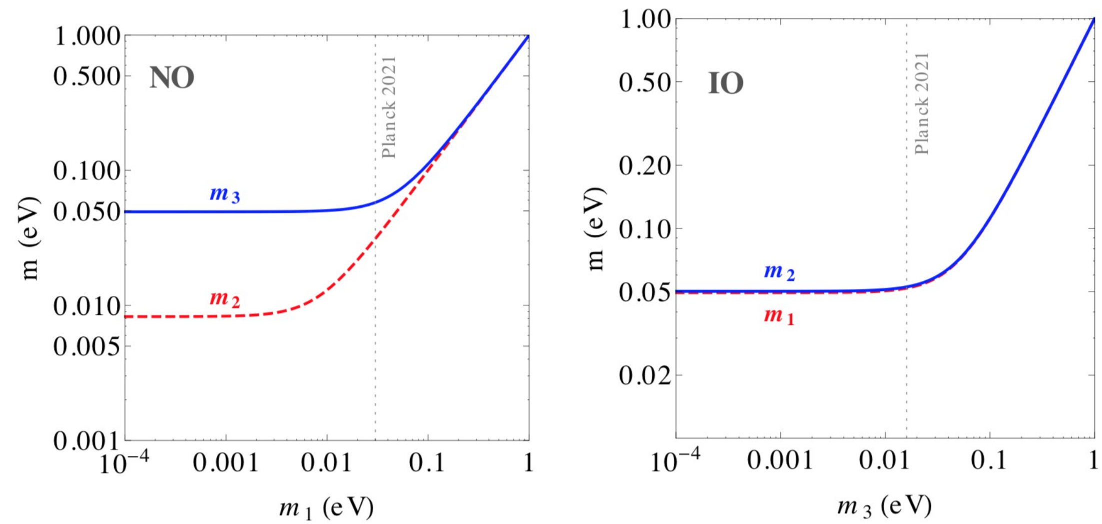
Neutrino masses are very small !

NuFIT5.1 (2021) 2007.14792

$$\begin{aligned} &\Delta m^2_{21} = 7.42^{+0.21}_{-0.20} \times 10^{-5}\,\mathrm{eV}^2, \\ \text{NO:}\quad &\Delta m^2_{31} = 2.515^{+0.028}_{-0.028} \times 10^{-3}\,\mathrm{eV}^2, \\ \text{IO:}\quad &\Delta m^2_{32} = -2.498^{+0.028}_{-0.029} \times 10^{-3}\,\mathrm{eV}^2, \end{aligned}$$

Planck (2021) 1807.06209

$$\sum_{i=1,2,3} m_i < 0.12\,\mathrm{eV}$$

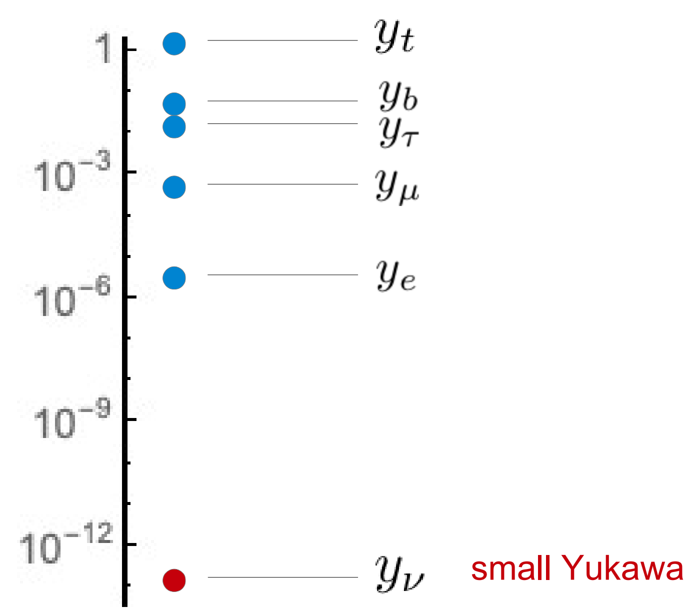


… either Dirac neutrino ...

RHN → Higgs mechanism → Small Yukawa

$$\mathcal{L}_D = -y_\nu^{ij} \nu_{R,i} (H^c)^\dagger L_j + \mathrm{H.c.}$$

$$m_\nu = \frac{y_\nu v_H}{\sqrt{2}}$$



Neutrino mass

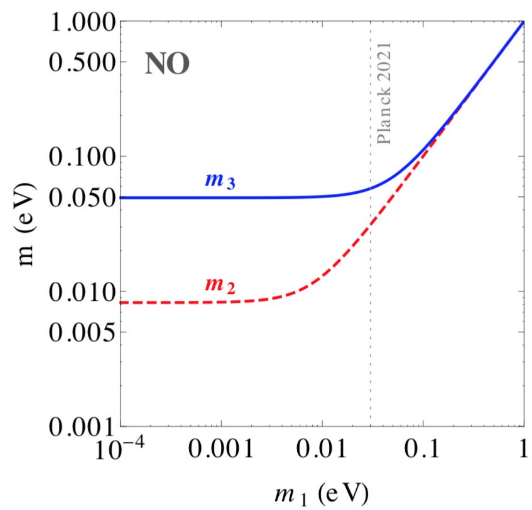
Neutrino masses are very small !

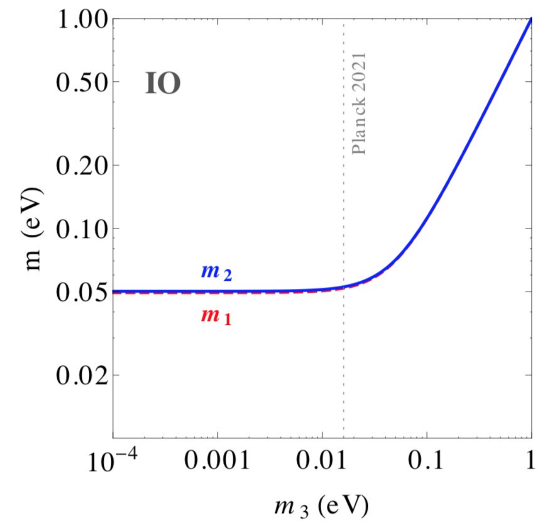
NuFIT5.1 (2021) 2007.14792

$$\begin{aligned} &\Delta m^2_{21} = 7.42^{+0.21}_{-0.20} \times 10^{-5}\,\mathrm{eV}^2, \\ \text{NO:}\quad &\Delta m^2_{31} = 2.515^{+0.028}_{-0.028} \times 10^{-3}\,\mathrm{eV}^2, \\ \text{IO:}\quad &\Delta m^2_{32} = -2.498^{+0.028}_{-0.029} \times 10^{-3}\,\mathrm{eV}^2, \end{aligned}$$

Planck (2021) 1807.06209

$$\sum_{i=1,2,3} m_i < 0.12\,\mathrm{eV}$$





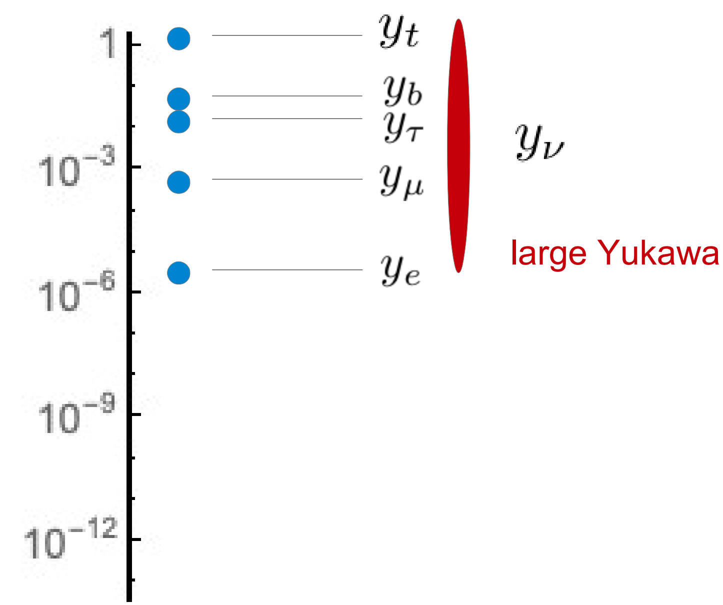
… or Majorana neutrino ...

<u>see-saw mechanism</u>

$$\mathcal{L}_M = \mathcal{L}_D - \frac{1}{2} M_N^{ij} \nu_{R,i} \nu_{R,j} + \mathrm{H.c.}$$

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D & M_N \end{pmatrix} \qquad m_\nu = \frac{y_\nu^2 v_H^2}{\sqrt{2} M_N} \longrightarrow$$

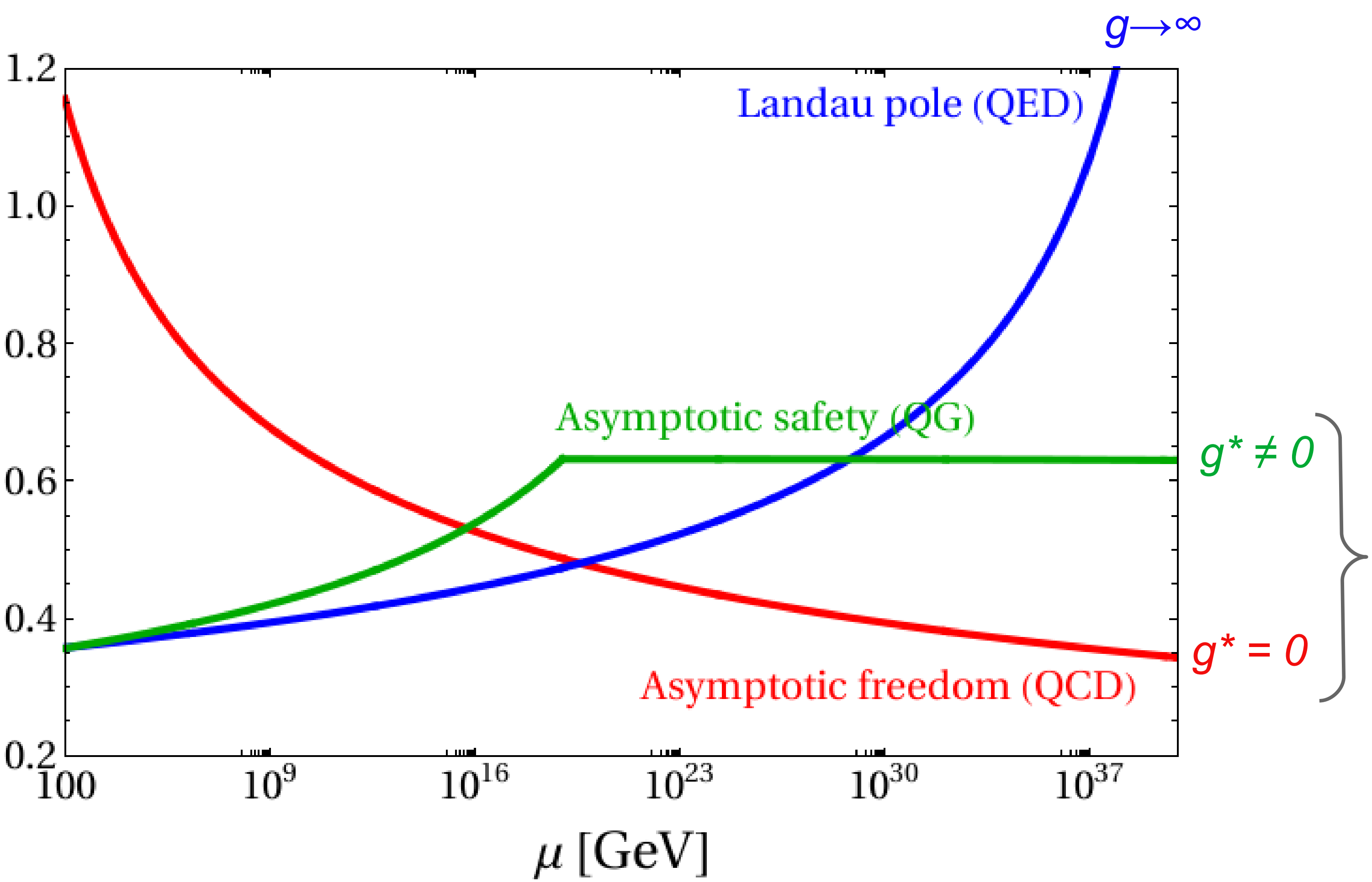
1 parameter M_N



An alternative dynamical mechanism and its signatures

Renormalization group flow

Charge-screening by quantum fluctuations → *running* coupling constants, $g(\mu)$



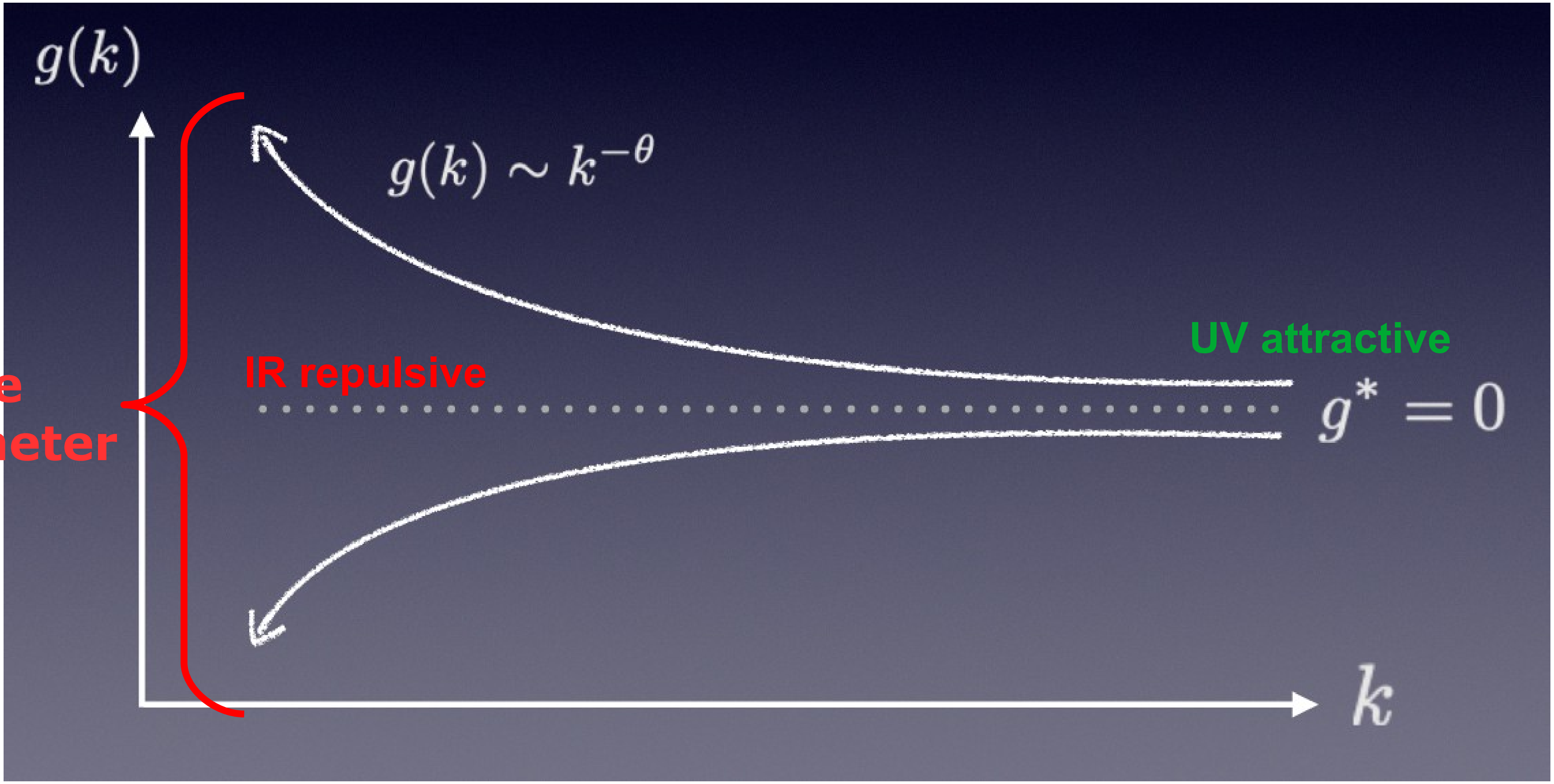
$$\beta_g = \frac{dg}{dt} = \frac{dg}{d\ln\mu}$$

$$\beta_g(g^*) = 0$$

fixed point g^*
in the RG flow

Scaling properties of *g*

$M_{ij} = \partial\beta_i/\partial\alpha_j|_{\{\alpha_i^*\}}$ (-) eigenvalue (critical exponent): θ > 0



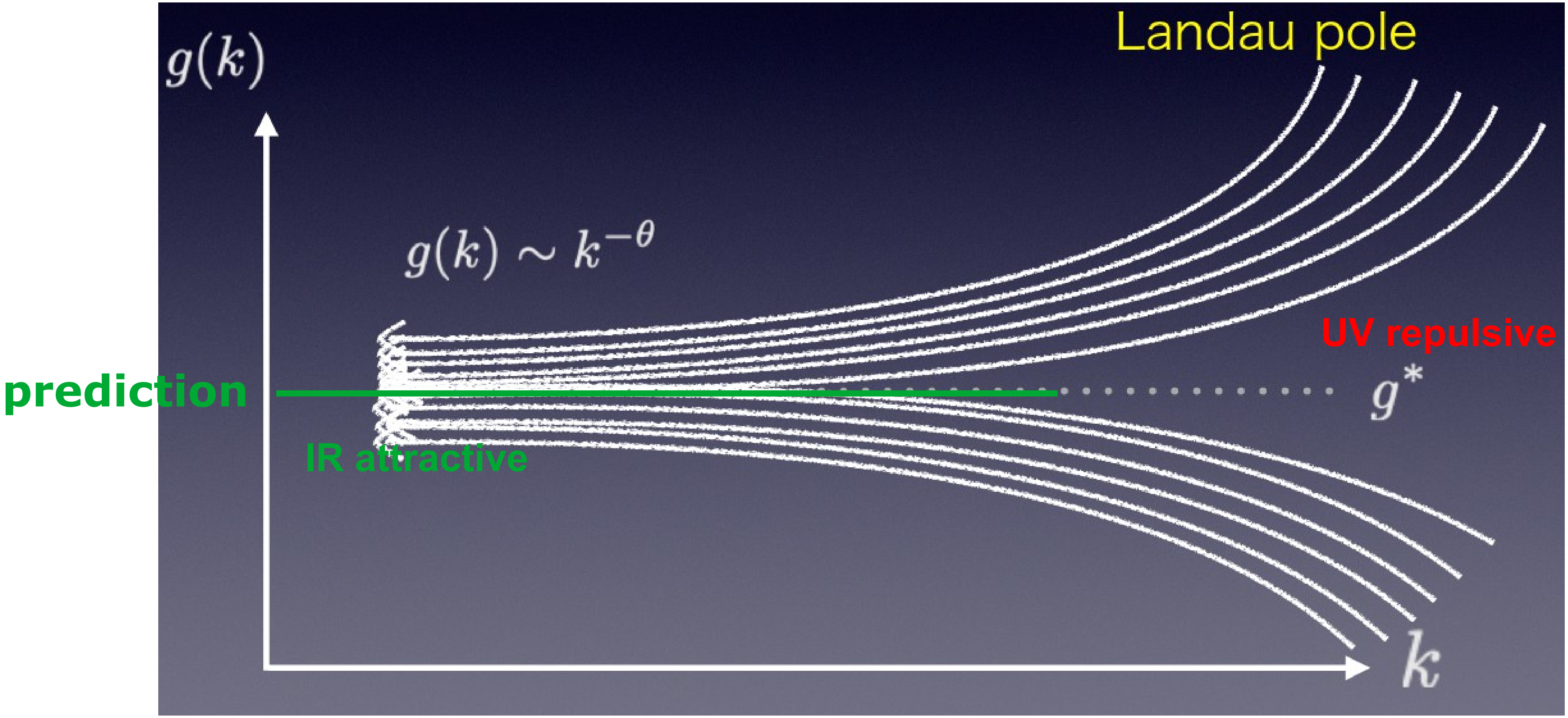
M.Yamada, HECA seminar, 08.10.2019

Relevant couplings are **free parameters**

Scaling properties of *g*

$$M_{ij} = \partial\beta_i/\partial\alpha_j|_{\{\alpha_i^*\}}$$

(-) eigenvalue (critical exponent): θ < 0



M.Yamada, HECA seminar, 08.10.2019

Irrelevant couplings provide **predictions**

Asymptotically safe gravity

Quantum gravity might feature interactive UV fixed points (functional renormalization group)

Reuter '96, Reuter, Saueressig '01, Litim '04, Codello, Percacci, Rahmede '06, Benedetti, Machado, Saueressig '09, Narain, Percacci '09, Manrique, Rechenberger, Saueressig '11, Falls, Litim, Nikolakopoulos '13, Dona', Eichhorn, Percacci '13, Daum, Harst, Reuter '09, Folkerst, Litim, Pawlowski '11, Harst, Reuter '11, Zanusso *et al.* '09 … many more

EAA *e.g.* Einstein-Hilbert action

$$\Gamma_k = \frac{1}{16\pi G}\int d^4x\sqrt{g}\,[-R(g)+2\Lambda]$$

FRG (Wetterich equation)

$$\partial_t\Gamma_k = k\,\partial_k\Gamma_k = \frac{1}{2}\mathrm{Tr}\left(\frac{1}{\Gamma_k^{(2)}+\mathcal{R}_k}\partial_t\mathcal{R}_k\right)$$

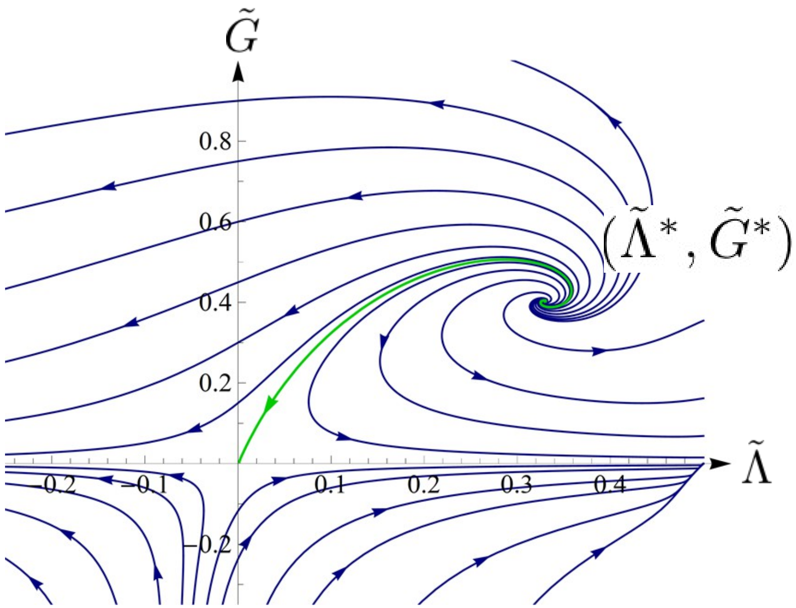
Beta functions of grav. couplings

$\tilde{G} = G(k)k^2$
$\tilde{\Lambda} = \Lambda(k)k^{-2}$
$t = \ln k$

$$\frac{d\tilde{G}}{dt} = \left[2+\tilde{G}\,\eta_1(\tilde{G},\tilde{\Lambda})\right]\tilde{G} = 0$$

$$\frac{d\tilde{\Lambda}}{dt} = -2\tilde{\Lambda}+\tilde{G}\,\eta_2(\tilde{G},\tilde{\Lambda}) = 0$$

Reuter, Saueressig, hep-th/0110054



2 relevant fixed points

... fixed points persist under the addition of gravity and matter interactions

Matter RGEs with quantum gravity

Christiansen, Eichhorn '17, Christiansen *et al.* '17, Shaposhnikov, Wetterich '09, Oda, Yamada '15, Eichhorn, Held, Pawlowski '16, Wetterich, Yamada '16, Hamada, Yamada '17, Pawlowski *et al.* '18, Eichhorn, Versteegen '17, Eichhorn, Held '17-'18, Pastor-Gutiérrez, Pawlowski, Reichert '22, ...

Trans-Planckian corrections of matter RGEs $k > M_{\rm Pl}$ (functional renormalization group)

SM gauge couplings

universal corrections depend on gravity fixed points

$$\frac{dg_Y}{dt} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} - f_g\, g_Y$$

$$\frac{dg_2}{dt} = -\frac{g_2^3}{16\pi^2}\frac{19}{6} - f_g\, g_2$$

$$\frac{dg_3}{dt} = -\frac{g_3^3}{16\pi^2}7 - f_g\, g_3$$

$$f_g = \tilde{G}^*\frac{1-4\tilde{\Lambda}^*}{4\pi\left(1-2\tilde{\Lambda}^*\right)^2}, \qquad f_y = -\tilde{G}^*\frac{96+\tilde{\Lambda}^*\left(-235+103\tilde{\Lambda}^*+56\tilde{\Lambda}^{*2}\right)}{12\pi\left(3-10\tilde{\Lambda}^*+8\tilde{\Lambda}^{*2}\right)^2}$$

A. Eichhorn, A. Held, 1707.01107
A. Eichhorn, F. Versteegen, 1709.07252

SM Yukawa couplings

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left(\frac{9}{2}y_t^2+\frac{3}{2}y_b^2+3y_s^2+3y_c^2-\frac{9}{4}g_2^2-8g_3^2-\frac{17}{12}g_Y^2+y_e^2+y_\mu^2+y_\tau^2\right) - f_y\, y_t$$

$$\frac{dy_b}{dt} = \frac{y_b}{16\pi^2}\left(\frac{9}{2}y_b^2+\frac{3}{2}y_t^2+3y_s^2+3y_c^2-\frac{9}{4}g_2^2-8g_3^2-\frac{5}{12}g_Y^2+y_e^2+y_\mu^2+y_\tau^2\right) - f_y\, y_b \quad \ldots$$

... same for other quarks and leptons

Matter RGEs with quantum gravity

Christiansen, Eichhorn '17, Christiansen *et al.* '17, Shaposhnikov, Wetterich '09, Oda, Yamada '15, Eichhorn, Held, Pawlowski '16, Wetterich, Yamada '16, Hamada, Yamada '17, Pawlowski *et al.* '18, Eichhorn, Versteegen '17, Eichhorn, Held '17-'18, Pastor-Gutiérrez, Pawlowski, Reichert '22, ...

Trans-Planckian corrections of matter RGEs $k > M_{\mathrm{Pl}}$ (functional renormalization group)

SM gauge couplings

universal corrections depend on gravity fixed points

$$\frac{dg_Y}{dt} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} - f_g\, g_Y = 0$$

$$\frac{dg_2}{dt} = -\frac{g_2^3}{16\pi^2}\frac{19}{6} - f_g\, g_2 = 0$$

$$\frac{dg_3}{dt} = -\frac{g_3^3}{16\pi^2}7 - f_g\, g_3 = 0$$

$$f_g = \tilde{G}^* \frac{1-4\tilde{\Lambda}^*}{4\pi\left(1-2\tilde{\Lambda}^*\right)^2}, \qquad f_y = -\tilde{G}^* \frac{96+\tilde{\Lambda}^*\left(-235+103\tilde{\Lambda}^*+56\tilde{\Lambda}^{*2}\right)}{12\pi\left(3-10\tilde{\Lambda}^*+8\tilde{\Lambda}^{*2}\right)^2}$$

A. Eichhorn, A. Held, 1707.01107
A. Eichhorn, F. Versteegen, 1709.07252

SM Yukawa couplings

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left(\frac{9}{2}y_t^2 + \frac{3}{2}y_b^2 + 3y_s^2 + 3y_c^2 - \frac{9}{4}g_2^2 - 8g_3^2 - \frac{17}{12}g_Y^2 + y_e^2 + y_\mu^2 + y_\tau^2\right) - f_y\, y_t = 0$$

$$\frac{dy_b}{dt} = \frac{y_b}{16\pi^2}\left(\frac{9}{2}y_b^2 + \frac{3}{2}y_t^2 + 3y_s^2 + 3y_c^2 - \frac{9}{4}g_2^2 - 8g_3^2 - \frac{5}{12}g_Y^2 + y_e^2 + y_\mu^2 + y_\tau^2\right) - f_y\, y_b = 0$$

... same for other quarks and leptons

get fixed points

Matter RGEs with quantum gravity

Christiansen, Eichhorn '17, Christiansen *et al.* '17, Shaposhnikov, Wetterich '09, Oda, Yamada '15, Eichhorn, Held, Pawlowski '16, Wetterich, Yamada '16, Hamada, Yamada '17, Pawlowski *et al.* '18, Eichhorn, Versteegen '17, Eichhorn, Held '17-'18, Pastor-Gutiérrez, Pawlowski, Reichert '22, ...

Trans-Planckian corrections of matter RGEs $k > M_{\rm Pl}$ (functional renormalization group)

SM gauge couplings

universal corrections depend on gravity fixed points

$$\frac{dg_Y}{dt} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} - f_g\, g_Y = 0$$

$$\frac{dg_2}{dt} = -\frac{g_2^3}{16\pi^2}\frac{19}{6} - f_g\, g_2 = 0$$

$$\frac{dg_3}{dt} = -\frac{g_3^3}{16\pi^2}7 - f_g\, g_3 = 0$$

Very large theory uncertainties!
(truncation in number of operators, cut-off scheme dependence, gauge fixing, etc.)

SM Yukawa couplings

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left(\frac{9}{2}y_t^2 + \frac{3}{2}y_b^2 + 3y_s^2 + 3y_c^2 - \frac{9}{4}g_2^2 - 8g_3^2 - \frac{17}{12}g_Y^2 + y_e^2 + y_\mu^2 + y_\tau^2\right) - f_y\, y_t = 0$$

$$\frac{dy_b}{dt} = \frac{y_b}{16\pi^2}\left(\frac{9}{2}y_b^2 + \frac{3}{2}y_t^2 + 3y_s^2 + 3y_c^2 - \frac{9}{4}g_2^2 - 8g_3^2 - \frac{5}{12}g_Y^2 + y_e^2 + y_\mu^2 + y_\tau^2\right) - f_y\, y_b = 0$$

get fixed points

... same for other quarks and leptons

Fixed points of SM + RHN:

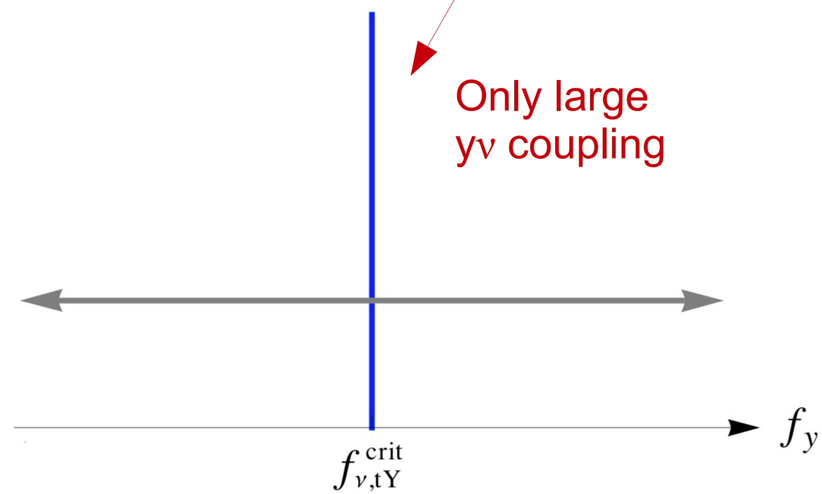
K.Kowalska, S.Pramanick, EMS, 2204.00866

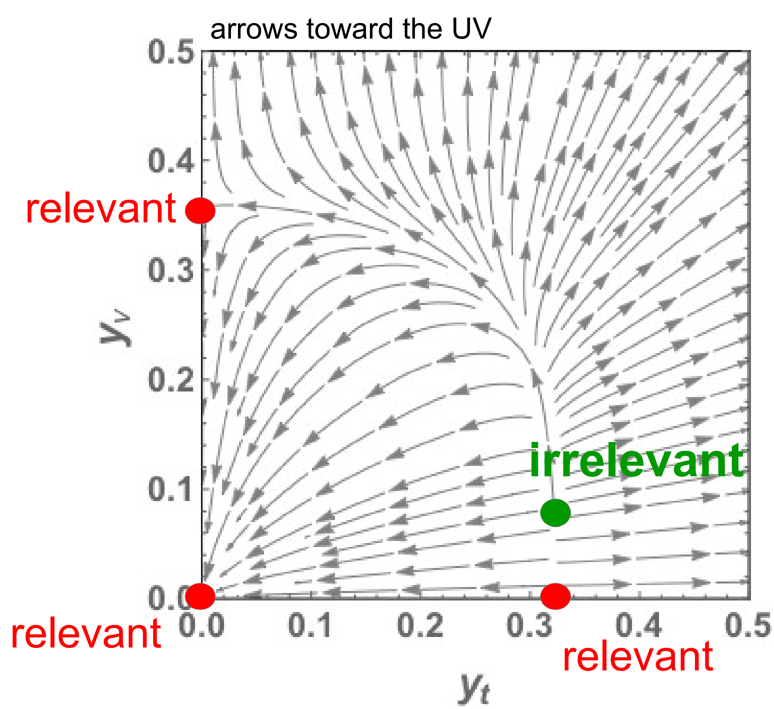
$$\frac{dg_Y}{dt} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} - f_g\, g_Y \quad \mathbf{= 0}$$

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left[\frac{9}{2}y_t^2 + y_\nu^2 - \frac{17}{12}g_Y^2\right] - f_y\, y_t \quad \mathbf{= 0}$$

$$\frac{dy_\nu}{dt} = \frac{y_\nu}{16\pi^2}\left[3y_t^2 + \frac{5}{2}y_\nu^2 - \frac{3}{4}g_Y^2\right] - f_y\, y_\nu \quad \mathbf{= 0}$$

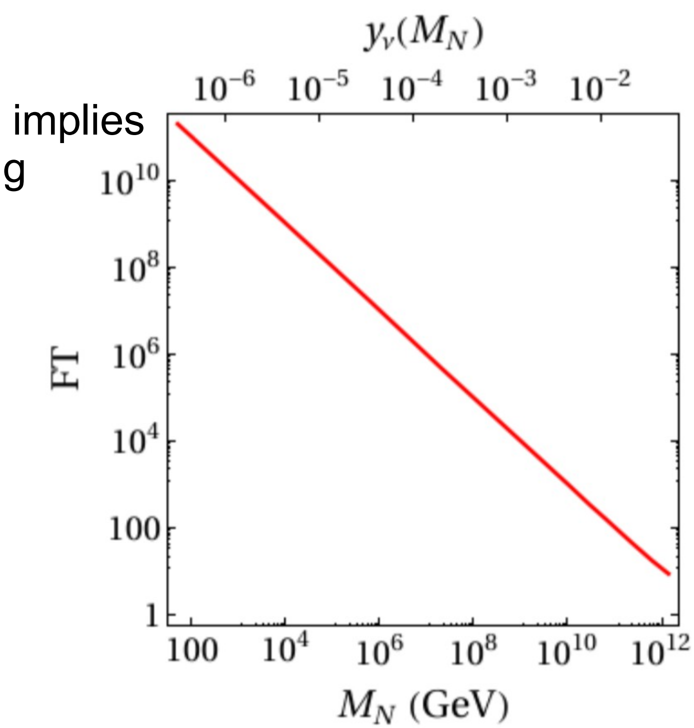
fy > fcrit $\sim 8 \times 10^{-4}$





$y_\nu^* \sim \mathcal{O}(1)$

Small coupling implies large fine tuning



Fixed points of SM + RHN:

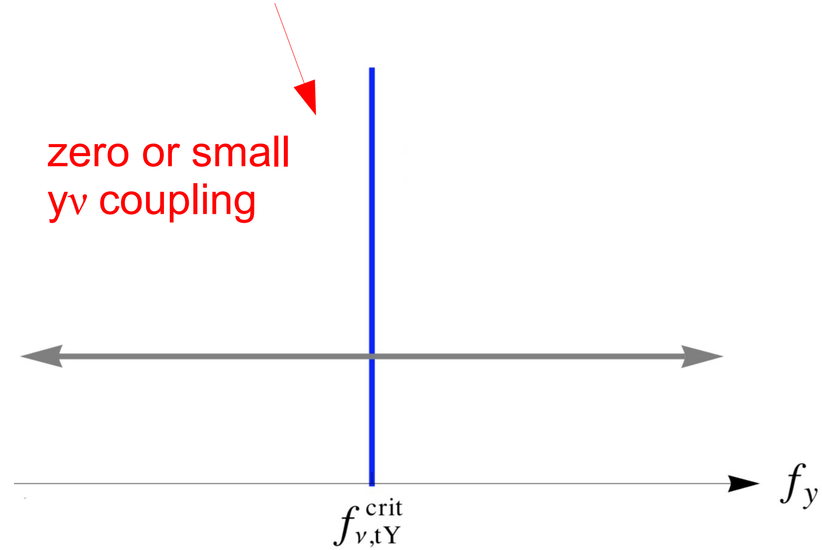
K.Kowalska, S.Pramanick, EMS, 2204.00866

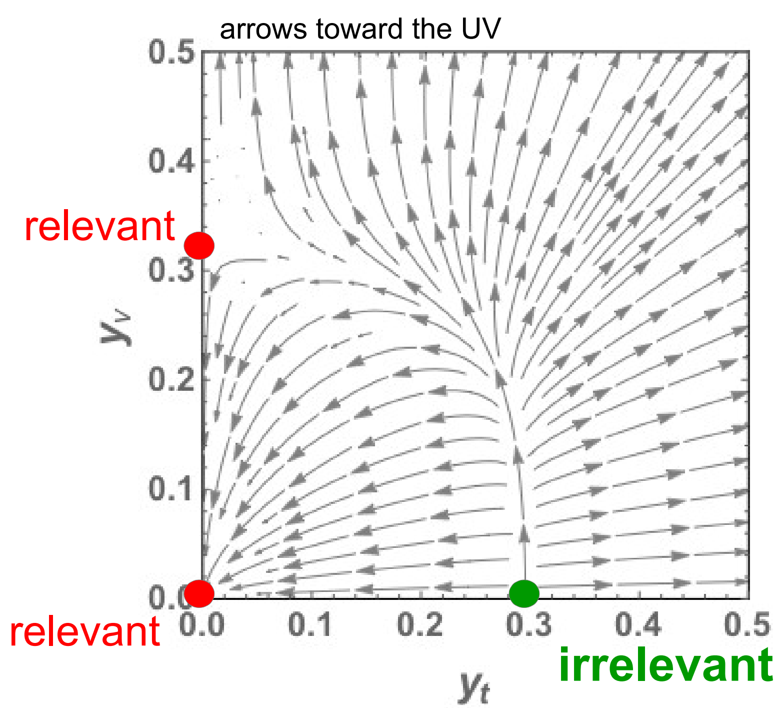
$$\frac{dg_Y}{dt} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} - f_g\, g_Y \quad \mathbf{= 0}$$

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left[\frac{9}{2}y_t^2 + y_\nu^2 - \frac{17}{12}g_Y^2\right] - f_y\, y_t \quad \mathbf{= 0}$$

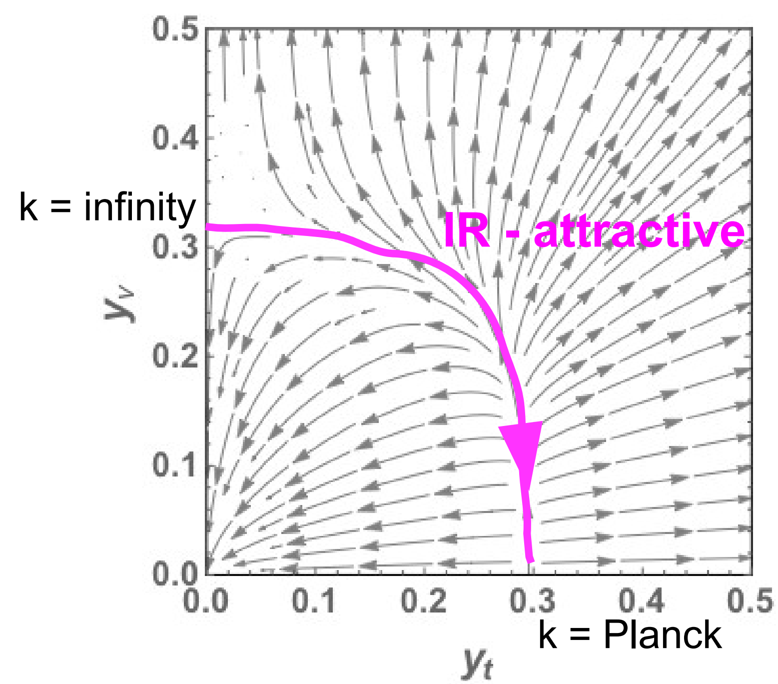
$$\frac{dy_\nu}{dt} = \frac{y_\nu}{16\pi^2}\left[3y_t^2 + \frac{5}{2}y_\nu^2 - \frac{3}{4}g_Y^2\right] - f_y\, y_\nu \quad \mathbf{= 0}$$

fy < fcrit $\sim 8 \times 10^{-4}$





$y_\nu^* = 0$



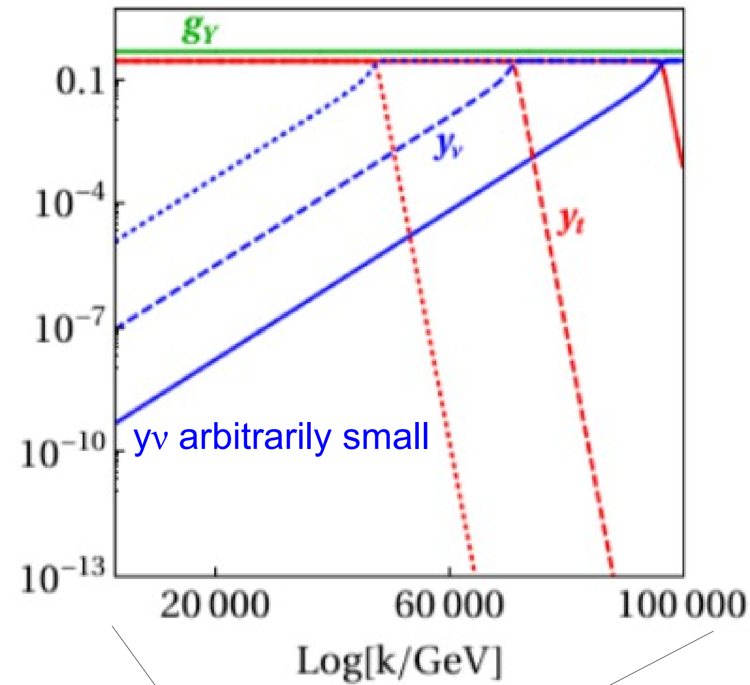
A dynamical mechanism!

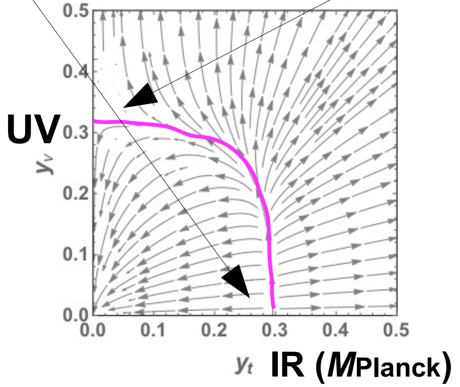
Integrate the curve:

$$y_\nu(t,\kappa) \approx \sqrt{\frac{16\pi^2(f_{\text{crit}} - f_y)}{e^{(f_{\text{crit}}-f_y)(16\pi^2\kappa - t)} + 5/2}}$$

$16\ \pi^2\kappa$ = “distance” in e-folds

K.Kowalska, S.Pramanick, EMS, 2204.00866





No fine tuning:

Smallness of the neutrino Yukawa due to the “distance” of the Planck scale from infinity

Neutrinos can be Dirac naturally

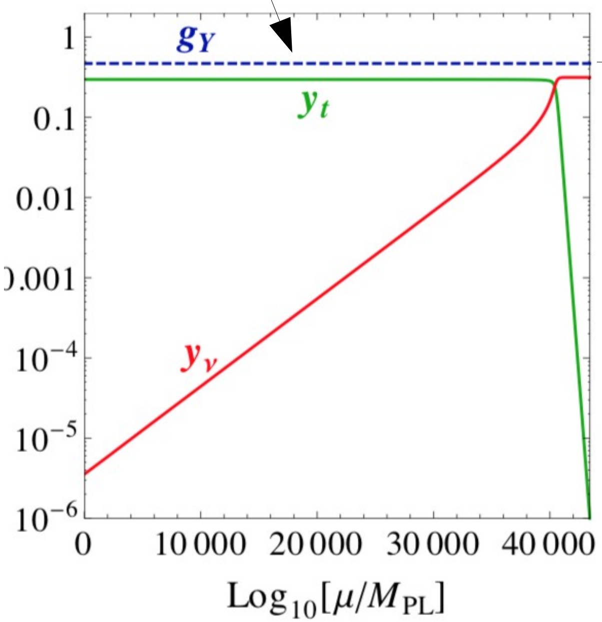
Alternative to the see-saw mechanism

Connections to quantum gravity

SM+RHN+QG:

neutrino crit. exponent must be negative

$$16\pi^2\theta_\nu \approx -\frac{2}{3}g_Y^{*2} + \frac{3}{2}y_t^{*2} < 0$$ for the mechanism to work

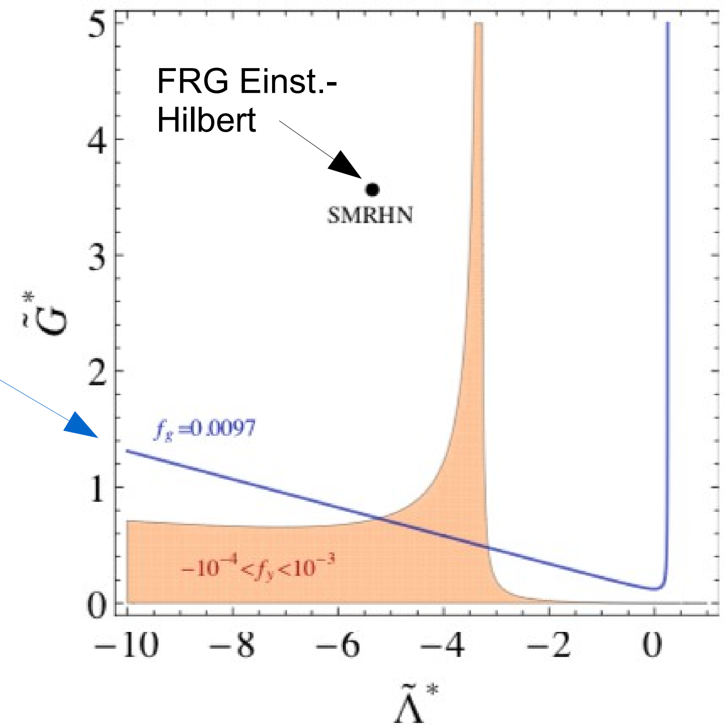


$f_g \approx 0.0097$

to match SM value ...

... It is a line in space of gravity fixed points

A. Chikkaballi, K.Kowalska, EMS, 2308.06114



(FRG calculation following
A. Eichhorn, F.Versteegen, 1709.07252)

Quantum gravity calculation should eventually match the blue line

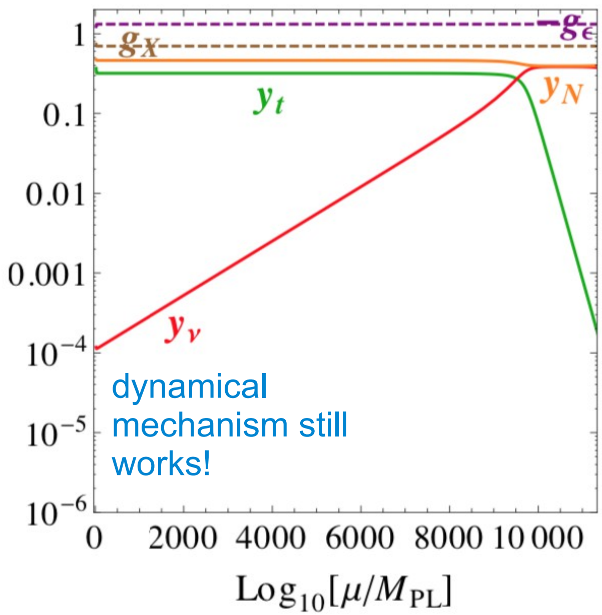
Connections to quantum gravity

gauged U(1)$_{B-L}$ + QG

$$g_Y^* = 0 \text{ (rel.)}$$

and g_Y role now played by...

$$g_X = \frac{g_{B-L}}{\sqrt{1-\epsilon^2}}, \quad g_\epsilon = -\frac{\epsilon\, g_Y}{\sqrt{1-\epsilon^2}}$$

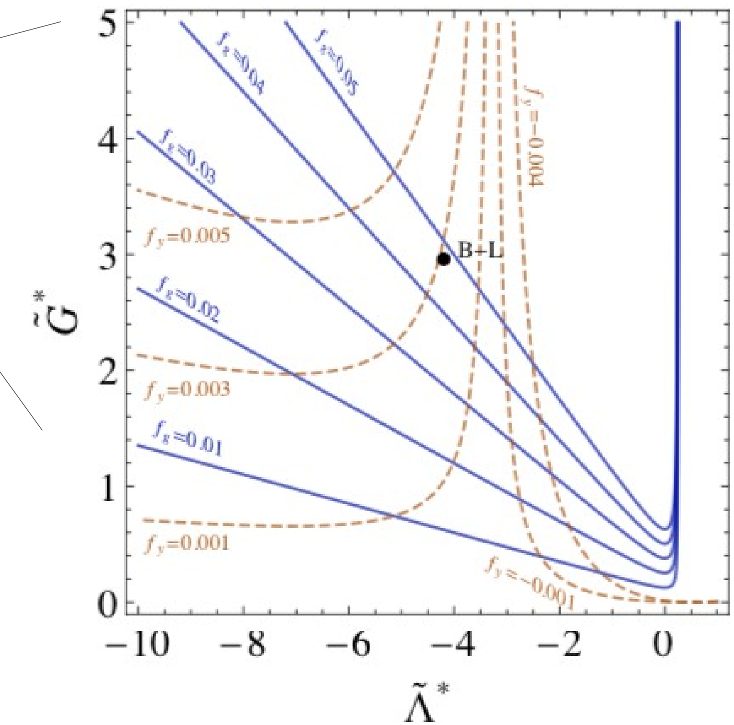


$$\to f_g = \text{any}$$

extended gauge sector:

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu} + i\bar{f}\left(\partial^\mu - ig_Y Q_Y \tilde{B}^\mu - ig_{B-L}Q_{B-L}\tilde{X}^\mu\right)\gamma_\mu f$$

A. Chikkaballi, K.Kowalska, EMS, 2308.06114



Quantum gravity calculation provides predictions for g_X, g_ε

(FRG calculation following *A. Eichhorn, F.Versteegen,* 1709.07252)

Predictions *B-L*

- New gauge sector ***g_X*, *g_ε*** (irr.)
- New Yukawa coupling ***y_N*** $\mathcal{L}_M = -y_N^{ij} S\, \nu_{R,i}\, \nu_{R,j} + \mathrm{H.c.}$ (Majorana mass term) (irr.)
- New scalar vev ***v_S*** *breaking* $U(1)_{B-L}$ (rel.)

Different f_g, f_y lead to **predictive (irrel.) fixed points** for g_X, g_ε, y_N :

A. Chikkaballi, K.Kowalska, EMS, 2308.06114

	f_g	f_y	g_X^*	g_ϵ^*	y_N^*	g_X ($10^{5,7,9}$ GeV)	g_ϵ ($10^{5,7,9}$ GeV)	y_N ($10^{5,7,9}$ GeV)	
BP1	0.01	0.0005	0.10	−0.55	0.12	0.29, 0.29, 0.30	−0.26, −0.27, −0.28	0.16, 0.16, 0.16	**Majorana**
BP2	0.05	−0.005	0.70	−1.32	0.47	0.40, 0.41, 0.44	−0.52, −0.56, −0.61	0.42, 0.44, 0.45	**Majorana**
BP3	0.02	−0.0015	0.10	−0.75	0.0	0.12, 0.12, 0.12	−0.33, −0.35, −0.37	0.0	**Dirac**
BP4	0.03	−0.004	0.10	0.75	0.0	0.09, 0.09, 0.09	0.23, 0.25, 0.28	0.0	**Dirac**

(large kinetic mixing implies $v_S \gg v_H$)

Predictions *B-L*

Possible gravitational-wave (GW) signatures from FOPT?

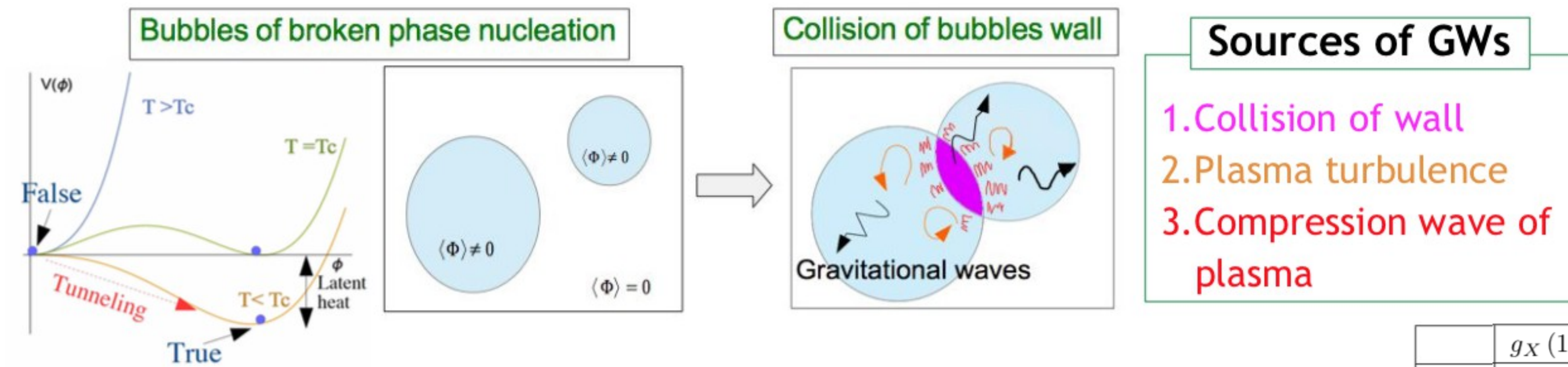


illustration
K. Hashino, Toyama 2018

	g_X ($10^{5,7,9}$ GeV)	y_N ($10^{5,7,9}$ GeV)
BP1	0.29, 0.29, 0.30	0.16, 0.16, 0.16
BP2	0.40, 0.41, 0.44	0.42, 0.44, 0.45
BP3	0.12, 0.12, 0.12	0.0

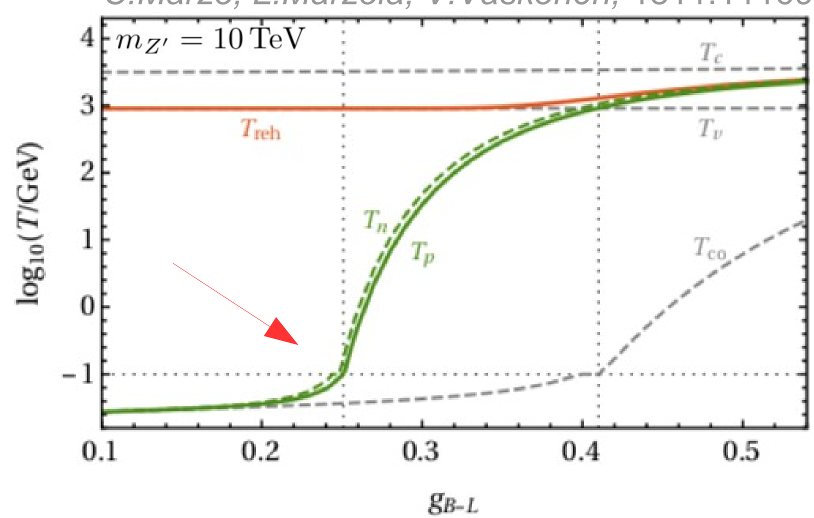
predictions have strong discriminating features... **may show up in GW amplitude!**

Signal dependence on pars. is well known if C-W pot. is "conformal" ...

$$V_{\rm CW} = \frac{1}{2} m_S^2 \phi^2 + \frac{1}{4}\lambda_S \phi^4 + \frac{1}{128\pi^2}\left(20\lambda_S^2 + 96 g_X^4 - 48 y_N^4\right)\phi^4 \left(-\frac{25}{6} + \ln\frac{\phi^2}{k^2}\right)$$

C.Marzo, L.Marzola, V.Vaskonen, 1811.11169

in the conformal limit
NO GW SIGNAL
due to small g_X
(or large y_N)



… nucleation/percolation *T* is too low
… FOPT stop-condition not satisfied

Scale-invariant potential confronts asymptotic safety...

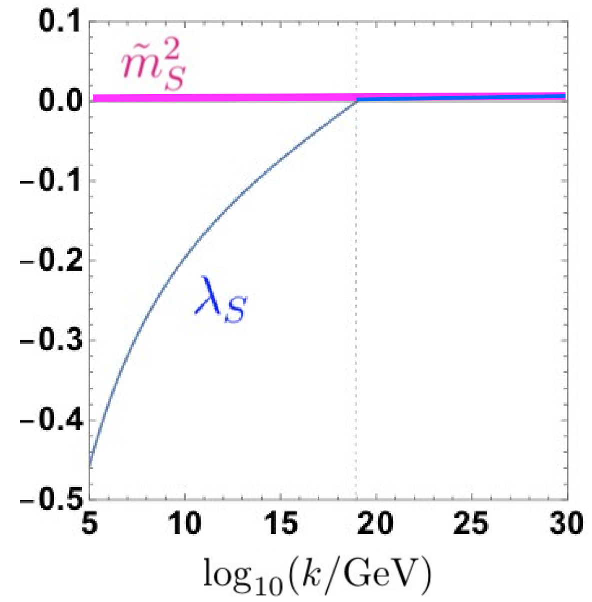
$$\frac{d\tilde{m}_S^2}{dt} \approx (-2 - f_\lambda)\,\tilde{m}_S^2$$

$$\frac{d\lambda_S}{dt} \approx -f_\lambda \lambda_S + \frac{6 g_X^{*4}}{\pi^2} + \ldots$$

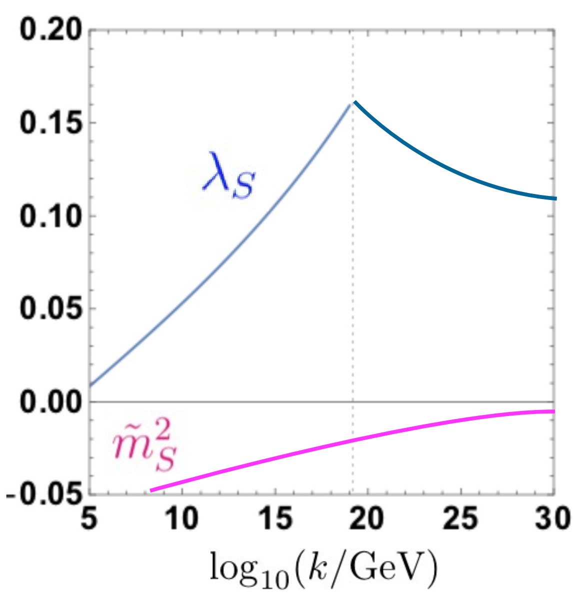
$\tilde{m}_S^{2*} = 0$ irrelevant

implies predictive $\lambda_S(t)$

… potential destabilized!



viceversa...



$\lambda_S(t)$ consistent with C-W

implies $\tilde{m}_S^{2*} = 0$ relevant

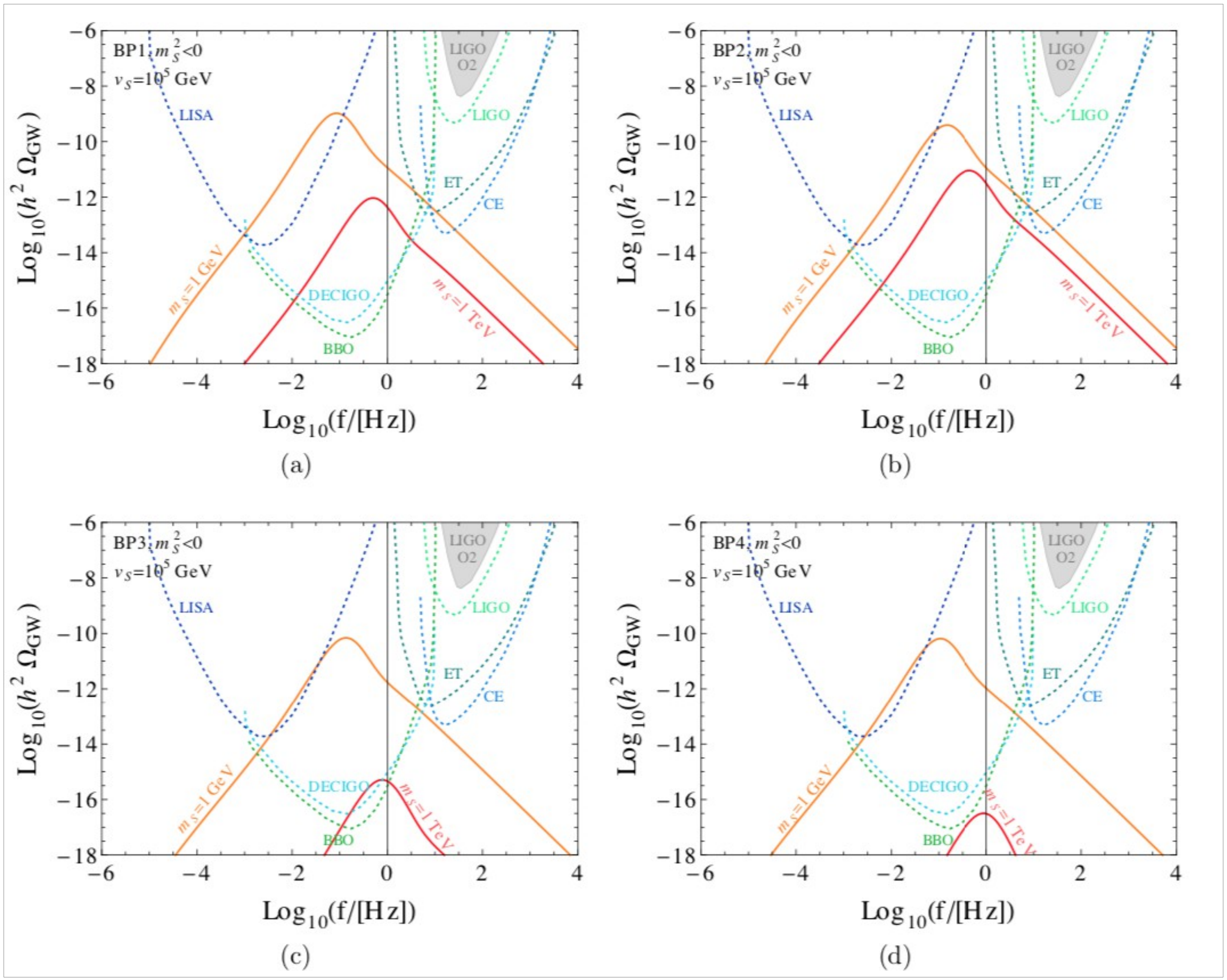
… tree-level mass is allowed

no conformal potential!

Gravitational waves

Signal is now visible...

A. Chikkaballi, K.Kowalska, EMS, 2308.06114



… but discriminating features washed-out by the scalar masses

To take home...

- AS was used to make the neutrino (or other) Yukawa coupling **arbitrarily small dynamically**

- Mechanism relies on an **irrelevant Gaussian fixed point** of the trans-Planckian RG flow of Yukawa coupling

- In the SM + QG **some tension** between the FRG results and phenomenology, but perhaps not so in gauged *B-L*

- Gravitational wave signatures from FOPTs

- Majorana/Dirac discrimination via gravitational waves from FOPTs is enticing but **not possible**.

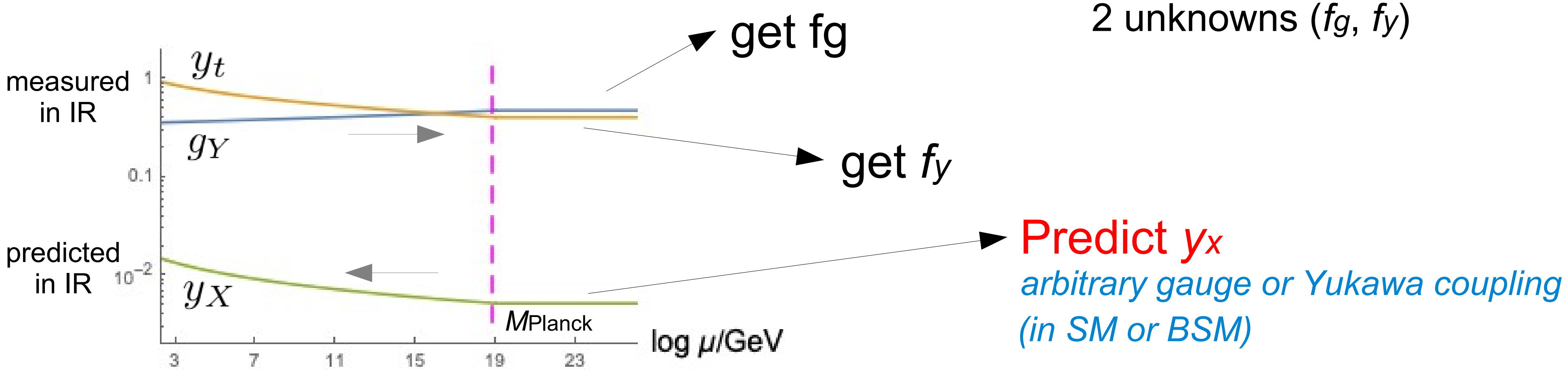
Backup

Predictions from trans-Planckian AS

- **FRG calculation of f_g, f_y has very large uncertainties...**
  (truncation in number of operators, cut-off scheme dependence, etc.)
  Lauscher, Reuter '02, Codello, Percacci, Rahmede '07-'08, Benedetti, Machado, Saueressig '09, Narain, Percacci '09, Dona', Eichhorn, Percacci '13, Falls, Litim, Schroeder '18 ...

- **FRG calculation is *not required* to get predictions...**
  Wetterich, Shaposhnikov '09, Eichhorn, Held '18, Reichert, Smirnov '19; Alkofer *et al.* '20, Kowalska, EMS, Yamamoto '20, Kowalska, EMS '21, Chikkaballi, Kotlarski, Kowalska, Rizzo, EMS '22 ...

… as the set of *irrelevant* couplings is overconstrained: 3 (or more) eqs (g_Y, y_t, y_x, ...)
2 unknowns (f_g, f_y)



AS leads to testable signatures ...

e.g. in flavor anomalies: *Kowalska, EMS, Yamamoto,* Eur.Phys.J.C 81 (2021) 4, 272
Chikkaballi, Kotlarski, Kowalska, Rizzo, EMS, JHEP 01 (2023) 164

in g-2 and DM: *Kowalska, EMS,* Phys. Rev. D 103, 115032 (2021)

… and neutrinos!
(this talk)

Lepton sector RGEs

$$\begin{aligned}\frac{dy_e}{dt} &= \frac{y_e}{16\pi^2}\left\{\frac{3}{2}y_e^2 - \frac{3}{2}\left[Xy_{\nu 1}^2 + Yy_{\nu 2}^2 + (1-X-Y)y_{\nu 3}^2\right] + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu 1}^2 + y_{\nu 2}^2 + y_{\nu 3}^2\right.\\ &\left.- \left(\frac{15}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_e \qquad \text{(A.9)}\end{aligned}$$

$$\begin{aligned}\frac{dy_\mu}{dt} &= \frac{y_\mu}{16\pi^2}\left\{\frac{3}{2}y_\mu^2 - \frac{3}{2}\left[Zy_{\nu 1}^2 + Wy_{\nu 2}^2 + (1-Z-W)y_{\nu 3}^2\right] + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu 1}^2 + y_{\nu 2}^2 + y_{\nu 3}^2\right.\\ &\left.- \left(\frac{15}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_\mu \qquad \text{(A.10)}\end{aligned}$$

$$\begin{aligned}\frac{dy_\tau}{dt} &= \frac{y_\tau}{16\pi^2}\left\{\frac{3}{2}y_\tau^2 - \frac{3}{2}\left[(1-X-Z)y_{\nu 1}^2 + (1-Y-W)y_{\nu 2}^2 + (X+Y+Z+W-1)y_{\nu 3}^2\right]\right.\\ &\left.+ y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu 1}^2 + y_{\nu 2}^2 + y_{\nu 3}^2 - \left(\frac{15}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_\tau \qquad \text{(A.11)}\end{aligned}$$

$$\begin{aligned}\frac{dy_{\nu 1}}{dt} &= \frac{y_{\nu 1}}{16\pi^2}\left\{\frac{3}{2}y_{\nu 1}^2 - \frac{3}{2}\left[Xy_e^2 + Zy_\mu^2 + (1-X-Z)y_\tau^2\right] + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu 1}^2 + y_{\nu 2}^2 + y_{\nu 3}^2\right.\\ &\left.- \left(\frac{3}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_{\nu 1} \qquad \text{(A.12)}\end{aligned}$$

$$\begin{aligned}\frac{dy_{\nu 2}}{dt} &= \frac{y_{\nu 2}}{16\pi^2}\left\{\frac{3}{2}y_{\nu 2}^2 - \frac{3}{2}\left[Yy_e^2 + Wy_\mu^2 + (1-Y-W)y_\tau^2\right] + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu 1}^2 + y_{\nu 2}^2 + y_{\nu 3}^2\right.\\ &\left.- \left(\frac{3}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_{\nu 2} \qquad \text{(A.13)}\end{aligned}$$

$$\begin{aligned}\frac{dy_{\nu 3}}{dt} &= \frac{y_{\nu 3}}{16\pi^2}\left\{\frac{3}{2}y_{\nu 3}^2 - \frac{3}{2}\left[(1-X-Y)y_e^2 + (1-Z-W)y_\mu^2 + (X+Y+Z+W-1)y_\tau^2\right]\right.\\ &\left.+ y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu 1}^2 + y_{\nu 2}^2 + y_{\nu 3}^2 - \left(\frac{3}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_{\nu 3} \qquad \text{(A.14)}\end{aligned}$$

$$\begin{aligned}\frac{dX}{dt} &= -\frac{3}{(4\pi)^2}\left[\left(\frac{y_e^2+y_\mu^2}{y_e^2-y_\mu^2}\right)\left\{(y_{\nu 1}^2-y_{\nu 3}^2)XZ + \frac{(y_{\nu 3}^2-y_{\nu 2}^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}\right.\\ &+ \left(\frac{y_e^2+y_\tau^2}{y_e^2-y_\tau^2}\right)\left\{(y_{\nu 1}^2-y_{\nu 3}^2)X(1-X-Z) + \frac{(y_{\nu 3}^2-y_{\nu 2}^2)}{2}[(1-Y)(1-Z)-X(1-2Y)-W(1-X)]\right\}\\ &+ \left(\frac{y_{\nu 1}^2+y_{\nu 2}^2}{y_{\nu 1}^2-y_{\nu 2}^2}\right)\left\{(y_e^2-y_\tau^2)XY + \frac{(y_\tau^2-y_\mu^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}\\ &\left.+ \left(\frac{y_{\nu 1}^2+y_{\nu 3}^2}{y_{\nu 1}^2-y_{\nu 3}^2}\right)\left\{(y_e^2-y_\tau^2)X(1-X-Y) + \frac{(y_\tau^2-y_\mu^2)}{2}[(1-Y)(1-Z)-X(1-2Z)-W(1-X)]\right\}\right]\end{aligned}$$

(A.15)

$$\begin{aligned}\frac{dY}{dt} &= -\frac{3}{(4\pi)^2}\left[\left(\frac{y_e^2+y_\mu^2}{y_e^2-y_\mu^2}\right)\left\{\frac{(y_{\nu 3}^2-y_{\nu 1}^2)}{2}[W(1-X)+X-(1-Y)(1-Z)] + (y_{\nu 2}^2-y_{\nu 3}^2)YW\right\}\right.\\ &+ \left(\frac{y_e^2+y_\tau^2}{y_e^2-y_\tau^2}\right)\left\{\frac{(y_{\nu 3}^2-y_{\nu 1}^2)}{2}[(1-Y)(1-Z)-W(1-X)-X(1-2Y)] + (y_{\nu 2}^2-y_{\nu 3}^2)Y(1-Y-W)\right\}\\ &+ \left(\frac{y_{\nu 2}^2+y_{\nu 1}^2}{y_{\nu 2}^2-y_{\nu 1}^2}\right)\left\{(y_e^2-y_\tau^2)XY + \frac{(y_\tau^2-y_\mu^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}\\ &\left.+ \left(\frac{y_{\nu 2}^2+y_{\nu 3}^2}{y_{\nu 2}^2-y_{\nu 3}^2}\right)\left\{(y_e^2-y_\tau^2)Y(1-X-Y) + \frac{(y_\mu^2-y_\tau^2)}{2}[W(1-X-2Y)+X-(1-Z)(1-Y)]\right\}\right]\end{aligned}$$

(A.16)

$$\begin{aligned}\frac{dZ}{dt} &= -\frac{3}{(4\pi)^2}\left[\left(\frac{y_\mu^2+y_e^2}{y_\mu^2-y_e^2}\right)\left\{(y_{\nu 1}^2-y_{\nu 3}^2)XZ + \frac{(y_{\nu 3}^2-y_{\nu 2}^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}\right.\\ &+ \left(\frac{y_\mu^2+y_\tau^2}{y_\mu^2-y_\tau^2}\right)\left\{(y_{\nu 1}^2-y_{\nu 3}^2)Z(1-X-Z) + \frac{(y_{\nu 2}^2-y_{\nu 3}^2)}{2}[W(1-X-2Z)+X-(1-Y)(1-Z)]\right\}\\ &+ \left(\frac{y_{\nu 1}^2+y_{\nu 2}^2}{y_{\nu 1}^2-y_{\nu 2}^2}\right)\left\{\frac{(y_e^2-y_\tau^2)}{2}[(1-Y)(1-Z)-X-W(1-X)] + (y_\mu^2-y_\tau^2)ZW\right\}\\ &\left.+ \left(\frac{y_{\nu 1}^2+y_{\nu 3}^2}{y_{\nu 1}^2-y_{\nu 3}^2}\right)\left\{\frac{(y_\tau^2-y_e^2)}{2}[(1-Z)(1-Y)-W(1-X)-X(1-2Z)] + (y_\mu^2-y_\tau^2)Z(1-Z-W)\right\}\right]\end{aligned}$$

(A.17)

$$\begin{aligned}\frac{dW}{dt} &= -\frac{3}{(4\pi)^2}\left[\left(\frac{y_\mu^2+y_e^2}{y_\mu^2-y_e^2}\right)\left\{(y_{\nu 2}^2-y_{\nu 3}^2)WY + \frac{(y_{\nu 3}^2-y_{\nu 1}^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}\right.\\ &+ \left(\frac{y_\mu^2+y_\tau^2}{y_\mu^2-y_\tau^2}\right)\left\{(y_{\nu 2}^2-y_{\nu 3}^2)W(1-Y-W) + \frac{(y_{\nu 3}^2-y_{\nu 1}^2)}{2}[(1-Y)(1-Z)-X-W(1-X-2Z)]\right\}\\ &+ \left(\frac{y_{\nu 2}^2+y_{\nu 1}^2}{y_{\nu 2}^2-y_{\nu 1}^2}\right)\left\{(y_\mu^2-y_\tau^2)WZ + \frac{(y_\tau^2-y_e^2)}{2}[(1-X)W+X-(1-Y)(1-Z)]\right\}\\ &\left.+ \left(\frac{y_{\nu 2}^2+y_{\nu 3}^2}{y_{\nu 2}^2-y_{\nu 3}^2}\right)\left\{(y_\mu^2-y_\tau^2)W(1-Z-W) + \frac{(y_\tau^2-y_e^2)}{2}[(1-Y)(1-Z)-X-W(1-X-2Y)]\right\}\right].\end{aligned}$$

(A.18)

Couple of comments...

1. Asymp. safe SM full fit works (with normal ordering)

PMNS parametrization

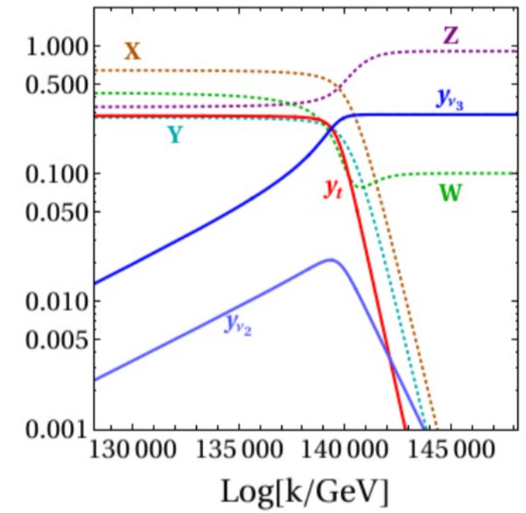
$$U_2 = |U_{\alpha i}|^2 = \begin{bmatrix} X & Y & 1-X-Y \\ Z & W & 1-Z-W \\ 1-X-Z & 1-Y-W & X+Y+Z+W-1 \end{bmatrix}$$

$$\theta_{12} = \arctan\sqrt{\frac{Y}{X}}$$
$$\theta_{13} = \arccos\sqrt{X+Y}$$
$$\theta_{23} = \arcsin\sqrt{\frac{1-W-Z}{X+Y}}$$

$$\delta = \arccos\frac{(X+Y)^2 Z - Y(X+Y+Z+W-1) - X(1-W-Z)(1-X-Y)}{2\sqrt{XY(1-X-Y)(1-Z-W)(X+Y+Z+W-1)}}$$

PMNS fit

$$X \in [0.64 - 0.71] \quad Y \in [0.26 - 0.34] \quad Z \in [0.05 - 0.26] \quad W \in [0.21 - 0.48]$$



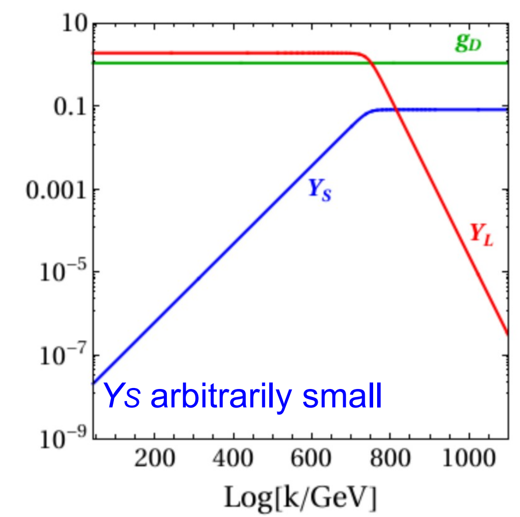
2. The mechanism is more generic than SM

e.g. dark gauge coupling g_D + Yukawa interactions

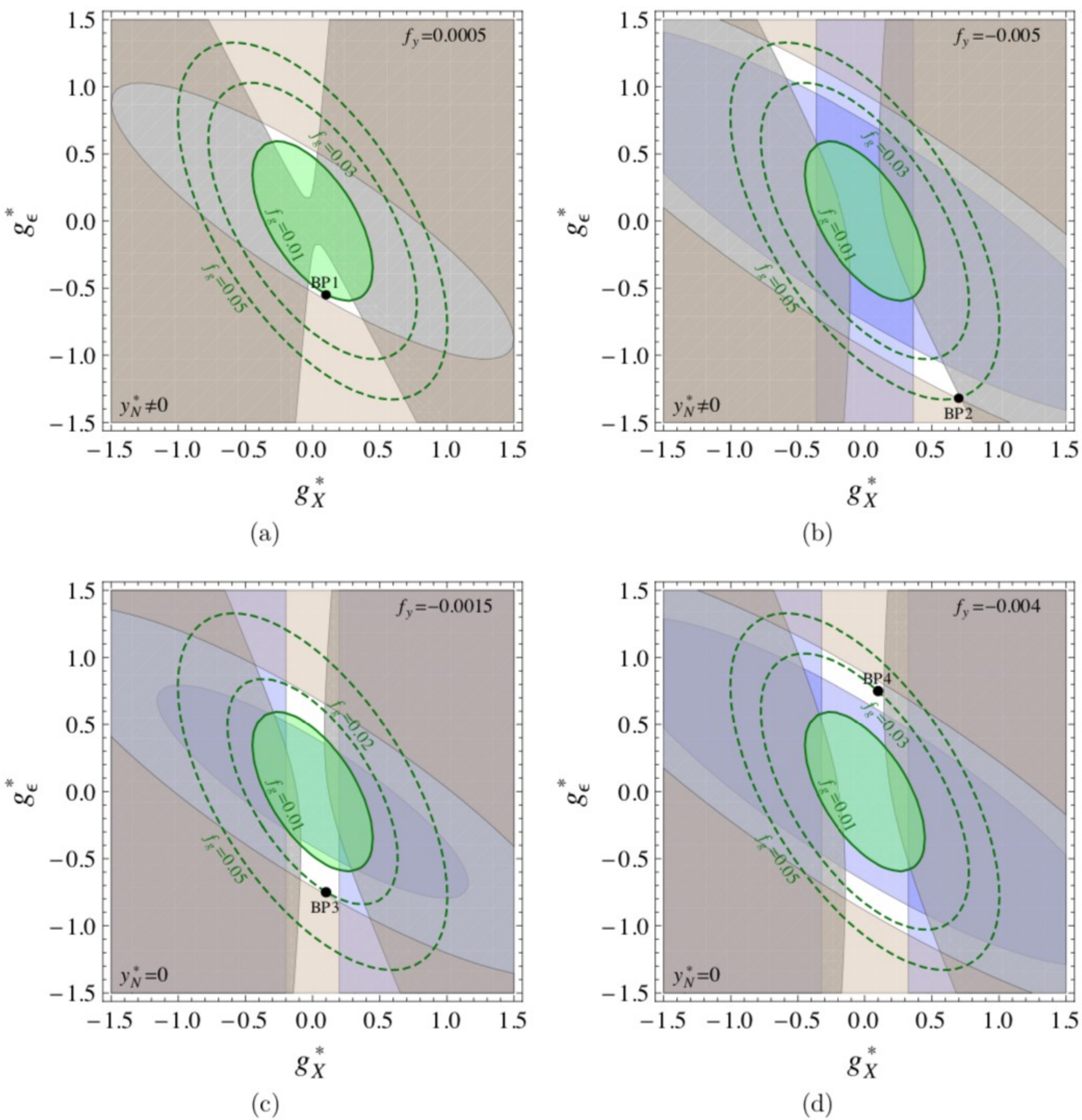
$$\mathcal{L} \supset Y_S\,\chi_R \Phi \chi_L + Y_L\,\psi_R \Phi \psi_L + \text{H.c.}$$

$$Q_\psi \gg Q_\chi$$ (dark abelian charge)

Can use it to justify freeze-in, feebly interacting models, etc...



Benchmark points of *B-L*



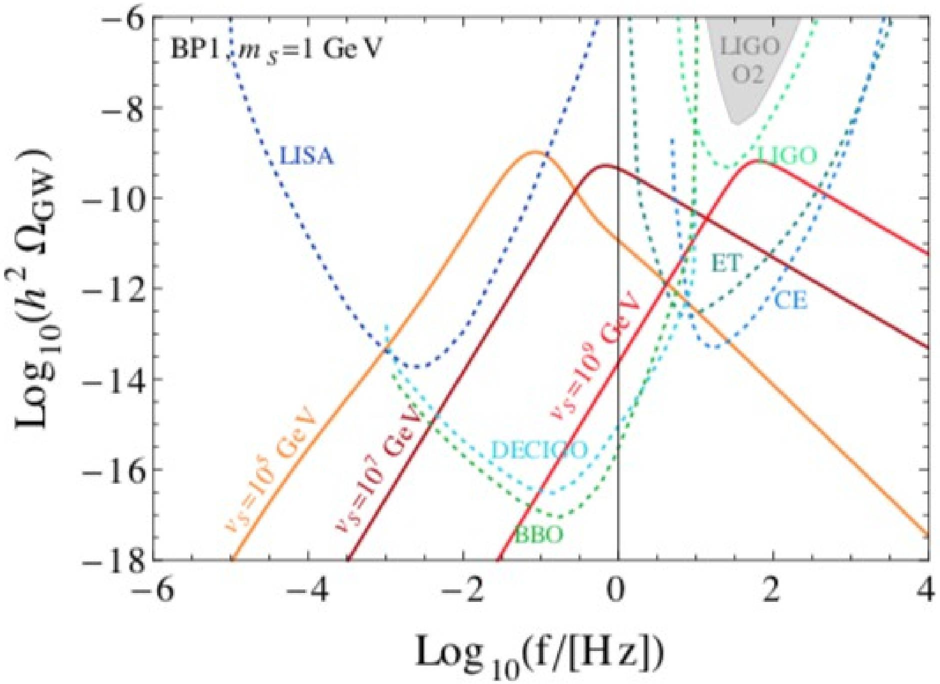
Gravitational wave signal

$$\alpha = \frac{\Delta V(T) - T\frac{\partial \Delta V(T)}{\partial T}}{\rho_R(T)}\Big|_{T_p} \qquad \frac{\beta}{H_*} = T_p \frac{\mathrm{d}(S_3/T)}{\mathrm{d}T}\Big|_{T_p} \qquad T_{\rm rh} = T_p[1+\alpha(T_p)]^{1/4}$$

$$\begin{aligned}
h^2\Omega^{\rm peak}_{\rm coll} &= 1.67\times10^{-5}\,\kappa^2_{\rm coll}\left(\frac{\alpha}{1+\alpha}\right)^2\left(\frac{v_w}{\beta/H_*}\right)^2\left(\frac{100}{g_*}\right)^{1/3}\left(\frac{0.11v_w}{0.42+v_w^2}\right)\\
h^2\Omega^{\rm peak}_{\rm sw} &= 2.65\times10^{-6}\,\kappa^2_{\rm sw}\left(\frac{\alpha}{1+\alpha}\right)^2\left(\frac{v_w}{\beta/H_*}\right)\left(\frac{100}{g_*}\right)^{1/3}\\
h^2\Omega^{\rm peak}_{\rm turb} &= 3.35\times10^{-4}\,\kappa^{3/2}_{\rm turb}\left(\frac{\alpha}{1+\alpha}\right)^{3/2}\left(\frac{v_w}{\beta/H_*}\right)\left(\frac{100}{g_*}\right)^{1/3},
\end{aligned}$$

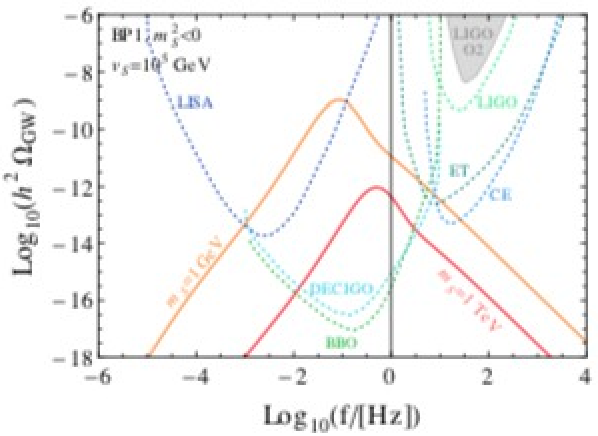
$$\begin{aligned}
f^{\rm peak}_{\rm coll} &= 1.65\times10^{-5}\,\mathrm{Hz}\left(\frac{v_w}{\beta/H_*}\right)^{-1}\left(\frac{100}{g_*}\right)^{-1/6}\left(\frac{T_*}{100\ \mathrm{GeV}}\right)\left(\frac{0.62v_w}{1.81-0.1v_w+v_w^2}\right)\\
f^{\rm peak}_{\rm sw} &= 1.90\times10^{-5}\,\mathrm{Hz}\left(\frac{v_w}{\beta/H_*}\right)^{-1}\left(\frac{100}{g_*}\right)^{-1/6}\left(\frac{T_*}{100\ \mathrm{GeV}}\right)\\
f^{\rm peak}_{\rm turb} &= 2.70\times10^{-5}\,\mathrm{Hz}\left(\frac{v_w}{\beta/H_*}\right)^{-1}\left(\frac{100}{g_*}\right)^{-1/6}\left(\frac{T_*}{100\ \mathrm{GeV}}\right).
\end{aligned} \tag{C.13}$$

GWs at different scales



Details of BP1 and BP2

BP1



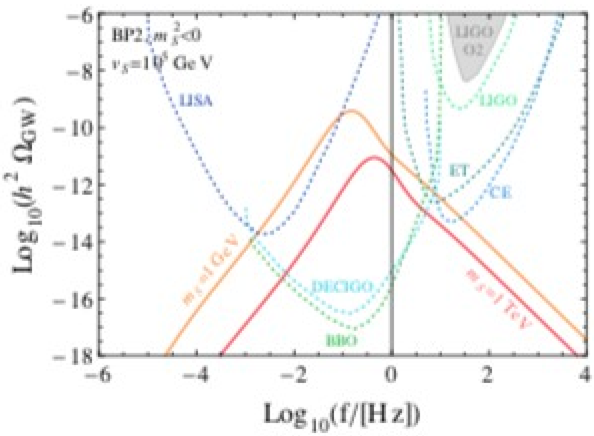
$m_S = 1\ GeV : \alpha = 10^{10}\,,\ \beta = 49.8$

$T_p = 14.6\ GeV$

$m_S = 1\ TeV : \alpha = 0.27\,,\ \beta = 185$

T_p ~ 10 TeV

BP2



$m_S = 1\ GeV : \alpha = 10^{11}\,,\ \beta = 78.9$

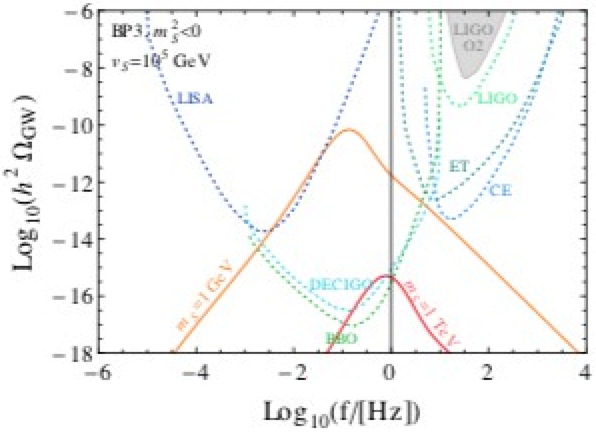
$T_p = 8\ GeV$

$m_S = 1\ TeV : \alpha = 0.88\,,\ \beta = 187$

T_p ~ 10 TeV

Details of BP3 and BP4

BP3



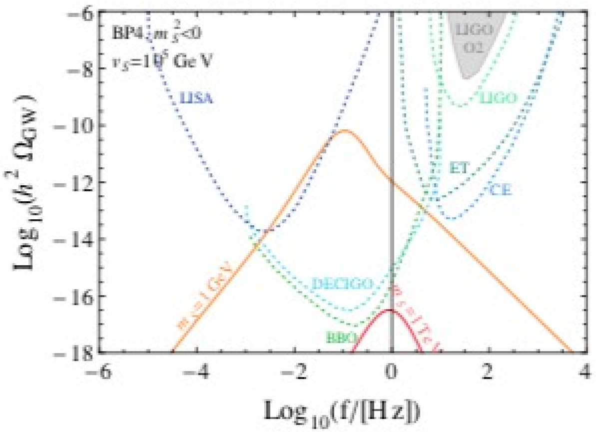
$m_S = 1\ GeV : \alpha = 10^9\, ,\ \beta = 189$

$T_p = 10.04\ GeV$

$m_S = 1\ TeV : \alpha = 0.02\, ,\ \beta = 227$

T_p ~ 10 TeV

BP4



$m_S = 1\ GeV : \alpha = 10^8\, ,\ \beta = 201$

$T_p = 11.5\ GeV$

$m_S = 1\ TeV : \alpha = 0.01\, ,\ \beta = 229$

$T_p =$ ~ 10 TeV