Phenomenology of flavor symmetric scoto-seesaw

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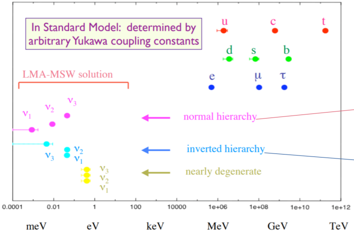
Based on 2209.08610, , 2311.xxxxx

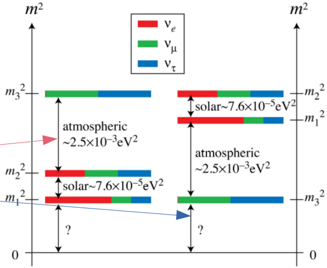
Co-authors: J. Ganguly, J. Gluza, S. Mahapatra

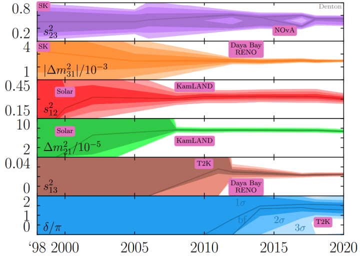
2PiNTS, Krakow

23.11.2023

Neutrino parameters and the known unknowns: 'Big' Data







	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.6$)	
	bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.77}_{-0.74}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \to 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \to 0.623$
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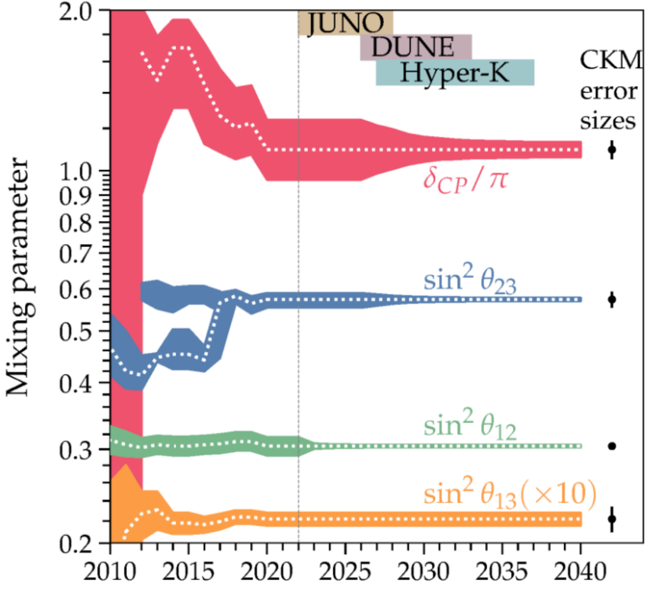
$\delta_{CP}/^\circ$	194^{+52}_{-25}	$105 \to 405$	287^{+27}_{-32}	$192 \to 361$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \to +2.599$	$-2.498^{+0.028}_{-0.029}$	$-2.584 \to -2.413$

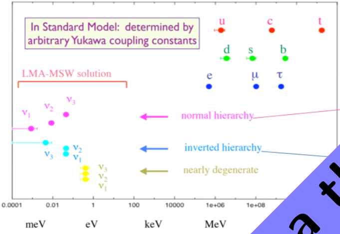
For a overview, see talk by Szymon Zieba

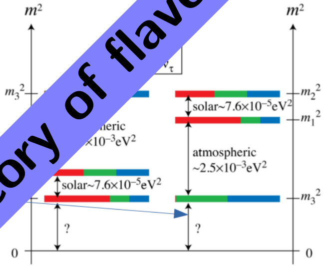
Neutrino parameters and the known unknowns: 'Big' Data



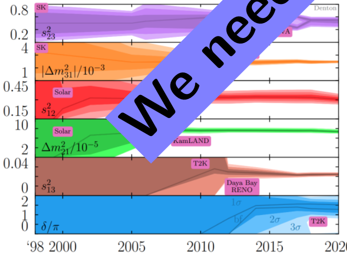
courtesy of Shirley Li

Neutrino parameters and the known unknowns: 'Big' Data





We need a theory of flavor!!



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Flavor symmetries, why?

$$U_{PMNS} = \begin{pmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{13}S_{23}e^{i\delta} & C_{12}C_{23} - S_{12}S_{13}S_{23}e^{i\delta} & C_{13}S_{23} \\ S_{12}S_{23} - C_{12}S_{13}C_{23}e^{i\delta} & -C_{12}S_{23} - S_{12}S_{13}C_{23}e^{i\delta} & C_{13}C_{23} \end{pmatrix}$$

$\Downarrow$

(Prior to 2012)

$s_{23} = 1/\sqrt{2}$ ($\theta_{23} = 45^\circ$) and $\theta_{13} = 0$

$\Downarrow$

$$U_0 = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

$\theta_{12} = 45^\circ (s_{12} = 1/\sqrt{2})$ — Bimaximal Mixing

$\theta_{12} = 35.26^\circ (s_{12} = 1/\sqrt{3})$ — Tribimaximal Mixing

$\theta_{12} = 31.7^\circ$ — Golden Ratio Mixing

$\theta_{12} = 30^\circ (s_{12} = 1/2)$ — Hexagonal Mixing

$$U_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ \frac{-1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & \frac{-\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{3}{4}} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Fukugita, Tanimoto, Yanagida PRD98; Harrison Perkins, Scott PLB02; Dutta,Ramond NPB03; Rodejohann et. al. EPJC10

(GR: tan $\theta_{12} = 1/\phi$ where $\phi = (1+\sqrt{5})/2$)

Flavor symmetries, why?

Simple example: $\mu - \tau$ permutation symmetry and TBM

$$m_\nu = U_0^\star \, \mathrm{diag}(m_1, m_2, m_3) U_0^\dagger ,$$

such a mixing matrices can easily diagonalize a $\mu - \tau$ symmetric (transformations $\nu_e \to \nu_e$, $\nu_\mu \to \nu_\tau$, $\nu_\tau \to \nu_\mu$ under which the neutrino mass term remains unchanged) neutrino mass matrix of the form

$$m_\nu = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix},$$

With $A + B = C + D$ this matrix yields tribimaximal mixing pattern where $s_{12} = 1/\sqrt{3}$ $i.e.$, $\theta_{12} = 35.26^\circ$

- Compatible Mixing Matrix :

$$U_{\rm TB} \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Flavor symmetries, why?

Simple example: $\mu - \tau$ permutation symmetry and TBM

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With $A + B = C + D$ this matrix yields tribimaximal mixing pattern where $s_{12} = 1/\sqrt{3}$ *i.e.*, $\theta_{12} = 35.26^\circ$

- Observed mixing matrix :

$$U_{\mathrm{PMNS}} \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \epsilon \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}(?) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}(?) \end{pmatrix}$$

General Framework

Anarchy

- Neutrino mixing anarchy is the hypothesis that the leptonic mixing matrix can be described as the result of a random draw from an unbiased distribution of unitary 3×3 matrices.
- Random analysis without imposing prior theories or symmetries on the mass and mixing matrices.
- This hypothesis does not make any correlation among the neutrino masses and mixing parameters

de Gouvea, Haba, Hall, Murayama : 9911341, 0009174, 1204.1249

Texture

- More specific studies with imposed mass or mixing textures for which models with underlying symmetries can be sought.
- It's an intermediate approach
- Some texture zeros of neutrino mass matrices can be eliminated.

Alejandro Ibarra, Graham Ross: Phys.Lett.B 2003

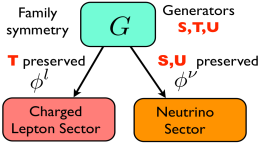
Symmetry

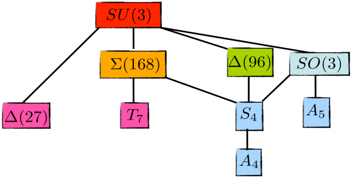
- Theoretical studies where some explicit symmetries at the Yukawa Lagrangian level are assumed and corresponding extended particle sector is defined.
- The symmetry-based approach to explain the non-trivial mixing in the lepton sector known as family symmetry or horizontal symmetry

Reviews: Tanimoto *et.al*. 1003.3552, Altarelli, Feruglio 1002.0211, King 1301.1340

General Framework: Symmetry based approach

- Fundamental symmetry in the lepton sector can easily explain the origin of neutrino mixing which is considerably different from quark mixing.
- Incidentally, both Abelian or non-Abelian family symmetries have potential to shade light on the Yukawa couplings.
- The Abelian symmetries (such as Froggatt-Nielsen symmetry) only points towards a hierarchical structure of the Yukawa couplings.
- Non-Abelian symmetries are more equipped to explain the non-hierarchical structures of the observed lepton mixing as observed by the oscillation experiments.





S. F. King 1301.1340

$\boxed{G_f \to G_e, G_\nu}$ typically, $G_e = Z_3$ and $G_\nu = Z_2 \times Z_2$.

Non-zero θ_{13}

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Bimaximal Mixing — Tribimaximal Mixing — Golden Ratio Mixing — Hexagonal Mixing

$$U_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ \frac{-1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & \frac{-\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{3}{4}} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Decendents of fixed pattern mixing schemes

Non-zero θ_{13}: Decendents of tribimaximal mixing

$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad U_{\mathrm{PMNS}} \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \epsilon \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}(?) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}(?) \end{pmatrix}$$

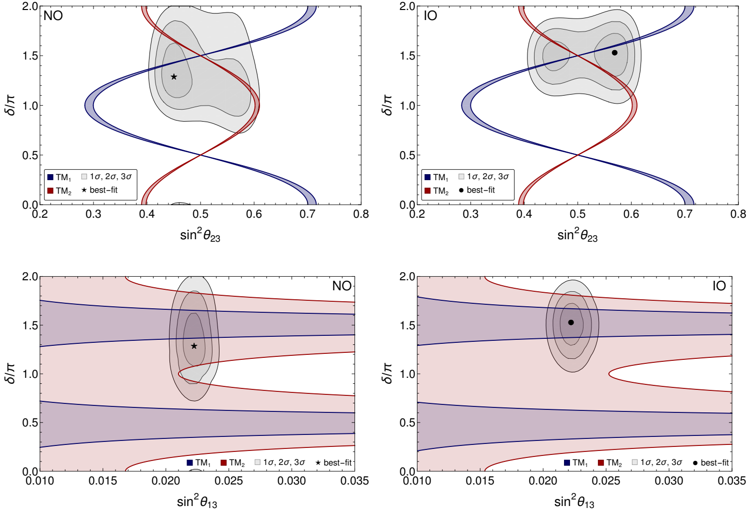
$$|U_{\mathrm{TM}_1}| = \begin{pmatrix} \frac{2}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \end{pmatrix} \qquad |U_{\mathrm{TM}_2}| = \begin{pmatrix} * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \end{pmatrix},$$

$$U_{\mathrm{TM}_1} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} & \frac{s_\theta}{\sqrt{3}} e^{-i\gamma} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s_\theta}{\sqrt{2}} e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} - \frac{c_\theta}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s}{\sqrt{2}} e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} + \frac{c_\theta}{\sqrt{2}} \end{pmatrix}, U_{\mathrm{TM}_2} = \begin{pmatrix} \frac{2c_\theta}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2s_\theta}{\sqrt{6}} e^{-i\gamma} \\ -\frac{c_\theta}{\sqrt{6}} + \frac{s}{\sqrt{2}} e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} - \frac{c_\theta}{\sqrt{2}} \\ -\frac{c_\theta}{\sqrt{6}} + \frac{s}{\sqrt{2}} e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} + \frac{c_\theta}{\sqrt{2}} \end{pmatrix}$$

Non-zero θ_{13}: Decendents of tribimaximal mixing

- TM_1, TM_2 Vs Current data:

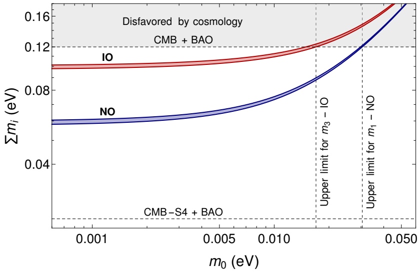
Gluza, Karmakar,Zieba et al 2310.20681

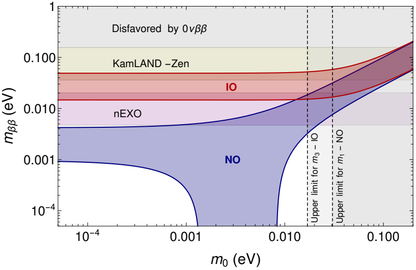


Dirac or Majorana Particle??

Neutrino Mass : Cosmology to $0\nu\beta\beta$

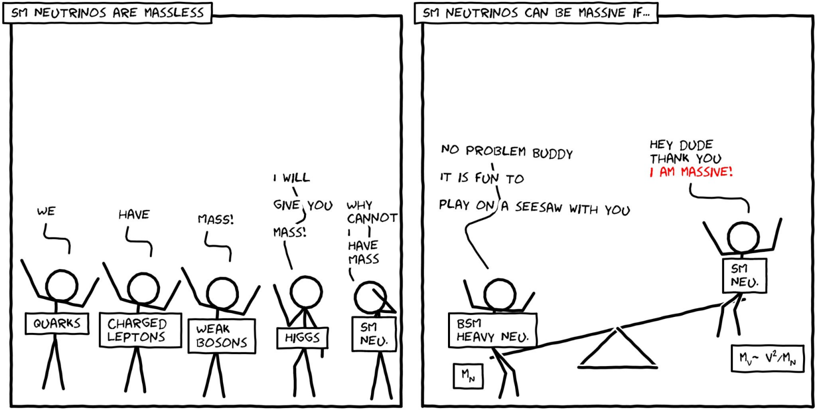
Gluza, Karmakar, Zieba et al 2310.20681





- Absolute neutrino mass : $m_\nu^2 < 0.9$ eV^2 (The KATRIN Collaboration 2022)

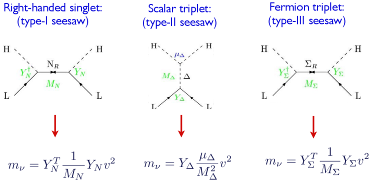
Neutrino Mass Generation



Cartoon by Sitian Qian

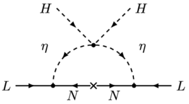
Neutrino Mass Generation

Seesaw frameworks



• **Type-I Seesaw, Type-II Seesaw, Type-III Seesaw, etc.:** Minkowski 77; Gellman, Ramond, Slansky 80; Glashow, Yanagida 79; Mohapatra, Senjanovic 80; Lazarides, Shafi; Schechter, Valle 81; Schechter, Valle 80; Mohapatra, Senjanovic 81; Lazarides, Shafi, Wetterich 81; Mohapatra Valle 86; Foot, Lew, He, Joshi 89; Ma 98; Bajc, Senjanovic 07....

Radiative neutrino mass



• **Radiative models, started in 80s:** Zee 80, Cheng, Li 80; Zee 86; Babu 88; Babu, Ma, Valle, 02; Ma 06;

• **For a review of radiative models:** Cai, Herrero-Garcia, Schmidt, Vicente, Volkas 17;

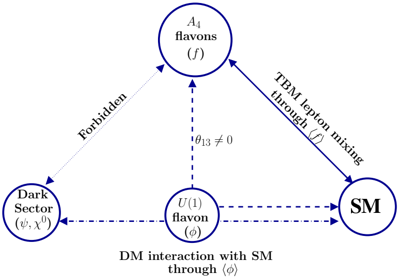
Hybrid Scenarios **In this talk we explore this less explored possibility**

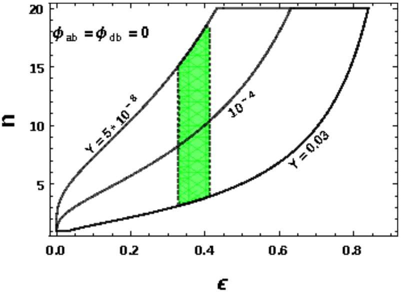
Are they connected?

Caldwell, Mohapatra 1993; Peltoniemi, Valle 1993, Asaka, Blanchet, Shaposhnikov 2005; Boehm 2008; Kubo, Ma, Suematsu 2006; Hambye, Kannike, Ma, Raidal 2007; Lindner, Schmidt, Schwetz 2011; Borah, Adhikari 2012; Restrepo, Zapata, Yaguna 2013; Huang, Deppisch 2014; Escudero, Rius, Sanz 2016; Borah, Karmakar, Nanda 2018;..many more..

- Example :

$$\mathcal{L}_{int} = \left(\frac{\phi}{\Lambda}\right)^n \bar{\psi}\tilde{H}\chi^0 + \frac{(HL^T LH)\phi\eta}{\Lambda^3} \text{ with } Y = \left(\frac{\phi}{\Lambda}\right)^n = \epsilon^n$$





- A schematic representation of dark matter (ψ, χ^0) interaction with SM to generate non-zero θ_{13} in the presence of the $U(1)$ flavor symmetry. The A_4 flavons help in generating base TBM mixing.

Bhattacharya , Karmakar, Sahu, Sil 1603.04776

Flavor Symmetry and Hybrid Mass Mechanisms: Why?

- Ratio of solar to atmospheric mass difference :

$$r = \frac{\Delta m^2_{\rm SOL}}{\Delta m^2_{\rm ATM}} \simeq \frac{7.41 \times 10^{-5}\ {\rm eV}^2}{2.51 \times 10^{-3}\ {\rm eV}^2} \simeq 3 \times 10^{-2}$$

- Two different mass scales that might originate from **entirely separate mechanisms !!**

- Minimal **Scoto** **Seesaw** scenario: Rojas, Srivastava, Valle 1807.11447

Greek word **'skótos'** $\rightarrow$ **'darkness'**

$$\mathcal{L} = -Y_N^k \bar{L}^k i\sigma_2 H^* N_R + \frac{1}{2} M_R \bar{N}_R^c N_R + Y_f^k \bar{L}^k i\sigma_2 \eta^* f + \frac{1}{2} M_f \bar{f}^c f$$

The number of right-handed neutrinos added to the SM is not fixed as they do not carry any anomaly
Schechter, Valle 1980

- The total neutrino mass reads:

$$M_\nu^{ij} = -\frac{v^2}{M_N} Y_N^i Y_N^j + \mathcal{F}(M_{\eta_R}, M_{\eta_I}, M_f) M_f \ Y_f^i Y_f^j .$$

where

$$\mathcal{F}(M_{\eta_R}, M_{\eta_I}, M_f) = \frac{1}{32\pi^2}\left[\frac{M_{\eta_R}^2 \log\left(M_f^2/M_{\eta_R}^2\right)}{M_f^2 - M_{\eta_R}^2} - \frac{M_{\eta_I}^2 \log\left(M_f^2/M_{\eta_I}^2\right)}{M_f^2 - M_{\eta_I}^2}\right],$$

where M_{η_R} and M_{η_I} are the masses of the neutral component of η.

Flavor Symmetry and Hybrid Mass Mechanisms: Why?

- Ratio of solar to atmospheric mass difference :

$$\Delta m^2_{\rm ATM} \sim \left(\frac{v^2}{M_N}\mathbb{Y}^2_{(N)}\right)^2, \quad \Delta m^2_{\rm SOL} \sim \left(\frac{1}{32\pi^2}\right)^2 \left(\frac{\lambda_5 v^2}{M_f^2 - m_\eta^{(R)2}} M_f \mathbb{Y}^2_{(f)}\right)^2$$

$$\frac{\Delta m^2_{\rm SOL}}{\Delta m^2_{\rm ATM}} \sim \left(\frac{1}{32\pi^2}\right)^2 \lambda_5^2 \left(\frac{M_N M_f}{M_f^2 - m_\eta^{(R)2}}\right)^2 \left(\frac{\mathbb{Y}^2_{(f)}}{\mathbb{Y}^2_{(N)}}\right)^2$$

- Benchmark Values :

$$M_N = 10^{12}\ \text{GeV},\ M_f = 10^4\ \text{GeV},\ M_{\eta_R} = 10^3\ \text{GeV},\ Y_N = 10^{-1},\ Y_f = 10^{-4}$$
$$M_N = 10^{6}\ \text{GeV},\ M_f = 10^6\ \text{GeV},\ M_{\eta_R} = 10^3\ \text{GeV},\ Y_N = 10^{-4},\ Y_f = 10^{-3}$$
$$M_N = 10^{4}\ \text{GeV},\ M_f = 10^4\ \text{GeV},\ M_{\eta_R} = 10^3\ \text{GeV},\ Y_N = 10^{-5},\ Y_f = 10^{-4}$$

- Scoto-seesaw scenarios : Accommodates DM candidates

- Flavor symmetric scoto-seesaw / discrete dark matter scenarios:
  Observed neutrino mixing
  Hierarchy of neutrino masses
  Accommodates DM candidates
  Low Energy Signatures

Flavor symmetric scoto-seesaw

- **Can we reproduce the neutrino mixing scheme?**

$$|U_{\mathrm{TM}_1}| = \begin{pmatrix} \frac{2}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \end{pmatrix} \qquad |U_{\mathrm{TM}_2}| = \begin{pmatrix} * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \end{pmatrix},$$

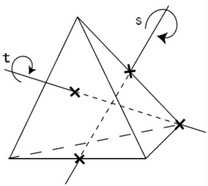
Flavor symmetric scoto-seesaw:

Standard Model with A_4 discrete flavor symmetry

Flavor symmetric scoto-seesaw:

Standard Model with A_4 discrete flavor symmetry

- A_4 is considered to be a favored symmetry in the neutrino sector
- Even permutation of 4 objects/invariant group of a tetrahedron
- Minimal group which contains 3 dim. representation (can accommodate three flavors of leptons)
- Product rule: $3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3_A \oplus 3_S$
- $1 \otimes 1 = 1,\ 1' \otimes 1' = 1'', 1' \otimes 1'' = 1$
  $1'' \otimes 1'' = 1'$ etc



Flavor symmetric scoto-seesaw : TM_2 mixing

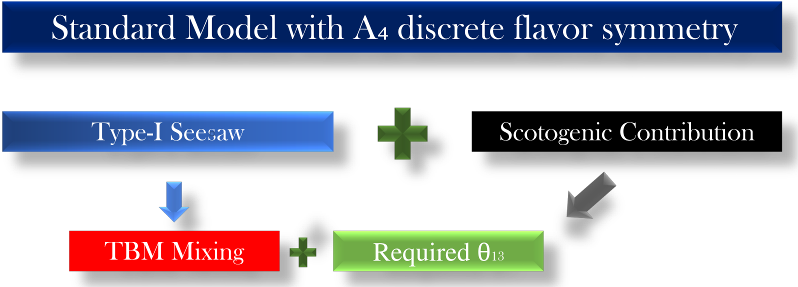
Standard Model with A_4 discrete flavor symmetry

Type-I Seesaw

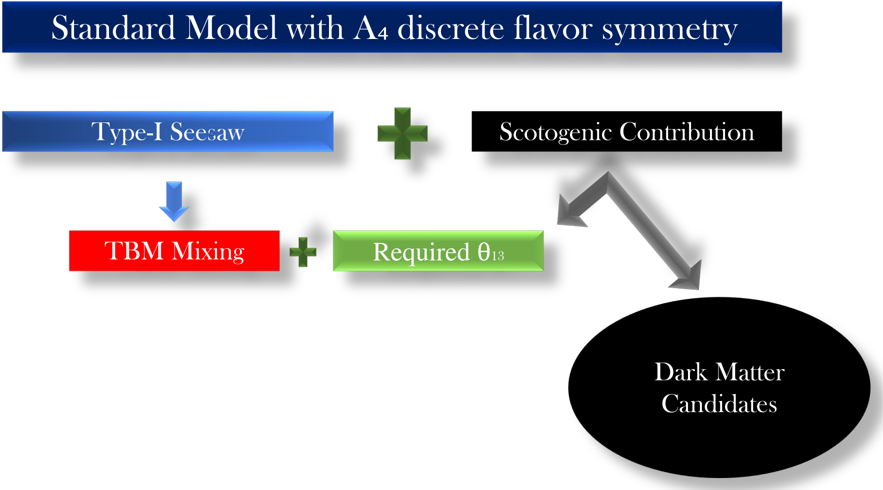
↓

TBM Mixing

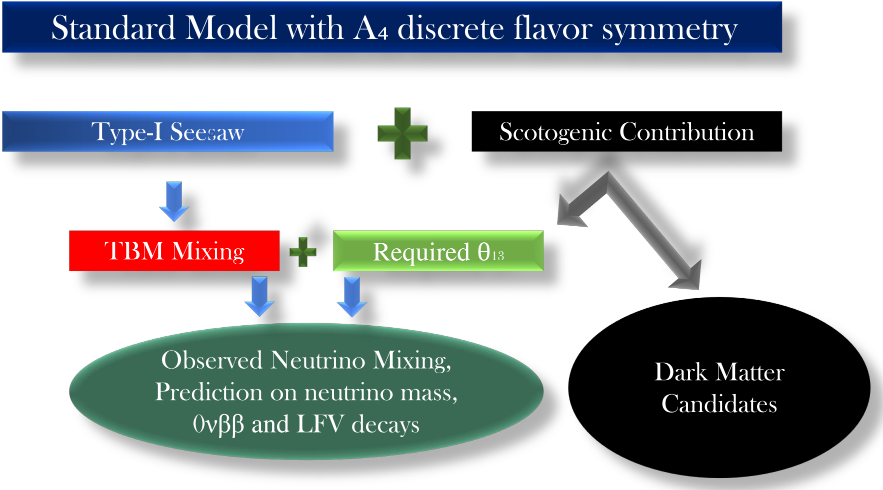
Flavor symmetric scoto-seesaw : TM_2 mixing



Flavor symmetric scoto-seesaw : TM_2 mixing



Flavor symmetric scoto-seesaw : TM_2 mixing



Flavor symmetric scoto-seesaw : TM_2 mixing

Type-I Seesaw contribution:

Ganguly, Gluza, Karmakar 2209.08610

$$\mathcal{L}_{\mathrm{TREE}} = \frac{y_{N_1}}{\Lambda}(\bar{L}\phi_s)\tilde{H}N_{R_1} + \frac{y_{N_2}}{\Lambda}(\bar{L}\phi_a)\tilde{H}N_{R_2} + \frac{1}{2}M_{N_1}\bar{N}^c_{R_1}N_{R_1} + \frac{1}{2}M_{N_2}\bar{N}^c_{R_2}N_{R_2} + h.c.,$$

- L, ϕ_a and ϕ_s $\rightarrow A_4$ triplets; H, N_{R_1}, N_{R_2} $\rightarrow A_4$ singlets

- A_4 multiplication rules: If we have two triplets (a_1, a_2, a_3) and (b_1, b_2, b_3), their products are given by $\Rightarrow 3 \otimes 3 = 1 + 1' + 1'' + 3_A + 3_S$

$$1 \sim a_1 b_1 + a_2 b_3 + a_3 b_2, \; 1' \sim a_3 b_3 + a_1 b_2 + a_2 b_1, \; 1'' \sim a_2 b_2 + a_3 b_1 + a_1 b_3,$$

$$3_S \sim \begin{bmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_2 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \\ 2a_2 b_2 - a_1 b_3 - a_3 b_1 \end{bmatrix}, \; 3_A \sim \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_1 b_2 - a_2 b_1 \\ a_3 b_1 - a_1 b_3 \end{bmatrix}.$$

- Flavon fields get VEVs along $\langle\phi_s\rangle = (0, v_s, -v_s)$, $\langle\phi_a\rangle = (v_a, v_a, v_a)$

$$\frac{y_{N_1}}{\Lambda}(\bar{L}\phi_s)_1\tilde{H}N_{R_1} = \frac{y_{N_1}}{\Lambda}(\bar{L_1}\phi_{s1} + \bar{L_2}\phi_{s3} + \bar{L_3}\phi_{s2})_1\tilde{H}N_{R_1} = \frac{y_{N_1}}{\Lambda}(0 - \bar{L_2}v_s + \bar{L_3}v_s)_1\tilde{H}N_{R_1}$$

$$\frac{y_{N_2}}{\Lambda}(\bar{L}\phi_a)_1\tilde{H}N_{R_2} = \frac{y_{N_2}}{\Lambda}(\bar{L}_1\phi_{a1} + \bar{L}_2\phi_{a3} + \bar{L}_3\phi_{a2})_1\tilde{H}N_{R_2} = \frac{y_{N_2}}{\Lambda}(\bar{L}_1 v_a + \bar{L}_2 v_a + \bar{L}_3 v_a)_1\tilde{H}N_{R_2}$$

- Dirac neutrino mass matrix :

$$M_D = \frac{v}{\Lambda}\begin{pmatrix} 0 & y_{N_2}v_a \\ -y_{N_1}v_s & y_{N_2}v_a \\ y_{N_1}v_s & y_{N_2}v_a \end{pmatrix} = vY_N, \quad M_R = \begin{pmatrix} M_{N_1} & 0 \\ 0 & M_{N_2} \end{pmatrix}.$$

Flavor symmetric scoto-seesaw : TM_2 mixing

Scotogenic contribution:

$$\mathcal{L}_{\rm LOOP} = \frac{y_s}{\Lambda^2}(\bar{L}\phi_s)\xi i\sigma_2\eta^* f + \frac{1}{2}M_f\bar{f}^c f + h.c.,$$

$$(M_\nu)_{\rm LOOP} = \mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f)M_f Y_f^i Y_f^j.$$

$$Y_F = (Y_F^e, Y_F^\mu, Y_F^\tau)^T = (y_s\frac{v_s}{\Lambda}\frac{v_\xi}{\Lambda}, 0, -y_s\frac{v_s}{\Lambda}\frac{v_\xi}{\Lambda})^T.$$

Therefore, the corresponding mass matrix takes the form

$$(M_\nu)_{\rm LOOP} = C\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix}, \quad C = \mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f)y_s^2\frac{v_s^2 v_\xi^2}{\Lambda^4}.$$

- **Effective neutrino mass matrix:**

$$\begin{aligned} M_\nu &= -M_D M_R^{-1} M_D^T + (M_\nu)_{\rm LOOP} \\ &= (M_\nu)_{\rm TREE} + (M_\nu)_{\rm LOOP} \\ &= \begin{pmatrix} -B+C & -B & -B-C \\ -B & -A-B & A-B \\ -B-C & A-B & -A-B+C \end{pmatrix}. \end{aligned}$$

- **After rotation by TBM matrix:**

$$\begin{aligned} M'_\nu &= U_{TB}^T M_\nu U_{TB} \\ &= \frac{1}{2}\begin{pmatrix} 3C & 0 & -\sqrt{3}C \\ 0 & -6B & 0 \\ -\sqrt{3}C & 0 & -4A+C \end{pmatrix}, \end{aligned}$$

Flavor symmetric scoto-seesaw : TM_2 mixing

- Effective neutrino mixing matrix (TM_2 mixing):

$$U_\nu = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}e^{i\phi}\sin\theta \\ -\frac{\cos\theta}{\sqrt{6}} + \frac{e^{i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\cos\theta}{\sqrt{2}} - \frac{e^{i\phi}\sin\theta}{\sqrt{6}} \\ -\frac{\cos\theta}{\sqrt{6}} - \frac{e^{i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{\cos\theta}{\sqrt{2}} - \frac{e^{i\phi}\sin\theta}{\sqrt{6}} \end{pmatrix} U_m.$$

- Corelations:

$$\tan\phi = \frac{\alpha\sin\phi_{AC}}{1-\alpha\cos\phi_{AC}}, \quad \tan 2\theta = \frac{\sqrt{3}}{\cos\phi + 2\alpha\cos(\phi_{AC}+\phi)}.$$

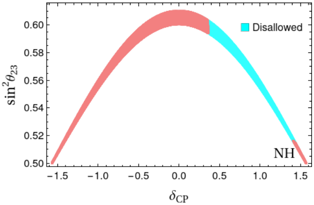
- Comparing with U_{PMNS}:

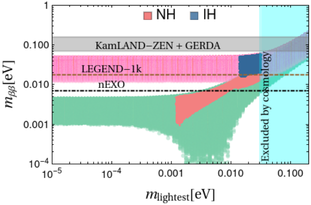
$$\sin\theta_{13}e^{-i\delta_{\mathrm{CP}}} = \sqrt{\frac{2}{3}}e^{-i\phi}\sin\theta, \quad \tan^2\theta_{12} = \frac{1}{2-3\sin^2\theta_{13}},$$

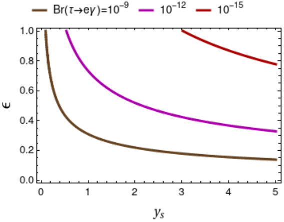
$$\tan^2\theta_{23} = \frac{\left(1+\frac{\sin\theta_{13}\cos\phi}{\sqrt{2-3\sin^2\theta_{13}}}\right)^2 + \frac{\sin^2\theta_{13}\sin^2\phi}{(2-3\sin^2\theta_{13})}}{\left(1-\frac{\sin\theta_{13}\cos\phi}{\sqrt{2-3\sin^2\theta_{13}}}\right)^2 + \frac{\sin^2\theta_{13}\sin^2\phi}{(2-3\sin^2\theta_{13})}}.$$

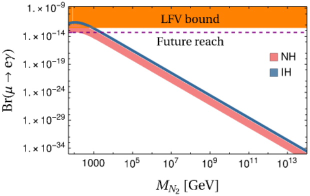
Flavor symmetric scoto-seesaw : TM_2 mixing

- Predictions:



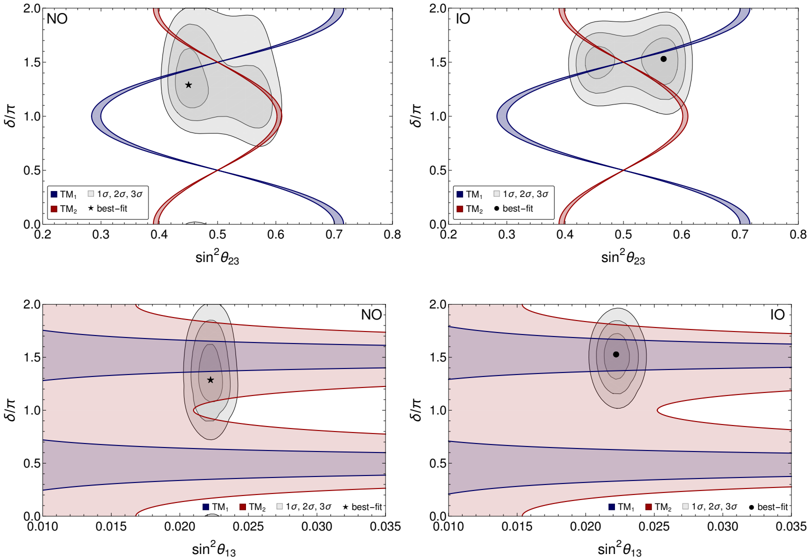






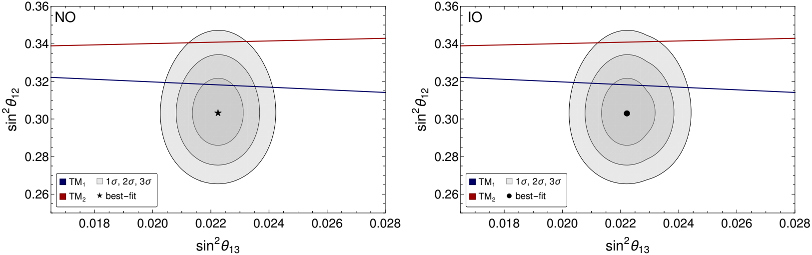
TM_1 vs TM_2

Gluza, Karmakar, Zieba et al 2310.20681



TM_1 vs TM_2

Gluza, Karmakar, Zieba et al 2310.20681



Flavor symmetric scoto-seesaw (FSS_1) : TM_1 mixing

- Contributions to the neutrino mass: Ganguly, Gluza, Karmakar, Mahapatra 2311.xxxxx

$$\mathcal{L} = \frac{y_N}{\Lambda}(\bar{L}\phi_s)\tilde{H}N_R + \frac{1}{2}M_N\bar{N}_R^cN_R + \frac{y_s}{\Lambda^2}(\bar{L}\phi_a)\xi i\sigma_2\eta^* f + \frac{1}{2}M_f\bar{f}^c f + h.c.,$$

$$M_\nu = -\frac{v^2}{M_N}Y_N^iY_N^j + \mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f)M_f Y_f^i Y_f^j$$

- Flavon fields get VEVs along $\langle\phi_s\rangle = (0, -v_s, v_s)$, $\langle\phi_a\rangle = (2v_a, v_a, 0)$

$$Y_N = (Y_N^e, Y_N^\mu, Y_N^\tau)^T = (0, y_N\frac{v_s}{\Lambda}, -y_N\frac{v_s}{\Lambda})^T, ; Y_F = (Y_F^e, Y_F^\mu, Y_F^\tau)^T = (y_s\frac{v_\xi}{\Lambda}\frac{v_a}{\Lambda}, y_s\frac{v_\xi}{\Lambda}\frac{2v_a}{\Lambda}, 0)^T$$

- Light neutrino mass matrix :

$$M_\nu = \begin{pmatrix} b & 2b & 0 \\ 2b & -a+4b & a \\ 0 & a & -a \end{pmatrix}, a = y_N^2\frac{v^2}{M_N}\frac{v_s^2}{\Lambda^2}, b = y_s^2\frac{v_\xi^2}{\Lambda^2}\frac{v_a^2}{\Lambda^2}\mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f)M_f = \kappa^2\mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f)M_f$$

- Diagonalizing matrix:

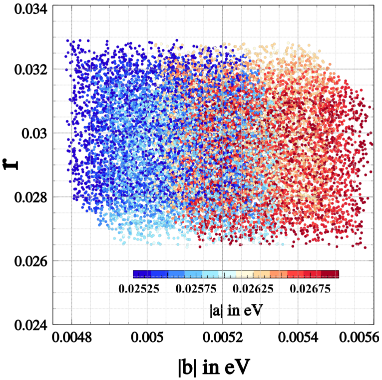
$$U_\nu = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{\cos\theta}{\sqrt{3}} & \frac{e^{-i\psi}\sin\theta}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{\cos\theta}{\sqrt{3}} + \frac{e^{i\psi}\sin\theta}{\sqrt{2}} & -\frac{\cos\theta}{\sqrt{2}} + \frac{e^{-i\psi}\sin\theta}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{\cos\theta}{\sqrt{3}} - \frac{e^{i\psi}\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} + \frac{e^{-i\psi}\sin\theta}{\sqrt{3}} \end{pmatrix} U_m$$

FSS_1 phenomenology : TM_1 mixing

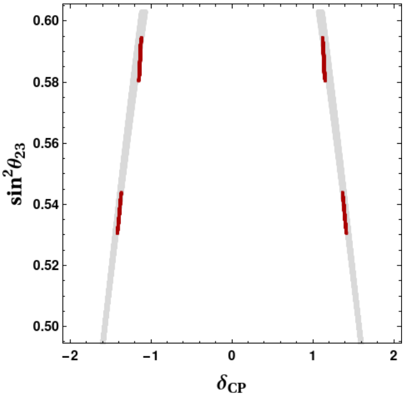
- Mass eigenvalues:

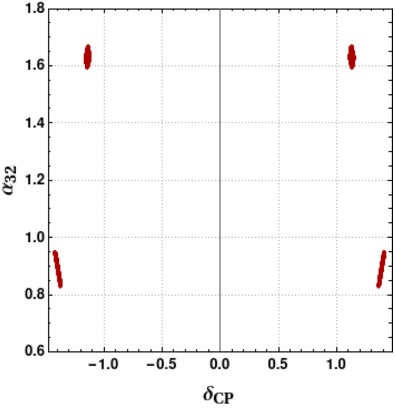
$$\widetilde{m}_1 = 0,$$
$$\widetilde{m}_2 = \frac{1}{2}\Big(-2a + 5b - \sqrt{4a^2 + 4ab + 25b^2}\Big),$$
$$\widetilde{m}_3 = \frac{1}{2}\Big(-2a + 5b + \sqrt{4a^2 + 4ab + 25b^2}\Big).$$

- Ratio of the solar to atmospheric mass-squared differences: $r \sim \frac{m_2^2}{m_3^2}$



FSS_1 phenomenology: neutrinos

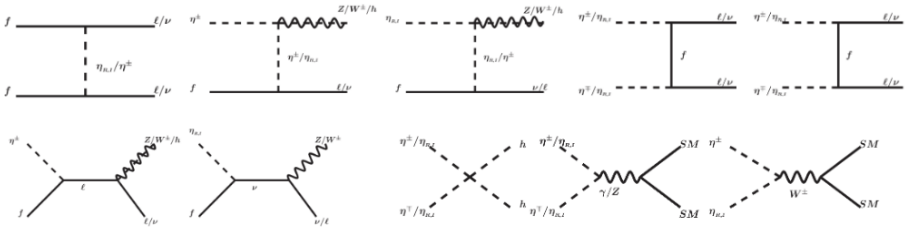


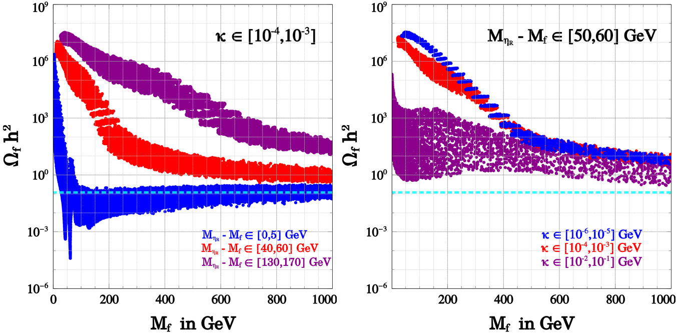


m_2 (meV)	m_3 (meV)	$\sum m_i$ (meV)	$m_{\beta\beta}$ (meV)
8.3 − 9.0	49.7 − 51.3	58.0 − 60.3	1.61 − 3.85

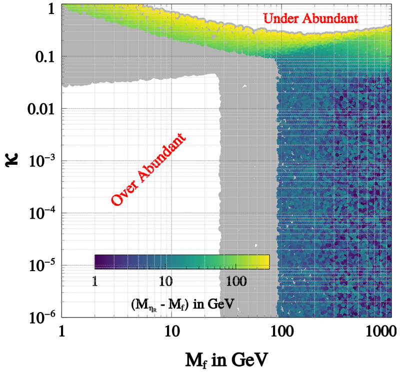
FSS_1 phenomenology: dark matter

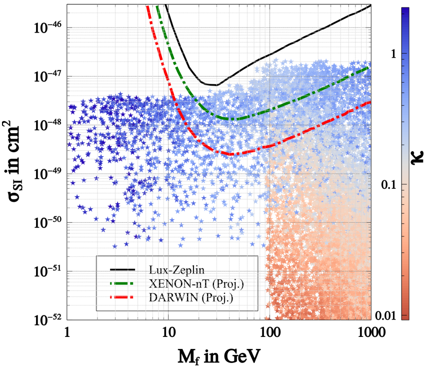
- 2 viable DM candidates ⇒ the **lightest neutral scalar** (Mandal, Srivastava, Valle, 2104.13401)
  ⇒ the **singlet fermion** (Ganguly, Gluza, Karmakar, Mahapatra 2311.xxxxx).





FSS_1 phenomenology: dark matter



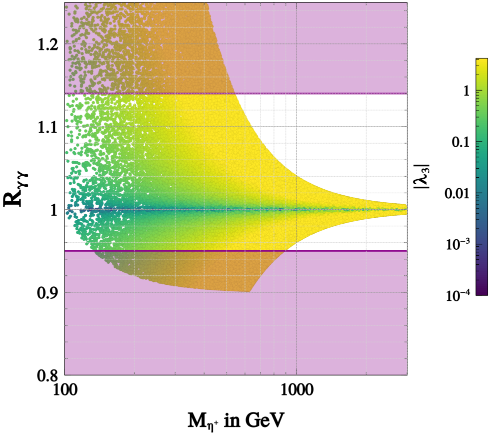


- Neutrino mixing dependence of dark matter phenomenology :

$$\Rightarrow \kappa^2 = \frac{|b|}{\mathcal{F}(M_{\eta_R}, M_{\eta_I}, M_f)M_f}$$

FSS_1 phenomenology: $h \to \gamma\gamma$

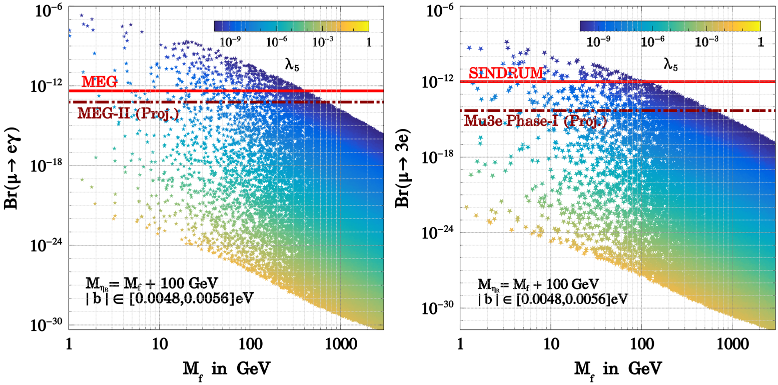
$$R_{\gamma\gamma} = \frac{\left[\sigma(gg \to h) \times \mathrm{Br}(h \to \gamma\gamma)\right]_{\mathrm{FSS}_1}}{\left[\sigma(gg \to h) \times \mathrm{Br}(h \to \gamma\gamma)\right]_{\mathrm{SM}}} = \frac{\Gamma^h_{\mathrm{SM}}}{\Gamma^h_{\mathrm{FSS}_1}} \frac{\Gamma(h \to \gamma\gamma)_{\mathrm{FSS}_1}}{\Gamma(h \to \gamma\gamma)_{\mathrm{SM}}}.$$



λ_3 is the coupling for the interaction $(H^\dagger H)(\eta^\dagger \eta)$

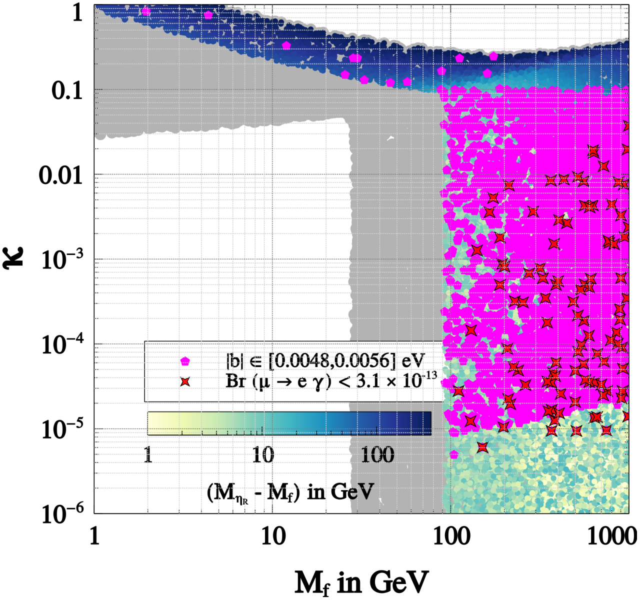
FSS_1 phenomenology: Lepton Flavor Violation

Decay Modes	Scotogenic contribution	Seesaw Contribution	Remarks
$\mu \to e\gamma$	✓	✗	$Y_N^e = 0$
$\tau \to e\gamma$	✗	✗	$Y_F^\tau = 0,\ Y_N^e = 0$
$\tau \to \mu\gamma$	✗	✓	$Y_F^\tau = 0$
$\mu \to 3e$	✓	✗	$Y_N^e = 0$
$\tau \to 3e$	✗	✗	$Y_F^\tau = 0,\ Y_N^e = 0$
$\tau \to 3\mu$	✗	✓	$Y_F^\tau = 0$



λ_5 is the coupling for the interaction $(H^\dagger \eta)(H^\dagger \eta)$

FSS_1 phenomenology: Summary



Conclusion

- Is there any guiding principle behind the observed pattern of lepton mixing?
- (Discrete) flavor symmetry is one such potential candidate.
- Tiny neutrino mass may originate from hybrid scoto-seesaw scenarios.
- It explains the hierarchy of the mass scales involved in neutrino oscillation
- Flavor symmetric scoto-seesaw (FSS) scenarios also explain neutrino mixing angles and CP phases involved.
- Possible frameworks : FSS_1 for TM_1 mixing and FSS_2 for TM_2 mixing.
- Rich phenomenology : $h \to \gamma\gamma$, potential DM candidates, LFV decays.....
- Not covered: Collider prospect of BSM states, leptogenesis, CP properties of heavy neutrinos (this afternoon by Janusz Gluza)..

- Advertisement: For a wide range of **phenomenological applications of flavor symmetric models in energy, intensity, and cosmic frontiers,** see **Phenomenology of Lepton Masses and Mixing with Discrete Flavor Symmetries** Chauhan, Dev, Dubovyk, Dziewit, Flieger, Grzanka, *Gluza, Karmakar, Zieba*. see, arXiv:2310.20681(under review, Progress in Particle and Nuclear Physics)

Thank you for your attention!!