KRAKOW SCHOOL OF INTERDYSCIPLINARY PHD STUDIES

Nuclear PDF Determination
Using Markov Chain Monte Carlo Methods

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nCTEQ collaboration

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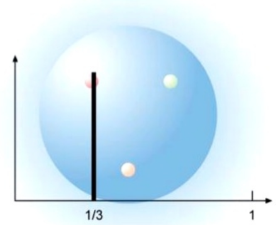
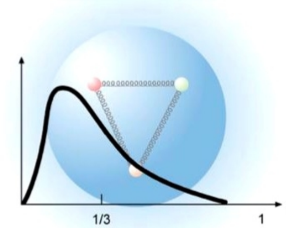
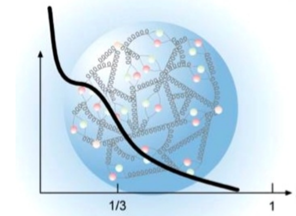


Free quarks



Bound quarks



Bound quarks + QCD effects

Parton Distribution Function (PDF):

The probability $\mathbf{f_{a/p}(x,\mu)}$ that a parton **a** carries fraction **x** of the proton's momentum

μ: Factorization scale
X: momentum fraction

Factorization Theorem

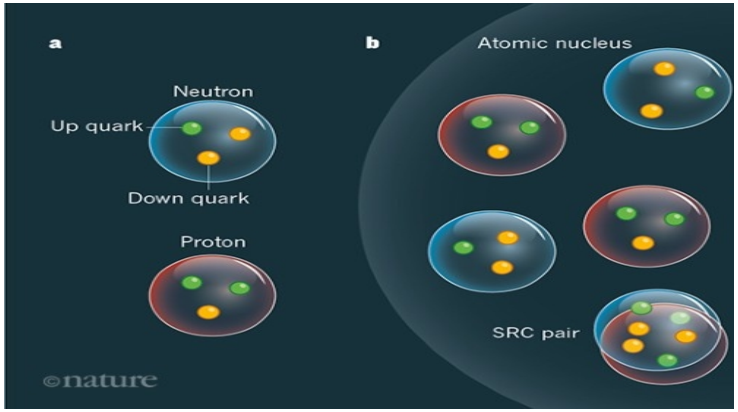
$$\sigma_{P\gamma\to c} = f_{P\to a} \otimes \hat{\sigma}_{a\gamma\to c}$$

Parton densities (long-distance) | Parton interaction (short-distance)

PDF properties:

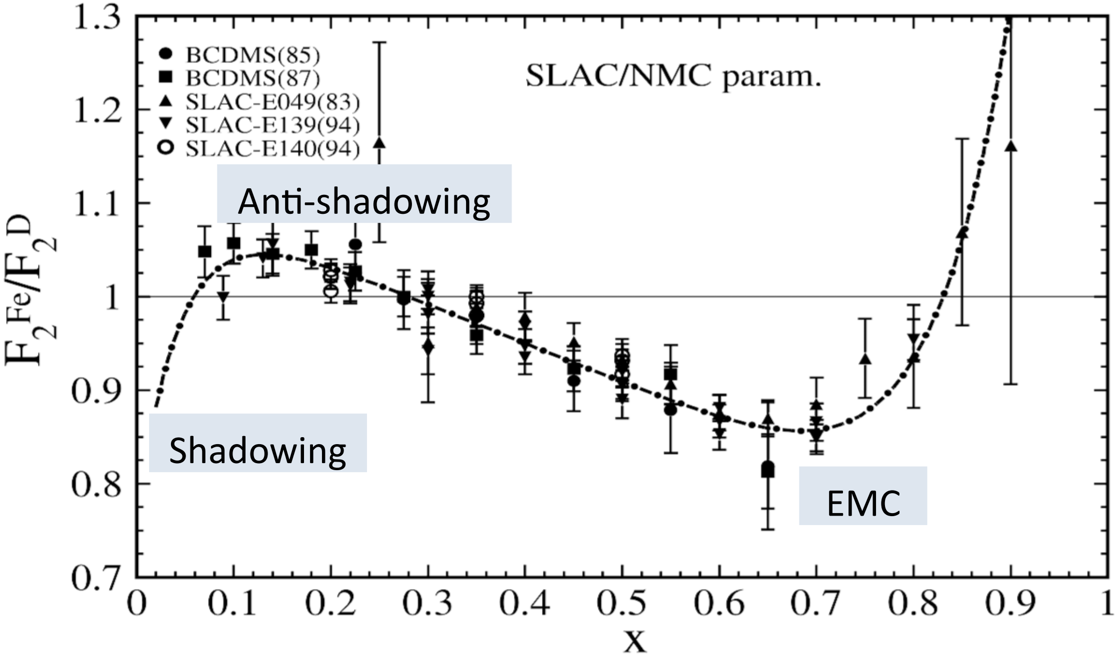
- Universal
- Constrained through momentum and number sum rules
- μ^2-dependence governed by DGLAP evolution equations
- Non-perturbative: x-dependence of PDF is NOT calculable in pQCD

→ **extracting from a fit to experimental data**



Nuclear PDFs(nPDFs):

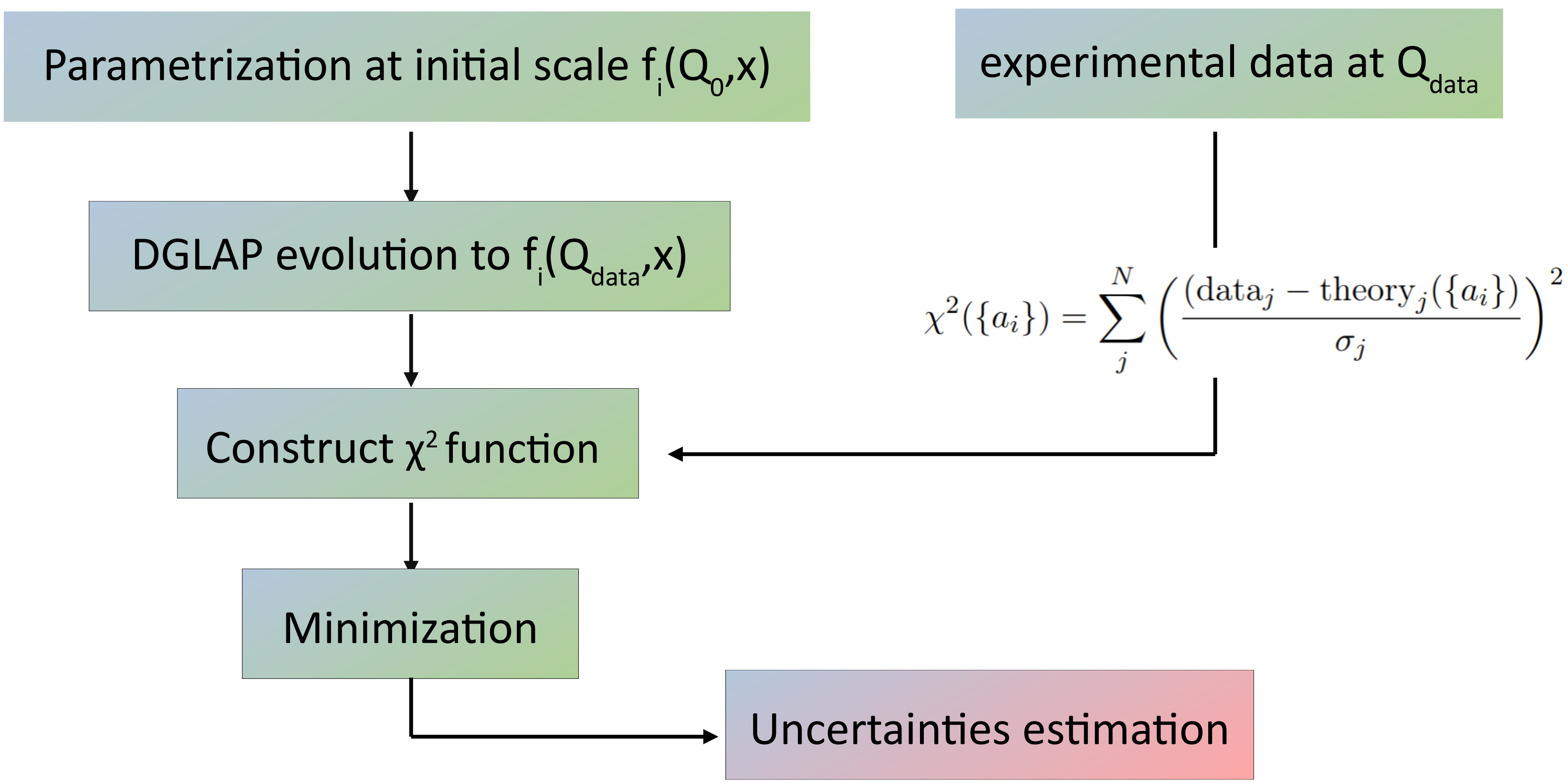
$$F_2^A(x) \neq ZF_2^p(x) + NF_2^n(x)$$



Motivations:

- Interpreting heavy-ion collision data
- Understanding Nuclear Structure

QCD Global analysis:



nCTEQ15 framework for nuclear PDF:

Kovarik et al., arXiv:1509.00792

Functional form for bound proton at Q_0:

$$x f_i^{p/A}(x, Q_0) = c_0\, x^{c_1} (1-x)^{c_2} e^{c_3 x} (1 + e^{c_4} x)^{c_5}$$

$$i = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s}$$

Atomic number dependence is characterized in the c_k coefficients as

$$c_k \to c_k(A) \equiv p_k + a_k(1 - A^{-b_k}), \qquad k = \{1, \ldots, 5\}.$$

PDF of a nucleus (A – mass, Z – charge):

$$f_i^{(A,Z)}(x, Q) = \frac{Z}{A} f_i^{p/A}(x, Q) + \frac{A-Z}{A} f_i^{n/A}(x, Q)$$

new nCTEQ global nPDF release: CJ15

Accardi et al., arXiv:1602.03154

Functional form for bound protons at Q_0:

$$x f_i^{p/A}(x, Q_0) \;=\; c_0 x^{c_1} (1-x)^{c_2} (1 + c_3 \sqrt{x} + c_4 x)$$

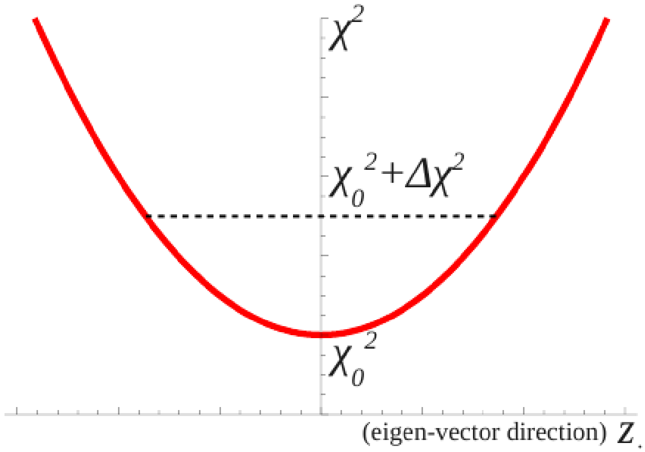
$$i = u_v, d_v, g, \bar{u}+\bar{d}, s, \bar{s}$$

Atomic number dependence is characterized in the c_k coefficients as

$$c_k \;\to\; p_k + a_k \ln(A) + b_k \ln^2(A). \qquad k = \{1, ..., 5\}.$$

PDF of a nucleus (A – mass, Z – charge):

$$f_i^{(A,Z)}(x, Q) = \frac{Z}{A} f_i^{p/A}(x, Q) + \frac{A-Z}{A} f_i^{n/A}(x, Q)$$



PDF uncertainties estimation:

Hessian method: Common method for estimating uncertainties in PDFs.

relying on the Gaussian approximation of $\Delta\chi^2$

Shortcomings:
- Non-gaussian errors
- Global minima judgment

nPDF difficulties
- Lacking data (need low-x & precise data, for several nuclei)
- Complexity and nature of nuclear effects

deeper insight

Markov Chain Monte Carlo method

Markov Chain Monte Carlo (MCMC)

A sequence of random variables where the current value is dependent on the value of the prior variable (Memory-less property)

A technique for randomly sampling a probability distribution and approximating a desired quantity.

Bayes theorem:

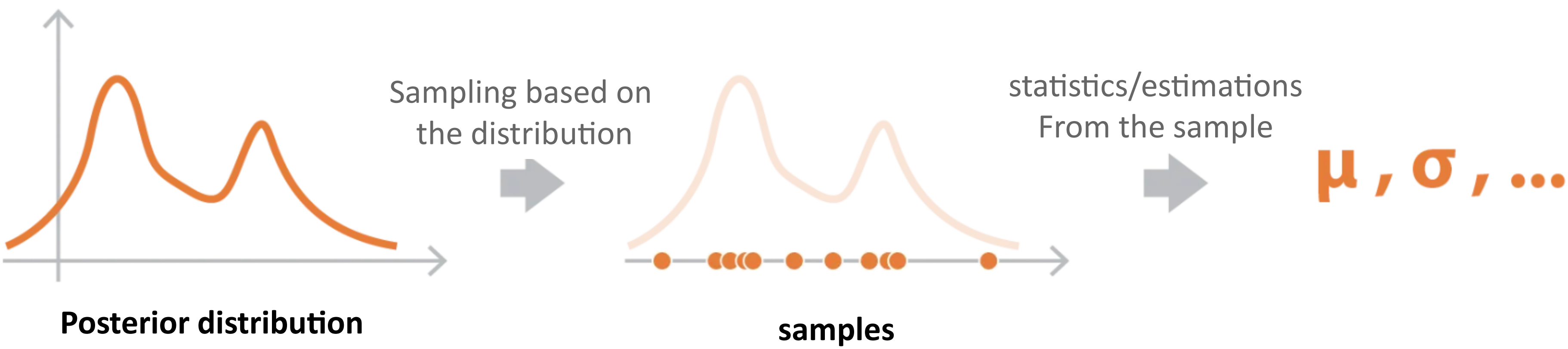
$$p(\theta \mid \text{data}) = \frac{p(\text{data} \mid \theta) \cdot p(\theta)}{p(\text{data})}$$

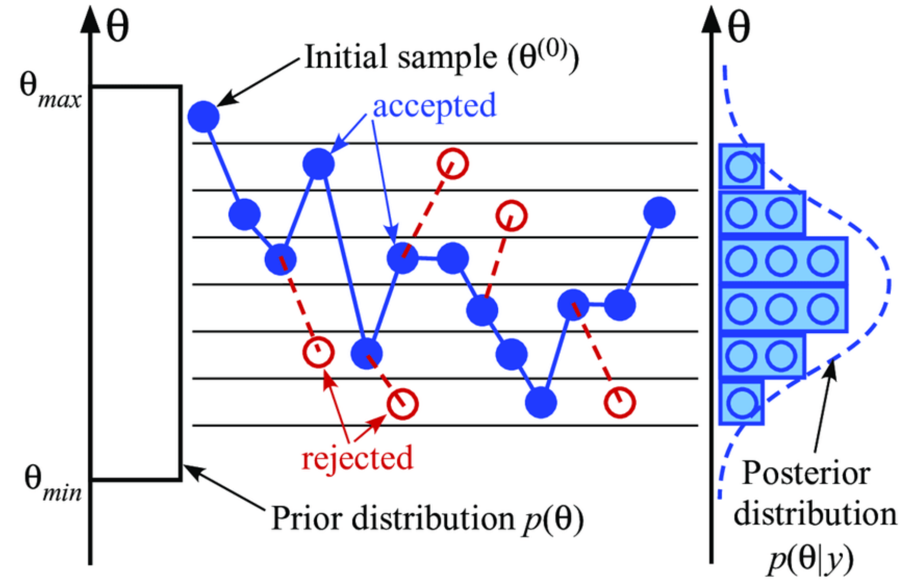
Posterior: $p(\theta \mid \text{data})$
Likelihood: $p(\text{data} \mid \theta)$
Prior: $p(\theta)$
Normalization: $p(\text{data})$

Prior: initial belief about the parameter before considering the data.
Likelihood: probability of observing the data given a specific value of the parameter.
Posterior: updated belief about the parameter given the data.

We aim to find the set of nPDF parameters that maximizes the posterior probability distribution given the experimental data.

Bayesian inference → **MCMC algorithms**





Metropolis algorithm:

- Initialize parameters
- for i=1 to i=N:

Propose a new sample

Generate proposed parameters via proposition function: $\theta^* \sim q(\theta^* \mid \theta_i)$

Sample from uniform distribution: $u \sim U(0,1)$

Compute acceptance ratio: $\alpha = p(\theta^* \mid D) / p(\theta_i \mid D)$

If $u < \min(1, \alpha)$ then $x_{i+1} = x^*$

Judgment of proposed sample

- Else $x_{i+1} = x_i$

Metropolis-Hasting: $x_{t+1} = \mathcal{N}(x_t, C_0)$

Adaptive Metropolis-Hasting: $x_{t+1} = \beta\mathcal{N}(x_t, C_0) + (1-\beta)\mathcal{N}(x_t, \hat{C}_n)$

PDF Global analysis:

Parametrization at initial scale $f_i(Q_0,x)$

DGLAP evolution to $f_i(Q_{data},x)$

Construct χ^2 function

Minimization

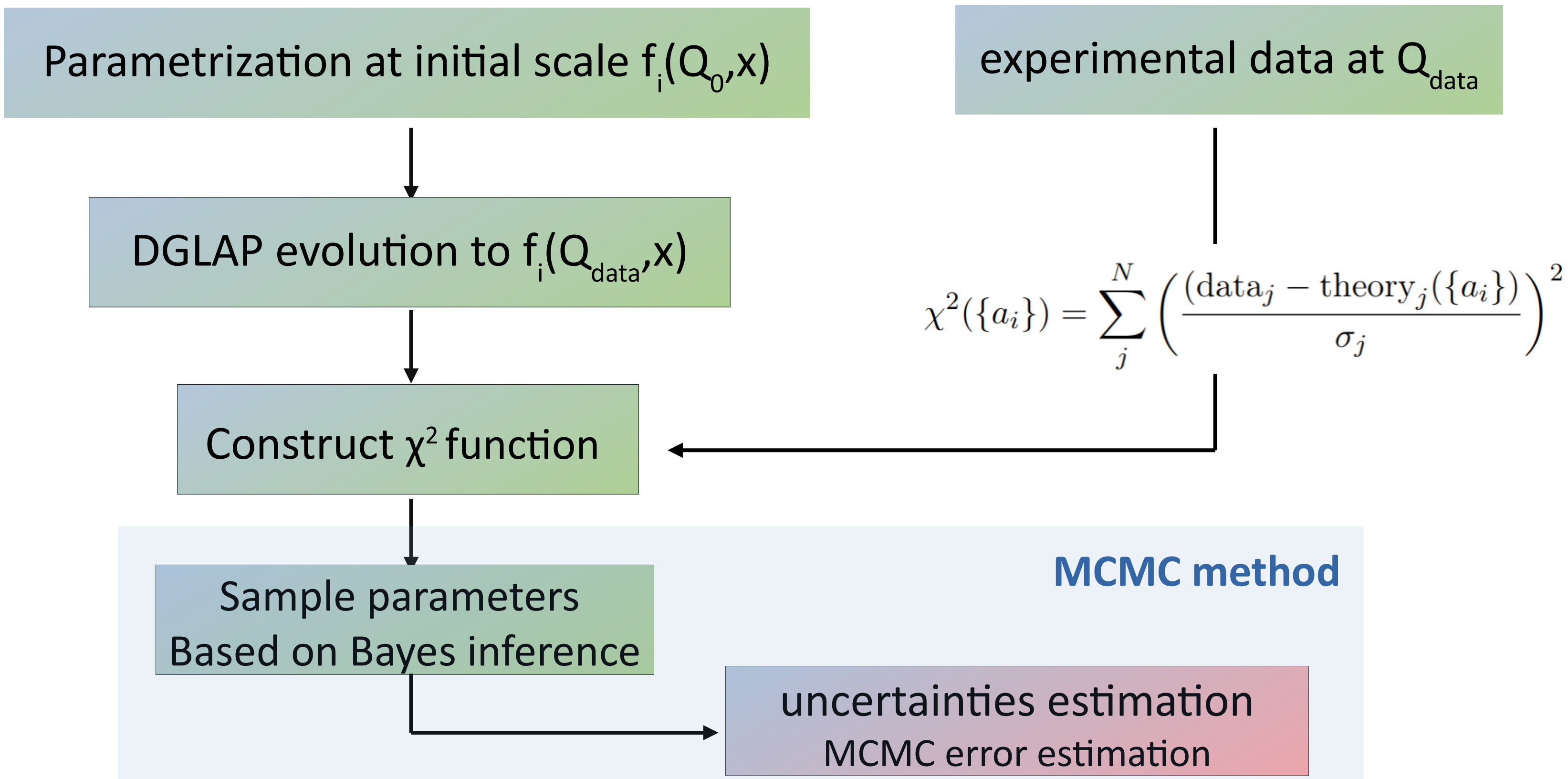
uncertainties estimation
Gaussian error propagation

experimental data at Q_{data}

$$\chi^2(\{a_i\}) = \sum_j^N \left(\frac{(\text{data}_j - \text{theory}_j(\{a_i\})}{\sigma_j} \right)^2$$

Hessian method

PDF Global analysis:



CJ15 nPDF parametrization

$$x f_i^{p/A}(x, Q_0) = c_0 x^{c_1}(1-x)^{c_2}(1 + c_3\sqrt{x} + c_4 x)$$

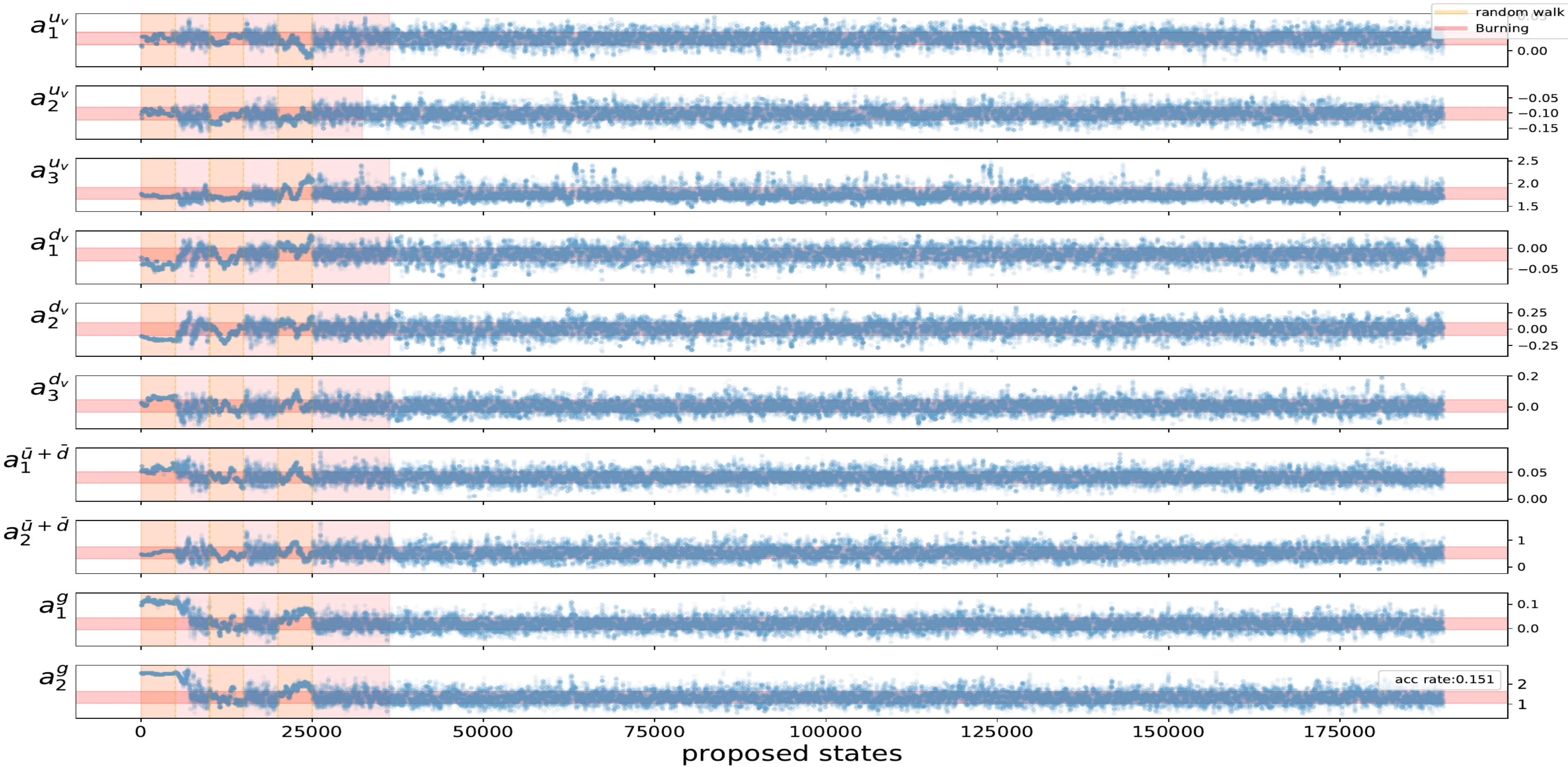
$$c_k \to p_k + a_k \ln(A) + b_k \ln^2(A).$$

Generating the Markov Chain of nPDF parameters:

Each point of the chain is representing a set of nPDF parameters

(6 valance, 2 sea quarks and 2 gloun)

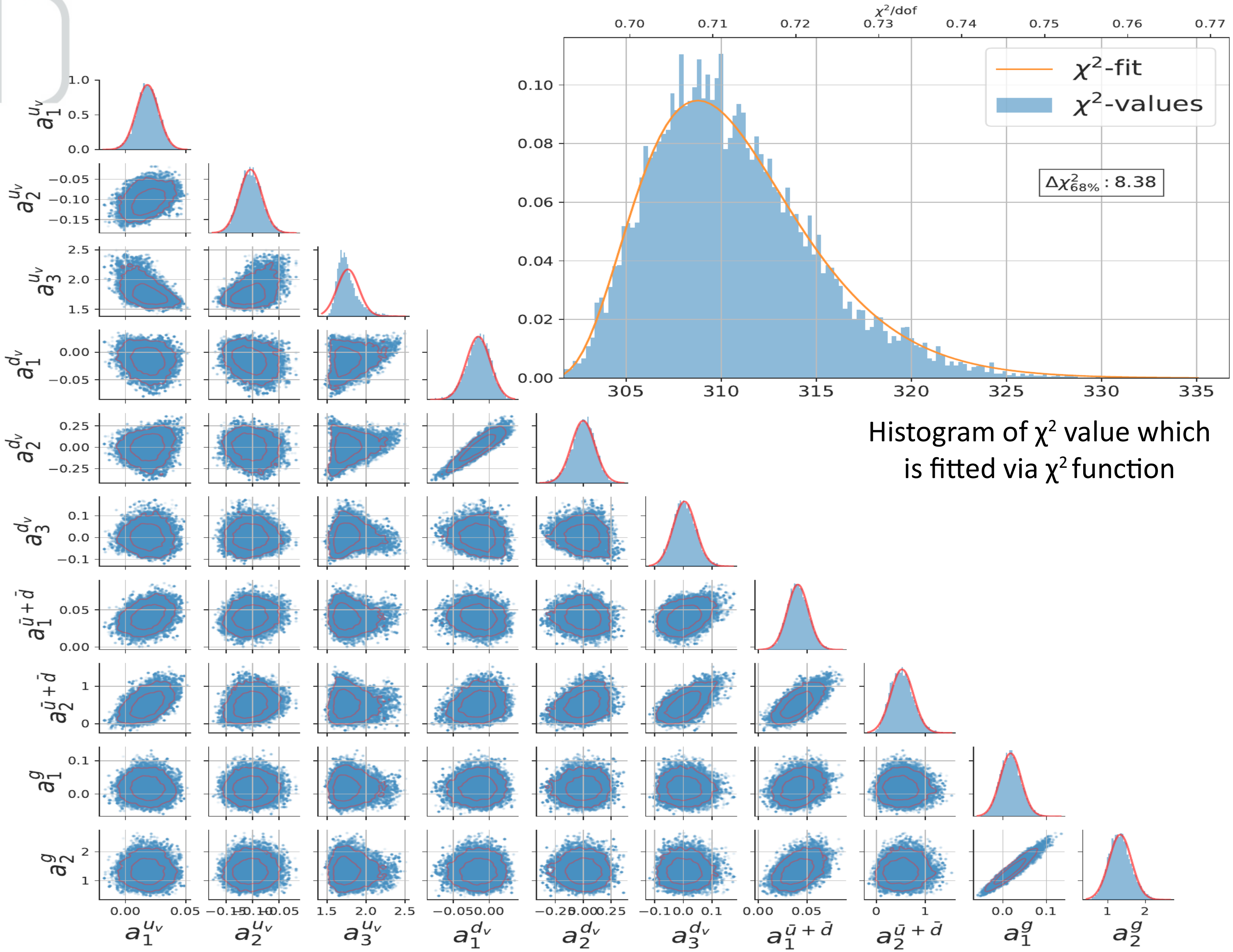
DIS and W/Z boson data:
436 data points



MCMC can reveal non-Gaussian features of the underlying distribution

Pairwise plot

diagonal: histogram of each parameter
off-diagonal: 2D correlation plots between parameters



Histogram of χ^2 value which is fitted via χ^2 function

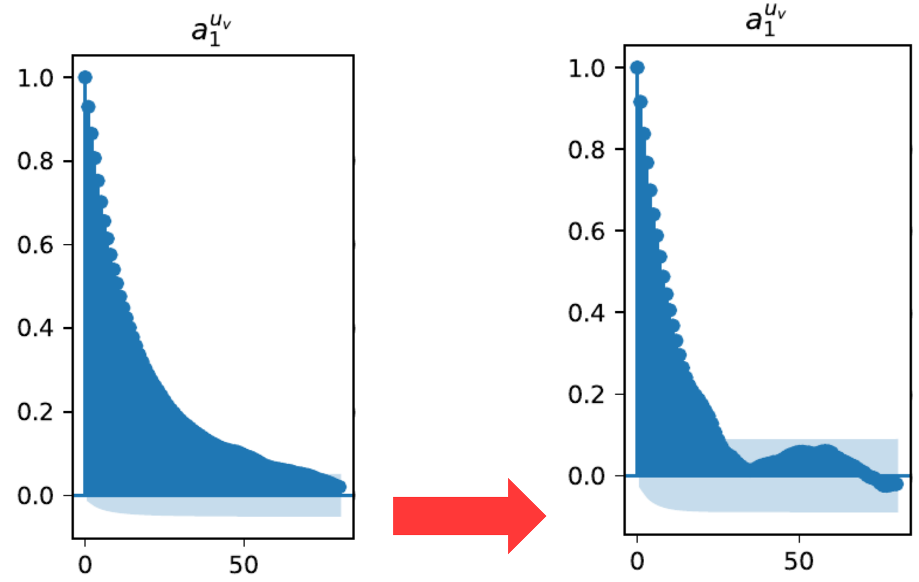
Error estimation:

Autocovariance: $\mathrm{Cov}(k) = \frac{1}{n}\sum_{t=1}^{n-k}(x_{t+k}-\bar{x})(x_t-\bar{x}),$

Autocorrelation: $\rho(k) = \frac{\mathrm{Cov}(k)}{\mathrm{Cov}(0)}$

Autocorrelation time: $\tau_{int} = \frac{1}{2}\sum_{-\infty}^{+\infty}\rho(k)$

Autocorrelation function versus time interval



Thinning by rate 40

discard all except every k-th point of the chain

MCMC error (correlated)

$$\sigma^2_{MCMC} = 2\,\tau_{int}\,\sigma^2_{MC}$$

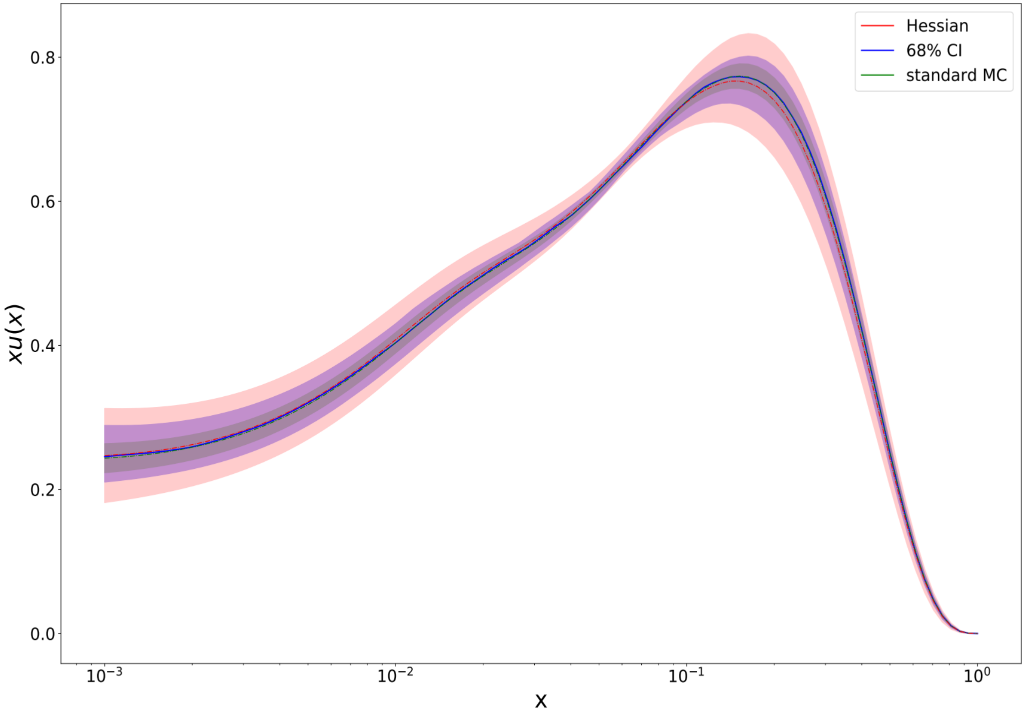
MC error (uncorrelated)

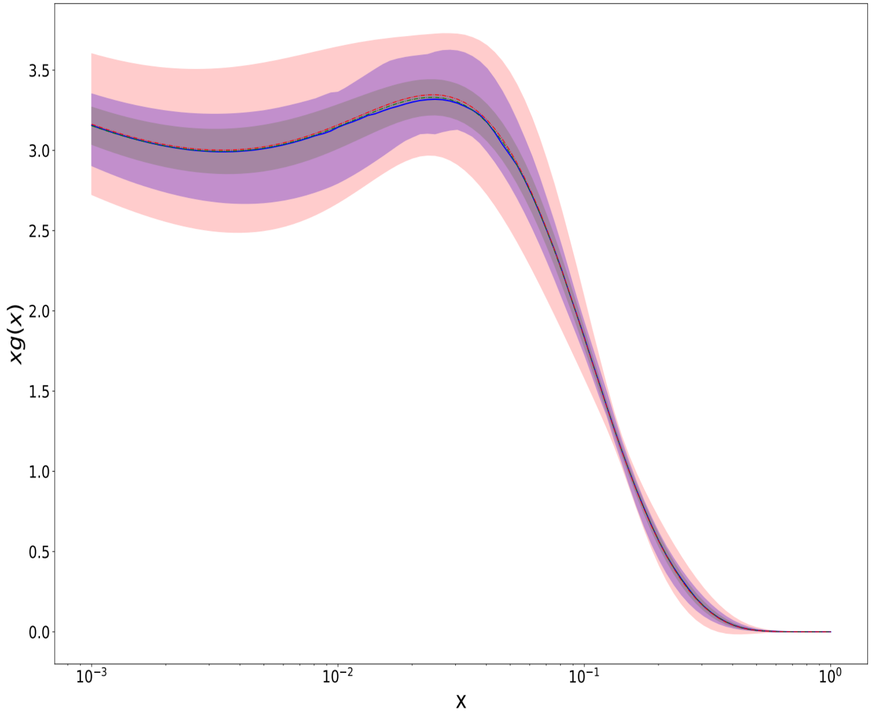
$$\sigma^2_{MC} = \frac{1}{n-1}\sum_{t=1}^{n}(X_t-\hat{\mu})^2$$

MCMC approach:

- Generating the Markov Chain
- Thinning the chain
- Dumping PDF corresponding to each unit of the thinned chain
- Evaluating the error band determined from Monte Carlo error

LHAPDF (set of nPDF grids):





Thank you for your attention

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