THE HENRYK NIEWODNICZAŃSKI
INSTITUTE OF NUCLEAR PHYSICS
POLISH ACADEMY OF SCIENCES

NLO matching with KrkNLO

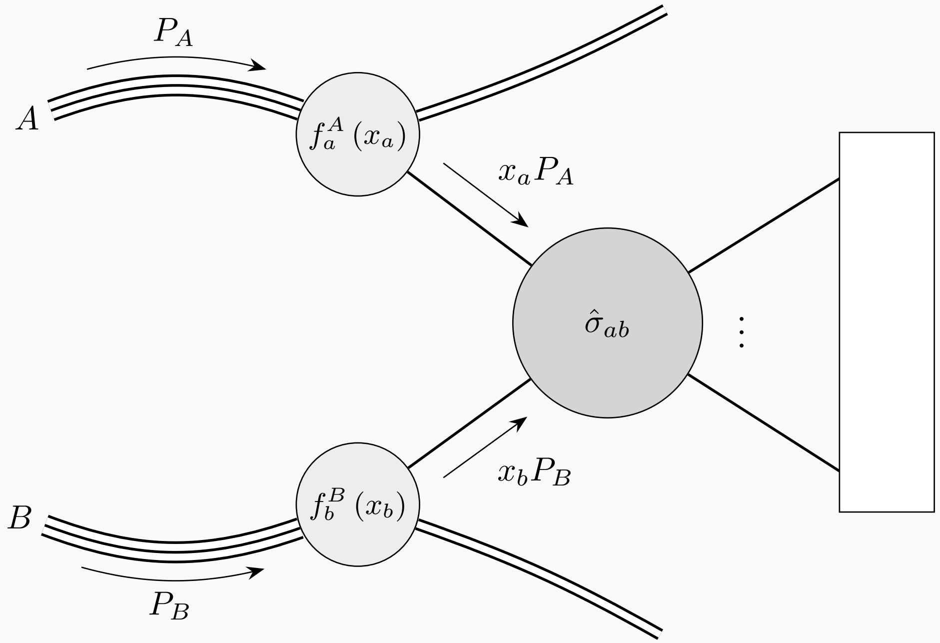
theory and progress

James Whitehead (IFJ PAN, Kraków)
with Wiesław Płaczek, Pratixan Sarmah, Andrzej Siódmok (UJ, Kraków)

2PiNTS 2023

Matching at NLO

NLO matching



Interested in some[1] function $\mathcal{O}$ of phase-space:

$$\mathrm{d}\sigma_{AB}\left[\mathcal{O}\right](\mu_\mathrm{F},\mu_\mathrm{R}) = \sum_{a,b} f_a^A(\xi_1;\mu_\mathrm{F}) \otimes_{\xi_1} \mathrm{d}\hat{\sigma}_{ab}\left[\mathcal{O}\right](\xi_1,\xi_2;\mu_\mathrm{F},\mu_\mathrm{R}) \otimes_{\xi_2} f_b^B(\xi_2;\mu_\mathrm{F}).$$

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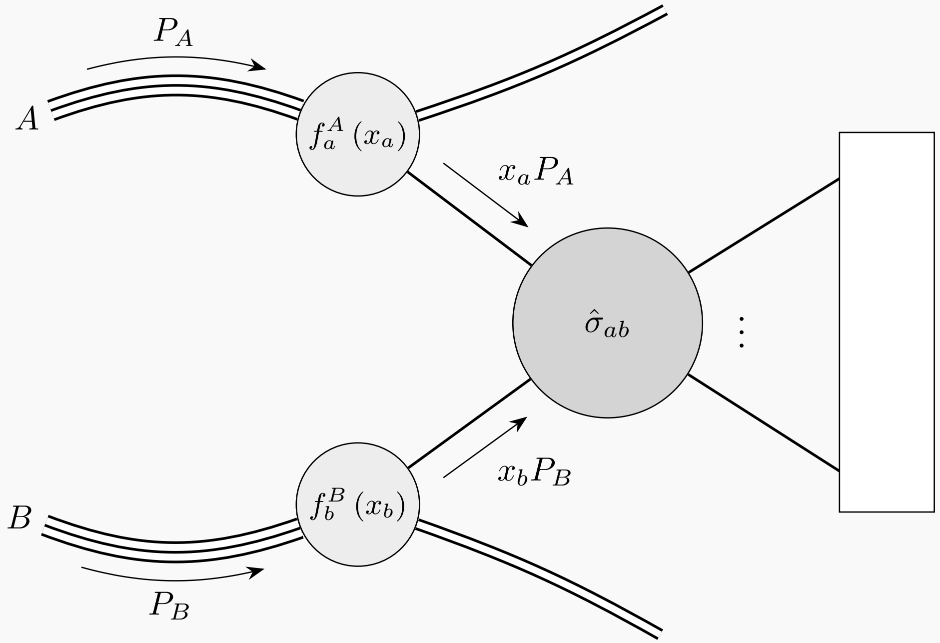

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[1] infrared and collinear safe

Building a cross-section

For fixed-order calculations: expand perturbatively (and subtract)

$$
\begin{aligned}
d\sigma_{ab}^{\mathrm{NLO}}\left[\mathcal{O}\right](\xi_1,\xi_2) = & \left(\frac{\alpha_s}{2\pi}\right)^k \left\{ d\Phi_m(\xi_1 P_1, \xi_2 P_2) \left[\mathrm{B}(\Phi_m)\right] \mathcal{O}(\Phi_m) \right\} \\
& + \left(\frac{\alpha_s}{2\pi}\right)^{k+1} \left\{ d\Phi_m(\xi_1 P_1, \xi_2 P_2) \left[\mathrm{V}(\Phi_m)\right] \mathcal{O}(\Phi_m) \right. \\
& \left. + \, d\Phi_{m+1}(\xi_1 P_1, \xi_2 P_2) \left[\mathrm{R}(\Phi_{m+1})\right] \mathcal{O}(\Phi_{m+1}) \right\}
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&+\left(\frac{\alpha_s}{2\pi}\right)^{k+1}\Bigg\{\mathrm{d}\Phi_m\left[\mathrm{V}(\Phi_m)+\mathrm{I}(\Phi_m)\right]\mathcal{O}(\Phi_m)\\
&\qquad+\mathrm{d}x\,\mathrm{d}\Phi_m(x\xi_1P_1,\xi_2P_2)\left[(\mathrm{P}+\mathrm{K})_{ac}(x)\cdot\mathrm{B}_{cb}(\Phi_m)\right]\mathcal{O}(\Phi_m)\\
&\qquad+\mathrm{d}x\,\mathrm{d}\Phi_m(\xi_1P_1,x\xi_2P_2)\left[(\mathrm{P}+\mathrm{K})_{bc}(x)\cdot\mathrm{B}_{ac}(\Phi_m)\right]\mathcal{O}(\Phi_m)\\
&\qquad+\mathrm{d}\Phi_{m+1}\left[\mathrm{R}(\Phi_{m+1})-\mathrm{S}(\Phi_{+1})\cdot\mathrm{B}(\tilde{\Phi}_m)\right]\mathcal{O}(\Phi_{m+1})\Bigg\}
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Phase Spaced-out

What is $\mathrm{d}\Phi_m$?

$$\mathrm{d}\Phi_m(\xi_1 P_1, \xi_2 P_2) = \left[\prod_{i=1}^{m} \frac{\mathrm{d}^4 p_i}{(2\pi)^3} \delta(p_i^2 - m^2)\right]$$

[2] e.g. R. Kleiss, W. James Stirling, and S. D. Ellis. “A New Monte Carlo Treatment of Multiparticle Phase Space at High-energies”. Simon Plätzer. “RAMBO on diet”. arXiv: 1308.2922 [hep-ph], Enrico Bothmann et al. “Efficient phase-space generation for hadron collider event simulation”.

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In practice for Monte Carlo we introduce a **phase-space generator**[2]

$$\Phi_m(\mathbf{r}; \xi_1 P_1, \xi_2 P_2) : [0,1]^{3m-4} \longrightarrow \Phi_m$$

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and integrate

$$\int_{[0,1]^{3m-4}} f(\Phi_m) \left| \frac{\partial \Phi_m}{\partial \mathbf{r}} \right| \mathrm{d}\mathbf{r} \equiv \int f(\Phi_m) \, \mathrm{d}\Phi_m$$

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Transporting between phase-spaces

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Can we go the other way?

projections

'integrating out information'

$$\Phi_{m+1} \longrightarrow \Phi_m \cdot \Phi_{+1}$$

just do the obvious$^?$ thing

define a subset to absorb the recoil and transform, i.e.

$$\tilde{p}_i = f_1(p_i, p_j, p_k)$$
$$\tilde{p}_k = f_2(p_i, p_j, p_k)$$

bonus: preserve ME factorisation in soft/collinear limits

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splittings

'generating new information'

$$\Phi_m \oplus \Phi_{+1}(\mathbf{r}) \longrightarrow \Phi_{m+1}$$

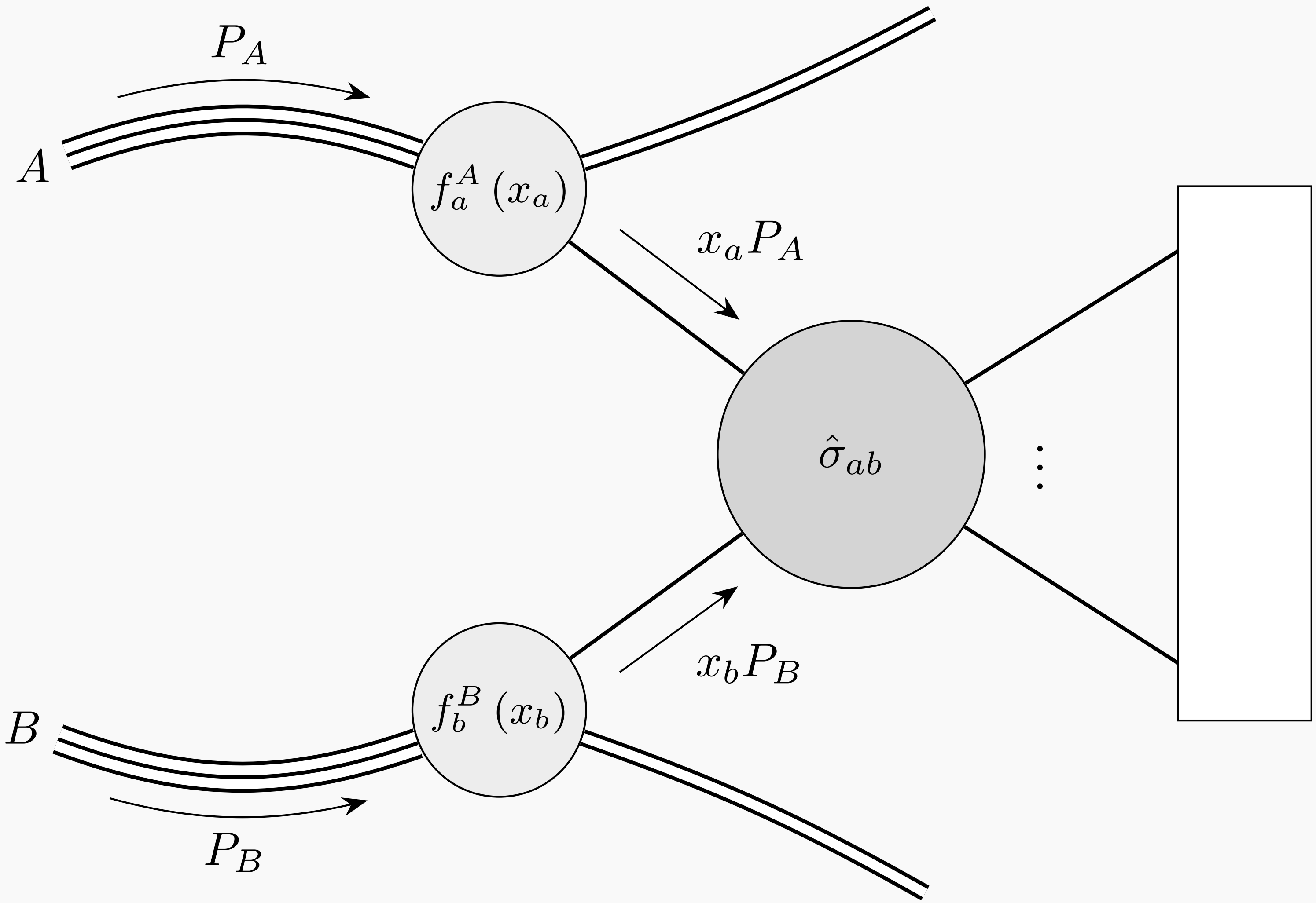
new problem: need to inject an extra 3 degrees of freedom, e.g.

$$\tilde{p}_i = z p_i + \alpha p_k + k_T$$

$$\tilde{p}_j = (1-z) p_i + \beta p_k - k_T$$

$$\tilde{p}_k = (1-\alpha-\beta) p_k$$

What is a parton shower?



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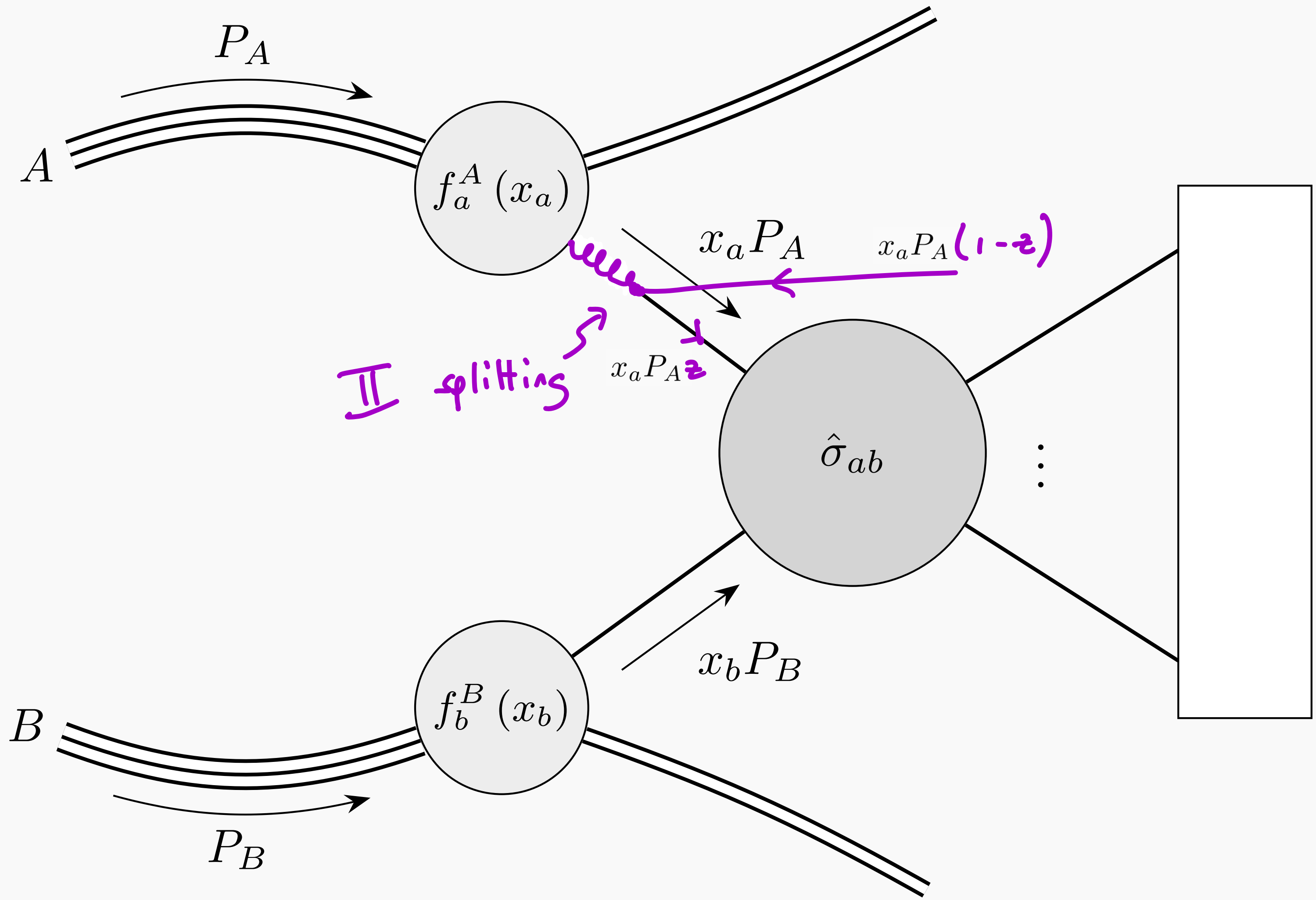

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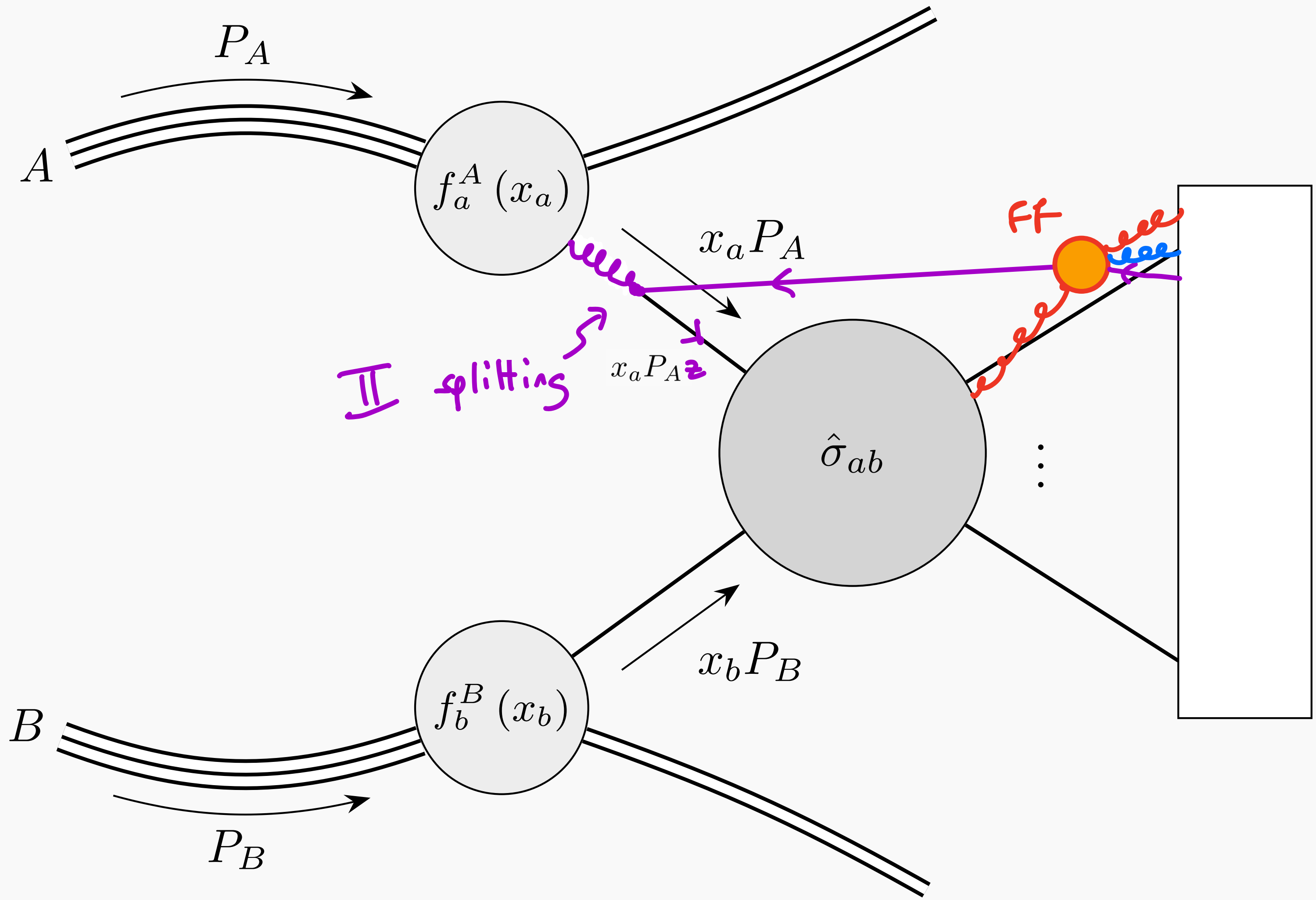

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...look familiar?

,

New legs from old[3]

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At its heart:

$$\mathrm{PS}\left[\mathcal{O}\right]\left(\Phi_m\right) = \Delta_{\mu_s}^{Q(\Phi_m)}\, \mathcal{O}(\Phi_m)$$

[3] Based on ongoing work with Andrzej Siódmok and Simon Plätzer.

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$$\begin{aligned}\mathrm{PS}\left[\mathcal{O}\right](\Phi_m) = \Delta_{\mu_s}^{Q(\Phi_m)}\, \mathcal{O}(\Phi_m) \\ + \sum_\alpha \int \mathrm{d}\Phi_{+1}^{(\alpha)}\, \Theta[\mu_s < \mu(\Phi_{+1}) < Q(\Phi_m)]\, \Delta_{\mu(\Phi_{+1})}^{Q(\Phi_m)} \\ \times P_m^{(\alpha)}(\Phi_{+1})\, \Theta_{\mathrm{PS}}^{(\alpha)}\left[\Phi_{m+1}^{(\alpha)}\right]\, \mathrm{PS}\left[\mathcal{O}\right](\Phi_{m+1}^{(\alpha)})\end{aligned}$$

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To be concrete: choose

1. ordering variable $\mu(\Phi_{+1})$
2. radiative splitting phase-space $\Phi_{m+1}^{(\alpha)}$
3. radiative splitting kernels $P_m^{(\alpha)}(\Phi_{m+1}^{(\alpha)})$
4. renormalisation (II: factorisation) scale choice $\mu_{\mathrm{R,F}}(\Phi_{m+1}^{(\alpha)})$
5. shower starting-scale $Q(\Phi_m)$, cut-off scale $\mu_s(\Phi_m)$

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To be concrete: choose

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2. radiative splitting kinematics $\Phi_{m+1}^{(\alpha)}$ — Catani–Seymour II/(IF/FI)/FF
3. radiative splitting kernels $P_m^{(\alpha)}(\Phi_{m+1}^{(\alpha)})$ — Catani–Seymour $\mathrm{D}^{(\alpha)}(\Phi_{+1}^{(\alpha)})$
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NLO matching criterion

The matched NLO cross-section shouldn't spoil the fixed-order result:

$$\hat{\sigma}^{\text{NLO+PS}}[\mathcal{O}] = \hat{\sigma}^{\text{NLO}}[\mathcal{O}]$$

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shower (α_s^{k+1})	$-\mathrm{B} \cdot \int \mathrm{d}\Phi_{+1} P_m^{(\alpha)}(\Phi_{+1})$	$+\mathrm{B} \cdot \mathrm{d}\Phi_{+1} P_m^{(\alpha)}(\Phi_{+1})$
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- collinear convolution terms can only go into the PDF
- where to put end-point contributions $\propto \delta(1-x)$?

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This provides the recipe for the PDF transformation (more later).

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Final recipe[7]

$$
\begin{aligned}
\mathrm{d}\Phi_m\, \Theta_{\text{cut}}[\Phi_m] \quad & \Bigg[\Bigg\{ \mathrm{B}(\Phi_m) + \mathrm{V}(\Phi_m) + I(\Phi_m) + \Delta_0^{\text{FS}} \Bigg\} \Delta_{\mu_s}^{Q(\Phi_m)}\, \mathcal{O}(\Phi_m) \\
& + \sum_\alpha \mathrm{d}\Phi_{+1}^{(\alpha)} \left\{ \frac{\mathrm{R}^{(\alpha)}(\Phi_{m+1}^{(\alpha)})}{\mathrm{PS}^{(\alpha)}(\Phi_{m+1}^{(\alpha)})}\, \Theta_{\text{PS}}^{(\alpha)} \left[\Phi_{m+1}^{(\alpha)}\right] \mathrm{PS}^{(\alpha)} \left[\Phi_{m+1}^{(\alpha)}\right] \Theta_{\mu_s}^{(\alpha)}\, \mathcal{O}(\Phi_{m+1}^{(\alpha)}) \right\} \Bigg]
\end{aligned}
$$

1. generate a Born phase-space point, ME and shower:
    - if an emission is generated, reweight to R
    - if not, reweight to B + V
2. matching complete; allow the shower to proceed!

7 S. Jadach et al. "Matching NLO QCD with parton shower in Monte Carlo scheme — the KrkNLO method". arXiv: 1503.06849 [hep-ph], Stanislaw Jadach et al. "New simpler methods of matching NLO corrections with parton shower Monte Carlo". arXiv: 1607.00919 [hep-ph].

Final recipe[7]

$$
\mathrm{d}\Phi_m\, \Theta_{\text{cut}}\,[\Phi_m] \quad \Bigg[\Big\{ \mathrm{B}(\Phi_m) + \mathrm{V}(\Phi_m) + I(\Phi_m) + \Delta_0^{\text{FS}} \Big\} \Delta_{\mu_s}^{Q(\Phi_m)}\, \mathcal{O}(\Phi_m)
$$

$$
+ \sum_\alpha \mathrm{d}\Phi_{+1}^{(\alpha)} \left\{ \frac{\mathrm{R}^{(\alpha)}(\Phi_{m+1}^{(\alpha)})}{\mathrm{PS}^{(\alpha)}(\Phi_{m+1}^{(\alpha)})}\, \Theta_{\text{PS}}^{(\alpha)} \left[\Phi_{m+1}^{(\alpha)}\right] \mathrm{PS}^{(\alpha)} \left[\Phi_{m+1}^{(\alpha)}\right] \Theta_{\mu_s}^{(\alpha)}\, \mathcal{O}(\Phi_{m+1}^{(\alpha)}) \right\} \Bigg]
$$

1. generate a Born phase-space point, ME and shower:
   - if an emission is generated, reweight to R
   - if not, reweight to B + V
2. matching complete; allow the shower to proceed!

This is NLO accurate, but differs from other methods at higher orders.

7 S. Jadach et al. “Matching NLO QCD with parton shower in Monte Carlo scheme — the KrkNLO method”. arXiv: 1503.06849 [hep-ph], Stanislaw Jadach et al. “New simpler methods of matching NLO corrections with parton shower Monte Carlo”. arXiv: 1607.00919 [hep-ph].

Krk PDF scheme

Krk (/MC/CS) factorisation scheme[8]

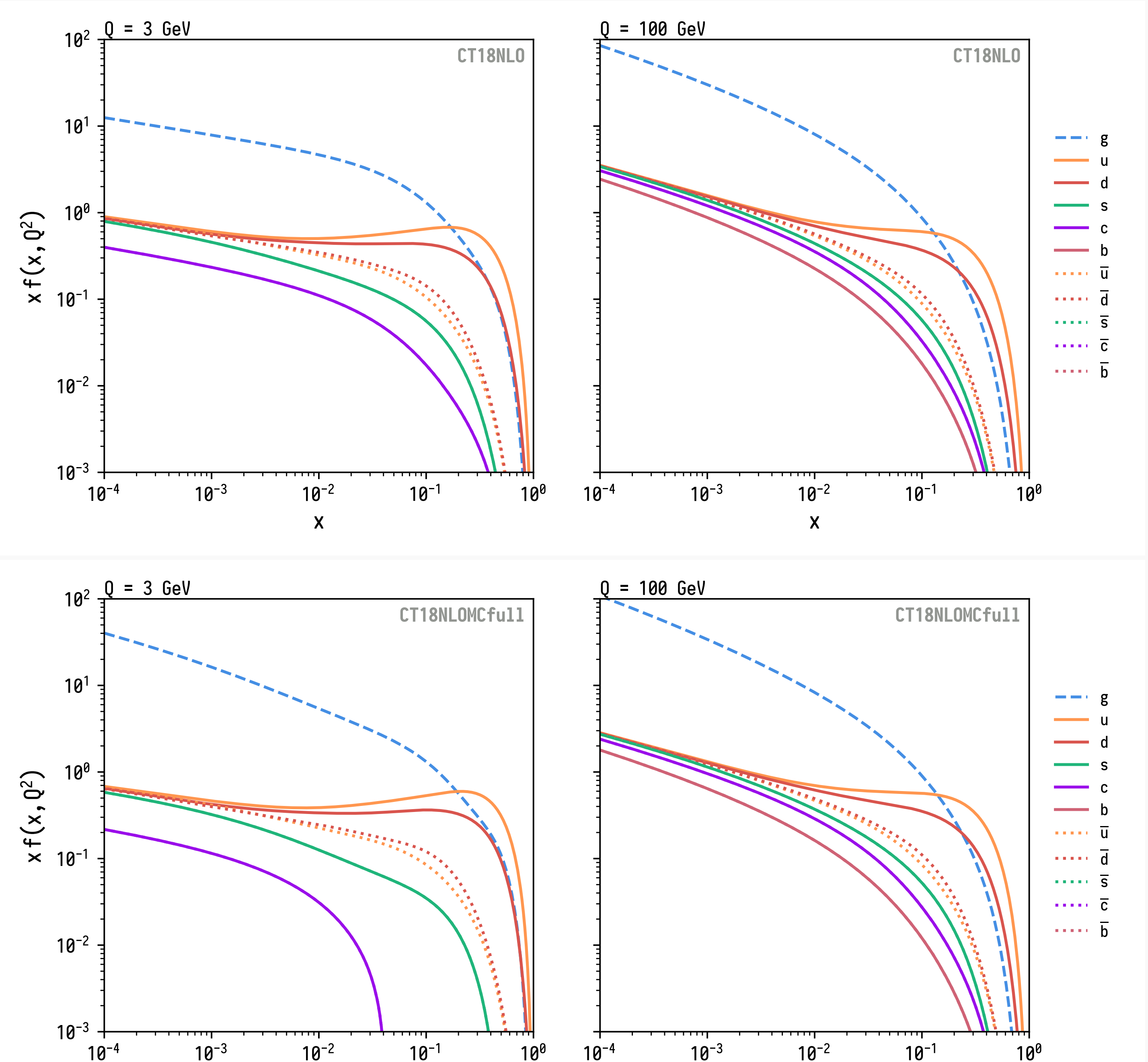
From the dipole operators, we can write down the convolution terms:

$$
\begin{aligned}
f_q^{\mathrm{Krk}}(x,\mu_\mathrm{F}) = {} & f_q^{\overline{\mathrm{MS}}}(x,\mu_\mathrm{F}) \\
& - \frac{\alpha_s(\mu_\mathrm{F})}{2\pi}\,\frac{3}{2} C_\mathrm{F}\, f_q^{\overline{\mathrm{MS}}}(x,\mu_\mathrm{F}) \\
& + \frac{\alpha_s(\mu_\mathrm{F})}{2\pi}\, C_\mathrm{F}\left[\int_x^1 \frac{\mathrm{d}z}{z} f_q^{\overline{\mathrm{MS}}}\left(\frac{x}{z},\mu_\mathrm{F}\right)\ \left[\frac{1+z^2}{1-z}\log\frac{(1-z)^2}{z}+1-z\right]_+\right] \\
& + \frac{\alpha_s(\mu_\mathrm{F})}{2\pi}\, C_\mathrm{A}\left[\int_x^1 \frac{\mathrm{d}z}{z} f_g^{\overline{\mathrm{MS}}}\left(\frac{x}{z},\mu_\mathrm{F}\right)\left[z^2+(1-z)^2\right]\log\frac{(1-z)^2}{z}+2z(1-z)\right] \\
f_g^{\mathrm{Krk}}(x,\mu_\mathrm{F}) = {} & f_g^{\overline{\mathrm{MS}}}(x,\mu_\mathrm{F}) \\
& - \frac{\alpha_s(\mu_\mathrm{F})}{2\pi}\, C_\mathrm{A}\left[\frac{\pi^2}{3}+\frac{341}{72}-\frac{59}{36}\frac{N_f T_\mathrm{R}}{C_\mathrm{A}}\right]\ f_g^{\overline{\mathrm{MS}}}(x,\mu_\mathrm{F}) \\
& + \frac{\alpha_s(\mu_\mathrm{F})}{2\pi}\, C_\mathrm{A}\left[\int_x^1 \frac{\mathrm{d}z}{z} f_g^{\overline{\mathrm{MS}}}\left(\frac{x}{z},\mu_\mathrm{F}\right)\left[4\left[\frac{\log(1-z)}{1-z}\right]_+ - 2\frac{\log z}{1-z}\right.\right. \\
& \qquad\qquad \left.\left. +2\left(\frac{1}{z}-2+z(1-z)\right)\ln\frac{(1-z)^2}{z}\right]\right] \\
& + \frac{\alpha_s(\mu_\mathrm{F})}{2\pi}\, C_\mathrm{F}\sum_{q_f,\bar{q}_f}\left[\int_x^1 \frac{\mathrm{d}z}{z} f_q^{\overline{\mathrm{MS}}}\left(\frac{x}{z},\mu_\mathrm{F}\right)\left[\frac{1+(1-z)^2}{z}\log\frac{(1-z)^2}{z}+z\right]\right]
\end{aligned}
$$

8 S. Jadach et al. "Parton distribution functions in Monte Carlo factorisation scheme". arXiv: 1606.00355 [hep-ph].

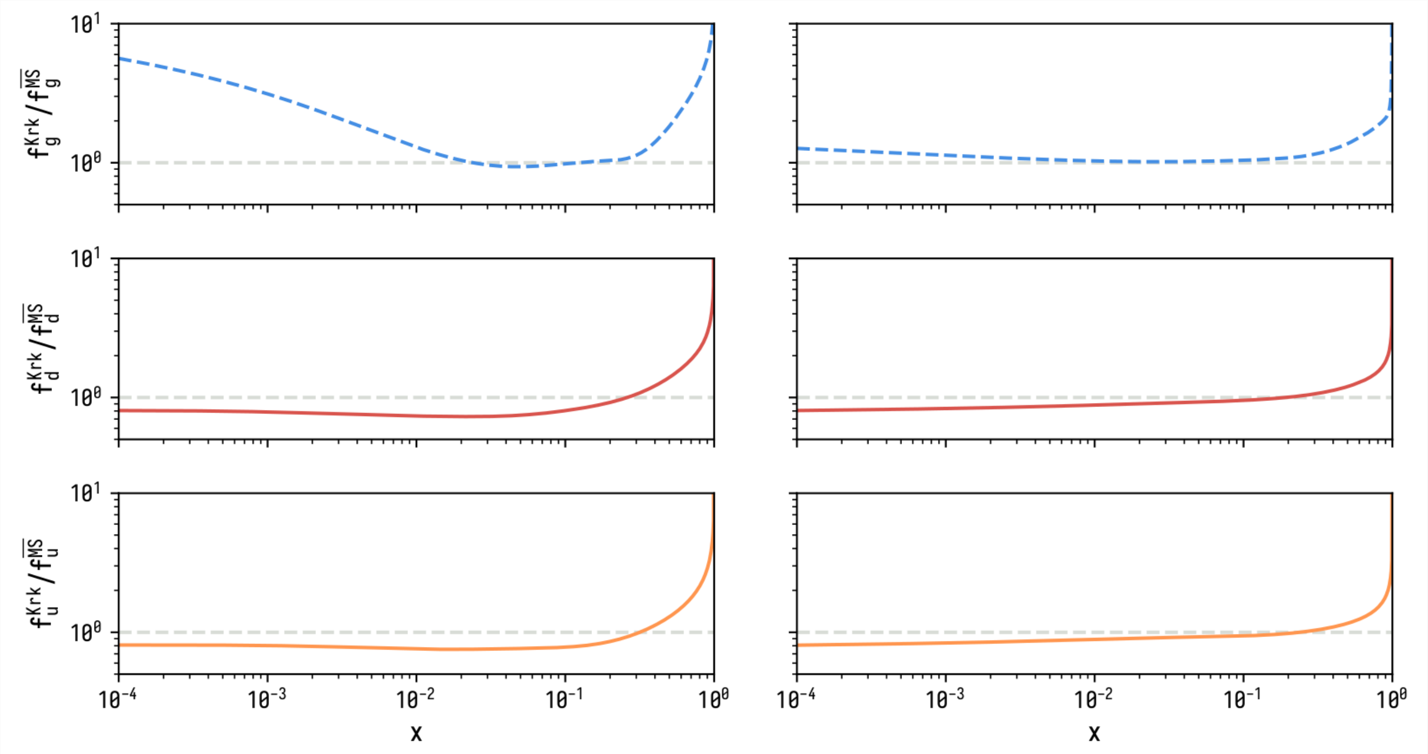
PDFs in MC scheme

Applied to LHAPDF6 grids:



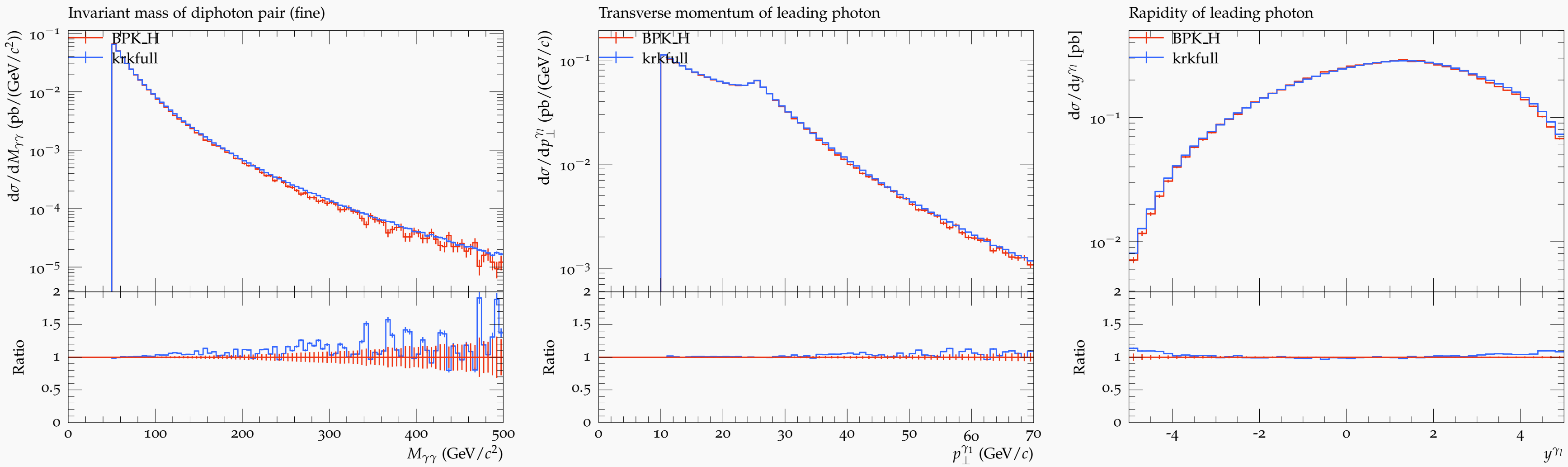
PDFs in MC scheme

Applied to LHAPDF6 grids:



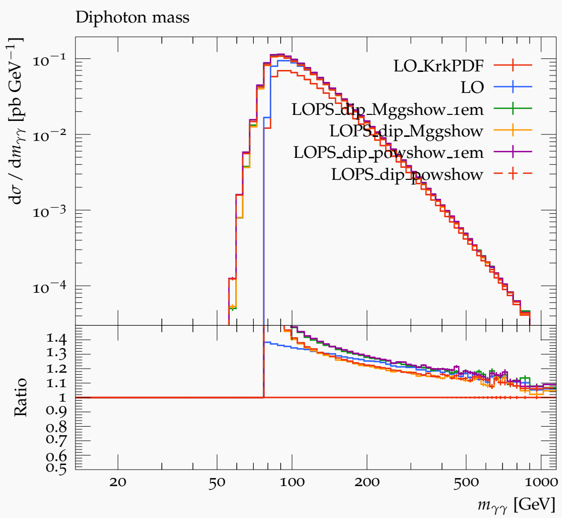
Validation

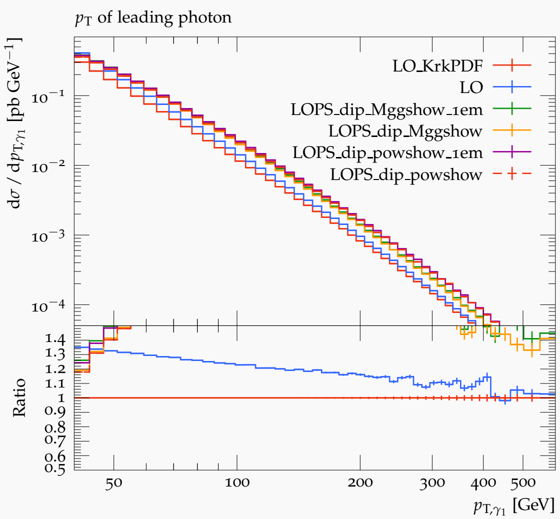
Do we reproduce the Herwig (Matchbox) automated P and K operators?

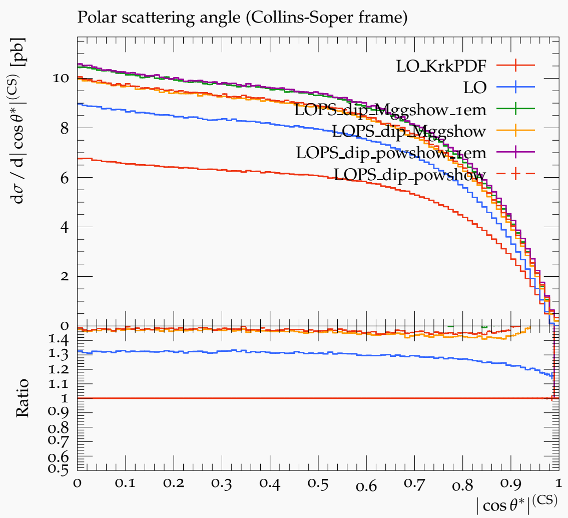


Normalisation

What is the numerical impact of the Krk scheme?







Validation

Seeing Sudakovs

To verify the real weight, we must *unweight* the Sudakov:

- numerical integration of dipole kernels considered in shower algorithm;
- over the same splitting phase-space/kinematic region used in the shower algorithm;
- with the same scales, PDF arguments, α_s etc

Seeing Sudakovs

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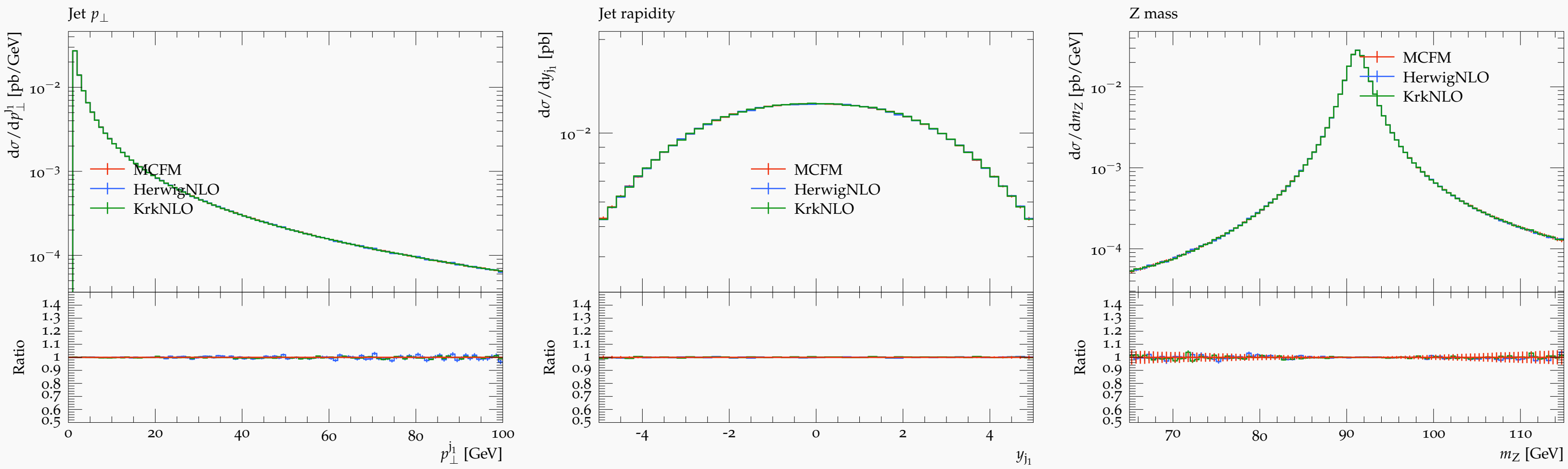
- with the same scales, PDF arguments, α_s etc

$$\Delta_{\mu_s}^{Q(\phi_m)} = \exp\left[-\sum_{\alpha}\int \mathrm{d}q(\phi_m)\ \Theta[\mu_s < \mu(q) < Q(\phi_m)]\ P_m^{(\alpha)}(q)\ \Theta_{\mathrm{PS}}^{(\alpha)}\right]$$

This is non-trivial!

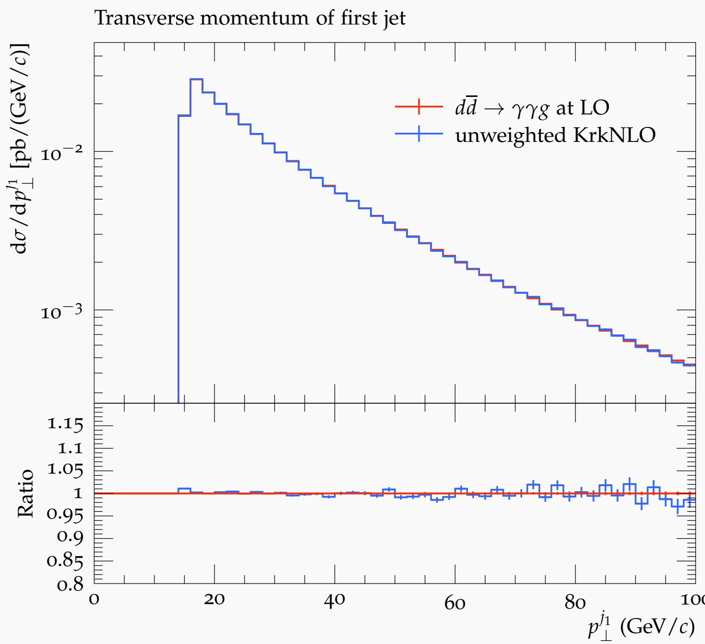
Does it work?

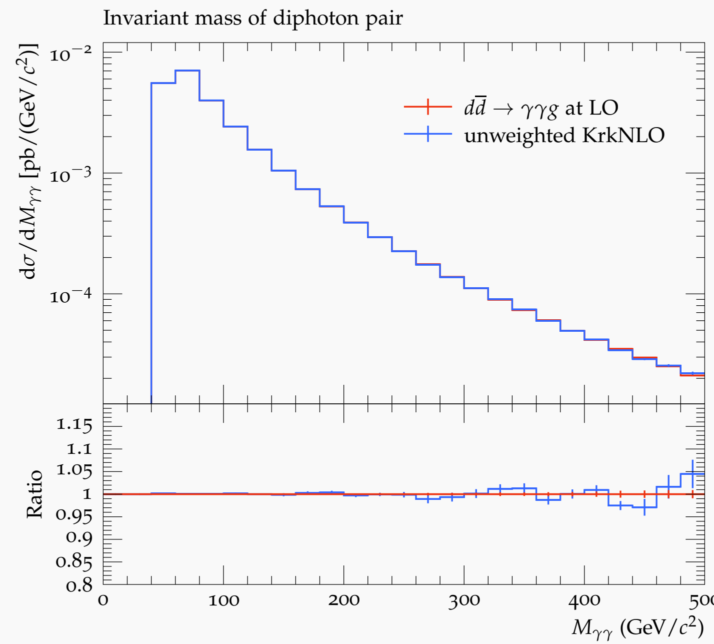
Drell-Yan:

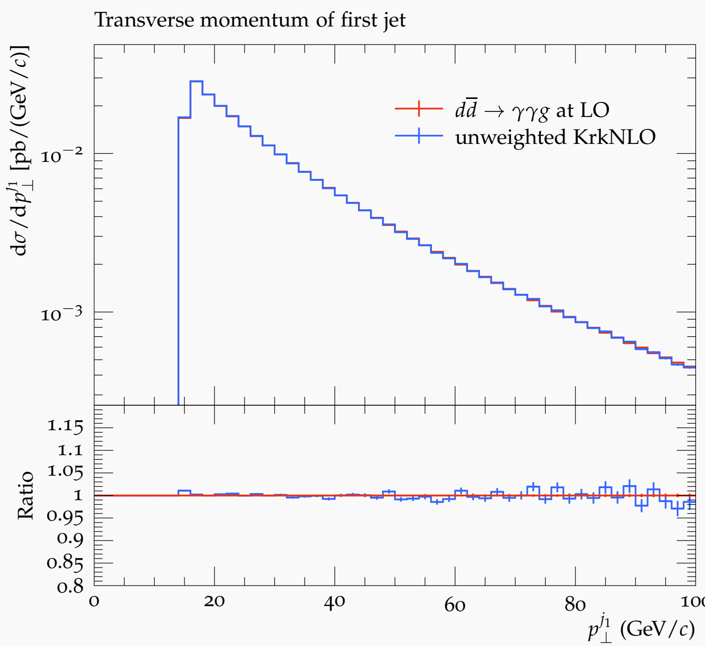


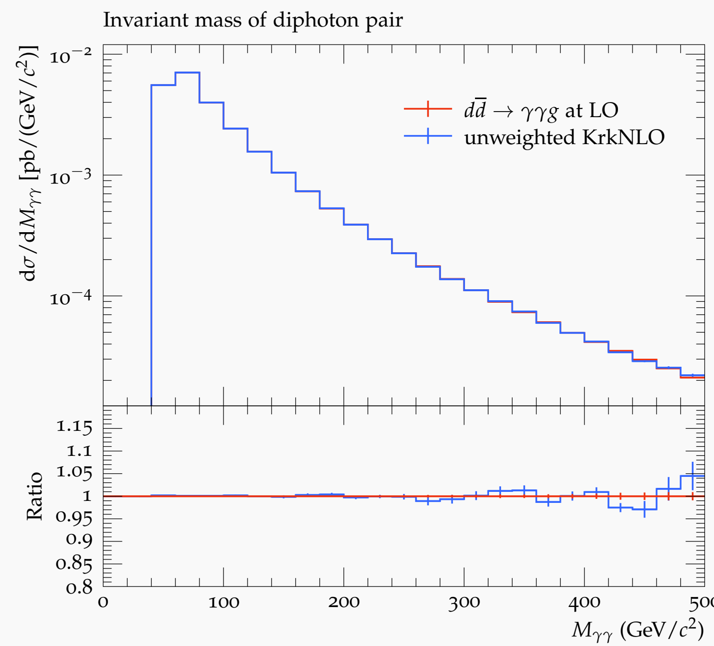
Does it work?

Diphoton:



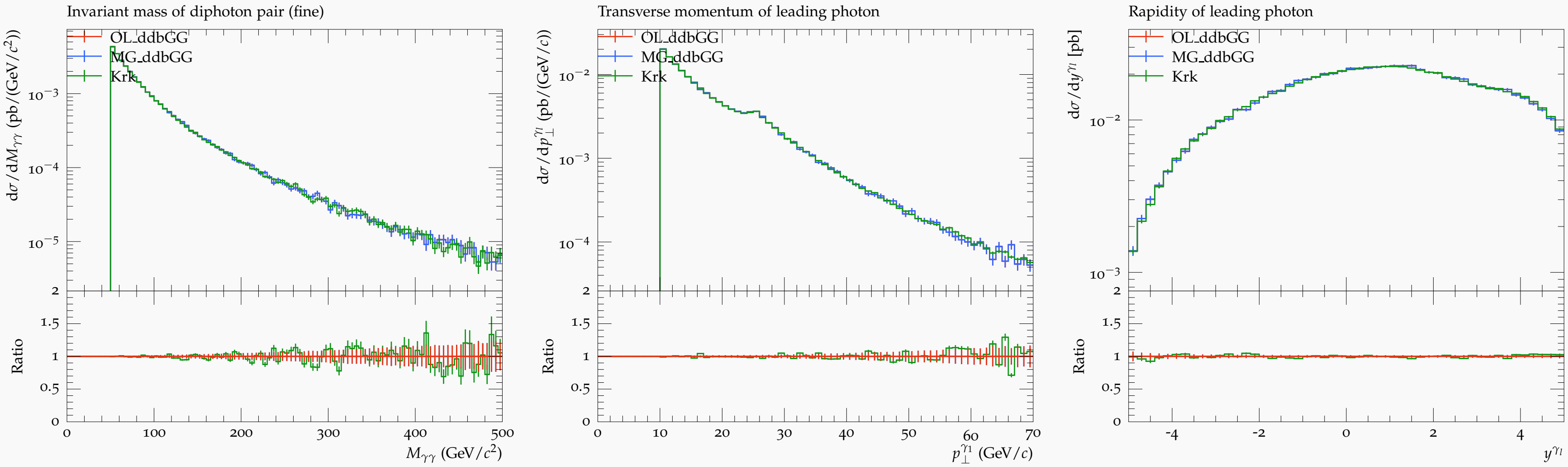






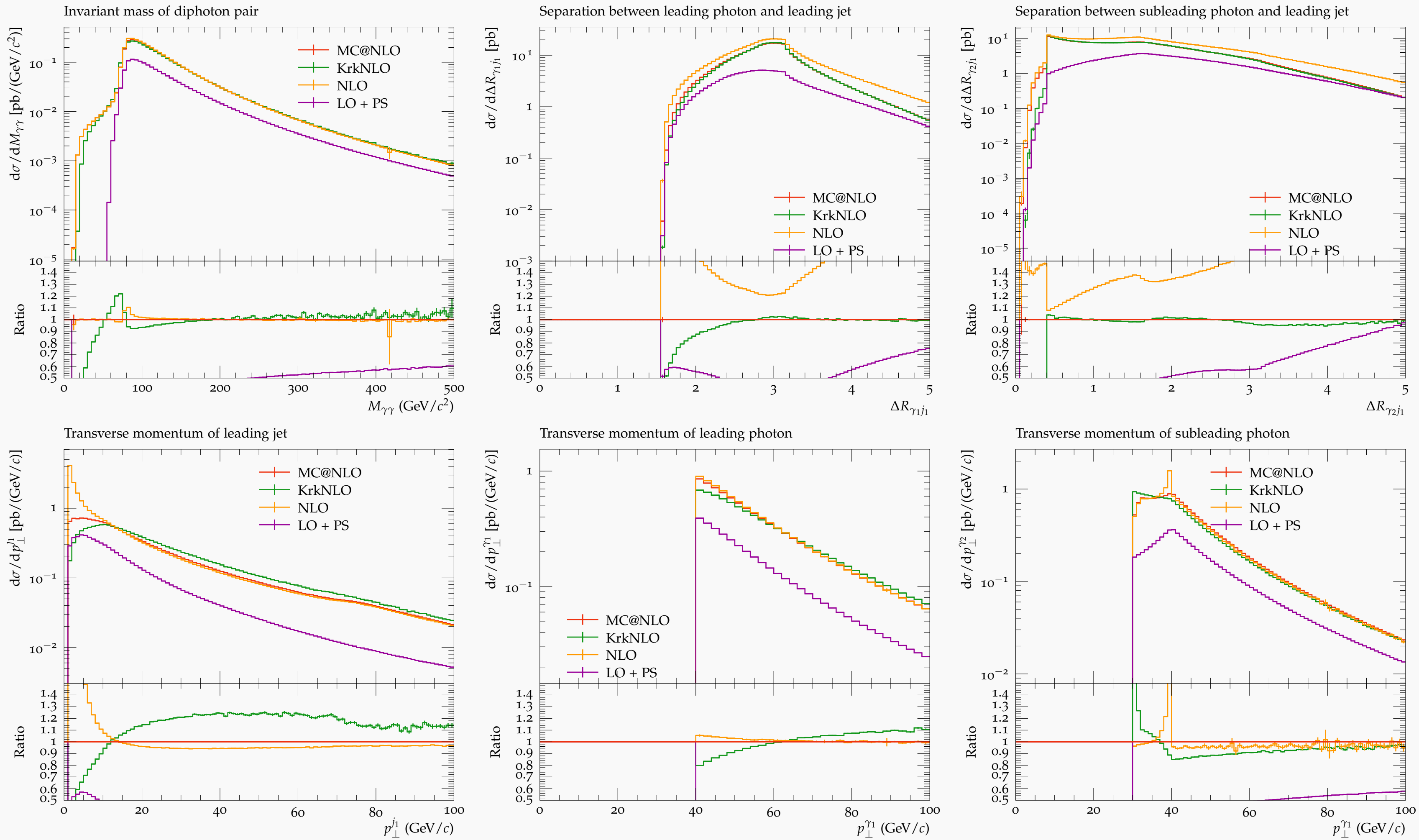
What about the virtuals?

Diphoton:

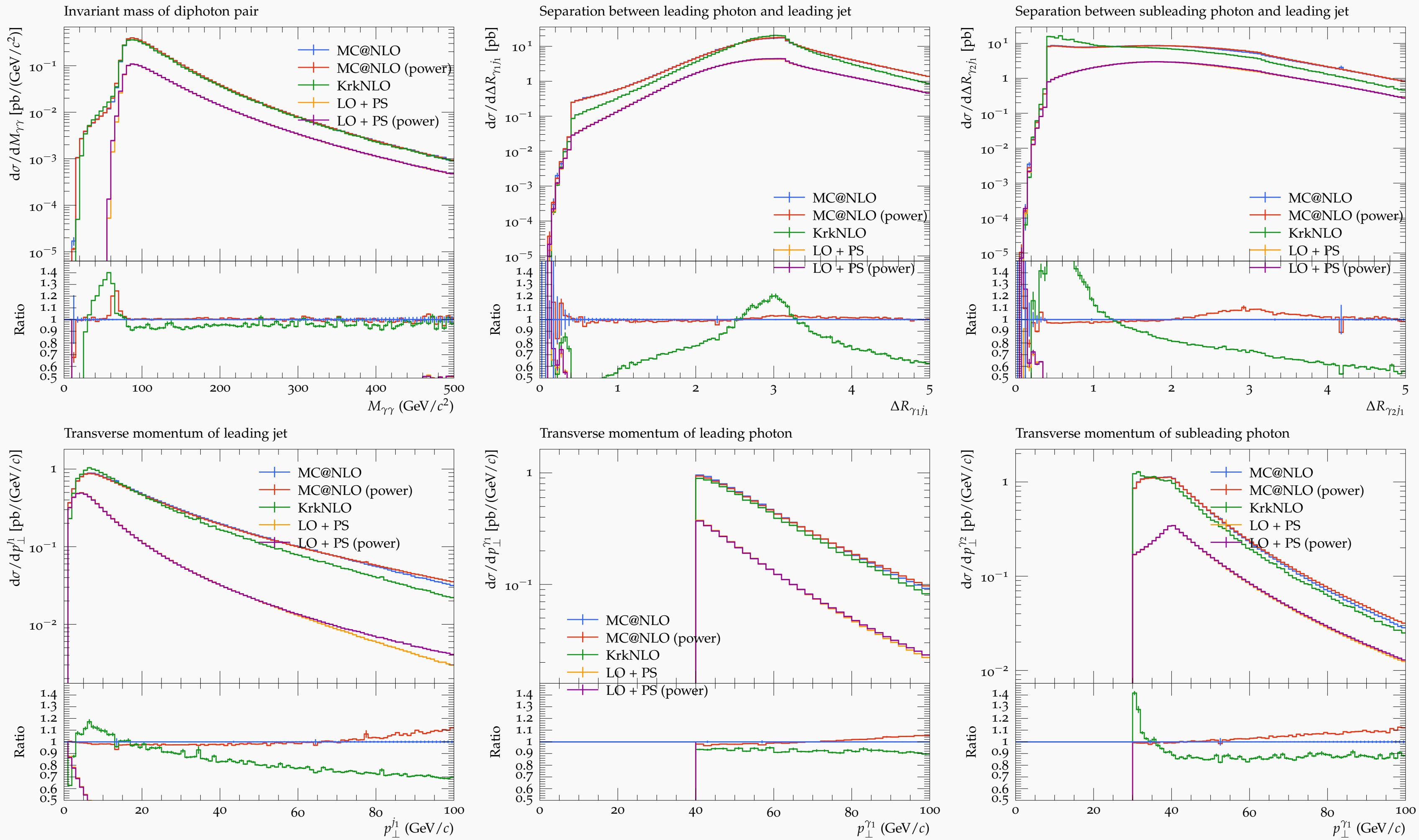


Results

Single emision

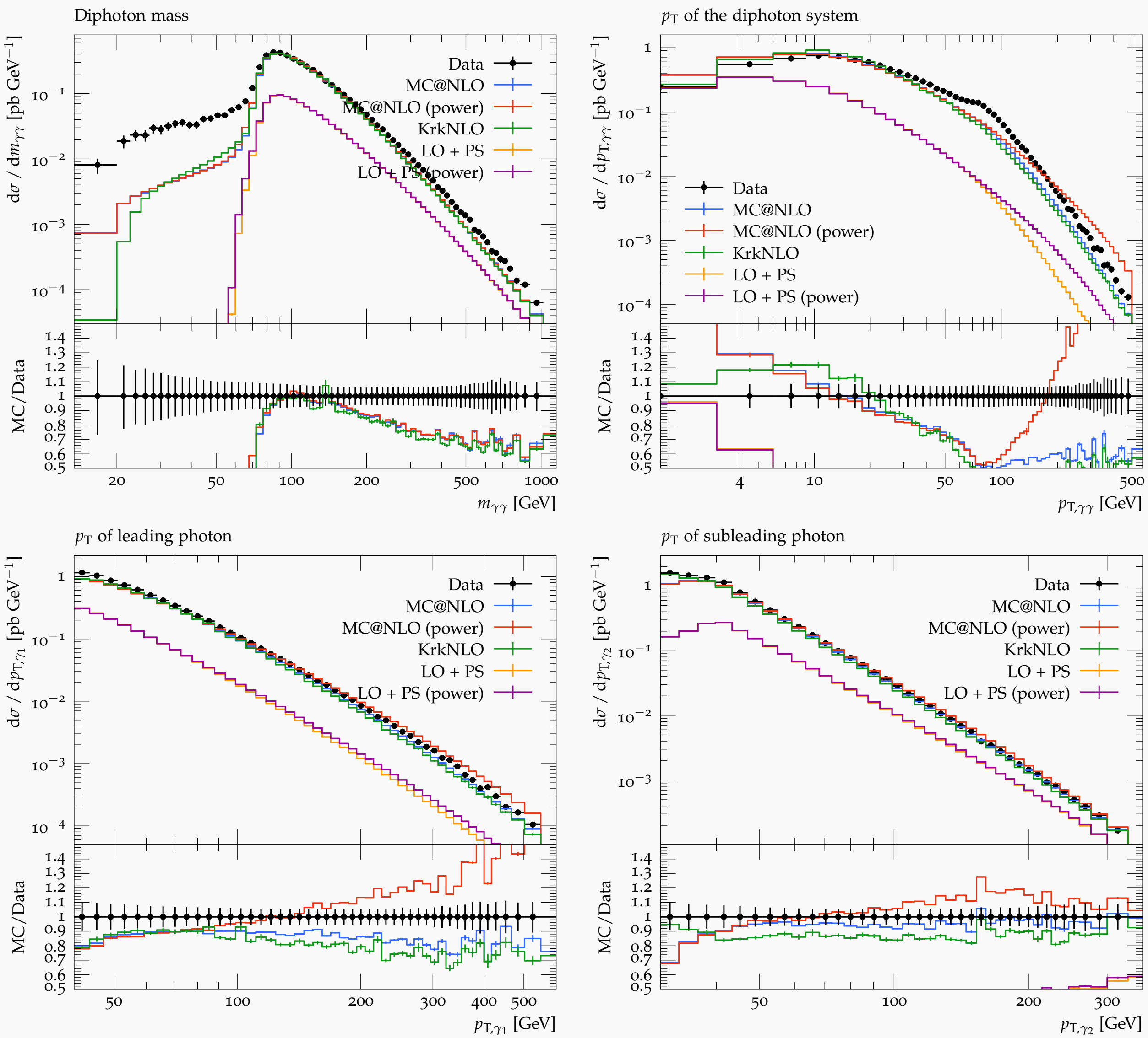


Full shower



Invariant mass of diphoton pair
MC@NLO
MC@NLO (power)
KrkNLO
LO + PS
LO + PS (power)
Ratio
$M_{\gamma\gamma}$ (GeV/c^2)
Separation between leading photon and leading jet
$\Delta R_{\gamma_1 j_1}$
Separation between subleading photon and leading jet
$\Delta R_{\gamma_2 j_1}$
Transverse momentum of leading jet
$p_\perp^{j_1}$ (GeV/c)
Transverse momentum of leading photon
$p_\perp^{\gamma_1}$ (GeV/c)
Transverse momentum of subleading photon
$p_\perp^{\gamma_1}$ (GeV/c)

Full shower: with data



Outlook

- more new processes in the pipeline
- PDF factorisation scheme[9]
- logs?
- automation!
- ...+jet?

[9] S. Jadach et al. "Parton distribution functions in Monte Carlo factorisation scheme". arXiv: 1606.00355 [hep-ph], S. Jadach. "On the universality of the KRK factorization scheme". arXiv: 2004.04239 [hep-ph].

Thank you!

Momentum mappings

require: four-momentum conservation & all particles remain on-shell

final-final

$$\tilde{p}_i = p_i + p_j - \frac{s_{ij}}{s_{ik} + s_{jk}} p_k \qquad \tilde{p}_k = \left(1 + \frac{s_{ij}}{s_{ik} + s_{jk}}\right) p_k$$

initial-final & final-initial

$$\tilde{p}_a = \left(1 - \frac{s_{jk}}{s_{aj} + s_{ak}}\right) p_a \qquad \tilde{p}_k = p_j + p_k - \frac{s_{jk}}{s_{aj} + s_{ak}} p_a$$

initial-initial

$$\tilde{p}_a = \left(1 - \frac{s_{aj} + s_{bj}}{s_{ab}}\right) p_a \qquad \tilde{p}_b = p_b$$

(in this case we further need to boost all FS particles)

Details of KrkNLO

Krk PDF scheme within KrkNLO

Krk PDFs compensate for the integrated shower radiation at $\mathcal{O}(\alpha_s)$ within the Sudakov factor. Schematically:

$$
\begin{aligned}
&\mathrm{d}\xi_1\,\mathrm{d}\xi_2 && \left\{\mathbf{f}^{\overline{\mathrm{MS}}} \otimes (\mathbb{I} + \mathbf{P} + \mathbf{K})\right\}_a \left\{\mathbf{f}^{\overline{\mathrm{MS}}} \otimes (\mathbb{I} + \mathbf{P} + \mathbf{K})\right\}_b \\
&\Bigg\{ \mathrm{d}\phi_m\, \Theta_{\mathrm{cut}}\left[\phi_m\right] && \Bigg[u(\phi_m)\, \mathrm{B}(\phi_m) \left\{ 1 + \frac{\mathrm{V}}{\mathrm{B}} + \sum_\alpha \mathrm{I}^{(\alpha)} - \mathrm{I}^{\mathrm{FS}}_{ab} \right\} \Delta_{\mu_s}^{Q_{\max}}(\phi_m) \\
& && + \sum_\alpha \mathrm{d}q^{(\alpha)}\; u(\Phi_{m+1}^{(\alpha)}) \left\{ \frac{\mathrm{R}}{\mathrm{PS}}\, \Theta_{\mathrm{PS}}^{(\alpha)}\left[\Phi_{m+1}^{(\alpha)}\right] \mathrm{PS}^{(\alpha)}\left[\Phi_{m+1}^{(\alpha)}\right] \Theta_{\mu_s}^{(\alpha)} \right\} \Bigg] \Bigg\}
\end{aligned}
$$

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\begin{aligned}
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&\Bigg\{\mathrm{d}\phi_m\,\Theta_{\text{cut}}\left[\phi_m\right] && \Bigg[u(\phi_m)\,\mathrm{B}(\phi_m)\left\{1+\frac{\mathrm{V}}{\mathrm{B}}+\sum_\alpha \mathrm{I}^{(\alpha)}-\mathrm{I}^{\text{FS}}_{ab}\right\}\Delta^{Q_{\max}(\phi_m)}_{\mu_s} \\
& && +\sum_\alpha \mathrm{d}q^{(\alpha)}\;u(\Phi^{(\alpha)}_{m+1})\left\{\frac{\mathrm{R}}{\mathrm{PS}}\,\Theta^{(\alpha)}_{\text{PS}}\left[\Phi^{(\alpha)}_{m+1}\right]\,\mathrm{PS}^{(\alpha)}\left[\Phi^{(\alpha)}_{m+1}\right]\,\Theta^{(\alpha)}_{\mu_s}\right\}\Bigg]\Bigg\}
\end{aligned}
$$

Additional convolutions define a PDF factorisation scheme: the ‘Krk scheme’.

Krk PDF transformation

Full details:[10]

Parton distribution functions in Monte Carlo factorisation scheme

S. Jadach[1], W. Płaczek[2], S. Sapeta[1,3], A. Siódmok[1,3,a], M. Skrzypek[1]

[1] Institute of Nuclear Physics, Polish Academy of Sciences, ul. Radzikowskiego 152, 31-342 Kraków, Poland
[2] Marian Smoluchowski Institute of Physics, Jagiellonian University, ul. Łojasiewicza 11, 30-348 Kraków, Poland
[3] Theoretical Physics Department, CERN, Geneva, Switzerland

Note additional imposition of sum rules:

10 S. Jadach et al. "Parton distribution functions in Monte Carlo factorisation scheme". arXiv: 1606.00355 [hep-ph].