Back-to-back DIS dijets at next-eikonal accuracy: from CGC to TMDs

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Context

The new Electron Ion Collider (EIC) at Brookhaven is expected to start in the next decade.

Among its main goals:

- Study of the 3D momentum distribution of partons inside protons or nuclei, using the TMD factorization formalism for semi-inclusive processes
- Study of the non-linear dynamics of partons with low momentum fraction inside protons or nuclei, using the CGC formalism for high energy (or low Bjorken x) processes.

$\Rightarrow$ *What about the consistency and interplay between these two very different approaches to QCD?*

Here: Study this issue on the example of back-to-back dijet production in DIS at low Bjorken x, in which both TMD and CGC formalisms should be valid.

TMD vs CGC approaches

For a process with a hard $\mathbf{P}$ and a not so hard $\mathbf{k}$ transverse momenta:

- TMD factorization: leading power (twist 2) in the limit $|\mathbf{k}| \ll |\mathbf{P}| \sim \sqrt{s}$
- CGC result: leading power (eikonal) in the limit $|\mathbf{k}| \sim |\mathbf{P}| \ll \sqrt{s}$

Consistency of both approaches shown in the double limit $|\mathbf{k}| \ll |\mathbf{P}| \ll \sqrt{s}$ at leading power (Dominguez, Marquet, Xiao, Yuan, 2011)

Power corrections in $|\mathbf{k}|/|\mathbf{P}|$ in the regime $|\mathbf{k}| \ll |\mathbf{P}| \ll \sqrt{s}$ studied from the CGC approach (Altinoluk, Boussarie, Kotko, 2019)

$\Rightarrow$ What about power corrections in $\mathbf{P}^2/s$ or $|\mathbf{P}||\mathbf{k}|/s$ beyond the eikonal limit?

Eikonal approximation in the CGC

High-energy dense-dilute scattering in the CGC : Semiclassical and Eikonal approx.

Dense target represented by a **strong semiclassical gluon field** $\mathcal{A}^\mu(x) \propto 1/g$
$\Rightarrow$ Perturbative expansion in g needs improvement by all order resummation of $(g\,\mathcal{A}^\mu(x))^n$

Eikonal approx. : limit of **infinite boost** of $\mathcal{A}^\mu(x)$ along x^-:

- $\mathcal{A}^\mu(x)$ independent on x^- (static limit) due to Lorentz time dilation
  $\Rightarrow$ No p^+ transfer from the target
- Lorentz contraction of $\mathcal{A}^\mu(x)$ (shockwave limit)
  $\Rightarrow$ Partons from the projectile interact instantly in x^+ with the target, without transverse motion within the target
- Under a boost of parameter γ_t along the "$-$" direction, $\mathcal{A}^-$ is enhanced and $\mathcal{A}^+$ is suppressed: $\mathcal{A}^- = O(\gamma_t) \gg \mathcal{A}_\perp = O(1) \gg \mathcal{A}^+ = O(1/\gamma_t)$

Background field in the eikonal limit: $\mathcal{A}^\mu(x^+, x^-, \mathbf{x}) \approx \delta^{\mu -} \mathcal{A}^-(x^+, \mathbf{x}) \propto \delta(x^+)$

$\Rightarrow$ Only $\big(g\mathcal{A}^-(x^+, \mathbf{x})\big)^n$ needs all orders resummation $\Rightarrow$ Wilson line along x^+

Next-to-Eikonal corrections to the CGC

Next-to-Eikonal (NEik) power corrections to the standard CGC formalism:

- Of order $1/\gamma_t$ at the level of the boosted background field
- Of order $1/s$ at the level of a cross section

$\rightarrow$ They arise from relaxing either of the 3 approximations:

1. x^- dependence of $\mathcal{A}^\mu(x)$ beyond infinite Lorentz dilation
   $\rightarrow$ Treated as gradient expansion around a common x^- value:
   $\frac{\partial_- \mathcal{A}^-(x)}{\mathcal{A}^-(x)} = O(1/\gamma_t)$
   $\Rightarrow$ Possibility of (small) p^+ exchange with the target
2. Target with finite width
   $\Rightarrow$ transverse motion of the projectile partons within the target
3. Interactions with $\mathcal{A}_\perp$ field taken into account, not only $\mathcal{A}^-$

Note: Background quark field of the target also relevant at NEik.
$\Rightarrow$ Separate contribution not included in this talk.

Full NEik quark propagator through a gluon background field

Propagator from y before the target to x after the target:

$$
\begin{aligned}
S_F(x,y) = & \int \frac{dq^+ d^2\mathbf{q}}{(2\pi)^3} \int \frac{dk^+ d^2\mathbf{k}}{(2\pi)^3}\; \theta(q^+)\,\theta(k^+)\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}\, \frac{(\check{\not{q}}+m)}{2q^+}\gamma^+ \\
& \times \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \bigg\{ \int dz^-\, e^{iz^-(q^+-k^+)}\, \mathcal{U}_F\Big(+\infty,-\infty;\mathbf{z},z^-\Big) \\
& -2\pi\delta(q^+-k^+)\, \frac{(\mathbf{q}^j+\mathbf{k}^j)}{2(q^++k^+)} \int dz^+\, \Big[\mathcal{U}_F\Big(+\infty,z^+;\mathbf{z},0\Big)\, \overleftrightarrow{\mathcal{D}_{\mathbf{z}^j}}\, \mathcal{U}_F\Big(z^+,-\infty;\mathbf{z},0\Big)\Big] \\
& -i\,\frac{2\pi\delta(q^+-k^+)}{(q^++k^+)} \int dz^+\, \Big[\mathcal{U}_F\Big(+\infty,z^+;\mathbf{z},0\Big)\, \overleftarrow{\mathcal{D}_{\mathbf{z}^j}}\, \overrightarrow{\mathcal{D}_{\mathbf{z}^j}}\, \mathcal{U}_F\Big(z^+,-\infty;\mathbf{z},0\Big)\Big] \\
& +\frac{2\pi\delta(q^+-k^+)}{(q^++k^+)}\, \frac{[\gamma^i,\gamma^j]}{4} \int dz^+\, \mathcal{U}_F\Big(+\infty,z^+;\mathbf{z},0\Big)\, gt\cdot\mathcal{F}_{ij}(z^+,\mathbf{z},0)\, \mathcal{U}_F\Big(z^+,-\infty;\mathbf{z},0\Big)\bigg\} \frac{(\check{\not{k}}+m)}{2k^+} \\
& +\text{NNEik}
\end{aligned}
$$

Altinoluk, G.B, Czajka, Tymowska (2021); Altinoluk, G.B (2022)

$$
\mathcal{U}_F(x^+,y^+;\mathbf{z},z^-) \equiv \mathbf{1} + \sum_{N=1}^{+\infty} \frac{1}{N!}\, \mathcal{P}_+ \left[-ig \int_{y^+}^{x^+} dz^+\, t\cdot\mathcal{A}^-(z)\right]^N
$$

- Generalized Eikonal contribution: also includes the NEik non-static corrections: overall z^- dependence of the Wilson line.

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\times & \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \bigg\{ \int dz^-\, e^{iz^-(q^+-k^+)}\, \mathcal{U}_F\Big(+\infty,-\infty;\mathbf{z},z^-\Big) \\
& -2\pi\delta(q^+-k^+)\, \frac{(\mathbf{q}^j+\mathbf{k}^j)}{2(q^++k^+)} \int dz^+ \Big[\mathcal{U}_F\Big(+\infty,z^+;\mathbf{z},0\Big) \overleftrightarrow{\mathcal{D}_{\mathbf{z}^j}}\, \mathcal{U}_F\Big(z^+,-\infty;\mathbf{z},0\Big)\Big] \\
& -i\, \frac{2\pi\delta(q^+-k^+)}{(q^++k^+)} \int dz^+ \Big[\mathcal{U}_F\Big(+\infty,z^+;\mathbf{z},0\Big) \overleftarrow{\mathcal{D}_{\mathbf{z}^j}}\, \overrightarrow{\mathcal{D}_{\mathbf{z}^j}}\, \mathcal{U}_F\Big(z^+,-\infty;\mathbf{z},0\Big)\Big] \\
& +\frac{2\pi\delta(q^+-k^+)}{(q^++k^+)}\, \frac{[\gamma^i,\gamma^j]}{4} \int dz^+\, \mathcal{U}_F\Big(+\infty,z^+;\mathbf{z},0\Big)\, gt\cdot\mathcal{F}_{ij}(z^+,\mathbf{z},0)\, \mathcal{U}_F\Big(z^+,-\infty;\mathbf{z},0\Big) \bigg\} \frac{(\check{\not{k}}+m)}{2k^+} \\
& +\text{NNEik}
\end{aligned}
$$

Altinoluk, G.B, Czajka, Tymowska (2021); Altinoluk, G.B (2022)

$$
\mathcal{U}_F(x^+,y^+;\mathbf{z},z^-) \;\equiv\; \mathbf{1} + \sum_{N=1}^{+\infty} \frac{1}{N!}\, \mathcal{P}_+ \left[-ig\int_{y^+}^{x^+} dz^+\, t\cdot\mathcal{A}^-(z)\right]^N
$$

- NEik contributions beyond the shockwave approx or due to $\mathcal{A}_\perp$.
  Last term: quark helicity coupling with longitudinal chromoelectric field of the target $\mathcal{F}_{ij}$.

Full NEik quark propagator through a gluon background field

Compact notations for the decorated Wilson lines:

$$\mathcal{U}_{F;j}^{(1)}(\mathbf{z}) = \int dz^+\, \mathcal{U}_F\Big(+\infty, z^+; \mathbf{z}\Big) \overleftrightarrow{\mathcal{D}_{\mathbf{z}^j}} \mathcal{U}_F\Big(z^+, -\infty; \mathbf{z}\Big)$$

$$\mathcal{U}_{F}^{(2)}(\mathbf{z}) = \int dz^+\, \mathcal{U}_F\Big(+\infty, z^+; \mathbf{z}\Big) \overleftarrow{\mathcal{D}_{\mathbf{z}^j}}\, \overrightarrow{\mathcal{D}_{\mathbf{z}^j}} \mathcal{U}_F\Big(z^+, -\infty; \mathbf{z}\Big)$$

$$\mathcal{U}_{F;ij}^{(3)}(\mathbf{z}) = \int dz^+\, \mathcal{U}_F\Big(+\infty, z^+; \mathbf{z}\Big) gt\cdot\mathcal{F}_{ij}(z^+, \mathbf{z}) \mathcal{U}_F\Big(z^+, -\infty; \mathbf{z}\Big)$$

Propagator from y before the target to x after the target:

$$\begin{aligned} S_F(x,y) \;=\; & \int \frac{dq^+ d^2\mathbf{q}}{(2\pi)^3} \int \frac{dk^+ d^2\mathbf{k}}{(2\pi)^3}\; \theta(q^+)\,\theta(k^+)\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}\, \frac{(\check{\not{q}}+m)}{2q^+}\gamma^+ \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \\ & \times \Bigg\{ \int dz^-\, e^{iz^-(q^+-k^+)}\, \mathcal{U}_F\Big(\mathbf{z}, z^-\Big) + 2\pi\delta(q^+-k^+) \Bigg[-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{2(q^++k^+)}\, \mathcal{U}_{F;j}^{(1)}(\mathbf{z}) \\ & \qquad - \frac{i}{(q^++k^+)}\, \mathcal{U}_F^{(2)}(\mathbf{z}) + \frac{[\gamma^i, \gamma^j]}{4(q^++k^+)}\, \mathcal{U}_{F;ij}^{(3)}(\mathbf{z}) \Bigg] \Bigg\} \frac{(\check{\not{k}}+m)}{2k^+} + \text{NNEik} \end{aligned}$$

Full NEik quark propagator through a gluon background field

Compact notations for the decorated Wilson lines:

$$\mathcal{U}^{(1)}_{F;j}(\mathbf{z}) = \int dz^+\, \mathcal{U}_F\Big(+\infty, z^+; \mathbf{z}\Big)\overleftrightarrow{\mathcal{D}_{\mathbf{z}j}}\mathcal{U}_F\Big(z^+, -\infty; \mathbf{z}\Big)$$

$$\mathcal{U}^{(2)}_{F}(\mathbf{z}) = \int dz^+\, \mathcal{U}_F\Big(+\infty, z^+; \mathbf{z}\Big)\overleftarrow{\mathcal{D}_{\mathbf{z}j}}\;\overrightarrow{\mathcal{D}_{\mathbf{z}j}}\mathcal{U}_F\Big(z^+, -\infty; \mathbf{z}\Big)$$

$$\mathcal{U}^{(3)}_{F;ij}(\mathbf{z}) = \int dz^+\, \mathcal{U}_F\Big(+\infty, z^+; \mathbf{z}\Big)gt\cdot\mathcal{F}_{ij}(z^+, \mathbf{z})\mathcal{U}_F\Big(z^+, -\infty; \mathbf{z}\Big)$$

Alternative expressions for the decorated Wilson lines:

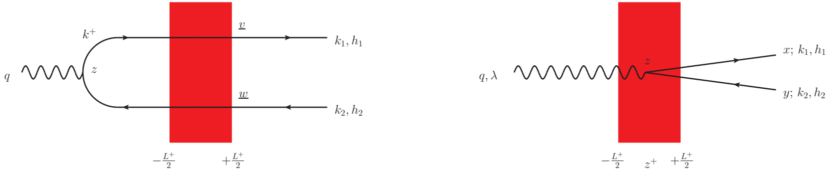
$$\mathcal{U}^{(1)}_{F;j}(\mathbf{z}) = -2\int dz^+\, z^+\, \mathcal{U}_F(+\infty, z^+; \mathbf{z})[-igt\cdot \mathcal{F}_j^{\;-}(z^+, \mathbf{z})]\mathcal{U}_F(z'^+, -\infty; \mathbf{z})$$

$$\mathcal{U}^{(2)}_{F}(\mathbf{z}) = \int dz^+ \int dz'^+\, (z^+ - z'^+)\,\theta(z^+ - z'^+)\mathcal{U}_F(+\infty, z^+, \mathbf{z})[-igt\cdot \mathcal{F}_j^{\;-}(z^+, \mathbf{z})]$$
$$\times\; \mathcal{U}_F(z^+, z'^+; \mathbf{z})[-igt\cdot \mathcal{F}_j^{\;-}(z'^+, \mathbf{z})]\mathcal{U}_F(z'^+, -\infty; \mathbf{z})$$

Thanks to the relation:

$$\partial_\mu \mathcal{U}_F(x^+, y^+; \mathbf{z}, z^-) + igt\cdot\mathcal{A}_\mu(x^+, \mathbf{z}, z^-)\mathcal{U}_F(x^+, y^+; \mathbf{z}, z^-) - ig\mathcal{U}_F(x^+, y^+; \mathbf{z}, z^-)t\cdot\mathcal{A}_\mu(y^+, \mathbf{z}, z^-)$$
$$= -ig\int_{y^+}^{x^+} dz^+ \mathcal{U}_F(x^+, v^+; \mathbf{z}, z^-)t\cdot\mathcal{F}_\mu^{\;-}(z)\mathcal{U}_F(v^+, y^+; \mathbf{z}, z^-) \qquad \text{for } \mu \neq +$$

DIS dijet at NEik accuracy: S-matrix for γ_L^*



DIS dijet cross calculated at NEik accuracy, at LO in α_s in the CGC.
(Altinoluk, G.B., Czajka, Tymowska, (2023))

- Only longitudinal photon contribution will be discussed for simplicity
- Second diagram vanishes in γ_L^* case, but matters in γ_T^* case.

S-matrix at NEik accuracy: $S_{q_1\bar{q}_2\leftarrow\gamma_L^*} = S^{\text{Gen. Eik}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dyn. target}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dec. on } q}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dec. on } \bar{q}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}$

$$S^{\text{Gen. Eik}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} = -\,2Q\,\frac{ee_f}{2\pi}\,\bar{u}(1)\gamma^+ v(2)\,\frac{(q^+ + k_1^+ - k_2^+)(q^+ + k_2^+ - k_1^+)}{4(q^+)^2}\int_{\mathbf{v},\mathbf{w}} e^{-i\mathbf{v}\cdot\mathbf{k}_1}\,e^{-i\mathbf{w}\cdot\mathbf{k}_2}$$
$$\times\ \mathrm{K}_0\left(\hat{Q}\,|\mathbf{v}-\mathbf{w}|\right)\int db^-\ e^{ib^-(k_1^+ + k_2^+ - q^+)}\left[\mathcal{U}_F\left(\mathbf{v}, b^-\right)\mathcal{U}_F^\dagger\left(\mathbf{w}, b^-\right) - 1\right]$$

$$\hat{Q}^2 = m^2 + \frac{(q^+ + k_1^+ - k_2^+)(q^+ - k_1^+ + k_2^+)}{4(q^+)^2}Q^2\,.$$

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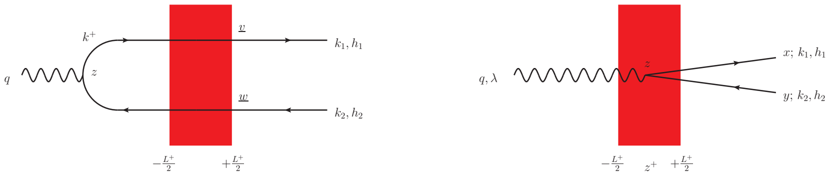

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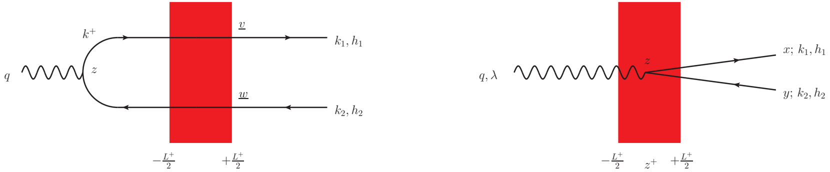
$$S^{\text{dyn. target}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\; iQ\,\frac{ee_f}{2\pi}\,\bar{u}(1)\gamma^+ v(2)\,\frac{(k_1^+ - k_2^+)}{(q^+)^2}\int d^2\mathbf{v}\, e^{-i\mathbf{v}\cdot\mathbf{k}_1}\int d^2\mathbf{w}\, e^{-i\mathbf{w}\cdot\mathbf{k}_2}$$
$$\times\left[\mathrm{K}_0\left(\bar{Q}\,|\mathbf{v}-\mathbf{w}|\right) - \frac{\left(\bar{Q}^2 - m^2\right)}{2\bar{Q}}\,|\mathbf{v}-\mathbf{w}|\,\mathrm{K}_1\left(\bar{Q}\,|\mathbf{v}-\mathbf{w}|\right)\right]\left[\mathcal{U}_F\left(\mathbf{v}, b^-\right)\overleftrightarrow{\partial_{b^-}}\,\mathcal{U}_F^\dagger\left(\mathbf{w}, b^-\right)\right]\Big|_{b^-=0}$$

$$\bar{Q}^2 = m^2 + Q^2\,\frac{k_1^+ k_2^+}{(q^+)^2}$$

DIS dijet at NEik accuracy: S-matrix for γ_L^*



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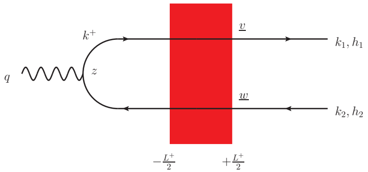
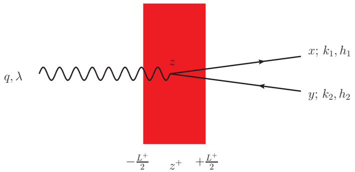
$$S^{\text{dec. on } q}_{q_1\bar{q}_2\leftarrow\gamma_L^*} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\,\frac{ee_f}{2\pi}\,(-1)Q\,\frac{k_2^+}{(q^+)^2}\int d^2\mathbf{v}\, e^{-i\mathbf{v}\cdot\mathbf{k}_1}\int d^2\mathbf{w}\, e^{-i\mathbf{w}\cdot\mathbf{k}_2}\,\mathrm{K}_0\left(\bar{Q}\,|\mathbf{v}-\mathbf{w}|\right)$$
$$\times\ \bar{u}(1)\gamma^+\left[\frac{[\gamma^i,\gamma^j]}{4}\,\mathcal{U}^{(3)}_{F;ij}(\mathbf{v}) - i\,\mathcal{U}^{(2)}_F(\mathbf{v}) + \mathcal{U}^{(1)}_{F;j}(\mathbf{v})\left(\frac{(\mathbf{k}_2^j - \mathbf{k}_1^j)}{2} + \frac{i}{2}\,\partial_{\mathbf{w}^j}\right)\right]\mathcal{U}_F^\dagger(\mathbf{w})\,v(2)$$

DIS dijet at NEik accuracy: S-matrix for γ_L^*





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$$\times\ \bar{u}(1)\gamma^+ \left[\mathcal{U}_F(\mathbf{v}) \left(\frac{[\gamma^i,\gamma^j]}{4}\, \mathcal{U}^{(3)\dagger}_{F;ij}(\mathbf{w}) - i\,\mathcal{U}^{(2)\dagger}_F(\mathbf{w}) + \left(\frac{i}{2}\, \overleftarrow{\partial_{\mathbf{v}^j}} - \frac{(\mathbf{k}_2^j - \mathbf{k}_1^j)}{2}\right)\mathcal{U}^{(1)\dagger}_{F;j}(\mathbf{w})\right)\right] v(2)$$

Change of variables and back-to-back limit

Back-to-back limit of dijets are conveniently expressed in terms of:

(dijet momentum imbalance) $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$ and (relative momentum) $\mathbf{P} = (z_2\mathbf{k}_1 - z_1\mathbf{k}_2)$

$z_1 = k_1^+/(k_1^+ + k_2^+)$ and $z_2 = k_2^+/(k_1^+ + k_2^+) = 1 - z_1$ such that

$$\boxed{\mathbf{k}_1 = \mathbf{P} + z_1\mathbf{k}} \qquad \boxed{\mathbf{k}_2 = -\mathbf{P} + z_2\mathbf{k}}$$

back-to-back correlation limit: $|\mathbf{k}| \ll |\mathbf{P}|$

In coordinate space:

(conjugate to $\mathbf{k}$) $\mathbf{b} = (z_1\mathbf{v} + z_2\mathbf{w})$ and (conjugate to P) $\mathbf{r} = \mathbf{v} - \mathbf{w}$

such that

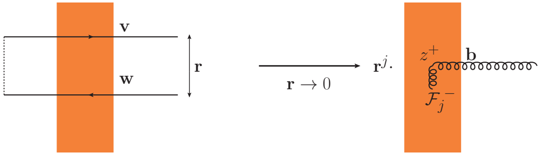
$$\boxed{\mathbf{v} = \mathbf{b} + z_2\,\mathbf{r}} \qquad \boxed{\mathbf{w} = \mathbf{b} - z_1\,\mathbf{r}}$$

back-to-back correlation limit: $|\mathbf{r}| \ll |\mathbf{b}|$

Small $\mathbf{r}$ expansion for the eikonal contribution (1)

Open dipole from the Generalized Eikonal term for $\mathbf{r} = \mathbf{v} - \mathbf{w} \to 0$:

$$\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big[\mathcal{U}_F\Big(\mathbf{b} + z_2\,\mathbf{r}, b^-\Big)\mathcal{U}_F^\dagger\Big(\mathbf{b} - z_1\,\mathbf{r}, b^-\Big) - 1\Big]$$
$$= \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big[z_2\mathbf{r}^j\left(\partial_j\mathcal{U}_F(\mathbf{b}, b^-)\right)\mathcal{U}_F^\dagger(\mathbf{b}, b^-) - z_1\mathbf{r}^j\mathcal{U}_F(\mathbf{b}, b^-)\left(\partial_j\mathcal{U}_F^\dagger(\mathbf{b}, b^-)\right) + O(\mathbf{r}^2)\Big]$$
$$= \mathbf{r}^j\, t^a \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \int_{z^+} \mathcal{U}_A\Big(+\infty, z^+; \mathbf{b}, b^-\Big)_{ab} (-ig)\mathcal{F}_j^{b\;-}(z^+, \mathbf{b}, b^-) + O(\mathbf{r}^2)$$



Note: 0th order in the $\mathbf{r}$ expansion trivial $\to$ first order in $\mathbf{r}$ is the leading power

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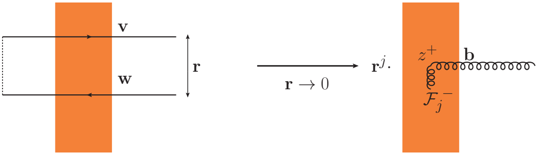
$$= \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \left[z_2\mathbf{r}^j\left(\partial_j\mathcal{U}_F(\mathbf{b}, b^-)\right)\mathcal{U}_F^\dagger(\mathbf{b}, b^-) - z_1\mathbf{r}^j\mathcal{U}_F(\mathbf{b}, b^-)\left(\partial_j\mathcal{U}_F^\dagger(\mathbf{b}, b^-)\right) + O(\mathbf{r}^2)\right]$$
$$= \mathbf{r}^j\, t^a \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \int_{z^+} \mathcal{U}_A\Big(+\infty, z^+; \mathbf{b}, b^-\Big)_{ab} (-ig)\mathcal{F}_j^{b\,-}(z^+, \mathbf{b}, b^-) + O(\mathbf{r}^2)$$



Note: 0th order in the $\mathbf{r}$ expansion trivial $\to$ first order in $\mathbf{r}$ is the leading power

However: the aim is to study the interplay between subleading power corrections
$\Rightarrow$ Terms of order $\mathbf{r}^2$ needed as well!

Small $\mathbf{r}$ expansion for the eikonal contribution (2)

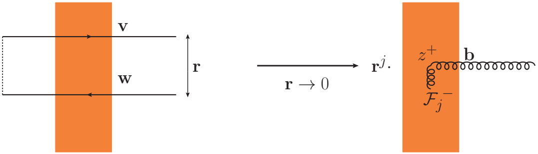
Open dipole from the Generalized Eikonal term for $\mathbf{r} = \mathbf{v} - \mathbf{w} \to 0$:

$$\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big[\mathcal{U}_F\Big(\mathbf{b}+z_2\,\mathbf{r}, b^-\Big)\mathcal{U}_F^\dagger\Big(\mathbf{b}-z_1\,\mathbf{r}, b^-\Big) - 1\Big]$$

$$= \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big[-i\Big(1+\frac{i(z_2-z_1)}{2}\mathbf{r}\cdot\mathbf{k}\Big)\mathbf{r}^j\, t^{a'} \int dv^+\, \mathcal{U}_A\left(+\infty, v^+; \mathbf{b}, b^-\right)_{a'a} g\mathcal{F}_j{}^-{}_a(v^+, \mathbf{b}, b^-)$$

$$-\frac{1}{2}\mathbf{r}^i\mathbf{r}^j\, t^{a'} t^{b'} \int dv^+ \int dw^+\, \mathcal{U}_A\left(+\infty, v^+; \mathbf{b}, b^-\right)_{a'a} g\mathcal{F}_i{}^-{}_a(v^+, \mathbf{b}, b^-)$$

$$\times \mathcal{U}_A\left(+\infty, w^+; \mathbf{b}, b^-\right)_{b'b} g\mathcal{F}_j{}^-{}_b(w^+, \mathbf{b}, b^-) \;+ O\left(|\mathbf{r}|^3\right) \Big]$$



$\Rightarrow$ Order $|\mathbf{r}|^2$ correction: contributions with either one or two field strength $\mathcal{F}_\perp^-$

Small $\mathbf{r}$ limit for the non-static NEik correction

For the open decorated dipole due to the dynamics of the target:

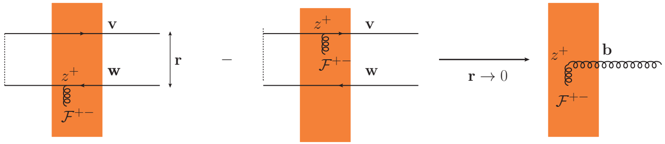
$$\left[\mathcal{U}_F\Big(\mathbf{b}+z_2\,\mathbf{r},b^-\Big)\overleftrightarrow{\partial_{b^-}}\mathcal{U}_F^\dagger\Big(\mathbf{b}-z_1\,\mathbf{r},b^-\Big)\right]\bigg|_{b^-=0}$$

$$= \int_{z^+}\Big\{\mathcal{U}_F(\mathbf{b})\mathcal{U}_F^\dagger\Big(z^+,-\infty;\mathbf{b}\Big)igt\cdot\mathcal{F}^{+-}(z^+,\mathbf{b})\mathcal{U}_F^\dagger\Big(+\infty,z^+;\mathbf{b}\Big)$$

$$-\mathcal{U}_F\Big(+\infty,z^+;\mathbf{b}\Big)(-ig)t\cdot\mathcal{F}^{+-}(z^+,\mathbf{b})\mathcal{U}_F\Big(z^+,-\infty;\mathbf{b}\Big)\mathcal{U}_F^\dagger(\mathbf{b})+O(|\mathbf{r}|)\Big\}$$

$$= 2it^{a'}\int_{z^+}\mathcal{U}_A\Big(+\infty,z^+;\mathbf{b}\Big)_{a'a}\,g\mathcal{F}_a^{+-}(z^+,\mathbf{b})+O(|\mathbf{r}|)$$

Involves the longitudinal chromoelectric field $\mathcal{F}^{+-}$ instead of the transverse field $\mathcal{F}_j^{\ -}$

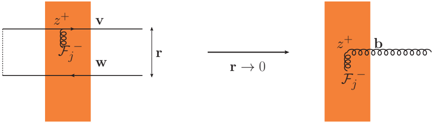


Note: Similar result for the NEik corrections with $\mathcal{U}^{(3)}_{F;ij}$, but with $\mathcal{F}_{ij}$ instead of $\mathcal{F}^{+-}$

Small $\mathbf{r}$ limit for the NEik corrections in $\mathcal{U}_{F;j}^{(1)}$ and $\mathcal{U}_F^{(2)}$

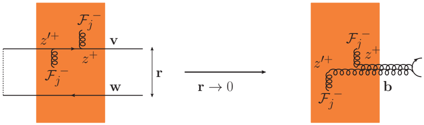
Terms with $\mathcal{U}_{F;j}^{(1)}$ and $\mathcal{U}_F^{(2)}$ decorating the quark line (remembering that $|\mathbf{r}| \sim 1/|\mathbf{P}|$):

$$\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big[-\Big(\mathbf{P}^j + \frac{(z_1-z_2)}{2}\mathbf{k}^j\Big) \mathcal{U}_{F;j}^{(1)}(\mathbf{b}+z_2\mathbf{r})\, \mathcal{U}_F^{\dagger}(\mathbf{b}-z_1\mathbf{r})$$
$$+ \frac{i}{2} \mathcal{U}_{F;j}^{(1)}(\mathbf{b}+z_2\mathbf{r})\, \partial_j \mathcal{U}_F^{\dagger}(\mathbf{b}-z_1\mathbf{r}) - i\, \mathcal{U}_F^{(2)}(\mathbf{b}+z_2\mathbf{r})\, \mathcal{U}_F^{\dagger}(\mathbf{b}-z_1\mathbf{r}) \Big]$$
$$= \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big\{ \Big[-\mathbf{P}^j + \frac{(z_2-z_1)}{2}\mathbf{k}^j - iz_2\mathbf{P}^j(\mathbf{r}\cdot\mathbf{k}) \Big] \mathcal{U}_{F;j}^{(1)}(\mathbf{b})\, \mathcal{U}_F^{\dagger}(\mathbf{b})$$
$$+ \Big[\frac{i}{2}\delta^{ij} + \mathbf{P}^j\mathbf{r}^i \Big] \mathcal{U}_{F;j}^{(1)}(\mathbf{b})\, \partial_i \mathcal{U}_F^{\dagger}(\mathbf{b}) - i\, \mathcal{U}_F^{(2)}(\mathbf{b})\, \mathcal{U}_F^{\dagger}(\mathbf{b}) + O(|\mathbf{r}|) \Big\}$$



$$\mathcal{U}_{F;j}^{(1)}(\mathbf{b})\mathcal{U}_F^{\dagger}\Big(\mathbf{b}\Big) = 2it^{a'} \int_{z^+} z^+\, \mathcal{U}_A\Big(+\infty, z^+; \mathbf{b}\Big)_{a'a}\, g\mathcal{F}_j^{a\,-}(z^+,\mathbf{b})$$

Small r limit for the NEik corrections in $\mathcal{U}^{(1)}_{F;j}$ and $\mathcal{U}^{(2)}_{F}$



$$\mathcal{U}_F^{(2)}(\mathbf{b})\mathcal{U}_F^{\dagger}(\mathbf{b}) = -t^{a'}t^{b'}\int_{z^+,z'^+}(z^+-z'^+)\,\theta(z^+-z'^+)\mathcal{U}_A\Big(+\infty,z^+;\mathbf{b}\Big)_{a'a}\,g\mathcal{F}_j^{a\;-}(z^+,\mathbf{b})$$
$$\times\,\mathcal{U}_A\Big(+\infty,z'^+;\mathbf{b}\Big)_{b'b}\,g\mathcal{F}_j^{b\;-}(z'^+,\mathbf{b})$$

$$\mathcal{U}_{F;j}^{(1)}(\mathbf{b})\,\partial_i\mathcal{U}_F^{\dagger}(\mathbf{b}) = -\,2t^{a'}t^{b'}\int dz^+\int dz'^+\,z^+\,\mathcal{U}_A(+\infty,z^+;\mathbf{b})_{a'a}\,g\mathcal{F}_j^{a\;-}(z^+,\mathbf{b})$$
$$\times\,\mathcal{U}_A(+\infty,z'^+;\mathbf{b})_{b'b}\,g\mathcal{F}_i^{b\;-}(z'^+,\mathbf{b})$$

Like in the GEik term: contributions with either 1 or 2 $\mathcal{F}_{\perp}^{\;-}$,
but now with an extra factor z^+ or $(z^+-z'^+)$: NEik suppression with the target width.

Similar results for decorations on the antiquark line instead.

Back-to-back cross section: (Generalized) Eikonal piece

Squaring the single $\mathcal{F}_\perp^-$ part of the Generalized Eikonal contribution in the back-to-back limit:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\Bigg|_{\mathrm{Gen.Eik}}^{\mathcal{F}_\perp^-\mathcal{F}_\perp^-} = g^2(ee_f)^2Q^2(q^+ + k_1^+ - k_2^+)^2(q^+ - k_1^+ + k_2^+)^2\frac{k_1^+k_2^+}{4(q^+)^6}\left[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\hat{Q}^2)^4} - 2(z_2-z_1)\frac{(\mathbf{P}^i\mathbf{k}^j+\mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\hat{Q}^2]^4}\right.$$
$$\left. + 16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\hat{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\right](2q^+)\int d(\Delta b^-)e^{i\Delta b^-(k_1^+ + k_2^+ - q^+)}\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}$$
$$\times\left\langle \mathcal{F}_i^{a\,-}\Big(z'^+,\mathbf{b}',-\frac{\Delta b^-}{2}\Big)\left[\mathcal{U}_A^\dagger\Big(+\infty,z'^+;\mathbf{b}',-\frac{\Delta b^-}{2}\Big)\mathcal{U}_A\Big(+\infty,z^+;\mathbf{b},\frac{\Delta b^-}{2}\Big)\right]_{ab}\mathcal{F}_j^{b\,-}\Big(z^+,\mathbf{b},\frac{\Delta b^-}{2}\Big)\right\rangle$$

Strict Eikonal result found by neglecting Δb^- in the fields:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\Bigg|_{\mathrm{Strict.Eik}}^{\mathcal{F}_\perp^-\mathcal{F}_\perp^-} = (2q^+)2\pi\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2g^24z_1^3z_2^3Q^2$$
$$\times\left[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4} - 2(z_2-z_1)\frac{(\mathbf{P}^i\mathbf{k}^j+\mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\bar{Q}^2]^4} + 16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\bar{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\right]$$
$$\times\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\left\langle \mathcal{F}_i^{a\,-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(+\infty,z'^+;\mathbf{b}')\mathcal{U}_A(+\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_j^{b\,-}(z^+,\mathbf{b})\right\rangle$$

- **Twist-2 gluon TMDs in the target** (both unpolarized and linearly polarized), with momentum fraction $\mathrm{x}=0$ and transverse momentum $\mathbf{k}$, with a *future staple* gauge link.
- **Kinematical twist 3 corrections**, suppressed by an extra $|\mathbf{k}|/|\mathbf{P}|$ in the back-to-back dijet limit $|\mathbf{k}| \ll |\mathbf{P}|$
- Not shown here: **Genuine twist 3 corrections**, involving a correlator of the type $\langle\mathcal{F}_\perp^-\mathcal{F}_\perp^-\mathcal{F}_\perp^-\rangle$
- Difference between Gen. Eik and strict Eik. : involves correlator $\langle\mathcal{F}_\perp^-\mathcal{F}_\perp^-\mathcal{F}^{+-}\rangle \Rightarrow$ twist 4 and NEik correction!

Back-to-back cross section: twist 3 TMDs from NEik

From the interference between the non-static NEik correction and the strict Eikonal amplitudes:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|_{NEik}^{\mathcal{F}_\perp^{-}\mathcal{F}^{+-}} = (2q^+)2\pi\delta(k_1^+ + k_2^+ - q^+)8Q^2e^2e_f^2g^2\,\frac{z_1^2z_2^2(z_2-z_1)}{q^+}\,\frac{\mathbf{P}^i(\mathbf{P}^2+m^2)}{(\mathbf{P}^2+\bar{Q}^2)^4}$$
$$\times\, 2\text{Re}\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\Big\langle \mathcal{F}_i^{a\;-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(\infty,z'^+;\mathbf{b}')\mathcal{U}_A(\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_b^{+-}(z^+,\mathbf{b})\Big\rangle$$

⇒ NEik. correction stemming from the dynamics of the target is a **twist-3 gluon TMD**, (Mulders, Rodrigues (2001)) with momentum fraction $\text{x} = 0$.

From the interference between the NEik correction with $\mathcal{U}_{F;ij}^{(3)}$ and the strict Eikonal amplitude:

- Vanishing result in the γ_L^* case due to Dirac algebra.
- An extra contribution to the cross section in the γ_T^* case:

$$\frac{d\sigma_{\gamma_T^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|_{NEik}^{\mathcal{F}_\perp^{-}\mathcal{F}_{ij}} \propto 2\text{Re}\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\Big\langle \mathcal{F}_l^{a\;-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(\infty,z'^+;\mathbf{b}')\mathcal{U}_A(\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_{ij}^{b}(z^+,\mathbf{b})\Big\rangle$$

⇒ The other **twist-3 gluon TMD** as found in Mulders, Rodrigues (2001), with momentum fraction $\text{x} = 0$.

Back-to-back cross section: x dependence from NEik

Including all contributions of the form $\langle \mathcal{F}_\perp^- \mathcal{F}_\perp^- \rangle$, of order Eik or NEik, and twist 2 or twist 3:

$$\frac{d\sigma_{\gamma_L^* \to q_1 \bar{q}_2}}{d\text{P.S.}}\Bigg|^{\mathcal{F}_\perp^- \mathcal{F}_\perp^-} = (2q^+) 2\pi \delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 4 z_1^3 z_2^3 Q^2$$
$$\times \left[\frac{4\mathbf{P}^i \mathbf{P}^j}{(\mathbf{P}^2 + \bar{Q}^2)^4} - 2(z_2 - z_1) \frac{(\mathbf{P}^i \mathbf{k}^j + \mathbf{k}^i \mathbf{P}^j)}{[\mathbf{P}^2 + \bar{Q}^2]^4} + 16(z_2 - z_1) \frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i \mathbf{P}^j}{[\mathbf{P}^2 + \bar{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right) \right]$$
$$\times \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+, z'^+} \left[1 + i(z^+ - z'^+) \frac{(\mathbf{P}^2 + \bar{Q}^2)}{2q^+ z_1 z_2} + NNEik \right] \Big\langle \mathcal{F}_i^{a\,-}(z'^+, \mathbf{b}') \left[\mathcal{U}_A^\dagger(+\infty, z'^+; \mathbf{b}') \mathcal{U}_A(+\infty, z^+; \mathbf{b}) \right]_{ab} \mathcal{F}_j^{b\,-}(z^+, \mathbf{b}) \Big\rangle$$

$\Rightarrow$ NEik corrections and kinematic twist 3 corrections to the $\langle \mathcal{F}_\perp^- \mathcal{F}_\perp^- \rangle$ contribution factorize from each other!

The " $-$ " momentum extracted from the target can be defined from the conservation relation (where the $\mathbf{k}^2$ term is a twist 4 correction):

$$\boxed{\text{x}P^-_{tar.} \equiv \check{k}_1^- + \check{k}_2^- - q^- = \frac{\mathbf{k}_1^2 + m^2}{2k_1^+} + \frac{\mathbf{k}_2^2 + m^2}{2k_2^+} + \frac{Q^2}{2q^+} = \frac{(\mathbf{P}^2 + \bar{Q}^2)}{2q^+ z_1 z_2} + \frac{\mathbf{k}^2}{2q^+}}$$

The NEik correction can be summed into a phase! $\Rightarrow$ dependence of the twist 2 gluon TMDs on x

$$\frac{d\sigma_{\gamma_L^* \to q_1 \bar{q}_2}}{d\text{P.S.}}\Bigg|^{\mathcal{F}_\perp^- \mathcal{F}_\perp^-} = (2q^+) 2\pi \delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 4 z_1^3 z_2^3 Q^2$$
$$\times \left[\frac{4\mathbf{P}^i \mathbf{P}^j}{(\mathbf{P}^2 + \bar{Q}^2)^4} - 2(z_2 - z_1) \frac{(\mathbf{P}^i \mathbf{k}^j + \mathbf{k}^i \mathbf{P}^j)}{[\mathbf{P}^2 + \bar{Q}^2]^4} + 16(z_2 - z_1) \frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i \mathbf{P}^j}{[\mathbf{P}^2 + \bar{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right) \right]$$
$$\times \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+, z'^+} e^{i(z^+ - z'^+)\text{x}P^-_{tar.}} \Big\langle \mathcal{F}_i^{a\,-}(z'^+, \mathbf{b}') \left[\mathcal{U}_A^\dagger(+\infty, z'^+; \mathbf{b}') \mathcal{U}_A(+\infty, z^+; \mathbf{b}) \right]_{ab} \mathcal{F}_j^{b\,-}(z^+, \mathbf{b}) \Big\rangle + NNEik$$

Summary

To further understand the interplay between CGC and TMD, we studied the NEik DIS dijet cross-section in the back-to-back jets limit, including twist 3 power corrections. Various types of contributions are obtained:

- $\langle \mathcal{F}_i^{\;-} \mathcal{F}_j^{\;-} \rangle$: twist 2 gluon TMDs
  - In that sector: factorization of kinematic twist 3 and of NEik correction
  - NEik correction reproduce the expansion of the phase defining the x dependence of the TMDs
- Twist 3 gluon TMDs: $\langle \mathcal{F}_i^{\;-} \mathcal{F}^{+-} \rangle$ and (for γ_T^*) $\langle \mathcal{F}_l^{\;-} \mathcal{F}_{ij} \rangle$, as further NEik corrections.
- 3-body twist 3 correlators $\langle \mathcal{F}_i^{\;-} \mathcal{F}_j^{\;-} \mathcal{F}_l^{\;-} \rangle$: beyond TMDs!
  Already appear in Eikonal contributions. NEik corrections partially resum into phase.

Remark: Odd (resp. Even) Twist terms typically proportional to Odd (resp. Even) powers of $\mathbf{P}\cdot\mathbf{k}$, for unpolarized target.

$\Rightarrow$ Odd (resp. Even) azimuthal harmonics in terms of $\mathbf{P}$ and $\mathbf{k}$