Single inclusive particle production in pA collisions at forward rapidities at NLO

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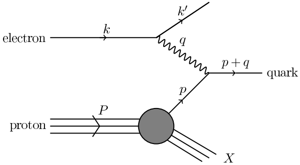
DIS in QCD :



Three Lorentz invariant quantities :

1. $q^2 = -Q^2 \equiv$ virtuality of the incoming photon
2. $x = \frac{Q^2}{2P \cdot Q} \equiv$ longitudinal momentum fraction carried by the parton
3. $s \simeq 2P \cdot Q \equiv$ energy of the colliding $\gamma - p$ system

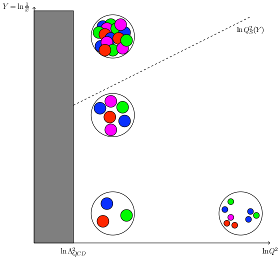
increasing the energy ($s = Q^2/x$) of the system:

Bjorken limit fixed x, $Q^2 \to \infty$

- density of partons decreases/DGLAP

Regge-Gribov limit fixed Q^2, $x \to 0$

- density of partons increases/Saturation!



- $Q_s \equiv$ saturation scale
- In the saturation regime, scattering processes are described by an effective theory: Color Glass Condensate:
  - fast partons : $k^+ > \Lambda^+$ $\to$ color sources: $J^\mu(x) = \delta^{\mu +}\rho(x^-, x_\perp)$
  - slow partons: : $k^+ < \Lambda^+$ $\to$ color fields $A^\mu(x)$
  - interaction: $\int d^4x J^\mu(x) A_\mu(x)$

Motivation to go from LO to NLO in the CGC

Leading Order in α_s CGC calculations:

↙ ↘

(pro) : CGC-based theoretical calculations are in qualitative agreement with the experimental data from all types of collisions

(con): LO CGC lacks precision in order to determine unambiguously whether saturation is exhibited by the experimental data.

increasing precision of theory predictions in order to perform precise quantitative studies:

⋆ relaxing the kinematical approximations performed at LO (see talk by Beuf)

⋆ going from LO to NLO in α_s:

There has been a lot activity to provide expressions of observables at NLO.

↙ ↘

eA collisions

- dipole factorization
- structure functions/ dijets

pA collisions

- hybrid factorization
- single inclusive hadron/jet

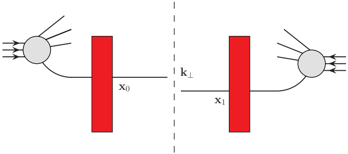
⋆ Many developments on the NLO corrections to the rapidity evolution equations (see talk by Korcyl)

Forward hadron production in pA collisions

[Dumitru, Hayashigaki, Jalilian-Marian - hep-ph/0506308]

Accepted calculation framework for forward production in pA collisions: Hybrid factorization

- The wave function of the projectile proton is treated in the spirit of collinear factorization (an assembly of partons with zero intrinsic transverse momenta) - DGLAP gives perturbative corrections.
- Target is treated as distribution of strong color fields which during the scattering event transfer transverse momentum to the propagating partonic configuration. (CGC like treatment)



$$\frac{d\sigma^{q\to H}}{d^2k\,d\eta} = \int_{x_F}^{1} \frac{d\zeta}{\zeta^2} D^q_{\mu_0^2}(\zeta) \frac{x_F}{\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right) \int e^{ik(x_0-x_1)} \langle s(x_0,x_1)\rangle$$

dipole operator: $s(x_0,x_1) = \frac{1}{N_c}\mathrm{tr}\Big[U(x_0)U^\dagger(x_1)\Big]$

high transverse momentum in the produced hadron is acquired from the interaction with the target.

Forward hadron production

Does LO "Hybrid" formula take into account all contributions at high $k_\perp$?

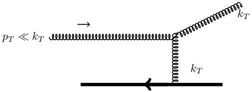
[TA, Kovner - arXiv:1102.5327]

For $k_\perp \gg Q_s$:

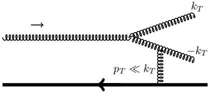
$$\frac{d\sigma}{d^2kd\eta} \propto \left[\frac{d\sigma}{d^2kd\eta}\right]_{el.} + \left[\frac{d\sigma}{d^2kd\eta}\right]_{inel.}$$

Real contributions at NLO.

"Elastic Scattering" (LO)



"Inelastic Scattering" (NLO)

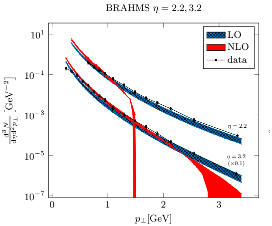


[Chirilli, Xiao, Yuan - arXiv:1112.1061 / arXiv:1203.6139] → Full NLO computation.

Collinear divergences: absorbed into DGLAP evolution of PDFs and FFs.

Rapidity divergences: absorbed into evolution of the target.

[Stasto, Xiao, Zaslavsky - arXiv:1307.4057] → Numerical studies of full NLO result.



cross sections turn out to be negative at large transverse momentum!

Several solutions proposed to fix the problem:

- kinematical constraints
- different choice of rapidity scales
- threshold/ Sudakov resummations

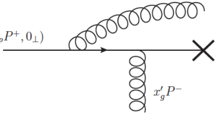
Revisiting NLO hybrid formula - kinematical constraints

[TA, Armesto, Beuf, Kovner, Lublinsky - arXiv:1411.2869]

• choice of frame:
(i) target moves fast and carries almost all the energy.
(ii) projectile moves fast enough: accommodates partons with momentum fraction x_p but does not develop a large low-x tail.
(iii) target is evolved to s from initial s_0 via BK.

• Ioffe time restriction: only pairs whose coherence time is greater than the propagation time through the target can be resolved.

$$\text{coherent scattering} \rightarrow \frac{(1-\xi)\xi x_p}{l_\perp^2} > \frac{1}{s_0}$$



New BK-like terms arise due to Ioffe time restriction.

[Watanabe, Xiao, Yuan, Zaslavsky - arXiv:1505.05183] → exact kinematical constraint.

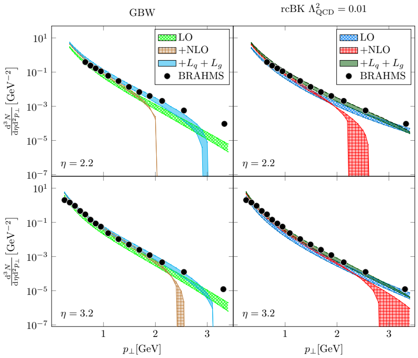
$$\int_0^{1-\frac{l_\perp^2}{x_p s}} \frac{d\xi}{(1-\xi)} = \ln\frac{1}{x_g} + \ln\frac{k_\perp^2}{l_\perp^2}$$

New terms $(L_q + L_g)$ arise from $l_\perp^2 < (1-\xi)k_\perp^2$

The new terms in both works are consistent and equivalent.

Revisiting NLO hybrid formula - kinematical constraints

[Watanabe, Xiao, Yuan, Zaslavsky - arXiv:1505.05183]



BRAHMS data with $\sqrt{s_{NN}} = 200$ *GeV.*

The negativity problem is shifted to higher transverse momentum but not cured!

[Xiao, Yuan - arXiv:1806.0352], [Shi, Wang, Wei, Xiao - arXiv:2112.06975]

A new proposal:

- resum soft logs → Sudakov resummation
- resum collinear logs → Threshold resummation (DGLAP of PDFs or FFs)

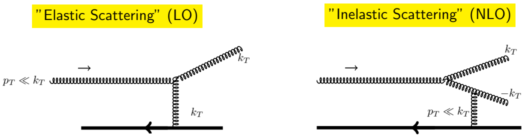
A new approach to forward pA scatterings

Common assumption in all these works: *large logs can be resummed within the collinear factorization.*

[TA, Armesto, Kovner, Lublinsky - arXiv: 2307.14922]

TMD-factorized framework is a natural choice to resum all large logs.

in [arXiv:1102.5327], the mechanisms that give rise to high transverse momentum hadrons:



– It is more natural to think the inelastic contribution in the TMD framework:
produced high k_T quark coming directly from quark TMD PDF.

⋆ another potential source to producing high transverse momentum hadron:

low k_T parton scatters softly, but subsequently fragments into a high transverse momentum hadron.

-*Hadron arising from TMD FF.*

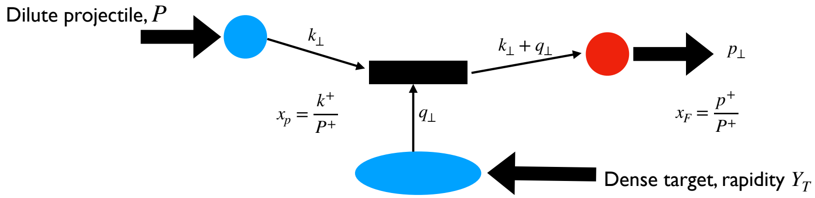
⋆ soft logs – we follow [arXiv:1411.2869]

The setup of the problem

[TA, Armesto, Kovner, Lublinsky - arXiv:2307.14922]

TMD-factorized parton model expression:

$$\frac{d\sigma^{LO+NLO}}{d^2p_\perp d\eta} \propto \int \frac{d\zeta}{\zeta^2} \int_{k_\perp q_\perp} \mathcal{T}(x_F/\zeta, k_\perp; \mu_T^2) P(k_\perp, q_\perp) \mathcal{F}(\zeta, p_\perp, (k_\perp + q_\perp); \mu_F^2) + \text{Gen. NLO}$$



$\mathcal{T}(x_F/\zeta, k_\perp; \mu_T^2) \to$ initial TMD PDF $\qquad$ $\mathcal{F}(\zeta, p_\perp, (k_\perp + q_\perp)) \to$ TMD FF

$P(k_\perp, q_\perp) \to$ differential probability to produce a parton with momentum $(k_\perp + q_\perp)$ from a parton with momentum $k_\perp$

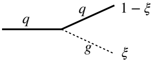
The factorization scales:

$$\mu_T^2 = \max\left\{k_\perp^2, q_\perp^2, Q_s^2\right\} \approx \max\left\{(k_\perp + q_\perp)^2, Q_s^2\right\}, \ \mu_F^2 = \left((q_\perp + k_\perp) - p_\perp/\zeta)^2\right) \approx \max\left\{(q_\perp + k_\perp)^2, (p_\perp/\zeta)^2\right\}$$

TMD distributions

- TMD PDFs are generated from the collinear ones (large k)

$$x\mathcal{T}_q(x,k^2,k^2;\xi_0) = \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{\xi_0}^{1} d\xi \frac{1+(1-\xi)^2}{\xi}\frac{x}{1-\xi} f^q_{k^2}\left(\frac{x}{1-\xi}\right)\frac{1}{k^2}$$



– The soft divergence of the gluon emission is regulated by the cut off ξ_0.
– Partons with high longitudinal momentum are produced from partons with lower longitudinal momentum by DGLAP splitting.
– Transverse resolution scale in these splittings is equal to the transverse momentum of the parton.

- Evolution of the TMDs

$$x\mathcal{T}_q(x,k^2;\mu^2;\xi_0) = \theta(\mu^2-k^2)\Big[x\mathcal{T}_q(x,k^2;k^2;\xi_0) - \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{k^2}^{\mu^2}\frac{\pi dl^2}{l^2}\int_{\xi_0}^{1} d\xi \frac{1+(1-\xi)}{\xi} x\mathcal{T}_q(x,k^2;l^2;\xi_0)\Big]$$

Increasing the transverse resolution $\Rightarrow$ number of q at a fixed transverse momentum decreases due to DGLAP splittings into qg pair with higher long. momentum given by the resolution scale.

With these definitions, collinear q-PDF andf TMD q-PDF are related via

$$xf^q_{\mu^2}(x) = \int_0^{\mu^2}\pi dk^2 x\mathcal{T}_q(x,k^2;\mu^2;\xi_0)$$

And it satisfies the DGLAP evolution equations...

Forward pA - quark channel

– start from the expressions obtained in LCPT (with Ioffe time restriction) in [arXiv:1411.2869] (no collinear subtraction and no + prescription)

– projectile contains quarks with transverse momentum smaller than μ_0, target sits at some rapidity with no need of further evolution.

– assumptions: large N_c, factorization of the dipoles, and translationally invariant dipoles.

After Including the fragmentation and FT to momentum space:

$$\frac{d\sigma^{q\to q\to H}}{d^2p d\eta} = \frac{d\sigma_0^{q\to q\to H}}{d^2p d\eta} + \frac{d\sigma_{1,\mathrm{r}}^{q\to q\to H}}{d^2p d\eta} + \frac{d\sigma_{1\mathrm{v.}}^{q\to q\to H}}{d^2p d\eta}$$

LO term

$$\boxed{\frac{d\sigma_0^{q\to q\to H}}{d^2p d\eta} = S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{\mu_0^2}(\zeta) \frac{x_F}{\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right) s(p/\zeta)}$$

$$\frac{d\sigma}{d^2p\, d\eta}\bigg|^{\mathrm{q}\to\mathrm{q}}_{\mathrm{NLO,r}} = \frac{g^2}{(2\pi)^3} S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{\mu_0^2}(\zeta) \int_{k^2,q^2>\mu_0^2} \int_{\xi_0} d\xi \frac{x_F}{\zeta(1-\xi)} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta(1-\xi)}\right) \frac{N_c}{2}\left[\frac{1+(1-\xi)^2}{\xi}\right] s(k)s(q)$$
$$\times \left\{\frac{1}{2}\frac{(q-k)^2}{(p/\zeta-k)^2(p/\zeta-q)^2} + \frac{1}{2}\frac{(1-\xi)^2(q-k)^2}{[p/\zeta-(1-\xi)k]^2[p/\zeta-(1-\xi)q]^2}\right\} + (\mathrm{Gen.\,NLO})_1$$

NLO real terms

The first term can be cast into

$$\frac{1}{2}\int_{k,q} s(k)s(q)\frac{(q-k)^2}{(p/\zeta-k)^2(p/\zeta-q)^2} = \int_{k,q}\frac{1}{k^2}s(-k+p/\zeta)\left[1-\frac{k\cdot q}{q^2}\right]s(-q+p/\zeta)$$

Second term (after rescaling $\zeta(1-\xi)\to\zeta'$) can be acts into the same form.
Using the definition of TMD PDF (analogously TMD FF)

$$x\mathcal{T}_q(x,k^2,k^2;\xi_0) = \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{\xi_0}^1 d\xi\frac{1+(1-\xi)^2}{\xi}\frac{x}{1-\xi}f^q_{k^2}\left(\frac{x}{1-\xi}\right)\frac{1}{k^2}$$

The real contribution reads

$$\frac{d\sigma_{1,\mathrm{r}}^{q\to q\to H}}{d^2p\,d\eta} = S_\perp\int_{x_F}^1\frac{d\zeta}{\zeta^2}\int_{k^2>\mu_0^2}\frac{x_F}{\zeta}\left\{D^q_{\mu_0^2}(\zeta)\mathcal{T}_q\left(\frac{x_F}{\zeta},k^2;k^2,\xi_0=\frac{k^2\zeta}{x_F s_0}\right)+f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)\mathcal{F}^q\left(\zeta,k^2;k^2,\xi_0=\frac{k^2\zeta}{x_F s_0}\right)\right\}$$
$$\times\int_q s(-k+p/\zeta)\left[1-\frac{k\cdot q}{q^2}\right]s(-q+p/\zeta)+(\mathrm{Gen.\,NLO})$$

– incoming quark with mom. k, scatters with mom exchange $-k+p/\zeta$, outgoing quark with mom. p/ζ collinearly fragments into a hadron with mom. p.

– (shift $k\to -q+p/\zeta$ and $q\to -k+p/\zeta$) incoming quark with vanishing mom., scatters with mom. transfer q, first perturbatively fragments into a quark with mom p/ζ, which then fragments into a hadron with momentum p.

NLO virtual contributions

Starting from the expressions in [arXiv:1411.2869], adopting the same assumptions:

$$\frac{d\sigma_{1,v}^{q\to q\to H}}{d^2p d\eta} = -2\frac{g^2}{(2\pi)^3} S_\perp \frac{N_c}{2} \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{H,\mu_0^2}(\zeta) \int_{k^2>\mu_0^2} \int_{k^2\zeta/(x_F s_0)}^1 d\xi \, \frac{x_F}{\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right) \frac{1+(1-\xi)^2}{\xi}$$
$$\times \int_q s\left(\frac{p}{\zeta}\right) s(q) \left\{ \left[\frac{\frac{p}{\zeta}-q-k}{(\frac{p}{\zeta}-q-k)^2}\frac{k}{k^2} + \frac{1}{k^2} \right] + \left[\frac{\frac{p}{\zeta}(1-\xi)-q-k}{(\frac{p}{\zeta}(1-\xi)-q-k)^2} - \frac{\frac{p}{\zeta}-q-k}{(\frac{p}{\zeta}-q-k)^2} \right] \frac{k}{k^2} \right\}$$

- incoming q $\rightarrow$ qg pair, pair scatters, recombines into q.
- NLO corr. to LO elastic q scattering.

- qg loop that appears either before or after the scattering.
- "proper" virtual diagram

Does not contain any large logs (a Gen. NLO correction)

in the first term one can perform the angular integration over the angle of vector k:

$$\int_{\mu_0^2} d^2k \left[\frac{\frac{p}{\zeta}-q-k}{(\frac{p}{\zeta}-q-k)^2}\frac{k}{k^2} + \frac{1}{k^2} \right] = \int_{\mu_0^2}^{(q-\frac{1}{\zeta}p)^2} \frac{d^2k}{k^2}$$

NLO virtual contributions

The virtual NLO contribution can be split into two intervals

$$-2\frac{g^2}{(2\pi)^3}S_\perp\frac{N_c}{2}\int_{x_F}^1\frac{d\zeta}{\zeta^2}\,D^q_{H,\mu_0^2}(\zeta)\int_q\left[\int_{\mu_0^2}^{\mu^2}+\int_{\mu^2}^{(q-\frac{1}{\zeta}p)^2}\right]\frac{d^2k}{k^2}\int_{k^2\zeta/(x_Fs_0)}^1 d\xi\,\frac{x_F}{\zeta}\,f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)\,\frac{1+(1-\xi)^2}{\xi}s\left(\frac{p}{\zeta}\right)\,s(q)$$

- the first term combines with LO to evolve the resolution scale of the TMD to μ^2.
- contribution from the pairs of the transverse size close to the resolution scale.

(no large logs & Gen. NLO correction)

LO + NLO virtual:

$$S_\perp\int_{x_F}^1\frac{d\zeta}{\zeta^2}\,D^q_{H,\mu_0^2}(\zeta)\,\frac{x_F}{\zeta}f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)s\left(\frac{p}{\zeta}\right)$$

$$-2\frac{g^2}{(2\pi)^3}S_\perp\frac{N_c}{2}\int_{x_F}^1\frac{d\zeta}{\zeta^2}\,D^q_{H,\mu_0^2}(\zeta)\int_q\int_{\mu_0^2}^{\mu^2}\frac{d^2k}{k^2}\int_{k^2\zeta/(x_Fs_0)}^1 d\xi\,\frac{x_F}{\zeta}\,f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)\,\frac{1+(1-\xi)^2}{\xi}s\left(\frac{p}{\zeta}\right)$$

$$=S_\perp\int_{x_F}^1\frac{d\zeta}{\zeta^2}\int_0^{\mu_0^2}d^2k\left[D^q_{H,\mu_0^2}(\zeta)\,\frac{x_F}{\zeta}\mathcal{T}_q\left(\frac{x_F}{\zeta},k^2;\mu^2;\xi_0=\frac{\zeta\mu^2}{x_Fs_0}\right)+\mathcal{F}^q_H\left(\zeta,k^2;\mu^2;\xi_0=\frac{\zeta\mu^2}{x_Fs_0}\right)\,\frac{x_F}{\zeta}f^q_{\mu_0^2}(\frac{x_F}{\zeta})\right]s\left(\frac{p}{\zeta}\right)$$

NLO virtual contributions

- evolve the factorization scale in the collinear PDFs and FFs up to μ^2

$$D^q_{H,\mu_0^2}(\zeta) \to \int_0^{\mu_0^2} d^2l \mathcal{F}^q_H\left(\zeta, l^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right); \quad f^q_{\mu_0^2}(\frac{x_F}{\zeta}) \to \int_0^{\mu_0^2} d^2k \mathcal{T}_q\left(\frac{x_F}{\zeta}, k^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right)$$

this introduces the term at $O(\alpha_s^2)$ therefore legitimate in our $O(\alpha_s)$ calculation.

- alter the scattering amplitude

$$s(\frac{p}{\zeta}) \to \int_q s\left(-(k+l) + \frac{p}{\zeta}\right)\left[1 - \frac{(k+l)\cdot q}{q^2}\right] s\left(-q + \frac{p}{\zeta}\right)$$

($|k+l|^2 \lesssim \mu_0^2 \ll p^2/\zeta^2$ & $q^2 \sim \max(Q_s^2, p^2/\zeta^2)$ & $\int_q s(q) = 1$) $\Rightarrow$ this modification only adds subleading power corrections of the order μ_0^2/Q_s^2

LO + NLO virtual:

$$S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2} \int_q \int_0^{\mu_0^2} d^2l \int_0^{\mu_0^2} d^2k \\ \times \mathcal{F}^q_H\left(\zeta, l^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) \frac{x_F}{\zeta}\mathcal{T}_q\left(\frac{x_F}{\zeta}, k^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) s\left(-(k+l) + \frac{p}{\zeta}\right)\left[1 - \frac{(k+l)\cdot q}{q^2}\right] s\left(-q + \frac{p}{\zeta}\right)$$

LO+NLO virtual+NLO real: Final TMD factorized expression

$$S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2} \int_q \int d^2l \int d^2k\, \mathcal{F}^q_H\left(\zeta, l^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) \frac{x_F}{\zeta}\mathcal{T}_q\left(\frac{x_F}{\zeta}, k^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) \\ \times s\left(-(k+l) + \frac{p}{\zeta}\right)\left[1 - \frac{(k+l)\cdot q}{q^2}\right] s\left(-q + \frac{p}{\zeta}\right) + (\mathrm{Gen.\,NLO})$$

Summary

• The progress continues in order to provide full NLO results which will provide the necessary precision for quantitive studies to determine whether saturation is exhibited by experimental data.

• We have discussed the issues and the suggested solutions for pA collisions at forward rapidities for NLO calculations.

• A new approach to forward pA scatterings is discussed.
Still have a lot of work to do:

- NLO calc. without TMD FFs, single jet production in forward pA collisions.

- NLO calc. without TMD PDFs, single inclusive hadron production in DIS.