

# Resolution of the $B \rightarrow \pi\pi, \pi K$ puzzles

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*Hsiang-nan Li and S.M., arXiv:0901.1272*

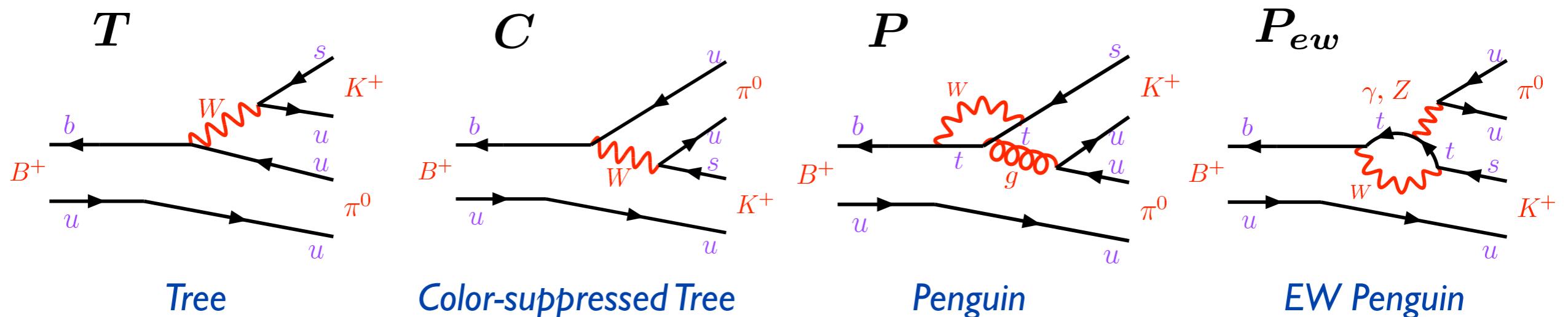
Outline:

1.  $B \rightarrow \pi K$  and  $B \rightarrow \pi\pi$  puzzles
  - Large color-suppressed tree amplitude
2. PQCD approach
3. Resolution in PQCD
  - Soft factor
  - Modified factorization formula
4. Summary

# 1. $B \rightarrow \pi K$ and $B \rightarrow \pi\pi$ Puzzles

## Topological decomposition:

Gronau, Hernandez, London, Rosner (94)



$$A(B^0 \rightarrow \pi^- K^+) = -P' - T' e^{i\phi_3}$$

$$\sqrt{2}A(B^+ \rightarrow \pi^0 K^+) = -(P' + P'_{ew}) - (T' + C') e^{i\phi_3}$$

$$A(B^0 \rightarrow \pi^+ \pi^-) = -T - P e^{i\phi_2}$$

$$\sqrt{2}A(B^+ \rightarrow \pi^+ \pi^0) = -(T + C) - P_{ew} e^{i\phi_2}$$

$$\sqrt{2}A(B^0 \rightarrow \pi^0 \pi^0) = -C + (P - P_{ew}) e^{i\phi_2}$$

# $B \rightarrow \pi K$ and $B \rightarrow \pi\pi$ Puzzles

- Naive estimate:

$$B \rightarrow \pi K: P' > T', P'_{ew} > C'$$

$$B \rightarrow \pi\pi: T > C, P > P_{ew}$$

$$A_{CP}(\pi^\mp K^\pm) \approx A_{CP}(\pi^0 K^\pm)$$

$$B(\pi^+ \pi^-) \gg B(\pi^0 \pi^0)$$

Gronau, Hernandez, London, Rosner (94)

- Theoretical Predictions vs. Data: HFAG (09)

[ $10^{-2}$ ]	Data	QCDF	SCET	PQCD
$A_{CP}(\pi^\mp K^\pm)$	$-9.8^{+1.2}_{-1.1}$	$4.5^{+1.1+2.2+0.5+8.7}_{-1.1-2.5-0.6-9.5}$	input	$-10^{+7}_{-8}$
$A_{CP}(\pi^0 K^\pm)$	$5.0 \pm 2.5$	$7.1^{+1.7+2.0+0.8+9.0}_{-1.8-2.0-0.6-9.7}$	$-18 \pm 8$	$-1^{+3}_{-6}$

[ $10^{-6}$ ]	Data	QCDF	PQCD
$B(\pi^\mp \pi^\pm)$	$5.16 \pm 0.22$	$8.9^{+4.0+3.6+0.6+1.2}_{-3.4-3.0-1.0-0.8}$	$6.5^{+6.7}_{-3.8}$
$B(\pi^\pm \pi^0)$	$5.59^{+0.41}_{-0.40}$	$6.0^{+3.0+2.1+1.0+0.4}_{-2.4-1.8-0.5-0.4}$	$4.0^{+3.4}_{-1.9}$
$B(\pi^0 \pi^0)$	$1.55 \pm 0.19$	$0.3^{+0.2+0.2+0.3+0.2}_{-0.2-0.1-0.1-0.1}$	$0.29^{+0.50}_{-0.20}$

QCDF: Beneke, Neubert(03)  
SCET: Bauer, Rothstein, Stewart(05)  
PQCD: Li, S.M., Sanda(05);  
Li, S.M.(06)

Theory errors are correlated.

Large C (or Pew) is required!

SM or NP ?

Gronau,Rosner(03); Yoshikawa(03); Buras et al.(04); Chiang et al.(04); Ciuchini et al.(04); He,Mckellar(04); S.M,Yoshikawa(04) and many others

## 2. $k_T$ Factorization (PQCD) Approach

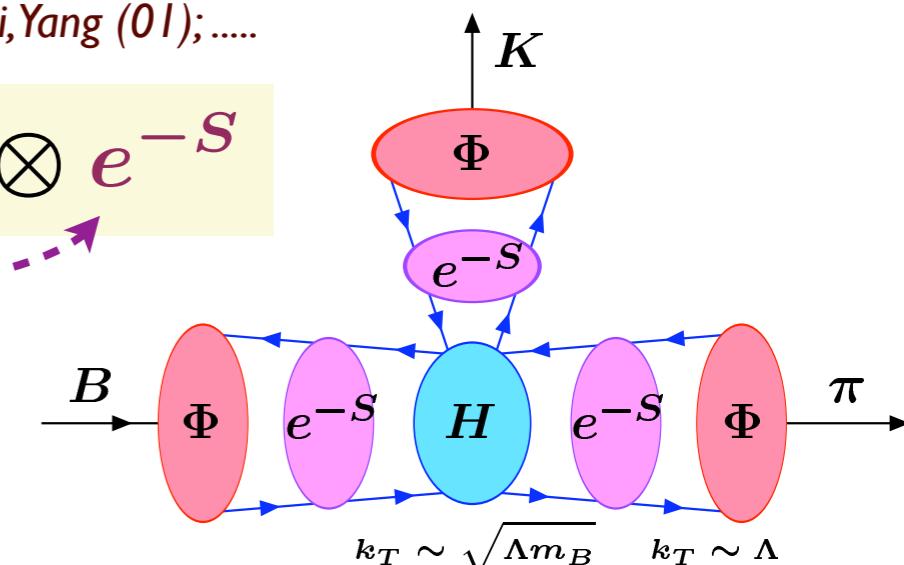
- **$k_T$  factorization** is an appropriate tool for the processes that involve significant contributions from small  $x$ . ➡ *applied to DIS, D-Y, etc.*

Botts, Sterman(89); Catani et al.(90,91);  
Ralston, Pire (90); Huang et al.(91);  
Collins, Ellis(91); Levin et al.(91);  
Li, Sterman(92); Jakob, Kroll (93); ....

- Factorization for B decays:

Chang, Li (97); Yeh, Li (97); Keum, Li, Sanda (01);  
Lu, Ukai, Yang (01); ....

$$\text{Amp.} \sim \Phi_{M_1} \otimes \Phi_{M_2} \otimes H \otimes \Phi_B \otimes e^{-S}$$

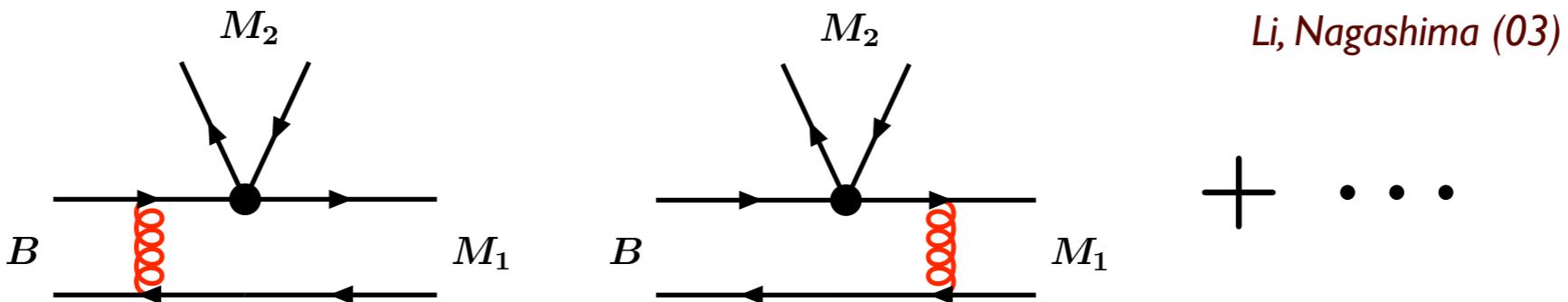


- $k_T$  is of  $O(\sqrt{m_b \Lambda})$  in a hard kernel  $H$  due to the Sudakov evolution.  
➡ ensures absence of end-point singularities in  $H$ .

### 3. Resolution in PQCD

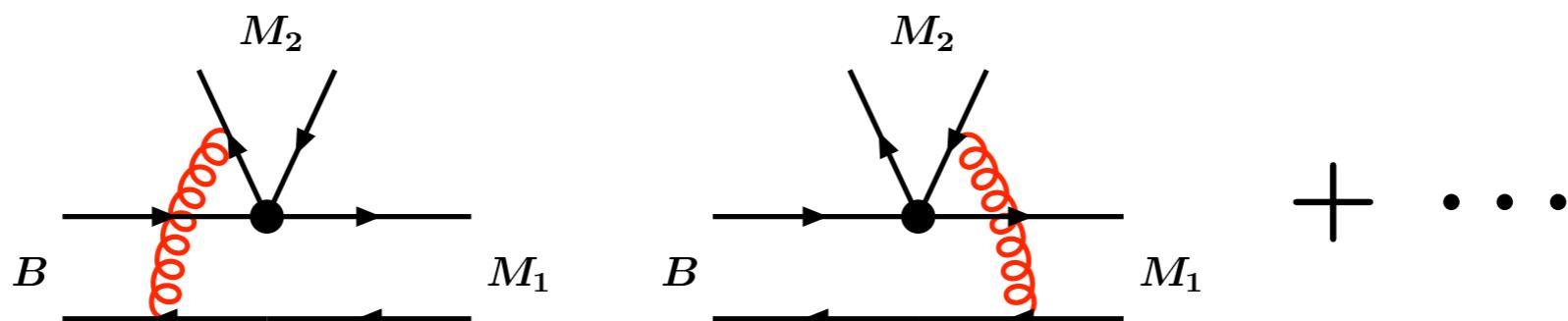
**Key: Factorization for spectator diagrams**

- In  $k_T$  factorization (PQCD), the factorization has been proved in the following diagrams:



$$\text{Amp.} \sim \Phi_{M_1} \otimes \Phi_{M_2} \otimes H \otimes \Phi_B \otimes (\text{Sudakov})$$

- However, the factorization has **not** been proved for

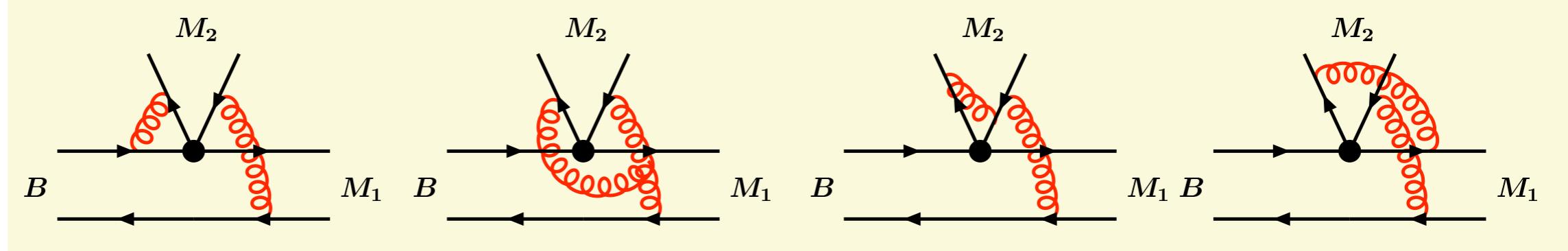
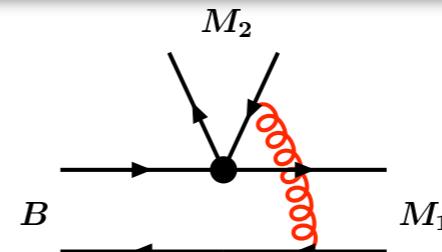


*So far, we have assumed the same factorization formula as the above.*

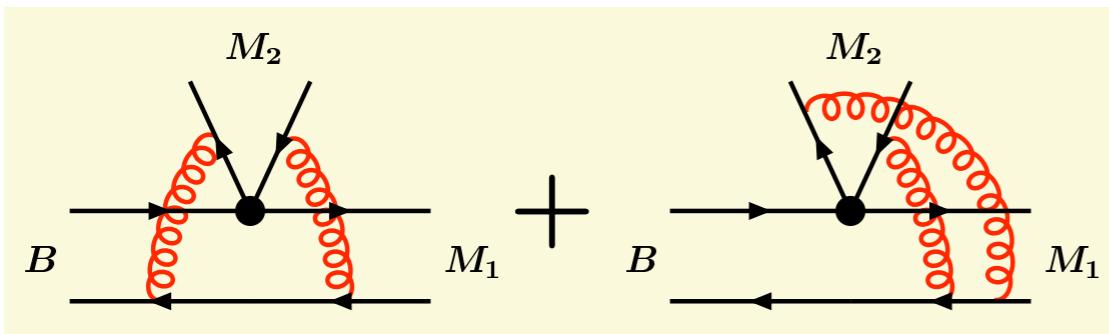
# Radiative Corrections

Li, S.M. (09)

- Radiative corrections to



- Collinear div's associated with M<sub>2</sub> are factorizable (eikonal lines) into the M<sub>2</sub> meson wave function.
- The sum of remaining corrections gives a **soft divergence** at  $\ell^\mu = (\ell^+, \ell^-, \ell_T) \sim (0, 0, 0_T)$ .



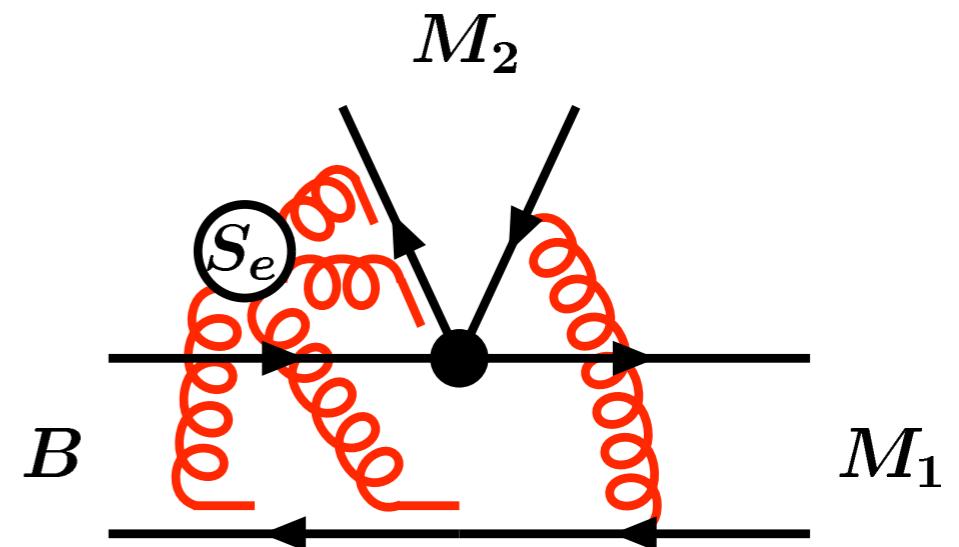
$$\propto i \frac{\alpha_s}{\pi} C_F \int \frac{d^2 \ell_T}{\ell_T^2} \mathcal{M}_a^{(\text{LO})}(\ell_T) \rightarrow \text{div.}$$

↗ LO amplitude

# Soft Factor

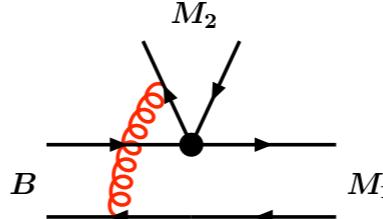
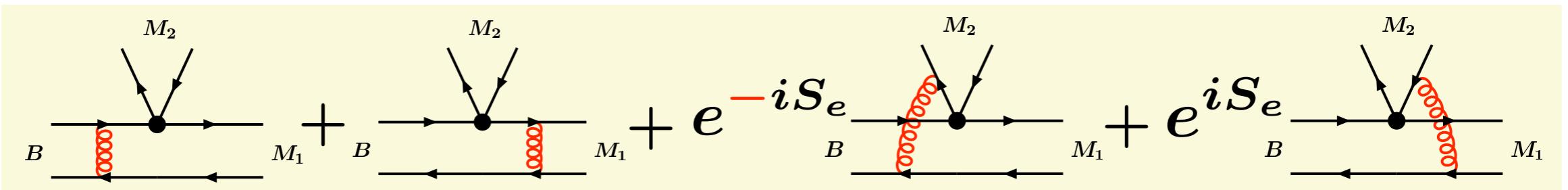
Li, S.M. (09)

- A similar soft divergence was found in hadron-hadron collisions. *Collins, Qiu (07); Collins (07)*
- The residual soft divergence can be factorized into a soft factor,  $S_e$ , using the eikonal approximation. *Li, S.M. (09); Chang, Li (09)*
- The soft factor has a dynamical origin similar to that of a meson wave function: the former (latter) absorbs the soft (collinear) gluons.
  - *can be studied by nonpert. methods or by using data, but we treat it as a parameter in this work.*



# Soft Contribution to C

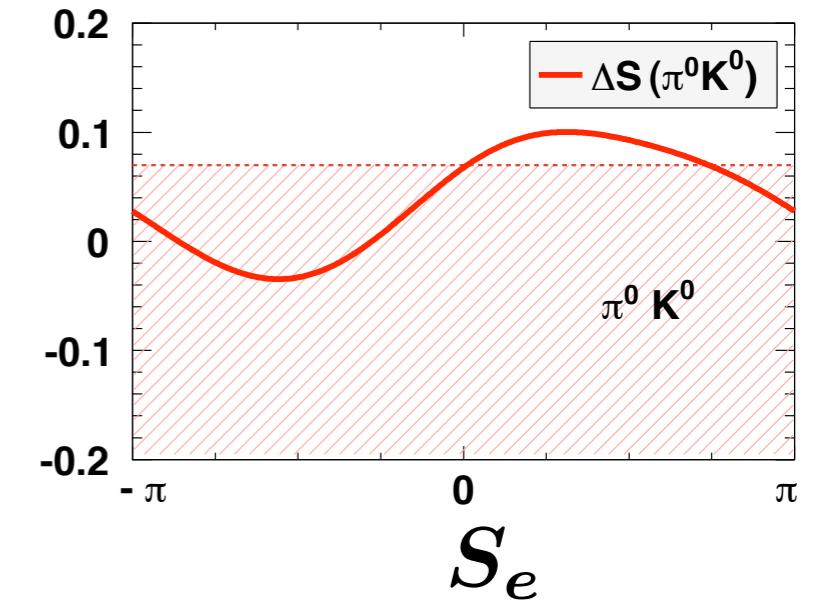
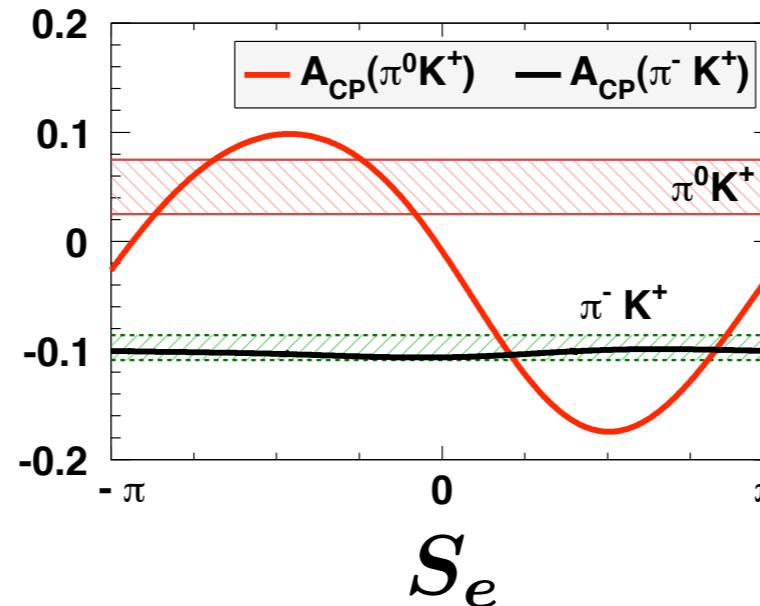
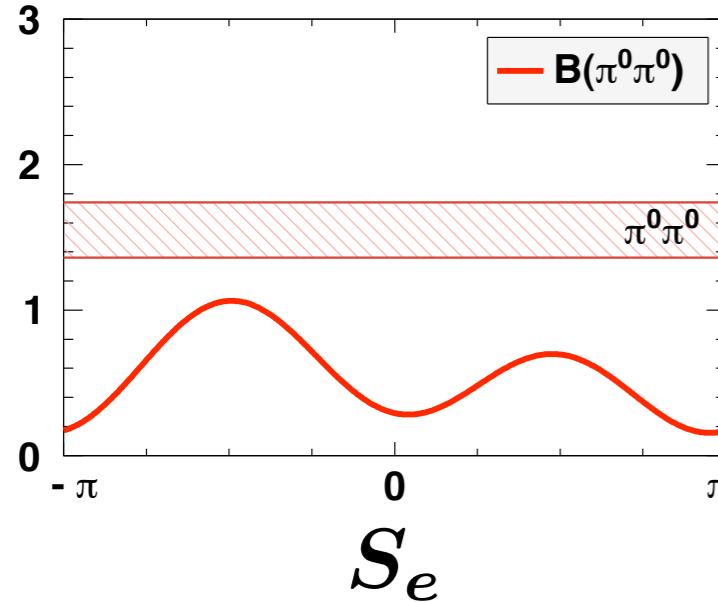
Li, S.M. (09)

- Calculating higher orders, the divergence can be summed into  $e^{iS_e} \mathcal{M}_a^{(\text{LO})}$ .
- Similarly, corrections to give  $e^{-iS_e} \mathcal{M}_b^{(\text{LO})}$ .
- “Modified” factorization formula:
$$\text{Amp.} \sim \Phi_{M_1} \otimes \Phi_{M_2} \otimes H \otimes \Phi_B \otimes (\text{Sudakov}) \otimes e^{\pm iS_e}$$
- The presence of  $S_e$  may convert a destructive interference into a constructive one.

small in C (large in T, P)      almost cancel if  $S_e=0$
- C could be enhanced, while T and P are unchanged.

# Resolution of the $B \rightarrow \pi\pi, \pi K$ puzzles

Li, S.M. (09)



- $S_e(\pi\pi) \approx S_e(\pi K)$ .
- $B(\pi^0\pi^0)$  can be enhanced.
- The difference between  $A_{CP}(\pi^\mp K^\pm)$  and  $A_{CP}(\pi^0 K^\pm)$  can be enlarged.
- The  $B \rightarrow \pi\pi, \pi K$  puzzles can be resolved simultaneously for  $S_e \sim -\pi/2$ .  $\xrightarrow{\text{---}} \frac{C}{T} \approx 0.5 e^{-2.2i}$
- We predict a bit smaller  $S(\pi^0 K_S)$ .

## 4. Summary

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- There exist uncanceled soft divergences in the spectator diagrams in  $k_T$  factorization (PQCD).
- The soft div. **can be factorized** in a soft factor.



The  $B \rightarrow \pi\pi, \pi K$  puzzles can be resolved simultaneously for  $S_e \sim -\pi/2$ .

- A difference between  $B \rightarrow \pi^0\pi^0$  and  $B \rightarrow \rho^0\rho^0$  is discussed in our paper [arXiv:0901.1272](https://arxiv.org/abs/0901.1272).

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# Backup Slides

# Sum Rules in $B \rightarrow \pi K$

Gronau,Rosner(05); Gronau(05)

$$A_{CP}(\pi^\mp K^\pm) \approx A_{CP}(\pi^0 K^\pm)$$

- Including C,

$$A_{CP}(\pi^\mp K^\pm) \approx A_{CP}(\pi^0 K^\pm) + A_{CP}(\pi^0 K^0)$$

- Including other subleading contributions,

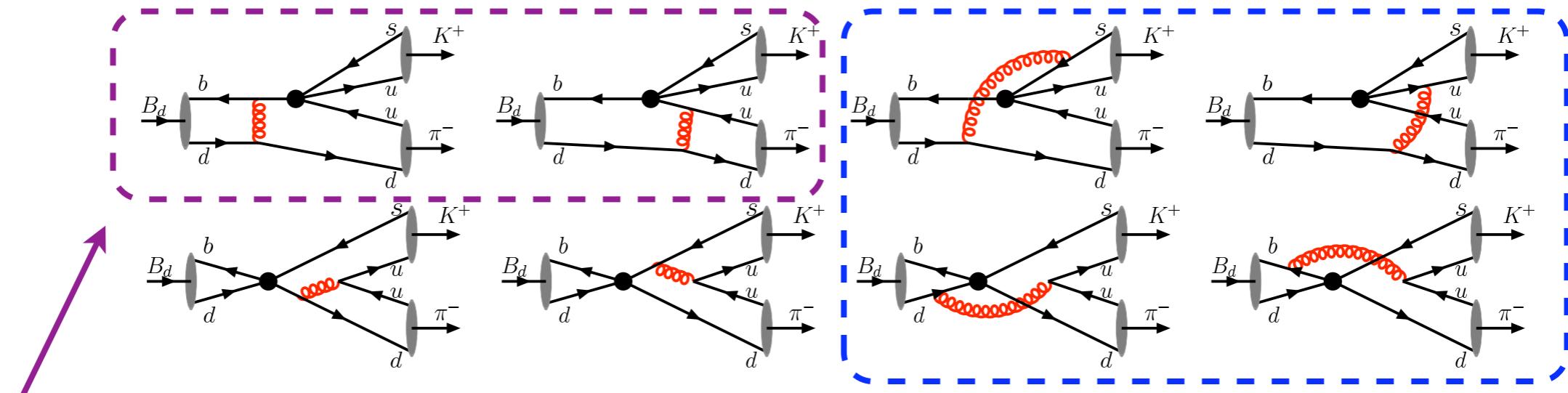
$$A_{CP}(\pi^\mp K^\pm) + A_{CP}(\pi^\pm K^0) \approx A_{CP}(\pi^0 K^\pm) + A_{CP}(\pi^0 K^0)$$

[10 <sup>-2</sup> ]	Data HFAG (09)
$A_{CP}(B^\pm \rightarrow \pi^\pm K^0)$	$0.9 \pm 2.5$
$A_{CP}(B^\pm \rightarrow \pi^0 K^\pm)$	$5.0 \pm 2.5$
$A_{CP}(B^0 \rightarrow \pi^\mp K^\pm)$	$-9.8^{+1.2}_{-1.1}$
$A_{CP}(B^0 \rightarrow \pi^0 K^0)$	$-1 \pm 10$

We need Super B factories.

# PQCD Approach

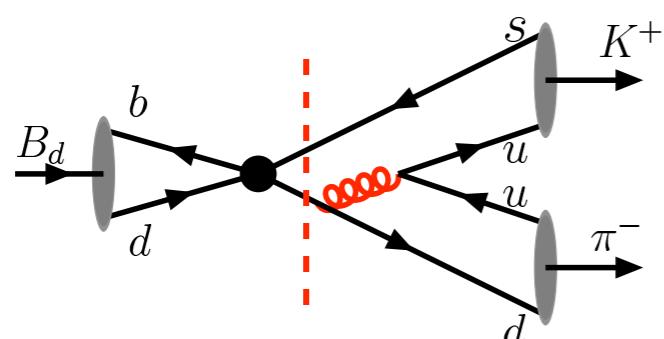
- LO diagrams:  $O(\alpha_s(\sqrt{m_b \Lambda}))$



Leading (naive factorization)

Subleading

- Penguin annihilation with  $(1 - \gamma_5) \otimes (1 + \gamma_5)$ -type operator generates a large strong phase.



Keum, Li, Sanda (01)

$$\frac{1}{xM_B^2 - |k_T|^2 + i\epsilon} = P \left[ \frac{1}{xM_B^2 - |k_T|^2} \right] - i\pi \delta(xM_B^2 - |k_T|^2)$$

- Main sources of uncertainties are non-pert. inputs, i.e. meson distribution amplitudes.

# Sudakov Suppression in PQCD

- Keeping  $k_T$ , large double logarithms are generated from the overlap of collinear and soft divergences:  $\alpha_s \ln^2(M_B/k_T)$
- The double logarithms can be resummed using a resummation technique. *Botts, Sterman (89)*

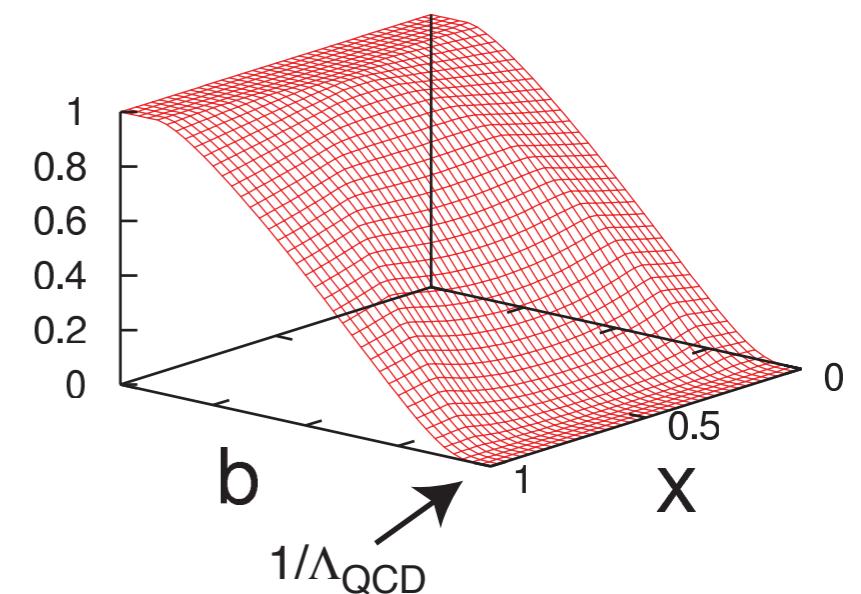


**Sudakov factor**

- Sudakov factor suppresses a small  $k_T$  (large  $b$ ) region.

*Li, Sterman (92)*

$b$  is the conjugate space of  $k_T$ .

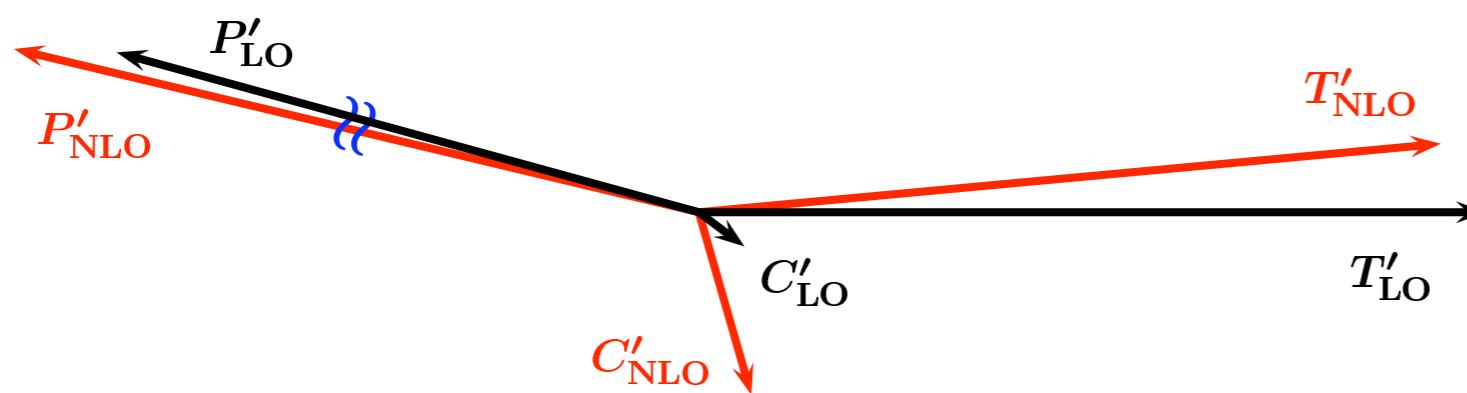


# PQCD predictions for $B \rightarrow \pi K$

## PQCD ( $k_T$ factorization) without the soft factor

- LO PQCD predicts  $A_{CP}(\pi^\mp K^\pm) \approx A_{CP}(\pi^0 K^\pm)$ .
- Including important NLO corrections,  $A_{CP}(\pi^0 K^\pm)$  approaches to the data, but not enough. *Li, S.M., Sanda (05)*

$$\begin{cases} A_{CP}(\pi^\mp K^\pm) \sim 2 \operatorname{Im} [T'/P'] \sin \phi_3 \sim -10\% \\ A_{CP}(\pi^0 K^\pm) \sim 2 \operatorname{Im} [(T' + C')/P'] \sin \phi_3 \sim -1\% \end{cases}$$



C' is still smaller than T,  
but  $\operatorname{Im}[(T+C)/P]$  vanishes!

# $B \rightarrow \rho^0 \rho^0$

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- $B \rightarrow \rho^0 \rho^0$  is similar to  $B \rightarrow \pi^0 \pi^0$  at the quark level, but its prediction is consistent with the data.

[ $10^{-6}$ ]	Data	QCDF	PQCD
$B(\rho^\mp \rho^\pm)$	$24.2^{+3.1}_{-3.2}$	$25.5^{+1.5+2.4}_{-2.6-1.5}$	$25.3^{+25.3}_{-13.8}$
$B(\rho^\pm \rho^0)$	$24.0^{+1.9}_{-2.0}$	$20.0^{+4.0+2.0}_{-1.9-0.9}$	$16.0^{+15.0}_{-8.1}$
$B(\rho^0 \rho^0)$	$0.73^{+0.27}_{-0.28}$	$0.9^{+1.5+1.1}_{-0.4-0.2}$	$0.92^{+1.10}_{-0.56}$

*HFAG (09)*      *Cheng, Yang (08)*      *Li, S.M. (06)*



Can we explain the large C,  
under the constraint from  $B(\rho^0 \rho^0)$ ?

# Pion vs. Rho meson

Nussinov, Shrock (08); Duraisamy, Kagan (08)

- **Key:** How to accommodate the simultaneous role of the pion as a  $q\bar{q}$  state like  $\rho$  and an almost massless NG boson?
- The valence  $q\bar{q}$  pair must be close enough to reduce the confinement effect. ( $r < 1/\Lambda_{\text{QCD}}$ ) If the pair is separated far apart, the linear potential will give the pion a high mass.
- $r_\pi \approx 1/\Lambda_{\text{QCD}}$  is accounted for by a soft cloud of higher Fock states:
$$|\pi\rangle \sim |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}q\bar{q}\rangle + \dots \Leftrightarrow \text{NG boson}$$
- **Assumption:** The soft effect in  $S_e$  is significant (negligible) in the decays with  $\pi(\rho)$ .