

## Towards a unitary Dalitz plot analysis of three-body hadronic $B$ decays \*

---

**L. Leśniak**<sup>a†</sup>

<sup>a</sup> *Division of Theoretical Physics, The Henryk Niewodniczański Institute of Nuclear Physics, Polish Academy of Sciences, 31-342 Kraków, Poland*  
E-mail: Leonard.Lesniak@ifj.edu.pl

**B. El-Bennich**<sup>b</sup>, **A. Furman**<sup>c</sup>, **R. Kamiński**<sup>a</sup>, **B. Loiseau**<sup>d</sup>, **B. Moussallam**<sup>e</sup>

<sup>b</sup> *Physics Division, Argonne National Laboratory, Argonne, Illinois, 60439, USA*

<sup>c</sup> *ul. Bronowicka 85/26, 30-091 Kraków, Poland*

<sup>d</sup> *Laboratoire de Physique Nucléaire et de Hautes Énergies (IN2P3–CNRS–Universités Paris 6 et 7), Groupe Théorie, Université Pierre et Marie Curie, 4 place Jussieu, 75252 Paris, France ‡*

<sup>e</sup> *Groupe de Physique Théorique, Institut de Physique Nucléaire (IN2P3–CNRS), Université Paris-Sud 11, 91406 Orsay Cedex, France*

A unitary model of the final state  $K\pi$  interaction amplitudes in the  $B \rightarrow K\pi\pi$  decays is constructed. The weak decay penguin amplitudes, derived in QCD factorization, are supplemented by phenomenological contributions. The strange  $K\pi$  scalar and vector form factors are used to calculate the  $K\pi$  effective mass and helicity angle distributions, branching ratios, CP asymmetries and the phase difference between the  $B^0$  and  $\bar{B}^0$  decay amplitudes to  $K^*(892)\pi$ . The fit on the phenomenological parameters leads to a good agreement with the experimental data, particularly for the  $B \rightarrow K^*(892)\pi$  decays. However, our predicted  $B^\pm \rightarrow K_0^*(1430)\pi^\pm$ ,  $K_0^*(1430) \rightarrow K^\pm\pi^\mp$  branching fraction is smaller than the results of the Belle and BaBar collaborations, obtained from isobar model analyses. A new parameterization of the  $S$ -wave  $K\pi$  effective mass distribution, which can be used in future experimental Dalitz plot analyses, is proposed.

*The 2009 Europhysics Conference on High Energy Physics,  
July 16 - 22 2009  
Krakow, Poland*

---

\*This work has been supported in part by the Polish Ministry of Science and Higher Education (grant No N N202 248135) and by the IN2P3-Polish Laboratories Convention (project No 08-127).

†Speaker.

‡Unité de Recherche des Universités Paris 6 et Paris 7, associée au CNRS

## 1. Introduction

Studies of three-body charmless hadronic decays of  $B$  mesons are very useful not only in standard model tests and in searches for "new physics" effects but also in the determination of strong interaction amplitudes. Both weak and strong interactions can create structures seen on Dalitz plots. Analyses of these diagrams should be performed in a unitary approach which allows for a proper construction of  $B$ -decay amplitudes. Usually a suitable partial wave analysis of final state amplitudes should be done. Then an adequate determination of branching fractions and CP asymmetries for different quasi-two body decay reactions is possible.

Construction of a fully unitary three-body strong interaction amplitude is, however, a difficult task. A first step towards this goal is to enforce two-body unitarity. Here we apply this concept to the  $K\pi$  channel for the  $B \rightarrow K\pi\pi$  decays by using a unitary coupled channel model. We study the  $K\pi$  amplitudes in the limited  $K\pi$  effective mass range smaller than about 1.8 GeV. Our aim is to describe such physical quantities as differential effective mass and helicity angle distributions, integrated branching fractions and direct CP asymmetries.

In experimental analyses of  $B$ -decays the isobar model is very frequently applied. Within that model quasi-two body branching fractions are determined. However, the decay amplitudes commonly used in the isobar model are not unitary neither in three-body decay channels nor in two-body subchannels. This lack of unitarity can create severe problems in the determination of branching fractions in the case of wide overlapping resonances. In  $B \rightarrow K\pi\pi$  decays one observes a wide  $S$ -wave resonance  $K_0^*(1430)$ . Its width equals to about 270 MeV and the postulated  $K_0^*(800)$  state can have even larger width of 500 MeV. Thus an important source of model errors in extraction of the branching ratio for the decay  $B^\pm \rightarrow K_0^*(1430)\pi^\pm$  is a possible wrong attribution of a part of Dalitz plot density to a background amplitude and to its interference with other amplitudes, mostly with the  $S$ -wave. Below, we shall briefly discuss that issue.

## 2. Theoretical model

The weak decay amplitudes of  $B^\pm$ ,  $B^0$  and  $\bar{B}^0$ , which are derived in QCD factorization, are supplemented by phenomenological contributions to the penguin amplitudes. Strong interaction amplitudes are constrained by chiral symmetry, QCD and experimental data on meson-meson interactions. The matrix elements of the effective weak Hamiltonian involve the strange  $K\pi$  scalar and vector form factors. The introduction of form factors, constrained by theory and other experiments than  $B$  decays, is an alternative to the use of the isobar model. The  $K\pi$   $S$ -wave contribution to the  $B^- \rightarrow K^- \pi^+ \pi^-$  amplitude reads:

$$\begin{aligned} \mathcal{M}_S^- \equiv \langle \pi^- (K^- \pi^+) | H_{eff} | B^- \rangle = & \frac{G_F}{\sqrt{2}} (M_B^2 - m_\pi^2) \frac{m_K^2 - m_\pi^2}{q^2} f_0^{B^- \pi^-}(q^2) f_0^{K^- \pi^+}(q^2) \\ & \times \left\{ \lambda_u \left( a_4^u(S) - \frac{a_{10}^u(S)}{2} + c_4^u \right) + \lambda_c \left( a_4^c(S) - \frac{a_{10}^c(S)}{2} + c_4^c \right) \right. \\ & \left. - \frac{2q^2}{(m_b - m_d)(m_s - m_d)} \left[ \lambda_u \left( a_6^u(S) - \frac{a_8^u(S)}{2} + c_6^u \right) + \lambda_c \left( a_6^c(S) - \frac{a_8^c(S)}{2} + c_6^c \right) \right] \right\}. \end{aligned}$$

Expressions for other contributions to the  $B$  decay amplitudes can be found in [1]. In the above equation  $G_F$  denotes the Fermi coupling constant,  $M_B$ ,  $m_K$  and  $m_\pi$  are the masses of the charged  $B$  mesons, kaons and pions,  $f_0^{B^- \pi^-}(q^2)$  and  $f_0^{K^- \pi^+}(q^2)$  are the  $B^-$  to  $\pi^-$  and the  $K\pi$  scalar form factors. The symbols  $\lambda_u = V_{ub}V_{us}^*$ ,  $\lambda_c = V_{cb}V_{cs}^*$ , are products of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix elements  $V_{qq'}$ ;  $a_j^{u,c}(S)$ ,  $j = 4, 6, 8, 10$  are the coefficients of the effective Hamiltonian  $H_{eff}$ ,  $q^2$  is the  $K\pi$  effective mass squared and  $m_b$ ,  $m_s$  and  $m_d$  are  $b$ ,  $s$  and  $d$  quark masses. Finally the  $c_4^u, c_4^c, c_6^u$  and  $c_6^c$  are the phenomenological complex parameters which are fitted to the experimental data.

The scalar and vector  $K\pi$  form factors are connected to scattering amplitudes in the  $S$  and  $P$  waves via unitarity relations. In the  $S$  wave we treat two coupled  $K\pi$  and  $K\eta'$  amplitudes. Three coupled channels:  $K\pi$ ,  $K^*\pi$  and  $K\rho$  appear in the  $P$  wave. The corresponding scattering amplitudes are constrained by experimental data, especially by the LASS results obtained at SLAC. The Muskhelishvili-Omnès equations are used to calculate the  $K\pi$  form factors.

### 3. Results and discussion

The  $K\pi$  effective mass and helicity angle distributions, branching ratios, CP asymmetries and the phase difference between the  $B^0$  and  $\bar{B}^0$  decay amplitudes to  $K^*(892)\pi$  are calculated using the minimization program which serves to fit the set of 319 data from the Belle and BaBar collaborations with the four phenomenological parameters  $c_{4,6}^{u,c}$ . As input we use only the well measured branching fractions for the  $B \rightarrow K^*(892)\pi$ . We do not use the experimental branching fractions for the  $B \rightarrow K_0^*(1430)\pi$ , which are not well determined due to the large width of the  $K_0^*(1430)$  resonance. Some of our model predictions for the  $S$ -wave part of the branching fraction for the  $B \rightarrow K\pi\pi$  decays are given in Table 1. One can notice that they are lower than the experimental data, being substantially smaller than the Belle results and closer to the BaBar numbers. If the parameters  $c_{4,6}^{u,c}$  are all put equal to zero then the  $P$ -wave part of the branching fraction is underestimated by a factor of 4 to 5 and the  $S$ -wave part by a factor of 2. The fit on the model parameters leads to a good agreement with the experimental data, particularly for the kaon-pion effective mass and helicity angle distributions. Some results are presented in Fig.1.

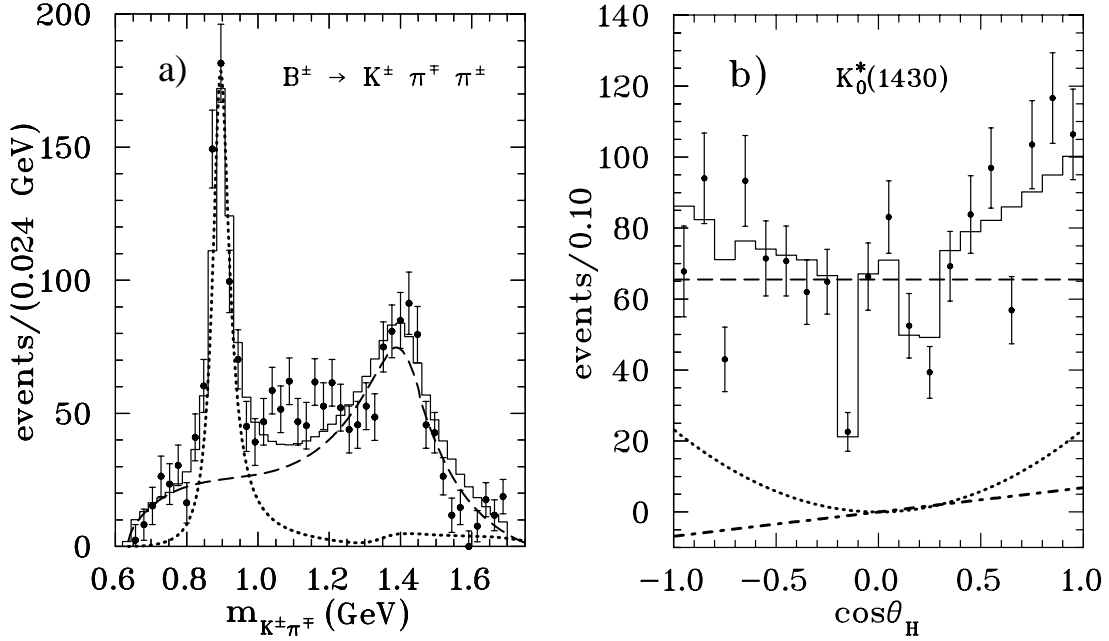
Based on a good description of data we propose to choose in the future experimental analyses the following parameterization of the  $S$ -wave  $B \rightarrow K\pi\pi$  amplitude:

$$\mathcal{M}_S^- = f_0^{K\pi}(m_{K\pi}^2)(c_0/m_{K\pi}^2 + c_1), \quad (3.1)$$

where  $f_0^{K\pi}$  is the scalar  $K\pi$  form factor while  $c_0$  and  $c_1$  are complex numbers to be fitted from the data. Numerical values of the complex scalar form factor can be provided on request.

**Table 1:** Branching ratios averaged over charge conjugate reactions  $B \rightarrow K\pi\pi$  in units of  $10^{-6}$ .  $(K^+\pi^-)_P$  and  $(K^+\pi^-)_S$  denote the  $(K^+\pi^-)$  pair in  $P$ -wave and  $S$ -wave, respectively. In the first two lines, the values of the model, calculated by the integration over the given  $m_{K\pi}$  range, are compared to the corresponding Belle [2, 3] and BaBar [4, 5] results written in the third and fourth column. In the last two lines, the Belle branching fractions [2, 3], calculated with a  $K_0^*(1430)$  Breit-Wigner amplitude, and the BaBar branching fractions [4, 5], calculated in their parametrization of the  $K\pi$   $S$ -wave, are compared to our model predictions. The model errors are the phenomenological parameter uncertainties found in the minimization procedure.

decay mode	$m_{K\pi}$ range (GeV)	Belle	BaBar	model
$B^+ \rightarrow (K^+\pi^-)_P \pi^-$	(0.82, 0.97)	$5.35 \pm 0.59$	$5.98 \pm 0.75$	$5.73 \pm 0.14$
$B^0 \rightarrow (K^0\pi^+)_P \pi^-$	(0.82, 0.97)	$4.65 \pm 0.77$	$6.47 \pm 0.75$	$5.42 \pm 0.16$
$B^+ \rightarrow (K^+\pi^-)_S \pi^-$	(0.64, 1.76)	$27.0 \pm 2.5$	$22.5 \pm 4.6$	$16.5 \pm 0.8$
$B^0 \rightarrow (K^0\pi^+)_S \pi^-$	(0.64, 1.76)	$26.0 \pm 3.4$	$17.3 \pm 4.6$	$15.8 \pm 0.7$



**Figure 1:** a) The  $K^\pm\pi^\mp$  effective mass distributions in the  $B^\pm \rightarrow K^\pm\pi^\mp\pi^\pm$  decays. Data points are from Ref. [4]. The dashed line represents the  $S$ -wave contribution of our model, the dotted line that of the  $P$ -wave and the histogram corresponds to the coherent sum of the  $S$ - and  $P$ -wave contributions. b) Helicity angle distribution for  $B^\pm \rightarrow K^\pm\pi^\mp\pi^\pm$  decays calculated from the averaged double differential distribution integrated over  $m_{K^\pm\pi^\mp}$  mass from 1.0 to 1.76 GeV. Data points are from Ref. [6]. The dashed line represents the  $S$ -wave contribution of our model, the dotted line that of the  $P$ -wave and the dot-dashed line that of the interference term. The histogram corresponds to the sum of these three contributions.

**References**

- [1] B. El-Bennich, A. Furman, R. Kamiński, L. Leśniak, B. Loiseau and B. Moussallam, Phys. Rev. D **79**, 094005 (2009).
- [2] A. Garmash *et al.* (Belle Collaboration), Phys. Rev. Lett. **96**, 251803 (2006),  
A. Garmash *et al.* (Belle Collaboration), Phys. Rev. **D 71**, 092003 (2005).
- [3] A. Garmash *et al.* (Belle Collaboration), Phys. Rev. D **75**, 012006 (2007).
- [4] B. Aubert, *et al.* (BaBar Collaboration), Phys. Rev. D **78**, 012004 (2008).
- [5] B. Aubert, *et al.* (BaBar Collaboration), Phys. Rev. D **78**, 052005 (2008).
- [6] K. Abe *et al.* (Belle Collaboration), arXiv:hep-ex/0509001.