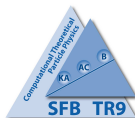


*The MSSM with large  $\tan\beta$   
beyond the decoupling limit*

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## The pattern of $\tan\beta$ -enhancement

- ▶ The MSSM contains two Higgs doublets  $H_u, H_d$ .

Both acquire vevs:  $v_u, v_d \rightarrow \tan\beta \equiv \frac{v_u}{v_d}$

- ▶ large  $\tan\beta \Leftrightarrow$  small  $v_d$

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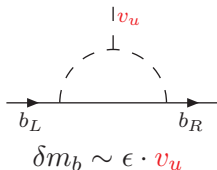
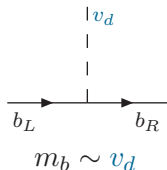
- ▶ The MSSM contains two Higgs doublets  $H_u, H_d$ .

Both acquire vevs:  $v_u, v_d \rightarrow \tan\beta \equiv \frac{v_u}{v_d}$

- ▶ large  $\tan\beta \Leftrightarrow$  small  $v_d$
- ▶ Consider tree-level amplitude with suppression  $v_d$ .  
One-loop corrections may involve  $v_u$  instead.

[Hall,Rattazzi,Sarid; Blazek,Raby,Pokorski]

- ▶ Example:  $b$ -quark mass



$$\frac{\delta m_b}{m_b} \sim \epsilon \cdot \tan\beta$$
$$\sim \mathcal{O}(1)$$

# Effective Lagrangian vs. full MSSM

- ▶ Two possibilities to deal with such  $\mathcal{O}(1)$  corrections
  1. Effective Lagrangian for  $M_{\text{SUSY}} \gg v, M_{A^0}, M_{H^0}, M_{H^\pm}$
  2. Calculation in the full MSSM **beyond decoupling**

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  1. Effective Lagrangian for  $M_{\text{SUSY}} \gg v, M_{A^0}, M_{H^0}, M_{H^\pm}$
  2. Calculation in the full MSSM **beyond decoupling**
- ▶ Why go beyond decoupling limit?
  - ▶  $M_{\text{SUSY}} \sim v$  is natural.
  - ▶ Test accuracy of calculations done with the effective Lagrangian approach .
  - ▶ Study  $\tan\beta$ -enhanced effects in couplings of SUSY-particles like  $\tilde{g}, \tilde{\chi}^0$ 

**Impossible in the decoupling limit where these particles are integrated out!**

# Summary of large- $\tan\beta$ effects

effect	decoupling limit	beyond
modified relation $y_{d_i} \leftrightarrow m_{d_i}$	[Hall,Rattazzi,Sarid; Carena,Olechowski, Pokorski,Wagner]	[Carena,Garcia, Nierste,Wagner], <span style="border: 1px solid blue; padding: 2px;">1</span>
corrections to CKM matrix	[Blazek,Raby,Pokorski]	[Buras,Chankowski, Rosiek,Slawianowska], <span style="border: 1px solid blue; padding: 2px;">2</span>
enhanced FCNCs $d_i d_j H^0/A^0$	[Hamzaoui,Pospelov,Toharia; Babu,Kolda; Buras,Chankowski,Rosiek, Slawianowska]	[Buras,Chankowski, Rosiek,Slawianowska], <span style="border: 1px solid blue; padding: 2px;">3</span>
enhanced FCNCs $d_i \tilde{d}_j \tilde{g}/\tilde{\chi}^0$	<b>not accessible</b>	<span style="border: 1px solid blue; padding: 2px;">3</span>
vertex corrections $\bar{u}_{i,R} d_{j,L} H^+$	[Degrassi,Gambino,Giudice; Carena,Garcia, Nierste,Wagner]	process-dependent (non-universal)

1 – 3 = this talk

Beyond the decoupling limit:

- 1 Scheme dependence of the resummation formula for the Yukawa coupling
- 2 Resummation of flavour-changing self-energies
- 3 New effects in FCNC processes

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# Input schemes for bottom-squark mixing

► Bottom-squark mass matrix:  $\mathcal{M}_b^2 = \begin{pmatrix} m_{b_L}^2 & -y_b^* v_u \mu \\ -y_b v_u \mu^* & m_{b_R}^2 \end{pmatrix}$

► Mixing matrix:  $\tilde{R}_b \mathcal{M}_b^2 \tilde{R}_b^\dagger = \text{diag}(m_{b_1}^2, m_{b_2}^2),$

$$\tilde{R}_b = \begin{pmatrix} \cos \tilde{\theta}_b & \sin \tilde{\theta}_b e^{i\tilde{\phi}_b} \\ -\sin \tilde{\theta}_b e^{-i\tilde{\phi}_b} & \cos \tilde{\theta}_b \end{pmatrix}$$

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- ▶ What to choose as input? → different possibilities, e.g.
- ▶ elements of  $\mathcal{M}_b^2$ :  $m_{\tilde{b}_L}, m_{\tilde{b}_R}, \mu, \tan \beta$
  - ▶ mass eigenvalues and mixing angle:  $m_{\tilde{b}_1}, m_{\tilde{b}_2}, \tilde{\theta}_b, \tilde{\phi}_b$
  - ▶ eigenvalues and off-diag. entries of  $\mathcal{M}_b^2$ :  $m_{\tilde{b}_1}, m_{\tilde{b}_2}, \mu, \tan \beta$

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  - ▶ eigenvalues and off-diag. entries of  $\mathcal{M}_b^2$ :  $m_{\tilde{b}_1}, m_{\tilde{b}_2}, \mu, \tan \beta$
- ▶ **Note:**  $\tilde{\theta}_b$  vanishes for  $v/M_{\text{SUSY}} \rightarrow 0$   
→ No different input schemes in the decoupling limit.

## *Scheme dependence of the resummation formula*

- ▶ Write  $\Sigma_b^{RL} = m_b \Delta_b = m_b \epsilon_b \tan \beta$
- ▶ Modified relation  $y_b \leftrightarrow m_b$  in the decoupling limit:

$$y_b = \frac{m_b}{v_d(1 + \Delta_b)}$$

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- ▶ Beyond decoupling: **Formula depends on renormalization scheme (choice of input)!!!**

- ▶ Example: Gluino-contribution  $\Sigma_{b,\tilde{g}}^{RL} = m_b \Delta_b^{\tilde{g}}$

(i) Input:  $m_{\tilde{b}_1}, m_{\tilde{b}_2}, \mu, \tan \beta$   $\rightarrow$   $y_b = \frac{m_b}{v_d(1 + \Delta_b^{\tilde{g}})}$

(ii) Input:  $m_{\tilde{b}_1}, m_{\tilde{b}_2}, \tilde{\theta}_b, \tilde{\phi}_b$   $\rightarrow$   $y_b = \frac{m_b}{v_d} \left(1 - \Delta_b^{\tilde{g}}\right)$

(iii) Input:  $m_{\tilde{b}_L}, m_{\tilde{b}_R}, \mu, \tan \beta$

$\rightarrow$  analytic resummation impossible, use (i) iteratively.

Beyond the decoupling limit:

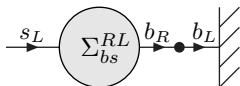
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## *Flavour-changing self-energies in external legs*

- ▶ **Naive MFV:** Only chargino-loops are flavour-changing

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- ▶ Consider flavour-changing self-energies in external quark-legs:



- ▶ New source of  $\tan \beta$ -enhancement:

$$\Sigma_{bs}^{RL} \propto \epsilon_{FC} m_b \tan \beta \quad \text{and} \quad \mathcal{M} \propto \frac{\Sigma_{bs}^{RL}}{m_b} \propto \epsilon_{FC} \tan \beta$$



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- ▶ Subtract self-energies by non-diagonal wave-function CTs:

$$\delta Z_{bi}^L \propto \epsilon_{FC} \tan \beta, \quad \delta Z_{bi}^R \propto \frac{m_i}{m_b} \epsilon_{FC} \tan \beta \quad (i = d, s)$$

→  $\delta Z_{L/R}$  contain the  $\tan \beta$ -enhanced effects!

## Resummed results

- ▶  $(\epsilon_{FC} \tan \beta)^n$ -effects can be analytically resummed to all orders:

$$\frac{\delta Z_{bi}^L}{2} = -V_{tb}^* V_{ti} \frac{\epsilon_{FC} \tan \beta}{1 + (\epsilon_b - \epsilon_{FC}) \tan \beta},$$

$$\frac{\delta Z_{bi}^R}{2} = -V_{tb}^* V_{ti} \frac{m_{d_i}}{m_b} \left[ \frac{\epsilon_{FC} \tan \beta}{1 + \epsilon_b \tan \beta} + \frac{\epsilon_{FC}^* \tan \beta}{(1 + \epsilon_i^* \tan \beta)} \right] \frac{1 + \epsilon_b \tan \beta}{1 + (\epsilon_b - \epsilon_{FC}) \tan \beta}$$

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- ▶ results in corrections to the CKM matrix:

$$V^0 = \begin{pmatrix} V_{ud} & V_{us} & K^* V_{ub} \\ V_{cd} & V_{cs} & K^* V_{cb} \\ KV_{td} & KV_{ts} & V_{tb} \end{pmatrix}, \quad K = \frac{1 + \epsilon_b \tan \beta}{1 + (\epsilon_b - \epsilon_{FC}) \tan \beta}$$

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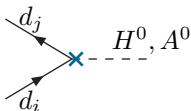
- ▶ These results
  - ▶ are of the **same form** as in the decoupling limit but with **different**  $\epsilon_b, \epsilon_{FC}$ .
  - ▶ are the **analytic expressions** for the limit to which the **iterative calculation** of BCRS converges.

Beyond the decoupling limit:

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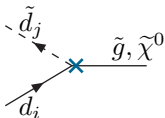
# FCNC-couplings at large $\tan\beta$

- ▶  $\delta Z_{ij}^L$  induce FCNC-couplings of order  $\epsilon_{FC} \tan\beta$ :



known in the decoupling limit

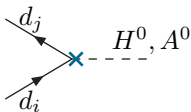
**new:** generalized to  $M_{\text{SUSY}} \sim v$



**new!** (not accessible in the decoupling limit)

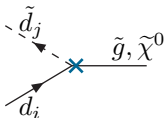
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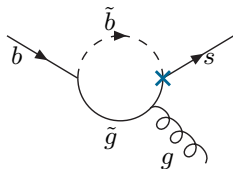
- ▶  $\delta Z_{bi}^L \propto \kappa V_{tb}^* V_{ti} \Rightarrow$  CKM structure of MFV preserved

- ▶ Coupling strength  $\kappa \propto \frac{\epsilon_{FC} \tan\beta}{1 + (\epsilon_b - \epsilon_{FC}) \tan\beta}$

Estimate for equal SUSY-Masses:  $|\kappa| \sim 0.08$ , for  $\mu > 0$   
(larger values for large  $A_t$ )  $|\kappa| \sim 0.24$ , for  $\mu < 0$

## Sizable effect in $C_8$

- ▶ Flavour-changing gluino-coupling enters  $\mathcal{H}_{\text{eff}}^{\Delta B=1}$  :
  - ▶ **small effects** in Wilson coefficients of **four-quark operators** and  $C_7$ .
  - ▶ **large effect** in  $C_8$  possible

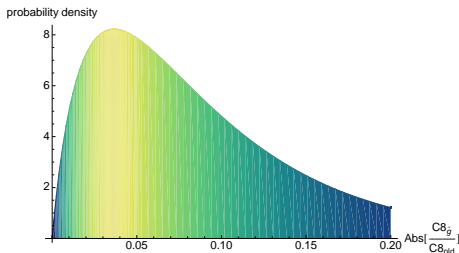
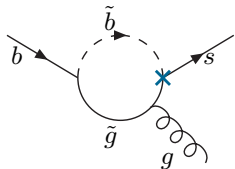




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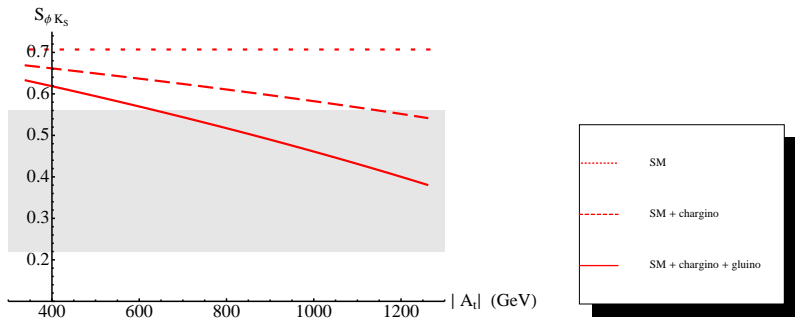
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  - ▶ **small effects** in Wilson coefficients of **four-quark operators** and  $C_7$ .
  - ▶ **large effect** in  $C_8$  possible
- ▶ Estimate for equal SUSY-masses:

$$|C_8^{\tilde{g}}/C_8^{\tilde{\chi}^\pm}| \sim 0.42, \text{ for } \mu > 0; \quad |C_8^{\tilde{g}}/C_8^{\tilde{\chi}^\pm}| \sim 1.3, \text{ for } \mu < 0$$



# Mixing-induced CP asymmetry in $B^0 \rightarrow \phi K_S$

$S_{\phi K_S}$  in naive factorization,  
including  $\tan\beta$ -enhanced corrections to  $C_8$ :



Here a rather large value  $\mu = 800$  GeV is used,  
parameter point is compatible with  $\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$ .

# Conclusions

- ▶ Effects of  $\tan\beta$ -enhanced self-energies can be **resummed analytically beyond the decoupling limit**, also in the flavour-non-diagonal case.
- ▶ The **resummation formula** for the Yukawa coupling depends on the **renormalization scheme**.
- ▶ Not only  $H^0, A^0$  but also  $\tilde{g}, \tilde{\chi}^0$  develop **flavour-changing couplings** at large  $\tan\beta$ .
- ▶ These couplings lead to a **sizable modification of  $C_8$** .

# Backup slides

## Backup: Parameter points

Scan ranges for  $C_8$ :  $\tan \beta = 40 - 60$ , any value for  $\varphi_{A_t}$ ,

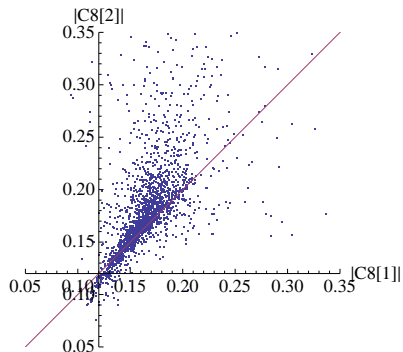
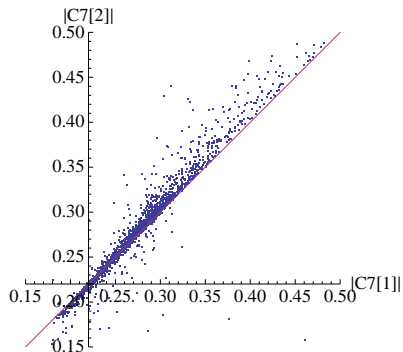
	min (GeV)	max (GeV)
$\tilde{m}_{Q_L}, \tilde{m}_{u_R}, \tilde{m}_{d_R}$	200	1000
$ A_t $	100	1000
$\mu, M_1, M_2$	200	1000
$M_3$	300	1000
$m_{H^+}$	200	1000

Parameter point used for  $S_{\phi_{K_S}}$ :

$\tilde{m}_{Q_L}, \tilde{m}_{u_R}, \tilde{m}_{d_R}$	600 GeV	$\tan \beta$	50
$\mu$	800 GeV	$m_{A^0}$	350 GeV
$M_1$	300 GeV	$M_2$	400 GeV
$M_3$	500 GeV	$\varphi_{A_t}$	$3\pi/2$

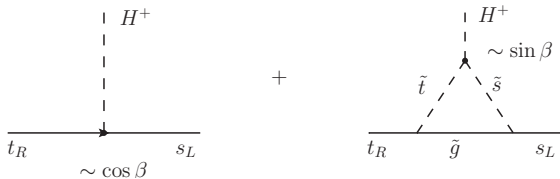
## The Wilson coefficients $C_7$ and $C_8$

- ▶  $C_{7,8}[1] = C_{7,8}^{\tilde{\chi}^\pm} + C_{7,8}^{H^+}$ ,      $C_{7,8}[2] = C_{7,8}^{\tilde{\chi}^\pm} + C_{7,8}^{H^+} + C_{7,8}^{\tilde{g}}$
- ▶ Scan over relevant SUSY parameter space with  
 $(\mu, M_1, M_2, m_{\tilde{g}}, M_{H^+}, m_{\tilde{t},LL}, m_{\tilde{t},RR}, m_{\tilde{b},LL}, m_{\tilde{b},RR}) \leq 1\text{TeV}$ ,  
 $|A_t| \leq 3\text{TeV}$ ,      $0 \leq \phi_{A_t} \leq 2\pi$ ,      $\tan\beta = 50$



## Backup: Non-local $\tan\beta$ -enhanced effects

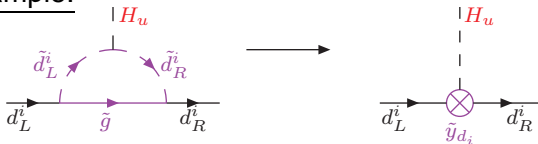
- ▶ Some couplings of  $H^+$  and  $h^0$  are suppressed by  $\cos\beta$  at tree-level.
- ▶ They obtain enhanced vertex corrections  $\sim \sin\beta$ , e.g.



- ▶ This effect is local only in the decoupling limit, but cannot be cast into a Feynman rule in the full calculation.

## Backup: Effective Lagrangian in the decoupling limit

- ▶ Integrate out all particles with masses  $M_{\text{SUSY}} \gg v$ , keep only SM particles and Higgs fields
- ▶ Example:



$$\mathcal{L}_{d,y}^{eff} = -y_{d_i} \bar{d}_i Q_i H_d - \tilde{y}_{d_i} \bar{d}_i Q_i H_u$$

- ▶ Consequence: Modified relation between  $y_{d_i}$  and  $m_{d_i}$

$$m_{d_i} = y_{d_i} v_d + \tilde{y}_{d_i} v_u$$

$\Rightarrow$

$$y_{d_i} = \frac{m_{d_i}}{v_d(1 + \epsilon_i \tan \beta)}$$

contains contributions of the form  $(\epsilon \tan \beta)^n$  to all orders  
→ resummation?



## Backup: Resummation beyond decoupling

- ▶ Subtract  $\tan \beta$ -enhanced corrections to all orders by appropriate finite counterterms

- ▶ Example:  $\Sigma_{b,\tilde{\chi}^\pm}^{RL}(y_b) = y_b v_d \Delta_b^{\tilde{\chi}^\pm}$ ,  $\Delta_b^{\tilde{\chi}^\pm} = \epsilon_b^{\tilde{\chi}^\pm} \tan \beta$

1 loop

$$v_d \delta y_b^{(1)} = - \text{[Diagram]}$$

$$\begin{aligned} \delta y_b^{(1)} &= -\Delta_b^{\tilde{\chi}^\pm} y_b \\ &= -\Delta_b^{\tilde{\chi}^\pm} \frac{m_b}{v_d} \end{aligned}$$

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2 loops

$$\begin{array}{c}
 v_d \delta y_b^{(2)} \\
 \text{---} \times \text{---} \\
 b_L \qquad b_R
 \end{array}
 = - \begin{array}{c}
 \tilde{t}_{1,2} \\
 \text{---} \text{---} \text{---} \\
 \text{---} \text{---} \text{---} \\
 \tilde{\chi}_{1,2}^\pm \\
 b_L \qquad b_R \\
 \text{---} \times \text{---} \\
 \delta y_b^{(1)}
 \end{array}$$

$$\begin{aligned}
 \delta y_b^{(2)} &= -\Delta_b^{\tilde{\chi}^\pm} \delta y_b^{(1)} \\
 &= (-\Delta_b^{\tilde{\chi}^\pm})^2 \frac{m_b}{v_d}
 \end{aligned}$$

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n loops

$$\begin{array}{c} v_d \delta y_b^{(n)} \\ \longrightarrow \times \longrightarrow \\ b_L \qquad b_R \end{array} = - \begin{array}{c} \tilde{t}_{1,2} \\ \text{---} \nearrow \text{---} \\ \text{---} \circlearrowright \text{---} \\ \tilde{\chi}_{1,2}^\pm \\ \text{---} \searrow \text{---} \\ b_L \qquad b_R \\ \delta y_b^{(n-1)} \end{array} \quad \delta y_b^{(n)} = -\Delta_{\tilde{\chi}^\pm} \delta y_b^{(n-1)} \\ = (-\Delta_{\tilde{\chi}^\pm})^n \frac{m_b}{v_d}$$

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n loops

$$v_d \delta y_b^{(n)} = - \delta y_b^{(n-1)} = - \Delta_b^{\tilde{\chi}^\pm} \delta y_b^{(n-1)}$$

$$= (-\Delta_b^{\tilde{\chi}^\pm})^n \frac{m_b}{v_d}$$

$$y_b^0 = \frac{m_b}{v_d} \left( 1 - \Delta_b^{\tilde{\chi}^\pm} + \Delta_b^{\tilde{\chi}^\pm 2} - \dots \right) = \frac{m_b}{v_d (1 + \Delta_b^{\tilde{\chi}^\pm})}$$

## Backup: Resummation beyond decoupling

- ▶ Subtract  $\tan \beta$ -enhanced corrections to all orders by appropriate finite counterterms

▶ Example:  $\Sigma_{b,\tilde{\chi}^\pm}^{RL}(y_b) = y_b v_d \Delta_b^{\tilde{\chi}^\pm}$ ,  $\Delta_b^{\tilde{\chi}^\pm} = \epsilon_b^{\tilde{\chi}^\pm} \tan \beta$

n loops

$$\begin{aligned}
 \begin{array}{c} v_d \delta y_b^{(n)} \\ \xrightarrow{b_L} \times \xrightarrow{b_R} \end{array} &= - \begin{array}{c} \tilde{t}_{1,2} \\ \xrightarrow{b_L} \text{loop} \xrightarrow{b_R} \\ \tilde{\chi}_{1,2}^\pm \end{array} \delta y_b^{(n-1)} \quad \delta y_b^{(n)} = -\Delta_b^{\tilde{\chi}^\pm} \delta y_b^{(n-1)} \\
 &= - \left( -\Delta_b^{\tilde{\chi}^\pm} \right)^n \frac{m_b}{v_d}
 \end{aligned}$$

$$y_b^0 = \frac{m_b}{v_d} \left( 1 - \Delta_b^{\tilde{\chi}^\pm} + \Delta_b^{\tilde{\chi}^\pm 2} - \dots \right) = \frac{m_b}{v_d (1 + \Delta_b^{\tilde{\chi}^\pm})}$$

- ▶ Explicit resummation of contributions of the form  $\Delta_b = \epsilon_b \tan \beta$

# Backup: Resummation of $\delta Z_{ij}^L$

- ▶ Subtract external leg contributions by matrix-valued **wave function renormalization**:

$$m_b \delta Z_{bi}^L \text{ (tree)} = - \text{ (top loop)} - \text{ (gluon loop)}$$

$$\frac{\delta Z_{bi}^L}{2} = - \frac{\Sigma_{bi}^{RL}(\delta Z)}{m_b}, \quad \delta Z_{bi}^L = \mathcal{O}(\epsilon_{FC} \tan \beta)$$

- ▶ **Resum**  $\delta Z_{ij}^L$ -insertions:

$$\boxed{\frac{\delta Z_{bi}^L}{2} = - \frac{\Sigma_{bi}^{RL}(\delta Z)}{m_b}}$$

- ▶ **Result:** 
$$\frac{\delta Z_{bi}^L}{2} = -V_{ti}V_{tb}^* \frac{\epsilon_{FC} \tan \beta}{1 + (\epsilon_b - \epsilon_{FC}) \tan \beta}$$