The MSSM with large \( \tan \beta \)

beyond the decoupling limit

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The pattern of $\tan\beta$-enhancement

- The MSSM contains two Higgs doublets $H_u, H_d$.
  Both acquire vevs: $v_u, v_d \rightarrow \tan \beta \equiv \frac{v_u}{v_d}$

- Large $\tan \beta \iff$ small $v_d$
The pattern of $\tan \beta$-enhancement

- The MSSM contains two Higgs doublets $H_u, H_d$.
  Both acquire vevs: $v_u, v_d \rightarrow \tan \beta \equiv \frac{v_u}{v_d}$

- large $\tan \beta \iff$ small $v_d$

- Consider tree-level amplitude with suppression $v_d$.
  One-loop corrections may involve $v_u$ instead.
  \[ [\text{Hall}, \text{Rattazzi}, \text{Sarid}; \text{Blazek}, \text{Raby}, \text{Pokorski}] \]

- Example: $b$-quark mass

  \[
  \begin{align*}
  m_b &\sim v_d \\
  \delta m_b &\sim \epsilon \cdot v_u \\
  \frac{\delta m_b}{m_b} &\sim \epsilon \cdot \tan \beta \\
  &\sim \mathcal{O}(1)
  \end{align*}
  \]
Two possibilities to deal with such $\mathcal{O}(1)$ corrections

1. Effective Lagrangian for $M_{\text{SUSY}} \gg v, M_A^0, M_{H^0}, M_{H^+}$

2. Calculation in the full MSSM beyond decoupling
Effective Lagrangian vs. full MSSM

- Two possibilities to deal with such $O(1)$ corrections
  1. Effective Lagrangian for $M_{\text{SUSY}} \gg v, M_{A^0}, M_{H^0}, M_{H^+}$
  2. Calculation in the full MSSM beyond decoupling

- Why go beyond decoupling limit?
  - $M_{\text{SUSY}} \sim v$ is natural.
  - Test accuracy of calculations done with the effective Lagrangian approach.
  - Study $\tan\beta$-enhanced effects in couplings of SUSY-particles like $\tilde{g}, \tilde{\chi}^0$

    Impossible in the decoupling limit where these particles are integrated out!
## Summary of large-\(\tan\beta\) effects

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<th>decoupling limit</th>
<th>beyond</th>
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<td>[Hall, Rattazzi, Sarid; Carena, Olechowski, Pokorski, Wagner]</td>
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<td>corrections to CKM matrix</td>
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<td>[Buras, Chankowski, Rosiek, Slawianowska], 3</td>
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<tr>
<td>enhanced FCNCs (d_i \tilde{d}_j \tilde{g}/\tilde{\chi}^0)</td>
<td>not accessible</td>
<td>3</td>
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<tr>
<td>vertex corrections (\bar{u}<em>{i,R} d</em>{j,L} H^+)</td>
<td>[Degrassi, Gambino, Giudice; Carena, Garcia, Nierste, Wagner]</td>
<td>process-dependent (non-universal)</td>
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</table>

\[1 - 3 = \text{this talk}\]
Three new results

Beyond the decoupling limit:

1. Scheme dependence of the resummation formula for the Yukawa coupling
2. Resummation of flavour-changing self-energies
3. New effects in FCNC processes
Three new results

Beyond the decoupling limit:

1. Scheme dependence of the resummation formula for the Yukawa coupling

2. Resummation of flavour-changing self-energies

3. New effects in FCNC processes
Input schemes for bottom-squark mixing

- Bottom-squark mass matrix: 
  \[ M_b^2 = \begin{pmatrix} m_{bL}^2 & -y_b v_u \mu^* \\ -y_b v_u \mu & m_{bR}^2 \end{pmatrix} \]

- Mixing matrix: 
  \[ \tilde{R}_b M_b^2 \tilde{R}_b^\dagger = \text{diag}(m_{b1}^2, m_{b2}^2), \]

  \[ \tilde{R}_b = \begin{pmatrix} \cos \tilde{\theta}_b & \sin \tilde{\theta}_b e^{i\tilde{\phi}_b} \\ -\sin \tilde{\theta}_b e^{-i\tilde{\phi}_b} & \cos \tilde{\theta}_b \end{pmatrix} \]
Input schemes for bottom-squark mixing

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- What to choose as input? → different possibilities, e.g.
  - elements of \( \mathcal{M}_{\tilde{b}}^2 \): \( m_{\tilde{b}L}, m_{\tilde{b}R}, \mu, \tan \beta \)
  - mass eigenvalues and mixing angle: \( m_{\tilde{b}_1}, m_{\tilde{b}_2}, \tilde{\theta}_b, \tilde{\phi}_b \)
  - eigenvalues and off-diag. entries of \( \mathcal{M}_{\tilde{b}}^2 \): \( m_{\tilde{b}_1}, m_{\tilde{b}_2}, \mu, \tan \beta \)
Input schemes for bottom-squark mixing

- Bottom-squark mass matrix: \[ M^2_{\tilde{b}} = \begin{pmatrix} m^2_{\tilde{b}_L} & -y_b v_u \mu \\ -y_b v_u \mu^* & m^2_{\tilde{b}_R} \end{pmatrix} \]

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- What to choose as input? → different possibilities, e.g.
  - elements of \( M^2_{\tilde{b}} \): \( m_{\tilde{b}_L}, m_{\tilde{b}_R}, \mu, \tan \beta \)
  - mass eigenvalues and mixing angle: \( m_{\tilde{b}_1}, m_{\tilde{b}_2}, \tilde{\theta}_b, \tilde{\phi}_b \)
  - eigenvalues and off-diag. entries of \( M^2_{\tilde{b}} \): \( m_{\tilde{b}_1}, m_{\tilde{b}_2}, \mu, \tan \beta \)

- Note: \( \tilde{\theta}_b \) vanishes for \( v/M_{\text{SUSY}} \rightarrow 0 \)
  → No different input schemes in the decoupling limit.
Scheme dependence of the resummation formula

- Write \[ \Sigma^R_L = m_b \Delta_b = m_b \epsilon_b \tan \beta \]

- Modified relation \( y_b \leftrightarrow m_b \) in the decoupling limit:

\[
y_b = \frac{m_b}{v_d (1 + \Delta_b)}
\]
Scheme dependence of the resummation formula

- Write \( \Sigma^{RL}_b = m_b \Delta_b = m_b \epsilon_b \tan \beta \)

- Modified relation \( y_b \leftrightarrow m_b \) in the decoupling limit:
  \[
y_b = \frac{m_b}{v_d (1 + \Delta_b)}
  \]

- Beyond decoupling: Formula depends on renormalization scheme (choice of input)!!!

- Example: Gluino-contribution \( \Sigma^{RL}_{b, \tilde{g}} = m_b \Delta_{\tilde{g}}_b \)
  
  (i) Input: \( m_{\tilde{b}_1}, m_{\tilde{b}_2}, \mu, \tan \beta \) \( \rightarrow \) \( y_b = \frac{m_b}{v_d (1 + \Delta_{\tilde{g}}_b)} \)

  (ii) Input: \( m_{\tilde{b}_1}, m_{\tilde{b}_2}, \tilde{\theta}_b, \tilde{\phi}_b \) \( \rightarrow \) \( y_b = \frac{m_b}{v_d} \left( 1 - \Delta_{\tilde{g}}_b \right) \)

  (iii) Input: \( m_{\tilde{b}_L}, m_{\tilde{b}_R}, \mu, \tan \beta \)
    \( \rightarrow \) analytic resummation impossible, use (i) iteratively.
Three new results

Beyond the decoupling limit:

1. Scheme dependence of the resummation formula for the Yukawa coupling

2. Resummation of flavour-changing self-energies

3. New effects in FCNC processes
Naive MFV: Only chargino-loops are flavour-changing
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Consider flavour-changing self-energies in external quark-legs:

\[ \Sigma_{bs}^{RL} \]

\[ \Sigma_{bs}^{RL*} \]

New source of \( \tan \beta \)-enhancement:

\[ \Sigma_{bs}^{RL} \propto \epsilon_{FC} m_b \tan \beta \quad \text{and} \quad \mathcal{M} \propto \frac{\Sigma_{bs}^{RL}}{m_b} \propto \epsilon_{FC} \tan \beta \]
Flavour-changing self-energies in external legs

- **Naive MFV:** Only chargino-loops are flavour-changing

- **Consider** flavour-changing self-energies in external quark-legs:

$$\Sigma_{RL}^{bs} \propto \epsilon F C m_b \tan \beta$$

- **New source of** $\tan \beta$-enhancement:

$$\Sigma_{RL}^{bs} \propto \epsilon F C m_b \tan \beta \quad \text{and} \quad M \propto \frac{\Sigma_{bs}^{RL}}{m_b} \propto \epsilon F C \tan \beta$$

- **Subtract** self-energies by non-diagonal wave-function CTs:

$$\delta Z_{bi}^L \propto \epsilon F C \tan \beta, \quad \delta Z_{bi}^R \propto \frac{m_i}{m_b} \epsilon F C \tan \beta \quad (i = d, s)$$

$$\rightarrow \delta Z_{L/R} \text{ contain the } \tan \beta \text{-enhanced effects!}$$
Resummed results

- $(\epsilon_{FC} \tan \beta)^n$-effects can be analytically resummed to all orders:

\[
\frac{\delta Z_{bi}^L}{2} = -V_{tb}^* V_{ti} \frac{\epsilon_{FC} \tan \beta}{1 + (\epsilon_b - \epsilon_{FC}) \tan \beta},
\]

\[
\frac{\delta Z_{bi}^R}{2} = -V_{tb}^* V_{ti} \frac{m_{d_i}}{m_b} \left[ \frac{\epsilon_{FC} \tan \beta}{1 + \epsilon_b \tan \beta} + \frac{\epsilon_{FC}^* \tan \beta}{(1 + \epsilon_i^* \tan \beta)} \right] \frac{1 + \epsilon_b \tan \beta}{1 + (\epsilon_b - \epsilon_{FC}) \tan \beta}.
\]
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\[
\frac{\delta Z_{bi}^R}{2} = -V_{tb}^*V_{ti} \frac{m_{di}}{m_b} \left[ \frac{\epsilon_{FC} \tan \beta}{1 + \epsilon_b \tan \beta} + \frac{\epsilon_{FC}^* \tan \beta}{(1 + \epsilon_i^* \tan \beta)} \right] \frac{1 + \epsilon_b \tan \beta}{1 + (\epsilon_b - \epsilon_{FC}) \tan \beta}
\]

- results in corrections to the CKM matrix:

\[
V^0 = \begin{pmatrix}
V_{ud} & V_{us} & K^*V_{ub} \\
V_{cd} & V_{cs} & K^*V_{cb} \\
KV_{td} & KV_{ts} & V_{tb}
\end{pmatrix}, \quad K = \frac{1 + \epsilon_b \tan \beta}{1 + (\epsilon_b - \epsilon_{FC}) \tan \beta}
\]
Resummed results

- \((\epsilon_{FC}\tan\beta)^n\)-effects can be analytically resummed to all orders:

\[
\frac{\delta Z_{bi}^L}{2} = -V_{tb}^*V_{ti}\frac{\epsilon_{FC}\tan\beta}{1 + (\epsilon_b - \epsilon_{FC})\tan\beta},
\]

\[
\frac{\delta Z_{bi}^R}{2} = -V_{tb}^*V_{ti}\frac{m_{di}}{m_b}\left[\frac{\epsilon_{FC}\tan\beta}{1 + \epsilon_b\tan\beta} + \frac{\epsilon_{FC}^*\tan\beta}{(1 + \epsilon_i^*\tan\beta)}\right]\frac{1 + \epsilon_b\tan\beta}{1 + (\epsilon_b - \epsilon_{FC})\tan\beta}
\]

- These results

  - are of the same form as in the decoupling limit but with different \(\epsilon_b, \epsilon_{FC}\).
  - are the analytic expressions for the limit to which the iterative calculation of BCRS converges.
Three new results

Beyond the decoupling limit:

1. Scheme dependence of the resummation formula for the Yukawa coupling
2. Resummation of flavour-changing self-energies
3. New effects in FCNC processes
FCNC-couplings at large $\tan\beta$

- $\delta Z^L_{ij}$ induce FCNC-couplings of order $\epsilon_{FC} \tan\beta$:

- $d_j \rightarrow d_i \rightarrow H^0, A^0$ known in the decoupling limit
- New: generalized to $M_{SUSY} \sim v$

- $\tilde{d}_j \rightarrow d_i \rightarrow \tilde{g}, \tilde{\chi}^0$ new! (not accessible in the decoupling limit)
**FCNC-couplings at large $\tan\beta$**

- $\delta Z_{ij}^L$ induce FCNC-couplings of order $\epsilon_{FC} \tan\beta$:
  
  \[
  d_j \times H^0, A^0 \quad \text{known in the decoupling limit}
  \]
  
  new: generalized to $M_{SUSY} \sim v$

- $\delta Z_{bi} \propto \kappa V_{tb}^* V_{ti}$
  
  new! (not accessible in the decoupling limit)

- Coupling strength
  
  $\kappa \propto \frac{\epsilon_{FC} \tan\beta}{1 + (\epsilon_b - \epsilon_{FC}) \tan\beta}$

Estimate for equal SUSY-Masses:

- $|\kappa| \sim 0.08$, for $\mu > 0$
- (larger values for large $A_t$) $|\kappa| \sim 0.24$, for $\mu < 0$
Sizable effect in $C_8$

- Flavour-changing gluino-coupling enters $\mathcal{H}^{\Delta B=1}_{\text{eff}}$:
  - small effects in Wilson coefficients of four-quark operators and $C_7$.
  - large effect in $C_8$ possible
**Sizable effect in $C_8$**

- Flavour-changing gluino-coupling enters $\mathcal{H}_{\Delta B = 1}^{\text{eff}}$:
  - small effects in Wilson coefficients of four-quark operators and $C_7$.
  - large effect in $C_8$ possible

- Estimate for equal SUSY-masses:
  
  \[
  |C_8^{\tilde{g}}/C_8^{\tilde{\chi}^\pm}| \sim 0.42, \text{ for } \mu > 0; \quad |C_8^{\tilde{g}}/C_8^{\tilde{\chi}^\pm}| \sim 1.3, \text{ for } \mu < 0
  \]
Mixing-induced CP asymmetry in $B^0 \rightarrow \phi K_S$

$S_{\phi K_S}$ in naive factorization, including $\tan \beta$-enhanced corrections to $C_8$:

Here a rather large value $\mu = 800 \text{ GeV}$ is used, parameter point is compatible with $\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$. 

[Graph showing the variation of $S_{\phi K_S}$ with $|A_t|$ (GeV) for SM, SM + chargino, and SM + chargino + gluino scenarios.]

| $|A_t|$ (GeV) | 400 | 600 | 800 | 1000 | 1200 |
|-------------|-----|-----|-----|------|------|
| $S_{\phi K_S}$ | 0.7 | 0.6 | 0.5 | 0.4 | 0.3 |
Conclusions

- Effects of $\tan \beta$-enhanced self-energies can be resummed analytically beyond the decoupling limit, also in the flavour-non-diagonal case.

- The resummation formula for the Yukawa coupling depends on the renormalization scheme.

- Not only $H^0$, $A^0$ but also $\tilde{g}$, $\tilde{\chi}^0$ develop flavour-changing couplings at large $\tan \beta$.

- These couplings lead to a sizable modification of $C_8$. 
Backup slides
Backup: Parameter points

Scan ranges for $C_8$: $\tan \beta = 40 - 60$, any value for $\varphi_{A_t}$,

<table>
<thead>
<tr>
<th>Parameter</th>
<th>min (GeV)</th>
<th>max (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{m}<em>{Q_L}, \tilde{m}</em>{u_R}, \tilde{m}_{d_R}$</td>
<td>200</td>
<td>1000</td>
</tr>
<tr>
<td>$</td>
<td>A_t</td>
<td>$</td>
</tr>
<tr>
<td>$\mu, M_1, M_2$</td>
<td>200</td>
<td>1000</td>
</tr>
<tr>
<td>$M_3$</td>
<td>300</td>
<td>1000</td>
</tr>
<tr>
<td>$m_{H^+}$</td>
<td>200</td>
<td>1000</td>
</tr>
</tbody>
</table>

Parameter point used for $S_{\phi K_S}$:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{m}<em>{Q_L}, \tilde{m}</em>{u_R}, \tilde{m}_{d_R}$</td>
<td>600 GeV</td>
</tr>
<tr>
<td>$\mu$</td>
<td>800 GeV</td>
</tr>
<tr>
<td>$M_1$</td>
<td>300 GeV</td>
</tr>
<tr>
<td>$M_3$</td>
<td>500 GeV</td>
</tr>
<tr>
<td>$\tan \beta$</td>
<td>50</td>
</tr>
<tr>
<td>$m_{A^0}$</td>
<td>350 GeV</td>
</tr>
<tr>
<td>$M_2$</td>
<td>400 GeV</td>
</tr>
<tr>
<td>$\varphi_{A_t}$</td>
<td>$3\pi/2$</td>
</tr>
</tbody>
</table>
The Wilson coefficients $C_7$ and $C_8$

$C_{7,8}[1] = C_{7,8}^{\tilde{\chi}^\pm} + C_{7,8}^{H^+}$, \hspace{1cm} $C_{7,8}[2] = C_{7,8}^{\tilde{\chi}^\pm} + C_{7,8}^{H^+} + C_{7,8}^{\tilde{g}}$

Scan over relevant SUSY parameter space with

$(\mu, M_1, M_2, m_{\tilde{g}}, M_{H^+}, m_{\tilde{t},LL}, m_{\tilde{t},RR}, m_{\tilde{b},LL}, m_{\tilde{b},RR}) \leq 1\text{TeV}$,

$|A_t| \leq 3\text{TeV}$, \hspace{0.5cm} $0 \leq \phi_{A_t} \leq 2\pi$, \hspace{0.5cm} $\tan \beta = 50$
Some couplings of $H^+$ and $h^0$ are suppressed by $\cos \beta$ at tree-level.

They obtain enhanced vertex corrections $\sim \sin \beta$, e.g.

This effect is local only in the decoupling limit, but cannot be cast into a Feynman rule in the full calculation.
Integrate out all particles with masses $M_{\text{SUSY}} \gg v$, keep only SM particles and Higgs fields

Example:

\[ \mathcal{L}^{\text{eff}}_{d,y} = -y_d \bar{d}_i Q_i H_d - \tilde{y}_d \bar{d}_i Q_i H_u \]

Consequence: Modified relation between $y_d$ and $m_{d_i}$

\[ m_{d_i} = y_d v_d + \tilde{y}_d v_u \quad \Rightarrow \quad y_d = \frac{m_{d_i}}{v_d (1 + \epsilon_i \tan \beta)} \]

contains contributions of the form $(\epsilon \tan \beta)^n$ to all orders \rightarrow \text{resummation?}
Subtract $\tan \beta$-enhanced corrections to all orders by appropriate finite counterterms.

**Example:**

$$\Sigma_{b,\tilde{\chi}^\pm}^{RL}(y_b) = y_b v_d \Delta_{b}^{\tilde{\chi}^\pm}, \quad \Delta_{b}^{\tilde{\chi}^\pm} = \epsilon_b^{\tilde{\chi}^\pm} \tan \beta$$

1 loop

$$\nu_d \delta y_b^{(1)}$$

$$b_L \times b_R$$

$$\delta y_b^{(1)} = -\Delta_{b}^{\tilde{\chi}^\pm} y_b$$

$$= -\Delta_{b}^{\tilde{\chi}^\pm} \frac{m_b}{v_d}$$
Backup: Resummation beyond decoupling

- Subtract $\tan \beta$-enhanced corrections to all orders by appropriate finite counterterms

**Example:**

\[
\sum_{b,\tilde{\chi}^\pm}^{RL} (y_b) = y_b v_d \Delta_b \tilde{\chi}^\pm, \quad \Delta_b \tilde{\chi}^\pm = \epsilon_b \tilde{\chi}^\pm \tan \beta
\]

2 loops

\[
v_d \delta y_b^{(2)}
\]

\[
\begin{align*}
\sum_{b,\tilde{\chi}^\pm}^{RL} (y_b) &= y_b v_d \Delta_b \tilde{\chi}^\pm, \\
\Delta_b \tilde{\chi}^\pm &= \epsilon_b \tilde{\chi}^\pm \tan \beta
\end{align*}
\]
Subtract $\tan \beta$-enhanced corrections to all orders by appropriate finite counterterms

**Example:**

$$\sum_{b, \tilde{\chi}^\pm}^{RL} (y_b) = y_b v_d \Delta_{b} \tilde{\chi}^\pm, \quad \Delta_{b} \tilde{\chi}^\pm = \epsilon_b \tan \beta$$

n loops

$$v_d \delta y_b^{(n)}_{b_L \times b_R} = - \delta y_b^{(n-1)}_{b_L \times b_R} \Delta_{b} \tilde{\chi}^\pm \delta y_b^{(n-1)}_{b_L \times b_R} = (-\Delta_{b} \tilde{\chi}^\pm)^n \frac{m_b}{v_d}$$
Backup: Resummation beyond decoupling

- Subtract $\tan \beta$-enhanced corrections to all orders by appropriate finite counterterms

**Example:**

$$\sum_{b, \tilde{\chi}^\pm}^{RL}(y_b) = y_b v_d \Delta_{b} \tilde{\chi}^\pm,$$

$$\Delta_{b} \tilde{\chi}^\pm = \epsilon_b \tan \beta$$

n loops

$$v_d \delta y_b^{(n)}_{b_L} b_L b_R = - t_{1,2} \delta y_b^{(n-1)}_{b_L} b_L b_R \tilde{\chi}_{1,2} \delta y_b^{(n)}_{b_R} = - \Delta_{b} \delta y_b^{(n-1)}_{b_R} = (- \Delta_{b} \tilde{\chi}^\pm)^n m_b v_d$$

$$y_b^0 = \frac{m_b}{v_d} \left( 1 - \Delta_{b} \tilde{\chi}^\pm + \Delta_{b} \tilde{\chi}^\pm^2 - \ldots \right) = \frac{m_b}{v_d(1 + \Delta_{b} \tilde{\chi}^\pm)}$$
Backup: Resummation beyond decoupling

- Subtract $\tan \beta$-enhanced corrections to all orders by appropriate finite counterterms

**Example:**

$$\Sigma_{b,\tilde{\chi}^\pm}^{RL}(y_b) = y_b v_d \Delta_b^{\tilde{\chi}^\pm}, \quad \Delta_b^{\tilde{\chi}^\pm} = \epsilon_b^{\tilde{\chi}^\pm} \tan \beta$$

$n$ loops

$$v_d \delta y_b^{(n)}_{b_L} \times_{b_R} = - \Delta_b^{\tilde{\chi}^\pm} \delta y_b^{(n-1)} = (\Delta_b^{\tilde{\chi}^\pm})^n m_b \frac{m_b}{v_d}$$

$$y_b^0 = \frac{m_b}{v_d} \left(1 - \Delta_b^{\tilde{\chi}^\pm} + \Delta_b^{\tilde{\chi}^\pm} - ... \right) = \frac{m_b}{v_d(1 + \Delta_b^{\tilde{\chi}^\pm})}$$

- Explicit resummation of contributions of the form $\Delta_b = \epsilon_b \tan \beta$
Backup: Resummation of $\delta Z_{ij}^L$

- Subtract external leg contributions by matrix-valued wave function renormalization:

\[
\frac{m_b}{2} \delta Z_{bi}^L = - \frac{\Sigma_{bi}^{RL} (\delta Z)}{m_b},
\]

\[
\delta Z_{bi}^L = \mathcal{O} (\epsilon_{FC} \tan \beta)
\]

- Resum $\delta Z_{ij}^L$-insertions:

\[
\frac{\delta Z_{bi}^L}{2} = - \frac{\Sigma_{bi}^{RL} (\delta Z)}{m_b}
\]

- Result:

\[
\frac{\delta Z_{bi}^L}{2} = - V_{ti} V_{tb}^* \frac{\epsilon_{FC} \tan \beta}{1 + (\epsilon_b - \epsilon_{FC}) \tan \beta}
\]